Labor Market Effects of Trade Induced Technological Change under Intellectual Property Rights Protection

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Abstract

In this paper I examine the effect of trade liberalization on relative prices and technology in a model with two types of labor and intellectual property rights (IPR) protection. I argue that IPR protection is crucial to determine labor market outcomes of trade liberalization. With IPR protection firms will fully internalize the greater market for their innovation that is created by trade liberalization, which benefits low-skill complementary technology. This low-skill-biased technological change countervails the negative effect of relative price changes (Stolper-Samuleson theorem.) I calibrate the model using data on the US in 1985 and demonstrate that under IPR protection trade liberalization leads to a higher wage gap, lower unemployment, and to higher wages for all workers. Without IPR protection the wages for low skilled workers decrease and their unemployment rate increase after trade liberalization. Moreover, I use CPS (current population survey) data to show that low-workers in industries more exposed to IPR violations face more negative labor market outcomes.

JEL classification : F13, F16, J31, O31, O34

Key Words : R&D, trade induced technological change, search unemployment, wage gap; intellectual property rights

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1 Introduction

In an economy with high and low skilled workers the standard Heckscher-Ohlin model suggests that trade liberalization will lead to higher (relative) wages of high skilled workers in high skill worker abundant countries (Stolper-Samuelson theorem) and hence the inequality in such countries increases, i.e., low skilled workers lose from trade liberalization. Consistently Autor et al. (2013) show evidence that trade between the United States (US) and China leads to negative labor market effects for low skilled worker – an increasing unemployment rate and wage inequality. On the other hand Goldberg and Pavcnik (2007) and Davis and Mishra (2007) argue that the Stolper-Samuelson theorem cannot explain the globally increasing inequality especially. One reason might be that the classical Heckscher-Ohlin model does not consider effects of trade on (relative) technology and hence omits an important factor to explain the wage gap.\footnote{Throughout this paper the notion wage gap refers to the relative wage of high skilled to low skilled workers, where workers with at least a college degree are high skilled and workers with only a high school diploma are low skilled.} Acemoglu (1998), Dinopoulos and Segerstrom (1999), Acemoglu (2003), and Burstein et al. (2013) develop models of trade induced technological change to analyze within country inequality, but do not consider different intellectual property rights (IPR) regimes nor changes in unemployment.

In this paper I analyze the wage gap and the unemployment rates of high and low skilled workers in the context of trade liberalization and trade induced technological change under different intellectual property rights (IPR) regimes. I argue that trade between two countries with different skill ratios (different relative supplies of high and low skilled workers) leads to an increasing wage gap independent of the IPR regime. If IPRs are not enforced, the unemployment rate of high skilled workers declines and of low skilled workers increases in the country with the higher skill ratio after trade liberalization. If IPRs are enforced, the unemployment rates for both skill groups decline. The main reason for these opposing effects is the reactions of R&D firms on trade liberalization. With IPR enforcement in both countries R&D firms will consider the aggregate demand of both markets for technology, which leads to changes in the relative demand of high and low skilled complementary technology.\footnote{This channel is similar to Burstein et al. (2013) and Parro (2013) where countries import capital with different technology levels and the wage gap arise due to capital-skill complementarity.} More specifically the demand for technology is related to the supply of high and low skilled workers, trade liberalization with IPR protection will lead to low skilled biased technological change in the country with the higher skill ratio and to high skilled biased technological change in the country with the lower skill ratio. With IPR protection and trade the aggregated skill ratio declines from the perspective of the high skill abundant country and increases for the low skill abundant country. Nevertheless, the (positive) price effect of trade liberalization still favors high skilled workers in the high skill abundant...
country which can lead to an increasing wage gap. In an numerical exercise I show that due to non-linearities in the production function the price effect dominates in the high skill abundant country and the technology effect in the low skill abundant country if IPRs are enforced. Thus, trade liberalization along with IPR enforcement can explain a significant increase of within country inequality. Moreover, due to the market size effect for technology, trade liberalization leads to faster technological progress and high and low skilled workers become more productive, which is an additional gain from trade. In the context of a search unemployment model this implies that the unemployment rates for both skill groups declines. Hence, policy makers should favor trade liberalization under IPR protection, as it has higher gains from trade (in terms of real wages) and lower unemployment rates relative to trade liberalization without IPR protection.

Specifically, I combine models of trade induced technological change and imperfect labor markets based on the concepts of Acemoglu (2003) and Moore and Ranjan (2005). I introduce a simple search unemployment model along the lines of Pissarides (2000) to analyze unemployment rates for each skill group separately. Davidson and Matusz (2000) and Davidson et al. (2014) show that globalization will have important implications on labor market outcomes in search unemployment models. The paper contributes to the literature along three lines. First, it links bilateral trade to technology as well as labor market outcomes including unemployment rates. Thereby, introduces a channel through which trade effects wage inequality and unemployment rates. Second, it highlights the importance of intellectual property rights in the context of international trade and its effect for low skilled workers.\footnote{Recent trade negotiations include often IPR, for example the Transatlantic Trade and Investment Partnership (TTIP) or the Trans-Pacific Partnership (TTP).} Free trade will only have negative effects for low skilled workers in terms of higher unemployment rates and lower (real) wages, if IPRs are not enforced in low skill abundant countries. Last, it shows that the effect of free trade is quantitatively important to explain the increasing wage gap. The calibrated model predicts that free trade between the United States and Japan in 1985 would have increased the wage gap by about 14 and the aggregated unemployment rate decreased by about 2.5 percentage points.

The reminder of the paper is organized as follows. Section 2 briefly summarizes the related literature. Section 3 describes the model. Section 4 presents some numerical solutions. Some empirical findings are presented in section 5. Finally, section 6 concludes.

2 Related Literature

In the literature the two most prominent explanations for the rising college wage gap are skill biased technological change (SBTC) and trade liberalization. Since the 1990s the SBTC
explanation was dominant, especially due to the analysis of Katz and Murphy (1992) and Autor et al. (2008). They found the puzzling fact that since the 1960s the supply of skilled workers has increased steadily in the US and at the same time the wage gap increased as well. Considering only the supply side we would expect a decreasing wage gap. Autor et al. (2008) identify, among others, skill biased technological change as the reason for this counter-intuitive development. They argue that the productivity of high skilled workers increased strongly and outweighted the negative supply effect. Their analysis led to many studies about skill biased technological change, especially in the form of information technology (IT), see Krueger (1993) and Jorgenson (2001). Still, Card and DiNardo (2002) critically remarked that, whenever the changes in the relative wages are not fully explained by changes in the relative skill supply, it is claimed that skill biased technological change has caused the opposing wage development. Hence, SBTC is a residual explanation for the increasing wage gap which is hard to quantify empirically.

Still the strong increase of international trade might be an important factor for the increasing inequality in the US. Trade theory models would suggest such negative outcomes only if countries with very different endowments of (high and low skilled) labor or comparative advantages trade, see Burstein and Vogel (2016). Most of the US trading partner in the 1980s were developed countries and very similar in their industry structure and hence common trade theory does not explain the increasing wage gap, see the review of Kurokawa (2014). Heckscher-Ohlin trade models rely on changes in the relative price of high skilled and low skilled intensive goods to explain changes in the relative wage of high skilled and low skilled workers (Stolper-Samuelson theorem). Lawrence and Slaughter (1993) found a small decline of the relative price of high skilled and low skilled intensive goods while Sachs and Shatz (1996) find only a small increase of the relative price in the 1980s. They concluded that trade liberalization cannot explain the increasing wage gap as the relative price change was too small. Moore and Ranjan (2005) compared changes in the unemployment rates of high skilled and low skilled workers to identify trade or SBTC as source of an increasing wage inequality. Both increase the wage gap, but only SBTC would be consistent with the decreasing unemployment rates of high and low skilled workers in the US in the 1980s.

4A recent strand of literature analyzes between firm wage differentials of ex ante homogenous workers, see Egger and Kreickemeier (2009), Helpman et al. (2010), Felbermayr et al. (2011a), Felbermayr et al. (2011b) and Egger et al. (2011). This paper is more concerned with wage differentials as well as difference in the unemployment rates between different types of workers (high skilled vs. low skilled workers) working in different sectors (high skill intensive and low skill intensive sector.
3 Model

In this section I outline the theoretical model that combines different IPR regimes with trade induced technological change. I show I focus on the production of the firm and derive the optimal technology levels in the economy. Moreover, I introduce imperfect labor markets in the form of search frictions and analyze how changes in the supply of high and low skilled workers effect the technology level, the wage rates (wage gap), and the unemployment rates under different IPR regimes.

3.1 Production

The world consists of two countries, domestic (D) and foreign (F). Individuals in each country are either high or low skilled, and both types are present in each country. All individuals have equal lifetime preferences that depend on their consumption of a final good c. The utility of individuals in country $j = D, F$ is given by

\[ u^j = \int_{\tau=0}^{\infty} c^j \exp(-\rho \tau) d\tau \quad \text{for } j = D, F, \]

where $\rho$ is the discount factor and $\tau$ is a time index. I omit the time index $\tau$ and country index $j$ where it does not lead to any confusion. The final good $c$ is produced by a firm with constant elasticity of substitution (CES) technology using a high skilled intensive intermediate good, $y_h$, and a low skilled intermediate good, $y_l$, so that

\[ c = \left( \omega y_l^{1-\frac{1}{\epsilon}} + (1-\omega) y_h^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{1-\frac{1}{\epsilon}}}, \]

where the parameter $\epsilon > 1$ defines the elasticity of substitution between the two goods and $\omega$ is a share parameter. The subscript $i = h, l$ indicates high and low skilled variables. Labor is not necessary for the production of the final good.

In each country two intermediate goods are produced separately by a high and a low skilled intensive (representative) firm using the corresponding kind of local labor, $N_h$ and $N_l$, and technology, $A_h$ and $A_l$, respectively. Each intermediate good uses its own specific factor (machines), $x_i$, which is complementary to the sector specific labor, i.e., skill intensive machines can only be used by high skilled workers and similarly for the low skilled intensive sector. The intermediate good firms decide about the optimal machine and labor usage and take prices and the technology level as given. Intermediate goods are produced with a Cobb-Douglas production function with labor augmenting technology:
\( y_i = A_i^\beta x_i^{1-\beta} N_i^\beta \) for \( i = h, l, \)

where \( \beta \in (0, 1) \) is a common technology parameter. \( N_i \) gives the high and low skilled labor employed in the corresponding sector, which is different from total \( i \)-type labor supply, \( N_i \).

Employed labor is given by \( N_i = (1-u_i)\bar{N}_i \), where \( u_i \) is the sector and skill type specific unemployment rate in a given country.

### 3.2 Prices

The relative price of high to low skill intensive intermediate goods is defined as \( p^s = p^s_h/p^s_l \).

This relative price can be either the relative price of a country in autarky, \( s = A \), or the relative price under free trade, \( s = T \), common to both countries. In the optimum, the marginal rate of substitution between the two intermediate goods has to be equal to the relative price. Without trade the price is determined by the local production (and consumption) of the intermediate goods:

\[
(4) \quad p^A = \frac{p^A_h}{p^A_l} = \frac{\partial c}{\partial y_h} = \frac{1 - \omega}{\omega} \left( \frac{y_h}{y_l} \right)^{-\frac{1}{\epsilon}},
\]

where \( y_i \) is the local production in sector \( i \). Under free (costless) trade the relative price is a function of the relative world production of intermediate inputs in each sector.

\[
(5) \quad p^T = \frac{p^T_h}{p^T_l} = \frac{1 - \omega}{\omega} \left( \frac{y^D_h + y^F_h}{y^D_l + y^F_l} \right)^{-\frac{1}{\epsilon}}.
\]

The price of the final consumption good, \( p^c \), depends on the prices of both inputs. For a CES production function the price (index) is given by

\[
(6) \quad p^c = (\omega^\epsilon (p^c_l)^{1-\epsilon} + (1 - \omega)^\epsilon (p^c_h)^{1-\epsilon})^{\frac{1}{1-\epsilon}}.
\]

Hence, the real prices of intermediate goods, \( \bar{p}^s = \frac{p^s}{p^c} \), in a country can be written as function of the relative price given by equation (4) or (5):

\[
(7) \quad \bar{p}^s_h = \frac{p^s_h}{p^c} = \frac{\bar{p}^s}{(\omega^\epsilon + (1 - \omega)^\epsilon (\bar{p}^s)^{1-\epsilon})^{\frac{1}{1-\epsilon}}},
\]
The price of the high skilled good increases with the relative price while the price of the low skilled good decreases, \( \partial p^h_s / \partial p^s > 0 \) and \( \partial p^l_s / \partial p^s < 0 \). I drop the superscript \( s \) for notional reasons, but it is important to keep in mind that the real intermediate good prices, \( \bar{p}^s \), differ under different trade regimes as the relative price, \( \bar{p}^s \), differs.

### 3.3 Search unemployment

Labor demand, unemployment and wage rates in each sector and country are determined with a simple search model along the lines of Pissarides (2000). In a search unemployment model intermediate firms only create a vacant position if it is profitable. Workers only accept a job offer if the wage paid is higher than their reservation wage. Exogenous shocks destroy filled positions. I derive all equations in detail in the Appendix A.

The matching functions are symmetric for the high and low skill intensive sector and between the two countries.\(^5\)

\[
M(v_i; u_i) = kv_i^\gamma u_i^{1-\gamma} \quad \text{for } i = h, l.
\]

\( \theta_i = v_i / u_i \) reflects the labor market tightness, where \( v_i \) is the vacancy rate and \( u_i \) is the unemployment rate in sector \( i \), \( \gamma \in (0, 1) \) is a matching coefficient and \( k \) is a scale parameter. The equilibrium unemployment rate is determined as

\[
u_i = \frac{\psi}{\psi + k\theta_i^\gamma},
\]

where \( \psi \) is the exogenous job destruction rate.

**Firms**

To determine the wage rate in each sector firms consider the value of a filled and a vacant position. \( F_i \) represents the discounted value of a vacancy in a firm and \( J_i \) the discounted value of a filled position in a firm. A vacant position is an asset for the firm. If capital markets are perfect, the valuation of this asset will be \( \rho F_i \) and equal to the expected gains from filling a position less the recruitment costs, \( \delta \cdot k\theta_i^{\gamma-1}(J_i - F_i) - \delta \). \( k\theta_i^{\gamma-1} \) is the probability of filling a vacancy, \( J - F \) are the discounted profits of a filled vacancy and \( \delta \) are the initial

\(^5\)Weiss and Garloff (2009) and Davidson et al. (1999) show that the parametrization of search unemployment models is an important aspect when considering trade effects. This paper focuses how the interaction of IPR and trade affects labor market outcomes and less on the search frictions itself. Thus, using symmetric matching technology allows a clearer view on the mechanism.
costs of creating a vacancy or recruitment costs. In equilibrium the value of a vacant position has to satisfy

\begin{equation}
\rho F_i = k \theta_i^{-1} (J_i - F_i) - \delta.
\end{equation}

Similarly, a filled position has a value for the firm, which is equal to marginal profits of an additionally employed worker plus the discounted expected profits until the match is resolved. We can write the value of a filled position as

\begin{equation}
\rho J_i = t_i - w_i - r_i + \psi (F_i - J_i),
\end{equation}

where the right hand side are the instantaneous profits of an additional employed worker, \( t_i - w_i - r_i \), plus the expected profits from the match in the future, \( \psi (F_i - J_i) \), where \( \psi \) is the exogenous job destruction rate. The instantaneous marginal profits of an additionally employed worker are calculated by subtracting the wage, \( w_i \), and marginal costs of machines with respect to labor, \( r_i \), from the marginal revenues gained from employing a worker, \( t_i \). The marginal rental costs are defined as the costs to optimally adjust the capital stock once a vacancy is filled. They do not correspond to the unit costs of a machine, but to the amount of machines needed to maximize output when employing an additional worker, thus I refer to these costs as marginal cost of machines with respect to labor. The (marginal) profits for an additional worker employed are derived explicitly in the Appendix B.

In equilibrium the value of a vacancy has to be zero, \( F_i = 0 \), otherwise firms would create more or less vacancies and the unemployment rate would not be in its steady state. To obtain the free entry condition I combine equations (11) and (12) using that \( F_i = 0 \):

\begin{equation}
k \theta_i^{-1} (t_i - r_i - w_i) = k \theta_i^{-1} (\beta \bar{p}_i A_i - w_i) = \delta (\psi + \rho),
\end{equation}

where I have substituted equations (60) and (61) in equation (12) to receive the second equality.

**Workers**

Workers accept any job that pays a higher wage than their reservation wage. As workers receive social benefits, the discounted value of being unemployed is equal to the social benefits, \( b \), plus the expected gain from finding a job. On the other hand, the discounted value of being employed is equal to the wage plus the expected loss when a match is destroyed. These considerations lead to the following two standard Bellman equations in the Pissarides
\begin{equation}
\rho U_i = b + k \theta_i^\gamma (W_i - U_i),
\end{equation}

\begin{equation}
\rho W_i = w_i + \psi (U_i - W_i),
\end{equation}

where $U_i$ is the discounted value of unemployment and $W_i$ is the discounted value of employment. The worker receives social benefits, $b$, if unemployed. $k \theta_i^\gamma$ gives the rate at which workers find a job and $\psi$ the rate at which workers lose their job. Note that all exogenous parameters, such as recruitment costs, $\delta$, or social benefits, $b$, and scale parameter, $k$, are the same for high and low skilled workers and firms. Different values of the exogenous parameters for the high and low skill intensive sector do not change the qualitative results of the model. For example higher recruitment costs for high skilled workers increase the high skilled unemployment rate and decreases the high skilled wages and consequently the wage gap. This is in line with the behavior of unemployment and wages in a search unemployment models. Nevertheless, this paper focuses less on the labor market institutions, but rather on the interaction of trade, labor markets and technological progress. See Weiss and Garloff (2009) for an analyze of the behavior of SBTC and unemployment with different labor market parameters as well as Davidson and Matusz (2000) and Davidson et al. (2014) for search models and their respond to globalization.

**Wage bargaining**

Wages are determined by Nash bargaining over the profits of a filled position. The parameter $\eta$ defines the bargaining power of workers and firms. A higher $\eta$ gives more weight to the workers and $\eta = 0.5$ implies symmetric bargaining.

\begin{equation}
w_i = \arg \max (W_i - U_i)^\eta (J_i - F_i)^{1-\eta}.
\end{equation}

The first order condition for equation (16) is

\begin{equation}
W_i - U_i = \frac{\eta}{1-\eta} (J_i - F_i).
\end{equation}

From this expression, the wage equation can be derived analogously to Pissarides (2000).\footnote{The complete mathematical derivation is shown in the Appendix A.}
I combine the wage equation (18) and the free entry condition (13) to obtain an implicit function of the labor market tightness $\theta_i$:

\[
(1 - \eta)(\beta \bar{P}_i^\frac{1}{\beta} A_i - b) - \eta \delta \theta_i - \frac{\delta(\rho + \psi)}{\theta_i^{\gamma - 1}} = 0.
\]

### 3.4 Technology

#### Machine demand

The (representative) intermediate good firms sequentially maximize their profits over labor, $N_i$, and machine usage, $x_i$. First, firms post vacancies. Second, vacancies are filled or no match is found in the period. Third, given the new employment level firms (optimally) adjust their capital stock. Fourth, production takes place. In the Appendix B I show in detail how the firm considers the effect of changes in employment on the marginal capital costs. When deciding about the optimal machine demand intermediate good firms take the rental price, $\chi_i$, and the technology level of the machines, employment in the economy, the wage rate, and the relative price as given. Thus, they face the following maximization problem:

\[
\max_{N_i, x_i} \pi_i = y_i \bar{P}_i - \chi_i x_i - N_i w_i,
\]

subject to

\[
y_i = A_i^{\beta} x_i^{1-\beta} N_i^\beta,
\]

where $w_i$ and $\chi_i$ are the real wage and the real rental price, respectively. This optimization yields the machine demand as a function of the real rental price, the real intermediate good price, the technology level and the employed labor.

\[
x_i = \left(\frac{(1 - \beta)\bar{P}_i}{\chi_i}\right)^\frac{1}{\beta} A_i N_i.
\]

Machines are produced by IP owning R&D firms that will gain monopoly rents or by counterfeiters that behave like IP owning R&D firms and charge the same price.\(^7\) As the demand is iso-elastic, the common expression of the Lerner index is used to determine the optimal monopoly mark up of the firm:

\(^7\)For the optimization of the intermediate good firms the type of supplier is not relevant.
The (compensated) elasticity of demand is \( \epsilon_{x_i} = \frac{1}{\chi_i} \). The R&D firm’s real marginal costs (MC) are constant and assumed to be \((1 - \beta)^2\), hence the monopoly price for each type of machine is constant

\[
\chi_i = (1 - \beta).
\]

Replace the monopoly rental price, \( \chi_i \), in equation (21) to obtain machine demand of an intermediate producer as

\[
x_i = (\bar{p}_i)^{\frac{1}{\beta}} A_i N_i.
\]

Machine demand increases with the employed labor, the intermediate good price and the technology level. Substituting the optimal machine usage in the intermediate good production function yields

\[
y_i = (\bar{p}_i)^{\frac{1-\beta}{\beta}} A_i N_i.
\]

**Research and Development**

High and low skilled intensive (IP owning) R&D firms produce h-type and l-type machines. Due to their IP they are monopolists. To secure their monopoly status R&D firms have to invest in each period into technology. The technology production function for each (IP owning) R&D firm is given by

\[
A_i = z_i^\mu q_i^{1-\mu},
\]

where \( z_i \) is the research effort in final good units, \( q_i > 0 \) is a scale parameter and \( \mu \in (0, 1) \) is a production coefficient. Technology production has diminishing returns to research effort and hence higher levels of technology are more costly to achieve.

Intermediate good firms rent machines in each period and pay the real rental price \( \chi_i \). When an intermediate good firm rents a machine it produces with the corresponding technology level, \( A_i \). For simplicity, only the newest vintage can be rented. If the technology level drops, intermediate good firms would want to rent older machines, as they have a higher technology level and productivity. This assumption excludes this possibility.
R&D firms will invest all monopoly profits into research and development and have zero profits. The logic behind this assumptions is simple. The firm with the newest vintage is able to extract monopoly profits. If the firm does not invest all its profits into research and development, there is an incentive for other firms to supply a newer machine and become the monopolist. Thus, in equilibrium all profits are invested into R&D. To determine the optimal technology level, it is sufficient to determine the profits of the R&D firms.

Considering the constant markup price from equation (22) and the demand function in equation (23) the zero profit condition is given by

\[
\pi_i^{RD} = 0, \\
\chi_i(x_i^D + \sigma x_i^F) - MC(x_i^D + \sigma x_i^F) - z_i\beta(1 - \beta) = 0, \\
\beta(1 - \beta)((\overline{p}_i)^{1-\beta} (N_i^D + \sigma N_i^F)) - z_i\beta(1 - \beta) = 0,
\]

where research effort costs, \(z_i\), are scaled by the expression \(\beta(1 - \beta)\) to simplify notation. \(x_i^D\) and \(x_i^F\) are the demands for machines in country D and F respectively. \(N_i^D\) and \(N_i^F\) is the employment in sector \(i\) in the domestic and foreign firm. \(\sigma \in [0, 1]\) indicates the IPR protection in the foreign country, i.e., how much of the foreign demand is fulfilled by IP owning R&D firms. \(\sigma = 1\) implies full enforcement in the foreign country and all demand is satisfied by IP owning R&D firms, while \(\sigma = 0\) implies no enforcement at all and consequently only counterfeiting firms provide machines in the foreign country. Without loss of generality I assume that IPR are always fully enforced at home. Substituting the marginal costs from the equation (22) the optimal technology level under free trade between two IPR enforcing countries is given by

\[
(27) \quad z_i = q_i \left( (\overline{p}_i)^{1-\beta} (N_i^D + \sigma N_i^F) \right)^{\frac{1}{1-\mu}},
\]

which implies a technology level of

\[
(28) \quad A_i = q_i \left( (\overline{p}_i)^{1-\beta} (N_i^D + \sigma N_i^F) \right)^{\frac{\mu}{1-\mu}}.
\]

Clearly, the technology in sector \(i\) increases with the employment in the sector and the IP enforcement, i.e., it increases with the market size for machines of type \(i\). The higher is the intermediate good price, \(\overline{p}_i\), the more profitable is the production of the intermediate good firm and the higher is the demand for machines and workers. In general higher machine demand increases the profits of R&D firms and hence the technology level.

In the Appendix C I outline a model similar to Aghion and Howitt (1992) in which indi-
3.5 Relative prices and trade

Until this point I have not considered autarky or free trade relative prices. Thus, \( p_i \) could be either \( p^A_i \) from equation (4) or \( p^T_i \) from equation (5). In the autarky case the consumption of high and low skill intensive goods equals the local production. Machines cannot be traded either and I assume that \( \sigma = 0 \) in the case of autarky, i.e., intermediate firms in the foreign country source all their machines from counterfeiting suppliers. Under autarky the relative price, \( p^A_i \) can be written as a function of the domestic labor supply ratio:

\[
\begin{align*}
\bar{p}^A &= \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \frac{y_h}{y_l} \right)^{-\frac{1}{\epsilon}} = \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \bar{p}^A \right)^{\frac{1}{\epsilon}} \left( \frac{A_h N_h}{A_l N_l} \right)^{-\frac{1}{\epsilon}} \\
&= \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \bar{p}^A \right)^{\frac{(1 - \mu)(1 - \beta) + \mu}{(1 - \rho)(1 - \beta)}} \left( \frac{q_h}{q_l} \right) \left( \frac{N_h}{N_l} \right)^{\frac{1}{1 - \rho}} \left( 1 - \frac{\frac{\sigma}{N_F} (1 - \rho)}{1 - \mu} \right)^{-\frac{1}{\epsilon}}.
\end{align*}
\]

Solving for \( \bar{p}^A \) yields

\[
\bar{p}^A = \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \frac{q_h}{q_l} \right) \left( \frac{N_h}{N_l} \right)^{\frac{1}{1 - \rho}} \left( \frac{\sigma}{N_F} \right)^{\frac{1}{1 - \rho}} \left( \frac{N_F^D + N_h^D}{N^D_F + N^D_h} \right)^{\frac{1}{1 - \rho}}.
\]

In autarky a higher employment in the skill intensive sector increases the supply of skill intensive intermediate goods and, hence, reduces its (relative) price. Note that the relative technology, \( \frac{A_h}{A_l} \), depends only on the domestic demand for machines, due to the assumption that \( \sigma = 0 \), i.e., IP owning R&D firms only consider the domestic demand for machines.

Under free trade the relative price changes in two ways. First, now it depends on the relative world supply of intermediate goods. Second, \( \sigma \) could be different from zero and hence relative technology depends on the foreign market size, see equation (28).

Substitute the post-trade technology level from equation (28) in the production function of the domestic and foreign countries. Use this to calculate the \( p^T \) from equation (5):

\[
\begin{align*}
\bar{p}^T &= \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \bar{p}^T \right)^{\frac{(1 - \mu)(1 - \beta) + \mu}{(1 - \rho)(1 - \beta)}} \left( \frac{q_h}{q_l} \right) \left( \frac{N_h^D + \sigma N_F^D}{N^D_h + \sigma N_F^D} \right)^{\frac{1}{1 - \rho}} \left( \frac{N_F^D + N_h^D}{N^D_F + N^D_h} \right)^{-\frac{1}{\epsilon}}.
\end{align*}
\]

Solving the above equation for \( \bar{p}^T \) yields an analogous expression to equation (30):
Proposition 1: For given high and low skilled unemployment rates, trade opening to a country with a lower skill ratio that does not respect IPR \((\sigma = 0)\) will increase the relative price, \(\bar{p}^T > \bar{p}^A\), and induces skill biased technological change in the skill abundant country. In response the wage gap will increase in this country.

Proof: Note that \(\bar{p}^A\) and \(\bar{p}^T\) in equations (32) and (30), respectively, differ only by the additional term \(\left( \frac{N_h^D + N_h^F}{N_h^D + N_l^D} \right)^\frac{1}{1-\mu} \left( \frac{N_h^D + N_h^F}{N_l^D + N_l^D} \right)^{-\frac{\beta(1-\mu)}{(1-\rho)(\sigma+1-\beta)+\mu}}\). By assumption the exponent is negative and hence for \(\bar{p}^T\) to be greater than \(\bar{p}^A\) it is sufficient to show that \(\frac{N_h^D + N_h^F}{N_l^D + N_l^D} < 1\), which is always the case if the domestic country is relatively skill abundant.

The post-trade relative price will be greater relative to the autarky relative price in the skill abundant country, the higher is the difference between the skill ratios of the two countries. In the skill abundant country the real price of the high skill intensive intermediate good increases and the one of the low skill intensive good declines, see equations (7) and (8). The change in the relative price reflects changes in the relative demand, and hence the R&D firm complementary to the high skill intensive sector becomes more profitable. Thus, the price effect directly translates into skill biased technological change. Which can be seen from equation (33) given that \(\sigma = 0\).

Finally, the wage gap will be effected by both: the technological change and the price effect. The wage rate increases with the technology level and the real intermediate good price, see equation (18). For the (skill abundant) domestic country the price and technology effect go in the same direction and hence the real wages of high skilled workers increase faster than for low skilled workers, which implies an increasing wage gap. For a given unemployment rate and with \(\sigma = 0\) changes in the technology level are only driven by changes in relative price in the domestic (IP enforcing) country. The real price of the high (low) skill intensive intermediate good in the foreign country decreases (increases) while the technology level
\(A_k\) increases \((A_l\) decreases). Thus, in the foreign country price and technology effect have opposite signs and the affect on the wage gap is ambiguous.

With IPR enforcement in both countries \(\sigma > 0\) R&D firms face greater demand for high and low skill complementary machines which increases the profits and the technology levels (given prices). In the high skill abundant country the price effect and the market size effect for technology will increase the wage of high skilled workers and decrease their unemploy-ment rate. Although low skilled workers in the high skill abundant country face a negative price effect, increased (foreign) demand for low skilled complementary machines might induces sufficient technological progress to offset the negative price effect. Consequently it is not clear if the wage gap in the high skill abundant country increases. In contrast, in the foreign country, low skilled workers gain due to a price effect and the market size effect.\(^8\)

**Proposition 2:** Free trade between a high skill abundant country and a low skill abundant country that enforces IPRs \((\sigma > 0)\) will directly favor skilled workers in the high skill abundant country (Stolper-Samuelson effect). It will increase the technology levels in both sectors via a price effect and a market size effect. The effect on the wage gap is ambiguous due to the technological change. The raising technology levels in the low skill intensive sector might be sufficient to compensate for the negative price effect and even lead to a declining wage gap. If the technology effect is sufficiently great it can have a positive effect on the unemployment rates in both countries and sectors.

A special case is noteworthy. If both countries are completely symmetric and \(\sigma > 0\), then trade opening will not change the relative price for given unemployment rates and only a market size effect for technology can be observed. By equation (33) the relative technology stays constant given relative supply of labor and relative prices, thus the relative wages will stay constant as well. The pure market size effect without a price effect only implies higher real wages but no changes in inequality.

### 3.6 Equilibrium

To find the equilibrium of the model the labor market tightness has to be determined. Equation (19) defines the labor market tightness in each sector, where the real intermediate price is either given by equations (7) and (8). As the real intermediate good price is a function of the relative price, the labor market tightness of both sectors are interdependent. In equilibrium the equation (19) have to hold simultaneously for the low and high skilled sector. For a country in autarky the equilibrium is defined by

\(^8\)Recall that in autarky \(\sigma = 0\) and only the supply of high and low skilled workers in the domestic country determined technology levels.
\[(1 - \eta) \left( \beta \left( \frac{p_i^A}{p_i} \right)^{\frac{1}{\tau}} A_i^A - b \right) - \eta \delta \theta_i - \frac{\delta(\rho + \psi)}{k\theta_i^{\gamma-1}} = 0 \quad \text{for } i = h, l,
\]

where

\[
p_i^A = \left( \frac{1 - \omega}{\omega} \right)^{-\epsilon} \left( \frac{q_i}{q_l} \right) \left( \frac{(1 - \omega_i)N_i}{(1 - \omega_l)N_l} \right)^{\frac{1}{1-\epsilon}} \bar{p}_i
\]

\[
p_h^A = \frac{p_i^A}{(\omega^\epsilon + (1 - \omega)^\epsilon (p_i^A)^{1-\epsilon})^{\frac{1}{1-\epsilon}}},
\]

\[
p_l^A = \frac{1}{(\omega^\epsilon + (1 - \omega)^\epsilon (p_i^A)^{1-\epsilon})^{\frac{1}{1-\epsilon}}},
\]

\[
A_i^A = q_i \left( \frac{p_i^A}{\bar{p}_i} \right)^{\frac{1}{\tau}} (1 - \omega_i)N_i^{\frac{1}{1-\epsilon}},
\]

\[
u_i = \frac{\psi}{\psi + k\theta_i^\gamma}.
\]

For the moment consider the prices and technology levels as exogenous. The labor market tightness, \(\theta_i\), is an increasing function of technology, \(A_i\), and the real intermediate good prices, \(p_i\).

\[
\frac{\partial \theta_i}{\partial A_i} > 0 \quad \frac{\partial \theta_i}{\partial p_i} > 0.
\]

Higher intermediate good prices and technological progress reduce the unemployment, see equation (10), as both factors make the employment of more workers more profitable for an intermediate producer. The wage in each sector depends on the sectoral technology level, the intermediate good price and the labor market tightness, see equation (18). An increase in the skill supply will have a positive impact on the labor market tightness and the technology, but at the same time the intermediate good price will decline. Which effect dominates is not clear a priori.\(^9\)

For a trading economies the equilibrium is defined by

\[(1 - \eta) \left( \beta \left( \frac{p_i^T}{p_i} \right)^{\frac{1}{\tau}} A_i^T - b \right) - \eta \delta \theta_i - \frac{\delta(\rho + \psi)}{k\theta_i^{\gamma-1}} = 0 \quad \text{for } i = h, l,
\]

where \(p_i^T\), \(p_h^T\) and are \(p_l^T\) defined using equation (32), (7) and (8), respectively. The technology level \(A_i^T\) is given by equation (28) using the trade relative price \(p_i^T\). Last, the expression for the unemployment rate is the same as under autarky.

\(^9\)In the Appendix D I show that a labor supply shock of skilled workers leads to skill biased technological change if \(\epsilon > 1\) as well as decreasing unemployment rates.
4 Numerical Results

The equilibrium conditions in equations (34) and (35) have no closed form solution, which makes it hard to determine equilibrium effects of price changes on the unemployment rates. I solve the model numerically to fully consider these general equilibrium effects. I present two different scenarios. First, I show the effects of trade liberalization with a country that respects IPR. Second, I investigate the effects of trade liberalization with an IPR violating country. For illustrative reasons I calibrated to domestic country to reflect the United States, while the foreign country is either Japan or Mexico.10

4.1 Calibration

I calibrate the model to match the wage gap and unemployment rate of the US economy. The values for the exogenous variables presented in Table 1 are in line with the values used in Pissarides (2007, 2009). A key parameter is the elasticity of substitution, $\epsilon$, in the production function of the final good. The literature suggests an elasticity of substitution $\epsilon \approx 1.5$, see Hamermesh and Grant (1979), Katz and Murphy (1992), Krusell et al. (2000), Autor et al. (2008) or Epifani and Gancia (2008). The parameter $b$ for social benefits and the recruitment costs $\delta$ have to be sufficiently low relative to the wage to ensure a solution.11

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.66</td>
<td>Cobb-Douglas production coefficient</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
<td>Matching parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5</td>
<td>Recruitment costs</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>1.5</td>
<td>Elasticity of substitution for the final good production</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.5</td>
<td>Bargaining power parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.25</td>
<td>Technology production parameter</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.004</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.019</td>
<td>Job destruction rate</td>
</tr>
<tr>
<td>$b$</td>
<td>1.74</td>
<td>Social benefits</td>
</tr>
<tr>
<td>$k$</td>
<td>0.25</td>
<td>Unemployment scale parameter</td>
</tr>
</tbody>
</table>

Parameter values following Pissarides (2007, 2009).

4.2 Trade with strong IPR protection

Analyzing to trade between the US (domestic) and a foreign IPR respecting country, taking Japan as an example. The supply of low skilled workers is 70% of the supply in the domestic...
country (US) and the supply of skilled workers is 60% of the US supply. The values for the foreign country are chosen to match the skill supply and the size of the Japanese population in the 1985 census. The lower relative labor supply ensures that the relative price in the US increases after trade liberalization.

The graphical results are shown in Figure 1. At $t = 20$ the two countries start trading. As a result the wage gap increases significantly, unemployment rates decline and real wage rates increase for both skill groups. Similar to Parro (2013) both skill groups gain from trade as their real wages increase. The wage inequality rises as high skilled wages increase faster than low skilled wages, while the unemployment rates for both skill groups decrease after trade opening. This example illustrates that trade liberalization increases the wage gap and decrease the unemployment rate of low skilled workers at the same time.\footnote{Note that the number of vacancies is a forward looking variable, it adapts immediately to trade opening. This leads to the big jump at $t = 20$, where countries switch from autarky to free trade by assumption. If the two countries opened only gradually, the transition would be smoother.}

The numerical result can be seen as evidence that the increasing trade, especially with Japan, in the 1980s contributed to the development of the wage gap in the US. In my calibration a relatively small price change is able to explain a significant increase of the wage gap: the relative price increases by less than 4% while the average wage gap increases by about 14%. This squares with the empirical findings of Lawrence and Slaughter (1993) and Sachs and Shatz (1996). Thus, in the model a combination of trade liberalization with countries that enforce IPRs and an increase of the US skill ratio is consistent with the increasing wage gap and the declining unemployment rates in the US.

Furthermore, the wage gap in the foreign country increases as well, as technological change in the foreign country is skill biased, $A_h$ increases faster than $A_l$ in Japan given that it

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Numerical solution for the domestic country trading with smaller developed country that respects IPR, for example Japan. $\epsilon = 1.5$.}
\end{figure}
initially has a relative higher level of low skill complementary technology. Additionally, the price effect works in the same direction, low skill intensive intermediate goods become relatively more expensive. Figure 2 summarizes these findings. Trade liberalization with IPR enforcement is consistent with the globally increasing within country inequality since the 1980s. This example shows that the relative technology increase faster in the foreign country than it decreases in the domestic country. Similarly the relative price increases stronger in the domestic country and decreases less in the foreign country. In the calibrated model this leads to a dominating price effect in the domestic country and a dominating market size effect for technology in the foreign country.

Figure 2: Numerical solution for the foreign country that respects IPR trading with bigger developed country. $\epsilon = 1.5$.

4.3 Trade with weak IPR protection

Figure 3 graphically summarizes the numerical results for trade with a technology imitating foreign country. The supply of high skilled workers in the foreign country is only 25% of the supply in the domestic country and 50% for low skilled workers, taking Mexico as an example. This implies that the price change is considerably greater than in the case of trade with a developed country.

Trade liberalization leads to an increasing wage gap and skill biased technological change in the domestic country (US). The high skilled unemployment rate decreases while the low skilled unemployment rate increases. This behavior is consistent with the findings of Moore and Ranjan (2005) who use the increasing unemployment rate of low skilled workers to identify the effect of trade on the wage gap. Similar to Moore and Ranjan, trade causes the low skilled wages to decrease and the high skilled wages to increase.

13Assume that Japan as well switched from autarky to free trade and neither machines were traded before.
Empirical findings

The theoretical model above shows that trade should have different effects on the unemployment rates for different IPR regimes.

To test the model predictions I analyse the relationship between unemployment rates and trade openness to countries with varying IPR enforcement regimes. I construct a data set of 141 exporting countries and 106 importing countries between 2007 and 2010 for which unemployment and IPR protection data is available. Unemployment rates for high and low skilled workers are not available for a large set of countries, thus I use the aggregate unemployment rates provided by the International Labor Organization (ILO), which should be mainly driven by low skilled unemployment as low skilled workers present a large fraction of the population in all countries. To proxy for openness between two countries I use the bilateral aggregate import shares in GDP, where imports are provided by UN Comtrade. As an indicator for IPR protection I use the classification of the WEF (World Economic Forum), which ranges between 1 (no IPR protection) to 7 (full IPR protection). As additional controls GDP and population size were obtained from Feenstra et al. (2013). Table 2 shows the descriptive statistics for the variables used.

Specifically, I estimate the following equation:

\begin{equation}
U_{st} = \alpha_0 + \alpha_1 \log(X_s) + \alpha_2 \log(X_v) + \beta_1 \log(OPEN_{s,t}) + \beta_2 \log(IPR_{v,t}) + \\
\gamma_1 \log(OPEN_{s,t}) \times \log(IPR_{v,t}) + z_s + z_v + z_t + \zeta_{s,t},
\end{equation}

Figure 3: Numerical solution for the domestic country trading with a small developing country (no IPR) with a low skill ratio. $\epsilon = 1.5$
Table 2: Descriptive statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>20,864</td>
<td>1.935</td>
<td>.450</td>
<td>.833</td>
<td>3.367</td>
</tr>
<tr>
<td>Dom. GDP</td>
<td>20,864</td>
<td>26.416</td>
<td>1.782</td>
<td>23.214</td>
<td>29.335</td>
</tr>
<tr>
<td>Foreign GDP</td>
<td>20,864</td>
<td>25.103</td>
<td>1.945</td>
<td>20.619</td>
<td>30.300</td>
</tr>
<tr>
<td>Dom. GDP</td>
<td>20,864</td>
<td>16.428</td>
<td>1.829</td>
<td>12.668</td>
<td>20.910</td>
</tr>
<tr>
<td>Foreign GDP</td>
<td>20,864</td>
<td>16.349</td>
<td>1.612</td>
<td>12.534</td>
<td>21.014</td>
</tr>
<tr>
<td>IPR protection</td>
<td>20,864</td>
<td>1.298</td>
<td>.308</td>
<td>.529</td>
<td>1.869</td>
</tr>
</tbody>
</table>

All variables in logs. Years 2007 - 2010.

where $U_{st}$ is the unemployment rate in the importing country $s$ in year $t$. $X_s$ and $X_v$ is a vector of covariates (GDP and GDP per capita) in the importing country and the exporting country $v$. $OPEN$ is the share of imports from $v$ to $s$ on $s$’ GDP. $IPR_v$ indicates the IPR protection in country $v$. $z$ are country and time fixed effects. $\zeta_{st}$ is an error term.\textsuperscript{14} The theoretical model predicts that with trade under IPR protection the unemployment rates decrease. Thus, I expect that the $\gamma$ coefficient to be negative.

Table 3 shows the results of the fixed effects regression using the unemployment rate (in levels) as dependent variable. The interaction of IPR protection and trade openness has a significant negative effect on the unemployment rate. Thus, trading with a country with weak IPRs protection is likely to increase the unemployment rate in the importing country, which squares with the model predictions.

6 Conclusion

Within country inequality has been raising globally in the last decades. The literature has identified two main sources for this development: Skill biased technological change and international trade. In this paper I link both explanation using a model of trade induced technological change. Moreover, including a search unemployment model allows to analyze unemployment rates for high and low skilled workers. I show that once we allow for trade induced technological change intellectual property rights are crucial to explain labor market outcomes. Free trade harms low skilled workers in terms of lower wages, higher unemployment rates and greater wage differentials if IPR are not enforced. This is consistent with the findings of Moore and Ranjan (2005). Once IPR are enforced the greater market for technology increases the incentives for innovations. Trade liberalization changes the relative wages through changes in the relative prices and changes in the relative technology level. With IPR protection trade liberalization still will increase wage differentials, but low skilled

\textsuperscript{14}This simple specification does not consider any spill-over effects between two countries on a third country.
Table 3: Fixed effect regression: Unemployment

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imports/GDP</td>
<td>-0.000547**</td>
<td>-0.000646**</td>
<td>-0.000643**</td>
<td>0.00105</td>
</tr>
<tr>
<td></td>
<td>(0.000328)</td>
<td>(0.000267)</td>
<td>(0.000267)</td>
<td>(0.000783)</td>
</tr>
<tr>
<td>Domestic GDP</td>
<td>-1.090***</td>
<td>-1.090***</td>
<td>-1.090***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.155)</td>
<td>(0.155)</td>
<td></td>
</tr>
<tr>
<td>Foreign GDP</td>
<td>0.000536</td>
<td>0.00161</td>
<td>0.00145</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00310)</td>
<td>(0.00292)</td>
<td>(0.00290)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.078)</td>
<td>(2.078)</td>
<td>(2.078)</td>
<td></td>
</tr>
<tr>
<td>Foreign Pop</td>
<td>-0.0617***</td>
<td>-0.0585***</td>
<td>-0.0569***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0202)</td>
<td>(0.0193)</td>
<td>(0.0195)</td>
<td></td>
</tr>
<tr>
<td>IPR</td>
<td></td>
<td>-0.0158**</td>
<td>-0.0426***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00643)</td>
<td>(0.0119)</td>
<td></td>
</tr>
<tr>
<td>IPR X (Imports/GDP)</td>
<td></td>
<td></td>
<td></td>
<td>-0.00143**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.00631)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.552***</td>
<td>101.8**</td>
<td>101.7**</td>
<td>101.7**</td>
</tr>
<tr>
<td></td>
<td>(0.0372)</td>
<td>(39.82)</td>
<td>(39.79)</td>
<td>(39.79)</td>
</tr>
<tr>
<td>Observations</td>
<td>20864</td>
<td>20864</td>
<td>20864</td>
<td>20864</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.871</td>
<td>0.882</td>
<td>0.902</td>
<td>0.933</td>
</tr>
</tbody>
</table>

All independent variables in logs. Country and year fixed effects. Standard errors in parentheses. Standard errors clustered at country-year level. * p<0.10, ** p<0.05, *** p<0.01

Workers will gain from higher real wages and a lower unemployment rate. Although the increased wage gap might been seen as a not desirable outcome of trade liberalization, policy makers are able to create lower unemployment rates and hence decrease the population share on social benefits and at the same time increase the real wage if trade liberalization is combined with IPR protection.

The model presented in the paper is consistent with many empirical observations in the United States. First, during the 1980s the United States traded mainly with developing countries which enforced IPR. In the context of the model this explains the increasing wage gap and the general declining unemployment rate. Last, more recent trade liberalization of the US with countries that only weakly enforce IPR had negative effects for low skilled workers, see Autor et al. (2013).

References


Appendix A  Search unemployment

This section derives the explicit equations for the search model used in Section 3.3.

Matching functions

\[(37) \quad M(v_i; u_i) = k \nu_i \gamma_i u_i^{1-\gamma_i} N_i = k \theta_i u_i N_i \quad \text{for } i = h, l.\]

The labor market tightness, $\theta_i$, is the ratio of vacancies to unemployed workers, $\frac{v_i}{u_i}$. The rate at which a worker finds a job is defined as

\[(38) \quad \frac{M(v_i; u_i)}{u_i N_i} = k \theta_i^\gamma \quad \text{for } i = h, l,\]

which is increasing in the labor market tightness. The rate at which a vacant position is filled is given by

\[(39) \quad \frac{M(v_i; u_i)}{v_i N_i} = k \theta_i^{\gamma-1} \quad \text{for } i = h, l,\]

which is decreasing in the labor market tightness. The flow rate into unemployment per unit of time, $\dot{u}_i$, is given as the rate of exogenously destroyed matches less the rate of workers newly employed.

\[(40) \quad \dot{u}_i = \psi (1 - u_i) - k \theta_i^{\gamma} u_i \quad \text{for } i = h, l,\]

where the parameter $\psi$ reflects an exogenous break up rate for filled positions. In the equilibrium, $\dot{u}_i = 0$ will be satisfied. This gives the steady state unemployment rate:

\[(41) \quad u_i = \frac{\psi}{\psi + k \theta_i^{\gamma}} \quad \text{for } i = h, l.\]

A vacant position is an asset for the firm. If capital markets are perfect, the capital costs $\rho F_i$ have to be equal to the rate of return on assets. The latter is given as the expected gains from filling a position less the recruitment costs, The expected gains are calculated using the marginal revenues, $t_i$ and subtract the (marginal) rental costs, $r_i$ and the wage $w$.

\[(42) \quad \rho F_i = k \theta_i^{\gamma-1} (J_i - F_i) - \delta\]
Similarly, a filled position has a capital cost of $\rho J_i$ to the firm. This has to be equal to the current marginal revenues of a worker less the wage and the (marginal) rental costs of machines less the expected loss if the match is destroyed at some point in time. In equilibrium, all profit opportunities from new jobs are exploited, driving rents from a vacant position to zero. Therefore, the equilibrium condition for the supply of vacant jobs is zero. Given a non-zero discount factor $\rho$, this is satisfied if $F_i = 0$. This implies that firms can enter and exit the market freely, and

$$J_i = \frac{t_i - w_i - r_i + \psi(F_i - J_i)}{\rho + \psi}.$$  

(44)

After substituting $J_i$ from equation (42) the above equation can be written as

$$k \theta_i^{-1} (t_i - w_i - r_i) = \delta (\psi + \rho).$$  

(45)

Workers face a similar problem as firms. $U_i$ is the discounted value of unemployment and $W_i$ is the discounted value of employment. Workers receive social benefits $b$ if unemployed. $k \theta_i^\gamma$ gives the rate at which workers are employed and $\psi$ the rate at which workers lose their job. These considerations lead to the following two Bellman equations:

$$\rho U_i = b + k \theta_i^\gamma (W_i - U_i)$$  

(46)

$$\rho W_i = w_i + \psi (U_i - W_i).$$  

(47)

Note that the permanent income $W_i$ is different from the actual wage rate $w_i$. This is caused by the risk of unemployment and hence a lower income. It is assumed that the wage will be higher than social benefits, i.e., $w_i > b$, so that an incentive to work exists.

Solving the above equation for $W_i$ yields

$$W_i = \frac{w_i + \psi U_i}{\rho + \psi}.$$  

(48)

To derive the wage as given in equation (18) a common Nash bargaining model is used by way of which
\[(49) \quad w_i = \arg \max (W_i - U_i)^\eta (J_i - F_i)^{1-\eta}.\]

The corresponding first order condition yields

\[(50) \quad W_i - U_i = \frac{\eta}{1-\eta} (J_i - F_i).\]

First, substitute (44) and (48) in (50) and use \(F_i = 0\) to obtain

\[(51) \quad w_i = \rho U_i + \eta (t_i - r_i - \rho U_i).\]

From equation (42) and \(F_i = 0\) it follows that

\[(52) \quad J_i = \frac{\delta}{k\theta_i^{-1}}.\]

Replace \(J_i\) in equation (50) by equation (42) to obtain

\[(53) \quad W_i - U_i = \frac{\eta}{1-\eta} \left( \frac{\delta}{k\theta_i^{-1}} \right).\]

Now substitute equation (53) in (46) to derive

\[(54) \quad \rho U_i = b + \frac{\eta}{1-\eta} \delta \theta_i.\]

Use (54) with (51)

\[(55) \quad w_i = b + \frac{\eta}{1-\eta} \delta \theta_i + \eta (t_i + r_i - b - \frac{\eta}{1-\eta} \delta \theta_i) \]

\[= (1-\eta)b + \frac{\eta}{1-\eta} \delta \theta_i + \eta t_i - \eta r_i - \frac{\eta^2}{1-\eta} \delta \theta_i\]

which then simplifies to the wage equation as given by equation (18).

\[(56) \quad w_i = (1-\eta)b + \eta (t_i - r_i + \delta \theta_i)\]

The wage depends on three endogenous parameters: marginal revenues, \(t_i\), (marginal) rental
costs, \( r_i \), and labor market tightness, \( \theta_i \). Substituting the expression for the \( t_i \) and \( r_i \) from equations (60) and (61) in the equations (56) yields

\[
(57) \quad w_i = (1 - \eta)b + \eta(\beta p_i^{1/\beta} q_i A_i + \delta \theta_i).
\]

\( \theta_i \) can be defined as an implicit function by using equations (56) and (45). The labor market tightness depends on marginal revenues, \( t_i \), and marginal rental costs, \( r_i \), as well:

\[
(58) \quad t_i - r_i - [(1 - \eta)b + \eta(t_i - r_i + \delta \theta_i)] - \frac{\delta(\rho + \psi)}{k\theta_i^{\gamma-1}} = 0.
\]

After substituting \( t_i \) and \( r_i \) from equations (60) and (61), this simplifies to

\[
(59) \quad (1 - \eta)[\beta p_i^{1/\beta} q_i A_i - b] - \eta \delta \theta_i - \frac{\delta(\rho + \psi)}{k\theta_i^{\gamma-1}} = 0.
\]

### Appendix B  Marginal profits

I derive the marginal revenues from employing an additional worker using the profit maximization problem of each intermediate good producer, see equation (20). The (representative) intermediate producer takes the technology level and prices as given and so that marginal revenues can be derived using equation (24):

\[
(60) \quad t_i = \left( \frac{\partial y_i}{\partial N_i} \right) p_i = \frac{1}{\beta} p_i^{1/\beta} A_i.
\]

The costs of optimally adjusting capital if one additional worker is employed are calculated using equations (22) and (23), hence the marginal capital costs are

\[
(61) \quad r_i = \frac{\partial (X_i x_i)}{\partial N_i} = \frac{1}{\beta} p_i^{1/\beta} A_i (1 - \beta).
\]

Although the representative firm does not optimize over the technology level, in the equilibrium the technology level will depend on the unemployment rate, i.e., the number of workers actually employed, and hence technology will be endogenous. If the job destruction rate is zero, no unemployment exists in the model. Hence, equation (12) would simplify to \( \rho J_i = t_i - w_i - r_i \). In this setting the value of an additionally filled position, \( J_i \), would be zero and the above equation would correspond to the first order condition with respect to labor in the firm’s maximization problem in expression (20).
Appendix C  Aghion and Howitt innovation model

I briefly outline an innovation model similar to Aghion and Howitt. While the above described model assumes firms to be monopolists and to reinvest profits in below described model the firms’ monopoly model arise endogenously. In this model technological progress will be derived in terms of growth rates and the model does not allow for comparative statics as it fully relies on numerical solutions.

Assume that individuals are able to invest into research and development. Hence the budget constraint will be

\[
\begin{align*}
 c_\tau &\leq w_{i\tau} + z_{h\tau} + z_{l\tau} + \lambda z_{h\tau-1} V_{h\tau} + \lambda z_{l\tau-1} V_{l\tau} \quad \forall \tau, i = h, l,
\end{align*}
\]

where \( c_\tau \) is the individual consumption and I omit the individual subscript, \( w_{i\tau} \) is the wage of individuals with skill \( i \), \( z_{i\tau} \) is the investment in final good units into research and development in sector \( i \), and \( \lambda z_{i\tau-1} V_{i\tau} \) is the expected value of the investment in sector \( i \) at time \( \tau \) based on the previous investment \( z_{i\tau-1} \) and the (exogenous) arrival rate of new innovation per unit of investment \( \lambda \). And innovation increases the technology level by \( \vartheta > 1 \).

Preferences are still given by equation (1) and are linear in consumption, i.e., individuals are risk-neutral and income effects do not matter. Thus, in the optimum the expected returns in the next period has to be equal to the investment in the current period:

\[
\begin{align*}
 z_\tau \lambda V_{i\tau+1} d &= z_{i\tau},
\end{align*}
\]

where \( d \) is the discount rate between two periods. The value of an innovation can be written as

\[
\begin{align*}
 r V_{i\tau+1} = \pi_{i\tau+1}^{RD} - \lambda z_{i\tau} V_{i\tau+1},
\end{align*}
\]

where \( r \) is the exogenous interest rate and \( \pi_{i\tau+1} \) are the profits generate for an successful innovation in the next period given by equation (26). Combining both equation yields the optimal investment for each sector.

\[
\frac{\pi_{i\tau+1}^{RD}}{r + \lambda z_{i\tau}} = V_{i\tau+1} = \frac{1}{\lambda d},
\]
The next periods profits depend on the demand for machines in the next period given in equation (21), and hence are a function of the future price $\bar{p}_{\tau+1}$.\textsuperscript{15} This implies that the Aghion and Howitt innovation model has a similar mechanism as the simplified model, i.e., more workers will increase the demand for machines and hence the incentives to invest into R&D. On the other hand the price effect will counter-veil. The simplified model has two major advantages. First, in the Aghion and Howitt style model technology levels have to be constructed based on a initial technology level $A_{\tau} = 0$ using the growth dynamics. Second, growth dynamics are highly non-linear and hard to solve for. For example, increasing the high skilled labor supply will directly increase the investment into high-skill complementary technology and the high-skill technology growth rate will increase. The price effect goes in the opposite direction, as the output in the high-skill sector increases the price decreases in the high skill sector and increases in the low-skill sector, thus investment in the high-skill technology decreases and in the low-skill technology increases. Last, the employment effects need to be factored in, which adds even more dynamic adjustment. In the light of the outlined Aghion and Howitt innovation model the simplified model is more tractable and gives more insights in the mechanism.

Appendix D  Labor supply shock and SBTC

**Proposition 3:** For a given $\theta_1 (\theta_h)$, an increase in the labor supply $N_h (N_l)$, will raise the labor market tightness $\theta_h (\theta_l)$ and reduce the unemployment rate $u_h (u_l)$.

Assuming that the labor market tightness of the other sector is constant, the labor market tightness of each sector is increasing with its labor supply. This implies that the unemployment rate in each sector is a decreasing function of the respective labor supply, since

$$\frac{\partial \theta_i}{\partial N_i} > 0 \quad \text{for } i = h, l,$$

which can be seen after substituting $p$ into equation (34). The decreasing unemployment rate is (mainly) driven by the increasing technology, which can be interpreted as a capitalization effect, i.e., recruitment costs become less and less important and new matches are formed easier. This is consistent with the empirical evidence for a negative relationship between the unemployment rate and labor productivity, see Pissarides and Vallanti (2007).

\textsuperscript{15}Note that the price $\bar{p}_{\tau+1}$ depends on the relative supply in the two sectors which is not effected by the investment decision.
Nevertheless, a high skilled labor supply shock will have two opposing effects on the technology level. First, it reduces the relative price by increasing the production of the high skilled good. This will diminish the profit incentives of the high skilled complementary R&D firm and will lower the technology level. Second, the demand for high skilled machines will increase as more high skilled workers are employed. This in turn increases the profits of the high skilled complementary R&D firm and the technology level. The net effect is of the negative price change and the positive labor market size effect on the relative technology level can be evaluated by using equation (28) and replacing the relative price by equation (30). Assuming that the labor market tightness in both sectors does not change, technology will always increase with the labor supply of the complementary worker, i.e., \( \frac{\partial A_i}{\partial N_i} > 0 \) for \( i = h, l \).

Similarly, the relative technology can be expressed as a function of the labor supply (and the unemployment rates).

\[
\frac{A_h}{A_l} = \left( \frac{q_h}{q_l} \right) \left( \frac{\bar{N}_h}{\bar{N}_l} \right)^{\frac{\beta}{\mu}} = \kappa \left( \frac{q_h}{q_l} \right) \left( \frac{(1 - u_h)N_h}{(1 - u_l)N_l} \right)^{\frac{\mu(\epsilon - 1)}{(1 - \mu)(\epsilon + 1)\epsilon + \mu}},
\]

where

\[
\kappa = \left( \frac{\omega}{1 - \omega} \right) \left( \frac{q_h}{q_l} \right)^{\frac{\beta}{\epsilon + \mu}}.
\]

The relative technology increases in \( \bar{N}_h \) and decreases in \( \bar{N}_l \) for given unemployment rates if \( \epsilon > 1 \). Hence, technological change will always be skill biased if the skill ratio increases.

**Proposition 4:** For a given labor market tightness in the two sectors, an increase in the labor supply \( \bar{N}_h \) (\( \bar{N}_l \)) raises (reduces) the relative technology if \( \epsilon > 1 \). An increasing skill ratio implies skill biased technological change.

**Appendix E   US historical labor supply shock**

For the supply of high skilled and low skilled workers, I use data from the US Census of Population Educational Attainment from 1963 to 2003, where skilled workers are defined by having at least a college degree. The raising supply of high skilled workers in the US increased the skill ratio from 0.09 to 0.37 between 1963 and 2003. Figure 4 presents the associated consequences in the calibrated model graphically.

The wage gap decreases for elasticities of substitution between 1 and 2.5, but for higher values of \( \epsilon \) the wage gap increases. In contrast to Acemoglu (2003) and Moore and Ranjan
Figure 4: Numerical solution for the US labor supply of high skilled and low skilled workers using various values of $\epsilon$.

(2005) an $\epsilon$ in excess of 2.5 is needed to explain a slight increase of the wage gap. The increasing skill ratio will lead to skill biased technological progress for any $\epsilon > 1$. In the context of the present model the commonly estimated elasticity of substitution between high and low skill intensive intermediate goods, $\epsilon$, is too low to explain the increasing wage gap in autarky due to an increasing supply of skilled workers although we observe skill biased technological change. As both the absolute number of low and high skilled workers increase the technology level increases in the country, i.e., not only high skilled workers become more productive but as well low skilled workers. As the production function of intermediate goods has decreasing returns to technology, the productivity of low skilled workers increases faster than for high skilled workers, which leads to an decreasing wage gap. If high and low skill intensive intermediate goods are closed substitutes, $\epsilon$ is low, than effect becomes even stronger. Only if high and low skill intensive intermediate goods the price effect will be sufficiently strong to increase the relative wages in the model.