Vertically Related Markets, Public Utility Ownership and In-kind Trade and Industrial Policy

Han Eol Ryu†
Korea Institute for Industrial Economics and Trade, Seoul, South Korea
August 7, 2016

Abstract

This paper examines the optimal government ownership level of an upstream public utility with the consideration of trade and industrial policy. It considers a simple two country model in which a home public firm and a foreign private firm in the upstream market supply a produced input solely for their own local downstream firms competing in the home market. The paper shows that the optimal government ownership level depends on the net welfare effects of an intermediate input price that is, in turn, affected by the tariff and the subsidy. The upstream public utility will likely be fully (partly) nationalized if the tariff and the subsidy are sufficiently small (large). It will be fully privatized if they are moderate. Simulation results reveal the diverse optimal public ownership regimes with varying tariff and subsidy levels. The paper sheds light on the possibility of using government ownership of an upstream public utility as in-kind trade and industrial policy.

Keywords: Optimal Privatization, Vertically Related Market, Trade Policy, Industrial Policy, Imperfect Competition, Intermediate Input, Market Structure.

JEL classification: D43, D49, L13, L22, M21

*Preliminary Draft, Please do not cite or circulate without authors’ permission
†E-mail: heryu@kiet.re.kr; Tel: 82-44-287-3248; Fax: 82-44-287-3500
1. Introduction

Since WTO regime started to open the door for free trade for the world, trade liberalization gained worldwide momentum. The direct use of trade and industrial policy as a tool for promoting exports or protecting domestic industries have been strictly restricted. It follows that tariffs and subsidies have been dropping steadily. Many countries, under such conditions, have to alter their economic policies relating with international trade since their welfare effects would be changed as trade liberalization intensified. Privatization policy will also be affected and therefore the optimal privatization level, when the tariffs and subsidies are altered, becomes an important government concern.

A number of studies have considered the relationship between privatization and trade and industrial policy. Pal and White (1998) researched on the relationship between full privatization and the optimal tariff and subsidy policy. They considered an international mixed oligopoly model in which the foreign private firms compete with home private and public firms in the domestic market. They showed that full privatization of public firm will always raise domestic social welfare if the home government subsidizes all domestic firms, and it will raise the welfare over much of the parameter space if instead it utilizes an import tariff. Chang (2005) analyzed the optimal privatization and trade policies under general demand and cost structure using a model in which a home public firm competes with a more efficient foreign private firm. He showed that the optimal privatization level depends on the competition characteristics of the two firms. In addition, he showed that in the special case of using linear demand and quadratic cost function, partial privatization of the public firm is always optimal under Cournot competition, but full nationalization is optimal under Stackelberg competition. Chao and Yu (2006) considered a mixed oligopoly model with one domestic public firm and a number of competing foreign firms and examined the effect of privatization or foreign competition on optimal tariffs. They showed that if the number of foreign firms increased or if the level of privatization is decreased, then the optimal tariff rate will become lower. Long and Stähler (2009) examined the effect of privatization on the optimal tariff and subsidy. They showed that if a domestic public firm competes with a foreign private rival in the home market, the optimal
The tariff rate is independent with the privatization level of the public firm, and the optimal subsidy decreases with privatization under the positive tariff.

In this paper, we consider the optimal privatization policy of public utilities with the consideration of trade and industrial policy. The public utilities, such as water, gas, and electricity, have exhibited some important characteristics. First, they are usually categorized as the upstream industries. Most of the existing literature analyzing privatization, trade and industrial policies were confined to a horizontal model in which both public and private firms produce final goods only and the results of those are not relevant for the public utilities. Second, they are natural monopoly in a particular region. There are a number of reasons for them being a monopoly. Since large capital investments are required to establish public utilities, small investors are unable to enter. In addition, the governments support the monopoly position because overlapping investments are unnecessary. Lastly, provided goods are untradeable among nations and thus foreign entry is limited.

In view of the above observations, this paper choose to consider a simple vertical model in which a home public firm and a foreign private firm in the upstream market supply a produced input solely for their own local downstream firms. The home and foreign downstream firms are competing in the home market. The home trade and industrial policies are in the form of import tariff and production subsidy and exogenously given. Under this structure, the home government chooses the optimal privatization policy. The paper shows various conditions for determining the optimal profit margin and privatization level. They depend on, among others, the net welfare effects of an intermediate input price which is determined by the tariff and the subsidy. If the tariff and the subsidy are sufficiently small (large), then the upstream public utility will likely be fully (partly) nationalized and its profit margin will be lower (higher) than the fully privatized one. It will be fully privatized if they are moderate. To obtain further insight, a couple of simulations are implemented. Simulation results reveal the diverse optimal public ownership regimes with varying tariff and subsidy levels. One major implication of these results is that the possibility of using government ownership level of an upstream public utility as in-kind trade and industrial policy.

\[\text{Some papers researched privatization in a vertical structure. For example, Lee (2006), Wen and Yuan (2010), and Chang and Ryu (2015, 2016). However, they did not consider trade and industrial policy.}\]
The remainder of the paper is organized as follows. Section 2 introduces the model and Section 3 characterizes the optimal profit margin and privatization level of an upstream public utility. Section 4 provides numerical simulation results. Concluding remarks are followed in Section 5.

2. The Model

Consider a two-country model consisting of a home (H) and a foreign (F). H has an upstream firm U1 and a downstream firm D1. Similarly, the foreign counterparts are denoted by U2 and D2. U1 can be a pure public firm or be partially or fully privatized, but U2 is always a private firm. Both upstream firms produce an intermediate input which is untradeable between the two countries and is only sold within a country for the local downstream firm. D1 and D2, with their purchased intermediate inputs, produce a homogeneous final good while competing as Cournot players in the home country. The home government uses trade and industrial policy in the form of import tariff and production subsidy.

Assume that U1 maximizes a weighted average of its profit and the domestic social welfare while U2, being a purely private, always maximizes its own profit. For the notational simplicity, we use the lower case letters to denote the home country’s variables and the upper case for the foreign. Let the outputs of U1 and U2 be $x$ and $X$, respectively and that of D1 and D2 be $q$ and $Q$. To incorporate the technology differences between U1 and U2, this paper assumes that they have different cost functions. Let U1’s total cost function be $c(x)$ and U2’s be $C(X)$, where $c' > 0$, $C' > 0$, $c'' \geq 0$ and $C'' \geq 0$. Assume that one unit of intermediate input is required to produce one unit of the final good, and the downstream firms could convert the inputs to the final goods without incurring any other costs. Let $p$ be the price of the final good and let also $v$ and $V$ be the prices of the intermediate goods produced by U1 and U2, respectively. Thus, the cost function of D1 is $vq$ and that of D2 is $VQ$. We assume further that the downstream firms face a linear inverse demand $p = a - bz = p(z)$, where $z = q + Q$, $a > 0$, and $b > 0$.

2Utilities in some developed countries, such as British, America, has already been privatized.
3Utilities, such as water, electricity, usually can not be traded among nations.
The game consists of three stages. At stage 1, the home government chooses the optimal privatization level of U1. At stage 2, U1 and U2 determine their own intermediate input price. At stage 3, D1 and D2 compete in the final good market, taking the price of intermediate good as given. To solve the game by backward induction, we begin by considering the final good market.

2.1 The Final Good Market

In the final good market, both D1 and D2 compete in the home country. Let $t$ and $s$ be the home country’s specific import tariff and production subsidy, respectively. The profit functions of downstream firms are

$$\pi = p(z) q - vq + sq$$
$$\Pi = p(z) Q - VQ - tQ.$$

Let a subscript denote a partial derivative. Taking the intermediate input price $v$ and $V$ as given, the first-order conditions for their profit maximization are

$$\pi_q (q, Q) = p'(z) q - v + s = 0, \quad (1a)$$
$$\Pi_Q (q, Q) = p'(z) Q - V - t = 0, \quad (1b)$$

which can be solved for the Cournot equilibrium outputs as functions of $v$, $V$, $t$, and $s$.

To examine the effects of a change in $v$, $V$, $t$ and $s$ on the equilibrium final outputs, we obtain from (1):

$$\begin{pmatrix} dq \\ dQ \end{pmatrix} = \frac{1}{|H|} \begin{pmatrix} \Pi_{22} & -\pi_{12} \\ -\Pi_{21} & \pi_{11} \end{pmatrix} \begin{pmatrix} dv - ds \\ dV + dt \end{pmatrix},$$

where $H$ is the determinant of the matrix in the equation.
where $|H| = \begin{vmatrix} \pi_{11} & \pi_{12} \\ \Pi_{21} & \Pi_{22} \end{vmatrix} > 0$. It follows that $q_v = 2/3p' < 0$, $q_V = -1/3p' > 0$, $q_s = -2/3p' > 0$, $q_t = -1/3p' > 0$, $Q_v = -1/3p' > 0$, $Q_V = 2/3p' < 0$, $Q_s = 1/3p' < 0$, and $Q_t = 2/3p' < 0$. Thus, an increase in $v$, $V$, or $t$ lowers the industry output $z$ while an increase in $s$ raises it.  

### 2.2 The Intermediate Good Market

Since one unit of final output requires one unit of the intermediate input and the upstream firms supply their produced inputs only to their own domestic downstream firms, the demand for the U1’s product is $x = q(v, V; t, s)$ and U2’s $X = Q(v, V; t, s)$. Thus, the profit functions of U1 and U2 are

$$\pi^u(v, V) = vx(v, V; t, s) - c(x(v, V; t, s)),$$  \hfill (2a)

$$\Pi^u(v, V) = VX(v, V; t, s) - C(X(v, V; t, s)),$$  \hfill (2b)

respectively.

The social welfare function of the Home country is the sum of consumer surplus, the profits of all home firms, and the government surplus:

$$W = CS + PS + GS$$
$$= \left( \int_0^z p(s) ds - p \cdot z \right) + \pi^u + \pi + tX - sx$$  \hfill (3)

We assume that the manager of U1 chooses $v$ to maximize a weighted average of domestic welfare and U1’s profit:

$$M = gW + (1 - g) \pi^u,$$  \hfill (4)

\footnote{Since $\pi_{11} = -2b < 0$, $\Pi_{22} = -2b < 0$ and $|H| = \pi_{11} \Pi_{22} - \pi_{12} \Pi_{21} = 3b^2 > 0$, the trace of $H$ is negative and its determinant is positive. The stability conditions are thus satisfied.}

\footnote{It can be shown that $z_v = 1/3p' < 0$, $z_V = 1/3p' < 0$, $z_s = -1/3p' > 0$, and $z_t = 1/3p' < 0$.}
where $g$ and $1 - g$ is the public and private ownership share of the home firm, respectively. As the public ownership level increases, $g$ becomes larger and the firm becomes more welfare-oriented.

$M$ can be expressed as a function of $v$ and $V$\footnote{Since $x = q(v, V; t, s)$, $X = Q(v, V; t, s)$, $z = q + Q = x + X$. The consumer surplus $\left( \int_0^x p(s) \, ds - p \cdot z \right)$ is a function of $z$, and hence a function of $v$ and $V$. $\pi^u(v, V)$ is shown in (2a) and the D1’s profit is $\pi = \pi(q, Q) = \pi(x, X)$.} From (4), we obtain

$$M_v = g \left[ \frac{\partial}{\partial v} \left( \int_0^x p(s) \, ds - p \cdot z \right) + \frac{\partial \pi}{\partial v} + tX_v - sx_v \right] + \frac{\partial \pi^u}{\partial v}, \quad (5)$$

Since $z_v = 1/3p'$ and $(p - v) = -p'x - s$ from Footnote 5 and (1a), $\pi_v = p'xz_v + (p - v) x_v + sx_v - x = -4x/3$. In addition, $\pi^u_v = (v - c') x_v + x = -2(v - c') / 3b + x$ and $\Pi^u_V(v, V) = (V - C') X_V + X = -2(V - C') / 3b + X$. Thus, the first-order conditions are:

$$M_v(v, V) = \frac{1}{3b} \left( g(-bz - 4bx + t + 2s) - 2(v - c') + 3bx \right) = 0, \quad (6a)$$

$$\Pi^u_V(v, V) = -\frac{2}{3b} (V - C') + X = 0. \quad (6b)$$

The above system of equations yield the optimal intermediate input prices $v$ and $V$ as functions of $g$, $t$ and $s$.

To precisely examine U1’s profit margin, we obtain from (6a)

$$v - c' \geq \theta \text{ according to } g \Phi \geq 0, \quad (7)$$

where $\theta = 3bx/2$ which is the profit margin of the fully privatized U1 and $\Phi = (-bz - 4bx + t + 2s) / 3b$. Note that $\Phi$ summarizes the net effect of an increase in $v$ on the home consumer surplus, downstream profit and government surplus\footnote{It can be shown from (6a) that an increase in $v$ serves to decrease the consumer surplus $(CS_v = -z/3 < 0)$ and D1’s profit $(\pi_v = -4x/3 < 0)$, while to increase the government surplus $(GS_v = (t + 2s) / 3b > 0)$.} \footnote{It can be shown from (6a) that an increase in $v$ serves to decrease the consumer surplus $(CS_v = -z/3 < 0)$ and D1’s profit $(\pi_v = -4x/3 < 0)$, while to increase the government surplus $(GS_v = (t + 2s) / 3b > 0)$.} shows that if $g > 0$ and $\Phi > (\theta)$, then $v - c' > (\theta)$. In addition, under such condition, if either $t$ or $s$ is large (small) enough, then $\Phi > (\theta)$, and hence $v - c' > (\theta)$. However, if the home government could not use the industrial and trade
policies, i.e., \( t = 0 \), and \( s = 0 \), then the profit margin of \( U1 \) will always be lower than that of the fully privatized one—not necessarily positive.

**Proposition 1** Consider the vertically-related oligopoly model described above:

(i) If \( U1 \) is full or partial public firm, then its profit margin will be higher or lower than that of the pure private one depending on whether the welfare effect of an increase in \( v \) on consumer and downstream firm is larger or smaller than that on government.

(ii) In the above case, if either the tariff or the subsidy is sufficiently large (small), \( U1 \) will charge a higher (lower) profit margin than the pure private one; if the use of tariff and subsidy policy is not allowed for the home government, the profit margin of \( U1 \) is always lower than that of the pure private one.

Next, we analyze some properties of comparative statics. The first-order conditions (6) yield upstream firms’ equilibrium intermediate input prices as functions of \( g, t, \) and \( s \). Thus, we have:

\[
\begin{pmatrix}
\frac{dv}{dV} \\
\frac{dV}{dv}
\end{pmatrix} = \frac{1}{|H^u|} \begin{pmatrix}
\Pi_{VV}^u & -M_{vV} \\
-P_{VV}^u & M_{vv}
\end{pmatrix} \begin{pmatrix}
-M_{vg}dg - M_{vt}dt - M_{vs}ds \\
-P_{Vg}dg - P_{Vt}dt - P_{Vs}ds
\end{pmatrix},
\tag{8}
\]

where, \(|H^u| = \begin{pmatrix} M_{vv} & M_{vV} \\ \Pi_{VV}^u & \Pi_{VV}^u \end{pmatrix} |,

\begin{align}
M_{vv} &= [3b(4 - 3g) - 4c'']/9b^2 < 0, \\
M_{vV} &= [3b(1 - g) + 2c'']/9b^2 > 0, \\
\Pi_{VV}^u &= (3b + 2C'')/9b^2 > 0, \quad \Pi_{VV}^u = -(12b + 4C'')/9b^2 < 0.
\end{align}
\tag{9a-9c}

It can be shown from (9) that \(|M_{vv}| > |M_{vV}| \) and \(|\Pi_{VV}^u| > |\Pi_{VV}^u|\). Thus, \(|H^u| > 0\) and the stability
conditions are satisfied. In addition, from (6), we obtain

\[ M_{vg} = CS_v + \pi_v + GS_v = \Phi \geq 0, \]

\[ M_{vt} = [3b + 2c'] / 9b^2 > 0, M_{vs} = [3b(2 - g) + 4c'] / 9b^2 > 0, \]

\[ \Pi_{Vg}^u = 0, \Pi_{Vt}^u = -[6b + 4C''] / 9b^2 < 0, \Pi_{Vs}^u = -[3b + 2C''] / 9b^2 < 0, \]

where \( \Phi = (-b_t - 4bx + t + 2s) / 3b \). It follows that

\[ v_g = -\Pi_{Vv}^u M_{vg} / |H^u| \geq 0, V_g = \Pi_{Vv}^u M_{vg} / |H^u| \geq 0, \]

\[ v_t = (-\Pi_{Vv}^u M_{vt} + \Pi_{Vt}^u M_{vV}) / |H^u| > 0, \]

\[ v_s = (-\Pi_{Vv}^u M_{vs} + \Pi_{Vs}^u M_{vV}) / |H^u| > 0, \]

\[ V_t = (\Pi_{Vv}^u M_{vt} - \Pi_{Vt}^u M_{vV}) / |H^u| < 0, \]

\[ V_s = (\Pi_{Vv}^u M_{vs} - \Pi_{Vs}^u M_{vV}) / |H^u| < 0, \]

and use has been made of \( |M_{vv}| \geq |M_{vs}| > |M_{vV}|, |M_{ve}| > |M_{vt}| \geq |M_{vV}|, |\Pi_{Vv}^u| > |\Pi_{Vt}^u| > |\Pi_{Ps}^u|, \) and \( |\Pi_{Vv}^u| > |\Pi_{Vt}^u| = |\Pi_{Vs}^u| \). If the total welfare effects of changing \( v \) on \( CS \) and \( \pi \) is larger (smaller) than that on the \( GS \), i.e. \( \Phi < (>) 0 \), then \( M_{vg} \) is negative (positive), and hence both \( v_g \) and \( V_g \) are negative (positive).

**Lemma 1** In the vertically related oligopoly model described above:

(i) An increase in the government ownership share of \( U1 \) increases (decreases) domestic and foreign intermediate input price if the net effects of a change in intermediate input price on the consumer surplus, the downstream’s profit and the government surplus is positive (negative).

(ii) An increase in domestic subsidy or tariff always increases domestic intermediate input price, while decreases foreign intermediate input price.
3. The Optimal Public Ownership Policy

This section examines the home country’s optimal public ownership policy. In the first stage, the government determines \( g \) to maximize the domestic social welfare in (3). We assume that \( W \) is strictly concave in \( g \) and therefore has a unique solution. From (3), it can be shown that

\[
W_g = \frac{d}{dg} \left( \int_{0}^{z} p(s) ds - p^* z \right) + \left( \frac{d \pi_u}{dg} + \frac{d \pi}{dg} \right) + \left( t \frac{d X}{dg} - s \frac{dx}{dg} \right)
\]

where \( |H^u| = \begin{bmatrix} M_{vv} & M_{vV} \\ \Pi_{Vv}^v & \Pi_{VV}^v \end{bmatrix} \) and \( \Phi = (-bz - 4bx + t + 2s) / 3b \) as defined in (7).

The effect of an increase in \( g \) on \( W \) comes from four sources as shown in (12)—consumer surplus, U1’s profit, D1’s profit, and government surplus. Let us show first the case where \( \Phi < 0 \). Clearly, the first and third effects are positive and the fourth effect is negative. However, the second one is ambiguous in sign since \( v - c' \) can be positive. Intuitively, as has been shown in Lemma 1, when the government share of U1 increases, \( v \) and \( V \) goes down and \( z \) rises, causing \( p \) to decrease.

Thus, the consumer surplus increases. In addition, since \( v_g \) is smaller than \( V_g \), a decrease of \( v \) and \( V \) works as an in-kind subsidy and will always benefit D1 \( (\pi_g > 0) \). Regarding the effect on government surplus, an increase in \( g \) lowers \( v \) that in turn raises \( x \) and lowers \( X \). In addition, it also lowers \( V \) and hence lowers \( x \) and raises \( X \). In total, \( x \) increases while \( X \) decreases, causing the total amount of the subsidy to increase but the tariff to decrease. Thus, the government surplus decreases.

Finally, an increase in \( g \) on U1’s profit in two ways—the price effect and the quantity effect. If \( g \) increases, the quantity effect can be positive or negative depending on the sign of \( v - c' \) (the first term in the bracket), but the price effect is always negative (the second term in the
bracket). In the case of \( \Phi > 0 \), the effects of an increase in government’s ownership of \( U_1 \) on the consumer surplus, \( D_1 \)’s profit and the government surplus would have the opposite sign compared with the former case but the sign of the effect on \( U_1 \)’s profit is still indeterminate.

To characterize the optimal public ownership policy, we evaluate \( W_g \) at \( g = 0 \) and \( g = 1 \):

\[
W_g|_{g=1} = -\frac{1}{18b^3} \frac{1}{|H^a|} (bX + t) (3b + 2C''') \Phi \tag{13a}
\]

\[
W_g|_{g=0} = -\frac{1}{18b^3} \frac{1}{|H^a|} [(10bz + 25bx - 14s - 4t) b + 2 (2bz + 3bx - 2s) C'''] \Phi \tag{13b}
\]

The form of the optimal government ownership would be different depending on \( t \) and \( s \). Given that \( W \) is strictly concave in \( g \), we know that if \( W_g = 0 \) occurs in the range of \( 0 < g < 1 \), then partial public ownership is optimal; if either \( W_g = 0 \) at \( g = 1 \) or \( W_g > 0 \) in all range of \( g \), then full public ownership is optimal; but if either \( W_g = 0 \) at \( g = 0 \) or \( W_g < 0 \) in all range of \( g \), then full privatization is optimal. We thus can obtain from (13) the following results: (1) If \( t \) and \( s \) are sufficiently small, then \( \Phi < 0 \), \( W_g|_{g=0} > 0 \) and \( W_g|_{g=1} > 0 \). It follows that full public ownership of \( U_1 \) is optimal. (2) If \( t \) and \( s \) are sufficiently large, then \( \Phi > 0 \), \( W_g|_{g=0} > 0 \) and \( W_g|_{g=1} < 0 \). Partial privatization of \( U_1 \) is thus optimal in this case. (3) In a moderate level of \( t \) and \( s \), which serves to make \( \Phi \) and the bracketed term in (13b) positive, \( W_g|_{g=0} < 0 \) and \( W_g|_{g=1} < 0 \). Thus, full privatization of \( U_1 \) should be pursued. (4) If \( \Phi = 0 \), then both \( W_g|_{g=0} \) and \( W_g|_{g=1} \) are zero, and hence any public ownership level is optimal.

**Proposition 2** In the vertically related oligopoly model described above:

(i) If the tariff and the subsidy are sufficiently small, then complete public ownership of the domestic upstream firm is optimal.

(ii) If the tariff and the subsidy are sufficiently large, then partial privatization of the domestic upstream firm is optimal.

(iii) In a moderate level of the tariff and the subsidy, which makes \( \Phi \) and the bracketed term in (13b) positive, then full privatization of the domestic upstream firm is optimal.

\(^{10}\)The derivation of (13) can be shown in the Appendix.
(iv) If $\Phi = 0$, then any kind of ownership of the domestic upstream firm will be optimal.

Suppose that the tariff and the subsidy is small enough. Then, given $U_1$ is fully privatized, a small degree of nationalization will cause to decrease $v$ and $V$, as shown in Lemma 1. This will increase the consumer surplus and the $D_1$’s profit, and decrease the government surplus. However, $U_1$’s profit may increase or decrease. It turns out that the total effect is always positive. In like manners, given $U_1$ is fully nationalized, a small degree of privatization will raise $v$ and $V$. It follows that the consumer surplus and the $D_1$’s profit will decrease, but the government surplus will increase. The direction of $U_1$’s profit is still uncertain but the aggregated welfare will increase. Overall, complete nationalization of $U_1$ is an optimal policy in this case. Suppose instead that the tariff and the subsidy is large enough. If $U_1$ is fully privatized and the government contemplates a small increase in its ownership of $U_1$, then $v$ and $V$ will increase. This in turn decreases the consumer and the producer surplus, but increases the government surplus. In total, the welfare effect will be positive. Similarly, if $U_1$ is fully nationalized and the government considers lowering its ownership of $U_1$, then it raises the consumer and the producer surplus, but lowers the government surplus. The total effect will be positive. Thus, partial privatization should be pursued.

Since our earlier analysis on $U_1$’s profit margin is closely tied to the optimal public ownership condition, the following corollary can be obtained from Propositions 1 and 2:

**Corollary 1** In the vertically related oligopoly model described above:

(i) If both the tariff and the subsidy is sufficiently small, then complete nationalization is optimal and $U_1$’s profit margin is lower than the fully privatized one.

(ii) If both the tariff and the subsidy is sufficiently large, then partial privatization is optimal and $U_1$’s profit margin is higher than the fully privatized one.

(iii) In a moderate level of the tariff and the subsidy, which makes $\Phi$ and the bracketed term in (13b) positive, then full privatization is optimal.

---

11 Since $\pi_u + \pi_g = -\left[\pi^u - (v - c') (7b + 2C''') + 2b^2 x\right] \Phi / 9b^3 |H^u|$, and $(v - c') = 3b (g \Phi + x) / 2$ from (16a), if $\Phi > 0$, then $\pi_u + \pi_g$ is negative.

12 In a moderate level of $t$ and $s$, which makes $\Phi$ and the bracketed term in (13b) positive, the total welfare effect will be negative in the current case.
(iv) If $\Phi = 0$, any kind of ownership of $U1$ can be optimal and $U1$'s profit margin is equal to the fully privatized one.

Under the condition in Corollary 1(i), since the level of tariff and subsidy is too low, an increase in $g$ will cause downstream in-kind subsidy effect and that will always result in an increase in the domestic welfare. Thus, home government will fully nationalize $U1$. However, in Corollary 1(ii), since the level of tariff and subsidy is too high, as $g$ increases the domestic welfare starts to decrease at a certain point of $g$. At the optimal point, $U1$ will charge its profit margin higher than the private one causing downstream in-kind tax effect in order to alleviate excessive support on $D1$.

4. Optimal Public Ownership Regimes and Industrial and Trade Policy

This section examines the optimal government ownership level under different trade and industrial policy. The ambiguity and complexity of some of our earlier outcomes are partly due to the cost generality of the two upstream firms. In this section, we therefore consider the simplified upstream cost functions with the following quadratic forms:

$$c(x) = \frac{1}{2}x^2, \quad C(X) = \frac{1}{2}kX^2,$$

where $k \leq 1$.

From $D1$ and $D2$’s first-order conditions in (1), the equilibrium downstream outputs and price

---


14 The value of $k$ serves as a proxy for the efficiency gap between public and private firms.
can be obtained as follows:

\[
q(v; V; t; s) = \frac{1}{3b} (a - 2v + V + t + 2s), \quad Q(v; V; t; s) = \frac{1}{3b} (a + v - 2V - 2t - s),
\]

\[
z(v; V; t; s) = \frac{1}{3b} (2a - v - V - t + s), \quad p(v; V; t; s) = \frac{1}{3} (a + v + V + t - s).
\]  

(14a)

Since one unit of final output requires one unit of the intermediate input and the upstream firms can supply only to their own domestic downstream firms, the inverse intermediate input demands for U1 and U2 are \( x = q(v; V; t; s) \) and \( X = Q(v; V; t; s) \), respectively.

In the second stage, the optimal upstream outputs and prices can be obtained from (6):

\[
v(g; t; s) = \left[ \frac{(3b + 2) (5ab + 2ak + 2bt + 7bs + 2ks)}{-bg (27ab + 10ak - 6bt + 9bs - 4kt + 2ks)} \right] / A,
\]  

(15a)

\[
V(g; t; s) = \left[ (3b + 2k) (2a + 5ab - 7bt - 2bs - 2t - 5abg + 6bg + 2bg) \right] / A,
\]  

(15b)

\[
x(g; t; s) = \left[ \frac{2 (5ab + 2ak + 2bt + 7bs + 2ks)}{+g (2ab - 9bt - 14bs - 2kt - 4ks)} \right] / A,
\]  

(15c)

\[
X(g; t; s) = \left[ 2 (2a + 5ab - 7bt - 2bs - 2t) - 2bg (5a - 6t - 2s)] / A, \right.
\]  

(15d)

where \( A = 45b^2 + 14b + 14bk + 4k - 33b^2g - 10bgk \) which is always positive.

To obtain the optimal \( g \) explicitly, by substituting (14) into (5), we obtain

\[
W = \frac{1}{2} b (x + X)^2 + \frac{1}{18b^2} (a - 2v + V + t + 2s) (-a + 2v + 6bv - V - t - 2s)
\]

\[
+ \frac{1}{9b} (a - 2v + V + t + 2s)^2 + \frac{1}{3b} (at - as + tv + 2sv - 2Vt - Vs - 2t^2 - 2s^2 - 2st),
\]

where \( v \) and \( V \) are functions of \( g \) as shown in (15a) and (15b). Maximization of \( W \) with respect to \( g \) yields the optimal public ownership level

\[
g = \frac{b (5b + 2k) A + 16b (2b + k) t - 2 (7b + 2k)^2 s}{b (5b + 2k) B + (7b + 2k) C},
\]
where \( A = 4a - 16t + 45ab + 10ak - 18bs - 18bt - 4ks, \)
\( B = 45ab + 10ak - 18bs - 21bt - 4ks - 2kt, \)
and \( C = 2ab - 14bs - 9bt - 4ks - 2kt. \)

In what follows, we demonstrate our results with numerical simulation. Figure 1 illustrates diverse optimal public ownership regimes with varying tariff and subsidy levels.

Figure 1

![Figure 1. The Effect of Trade and Industrial Policy on \( g \) (\( a = 1, b = 1 \) and \( k = 1 \))](image)

(To be completed......)

5. Concluding remarks

This paper has examined the optimal privatization policy of an upstream public utility in the presence of tariff and subsidy. Both upstream firms supply its produced input solely for their own downstream firms which are competing in the domestic market. Different cost structures between the two upstream firms and different levels of tariff and subsidy are considered. It is shown that the optimal profit margin and privatization level of the public utility depends on the net welfare effects of an intermediate input price and that is changed by the tariff and the subsidy. To further
characterize the derived general results, a number of simulation analyses are performed. It is found that there exist diverse optimal privatization regimes in terms of the tariff and subsidy.

This paper has laid a foundation for studying the privatization of public utility issues using a vertical and a secluded upstream monopoly structure. This paper sheds light on the possibility of using government ownership of an upstream public utility as in-kind trade and industrial policy.

6. Appendix

6.1 Derivation of $W_g$ in (12)

We have

$$W_g = \frac{d}{dg} \left( \int_0^z p(s) \, ds - p \cdot z \right) + \left( \frac{d\pi^u}{dg} + \frac{d\pi}{dg} \right) + (tX_g - sx_g).$$

Using (9) and (11), we obtain

$$\frac{d}{dg} \left( \int_0^z p(s) \, ds - p \cdot z \right) = bz (z_v v_g + z_V V_g) = -\frac{z}{3} (v_g + V_g)$$

$$= -\frac{z}{9b^3 \lvert H^u \rvert} (5b + 2C'') M_{vg},$$

$$\frac{d\pi^u}{dg} + \frac{d\pi}{dg} = \left\{ -\frac{1}{9b^3 \lvert H^u \rvert} [(v - c') (7b + 2C'') - 4bx (3b + C'')] M_{vg} \right\}$$

$$+ \left\{ -\frac{2x}{9b^3 \lvert H^u \rvert} (7b + 2C'') M_{vg} \right\}$$

$$= -\frac{1}{9b^3 \lvert H^u \rvert} [(v - c') (7b + 2C'') + 2b^2 x] M_{vg},$$

$$tX_g - sx_g = t (X_v v_g + X_V V_g) - s (x_v v_g + x_V V_g)$$

$$= \left( \frac{2}{3} bs + \frac{1}{3} bt \right) v_g - \left( \frac{1}{3} bs + \frac{2}{3} bt \right) V_g$$

$$= \frac{1}{9b^3 \lvert H^u \rvert} (2bt + 7bs + 2sC'') M_{vg},$$
where $|H^u| = \begin{vmatrix} M_{vv} & M_{vV} \\ \Pi_{Vv} & \Pi_{VV} \end{vmatrix}$.

Since $M_{vg} = \Phi$ from (10), $W_g$ in (12) then follows.

6.2 Derivation of $W_g|_{g=1}$ and $W_g|_{g=0}$ in (13)

We have $W_g = W_v v_g + W_V V_g$ from (12a) and $\Phi = CS_v + \pi_v + GS_v$. Use has been made of $(v - c') = 3b (g \Phi + x) / 2$. Since $M_v = W_v = 0$ if $g = 1$ and $M_v = \pi_v^u = 0$ if $g = 0$ by (6a), we obtain:

$$W_g|_{g=1} = W_V V_g = -(bX + t) V_g / 2b$$

$$W_g|_{g=0} = \Phi v_g + W_V V_g = \Phi v_g - (5bx - 2bX - 4t - 2s) V_g / 6b.$$  \hspace{1cm} (16a) (16b)

Substituting $v_g$ and $V_g$ in (11) into (16), two equations in (13) can thus be obtained.


**References**


