GROWING THROUGH TRADE IN INTERMEDIATE GOODS:
THE ROLE OF FOREIGN GROWTH AND DOMESTIC TARIFFS

Carmen D. Álvarez-Albelo*, Antonio Manresa† and Mònica Pigem-Vigo‡

ABSTRACT

We show that pure Ricardian trade can account for the empirical evidence that domestic growth is more affected by foreign growth than by trade openness. To this purpose, we develop a two-country model including a backward economy that exchanges intermediate goods with a faster growing country. We obtain three main results regarding growth and welfare of the backward economy: (i) the growth-enhancing comparative advantage is facilitated by faster foreign growth; (ii) the growth rate may be negatively affected or unaffected by a domestic tariff, while it is always positively impacted by foreign growth; (iii) a domestic tariff could be welfare-improving.

JEL classification: F10, O24, O41

Keywords: Growing through trade; pure Ricardian trade; foreign growth; domestic tariff

* Corresponding author: Department of Economics, Accounting and Finance, University of La Laguna, Campus de Guajara, c./ Rector Ángel Gutiérrez Navarro 19, 38205 La Laguna, Spain, calbelo@ull.edu.es
† Department of Economic Theory and Center for Research in Welfare Economics (CREB), University of Barcelona, Av. Diagonal 690-696, 08034 Barcelona, Spain, manresa@ub.edu
‡ Department of Economic Theory, University of Barcelona, Av. Diagonal 690-696, 08034 Barcelona, Spain, pigem@ub.edu

We gratefully acknowledge comments and suggestions from Jaume Ventura. The computing codes are available upon request to the authors. Antonio Manresa acknowledges financial support from Spanish Ministry of Economy and Competitiveness (ECO 2012-34046 and ECO 2013-41917-P) and Generalitat de Catalunya (2014 SGR-493 and XREAP2015-308149).
I INTRODUCTION

Despite the profusion of studies on the connection between trade and economic growth, no conclusive results have been obtained on this issue (e.g. Rodríguez and Rodrik, 2001; Singh, 2010). However, some of the empirical literature indicates that domestic features are not the only ones playing a role, but also foreign conditions. Indeed, the econometric findings in Arora and Vamvakidis (2005) reveal that the trading partners’ growth rate has a greater impact on domestic growth than trade openness. More specifically, they separated countries between closed and open ones, according to Sachs and Warner’s (1995) definition, and in both cases obtained that a country’s growth is positively affected by their trading partners’ growth rate. While empirical studies have found a positive linkage between domestic and foreign growth (e.g. Easterly, 2001; Calderón, Loayza and Schmidt-Hebbel, 2006; Arora and Vamvakidis, 2006, 2011), the findings regarding the relationship between trade policy and growth are mixed (e.g. Rodríguez and Rodrik, 2001; Yanikkaya, 2003; Clements and Williamson, 2004; DeJong and Ripoll 2006; Madsen, 2009). Any explanation to these empirical results needs an approach that focuses “on how much economic conditions in trading partners matter for growth” (Arora and Vamvakidis, 2005, p. 27), besides how much trade openness matters for growth.

In this paper, we adopt this approach to show that pure Ricardian trade can account for the aforementioned empirical evidence. As argued by Yenokyan, Seater and Arabshahi, (2014), a pure Ricardian approach is valuable for the study of the trade-growth linkage, since the empirical results regarding the significance of scale economies are mixed (e.g. Backus, Kehoe and Kehoe, 1992; Hanson and Xiang, 2004;
Head and Mayer, 2004), and international spillovers, though empirically relevant (e.g. Liu and Buck, 2007; Keller, 2010), cannot be considered as a trade mechanism.

To this purpose, we build on Ventura’s (1997) model and develop a theoretical framework with two technologically different countries: a backward economy (country B) that may boost its growth rate simply by exchanging intermediate goods with a faster growing country (country P). It should be highlighted that our model considers trade in intermediate goods as it represents the largest share of world trade flows. Indeed, as reported by Miroudot, Lanz and Ragoussis, (2009, p. 48), for OECD countries, trade in intermediate inputs represents 56.2% (in 2006) and 73.19% (in 2005) of trade flows of goods and services, respectively.¹ We assume that country P grows at an exogenous rate, while for country B trade becomes the only possibility of achieving faster growth in the long-run by boosting investment in physical capital. Thus, our framework captures the empirical result that trade impacts growth mainly via capital investment (e.g. Levine and Renelt, 1992; Baldwin and Seghezza, 1996; Wacziarg, 2001; Wacziarg and Welch, 2008). In this context, we consider an import tariff established by country B that can never be growth-enhancing. In the theoretical literature on this topic, the positive relationship between tariffs and growth relies on scale economies and international spillovers (e.g. Grossman and Helpman, 1990; Rivera-Batiz and Romer, 1991; Lee, 2011).

Countries B and P produce a non-traded final good with two traded intermediate inputs, goods \( x \) and \( z \). In the same vein as Yenokyan et al. (2014), the final good technologies in countries may differ in input shares, reflecting different input intensities. This assumption allows any growth outcome in country B, going from autarky growth

---

¹ The figures for the emerging economies of Brazil, China and India are 72.7% and 67.14%, 75.3% and 86.99%, 79.5% and 47.85%, respectively.
to convergence in growth rate with the trading partner. The production of intermediate goods uses capital and labor. In sector \( x \) there is exogenous labor-augmenting technological progress, with productivity gains being greater in country \( P \) than in country \( B \).\(^2\) The countries have the same AK technology in sector \( z \), which is the result of an external learning-by-doing (LBD) process à la Arrow (1962). We analyze the long-run equilibrium, and also perform numerical exercises to evaluate the temporary tariff impacts on growth and welfare.

The comparative advantages of countries in the long-run rely on exogenous productivity gains in sector \( x \) and input intensities in the final good sector. Since relative prices reflect the relative scarcity of intermediate goods, identical input shares in countries would result in country \( P \) (country \( B \)) having comparative advantage in good \( x \) (good \( z \)). However, comparative advantages may reverse when country \( P \) is less intensive in good \( x \) than country \( B \).

We show that for benefiting from trade in terms of faster growth, country \( B \) must get rid of sector \( x \) and be specialized in sector \( z \), the one with learning opportunities (Young, 1991; Reeding, 1999).\(^3\) Remarkably, we obtain that the growth-enhancing comparative advantage is facilitated by faster growth in country \( P \). Otherwise, country \( B \) would not benefit from foreign productivity gains, so its long-run growth rate remains unchanged.

---

\(^2\) Empirical evidence has documented differences in sectoral total factor productivity between countries (e.g. Fadinger and Fleiss, 2011).

\(^3\) Literature has shown that the specialization in sectors with higher learning opportunities fosters long-run growth, and vice versa. Moreover, the presence of dynamic productivity gains linked to externalities may result in a “wrong” specialization pattern, since the comparative advantage in a particular period depends on knowledge accumulated until then. In our analysis, we discard this possibility by assuming that countries specialize according to their long-run comparative advantage.
Thus, faster growth emerges in country B because of specialization and more favorable relative prices than in autarky, which rises the interest rate and hence the growth rate. Yet, the import tariff may be growth-impairing, since it introduces a wedge between international and domestic relative prices. When country B has the growth-enhancing comparative advantage, the trade equilibrium can be characterized by either complete or incomplete specialization of country P, while the backward economy only produces good z.

Under incomplete specialization, country B eventually faces the more favorable autarky price of the trading partner. Nonetheless, the backward economy cannot reach the partner’s growth rate as long as both countries have an AK technology in sector z, which prevents the equalization of interest rates. Moreover, the growth rate of country B is affected positively by foreign growth and negatively by a domestic import tariff. Even so, we can establish a sufficient condition on the input intensities for the foreign growth effect to outweigh the tariff effect.

Complete specialization allows the equalization of interest rates and hence of growth rates. The reason lies in the combination of an AK technology in sector z and a Cobb-Douglas technology in sector x, so country P’s capital stock can be adjusted to allow convergence. Owing to the AK technology in sector z, the domestic relative price of country B does not depend on the import tariff, so there could be room for placing the international relative price of good z above the one arising under free trade. Consequently, faster growth in country P always increases the growth rate of country B, but changes in the import tariff have no impact on domestic growth.

Moreover, under complete specialization there might be a rationale for setting a tariff, since the policy could be welfare-improving for the backward economy. This possibility arises when the tariff has a slight impact on the capital accumulation of
country P, which occurs when this economy is highly intensive in good $x$. Nonetheless, we show that although a tariff does not affect long-run growth, it causes temporary growth reductions in both countries because of a lower capital accumulation.

Our model belongs to the rather scarce literature where the impacts of trade on long-run growth operate solely via the comparative advantage and specialization, and the countries exchange increasing amounts of goods at constant terms of trade. Manresa and Pigem-Vigo (1999) and Álvarez-Albelo et al. (2009) showed that a stagnated economy can converge in growth rate solely by trading in intermediate goods with a growing economy. However, their models involve the same technology to accumulating capital in countries. With a multi-country model of AK economies, Acemoğlu and Ventura (2002) obtained that trade and specialization yield convergence in growth rates even in the absence of decreasing returns. Yenokyan et al. (2014) built a two-country model with different factor intensities in technologies, and showed that trade in factors of production can result in either faster growth and convergence in growth rates –under complete specialization– or in uneven growth –under incomplete specialization–. Ji and Seater (2014) developed a model involving R&D and endogenous market structures that can deliver any growth outcome in countries. However, their model does not include capital accumulation.

Our work is also related to the literature on the tariffs effects on growth and welfare. Osang and Pereira (1996) studied these effects with a model of human capital accumulation representing a small open economy. They found that most tariffs reduce growth, but a revenue-neutral tariff reform\(^4\) may enhance welfare and long-run growth. Osang and Turnovsky (2000) obtained the same result using a similar framework, but

---

\(^4\) Starting from a uniform tariff structure, this reform must keep the present value of public revenues constant.
including LBD, endogenous labor supply and access restrictions to international capital markets. In a framework of small open economy and LBD, Naito (2006) obtained growth and welfare increases with a combination of a consumer-price-neutral tariff, a growth-enhancing tax reform and a tax increase on the less distorted good. By contrast, our work is not just concerned with studying the effects of a tariff on growth and welfare, but with offering an explanation based on pure Ricardian trade to the empirical evidence by Arora and Vamvakidis (2005). To this aim, we develop a two-country model where faster long-run growth may only emerge from trade and never from an import tariff.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 solves the autarky equilibrium. Section 4 shows the role of comparative advantage for growing through trade. Section 5 identifies the conditions for country B to have the growth-enhancing comparative advantage, and analyzes the trade equilibrium in the long-run. Section 6 studies the impacts of foreign growth and an import tariff on domestic growth. Section 7 summarizes and concludes.

II THE MODEL

The model involves two technologically different countries: country B, a backward economy that may increase its long-run growth rate simply by trading with a faster growing partner, country P. In an autarky situation they grow at the exogenous rates $\theta_i$, $i = B, P$, with $0 \leq \theta_B < \theta_P$. In this context, the countries engage in trade relationships according to their long-run comparative advantages, which might only affect the growth potential of the backward economy. In addition, country B sets an import tariff that can never be growth-enhancing.
In each period, \( t \in \left[0, \infty \right) \), both economies are inhabited by a continuum of identical households that is normalized to one. The households are endowed with one unit of time that can be only allocated to work, so the labor input is equal to one. The countries produce a non-traded final good with two traded intermediate inputs. The final good can be used for consumption and investment in physical capital. The factor inputs labor and capital are allocated to the production of intermediate goods. In addition, markets are competitive, international factor flows are not allowed and foreign and domestic intermediate goods are homogeneous.

The non-traded final good, \( y^i(t) \), is produced with the Cobb-Douglas technology:

\[
y^i(t) = \left( x^i(t) \right)^{\alpha_i} \left( z^i(t) \right)^{1-\alpha_i}, \quad \alpha_i \in (0,1),
\]

where \( x^i(t) \) and \( z^i(t) \) denote the total productions of intermediate inputs. However, this notation will adequately change in a trade situation. We purposely assume that input shares may be different in countries, i.e. \( \alpha_\beta \neq \alpha_\rho \). As shown by Yenokyan et al. (2014), dissimilar input intensities may impede the equalization of countries’ interest rates, so convergence in growth rates is not guaranteed. Henceforth, it is convenient to keep in mind that good \( x \) is chosen as the numeraire. Thus, one unit of final good costs \( p^i(t) \) units of good \( x \), while one unit of good \( z \) costs \( p^z(i)(t) \) units of good \( x \).

We consider the following technologies for intermediate goods:

\[
x^i(t) = \left( k^x_i(t) \right)^{\beta} \left( TP^x(i)l^i(t) \right)^{1-\beta},
\]

\[
\beta \in (0,1), \quad TP^x(t) = TP^x(0)e^{\theta^x}, \quad TP^x(0) > 0, \quad \theta^x \in [0,\theta^x), \quad \theta^x > 0.
\]

\[
z^i(t) = \left( k^z_i(t) \right)^{\beta} \left( k^i(t)(1-l^i(t)) \right)^{1-\beta},
\]

\[
\theta^z \in [0,\theta^z), \quad \theta^z > 0.
\]
where $k^i_x(t)$ and $l^i(t)$ denote capital and labor allocated to sector $x$, respectively, and $k^i_z(t)$ and $l^i(t) - l^i(t)$ represent the amounts of factor inputs used in sector $z$. In sector $x$ the variable $TP^i(t)$ represents labor-augmenting technological progress, with initial value $TP^i(0)$ and exogenous growth rate $\theta_i$. In sector $z$ productivity gains come from an external LBD process à la Arrow (1962) linked to capital per capita of the economy, $k^i(t) = k^i_x(t) + k^i_z(t)$, and hence the technologies become of AK type. The LBD process entails an engine of productivity gains that may allow country B to take advantage of trade gains in terms of growth. Since $0 \leq \theta_B < \theta_P$, in autarky country P enjoys a higher long-run growth rate than country B. We consider identical factor shares between countries and across sectors. This simplifying assumption allows us to identify $\beta$ and $1 - \beta$ with aggregate capital and labor shares which, according to some of the empirical literature, do not differ much between countries (e.g. Gollin, 2002).

Furthermore, in a trade situation country B sets an ad-valorem import tariff with tax rate $\tau \geq 0$, and distribute the tax revenues among the households in the form of lump sum transfers, $T^B(t) \geq 0$.

Preferences are identical in both countries. The representative household derives utility from consumption, $c^i(t)$, and maximizes its intertemporal utility discounted at the rate $\rho$:

$$U^i(0) = \int_0^\infty e^{-\rho t} \left( \frac{(c^i(t))^{1-\sigma}}{1-\sigma} - 1 \right) dt, \quad \sigma > 0,$$  \hspace{1cm} (4)

subject to the budget constraint:

---

*For notational simplicity, the equilibrium condition of the labor market has been introduced.*
\[ a'(t) \leq r'(t)a'(t) + w'(t) + T'(t) - p'(t)c'(t), \]  

(5)

and the initial condition, \( a'(0) > 0 \). The variable \( a'(t) \) denotes wealth, \( r'(t) \) is the interest rate and \( w'(t) \) is the wage. Moreover, in a trade situation it holds that \( T^a(t) \geq 0 \) and \( T^p(t) = 0 \).

### III Autarky Equilibrium

The maximization of profits in the final good sector implies that the prices of intermediate goods are equal to their respective values of marginal productivities. Thus, \( p^i_z(t) \) can be expressed in terms of the relative production of intermediate goods:

\[ 1 = p'(t)\alpha \frac{y'(t)}{x'(t)}, \quad p^i_z(t) = p'(t)(1-\alpha) \frac{y'(t)}{z'(t)} \rightarrow p^i_z(t) = \frac{1-\alpha}{\alpha} \frac{x'(t)}{z'(t)}. \]  

(6)

From the maximization of profits in the intermediate good sectors, interest rate and the wage can be written as:

\[ r'(t) = \beta \frac{1}{p'(t)k_z'(t)} - \delta + \rho - \frac{\beta p^i_z(t) z'(t)}{p'(t) k_z'(t)} - \delta + \frac{\rho}{p'(t)}, \]  

(7)

\[ w'(t) = (1-\beta) \frac{x'(t)}{I'(t)} = (1-\beta) p^i_z(t) \frac{z'(t)}{I'(t)}, \]  

(8)

where \( \delta > 0 \) is the depreciation rate of capital. Moreover, the equation driving consumption through time comes from solving the household’s problem:

\[ \frac{c'(t)}{c'(t)} = \frac{1}{\sigma} \left( r'(t) - \rho - \frac{\rho}{p'(t)} \right). \]  

(9)

\[ ^6 \text{In equilibrium the household’s wealth } a'(t) \text{ will be equal to } \rho'(t)k'(t). \]
The competitive equilibrium is a set of allocations and prices that satisfy firms and household problems, and clear all markets in the economy. The relative prices in (6) and the factor price equalization in (7) and (8) yield the factor allocation:

$$\frac{k^i(t)}{k^l(t)} = l^i(t) = \alpha_i. \quad (10)$$

Using equations (2), (3), (6) and (10), $p_z^i(t)$ can be expressed as:

$$p_z^i(t) = \left(\frac{TP^i(0)e^{\theta t}}{k^l(t)}\right)^{\alpha - \beta}. \quad (11)$$

Also, using equations (1), (2), (3), (6), (7) and (10) we can write the interest rates as:

$$r_i(t) = \beta A_i \left(p_z^i(t)\right)^{\alpha - \beta} - \delta + (1 - \alpha_i) \frac{\beta^i(t)}{p_z^i(t)} = \beta A_i \left(p_z^i(t)\right)^{\alpha - \beta} - \delta + (1 - \alpha_i) \frac{\beta^i(t)}{p_z^i(t)}, \quad (12)$$

where $A_i = \alpha_i (1 - \alpha_i)^{\alpha_i}$. After introducing (12) in the growth rate of consumption in (9), we obtain:

$$\frac{c^i(t)}{c^i(t)} = \frac{1}{\sigma} \left(\beta A_i \left(p_z^i(t)\right)^{\alpha - \beta} - \delta - \rho\right) = \frac{1}{\sigma} \left(\beta A_i \left(p_z^i(t)\right)^{\alpha - \beta} - \delta - \rho\right). \quad (13)$$

The long-run equilibrium is a balanced growth path (BGP) where the growth rate is equal to $\theta_i$ and $p_z^i(t)$ becomes equal to:

$$p_z^i = \left(\frac{\sigma \theta_i + \delta + \rho}{\beta A_i}\right)^{\frac{1}{\alpha_i}}. \quad (14)$$
As expected, there is a positive relationship between productivity gains in sector $x$ and the autarky relative price of good $z$.

IV THE ROLE OF COMPARATIVE ADVANTAGE FOR GROWING THROUGH TRADE

From the autarky prices in equation (14) it follows that country B could have comparative advantage in good $z$ or in good $x$. By solving the trade equilibrium in country B at exogenously given terms of trade, we show that growing through trade with country P is only possible when the former economy has comparative advantage in good $z$.

Comparative advantage in good $z$

Trade may allow country B to get rid of lower productivity gains in sector $x$ by importing good $x$. When this is the case, the final good production can be written as $y^B(t) = (x^B_B(t))^{\alpha_x} (z^B_B(t))^{1-\alpha_x}$, where $x^B_B(t)$ is the imported production of good $x$ (produced in country P and used in country B). The amount of good $z$ used within country B is denoted as $z^B_B(t)$, and hence the exported production is denoted as $z^B_P(t)$.

We will maintain these notation criteria throughout the paper. The equilibrium in the trade balance implies that $x^B_B(t) = p^B_z(t) z^B_B(t)$, where the omission of the country superscript denotes international price. Moreover, the interest rate can be expressed as:

$$r^B(t) = \beta A_B \left( p^{B,D}_z \right)^{\alpha_z} - \delta + (1-\alpha_B) \frac{\dot{p}_z(t)}{p_z(t)},$$

where $p^{B,D}_z = p_z/(1+\tau)$ is the domestic relative price. After introducing the interest rate in equation (9), we obtain the growth rate of country B in the long-run, $\mathcal{g}^B$.
Equation (16) reveals that a growth increase will emerge from trade provided that country B enjoys a more favorable domestic relative price than in autarky. More remarkably, under this specialization pattern the terms of trade of country B are positively related to the partner’s growth rate, so faster foreign growth leads to higher interest rate and growth rate. However, an increase in the import tariff may impair domestic growth, unless country B can influence the international relative prices. This issue will be analyzed later on.

**Comparative advantage in good x**

The final good production can be written as $y^B(t) = \left(x^B_p(t)\right)^{\alpha_a} \left(z^B_x + z^B_{a}(t)\right)^{1-\alpha_a}$, while the equilibrium in the trade balance is $x^B_p(t) = p_x(t)z^B_{a}(t)$. Since the terms of trade become constant in the long-run, country B enjoys its own productivity gains in sector $x$ and hence grows at the same rate as in autarky:

$$g^B = \frac{1}{\sigma} \left[ \beta A_B \left(p_\zeta^{B,D}\right)^{1-\alpha_a} \left(TP^B(0)e^{\psi^B}\right)^{1-\beta} \right] - \delta - \rho = \theta_B,$$

where $p_\zeta^{B,D} = p_\zeta(1+\tau)$ is the domestic relative price.

V Growth-Enhancing Comparative Advantage and the Trade Equilibrium

In this section we address four questions. We first identify the conditions for country B to have the growth-enhancing comparative advantage. Then, we characterize the trade equilibrium under this specialization pattern. The trade equilibrium allows us to undertake the third task, namely, to study the potential impacts of trade on the long-run
growth rate of country B. Lastly, we assess the long-run welfare effects of a tariff and hence the question about the rationale of setting a tariff in this context.

The comparative advantages of countries

Figure 1 illustrates the determinants of comparative advantages, namely, input intensities and the growth rates of technological progress. The autarky price of country P as a function of $\alpha_p$, $p_z^p(\alpha_p, \theta_p)$, is indicated with a solid line. The dashed/dotted line represents country B’s autarky price as a function of $\alpha_B$, $p_z^B(\alpha_B, \theta_B)$. The functions reach a maximum at $\alpha^\text{max}_i \equiv 1 - (\sigma \theta + \delta + \rho)/\beta$, which is smaller than one for empirically plausible parameter values. We set $\alpha_B = \bar{\alpha}_B$, so choose a particular value for country B’s autarky price, $p_z^B(\bar{\alpha}_B)$. Thus, we can define a threshold value $\bar{\alpha}_p(\bar{\alpha}_B, \theta_p)$ such that $p_z^B(\bar{\alpha}_B) = p_z^p(\bar{\alpha}_p(\bar{\alpha}_B, \theta_p), \theta_p)$.\(^7\)

**FIGURE 1 ABOUT HERE**

Consequently, the condition $\alpha_p < \bar{\alpha}_p(\bar{\alpha}_B, \theta_p)$ implies that country B has comparative advantage in good $x$, while $\alpha_p > \bar{\alpha}_p(\bar{\alpha}_B, \theta_p)$ leads to the opposite result. Moreover, faster growth in country P ($\tilde{\theta}_p > \theta_p$) moves its autarky price upwards, $p_z^p(\alpha_p, \tilde{\theta}_p)$ (indicated with a dashed line), thus amplifying the range of values for $\alpha_p$ that allow country B to have comparative advantage in good $z$. Therefore, the growth-

\(^7\) The autarky prices of countries might also coincide for a higher $\alpha_p$. We leave aside this possibility since it involves extreme values for $\alpha_p$. For example, the parameter values in Table 1 (at the end of this section) yield $\alpha^\text{max}_B = 0.74$ and $p_z^B(\alpha^\text{max}_B) = 0.35$, with $p_z^p = p_z^B(\alpha^\text{max}_B)$ for $\alpha_p = 0.49$ and $\alpha_p = 0.94$. 

13
enhancing comparative advantage is facilitated by faster growth of the partner’s
economy.

The trade equilibrium

As commented earlier, country B will eventually be completely specialized in good \( z \),
since country P has absolute advantage in good \( x \). Country P, however, may produce
both goods (incomplete specialization), or just good \( x \) (complete specialization), which
relies on input intensities in the final good production of countries.

The production of final good in country B has been indicated in section IV, while
country P’s can be written as \( y^P(t) = \left( x^P(t) \right)^{\alpha_P} \left( z^P(t) + z^B_P(t) \right)^{1-\alpha_P} \). The maximization
of profits in the final good sector of countries B and P and the equilibrium in the trade
balance\(^8\) yield the equilibrium expression for the international relative price:

\[
p^*_c(t) = \frac{(1-\alpha_P)x^P(t)}{\Omega^B_\tau z^B(t) + \alpha_P z^P(t)},
\]

where \( \Omega^B_\tau = \alpha_B / \left( \alpha_B + (1-\alpha_B)(1+\tau) \right) \), and the exported proportions by countries:

\[
\frac{z^B_P(t)}{z^B(t)} = \Omega^B_\tau,
\]

\[
\frac{x^B_P(t)}{x^P(t)} = \frac{l^P(t) - \alpha_P}{l^P(t)}.
\]

The interest rate of country B appeared in (15), while the interest rate of country P
comes from the equation (12) after removing the country superscript in the relative price

\(^8\) Namely, \( p^*_c(t) = \frac{1-\alpha_P}{\alpha_P} \frac{x^P(t)}{z^P(t)} \), \( p^B_P(t) = \frac{1-\alpha_B}{\alpha_B} \frac{x^P(t)}{z^B(t)} \) and \( p^*_c(t) = \frac{x^P(t)}{z^P(t)} \), respectively.
of good \( z \). The equalization of interest rates and wages between sectors of country P
and the relative price in (18) yield the factor allocation in this economy:

\[
\frac{k^P_z(t)}{k^P(t)} = I^P(t) = \alpha P + \Omega^B \frac{k^B(t)}{k^P(t)}.
\]  

(21)

The long-run growth rate of country B

It is reasonable to assume that country B sets an import tariff that allows a growth
increase in the long-run which, according to (16), requires that:

\[
\frac{p^P_z}{I + \tau} > p^B_z \quad \Rightarrow \quad \tau \in \left[0, \tau^\text{max}\right], \quad \tau^\text{max} \equiv \frac{p^P_z - p^B_z}{p^B_z}
\]  

(22)

The growth possibilities of country B depend on whether or not trade leads to the
equalization of interest rates between the countries. The interest rates, in turn, hinge on
international and domestic relative prices. Thus, to study the growth potential outcomes
of country B we rely on the trade equilibrium and also on the example depicted in
Figure 2. In the figure, the autarky prices \( p^B_z(\tilde{\alpha}_B) \) and \( p^P_z(\alpha_P) \) are denoted with dotted
lines, while international and domestic relative prices \( p_z \) and \( p_z/(1+\tau) \) are indicated
with solid lines.

FIGURE 2 ABOUT HERE

Let us first consider the trade equilibrium with incomplete specialization of the
trading partner, where country B faces country P’s autarky price, i.e. \( p_z = p^P_z \). As both
economies produce good \( z \) with an AK technology, the growth rates of countries P and
B converge to:

\[
\mathcal{G}^P = \frac{\beta A^P (p^P_z)^{\sigma}}{\sigma} - \delta - \rho = \theta_P,
\]  

(23)
respectively. For better understanding the results in equation (24), it is convenient to notice that in a free trade (FT) equilibrium with no tariff the interest rates (growth rates) of countries would equalize when $\alpha_p = \bar{\alpha}_b$, yielding the international price:

$$p^{FT}_{z} = \left(\frac{\sigma \theta_p + \delta + \rho}{\beta \bar{A}_B}\right)^{\frac{1}{\bar{\alpha}_b}}.$$ (25)

However, since the tariff reduces the domestic price of country B, this equalization would require a higher $\alpha_p$ ($\hat{\alpha}_p > \bar{\alpha}_b$), which depends on the tariff size. The value $\hat{\alpha}_p$ yields an international price $(1 + \tau) p^{FT}_{z}$ and a domestic price $p^{FT}_{z}$. In this respect, it is worth noting that the needed $\hat{\alpha}_p$ will not exist if $\bar{\alpha}_b \geq \alpha_p^{\text{max}}$ and also when the tariff rate is high enough.

Let us assume that $\bar{\alpha}_p < \alpha_p < \hat{\alpha}_p$, so the autarky price of country P and the domestic price of country B are such that $p^{B}_z < p^{P}_z/(1 + \tau) < p^{P}_z < (1 + \tau) p^{FT}_{z}$. From equation (24) it follows that country B grows faster than in autarky, i.e. $\theta_B > \theta_p$, because its domestic price is higher than its autarky price. Nonetheless, country B would need a domestic price as $p^{FT}_{z}$ to converge in growth rate with the partner, so the backward economy grows more slowly than country P, i.e. $\theta_B < \theta_p$. Accordingly, the exported proportions by countries B and P in (19) and (20) asymptotically approach $\hat{\alpha}_B$ and zero, respectively, and the factor allocation in country P in equation (21) converges to that in autarky.
Under incomplete specialization of country P, convergence in growth rates (interest rates) is only possible when $\alpha_p = \hat{\alpha}_p$. In this case, the factor allocation in country P would be equal to $\alpha_p + \tilde{\Omega}_b < 1$, owing to the wage equalization between the countries.\(^9\)

Moreover, the exported proportions by countries B and P would be $\tilde{\Omega}_b$ and $\tilde{\Omega}_b / (\alpha_p + \tilde{\Omega}_b)$, respectively.

The condition $\alpha_p = \hat{\alpha}_p$ may also lead to complete specialization of country P provided that $\alpha_p + \tilde{\Omega}_b \geq 1$. So does the condition $\alpha_p > \hat{\alpha}_p$. Since there is a Cobb-Douglas technology in sector $x$, the capital stock of country P can be adjusted to enable the equalization of interest rates and hence of growth rates:

$$g^p = \frac{\beta A_p \left((1 + \tau) p^c \right)^{(1-\alpha_p)} \left(\frac{TP_p(\theta) e^{\theta y}}{k^p(t)}\right)^{1-\beta}}{\sigma} - \delta - \rho = \theta_p, \quad (26)$$

$$g^b = \frac{\beta \tilde{A}_b \left(p^c \right)^{\hat{\alpha}_b}}{\sigma} - \delta - \rho = \theta_p. \quad (27)$$

The exported proportions by countries B and P become equal to $\tilde{\Omega}_b$ and $1 - \alpha_p$, respectively.

*Long-run welfare impacts as a rationale for an import tariff*

A question that must be addressed refers to the welfare impacts as a rationale for setting a tariff in this context. Noticeably, this possibility relies on country B’s capability of affecting the international relative prices. Indeed, country B is price-acceptant when the trading partner produces both goods, so an import tariff will harm its domestic growth.

\(^9\) Note that $\omega^r(t) = p^c(t)(1 - \beta)k^c(t)$, $i = B.P$, so $k^g(t) = k^p(t)$.
and welfare. However, under complete specialization the international prices come from
the interplay between the countries, which yields convergence in growth rates. In this
section, we will show that under the latter trading regime an import tariff may improve
long-run welfare of country B, so there could be a rationale for such a policy.

We analyze this possibility by evaluating the dynamic system that appears in
Appendix A in the BGP, and computing long-run capital and the allocation of final
output between consumption and gross investment. The variables in the dynamic system
become constant in the long-run as they are expressed per efficiency unit
in sector $x$, i.e. divided by $TP^p(t)$. From now on, variables per efficiency unit will be
denoted with a bar.

The countries’ capital stocks in the BGP can be written as:

$$
\ddot{k}^p = \left(p^p_x\right)^{\alpha_p (1-\beta)} \left(\frac{1}{1-\beta} \frac{\partial \ddot{k}^p}{\partial \tau}\right) < 0, \quad (28)
$$

$$
\ddot{k}^B = \frac{1-\alpha_p}{\Omega_B} \left(p^p_x\right)^{\beta_a (1-\beta)} \left(\frac{1}{1-\beta} \frac{\partial \ddot{k}^B}{\partial \tau}\right) < 0. \quad (29)
$$

Moreover, consumption is equal to final output minus gross investment, $c^i$:

$$
\ddot{c}^p = \left(p^p_x\right)^{\alpha_p (1-\beta)} \left(\frac{1}{1-\beta} \frac{\partial \ddot{c}^p}{\partial \tau}\right) \frac{\partial \ddot{c}^p}{\partial \tau} < 0, \quad \ddot{c}^P > 0. \quad (30)
$$

The impact of a tariff on final output can be decomposed in an income effect and an
accumulation effect. Firstly, holding capitals constant, the income effect gives rise to a
permanent increase (decrease) of country B’s (country P’s) final output because of the
lower exported (imported) proportion of good \( z \), i.e. \( \Omega_{g} \). In other words, country B can import the same quantity of good \( x \) in exchange of a smaller quantity of good \( z \).

Secondly, the tariff worsens the terms of trade of country P, which lowers its interest rate and hence capital accumulation and final output in both economies. A look at equations (28) and (30) reveal that the tariff reduces country P’s final output and gross investment by the same proportion, so this economy experiences a fall in consumption and welfare. By contrast, equations (29) and (31) show quite different results for country B. Indeed, final output decreases by a lower proportion than gross investment, so a tariff has an ambiguous effect on consumption. An increase in consumption could take place if the accumulation effect is weakened, which occurs when country P is highly intensive in good \( x \), i.e. for high enough \( \alpha_{p} \). This is so because the interest rate of country P is barely affected by the terms of trade. Next, we explore this possibility through the numerical examples that appear in Table 1.

**TABLE 1 ABOUT HERE**

We compute numerically the long-run values of capital, consumption, final output and international and domestic prices under free trade and with an import tariff of 10%. To do so, we consider the calibration for the US by Cooley and Prescott (1995), which provides us with reliable parameter values reflecting the actual behavior of an economy. In addition, we set \( \alpha_{p} = 0.45 \) and \( \alpha_{p} = \{0.65, 0.85\} \). Consistently with our theoretical analysis, the tariff causes a greater decline in the countries’ capital stock and final output under \( \alpha_{p} = 0.65 \) than with \( \alpha_{p} = 0.85 \). Moreover, the former value yields a decrease in country B’s consumption, while an increase is obtained with the latter value.
VI THE GROWTH IMPACTS OF FOREIGN GROWTH AND A DOMESTIC IMPORT TARIFF

We have just shown that under incomplete specialization of the trading partner the long-run growth rate of country B is affected by both foreign growth and a domestic import tariff. In this section, we study the net impact of changes in these variables on domestic growth. Furthermore, under complete specialization the long-run growth rate of country B is affected by foreign growth, but unaffected by a domestic tariff. Even so, setting a tariff may have temporary growth and welfare impacts that we illustrate by computing numerically the transitional dynamics.

The impacts of foreign growth and a tariff on domestic long-run growth

Under incomplete specialization, the growth rate of country B in equation (24) depends on the domestic relative price of good \( z \), which in turn depends on foreign growth (via the terms of trade) and the import tariff. For assessing the relative growth impacts of changes in these two variables, we compute the percentage variation of country B’s domestic relative price in response to a percentage point change in the trading partner’s growth rate (foreign growth elasticity, \( \varepsilon_{p^*_{BD}, \theta_p} \)) and in the import tariff (tariff elasticity, \( \varepsilon_{p^*_{BD}, \tau} \)):

\[
\varepsilon_{p^*_{BD}, \theta_p} = \frac{\partial p_{z}^{B,D}}{\partial \theta_p} \frac{\theta_p}{p_{z}^{B,D}} = \frac{1}{\alpha_p} \frac{\sigma \theta_p}{\sigma \theta_p + \delta + \rho} > 0, \tag{32}
\]

\[
\varepsilon_{p^*_{BD}, \tau} = \frac{\partial p_{z}^{B,D}}{\partial \tau} \frac{\tau}{p_{z}^{B,D}} = \frac{\tau}{1 + \tau}. \tag{33}
\]

For \( \alpha_p \leq \alpha^*_p \equiv \sigma \theta_p / (\sigma \theta_p + \delta + \rho) \), the foreign growth effect is greater than the tariff effect provided that \( \varepsilon_{p^*_{BD}, \tau} \geq 1 \). Nonetheless, according to the calibrated parameter
values in Table 1, this case involves the narrow value range $\alpha_p < \alpha^*_p = 0.1304$. For $\alpha_p > \alpha^*_p$, the net impact relies on the tariff size:

$$
\epsilon_{p, p^0, \theta_p} \begin{cases}
\geq \left| \epsilon_{p, p^1, \tau} \right| & \text{if } \tau < \tau^* \equiv \frac{\sigma \theta_p}{\alpha_p (\sigma \theta_p + \delta + \rho) - \sigma \theta_p}.
\end{cases}
$$ (34)

Even so, we can establish a sufficient condition for the foreign growth effect to prevail over the tariff effect. More specifically, $\epsilon_{p, p^0, \theta_p} > \left| \epsilon_{p, p^1, \tau} \right|$ if $\bar{\alpha}_p < \alpha_p \leq \alpha_B \leq \alpha^*_p$; because the threshold tariff $\tau^*$ is greater than the maximum tariff rate $\tau^\text{max}$ defined in (22). Indeed, $\alpha_p = \bar{\alpha}_B$ and $\theta_B = 0$ yields $\tau^* > \tau^\text{max}$. So does $\alpha_p < \bar{\alpha}_B$ and $\theta_B > 0$ provided that both conditions reduce $\tau^\text{max}$, while the former one increases $\tau^*$.

By contrast, when the partner is completely specialized in good $x$, the growth rate of country B in equation (27) does not depend on the import tariff, and domestic and foreign growth rates exhibit a positive one-to-one relationship. It is also worth noting that, as illustrated in Figure 2, convergence in growth rates may take place in the presence of an import tariff and under free trade. In the former case the domestic price is lower than the international price, $p_{z}^{FT} < (1+\tau)p_{z}^{FT}$, while in the latter case both countries face the international price $p_{z}^{FT}$.

The short-run growth and welfare effects of a tariff

Under incomplete specialization country B becomes negligible in terms of income, and eventually behaves as an AK economy facing constant terms of trade. By contrast, short-run effects can be sizeable when both countries are completely specialized. This issue is relevant because, although a tariff does not affect the long-run growth rate, it does have temporary growth and welfare impacts. For analyzing these impacts, we
compute numerically the transitional dynamics in a neighborhood of the BGP using the parameter values in Table 1. The solution method is described in Appendix B.

The numerical exercises are aimed to compare the transitional dynamics when country B sets an import tariff of 10% with the dynamics under free trade. To do so, we construct the time paths of ratios of value with a tariff over value under free trade. Figures 3 and 4 depict the ratios of capital, consumption and final output, and also the ratios of international and domestic relative prices of good \( z \) for \( \alpha_p = 0.65 \) and \( \alpha_p = 0.85 \), respectively. We consider these two values for \( \alpha_p \) since, as shown in the previous section, they greatly affect capital accumulation and welfare.

**FIGURES 3 AND 4 ABOUT HERE**

The introduction of a tariff causes a permanent deterioration of country P’s terms of trade and a temporary decrease in the domestic price of country B, which reduces the interest rates and hence the incentives to accumulate capital. Consequently, the ratios of capital in both figures are lower than unity. Regarding final output, the figures show a decline in the ratios of both countries, which can be explained by the double effect of a tariff formerly studied. Both effects reduce country P’s final output, so the ratio of this economy is lower than unity. For country B, however, the effects go in opposite directions. Indeed, at the beginning of the transition the ratio of country B is greater than unity since the income effect dominates over the accumulation effect. As the transition advances, the latter effect becomes increasingly important, thus causing a decline in final output of the backward economy. Moreover, Figure 4 shows smaller reductions in capital and final output than Figure 3. These results are consistent with our previous analysis showing that the accumulation effect is weakened when the partner’s
economy is highly intensive in good $x$. All in all, the tariff impairs short-run growth in both countries.

The response of consumption is also related to the effects of a tariff on final output. The ratio of country P is lower than unity in both figures, so this economy experiences a fall in consumption and welfare. In country B, however, the tariff has an ambiguous impact on consumption. Indeed, the income effect entails a permanent increase in final output and consumption. Contrariwise, the accumulation effect reduces capital accumulation, output and consumption. The tariff becomes welfare-improving when the partner’s economy is highly intensive in good $x$, as long as this condition weakens the second effect. Consistently, in Figure 3 the ratio of consumption of country B eventually becomes lower than unity, while in Figure 4 this ratio holds above unity throughout the transition.

VII SUMMARY AND CONCLUSION

We have shown that pure Ricardian trade, in the absence of scale economies and international spillovers, can account for the empirical evidence that domestic growth is more affected by the trading partners’ growth rate than by trade openness. To do so, we have developed a two-country model based on Ventura’s (1997), where a backward economy may boost its long-run growth by trading in intermediate goods with a faster growing partner. In addition, the backward economy sets an import tariff that could never be growth-enhancing. The model captures two significant empirical facts, namely, world trade flows are largely of intermediate goods, and trade impacts growth mainly via capital investment.

Owing to the countries’ differences in technology, the model can deliver any growth outcome in the backward economy, ranging from autarky growth to convergence in
growth rate with the trading partner. We have obtained that growing through trade requires having comparative advantage in a sector with learning opportunities, which is facilitated by a higher growth rate of the partner. Moreover, under the growth-enhancing comparative advantage, growth acceleration in the partner’s economy always increases domestic growth, which is consistent with empirical findings. This is so because faster foreign growth improves the backward economy’s terms of trade, which boosts capital accumulation.

The net impact of changes in foreign growth and a tariff rate on domestic growth depends on the specialization regime of the partner. Under incomplete specialization the backward economy grows more slowly than the partner, and its growth rate is negatively affected by a tariff, since this economy cannot influence the international prices. Even so, we have identified a sufficient condition for the foreign growth effect to prevail over the tariff effect. By contrast, under complete specialization there is equalization of countries’ growth rates, and the domestic relative price of the backward economy does not depend on the tariff. Consequently, convergence in growth rates can take place with an import tariff and under free trade. Furthermore, under this specialization regime there may be a rationale for setting a tariff, since this policy could be welfare-enhancing for the backward economy. Yet, a tariff impairs short-run growth of both countries because of its negative impact on capital accumulation.

In line with Arora and Vamvakidis (2005b), we conclude that understanding the trade-growth linkage requires going beyond domestic conditions, such as the degree of openness. Indeed, we have shown that domestic growth can rely more on foreign growth than on a domestic import tariff, even under a pure Ricardian approach.
APPENDIX A: DYNAMIC SYSTEM UNDER COMPLETE SPECIALIZATION

Considering $\theta_b = 0$, both countries will be completely specialized provided that:

$$\left( \frac{TP^B (0)}{TP^A (0) e^{\rho^B (T)}} \frac{1}{1 - \alpha_p} \right)^{1-\beta} < \frac{p^B (t)}{1 + \tau} \frac{1 - \alpha_p}{\Omega_B} \left( \frac{\kappa^B (t)}{1 - \beta \alpha_p} \right)^{1-\beta} < \left( \frac{1}{\kappa^B (t)} \right)^{1-\beta} .$$

(A1)

In the computation of the transitional dynamics, we assume that $\bar{k}^P (0) = \bar{k}^B (0)$. The initial values of technological progress can be then adjusted to ensure complete specialization of both economies throughout the transition.

The dynamic system driving the time evolution of the two-country economy is composed of the two usual transversality conditions, and the following four differential equations:

$$\bar{c}^B (t) = \frac{1}{\sigma} \left( \beta A_p \frac{\Omega_B}{1 - \alpha_p} \left( \frac{\kappa^B (t)}{1 - \beta \alpha_p} \right)^{1-\beta} - \delta \right) \bar{c}^B (t) - \theta_p \bar{c}^B (t),$$

(A2)

$$\bar{c}^B (t) = \frac{1}{\sigma} \left( \beta A_p \frac{1 - \alpha_p}{\Omega_B (1 + \tau)} \left( \frac{\kappa^B (t)}{1 - \beta \alpha_p} \right)^{1-\beta} - \delta \right) \bar{c}^B (t) - \theta_p \bar{c}^B (t),$$

(A3)

$$\bar{\kappa}^B (t) = \alpha_p \frac{\Omega_B}{1 - \alpha_p} \left( \frac{\kappa^B (t)}{1 - \beta \alpha_p} \right)^{1-\beta} - \bar{c}^B (t) - (\delta + \theta_p) \bar{\kappa}^B (t),$$

(A4)

$$\bar{\kappa}^B (t) = \frac{1 - \alpha_p}{\Omega_B} \left( \frac{1}{1 - \beta \alpha_p} \right)^{1-\beta} \left( \frac{\kappa^B (t)}{1 - \beta \alpha_p} \right)^{1-\beta} - \bar{c}^B (t) - (\delta + \theta_p) \bar{\kappa}^B (t).$$

(A5)

where variables with a bar are expressed in terms of efficiency units of labor in sector $x$, i.e. divided by $TP^p (t)$, so they become constant in the BGP.
APPENDIX B: SOLUTION TO THE LINEARIZED DYNAMIC SYSTEM UNDER COMPLETE SPECIALIZATION

To construct Figures 3 and 4 in sub-section 6.2, we linearized the dynamic system defined in Appendix A in a neighborhood of the BGP:

\[
\begin{pmatrix}
\frac{\dot{c}^p}{c^p}(t) \\
\frac{\dot{c}^B}{c^B}(t) \\
\frac{\dot{k}^p}{k^p}(t) \\
\frac{\dot{k}^B}{k^B}(t)
\end{pmatrix} = 
\begin{pmatrix}
J_{11} & \cdots & J_{14} \\
\vdots & \ddots & \vdots \\
J_{41} & \cdots & J_{44}
\end{pmatrix}
\begin{pmatrix}
\frac{\dot{c}^p}{c^p}(t) - \frac{c^p}{c^p} \\
\frac{\dot{c}^B}{c^B}(t) - \frac{c^B}{c^B} \\
\frac{\dot{k}^p}{k^p}(t) - \frac{k^p}{k^p} \\
\frac{\dot{k}^B}{k^B}(t) - \frac{k^B}{k^B}
\end{pmatrix},
\]

(B1)

After finding eigenvalues \(\lambda_j\) and corresponding eigenvectors \(v_j\) of the Jacobian matrix, the solution of the linearized system can be expressed as:

\[
\begin{pmatrix}
\frac{c^p}{c^p}(t) \\
\frac{c^B}{c^B}(t) \\
\frac{k^p}{k^p}(t) \\
\frac{k^B}{k^B}(t)
\end{pmatrix} = 
\begin{pmatrix}
\frac{c^p}{c^p} \\
\frac{c^B}{c^B} \\
\frac{k^p}{k^p} \\
\frac{k^B}{k^B}
\end{pmatrix} + 
\begin{pmatrix}
v_{11} & \cdots & v_{14} \\
\vdots & \ddots & \vdots \\
v_{41} & \cdots & v_{44}
\end{pmatrix}
\begin{pmatrix}
b_1e^{\lambda_1t} \\
b_2e^{\lambda_2t} \\
b_3e^{\lambda_3t} \\
b_4e^{\lambda_4t}
\end{pmatrix},
\]

(B2)

where \(b_j\) are the constants of integration which are determined by the values at \(t=0\).

The BGP is locally stable in the four cases computed. The eigenvalues belong to real numbers; there are two negative eigenvalues, \(\lambda_1\) and \(\lambda_2\), and two positive eigenvalues; \(b_1 = b_2 = 0\); and the constants of integration \(b_1\) and \(b_2\) come from solving the system of equations:

\[
\begin{align*}
\frac{\dot{k}^p}{k^p}(0) - \frac{k^p}{k^p} &= v_{21}b_1 + v_{22}b_2, \\
\frac{\dot{k}^B}{k^B}(0) - \frac{k^B}{k^B} &= v_{41}b_1 + v_{42}b_2.
\end{align*}
\]

(B3)

REFERENCES


