Capital Tax Competition, Vertical Fiscal Externalities, and the Fiscal Gap in a Federation with Variable Factor Supplies
(Preliminary and Incomplete Draft)

Nikos Tsakiris,∗ Panos Hatzipanayotou,† Michael S. Michael‡

July 6, 2017

Abstract

This paper contributes to the literature on vertical fiscal externalities and on the fiscal gap by considering variable factor supplies, i.e., variable supply of capital due to international mobility and variable labor supply due to endogenous labor-leisure choice. Specific capital and labor taxes by both levels of governments finance the provision of a federal and of a state public consumption goods. Our main results are that, first, when capital and labor are substitutes in production, then, the vertical fiscal externality can be reversed from negative, implying inefficiently high Nash specific capital taxes, to positive, implying inefficiently low Nash specific capital taxes. Second, when factor supplies are variable, even with specific factor taxes, the fiscal gap can be positive, requiring resource transfers from the federal to the state governments.


Keywords: Fiscal Federalism, Vertical Fiscal Externalities, Fiscal Gap, Variable factor Supplies.

∗Department of Economics, University of Ioannina, P.O. Box 1186, 45110 Ioannina, Greece. E-mail: ntsak@cc.uoi.gr
†Corresponding Author: Department of International and European Economic Studies, Athens University of Economics and Business; 76, Patission str., Athens 104 34, Greece, and CESifo (Center for Economic Studies and the Ifo Institute of Economic Research). E-mail: hatzip@aueb.gr
‡Department of Economics, University of Cyprus; P.O. Box 20537 Nicosia, CY 1678, Cyprus, and CESifo. E-mail: m.s.michael@ucy.ac.cy
1 Introduction

A prevalent feature in federal, i.e., multileveled, economies is the co-sharing of tax-bases between the different levels of fiscal authority, e.g., between federal (high-level) and state (low-level) governments. This co-sharing of tax-bases potentially gives rise to so-called “common pool problems” whereby tax decisions by one level of government affect tax-payers decisions, which in turn affect the shared tax-bases. In the core of this common pool problem two issues of importance are the so-called vertical fiscal externalities and fiscal gap.

Vertical fiscal externalities arise when state governments levy (high) taxes, unduly discounting the harm done to federal tax revenues and consequently to other states, in so far that the latter are entitled to share of the federal tax revenues.\footnote{Vertical fiscal externalities are different and concurrent to horizontal fiscal externalities, i.e. tax competition among state governments resulting to horizontal mobility of tax-bases among them. While horizontal fiscal externalities are likely to lead to low state taxes, the vertical fiscal externalities, by and large, lead to high state taxes.} Contributions to the literature on vertical fiscal externalities include Boadway and Keen (1996), Boadway et al. (1998), Keen and Kotsogiannis (2002, 2004), Hauffer and Löffesmann (2015).\footnote{For a survey of vertical fiscal externalities see Keen (1998).} Boadway and Tremblay (2012) provides a general review of the fiscal federalism literature and survey, among others, the empirical literature on vertical fiscal externalities. The pivotal conclusion of this literature is that when taxation is specific the emerging vertical fiscal externalities are negative, implying ineffectively high Nash specific taxes. With ad valorem taxation, however, the vertical fiscal externality can be either positive or negative, e.g., Dahlby and Wilson (2003), Kotsogiannis and Martinez (2008), Karakosta (2010). The fiscal gap in a federal fiscal system arises as a result of resource transfers between the different levels of government. A negative (positive) fiscal gap entails the transfer of resources from (to) low-level, e.g., state, to (from) high-level governments. Boadway and Keen (1996) show that with specific taxes, resulting to negative vertical fiscal externality, the federal government sets a negative specific tax rate which implies a negative fiscal gap. Subsequent contributions to this to this literature, e.g., Dahlby and Wilson (2003), Kotsogiannis and Martinez (2008), Boadway and Tremblay (2012) demonstrate that with ad valorem taxation the fiscal gap is generally ambiguous depending on the underlying fundamentals of the federal economy.

The central objective of the paper is to revisit the interrelationship between vertical fiscal externalities and fiscal gap in a structure with federal-cum-state government factor taxes, and variable factor supplies. To this end we develop a model of a federal economy with many state governments or jurisdictions. Variable factor supplies are due to international mobility, e.g., capital, and due to endogenous labor-leisure decision, i.e., labor. Specific capital and labor taxes by both levels of governments finance the provision of a federal and of a state public consumption goods.

Its contribution to the literature of vertical fiscal externalities is that its sign can be reversed from negative to positive, implying inefficiently low, instead of high, Nash specific capital taxes. Intuitively, when state governments raise their own specific capital tax rates, on the one hand the capital tax base of the federal government contracts and as a result federal revenues fall. According
to the relevant literature this result tends to leave state taxes too low when taxation is per-unit or specific. On the other hand, however, a rise in state capital specific taxes increases labor supply when capital and labor are substitutes in production. This expands the labor tax base of the federal government, thus, federal tax revenues. Consequently, in the presence of variable labor supply the vertical fiscal externality, under plausible conditions, can be reversed from negative to positive.

In regards to the literature on fiscal gap, we conclude that in the present framework, even with specific factor taxes, the fiscal gap can be positive, requiring resource transfers from the federal to the state governments, and that under plausible conditions the optimal response policy of the federal government is a positive instead of negative specific tax rate.

2 The Model

The model is one of a federal economy with $N$ symmetric states, populated by $K \geq 1$ identical, in terms of their preferences, immobile households. Since households are assumed identical, following standard practice, we normalize $K = 1$, thus $N$ also denotes the overall population of the federal economy. The federal economy is assumed small in world commodity and factor markets, thus world commodity are fixed and exogenous. All commodity and factor markets are perfectly competitive.

Given the assumption of symmetric states in the federation, hereon we layout the characteristics of such a representative state. A number of freely tradable consumption goods are produced, and production technologies are assumed identical across all states. Several primary factors are used in the production of the traded goods, among which, capital $(K)$, and labor $(L)$, are in variable supply. Supply of $(K)$ is variable due to international mobility of the factor, while supply of $(L)$, an internationally immobile factor, is variable due to endogenous labor-leisure choice. All other factors are internationally immobile and in fixed endowment. As a result, the rate of return to all factors in fixed endowments and to labor are endogenously determined, while the rate of return to the internationally mobile capital is equal to the fixed world rate of return to the factor.

The production side is represented by the Gross Domestic Product (GDP) function $R(K, L)$ denoting the maximum value of domestic production of all traded goods given the supply of capital $(K)$ and the level of employment $(L)$. World commodity prices and supplies of all other factors are suppressed as arguments in the GDP function since they fixed and invariant. $K = \overline{K} + k$ is the supply of capital in the representative state, $\overline{K}$ is the state’s own capital endowment, and $k \geq 0$ is the amount of state capital employed "abroad" $(k < 0)$ or of "foreign" capital employed locally $(k > 0)$. Hereon, for the purposes of our analysis and without loss of generality, we assume that $k > 0$, implying that the representative state is a capital-importing one. The derivatives of the $R(.)$ function with respect to $K$ and $L$, i.e., $R_K = \partial R/\partial K$ and $R_L = \partial R/\partial L$, respectively, denote the marginal revenue products of capital and labor. Strict concavity is assumed of the $R(.)$ function with respect to $K$ and $L$, i.e., $R_{KK} (= \partial R_K/\partial K) < 0$ and $R_{LL} (= \partial R_L/\partial L) < 0$.

3International mobility of capital may refer to mobility on the factor across the states of the federation, and/or its mobility between the federation and the "rest of the world".
A representative household derives utility from the consumption of the traded goods, leisure, and two public consumption goods. A "state" public consumption good \((g)\) provided free of charge by the state authority to its residents, and a "federal" public consumption good \((G)\) provided free of charge by the federal authority to the residents of all states in the federation. The representative household’s consumption spending and preferences are depicted by the minimum expenditure function \(E(g, G, L, u)\). It denotes the minimum expenditure on traded goods required to attain a given level of utility \((u)\), given the levels of employment \((L)\), of consumption of the state and federal public goods, \(g\) and \(G\), and of fixed world commodity prices, which tacitly are omitted as arguments of the \(E(.)\) function. The derivatives of the \(E(.)\) function, \(-E_g\) and \(-E_G\), denote respectively the so called marginal willingness to pay for the consumption of the state and federal public goods, the derivative \(E_u\), by the properties of the \(E(.)\) function, is the inverse of the marginal utility of income, and \(E_L\) capture the representative household’s reservation wage, denoted by \(\tilde{w}\), i.e., \(E_L = \tilde{w}\), e.g., as recently modeled by Anderson and Neary (2016).\(^4\) The \(E(.)\) function is strictly convex in \(L\), i.e., \(E_{LL} > 0\), and strictly concave in \(g\) and \(G\), i.e., \(E_{gg} < 0\) and \(E_{GG} < 0\). Outputs of the private goods are costlessly used for \(g\) and \(G\).\(^5\)

Equilibrium in the representative state’s labor market requires that the reservation wage equals the net rate of return to workers, the latter being the marginal revenue product of labor minus a consolidated labor tax \(\sigma\), i.e.,:

\[
E_L (g, G, L, u) = R_L (K, L) - \sigma. \quad (1)
\]

Equilibrium in the state’s capital market requires that the net rate of return to the internationally mobile capital, being the marginal revenue product of capital minus the consolidated capital tax \(\tau\), must equal the fixed world rate of return to capital \((\rho)\). That is:

\[
R_K (K, L) - \tau = \rho. \quad (2)
\]

The consolidated labor and capital taxes are respectively denoted by \(\sigma = s + S\) and \(\tau = t + T\), where \(s\) and \(t\) are the state labor and capital tax rates, \(S\) and \(T\) are the corresponding federal ones.\(^6\) Equations (A.1) and (A.2) in the Appendix provide detailed calculations of the effects of changes in \(\tau\) and \(\sigma\) on the supply of capital and level of employment.

The state and federal governments finance the provision of the public consumption goods, \(g\) and \(G\), respectively. State tax rates \((t, s)\) are the strategic policy instruments of the state government,

---

4This modelling is quite prevalent in international trade literature that examines various issues, such as welfare effects from trade liberalization under edogenous labor supply within a perfectly competitive context (see among others Mayer 1991, and Mayer and Li 1990). Recently, Arkolakis and Esposito (2014) consider variable labor supply under monopolistic competition to re-examine the gains from international trade.

5For analytical tractability of the results we assume \(E_{Lu} = E_{Lg} = E_{LG} = 0\). Such an assumption is supported by considering a separable utility function, as in Kotsogiannis and Martinez (2008), of the type \(u(x, L) + b(g) + B(G)\), where \(x\) is a private-numeraire good, \(u(.)\) is quasi-concave, increasing in \(x\) and decreasing in \(L\), \(b(g)\) and \(B(G)\), respectively, are increasing and concave.

6In the present framework, like by and large in the relevant literature, state taxes are not deducted or credited against federal taxes. For this issue see, e.g., Dahlby et al (2000).
while federal taxes $(T, S)$ and resource (income) transfers $F$ are the strategic policy instruments of the federal government. The state government acts non-cooperatively, i.e., Nash, against all other state governments, and there are no horizontal, i.e., among states, resource transfers. Assuming that both levels of government maintain a balanced budget, their respective budget constraints are denoted by:

$$g = sL + tK + F, \quad \text{and}$$

$$G = N(SL + TK - F),$$

where $F$, the "vertical" lump-sum income transfer from the federal to the state government, can be either positive ($F > 0$) or negative ($F < 0$). Totally differentiating the budget constraints (3) and (4) we obtain the following comparative statics results, central to the analysis to follow:

$$g_t = \frac{dg}{dt} = K + \alpha, \quad g_T = \frac{dg}{dT} = a, \quad g_F = 1$$

$$G_t = NA, \quad G_T = N(K + A), \quad G_F = -N, \quad \text{and}$$

$$g_t = g_T + K, \quad \text{and} \quad G_T = NK + G_t,$$

where $\alpha = \Delta^{-1}(sR_{KL} + tZ_{LL})$ and $A = \Delta^{-1}(SR_{KL} + TZ_{LL})$, $\Delta(< 0)$ and $Z_{LL}(> 0)$ are defined in Appendix (see equation A.2). Equation (A.3) in the Appendix provides more analytical expressions of these results. Subscripts denote partial derivatives, e.g., $g_t = \partial g/\partial t$, $g_T = \partial g/\partial T$, etc. We adopt the plausible assumptions that $g_t > 0$ and $G_T > 0$, i.e., a higher state/federal capital tax increases the provision of the state/federal public consumption good. This assumption requires that $(K + \alpha) > 0$ and $(K + A) > 0$, respectively. $G_t$ captures the so-called vertical fiscal externality between the state and federal governments due to the tax setting behavior of the former. Contrary to the Boadway and Keen (1996) where the sign of the vertical fiscal externality is unambiguously negative when taxes are specific and factor supplies are fixed, here, with specific factor taxes but variable factor supplies, the sign of the vertical fiscal externality can be positive. Specifically, $G_t > 0$ if $R_{KL} > 0$, and $G_t \leq 0$ if $R_{KL} < 0$. When $R_{KL} > 0(< 0)$ we say that $K$ and $L$ are complement (substitute) factors in production.

To close the model, the representative state’s income-expenditure constraint requires that private spending on traded goods equals income from production minus capital and labor taxes and payments to foreign capital used domestically. That is:

$$E(g, G, L, u) = R(K, L) - \tau K - \sigma L - \rho k.$$
3 Benchmark Case: The Unitary Government

We first consider the equilibrium outcome pursued by a single, i.e., unitary, government, whose objective is to maximize the federal welfare $W = Nu$ by choosing $\tau$, subject to the consolidated government budget constraint:

$$Ng + G = N (\sigma L + \tau K) \quad (7)$$

This result serves as a benchmark case to the alternative equilibrium setting of capital taxes by both levels of government. Differentiating the federal welfare function $W = Nu$ with respect to the capital tax $\tau$, and using equations (1) and (2) we obtain:

$$\frac{1}{N} dW = E_u du - E_g \frac{dg}{d\tau} - E_G \frac{dG}{d\tau} - \sigma \frac{dL}{d\tau} - K,$$

Furthermore, from equation (7) we have:

$$N \frac{dg}{d\tau} + \frac{dG}{d\tau} = N \left( \sigma \frac{dL}{d\tau} + \tau \frac{dK}{d\tau} + K \right). \quad (9)$$

Combining equations (9) and (8), using equations (A.2), and assuming that $dG = 0$, the first order condition for the optimal provision of $g$ requires that the optimal capital tax is chosen so that:

$$\frac{1}{N} dW = E_u du = -E_g (\alpha + A) - (1 + E_g) K - \sigma \frac{dL}{d\tau} = 0. \quad (10)$$

Following a similar procedure, assuming $dg = 0$, the first order condition for the optimal provision of $G$ requires that the capital tax is set so that:

$$\frac{1}{N} dW = E_u du = -NE_G (\alpha + A) - (1 + NE_G) K - \sigma \frac{dL}{d\tau} = 0, \quad (11)$$

where we define $NE_G$ to be the global marginal willingness to pay for the federal public consumption good.

Combining the first-order conditions (10) and (11), yields the standard optimality rule for the provision of $g$ and $G$:

$$(E_g - NE_G) (g_\tau - G_\tau) = 0. \quad (12)$$

That is, in a second-best environment the unitary government is indifferent between spending a dollar for the provision of $g$ or $G$ when households’ marginal willingness to pay for the provision of the state or the federal public consumption goods are the same. The optimality rule (12) determines the unitary optimum capital tax ($\tau$).\footnote{The equivalent condition in Kotsogiannis and Martinez (2008), i.e., their equation (16), states that the second best unitary optimum capital tax is such that the sum of the marginal rate of substitution between both the state and the federal public goods and the private good are equal to the marginal cost of public funds (MCPF).} We can state:
Lemma 1 At the unitary optimum, the specific capital tax \( \tau \) is set so that the marginal willingness to pay for the state public consumption good equals the global marginal willingness to pay for the federal public consumption good.

4 Capital Taxes and Vertical Fiscal Externality

Now, we turn to the case where the two levels of government pursue separately the setting of fiscal policy, i.e., the choice of capital taxes \( \tau \) and \( T \). In this context, the vertical fiscal externality signifies the impact of the tax setting behavior of the representative state government on the provision of the federal public consumption good, i.e., \( \text{sign}(G_t) \) given in equations (5). As reviewed, Boadway and Keen (1996) demonstrate that with specific labor taxes this externality is unambiguously negative, while Kotsogiannis and Martinez (2008) extending the previous work to the case of ad valorem labor taxes, conclude that this externality can be positive. We revisit these results in the case of endogenous factor, i.e., \( K \) and \( L \), supplies, specific factor taxes, and capital tax competition among states of the federal economy. For expositional clarity of the results, we consider the case of a federal economy with two identical states \( (N = 2) \), e.g., Home and Foreign, and we examine the so called vertical fiscal externality due to the tax setting behavior of one state, e.g., say \( t^* \) of Foreign, on the provision of the federal public consumption good, i.e., \( \partial G/\partial t^* \), and on the welfare level of Home, i.e., \( du/dt^* \). By equations (5) the induced impact on \( G \) of setting \( t^* \) is given as follows:

\[
\frac{dG}{dt^*} = G_{t^*} = \frac{SR_{L^*K^*} + TZ_{L^*L^*}}{\Delta^*}.
\]

The effect on Home’s welfare due to the setting of \( t^* \) is given by totally differentiating equation (6) and making use of (13) to obtain:

\[
E_u \frac{du}{dt^*} = -E_G G_{t^*} = -E_G \frac{SR_{L^*K^*} + TZ_{L^*L^*}}{\Delta^*}.
\]

Equations (13) and (14) indicate that in the present context of variable factor supplies, even the setting of a specific capital tax by one state entails an ambiguous impact on the provision of the federal public good, i.e., the setting of a state capital tax entails an ambiguous vertical fiscal externality, and on the welfare of the other state. An increase in \( t^* \) by Foreign entails an unambiguously negative vertical fiscal externality if \( R_{L^*K^*}^L \) is positive, i.e., when \( K \) and \( L \) are substitutes in production. If, however, \( R_{L^*K^*}^L \) is negative, i.e., \( K \) and \( L \) are complements in production, then the setting of \( t^* \) by Foreign may lead to a positive vertical externality. Thus, the higher \( t^* \) has a positive (ambiguous) effect on Home’s welfare when \( R_{L^*K^*}^L \) is positive (negative).

We state the following Proposition:

Proposition 1 A higher specific capital tax \( \tau^* \) by Foreign entails a positive vertical fiscal externality and leads to a higher Home welfare if the following condition holds:

\[\text{Totally differentiating equation (6) with respect to } t^* \text{ yields } E_u \frac{du}{dt^*} = -E_G G_{t^*} = -E_G \frac{SR_{L^*K^*} + TZ_{L^*L^*}}{\Delta^*}, \text{ since } d\tau = 0. \text{ Also from equations (A.1), } \frac{dK}{dt^*} = \frac{dL}{dt^*} = 0, \text{ as a result of which } \frac{du}{dt^*} = 0. \text{ Hence, the result in equation (14)}\]
\[-\eta_{L^*K^*} > \frac{TK^*}{SL^*} \left( \gamma_{L^*L^*} - \eta_{L^*L^*} \right) \]  \hspace{1cm} (15)

**Proof.** see equations [A.4] in the Appendix. \hfill \blacksquare

### 5 Capital Taxes, Intergovernmental Transfers, and the Fiscal Gap

We now examine the case where fiscal policy is pursued by both the state and the federal governments. We consider a two-stage game, where we assume that the federal government holds a first mover’s advantage vis-a-vis all states. In the first stage, the federal government, in order to maximize the federal economy’s cumulative welfare level \( W = Nu \), chooses the rate of capital tax \( T \) and the level of the vertical income transfer \( F \), thus implicitly setting the level of \( G \). In the second stage, because of the symmetry assumption, a representative state government chooses its own specific capital tax \( t \), in order to maximize the state level of welfare \( u \), given \( T \), \( F \) and \( G \), and the (specific) capital taxes set by all other states. We further assume that both federal and state labor taxes \((S \text{ and } s)\) are fixed at all times. As standard practice calls for, the sub-game perfect equilibrium is solved via backwards induction.

Starting with the second stage of the game, the optimal specific capital tax for the state is determined by totally differentiating the income constraint (3) and setting the first-order-condition equal to zero. Doing so we obtain:

\[
E_u \frac{du}{dt} = H_t = -E_g \frac{dg}{dt} - K = -E_g \alpha - (1 + E_g) K = 0 \implies t^o = t^o (T, F). \hspace{1cm} (16)
\]

Changes in \( t^o \) due to changes in \( T \) and \( F \) are obtained by totally differentiating the first-order-condition \( H_t = 0 \) with respect to \( t \), \( T \) and \( F \) to get:

\[
\frac{dt}{dT} = t_T = -H_{tt}^{-1} H_{tT} \quad \text{and} \quad \frac{dt}{dF} = t_F = -H_{tt}^{-1} H_{tF}, \hspace{1cm} (17)
\]

where \( H_{tt}, H_{tT}, \text{ and } H_{tF} \) are derived in equations [A.5] in Appendix 5.

Equations (A.5) in the Appendix provide details of these derivations. For stability \( H_{tt}^{-1} < 0 \), and by the assumptions of our model \( H_{tF} > 0 \). Moreover, if capital and labor are complements in production, i.e., \( R_{LK} > 0 \), then \( H_{tT} > 0 \), thus \( t_T > 0 \). That is, a higher federal capital tax increases the corresponding state capital tax. A vertical transfer from the federal to the state government raises the latter’s capital tax rate.

In the first stage of the game, as previously noted, the federal government chooses \( T \) and \( F \) in order to maximize \( W = Nu \), accounting now for the adjustments in the state capital tax \( t \) due to the induced changes in \( T \) and \( F \). The first-order conditions for optimally choosing \( T \) and \( F \) are given by the following first-order conditions:

\[
\frac{dW}{dT} = 0 \implies W_T = N \frac{du}{dT} = 0 \implies -E_g = -NE_G - E_G \frac{G_t (1 + t_T)}{K}, \hspace{1cm} (18)
\]
\[ W_F = 0 \implies -E_g = -NE_G - E_G t_F. \tag{19} \]

Equations (A.6-A.9) in the Appendix provide the details of this derivation. Condition (18) shows how does the marginal willingness to pay for the provision of a state and the federal public good relate to each other, in the presence of the vertical fiscal externality, i.e., \( G_t (1 + t_T) \), variable factor supplies, and specific factor taxes. Based on the preceding analysis, when \( R_{LK} > 0 \), then \( G_t < 0 \) and \( t_T > 0 \), implying that \( -E_g < -NE_G \). If, however, \( R_{LK} > 0 \), then both \( G_t \) and \( t_T \) are of ambiguous sign, in which case we may have \( -E_g > -NE_G \). Condition (19) determines the direction of the inter-governmental transfer \( F \), in the presence of the vertical fiscal externality, variable factor supplies, and specific factor taxes. It states that the transfer should go from the government providing the public good with lower marginal willingness to pay for its provision to that providing the public good with higher marginal willingness to pay for its provision. Thus, if \( -E_g + NE_G < (>)0 \), then the transfer should go from (to) the state to (from) the federal government. Condition (19) indicates that the direction of the transfer, via \( G_t \) and \( t_F \), depends on the relationship between \( K \) and \( L \) in production, see discussion of equations (5).

Combining the first-order conditions (18) and (19) it is easily shown that the optimal federal capital tax \( (T^{op}) \) is determined by the condition \( G_t = 0 \) and is given by:

\[
T^{op} = -SR_{LK} Z_{LL}^{-1} = -\frac{SL \eta_{R_{LK}}}{K(\gamma \varepsilon_{LL} - \eta_{LL})}, \tag{20}
\]

where the right-hand side elasticities, \( \eta_{R_{LK}}, \varepsilon_{LL} \) and \( \eta_{LL} \), and the ratio \( \gamma \) are equivalently defined to the corresponding terms in optimality condition (15).

Following condition (20) three observations are in order. First, the above given optimal capital tax \( T^{op} \) replaces the one which is produced by the condition (12) for the unitary optimum. Second, \( T^{op} \) is a capital tax (subsidy) provided that capital and labor are substitutes (complements) in production. Last, inquiring when is this optimal federal capital tax or subsidy rate high or low, then, the size of the terms in the right-hand side of the optimality condition (20) for example assuming that \( T^{op} \) is a capital tax, then can offer an answer. For example, let \( T^{op} \) be a capital tax. Then, high \( S \), \( L \), and \( \eta_{LL} \), and/or low \( K \), \( \eta_{R_{LK}}, \varepsilon_{LL} \), and \( \gamma \) mandate a high federal capital tax. Equivalent conditions can be set for a high/low federal capital subsidy \( (T^{op} < 0) \).

References


10 In deriving the optimality conditions (18) and (19) we make use of equations (5) and of the first-order condition \( H_t = 0 \implies E_g (K + \alpha) = -K \).

11 In Kotsogiannis and Martinez (2008), the equivalent condition, i.e., equation (22) is stated in terms of MCPFs for the state and federal governments.


Appendix

Effects of changes in $\tau$ and $\sigma$ on $K$ and $L$

Totally differentiating the equilibrium conditions (1) and (2) we obtain the following matrix system

$$
\begin{bmatrix}
Z_{LL} & -R_{LK} \\
R_{KL} & R_{KK}
\end{bmatrix}
\begin{bmatrix}
dL \\
dK
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
1
\end{bmatrix} d\tau
+ 
\begin{bmatrix}
-1 \\
0
\end{bmatrix} d\sigma
, \quad (A.1)
$$

from which we obtain:

$$
\begin{align*}
\frac{dL}{d\tau} &= \Delta^{-1} R_{LK}, \\
\frac{dL}{d\sigma} &= -\Delta^{-1} R_{KK}, \\
\frac{dK}{d\tau} &= \Delta^{-1} Z_{LL}, \\
\frac{dK}{d\sigma} &= -\Delta^{-1} R_{KL},
\end{align*}
$$

where $\Delta = R_{KK} (Z_{LL} + R_{LK} R_{KK}^{-1} R_{KL})$ which for stability is negative, and $Z_{LL} = E_{LL} - R_{LL}$ is positive by the properties of the $E(.)$ and $R(.)$ functions. $R_{LK} = \partial R_L / \partial K$ and Moreover, $\text{sign} (R_{KL}) \geq 0$ depending on the relationship between $K$ and $L$ in production. The total differentiation of the equilibrium condition (2) gives $(dK/dL) = -R_{KK}^{-1} R_{KL}$. If $R_{KL} < (>) 0$, then $K$ and $L$ are substitutes (complements) in production. In a two factors model, firms’ profit maximization requires that $R_{KL} < 0$, i.e., $K$ and $L$ beign substitutes in production. In a model with more than two factors, however, we can have $K$ and $L$ beign complements in production, i.e., $R_{KL} > 0$.

Comparative statics results for equations (5)

Total differerntiation of the state and federal governments budget constraints with respect to the governments capital taxes ($t$ and $T$) gives :

$$
\begin{align*}
g_t &= K + s \frac{dL}{dt} + t \frac{dK}{dt}, \\
g_T &= s \frac{dL}{dT} + t \frac{dK}{dT}, \\
G_t &= N \left( S \frac{dL}{dt} + T \frac{dK}{dT} \right), \\
G_T &= N \left( K + S \frac{dL}{dT} + T \frac{dK}{dT} \right).
\end{align*}
$$

(A.3)

Using the results in (A.2), we obtain the expressions in the text.

Comparative statics results for equation (15)

The $\text{sign} \left( \frac{dG}{dt} \right)$ and $\text{sign} \left( \frac{du}{dt} \right)$, in equations (13) and (14) respectively, depend on the $\text{sign} (SR_{LR^*}^* + TZ_{LR^*}^*)$ in the right-hand-side of these equations. Recall that by equations (??), the determinant $\Delta^*$ is negative. Then, straightforward algebra produces the following result:
where \( \epsilon_{L^*L^*} = \frac{\partial E_{L^*}}{\partial L^* L^*} \) and \( \eta_{L^*L^*} = \frac{\partial R^*_{L^*}}{\partial L^* L^*} \) respectively are the elasticities of the reservation wage and of the marginal revenue product of labor with respect to employment, and using equation (1), \( 0 < \gamma^* = \frac{E_{L^*}}{R^*_{L^*}} = 1 - \frac{\sigma}{R^*_{L^*}} \leq 1 \), the ratio of the reservation wage to the marginal revenue product of labor is less than 1 in the presence of a labor tax. Then, the optimality condition (15) emerges.

**Derivation of equations (17)**

Totally differentiating the first order condition \( H_t = 0 \) with respect to \( t, T \) and \( F \) we get:

\[
dH_t = 0 = H_{tt} dt + H_{tT} dT + H_{tF} dF = 0,
\]

from which equations (17) in the text are obtained, and:

\[
\begin{align*}
H_{tt} &= \left\{ - (K + \alpha) E_{gg} g_t - E_g \alpha_t - (1 + K) K_t = \\
&= - (K + \alpha) E_{gg} (K + \alpha) - E_g \Delta^{-1} Z_{LL} - (1 + K) \Delta^{-1} Z_{LL} \right. \\
H_{tT} &= \left\{ - (K + \alpha) E_{gg} g_T - E_g (K_T + \alpha T) - K_T = \\
&= - (K + \alpha) E_{gg} \alpha - (1 + E_g) \Delta^{-1} Z_{LL} \right. \\
H_{tF} &= - (K + \alpha) E_{gg} g_F - E_g (K_F + \alpha F) - K_F = - (K + \alpha) E_{gg},
\end{align*}
\]

where \( \alpha_t = \alpha T = K_t = K_T = \Delta^{-1} Z_{LL} \) and \( \alpha_F = K_F = 0 \).

**1st-stage comparative statics: Derivation of equations (18) and (19)**

The first-order condition (18) can be written as:

\[
W_T = N \frac{du}{dT} = 0 \implies -E_g \frac{dg}{dT} - E_G \frac{dG}{dT} - (1 + t_T) K = 0,
\]

where by totally differentiating the two government budget constraints, equations (3) and (4), with respect to \( T \), accounting for the induced adjustments in \( t \) due to changes in \( T \), we have:

\[
\begin{align*}
\frac{dg}{dT} &= s \left( \frac{dL}{dT} + dL dt_T \right) + t \left( \frac{dK}{dT} + dK dt_T \right) + K t_T = g_T + g t_T, \\
\frac{dG}{dT} &= N \left( S \frac{dL}{dT} + S \frac{dL}{dT} t_T + T \frac{dK}{dT} + T \frac{dK}{dT} t_T + K \right) = N (G_T + G t_T).
\end{align*}
\]
Substituting equations (A.7) into equation (A.6) gives equation (18) in the text. Similarly, the first-order condition (19) can be written as:

\[ W_F = N \frac{du}{dF} = 0 \implies -E_g \frac{dg}{dF} - E_G \frac{dG}{dF} - Kt_F = 0 \implies (-E_g + NE_G) + (1 + E_g)K + E_g \alpha + NE_G A) t_F = 0 \]  

(A.8)

Totally differentiating the two government budget constraints, equations (3) and (4), with respect to \( F \), and accounting for the induced adjustments in \( t \) due to changes in \( F \), we have:

\[
\frac{dg}{dF} = s \left( \frac{dL}{dF} + \frac{dL}{dt} t_F \right) + t \left( \frac{dK}{dF} + \frac{dK}{dT} t_F \right) + K t_F + 1 = g_F + g_t t_F, \text{ and} \\
\frac{dG}{dF} = N \left( S \frac{dL}{dF} + S \frac{dL}{dt} t_F + T \frac{dK}{dF} + T \frac{dK}{dT} t_F - 1 \right) = G_F + G_t t_F. \]  

(A.9)

Substituting equations (A.9) into equation (A.8), and recalling that \( H_t = 0 \implies E_g \alpha = -(1 + E_g)K \), gives equation (19) in the text.