Abstract

We develop a novel theory to explain the recent phenomenon of reshoring, i.e. firms moving back their previously offshored business activities. Thanks to the access to a unique survey of American reshoring firms, we provide evidence for the importance of quality behind reshoring decision. We develop a dynamic heterogeneous firms model in which firms decide where to locate production and choose the quality of the produced variety. If a firm decides to offshore its production, it will face lower payroll costs yet higher quality production costs. In equilibrium reshoring decision arises as some firms initially offshore, exploit the rise in the profits due wages differentials and finally return to domestic country to further increase the quality. Moreover, the model delivers equilibrium sorting of firms: the most productive firms will never offshore, the least productive firms will always offshore and the firms with an intermediate productivity decide to reshore.

Keywords: Heterogeneous Firms, Quality Choice, Offshoring, Reshoring

JEL Classification: F12, O14, R3

1 Introduction

The reshoring of the manufacturing production has recently gained a lot of publicity in advanced economies. Although the aggregate offshoring trends do not seem to be yet reverted\(^1\), the increasing number of firms choosing to transfer back the manufacturing activities to their home countries caught the attention of both...
media and the experts. Especially in the United States the public debate on the topic is very lively as the most prominent examples of reshorners include General Electric transferring the production of water heaters from China to Louisville, Kentucky\(^2\), Ford Motor Company shifting its production of the newest EcoBoost engines from China to Cleveland\(^3\) or General Motors moving the production of the next-generation Cadillac SRX from Mexico to Spring Hill, Tennessee\(^4\). In his 2013 State of the Union speech president Obama stated: *So we have a huge opportunity, at this moment, to bring manufacturing back. But we have to seize it.*\(^5\). Despite broad public debate the academic discussion of the topic is scarce. Empirical investigation is suffering from lack of representative economy-wide data and relies on surveys conducted within reshoring companies. Kinkel (2014) and Kinkel and Maloca (2009) report the survey data for German firms, Dachs and Zanker (2014) report reshoring surveys for eight European countries, Bailey and Propris (2014) and Pricewaterhouse Coopers (2014) report on the reshoring trends in UK. The trends in US reshoring over recent years are thoroughly covered by different consultancy companies reports: The Boston Consulting Group (2011, 2013, 2014) and The Hackett Group (2012) with mixed conclusions on the prospects of reshoring. Attempts to measure the importance of reshoring on aggregate economy level are limited. Oldenski (2015) reports that in the period 1999 – 2012 imports by US-based multinational (MNE) affiliates were steadily increasing. DeBacker et al. (2016) study MNEs activity for a number of advanced countries and check whether there were any changes in the share of productive resources deployed in the home countries of those companies. In the sample of US MNEs they find no evidence of an increasing home share in employment, however they provide some evidence of a growing concentration of capital investments; they document this pattern also for some other high-income economies.

In spite of the obvious issue of the representativeness, the survey studies provide some interesting insights into drivers of reshoring decision. Kinkel (2014) report that 65% of reshorners in Germany in the period 2010 – 2012 quoted quality-related problems as the main reason behind production transfer. Similarly, EEF The Manufacturer’s Organization/GFK (2014) reports that the main motivation of UK reshorners surveyed in 2014 was intention to improve quality, mentioned by 49% of interviewed companies. Thanks to the access to a unique survey of American reshoring firms in the period 1995 – 2015, we provide preliminary evidence for the importance of quality and technology upgrade as the main drivers behind reshoring decision also for US-based companies. It turns out that also within the group of US reshorners the quality-related problems are the main push factor behind giving up on offshoring activity. Additionally, over 27% of those firms quote innovation possibilities and skilled workforce as main pull factors for locating the production back in US. Moreover, another 12% of firms quote access to skilled workforce as important reshoring driver. In this paper we embrace this quality-related evidence and we develop a novel theory that explains the recent growing reshoring activity.

To our best knowledge there is only one theoretical paper that generates reshoring patterns. Baldwin and


\(^4\)Associated Press, *GM Moving Cadillac SRX Production from Mexico to TN*, August 27, 2014

\(^5\)See *State of the Union 2013* and also Economist article
Venables (2013) analyze theoretically the location decision of a global firm, separating between a sequential (snake) and a more separated (spider) production processes. Location decision in their model is the outcome of the tradeoff between international differences in the production costs and the production co-location benefits. Reductions in international frictions (trade costs, communication or coordination costs) facilitate the relocation of production but can result in overshooting of offshoring and a subsequent reshoring pattern. They do not consider quality choice in the production process. Therefore, our approach to reshoring is complementary, as we put the quality-related factors at the heart of our analysis. Moreover, we conduct the analysis in heterogeneous firm framework, a margin which is absent in Baldwin and Venables (2013). This paper also contributes to the literature by developing a theory for the offshoring and the quality choice in the heterogeneous firm framework. To our best knowledge, Smeets et al. (2014) is the only one paper that considers this question. However, the model developed there is static and therefore does not admit reshoring possibility, which is in turn the core of our analysis.

In our setting each firm is producing a single good for the domestic country market, deciding the quantity and the quality supplied as well as the factory location. We build on Antoniades (2015), a model introducing the quality choice into seminal Melitz and Ottaviano (2008) framework and we enrich it in two steps. First, we add the offshoring possibility. Offshoring is reducing wage costs paid by firms, but it is increasing the quality production costs and entails transportation cost for the components (iceberg type). Introduction of the offshoring possibility into Antoniades (2015) leads to the following findings: i) the most productive firms produce only domestically, ii) the least productive firms offshore, iii) thanks to offshoring possibility some of the least productive firms, who would have to otherwise exit the market, produce. Second, we extend the enriched model into dynamic, two-period setting. High quality varieties yield higher revenues than the low quality ones, yet quality production is costly. Firms would be therefore facing a choice between setting a high quality upfront or smoothing quality upgrade across both periods. Since the fixed costs of quality innovation are convex, firms will find it optimal to set a given level of quality in the first period and upgrade it in the second period. Once we allow for offshoring, some firms in the first period produce abroad. Yet given the second period quality upgrade and increasing quality adaptation costs it entails, they transfer the production back to the domestic country.

We solve the model numerically. The equilibrium delivers a sorting pattern: the most productive firms always produce domestically, the least productive always offshore and the firms with an intermediate productivity reshore. We discuss the crucial parameters affecting the equilibrium interval of the productivity for which reshoring arises. Comparative statics exercises points the importance of the love for quality parameter. The increase in the consumers’ taste for quality increases the intensity of the reshoring activity in the equilibrium.

The reminder of this paper is organized as follows: section 2 presents some stylized fact about US reshoring firms. Section 3 presents the static model, section 4 develops the dynamic model and describes the solution method and the equilibrium outcomes. Section 5 concludes.

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6We assume that the domestic and offshore countries are advanced and developing, respectively.
2 US Reshorers: a brief view

Reshoring Initiative (RI)\(^7\) is a non-profit organization assisting US companies in reshoring process. One of the core assets of RI is its reshoring database, in which the organization collects the data on the events of reshoring among US companies from publicly available sources (press releases, companies white papers, media announcements, etc.) as well as directly from firms, and verifies their accuracy. In June 2015 RI kindly shared this database with us. Its full content covered 410 reshoring firms and another 231 classified as kept from offshoring. Each record comprises company name, the year of reshoring, the product reshored, industry classification and the main domestic and offshore factors behind the transfer decision.

Table 2 in the Appendix summarizes the timing of the observed reshoring events. Although there were occasional events of reshoring dating back to as early as 1995, the majority of reshoring decisions were taken in post-2010, with a clear concentration in the period 2012 – 2014.\(^8\) Figure 8 in the Appendix represents the sectoral composition of reshored companies: it is clearly dominated by manufacturing industry, which coupled with retail and wholesale trade, and professional services account for almost 90% of the sample.

<table>
<thead>
<tr>
<th>OFFSHORE FACTOR</th>
<th>% of firms quoting</th>
<th>DOMESTIC FACTOR</th>
<th>% of firms quoting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Issues</td>
<td>31.63</td>
<td>Technology and innovation difficulties</td>
<td>27.74</td>
</tr>
<tr>
<td>Freight costs</td>
<td>29.20</td>
<td>Other</td>
<td>20.19</td>
</tr>
<tr>
<td>Lead time, inventory</td>
<td>27.49</td>
<td>Skilled workforce</td>
<td>12.41</td>
</tr>
<tr>
<td>Wage costs</td>
<td>19.22</td>
<td>Government Incentives</td>
<td>9.00</td>
</tr>
<tr>
<td>Communication &amp; audit</td>
<td>10.46</td>
<td>U.S. price of natural gas</td>
<td>4.38</td>
</tr>
<tr>
<td>Intellectual property</td>
<td>6.33</td>
<td>Customer/demand issues</td>
<td>4.38</td>
</tr>
<tr>
<td>Loss of control</td>
<td>5.35</td>
<td>Eco-system synergies</td>
<td>3.89</td>
</tr>
<tr>
<td>Other</td>
<td>4.87</td>
<td>Infrastructure</td>
<td>2.92</td>
</tr>
<tr>
<td>Ethical/green considerations</td>
<td>4.14</td>
<td>Lower real-estate/construction costs</td>
<td>0.97</td>
</tr>
<tr>
<td>Difficulty of Innovation</td>
<td>2.92</td>
<td>Supplier issues</td>
<td>0.49</td>
</tr>
<tr>
<td>Currency variation</td>
<td>3.89</td>
<td></td>
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</tr>
<tr>
<td>Regulatory compliance</td>
<td>1.46</td>
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<tr>
<td>Political instability</td>
<td>1.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employee turnover</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Image/Brand</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Main offshore and domestic factors behind reshoring decision for US firms

Probably the most important aspect of RI data are survey questions in which reshoring firms quote the main drivers of reshoring, describing both offshore push factors and pull home country incentives. Table 1 summarizes this information\(^9\). Although some firms point to more than one factor (with the single top-scorer quoting 11 factors), the mode for the number of pull and push factors is 1. Similarly to the survey-based reshoring evidence in Germany and UK, the quality-related problems faced by offshore plants seem to be the

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\(^7\)www.reshorenow.org

\(^8\)Observation in year 2016 refers to the firms that declared reshoring scheduled to take place in 2016.

\(^9\)Note that in Table 1 the percentage do not sum up to 100 as each firm can quote one or more factors. The percentage is expressed in reference to the total number of factors quoted.
leading factor behind production transfer also for the American firms. The quality-related factors comprise problems with necessary rework, warranty issues, low product liability and alike. Overall, above 31% of the firms report quality problems followed by lead time and inventory and freight costs (29% and 27%, resp.). Increasing wage costs are quoted by 19% of firms. The prominent role of quality considerations is even more evident once we limit the analysis to the group of firms who quote only one main driver behind their reshoring decision (Figure 1): over 40% of firms point to quality issues with lead time and inventory costs being second factor, mentioned in less than 20% of the answers; wage costs are mentioned by less than 10% of firms. Complimentary to the quality issues domination in the offshore push factors, approximately half of the firms interviewed also point to a limited scope for product innovation to offshore production as the main domestic pull incentive for reshoring (Table 1).

![Figure 1: Offshore factors behind reshoring in the sample of firms quoting one reason only](image-url)
3 Static Model

Prior to developing a full dynamic model, we begin with a simple static framework in which we can highlight the relationships between the quality choice and offshoring. We base our setting on the closed economy version of Antoniades (2015) which we alter by adding the production location choice.

Preferences The economy is populated with L consumers, each supplying one unit of labor. The utility expression follows closely Antoniades (2015) and reads:

\[
U = q_c^c + \alpha \int_{\omega \in \Omega} q_c^\omega d\omega + \beta \int_{\omega \in \Omega} z_\omega q_c^\omega d\omega - \frac{1}{2} \gamma \int_{\omega \in \Omega} (q_c^\omega)^2 d\omega - \frac{1}{2} \eta \left\{ \int_{\omega \in \Omega} q_c^\omega \right\}^2
\]  

(1)

where \(q_c^\omega\) and \(q_c^c\) represent the consumption of the numeraire good and the variety \(\omega\), and \(z_\omega\) stands for quality of variety \(\omega\). \(\alpha\) and \(\eta\) capture the degree of substitution between each variety and the numeraire, \(\gamma\) describes the degree of differentiation among the varieties. Importantly, \(\beta\) is a taste for quality parameter.

The inverse demand for each variety is:

\[
p_\omega = \alpha - \gamma q_c^\omega + \beta z_\omega - \eta Q^c
\]  

(2)

Technology As in Antoniades (2015) a firm produces a given variety \(\omega\) with inelastically supplied labor input. Homogeneous good and labor markets are competitive. Upon payment of entry cost \(f_e\), a firm draws productivity which determines their marginal cost \(c\) (distributed accordingly to \(G(c)\) on the support \([0, c_M]\)). Firms that can cover their marginal cost survive and produce, those with the lowest productivity exit the market. The survivors maximize profits based on residual demand curve, taking average prices, average quality level and the number of firms, \(N\), as given. We allow firms to choose the production location: they decide whether to remain and produce at home or whether to offshore. For simplicity, we assume the extreme view of offshoring: once offshored, a firm will offshore all its production.\(^{10}\) We formulate the total cost structures by closely following Antoniades (2015), but we introduce a difference in total costs due to production location:

\[
TC_H^{\omega} = c_\omega q_\omega + \delta_H z_\omega q_\omega + \theta z_\omega^2
\]

\[
TC_O^{\omega} = w_\tau c_\omega q_\omega + \delta_O z_\omega q_\omega + \theta z_\omega^2
\]  

(3)

\(TC_H^{\omega}\) and \(TC_O^{\omega}\) stand for total cost of firm \(\omega\) located in the home country and offshore\(^{11}\). The first terms of the total cost functions capture the variable costs of production as in standard Melitz and Ottaviano (2008) setting. The second terms with parameters \(\delta_H\) and \(\delta_O\) capture the increases in marginal costs due to quality upgrades. Those quality adaptation costs are brought about by the implementation of quality innovations.

\(^{10}\)The model can be easily extended to a version where a firm combines a range of potentially offshorable tasks in the spirit of Grossman and Rossi-Hansberg (2008). Each firm would then decide on the fraction of tasks offshored. However, this complication would not qualitatively change the results of the model.

\(^{11}\)Wage in the domestic country is normalized to 1.
We assume that the quality-related production costs are always greater for the offshoring firm ($\delta_H \leq \delta_O$), *i.e.* the greater the geographical distance between the plants and the headquarters, the more costly is quality adaptation. Those variable costs entail for instance machines fine-tuning for the new technology processes, new materials, workers retraining, *etc.* The third terms, involving $\theta$'s account for fixed cost of quality innovation, invariant to quantity produced. They describe firms’ R&D investments, product re-design, invention of the new technology processes and so on. Following Antoniades (2015) we assume this cost to be convex. In principle, we could allow for differences in $\theta$'s across production locations. However, firms R&D activities are predominantly located in the headquarters, in particular if the main destination market is the domestic one, therefore we assume $\theta$'s to be equal across production locations. Additionally, we assume that the total wage costs are always lower offshore: $w_{\tau} < 1$.

In such a setting, the problem for a firm producing domestically is identical to the closed economy solution in Antoniades (2015). Therefore, we solve the problem only for the offshoring firm and present the equilibrium outcome.

Denote by $c_{D,O}$ the marginal cost value for which the offshoring firm’s demand is driven to zero, $q_{\omega}(c_{D,O}) = 0$ and $z_{D,O}$ stands for quality level relative to $z_{D,O}$. We can now express prices and quantities as functions of $c_{D,O}$, $c_\omega$ and qualities $z_\omega$ and $z_{D,O}$:

$$p_\omega = \frac{1}{2}(w_\tau)(c_{D,O} + c_\omega) + \frac{1}{2}(z_\omega(\beta + \delta_O) - z_{D,O}(\beta - \delta_O))$$

$$q_\omega = \frac{L}{2\gamma}(w_\tau)(c_{D,O} - c_\omega) + \frac{L}{2\gamma}(\beta - \delta_O)(z_\omega - z_{D,O})$$

$$\pi_\omega = \frac{L}{4\gamma}(w_\tau)(c_{D,O} - c_\omega) + (\beta - \delta_O)(z_\omega - z_{D,O})^2 - \theta(z_\omega)^2$$

Next, we find the optimal quality level, $z^*_\omega$, which is maximizing profit (6)\(^{13}\).

$$z^*_\omega = \lambda_0 \left((c_{D,O} - c_\omega)(w_\tau) - z_{D,O}(\beta - \delta_O)\right) = \lambda_0(c_{D,O} - c_\omega)(w_\tau)$$

$$\lambda_0 = \frac{L(\beta - \delta_O)}{4\gamma\theta - L(\beta - \delta_O)^2}$$

The last passage in (7) follows from the fact that for $c_\omega = c_{D,O}$ $\Rightarrow$ $z_{D,O} = \lambda_0((c_{D,O} - c_{D,O})(w_\tau) - z_{D,O}(\beta - \delta_O))$ $\Rightarrow$ $z_{D,O} = -z_{D,O}\lambda_0(\beta - \delta_O)$ $\Rightarrow$ $z_{D,O} = 0$. Given the optimal quality, we can express (4), (5) and (6)\(^{12}\) the earlier version of this paper assumed $\theta_H \leq \theta_O$. The qualitative results of both the static and the dynamic model are identical. The results are available upon request.

\(^{12}\)As in Antoniades (2015) firms here choose simultaneously price and quality for a given output level. Given linearity and separability of the model, we first solve for the optimal price and next, we find the optimal quality level.
dependent on \( c_\omega \) and cost cutoff \( c_{D,O} \):

\[
p_\omega = \frac{1}{2}(w\tau)(c_{D,O} + c_\omega) + \frac{1}{2}(w\tau)\lambda_\omega(\beta + \delta_O)(c_{D,O} - c_\omega)
\]

\[
q_\omega = \frac{L}{2\gamma}(w\tau)(c_{D,O} - c_\omega)(1 + \lambda_\omega(\beta - \delta_O))
\]

\[
\pi_\omega = (w\tau)^2(c_{D,O} - c_\omega)^2 \frac{L}{4\gamma}(1 + \lambda_\omega(\beta - \delta))
\]

This results lead to two parametric assumptions. First, to assure concavity of profit \( \pi_\omega \) in quality \( z_\omega \) it is required that

\[
L(\beta - \delta_O)^2 - 4\gamma > 0.
\]

Second, in order to impose non-negative \( z_\omega \) we must assume that \( \beta > \delta_O \). Each firm, given its marginal cost \( c_\omega \), will be choosing the location of its production by comparing the maximized profits under each of the scenarios:

\[
\pi^H_\omega = \frac{L}{4\gamma}(c_{D,H} - c_\omega)^2(1 + \lambda_H(\beta - \delta_H))
\]

\[
\pi^O_\omega = \frac{L}{4\gamma}(w\tau)^2(c_{D,O} - c_\omega)^2(1 + \lambda_O(\beta - \delta_O))
\]

As long as \( \pi^H_\omega \geq \pi^O_\omega \) a given firm with marginal cost \( c_\omega \) would prefer to produce domestically instead of offshoring. We find that the pivotal firm that is indifferent between producing domestically and offshoring is characterized by the following marginal cost \( c_1 \):

\[
c_1 = c_{D,O}w\tau \left( \frac{\Gamma_H + \Gamma_Ow\tau + (1 - w\tau)\sqrt{\Gamma_H\Gamma_O}}{\Gamma_H + \Gamma_Ow\tau^2} \right)
\]

where \( \Gamma_H = \frac{\theta(\beta + 4\gamma - \delta_h) + L(\delta_h - \beta)}{L(\beta - \delta_h) - 4\gamma} \) and \( \Gamma_O = \frac{\theta(\beta + 4\gamma - \delta_o) + L(\delta_o - \beta)}{L(\beta - \delta_o) - 4\gamma} \). It is easy to show that under model parametric restrictions, it is always the case that \( c_1 < c_{D,H} < c_{D,O} \). Figure 2 represents the equilibrium location choices. Firms with the marginal costs below the cost cutoff \( c_1 \) produce in the home country, whereas firms with the marginal costs above this threshold produce offshore. \( c_{D,H} \) is the critical cost cutoff originating from the closed economy model of Antoniades (2015), where firms with marginal costs above \( c_{D,H} \) exit the market. Introduction of the offshoring possibility results in a new critical cost cutoff, \( c_{D,O} \), \( c_{D,O} > c_{D,H} \). This implies that thanks to offshoring we observe in equilibrium some firms with very low productivity (with their marginal costs falling into \( [c_{D,H}, c_{D,O}] \) interval) that without offshoring option would not be able to survive. Moreover, for the firms whose marginal cost lays between \( c_1 \) and \( c_{D,H} \) offshore leads to higher profits.

The model is closed by free entry condition as firms \textit{ex ante} expect zero profits:

\[
\int_0^{c_1} \pi^H_\omega dG(c) + \int_{c_1}^{c_{D,O}} \pi^O_\omega dG(c) = f_e
\]

This condition determines the cost cutoff \( c_{D,O} \). Following Melitz (2003), Melitz and Ottaviano (2008) and Antoniades (2015) we assume that the firm cost draws are Pareto distributed on the support \([0, c_M]\) with
Figure 2: Static model equilibrium.

\[ G(c) = \left( \frac{c}{c_M} \right)^k \]

The cost cutoff in this economy is:

\[ c_{D,O} = \left( \frac{4\gamma f_e(k+1)(k+2)c_M^k}{Lw\tau(\lambda_H(\beta - \delta_H) - \lambda_O(\beta - \delta_O))\psi + ((4k^2 + 8k + 2)(1 + \lambda_O(\beta - \delta_O)))} \right)^{\frac{1}{k+2}} \]  \hspace{1cm} (15)

\[ \psi = \left( (k+1)(k+2)(\chi w\tau)^k + 2(k+2)k(\chi w\tau)^{k+1} + (k+1)k(\chi w\tau)^{k+2} \right), \]  \hspace{1cm} (16)

where \( \psi \) is the constant multiplying cutoff \( c_1 \) (equation (13)) and \( \lambda_H = \frac{L(\beta - \delta_H)}{4\gamma \theta - L(\beta - \delta_H)^2} \) and \( \lambda_O = \frac{L(\beta - \delta_O)}{4\gamma \theta - L(\beta - \delta_O)^2} \).
4 Dynamic Model

Let us now analyze firm location decision in the two-period setting. Analogously to the static formulation, offshoring comprises a tradeoff between lower wages costs and higher quality-related production costs. The timing of the events is as follows: firstly all firms pay the entry cost, $f_e$ and draw the marginal cost $c_ω$ from the common distribution $G(c)$. Firm productivity is invariant across the periods. Next, given the realized marginal value of $c_ω$ firms decide the quantities produced, the quality upgrades and the production location in both periods. Finally production takes place. Each firm can choose to always produce in the home country, always offshore, reshore in the second period or offshore in the second period. Given the realized marginal cost, $c_ω$ and the location choice firms experience different marginal costs of production. They choose the profit maximizing scenario.

Denote by $i \in \{\text{Home}(H), \text{Offshore}(O)\}$ a firm’s location decision in the first period and by $j$ analogous decision in the second period. The joint profit for the $ω$ firm reads:

$$\Pi_{ω}^{i,j} = \Pi_{ω,1}^{i,j} + \Pi_{ω,2}^{i,j} = q_{ω,1}^{i,j}(p_{ω,1}^{i,j} - c_ωT^i - \delta_i z_{ω,1}^{i,j}) - \theta_i(z_{ω,1}^{i,j})^2 + q_{ω,2}^{i,j}(p_{ω,2}^{i,j} - c_ωT^j - \delta_i(z_{ω,1}^{i,j} + \Delta^{i,j}_ω)) - \theta_j(\Delta^{i,j}_ω)^2 \tag{17}$$

where $q_{ω,1}^{i,j}$ and $q_{ω,2}^{i,j}$ stand for the quantity in the first and second period, $z_{ω,1}^{i,j}$ is the quality level in the first period and $\Delta^{i,j}_ω$ is the second period quality upgrade. The fixed costs of quality innovation are convex and paid only on the per period quality upgrade (i.e. the first period innovation cost is $\theta_i(z_{ω,1}^{i,j})^2$, whereas in the second period it equals $\theta_j(\Delta^{i,j}_ω)^2$). $T^i$ and $T_j$ are the payroll costs, conditional on location choice. For home production the wages are normalized to 1, $T^H = 1$. On the other hand, the offshore labor costs include offshore wages (assumed to be lower than the home wages, $w < 1$) and iceberg cost of shipping the goods back to home country ($τ > 1$) $T^O = wτ$. Denoting the period by $t \in \{1, 2\}$, the inverse demand function is expressed in the standard way:

$$p_{ω,t}^{i,j} = \alpha - γ q_{ω,t}^{i,j} + β z_{ω,t}^{i,j} - ηQ^i_t \quad \text{with} \quad Q^i_t = \int_{ω \in Ω_t} q_{ω,t}^{i,j} dω \tag{18}$$

As before, we can express the optimal quantities and prices, and the maximized profit as the functions of per period cost cutoffs, quality choices and the marginal cost, $c_ω$:

$$q_{ω,1}^{i,j} = \frac{L}{2γ}T^i(c_{D,1}^{i,j} - c_ω) + \frac{L}{2γ}(β - δ_i)(z_{ω,1}^{i,j} - z_{D,1}^{i,j}) \tag{19}$$

$$q_{ω,2}^{i,j} = \frac{L}{2γ}T^j(c_{D,2}^{i,j} - c_ω) + \frac{L}{2γ}(β - δ_j)(z_{ω,1}^{i,j} + Δ_{ω}^{i,j} - z_{D,2}^{i,j}) \tag{20}$$

$$p_{ω,1}^{i,j} = \frac{1}{2}T^i(c_{D,1}^{i,j} + c_ω) + \frac{1}{2}((β + δ_i)z_{ω,1}^{i,j} + (β - δ_i)z_{D,1}^{i,j}) \tag{21}$$

$$p_{ω,2}^{i,j} = \frac{1}{2}T^j(c_{D,2}^{i,j} + c_ω) + \frac{1}{2}((β + δ_j)(z_{ω,1}^{i,j} + Δ_{ω}^{i,j}) - (β - δ_j)z_{D,2}^{i,j}) \tag{22}$$

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14In principle the innovation costs are symmetric for both quality upgrades and downgrades, however, in equilibrium the latter choice is absent.
the optimal quality choice in every period can be found by maximizing (23) with respect to the highest quality upgrade at the lowest possible cost in the following period. As in the static formulation, despite no production in the first period, it engages in quality enhancing investments, as it would allow for producing in period 1 (period 2).

By imposing the convexity of quality innovation costs. Consider a firm’s with marginal cost in equations (19) - (23) which enables us to express the cutoffs \( \omega \) in period 1 and 2, respectively. A firm with a marginal cost \( c_\omega \) such that it is equal to \( c_{D,1}^{i,j} \) and lower than \( c_{D,2}^{i,j} \), it does not produce in the first period, but it produces in the second one. However, despite no production in the first period, it engages in quality enhancing investments, as it would allow for the highest quality upgrade at the lowest possible cost in the following period. As in the static formulation, the optimal quality choice in every period can be found by maximizing (23) with respect to \( z_{\omega,1}^{i,j} \) and \( \Delta_{\omega}^{i,j} \):

\[
\Pi_{\omega}^{i,j} = \frac{L}{4\gamma} \left( T^i (c_{D,1}^{i,j} - c_\omega) + (\beta - \delta_i)(z_{\omega,1}^{i,j} - z_{D,1}^{i,j}) \right)^2 + \frac{L}{4\gamma} \left( T^i (c_{D,2}^{i,j} - c_\omega) + (\beta - \delta_j)(z_{\omega,2}^{i,j} + \Delta_{\omega}^{i,j} - z_{D,2}^{i,j}) \right)^2 + \left( \theta_j (z_{\omega,1}^{i,j})^2 + \theta_j (\Delta_{\omega}^{i,j})^2 \right)
\]

In equations (19) - (23) \( c_{D,1}^{i,j} \) and \( c_{D,2}^{i,j} \) are the marginal cost cutoff values for a firm making a location decision \( \{i,j\} \) in period 1 and 2, respectively. A firm with a marginal cost \( c_\omega \), \( c_\omega > c_{D,1}^{i,j} \) (\( c_\omega > c_{D,2}^{i,j} \)) will not be producing in period 1 (period 2). \( z_{\omega,1}^{i,j} \) and \( z_{\omega,2}^{i,j} \) are the quality levels that are associated with marginal cost cutoffs \( c_{D,1}^{i,j} \) and \( c_{D,2}^{i,j} \), respectively. While in the static model \( z_D \) is zero in the equilibrium, in the dynamic model it is not necessarily the case. This is the dynamic feature due to two period horizon combined with the convexity of quality innovation costs. Consider a firm’s with marginal cost \( c_\omega \) such that it is equal to \( c_{D,1}^{i,j} \) and lower than \( c_{D,2}^{i,j} \), it does not produce in the first period, but it produces in the second one. However, despite no production in the first period, it engages in quality enhancing investments, as it would allow for the highest quality upgrade at the lowest possible cost in the following period. As in the static formulation, the optimal quality choice in every period can be found by maximizing (23) with respect to \( z_{\omega,1}^{i,j} \) and \( \Delta_{\omega}^{i,j} \):

\[
z_{\omega,1}^{i,j} = \Phi_{i,j} (\beta - \delta_i) \left( \frac{L(\beta - \delta_i)}{\lambda_j} \left( (c_{D,1}^{i,j} - c) T^i - z_{D,1}^{i,j} (\beta - \delta_i) \right) + 4\gamma \theta_j \left( T^j (c_{D,2}^{i,j} - c) - z_{D,2}^{i,j} (\beta - \delta_j) \right) \right)
\]

\[
\Delta_{\omega}^{i,j} = \Phi_{i,j} (\beta - \delta_i) (\beta - \delta_j) L \left( (\beta - \delta_i) \left( (c_{D,1}^{i,j} - c) T^i - z_{D,1}^{i,j} (\beta - \delta_i) \right) + \frac{1}{\lambda_i} \left( (c_{D,2}^{i,j} - c) T^j - z_{D,2}^{i,j} (\beta - \delta_j) \right) \right)
\]

\[
\Phi_{i,j} = \frac{\lambda_i \lambda_j}{(\beta - \delta_i)(L(\beta - \delta_i) - 4\gamma \theta_j \lambda_i \lambda_j (\beta - \delta_j))} \quad \lambda_j = \frac{L(\beta - \delta_j)}{4\gamma \theta_j - L(\beta - \delta_j)^2} \quad \lambda_i = \frac{L(\beta - \delta_i)}{4\gamma \theta_i - L(\beta - \delta_i)^2}
\]

By imposing \( c = c_{D,1}^{i,j} \) and \( c = c_{D,2}^{i,j} \) in equations (24) and (25). We are left with a system of two equations which enables us to express \( z_{D,1}^{i,j} \) and \( z_{D,2}^{i,j} \) as functions of \( c_{D,1}^{i,j} \), \( c_{D,2}^{i,j} \) and parameters. Therefore, we can rewrite equations (19) - (25) as follows:
For the sake of clarity of the exposition from now onwards we restrict attention only to the firms that are unrestricted and firms enter until the expected profit is driven to zero. Note that the maximum price a firm can quote is bounded and it is associated with zero quantity produced. It also must equal the marginal cost, thus we can write the following regularities:

\[ p_{i,j}^t = \frac{\Phi_{i,j}(\beta - \delta_j)}{\theta_i} \left( c_{D,2}^i \delta_i L(\beta - \delta_i) \left( \frac{T^i}{\lambda_j} + T^i(\beta - \delta_i) \right) + \right. \]
\[ \left. + c \delta_i \left( 2\gamma \theta_i + L(\beta - \delta_i) \right) \left( T^i(\beta + \delta_i) \right) + \beta T^i(\beta - \delta_i) \theta_i \left( T^i(\beta + \delta_i) \right) \right) + \in\left(2\gamma \theta_i + L(\beta - \delta_i) \right) \theta_i \left( T^i(\beta + \delta_i) \right) \right) + \in\left(2\gamma \theta_i + L(\beta - \delta_i) \right) \theta_i \left( T^i(\beta + \delta_i) \right) \right) + \right) \]

In each period \( t \) firms with marginal cost \( c_\omega \) above cost cutoff value \( c_{D,t} \) will not produce. They exit the market (i.e. neither engage in any production, nor in any quality investments) if \( c_\omega > \max\{c_{D,1}^i, c_{D,2}^i\} \). For the sake of clarity of the exposition from now onwards we restrict attention only to the firms that are producing in both periods, i.e. \( c_\omega \leq \min\{c_{D,1}^i, c_{D,2}^i\} \), for given \{\( i, j \)\} location choice. The entry of firms is unrestricted and firms enter until the expected profit is driven to zero. Note that the maximum price a firm can quote is bounded and it is associated with zero quantity produced. It also must equal the marginal cost, thus we can write the following regularities:

\[ c_{D,1}^i = \frac{1}{T^i} \left( \alpha - \eta Q_1 + (\beta - \delta_i) \right) \]
\[ c_{D,2}^i = \frac{1}{T^i} \left( \alpha - \eta Q_2 + (\beta - \delta_i) \right) \]
where $Q_t = \int_{e \in \Omega} q_{t,i,j}^e d\omega, t \in \{1, 2\}$ and it stands for the consumption level over all varieties in period $t$. As in our setup the only destination market is the home country market, in equilibrium $Q_1$ and $Q_2$ are unique and common for all production location scenarios. Considering all possible location choices, the equations (33) generate a system. Once the system is solved, we can express all performance measures (19) - (25) and the maximized profits as the functions of model parameters and $Q_1$ and $Q_2$.\[^{15}\]

We can write the conditions that fully specify the equilibrium as:

$$\Pi^{i^*,j^*}_{\omega} = \max_{i,j \in \{H, O\}} \left\{ \Pi^{i,j}_{\omega}(c_{\omega}, Q_1, Q_2; \Theta) \right\}$$

(34)

$$\int_{\omega_i}^e \Pi^{i,j}_{\omega}^*(c_{\omega})dG(c_{\omega}) + \int_{\omega}^e \Pi^{i,j}_{\omega}^*(c_{\omega})dG(c_{\omega}) + \int_{\omega}^e \Pi^{i,j}_{\omega}^*(c_{\omega})dG(c_{\omega}) + \int_{\omega}^e \Pi^{i,j}_{\omega}^*(c_{\omega})dG(c_{\omega}) = f_e$$

(35)

$$Q_t = \int_{0}^{\tilde{e}_i} q_{t,i,j}^* dG(c_{\omega}) + \int_{\tilde{e}_i}^{\tilde{e}_2} q_{t,i,j}^* dG(c_{\omega}) + \int_{\tilde{e}_2}^{\tilde{e}_3} q_{t,i,j}^* dG(c_{\omega}) + \int_{\tilde{e}_3}^{\tilde{c}_M} q_{t,i,j}^* dG(c_{\omega})$$

(36)

\[\text{s.t. } \bar{c}_k \leq \min \left\{ \min \left\{ c_{D,1}^*, (Q_1, Q_2), c_{D,2}^*, (Q_1, Q_2) \right\}, c_M \right\}, k \in \{1, 2, 3, M\}\]

where $i^*, j^*$ are the optimal location choices. $\bar{c}_k$ for $k \in \{1, 2, 3\}$ are the profit cutoffs between 4 potential location scenarios. $\tilde{c}_M$ is maximum value for the marginal cost. As profit functions are convex, there are at most 3 cutoffs, however in equilibrium we do not necessarily observe all of them. $\Pi^{i,j}_{k,\omega}^*(c_{\omega})$ for $k \in \{1, 2, 3, 4\}$ stand for the maximal profit for a given interval of marginal cost and $\Theta$ stands for the model parameter set. $G(c)$ is the common cost distribution, assumed to be Pareto for productivity $\frac{1}{\gamma}$, i.e. $G(c) = \left( \frac{c}{c_M} \right)^k$.

Equation (34) describes each firm’s optimal location decision $\{i, j\}$ as the choice of the scenario under which the maximized joint two period profit is the greatest. Equation (35) is the standard Free Entry condition, bounded by the restriction to the firms producing in the two periods (thus restrictions on $\bar{c}$). (36) is the condition closing the model, stating the aggregate equilibrium consumption levels of $Q_1$ and $Q_2$.

Because of the complex analytical form of the profit functions $\Pi^{i,j}_{\omega}(c_{\omega}, c_{D,1}^*, c_{D,2}^*, \Theta)$ and the large set of model parameters\[^{16}\] the model cannot be solved analytically. Instead, we solve it by means of the numeric methods. The numerical solution procedure is based on fixed point theorem. We proceed as follows: given a set of parameter values, we initially guess the values of $Q_1$ and $Q_2$ and we find the relative profit-maximizing location choices $\{i^*, j^*\}$ for each $c_{\omega} \in [0, c_M]$. Next, we compute the Free Entry condition (35) and verify whether the guessed values of $Q_1$ and $Q_2$ overlap with their model-based counterparts, i.e. whether (36) holds. If not, the guess on $Q_1$ and $Q_2$ is updated. We repeat this procedure by iterating over the combinations of the parameter values.

We would assume, similarly to the formulation in the static model in the previous section that a firm’s fixed cost of quality innovation is invariant both to production location and timing, i.e. $\theta_i = \theta_j = \theta$. As argued before, $\theta$’s stand for the R&D-related quality investments, that are most likely to take place in the

\[^{15}\text{To be found in the Appendix.}\]

\[^{16}\Theta \equiv \{\alpha, \beta, \delta_H, \delta_O, \eta, \gamma, \theta_H, \theta_Q, w, \tau, L\}\]
headquarters. Moreover, the reshoring phenomenon does not address the re-location of R&D activities, but it is concentrated in the component manufacturing business. Modeling the choice of R&D location is beyond the scope of this model. Moreover, it is easy to show that in the dynamic setting firm’s quality choice in the first period is always greater than the subsequent quality upgrade in the following period. Therefore, if \( \theta \)'s would differ accordingly to the production location, firms would always choose to remain in the first period in the location offering lower fixed quality costs. As a consequence, if the quality innovation costs are greater offshore, the firms initially choose to produce domestically, build-up the quality stock in the first period and finally offshore. We would not observe any reshoring activity whatsoever, which is at odds with the data.

### 4.1 Equilibrium Results

The numerical solution delivers reshoring in the equilibrium. The equilibrium is characterized by a sorting pattern into production location choices according to the individual firm productivity: the most productive firms (with the lowest marginal cost draws, \( c_\omega \)) always decide to produce in the home country, whereas the least productive (with the highest marginal cost draws) always offshore. Reshoring arises for the intermediate values of productivity. For illustration, in Figure 3 we present one parametrization that delivers a reshoring equilibrium. Firms within the area A choose production at home, firms from the region C choose production offshore, whereas the intermediate productivity firms (region B) are the reshorers. For the reshoring firms the first period benefits from lower offshore wages outweigh higher offshore quality adaptations costs. However, when the quality upgrade in the second period materializes, the quality adaptations costs abroad rise as well and those firms prefer to transfer the production back to the domestic country\(^{17}\).

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17 Arguably, production transfer across countries can entail important fixed costs, from which our framework abstracts. However, an introduction of fixed offshore or/and fixed reshoring costs would not alter qualitatively the main results of the model.
In Figure 4 we present the reshoring equilibrium sensitivity to the variations in the taste for quality parameter ($\beta$) and to the degree of product differentiation ($\gamma$). When the consumer’s love for quality increases two things happen (panel 4a). First, there is an increase of the interval of productivity where reshoring is an equilibrium outcome, *ceteris paribus*. Secondly, the equilibrium reshoring takes place for lower productivity firms, *ceteris paribus*. Intuitively, as the consumers in the home country value quality more and more, the scope for reshoring is also growing. The opposite effects happen for an increase in the degree of product differentiation, $\gamma$ (panel 4b). First, in the more differentiated sectors reshoring is less likely to occur and more and more firms choose to offshore the production in both periods. Secondly, in the more differentiated
sectors, reshoring takes place for more productive firms. Increase in the degree of product differentiation decreases the importance of quality in the choice of the consumers, more specifically the consumers are now more interested in consuming different goods than similar goods with different quality, thus there is less and less incentive for firms to increase quality. This fall in quality induces a decrease in profits and an increase in the number of firms deciding to offshore all the periods. Summing up, the model predicts that reshoring should be more prevalent in the sectors characterized by a lower degree of product differentiation and a higher taste for quality.

In Figure 5 we present equilibrium sensitivity to the variations in the quality cost structure. In the panel 5a there are plotted reshoring equilibrium changes due to an increase in the variable costs of producing quality,
$\delta_O$. The reaction pattern is non-monotonic. Initially, for low values of $\delta_O$ (when $\delta_H$ is close in value to $\delta_O$) the reshoring activity is more likely to occur, and it takes place for lower productivity firms. However, when the quality production becomes very costly (for $\delta_O$ sufficiently higher than $\delta_H$), the reshoring interval starts to shrink and eventually it vanishes. Increasing the quality adaptation costs decrease the net benefits from the offshore production, ceteris paribus. In the limiting case, when the quality production is prohibitively expensive, we would observe only the home producing firms. On the other hand, in the panel 5b there are plotted the equilibrium changes due to variations in $\theta$. It describes the cost of quality innovation, i.e. the new design expenses, R&D outlays, the machinery replacement costs, etc. Increase in $\theta$ results in the reshoring activity being less and less likely to occur and taking place for more and more productive firms. This is because rising $\theta$ reduces the net benefits from the investments in the quality and depresses the profits for all the firms, but most prominently for the home producers. In the limiting case, when the quality innovation is very costly we would observe all the firms producing only offshore.

In Figure 6 we present the comparative statics exercise for wages, $w$ (panel 6a) and transport cost parameter, $\tau$ (panel 6b). Qualitatively, the impact of an increase in wages or a rise in the transportation cost is similar, as those parameters jointly describe the effective unit labor cost of the offshore labor. Increase in $w$ or in $\tau$ decrease the profits of the firms producing at least one period abroad. When offshoring is more expensive, a higher number of firms will decide to produce in US and to reshore, also, alongside increasing $w$ and $\tau$, we observe less and less productive firms transferring their offshored production back because the most productive ones are incentivized to produce at home both the periods.

Figure 6: Reshoring equilibria. Comparative statics (III).
Finally, in Figure 7 we report how the reshoring equilibrium reacts to the changes in the market size, L. Similarly to the impact of an increasing offshore wages and a rise in the transportation costs, an increase in the market size results in the reshoring activity taking place for less and less productive firms. As the market size grows, the scope for quality differentiation increases, firms invest more in quality and experience higher profits.

18 The increase in the scope for quality differentiation leading to a higher optimal quality choice by firms is one of the main findings in Antoniades (2015).
5 Conclusions

We present a novel theory explaining the recent phenomenon of reshoring. We develop a dynamic model of heterogeneous firms choosing both the quantity and the quality of the good, and deciding on the production location. Quality production is attractive as consumers are willing to pay a higher price for the higher quality good, yet quality production is costly. Offshoring offers a way for reducing the payroll costs, however it comprises quality production costs greater than the domestic manufacturing. The model generates the equilibrium reshoring of production and yields an equilibrium sorting pattern with reshoring arising for the intermediate values of productivity. Comparative statics exercises suggest that reshoring activity is more prevalent in sectors with lower degrees of product differentiation and exhibiting higher love for quality. We leave empirical verification of those hypotheses for the future research.
6 References


20

THE HACKETT GROUP (2012): “Reshoring Global Manufacturing: Myths and Realities,”.


# Appendix

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<tr>
<th>Year</th>
<th>Full Sample</th>
<th>Reshorners</th>
<th>KFO</th>
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<td>No.</td>
<td>%</td>
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<td>1 0,44</td>
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Table 2: The year of reshoring, different samples

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<td>Other</td>
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<td>U.S. price of natural</td>
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<td>Eco-system synergies</td>
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<td>Supplier issues</td>
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</tr>
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</table>

Table 3: Main domestic factors behind reshoring decision for US firms
Reshoring firms by industry
NAICS 2012 2 digit class.

Source: own calculations

Figure 8
Noticing that $\lambda_k(\beta - \delta_k) = \frac{L(\beta - \delta_k)^2}{4\gamma k_L - L(\beta - \delta_k)^2}$ for $k \in \{H, O\}$ and $c_{D,O} = \frac{c_{D,H}}{w\tau}$:

\[
\frac{L}{4\gamma}(c_{D,H} - c_\omega)^2(1 + \lambda_H(\beta - \delta_H)) > \frac{L}{4\gamma}(w\tau)^2(c_{D,O} - c_\omega)^2(1 + \lambda_O(\beta - \delta_O))
\]

\[
(c_{D,O}^2 - 2c_{D,O}c_\omega + c_\omega^2)(\frac{4\theta_O\gamma}{4\theta_O\gamma - L(\beta - \delta_O)^2}) > (c_{D,O}^2 - 2c_{D,O}c_\omega(w\tau) + c_\omega^2(w\tau)^2)(\frac{4\theta_O\gamma}{4\theta_O\gamma - L(\beta - \delta_O)^2})
\]

\[
(c_{D,H}^2 - 2c_{D,H}c_\omega + c_\omega^2)(\frac{\theta_H}{4\theta_H\gamma - L(\beta - \delta_H)^2}) > (c_{D,H}^2 - 2c_{D,H}c_\omega(w\tau) + c_\omega^2(w\tau)^2)(\frac{\theta_O}{4\theta_O\gamma - L(\beta - \delta_O)^2})
\]

where $\Gamma_H \equiv \frac{\theta_H}{4\theta_H\gamma - L(\beta - \delta_H)^2}$, $\Gamma_O \equiv \frac{\theta_O}{4\theta_O\gamma - L(\beta - \delta_O)^2}$

\[
\Leftrightarrow c_{\omega}^2(\Gamma_H - (w\tau)^2\Gamma_O) - 2c_\omega c_{D,H}(\Gamma_H - (w\tau)\Gamma_O) + c_{D,H}^2(\Gamma_H - \Gamma_O) > 0
\]  (37)