Innovation and Manufacturing Offshoring with Fully Endogenous Productivity Growth

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Abstract

This paper investigates the relationship between net offshoring patterns for innovation and manufacturing and fully endogenous productivity growth in a two-country model that is not biased by scale effects. The occupational choice of skill-differentiated workers into low-skilled employment in production and high-skilled employment in innovation determines labor market allocations, and perfect capital mobility allows firms to shift innovation and manufacturing independently between countries. These mechanisms generate a tension between access to technical knowledge and low-cost high-skilled labor in the location decision for innovation, with innovation and manufacturing tending to concentrate in the asset-wealthy (poor) country when trade costs are high (low). The model exhibits a positive relationship between innovation costs and the concentration of industry and innovation, which ensures that a rise in the degree of knowledge diffusion between countries coincides with increases in net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country, and a faster rate of productivity growth, when the asset-wealthy countries has larger shares of production and innovation.

JEL Classifications: F12, F43, R11

Keywords: innovation offshoring, manufacturing offshoring endogenous productivity growth, process innovation, industry location, international trade

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1 Introduction

In recent years the range of activities being offshored has expanded to include high-skilled labor intensive activities. In particular, a rise in the offshoring of R&D services has attracted the attention of policymakers who are concerned about the implications for economic growth and who hope to attract firms offering the high wages associated with R&D employment (UNCTAD 2005a). Figure 1 shows a plot of U.S. net exports in R&D services. Despite the position of the U.S. as the largest producer of R&D services (NSF 2014), in recent years its trade balance in R&D services has shifted from a trade surplus to a trade deficit, a trend that appears to be driven by intra-firm trade between U.S. parent firms and their foreign affiliates, and by significant growth in the U.S. trade deficit in R&D services with China and India. On average, trade deficits with China and India grew at rates of 27.4% and 20.1% between 2008 and 2013 to become $1.8 billion and $2.4 billion in 2013 (BEA 2015), a trend that corresponds with survey data in which top firms in R&D spending ranked China and India within the top three locations for undertaking R&D (UNCTAD 2005b).

This paper investigates the relationship between offshoring patterns in innovation

![Figure 1: U.S. Net Exports in R&D Services (BEA 2015)](image)
and manufacturing and endogenous productivity growth driven by firm-level investment in process innovation. In particular, we extend the two-country model of endogenous market structure and endogenous growth of Davis and Hashimoto (2015) to include an occupational choice for skill-differentiated workers between low-skilled employment in production and high-skilled employment in R&D.\(^1\) Within our model, innovation costs are intrinsically linked with the geographic pattern of industry, as labor productivity in R&D depends on localized knowledge spillovers from the technologies employed in production. Consequently, the free movement of investment capital generates a tension in the R&D location decision between localized access to technical knowledge and the availability of low-cost high-skilled labor, as firms shift their innovation and manufacturing independently between countries with the aim of minimizing costs.\(^2\)

Market size is determined endogenously through the interaction of national labor allocations with a home market effect, whereby the country with the larger market hosts a greater share of industry (Krugman 1980). Importantly, there is a positive circular causality in the location patterns of innovation and manufacturing that manifests itself in two effects (Davis 2013). The first is a labor income effect that is inversely related with trade costs. In particular, a rise in a country’s share of innovation activity increases high-skilled employment, pushing up high-skilled wages and expanding the national market. As a result, the country is able to attract a greater share of manufacturing. The second effect is an innovation cost effect that is increasing in the degree of international knowledge diffusion. An expansion in a country’s share of manufacturing activity improves local spillovers of technical knowledge from production to innovation, leading to lower unit costs for process innovation and enabling the

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\(^1\)The endogenous market structure and endogenous growth framework has developed over a broad literature. See for example Peretto (1996), Aghion and Howitt (1998), Peretto and Connolly (2007), and Etro (2009).

\(^2\)The business literature emphasizes increased knowledge diffusion through falling communication costs and as a key factor in the offshoring of R&D activities to emerging countries where high-skilled labor is available at low cost (Manning et al. 2008; Lewin et al. 2009; Nieto and Rodríguez, 2011).
country to attract a greater share of innovation.

We consider countries with symmetric labor endowments and skill distributions, but allow national levels of firm ownership to differ, ensuring that the relatively asset wealthy country receives a higher level of investment income through an investment income effect. Given the degree of knowledge diffusion, we then identify three cases for the location and offshoring patterns associated with innovation and manufacturing activity. First, for a low level of trade costs the labor income effect dominates, and the asset-poor country has a larger market, hosts a greater share of industry, and becomes a net exporter of manufacturing goods and process innovations. Accordingly, net offshoring in innovation and manufacturing flows from the asset-wealthy country to the asset-poor country.

In the second case, for an intermediate level of trade costs, the investment income and innovation cost effects dominate, and the asset-wealthy country has a larger market, attracts a greater share of industry, and is the net exporter of manufacturing goods and process innovations. In this case, net offshoring in manufacturing flows from the asset-poor country to the asset-wealthy country. The pattern of innovation offshoring, however, depends on the degree of international knowledge diffusion. With a low degree of knowledge diffusion, net offshoring in innovation flows from the asset-poor country to the asset-wealthy country, as firms take advantage of knowledge spillovers from production to innovation in the asset-wealthy country. In contrast, with a high degree of knowledge diffusion, firms place more emphasis on accessing low-cost high-skilled labor in the asset-poor country, and net offshoring in innovation flows from the asset-wealthy country to the asset-poor country.

The third case arises when trade costs are high and the investment income and innovation cost effects continue to dominate. Although the asset-wealthy country has a larger market, its greater shares of industry and innovation are smaller than its share of firm ownership, and net offshoring in both industry and innovation flows from the
asset-wealthy country to the asset-poor country. This final case corresponds with the U.S. pattern of net exports in R&D services described in Figure 1.

An important feature of our framework is a positive relationship between the unit cost of process innovation and the geographic concentration of industry as the benefit of greater knowledge spillovers is offset by the cost of rising high-skilled wages. An increase in the unit cost of process innovation lowers optimal firm-level employment in innovation, resulting in a lower per-period labor costs and higher per-period profits that attract new firms into the market. As such, an increase in the concentration of industry induces a greater level of market entry while dampening the rate of productivity growth. Focusing on the case for which the asset-wealthy country has greater shares of innovation and industry, we investigate the effects of an improvement in the degree of knowledge diffusion between countries, and find that net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country increase as firms offshore innovation to the asset-poor country to take advantage of lower wages for high-skilled workers. As a consequence, the dispersion of industry and innovation activity away from the asset-wealthy country leads to lower unit-costs for process innovation and a higher rate of productivity growth.

Our paper is closely related to the international trade literature that builds upon the variety-expansion model developed by Grossman and Helpman (1991). Gao (1999) and Martin and Ottaviano (1999, 2001) consider the relationship between growth and industry location patterns when firms are free to shift production and innovation independently between countries. Greater economic integration through a reduction in trade costs causes industry to concentrate in the larger country, resulting in an improvement in knowledge spillovers from production to innovation and accelerating economic growth. Although these studies focus on production offshoring, they also implicitly model trade in R&D services. Given the localized nature of knowledge spillovers, R&D concentrates fully in the country with the larger market and the
greater share of industry, ensuring that net exports in R&D services always flow from the larger country to the smaller country. As such, these models are not able to produce the pattern of innovation offshoring presented in Figure 1. Davis (2013) attempts to investigate the trade-off between localized knowledge spillovers and high-skilled wage costs by extending Martin and Ottaviano (1999) to include an occupational choice for skill-differentiated workers. The link between industry location and economic growth is cut, however, and there is no relationship between growth and offshoring patterns.

An additional issue with the frameworks introduced above is that they all exhibit scale effects, whereby growth is positively related to the size of the labor force. Indeed, the positive effect of industry concentration on growth is closely linked with the scale effect, and therefore may be somewhat at odds with two strands of the empirical literature. First, empirical evidence for the relationship between industry concentration and GDP growth is generally mixed (Gardiner et al. 2011). Second, there is a growing body of empirical literature which concludes that there is no significant relationship between economic growth and population size (Dinopoulos and Thompson 1999; Barro and Sala-i-Martin 2004; and Laincz and Peretto 2006). The framework presented in this paper corrects for scale effects by focusing on the innovation associated with the production technologies of individual product lines, rather than considering R&D activity at the national level.

The remainder of the paper proceeds as follows. Section 2 introduces the model, and then Section 3 characterizes the industry and innovation location patterns that arise in long-run equilibrium. In Section 4 we investigate the directions of net offshoring flows, and in Section 5 we consider the implications of offshoring patterns for market entry and productivity growth. In Section 6 concludes.
2 The Model

Two countries, home and foreign, potentially employ labor in the four activities: traditional production, manufacturing, process innovation, and market entry. Home and foreign have symmetric labor endowments that are fully employed. There is no international migration, but skill-differentiated workers choose between low-skilled employment in production and high-skilled employment in innovation within each country. We focus on the home country while introducing the model. The setup for foreign is analogous, however, with an asterisk denoting variables associated with foreign.

2.1 Household Preferences

Dynastic households in each country select optimal expenditure-saving paths with the aim of maximizing utility over an infinite time horizon. The lifetime utility of a household residing in the home country is

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} \left( \alpha \ln C_X(\tau) + (1 - \alpha) \ln C_Y(\tau) \right) d\tau,$$

where at time $t$ the consumptions of a composite of manufacturing goods and a traditional good are $C_X$ and $C_Y$, the subjective discount rate is $\rho$, and $\alpha \in (0, 1)$. The manufacturing composite is

$$C_X(t) = \left( \int_0^{N(t)} c_i(t) \frac{\sigma - 1}{\sigma} \, di + \int_0^{N^*(t)} c_j(t) \frac{\sigma - 1}{\sigma} \, dj \right)^{\frac{\sigma}{\sigma - 1}},$$

where $c_i$ and $c_j$ are the demands for manufacturing varieties $i$ and $j$ of the $N$ and $N^*$ varieties produced in home and foreign, and $\sigma > 1$ is the elasticity of substitution. Lifetime utility is maximized subject to a national flow budget constraint:

$$\dot{B}(t) = r(t)B(t) + I(t) - E(t),$$
where $E$ is household expenditure, $I$ is labor income, $B$ is asset wealth, $r$ is the interest rate, and a dot over a variable denotes time differentiation. The solution to the household’s utility maximization problem is the optimal expenditure-saving path described by the Euler condition: $\dot{E}/E = \dot{E}^*/E^* = r - \rho$, with equal access to an international financial market ensuring a common interest rate on asset wealth, and common motions for household expenditure. To simply notation, we suppress time arguments whenever possible.

With identical investment opportunities, the asset wealths of home and foreign accumulate at the same rate ($\dot{B}/B = \dot{B}^*/B^*$), and national shares of asset wealth are constant across time. Adding the flow budget constraints (3) to obtain world expenditure $E_W \equiv E + E^*$ as a function of world asset wealth $B_W \equiv B + B^*$, and using the result to substitute the net return to investment $(r - \dot{B}_W/B_W)$ out of the national flow budget constraint yields home expenditure:

$$E = I + b(E_W - I_W),$$

with $I_W \equiv I + I^*$ and national shares of asset wealth determined by initial levels of asset wealth: $b \equiv B/B_W$ and $b^* \equiv B^*/B_W$.

At each moment in time, households allocate constant shares of expenditure to the purchase of the manufacturing composite and the traditional good: $P_X C_X = \alpha E$ and $P_Y C_Y = (1 - \alpha) E$, where $P_X$ is a price index for manufacturing goods and $P_Y$ is the traditional good price in home. The price index over manufacturing goods is

$$P_X = \left( \int_0^N p_{Xi}^{1-\sigma} \, di + \int_0^{N^*} (\zeta p_{Xj}^*)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}},$$

where $p_{Xi}$ and $p_{Xj}^*$ are the prices associated with goods produced in home and foreign. Iceberg trade costs are incurred on international shipments with a shipment of $\zeta > 1$ units required for every unit supplied to an export market (Samuelson 1954). Viewing
(5) as the unit expenditure function over manufacturing goods, Shephard’s Lemma yields the home country demands for home and foreign produced varieties:

\[ c_i = \alpha p_{X_i}^{-\sigma} P_X^{-1} E, \quad c_j = \alpha (\zeta p_{X_j})^{-\sigma} P_X^{-1} E. \]  

(6)

The demand conditions for foreign households are analogous.

2.2 Occupational Choice

Home and foreign have equal masses of workers \( Z = Z^* \) with heterogeneous skill levels \( z \) that follow continuous uniform distributions with support \([0, 1]\). Workers are free to choose between employment in production or innovation. A worker employed in production supplies one unit of low-skilled labor, regardless of skill level, earning the low-skilled wage rate \( w_L \). A worker employed in innovation supplies \( z \) units of high-skilled labor, earning income \( zw_H \), where \( w_H \) is the high-skilled wage rate.

National labor markets are competitive with all firms paying the same effective low-skilled and high-skilled wages. When there is positive employment in production and innovation, one marginal worker potentially earns the same incomes from low-skilled and high-skilled employment, and is therefore indifferent between employment type. The skill level of this marginal worker equals the relative wage rate \( z = \omega = w_L/w_H \), and separates the labor force into workers with skill levels \( z \in [0, \omega] \) who choose employment in production, and workers with skill levels \( z \in [\omega, 1] \) who choose employment in innovation. This national labor allocation leads to the following effective low-skilled and high-skilled labor supplies for the home country: \( L = \omega Z \) and \( H = (1 - \omega^2)Z/2 \). Expected national labor income, conditional on employment levels, is therefore \( I(\omega) = w_L L + w_H H = w_L(1 + \omega^2)Z/(2\omega) \).
2.3 Traditional Production

Traditional firms employ one unit of low-skilled labor with a constant returns to scale technology to produce one unit of output for supply to an international market characterized by free trade. Setting low-skilled labor as the model numeraire, we assume that the world demand for traditional goods is large enough that both countries continue to produce traditional goods at all moments in time. The traditional good price and low-skilled wage rates then equalize across countries, $P_Y = P_Y^* = w_L = w_L^* = 1$, and the home and foreign demands for traditional goods determine the world demand for low-skilled labor in traditional production:

$$L_Y + L_Y^* = (1 - \alpha)E_W.$$

(7)

2.4 Manufacturing

The manufacturing sector is monopolistically competitive (Dixit and Stiglitz 1977), with each firm producing a single unique product for supply to domestic and export markets. The production technology of firm $i$ with production located in home is

$$x_i = \theta^\gamma l_{Xi},$$

(8)

where $x_i$ and $l_{Xi}$ are firm-level output and employment in production, $\theta$ is firm-level productivity, and $\gamma \in (0, 1)$ is the productivity elasticity of output. Although each firm employs a unique production technology, we assume that productivity levels are symmetric across firms ($\theta = \theta^*$), regardless of the location of production.$^3$

Firms maximize operating profit by setting price equal to a constant markup over unit cost: $p_X = p_X^* = \sigma/((\sigma - 1)\theta^\gamma)$, where we now suppress the firm index $i$. Equating supply with the demands from home and foreign households (6), $x = c + \zeta c^*$, optimal

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$^3$See Davis and Hashimoto (2014) for a framework that considers the dynamics of productivity convergence between firms with production located in different countries.
operating profit on sales \((\pi = px - l_X)\) for a firm with production located in home is
\[
\pi = \frac{l_X}{\sigma - 1} = \frac{\alpha p_X^1}{\sigma} \left( \frac{E}{P_X^1} + \varphi E^* \right), \tag{9}
\]

where \(\varphi \equiv \zeta^{1-\sigma} \in (0, 1)\) describes the freeness of trade between countries.

Firms are free to shift production between countries, with the aim of maximizing profit on sales (Martin and Rogers 1995). Therefore, when there are active manufacturing sectors in both countries, operating profit is the same for all firms, regardless of production location; that is, \(\pi = \pi^*\). Combining the price indices (5) with operating profit (9), we solve for the share of firms locating production in home country is
\[
s_X \equiv \frac{N}{N_W} = \frac{E - \varphi E^*}{(1 - \varphi)E_W} = \frac{b - \varphi b^*}{1 - \varphi} + \frac{(1 + \varphi)(b^*I - bI^*)}{(1 - \varphi)E_W} , \tag{10}
\]

where \(N_W \equiv N + N^*\). Substituting the manufacturing shares for each country back into operating profit yields firm-level employment in production for all locations:
\[
l_X = \frac{\alpha(\sigma - 1)E_W}{\sigma N_W} , \tag{11}
\]

where we have used the production function (8).

### 2.5 Process Innovation

Firms invest in process innovation with the aim of lowering production costs. The evolution of productivity for a firm with process innovation located in home follows
\[
\dot{\theta} = k\theta h_I, \tag{12}
\]

where \(h_I\) is firm-level high-skilled employment and \(k\theta\) is labor productivity in process innovation. The productivity coefficient \(\theta\) represents the current stock of technical knowledge, and \(k\) determines the strength of intertemporal knowledge spillovers from
accumulated knowledge into current innovation activity.

Adapting the specification of Baldwin and Forslid (2000), the strength of knowledge spillovers from production into innovation located in home is

\[ k = s_X + \delta (1 - s_X), \] (13)

where the localized nature of knowledge spillovers is regulated by the degree of knowledge diffusion \( \delta \in (0, 1) \).\(^4\) Therefore, the labor productivity of high-skilled workers in innovation is determined as the weighted-average productivity of the stock of observable knowledge, with a stronger weighting for production technologies employed in proximity to the innovation department of the firm.

Accounting for fixed per-period management costs \((l_F)\) and process innovation costs \((w_H h_I)\), total per-period profits are \( \Pi = \pi - w_H h_I - l_F \) and firm value equals the presented discounted value of the expected future profit stream:

\[ V(t) = \int_t^\infty e^{-\int_t^{\tau'} (r(\tau') + \lambda) d\tau'} \Pi(\tau) d\tau, \] (14)

where \( \lambda > 0 \) is an exogenous default rate associated with the probability that a firm-specific shock forces the firm out of the market at each moment in time. A firm set the level of high-skilled employment in process innovation to maximize firm value subject to (12). We solve this optimization problem with the following current-value Hamiltonian function: \( F = \Pi + p_I k\theta h_I \), where \( p_I \) is the internal price of a unit mass of new innovations produced in home over the time interval \( dt \), and each firm perceives the price indices (5) as constant when considering the impact of changes in its price on profits, given its small market share. The first order conditions for optimization

\(^4\)See Keller (2004) for a survey of the various channels through which knowledge spillovers arise and a discussion of their localized nature.
provide the following static and dynamic efficiency conditions:

\[ p_I = \frac{w_H}{k\theta}, \quad p_I(r + \lambda) - \dot{p}_I = \frac{\partial \pi}{\partial \theta} = \frac{(\sigma - 1)\gamma\pi}{\theta}. \tag{15} \]

Firms are free to shift their innovation actives between countries, ensuring a common internal price for new process innovations when there is innovation located in both home and foreign; that is, \( p_I = p_I^* \). We combine this condition with \( \omega = 1/w_H \) to obtain the home production share required to equate the price of new process innovations across countries:

\[ s_X = \frac{w_H - \delta w_H^*}{(1 - \delta)(w_H + w_H^*)} = \frac{\omega^* - \delta\omega}{(1 - \delta)(\omega + \omega^*)}, \tag{16} \]

where we have used (13) and (15). Substituting (16) back into (13) yields the equilibrium level of knowledge spillovers into innovation in home: \( k = (1 + \delta)\omega^*/(\omega + \omega^*) \).

### 2.6 Market Entry

In order to focus on firm-level investment in process innovation, we consider a simple setup for market entry with new firms employing low-skilled labor as they prepare to enter the industry. Following Etro (2004) and Peretto and Connelly (2007), we assume that the cost of market entry is determined proportionally with the expected value of production after market entry, leading to higher entry costs for firms entering larger markets. Under these assumptions, the costs of market entry in home and foreign become \( V = p_X x \) and \( V^* = p_X^* x^* \), with positive levels of manufacturing in both countries ensuring that these costs equalize: \( V = V^* \).

The value of a new firm equals the present value of the future profit stream that is potentially earned after entry into the industrial sector (14). With free entry firm
value is driven down to the cost of market entry:

\[ V = p_x x = \frac{\alpha E_W}{N_W}. \]  

(17)

The time derivative of (14) combined with (17) yields the following no-arbitrage condition associated with investment in market entry (Grossman and Helpman 1991):

\[ (r + \lambda) V - \dot{V} = \pi - w_H h_I - l_F. \]  

(18)

The investment conditions (15) and (18) imply that \( \omega k = \omega^* k^* \), \( h_I \omega = h_I^* \omega^* \), and \( kh_I = k^* h_I^* \) when firms locate innovation in home and foreign.

Aggregating across countries, we obtain the following differential equation to describes the dynamics of market entry and exit:

\[ \dot{N}_W = \frac{L_N + L_N^*}{p_x x} - \lambda N_W = \frac{(L_N + L_N^*) N_W}{\alpha E_W} - \lambda N_W \]  

(19)

where we have used (8), (9) and (11), and \( L_N \) and \( L_N^* \) are the national levels of low-skilled labor employed in market entry. At each moment in time \( \lambda N_W \) firms default and are forced to exit the market.

3 Long-run Location Patterns

We now derive the long-run industry and innovation location patterns consistent with equilibrium in the labor and investment markets. In order to simplify the analysis we consider the level of market entry relative to market size: \( n \equiv N_W/(\alpha E_W) \). In addition, for the remainder of the paper, we assume that the home country has a greater share of asset wealth: \( b \geq 1/2 \).
3.1 National Labor Allocations

We first solve for a condition that determines the national labor allocations associated with common prices for goods $p_X = p_X^*$ and innovations $p_I = p_I^*$. Using the flow budget constraint for total expenditure (3) with $B_W = N_W V = \alpha E_W$ and $\dot{E}_W/E_W = r - \rho$, we find that total labor income is determined proportionately with to tal expenditure: $I_W = (1 - \alpha \rho) E_W$. Then, equating (10) and (16) and reorganizing the result yields

$$\alpha \rho (b - \varphi b^*) + \frac{(1 - \alpha \rho) (I - \varphi I^*)}{(1 - \varphi) I_W} = \frac{\omega^* - \delta \omega}{(1 - \delta) (\omega + \omega^*)},$$

which we refer to as the share locus, as it indicates the relative wage combinations consistent with national labor allocations and production shares that ensure equalized operating profits and innovation costs across countries at all moments in time.

Second, we derive the dynamics associated with national labor allocations. Common motions for household expenditures ($\dot{E}/E = \dot{E}^*/E^*$) result in constant production shares ($\dot{s}_X = 0$) and common motions for relative wages ($\dot{\omega}/\omega = \dot{\omega}^*/\omega^*$). Combining (15) with $I_W = (1 - \alpha \rho) E_W$, $h_I/\omega = (H/\omega + H^*/\omega^*)/N_W$, and $\dot{E}_W/E_W = -((H/\omega + H^*/\omega^*)/I_W)(\dot{\omega}/\omega) = r - \rho$, we obtain the following motion for the evolution of relative wages:

$$\frac{\dot{\omega}}{\omega} = \frac{(n_H - n)(\rho + \lambda) I_W}{(L + L^*) n}, \quad n_H = \frac{(\sigma - 1) \gamma \omega k}{\sigma (\rho + \lambda)} - \frac{(1 - \alpha \rho) (H/\omega + H^*/\omega^*) \omega k}{\alpha (\rho + \lambda) I_W},$$

where $n_H$ is the steady-state level of market entry consistent with optimal investment in process innovation and equilibrium in the high-skilled labor market.

Third, we derive an expression to describe the evolution of market entry. Substituting (7), (11), and (19) with $I_W = (1 - \alpha \rho) E_W$ into the low-skilled labor market
These stylized labor allocation patterns can be reproduced numerically using $\alpha = 0.95$, $\sigma = 2.5$, $\gamma = 0.5$, $\rho = 0.01$, $\lambda = 0.01$, $l_F = 0.05$, $b = 0.75$, $\delta = 0.45$, and $\varphi_B = 0.89$, for both panels, and with $\varphi = 0.75$ for Panel (a) and $\varphi = 0.95$ for Panel (b).
equilibria associated with the intersection between the investment locus and the upper and lower branches of the share locus are shown in Figures 2a and 2b. In Appendix A, we investigate the local dynamics of the model and find that \( \omega k > \rho + \lambda \) and a greater slope for the share locus than the investment locus are sufficient conditions for the saddle-path stability of a long-run equilibrium with dispersed industry and innovation.\(^6\) We limit our analysis to steady states that satisfy these conditions.

### 3.2 Industry and Innovation Location Patterns

National shares of industry are described by (16). In Figure 2, the full concentration of industry occurs along the \( \omega^*/\omega = 1/\delta \) dashed line \((s_X = 1)\), and in foreign along the \( \omega^*/\omega = \delta \) dashed line \((s_X = 0)\). Similarly, denoting the numbers of firms locating process innovation in home and foreign by \( m = H/h_I \) and \( m^* = H^*/h_I^* \), with high-skilled labor employed solely in process innovation, the home share of innovation output is

\[
s_I \equiv \frac{m \dot{\theta}}{m \dot{\theta} + m^* \dot{\theta}^*} = \frac{w_H H}{w_H H + w^*_H H^*} = \frac{(1 - \omega^2)\omega^*}{(1 - \omega^2)\omega^* + (1 - \omega^*^2)\omega},
\]

where we have used \( kh_I = k^* h_I^* \) and \( h_I/\omega = h_I^*/\omega^* \). National shares of innovation activity are described by the investment locus in Figure 2, with innovation fully concentrated in home at point \( a \) \((s_I = 1)\), and innovation fully concentrated in foreign at point \( a^* \) \((s_I = 0)\). The shaded areas indicate national labor allocations that are not consistent with labor market equilibrium.

We characterize industry and innovation location patterns according to the level of trade costs, and obtain the following proposition.

**Proposition 1** (i) If \( \varphi < \varphi_B \), the asset-wealthy country has larger shares of industry and innovation, with the full concentration of industry for \( \varphi \in (\varphi_X, \varphi_B) \) and the full

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\(^6\)Numerical simulations suggest that these sufficient conditions are actually necessary conditions for a saddle-path stable equilibrium, although we are unable to show this analytically.
concentration of innovation for $\varphi \in (\varphi_1, \varphi_B)$. (ii) If $\varphi > \varphi_B$, the asset-poor country has larger shares of industry and innovation, with the full concentration of industry for $\varphi \in (\varphi_B, \varphi_X)$ and the full concentration of innovation for $\varphi \in (\varphi_B, \varphi^*_I)$, where

$$
\varphi_B \equiv \frac{\delta + \omega \omega^* + (1 - \delta)(b^* - \omega \omega^*b)\alpha \rho}{1 + \delta \omega \omega^* - (1 - \delta)(b^* - \omega \omega^*b)\alpha \rho},
$$

with positive values for both the numerator and denominator since $1 > \alpha \rho$. Note that $\varphi_1 < \varphi_X$ and $\varphi^*_I > \varphi^*_X$ for $\delta < \underline{\delta}$, but $\varphi_1 \geq \varphi_X$ and $\varphi^*_I \leq \varphi^*_X$ for $\delta \geq \underline{\delta}$.

Proof: See Appendix B.

National shares of industry and innovation are determined through a home market effect (Krugman 1980) with greater shares of firms locating production and process innovation in the country with the larger market. Market size is determined endogenously, however, according to investment and labor income. Consider the adjustments in national labor markets that occur after a small increase in $b$ expands the home production share through the investment income effect described by the first term on the RHS of (20). The firm-level location decision for process innovation drives national labor markets adjustments as the economy returns to equilibrium through a shift along the investment locus, with the direction of the shift depending on the balance of two opposing effects (Davis 2013). The first is the labor income effect captured by the second term on the RHS of (20). With a rise in high-skilled labor income, the home share of industry expands, reducing innovation costs through improved knowledge spillovers and attracting a greater share of innovation to the home country. This effect is increasing in the freeness of trade. The second effect is the innovation cost effect expressed by the LHS of (20), and regulates how far high-skilled wages can rise in home relative to foreign, as high-skilled employment expands, before foreign becomes more attractive as a location for process innovation. This effect is increasing in the degree of knowledge diffusion.
While there is clearly a positive circular causality between the location patterns of production and innovation, with a higher share of production raising knowledge spillovers, and a higher share of innovation raising labor income and increasing market size, the initial direction of this causality depends on whether the labor income effect or the innovation cost effect dominates. For a given degree of knowledge diffusion, we use the freeness of trade to identify two cases for the direction of labor market adjustments after an increase in $b$ induces a relative increase in the knowledge spillovers of home and $w_H/w_H^* < k/k^*$. First, for $\varphi < \varphi_B$, the innovation cost effect dominates and $w_H/w_H^*$ increases at a faster rate than $k/k^*$ until the costs of process innovation are again equalized, with the home country maintaining greater levels of production, high-skilled employment, and innovation. In this case, home becomes a net exporter of manufacturing goods and process innovations. Second, for $\varphi > \varphi_B$, the labor income effect dominates, and $k/k^*$ decreases at a faster rate than $w_H/w_H^*$ until $p_I = p_I^*$. In this case, the foreign country has greater levels of production, high-skilled employment, and innovation, and becomes the net exporter of manufacturing goods and process innovations.

Whether or not the full concentration of production coincides with the full concentration of innovation depends on the degree of knowledge diffusion. If $\delta < \delta_*$, the knowledge spillover advantage of the larger national market discourages firms from locating innovation in the smaller country, and innovation concentrates fully before industry; that is, $\varphi_X < \varphi_I$ and $\varphi_I^* > \varphi_X^*$. In contrast, if $\delta > \delta_*$, a high degree of knowledge diffusion mitigates the benefits of knowledge spillovers, allowing the smaller country to attract innovation with its low-cost high-skilled labor, even with industry fully concentrated in the larger country; that is, $\varphi_X > \varphi_I$ and $\varphi_I^* < \varphi_X^*$. 

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These stylized labor allocation patterns can be reproduced numerically using $\alpha = 0.95$, $\sigma = 2.5$, $\gamma = 0.5$, $\rho = 0.01$, $\lambda = 0.01$, $l_F = 0.05$, $b = 0.75$ for both panels, and with $\delta = 0.45$ and $\varphi_B = 0.89$ for Panel (a), and $\delta = 0.85$ and $\varphi_B = 0.97$ for Panel (b).

3.3 Offshoring Patterns

The direction of net offshoring flows in the manufacturing sector is determined as the difference between national shares of asset wealth and production. For the home country we have

$$S_X \equiv b - s_X = \frac{(b + \delta b^*) \omega - (b^* + \delta b) \omega^*}{(1 - \delta) (\omega + \omega^*)}, \quad (24)$$

where we have used (16). Net offshoring flows from the asset-wealthy home country to the asset-poor foreign country when $S_X > 0$, and from foreign to home when $S_X < 0$. Characterizing offshoring patterns using the freeness of trade, we obtain the following:

**Proposition 2** Net offshoring in manufacturing flows from the asset-wealthy country to the asset-poor country for $\phi \notin (\phi_{XO}, \phi_B)$, and from the asset-poor country to the asset-wealthy country for $\phi \in (\phi_{XO}, \phi_B)$, where $\phi_{XO} \in (0, \phi_X)$.

Proof: See Appendix C.

Net offshoring flows for the manufacturing sector are illustrated by the $S_X$ curve in
Figure 3, where the vertical axis measures $S_X \in (-b^*, b)$ over the range $\varphi \in (0, 1)$ indicated by the horizontal axis. There are three cases for the direction of net offshoring. Starting from a high level of trade costs over the range $\varphi \in (0, \varphi_{XO})$, the investment income and innovation cost effects dominate the labor income effect ensuring a greater share of industry for home. The market of home is not sufficiently large to support the production of all home-owned firms, however, resulting in net offshoring flows from home to foreign ($S_X > 0$). For a mid-level of trade costs $\varphi \in (\varphi_{XO}, \varphi_B)$, the home market is sufficiently large to attract the production of a larger share of firms, including the production of firms with foreign owners, and net offshoring therefore flows from foreign to home ($S_X < 0$). For a low level of trade costs over the range $\varphi \in (\varphi_B, 1)$, the labor income effect dominates the investment income and innovation cost effects, and the larger market of foreign attracts the greatest share of production, with a share of home-owned firms also locating production in foreign, thereby generating net offshoring flows from home to foreign ($S_X > 0$).

The direction of net offshoring flows in innovation is calculated using the difference between national shares of asset wealth and innovation. For home we have

$$S_I \equiv b - s_I = \frac{b\omega(1 - \omega^2) - b^*\omega^*(1 - \omega^2)}{(1 - \omega^2)\omega^* + (1 - \omega^2)\omega^*},$$

where we have used (23). Net offshoring in innovation flows from the asset-wealthy home country to the asset-poor foreign country when $S_I > 0$, and from foreign to home when $S_I < 0$.

**Proposition 3** When $\delta < \delta$, net offshoring in innovation flows from the asset-wealthy country to the asset-poor country for $\varphi \notin (\varphi_{IO}, \varphi_B)$ and from the asset-poor country to the asset-wealthy country for $\varphi \in (\varphi_{IO}, \varphi_B)$, where $\delta < \delta$ and $\varphi_{IO} \in (0, \varphi_I)$. When $\delta > \delta$, net offshoring in innovation always flows from the asset-wealthy country to the asset-poor country.
The pattern of innovation offshoring depends on the degree of knowledge diffusion. As shown in Figure 3a, where net offshoring flows for innovation are measured on the vertical axis \( (S_I \in (-b^*, b)) \), when the degree of knowledge diffusion is relatively low \( (\delta < \delta) \), there are three cases. The first case occurs over the range \( \varphi \in (\varphi, \varphi_{IO}) \) where home’s greater share of industry provides it with a knowledge spillover advantage that allows it to attract a larger share of innovation. The knowledge spillover advantage is not sufficient, however, to prevent a share of home-owned firms from locating innovation in foreign with the aim of taking advantage of lower high-skilled wages. As a result, net offshoring in innovation flows from home to foreign \( (S_I > 0) \). The second case occurs for a mid-level of trade costs \( \varphi \in (\varphi_{IO}, \varphi_B) \) where home’s share of industry generates a knowledge spillover advantage that is great enough to ensure that all home-owned firms, and a share of foreign-owned firms, locate innovation in the home country, despite higher high-skilled wages. In this case case net offshoring flows from foreign to home \( (S_I < 0) \). The third case arises for a low level of trade costs \( \varphi \in (\varphi_B, 1) \) when foreign has a larger share of industry, and all foreign firms and a share of home-owned firms locate innovation in the foreign country to take advantage of greater knowledge spillovers, leading to net offshoring flows from home to foreign \( (S_I > 0) \).

In general, marginal increases in the degree of knowledge diffusion have ambiguous effects on national labor allocations. We can show, however, that for relatively high values of \( \delta \), we have \( \varphi < \varphi_B, s_I > 0, \) and \( s_X > 0, \) and improved knowledge spillovers induce a fall in the relative wage rate, and reductions in the home shares of innovation and production that induces upward shifts in the \( S_X \) and \( S_I \) curves in Figure 3. In particular, as shown in Figure 3b, for a sufficiently high degree of knowledge diffusion, the knowledge spillover advantage associated with concentrated industry is relatively weak creating an incentive for firms to focus on minimizing the cost of employing high-
skilled labor when choosing where to locate innovation. As such, process innovation never concentrates fully in one country. Indeed, with perfect knowledge diffusion ($\delta = 1$), exactly half of all innovation activity is located in each country: $S_I = b - 1/2 > 0$. Therefore, in the case for $\delta > \delta_0$, net offshoring in innovation always flows from the asset-wealthy home country to the asset-poor foreign country ($S_I > 0$).

## 4 Offshoring and Productivity Growth

This section compares the effects of increased industry concentration on net offshoring flows, market entry, and productivity growth. We combine (9), (11), (12), (15), and (18) to obtain the level of market entry and the rate of productivity growth as follows:

$$
n = \frac{(\nu - \rho - \lambda)\omega k}{\omega kl_F - \rho - \lambda} - \rho - \lambda, \quad g = \frac{(\sigma - 1)\gamma(\omega kl_F - \rho - \lambda)}{\sigma(\nu - \rho - \lambda)} - \rho - \lambda,
$$

where $\nu = (1 - (\sigma - 1)\gamma)/\sigma \in (0, 1)$ and $\nu > \rho + \lambda$ is necessary for a positive level of market entry, given that we assume $\omega k > \rho + \lambda$ to ensure saddle-path stability. Note that productivity growth is not biased by a scale effect as changes in overall population size ($2Z$) are fully absorbed by adjustments in the number of firms in the market ($N$), leaving the level of market entry ($n$) unchanged.

We use the expressions in (26) to examine how greater economic integration between countries affects market entry and productivity growth. A key attribute of the framework is a positive relationship between the unit cost of process innovation ($p_I = 1/(\omega k\theta)$) and the concentration of industry and innovation in either home or foreign, as the benefit of improved knowledge spillovers is outweighed by the cost of rising high-skilled wages. Returning to (26), an increase in the unit cost of process innovation leads to a greater level of market entry and a faster rate of productivity growth. Specifically, optimal firm-level employment in process innovation falls, resulting in lower per-period labor costs ($w_H h_I$) and higher per-period profits.
(Π = ν/n – (ρ + λ)/(ωk) – l_F) that attract new firms into the market (dn/d(ωk) < 0).
The decrease in firm-level employment in innovation, however, depresses the rate of productivity growth (dg/d(ωk) > 0). Thus, improved economic integration through a fall in trade costs or a rise in the degree of knowledge diffusion affects market entry and productivity growth through adjustments in the unit cost of process innovation.

Beginning with a decrease in trade costs we obtain the following proposition.

**Proposition 4** A reduction in trade costs increases market entry and dampens productivity growth for ϕ < ϕ_B, while decreasing market entry and accelerating productivity growth for ϕ > ϕ_B.

Proof: See Appendix D.

A decrease in trade costs affects the unit cost of process innovation indirectly through changes in national shares of production. Specifically, for ϕ < ϕ_B, a decrease in trade costs increases the concentration of industry in home, generating a higher cost for process innovation. As a result, optimal firm-level employment in innovation falls, leading to lower per-period labor costs and a greater level of operating profit that attracts new firms into the industry and raises the level of market entry. The decrease in firm-level employment in process innovation, however, depresses the rate of productivity growth. In contrast, for ϕ > ϕ_B, lower trade costs reduce the concentration of industry in foreign leading to lower unit costs for innovation, a lower level of market entry, and a faster rate of productivity growth.

Next, examining the effects of adjustments in the degree of knowledge spillovers we obtain the following result.

**Proposition 5** An improvement in the degree of knowledge diffusion reduces market entry and accelerates productivity growth for ϕ < ϕ_B, but has ambiguous effects on market entry and productivity growth for ϕ > ϕ_B.

Proof: See Appendix D.
An improvement in the degree of knowledge diffusion affects the unit cost of process innovation both directly through greater knowledge spillovers that raise labor productivity in innovation and indirectly though adjustments in national shares of production. Thus, on the one hand, as discussed in Section 3, for $\varphi < \varphi_B$ a sufficient increase in $\delta$ lower the concentration of industry home, and both the direct and indirect effects aline to decrease the unit cost of process innovation. The result is greater firm-level employment in innovation, a lower level of market entry, and accelerated productivity growth. On the other hand, for $\varphi > \varphi_B$, the increase in $\delta$ has an ambiguous effect on national shares of production, with ambiguous results for the unit cost of process innovation, the level of market entry, and the rate of productivity growth.

Focusing on the case for which industry and innovation concentrates in the asset-wealthy country ($\varphi < \varphi_B$), we now compare the effects of economic integration through lower trade costs on net offshoring flows and productivity growth. Returning to Figure 3, an increase in $\varphi$ reduces the home shares of innovation and production, thus decreasing $S_I$ and $S_X$, implying that a decrease in net offshoring flows in innovation and manufacturing coincides with a greater level of market entry and a slower rate of productivity growth after a fall in trade costs. In contrast, an improvement in the degree of knowledge diffusion shifts the $S_X$ and $S_I$ curves upwards in Figure 3. Accordingly, increases in net offshoring flows in manufacturing and innovation coincide with a decrease in the level of market entry and a faster rate of productivity growth.

With the evidence presented in Baldwin et al. (2001) suggesting that communication costs have fallen at a faster rate than trade costs, our results indicate that an improvement in the degree of knowledge diffusion is a plausible explanation for the recent rise in innovation offshoring. In addition, this trend may potential coincide with faster productivity growth as the unit costs associated with process innovation fall.
5 Concluding Remarks

This paper has developed a two-country model to examine the relationship between net offshoring patterns in innovation and manufacturing and fully endogenous productivity growth that is not biased by scale effects. Central to the model, monopolistically competitive firms invest in process innovation that lowers production costs and drives aggregate productivity growth. The occupational choice of skill-differentiated workers into low-skilled employment in production and high-skilled employment in innovation determines national labor allocations, while the free capital movement allows manufacturing firms to shift their production and innovation activities freely between countries. These two mechanisms create a tension between access to the technical knowledge contained in production processes and low-cost high-skilled labor as firms independently select the optimal locations for innovation and production. A key feature of the model is a positive relationship between the unit cost of process innovation and the geographic concentration of industry and innovation as the benefit of greater knowledge spillovers is offset by the cost of rising high-skilled wages.

Assuming symmetric skill distributions and labor endowments, market size is determined endogenously, and while a high level of trade costs leads to the concentration of industry and innovation in the asset-wealthy country, when trade costs are low industry and innovation concentrate in the asset-poor country. Given these location patterns, we use the level of trade costs to identify three cases for the directions of net offshoring in innovation and manufacturing. For a high level of trade costs, although the asset-wealthy country has greater shares industry and innovation, the domestic market is not sufficiently large to attract the innovation and production activities of all firms with domestic owners, and net offshoring thus flows towards the asset-poor country. For an intermediate level of trade costs, however, net offshoring flows from the asset-poor country towards the larger market of the asset-wealthy country. Finally, for a low level of trade costs, the net offshoring flows towards the asset-poor country.
as it maintains greater concentrations of industry and innovation.

Focusing on the case for which the asset-wealthy country has greater shares of industry and innovation activity, we investigate the effects of an improvement in the degree of knowledge diffusion between countries, and find that net offshoring flows in innovation and manufacturing from the asset-wealthy country to the asset-poor country increase as firms offshore innovation to the asset-poor country to take advantage of lower wages for high-skilled workers. The resulting increased dispersion of industry and innovation activity away from the asset-wealthy country leads to lower unit costs for process innovation and thus accelerates productivity growth.

Appendix A: Saddle-path Stability

With \( \omega k = (1 + \delta)\omega \omega^*/(\omega + \omega^*) \), \( L + L^* = (\omega + \omega^*)Z \), \( H/\omega + H^*/\omega^* = (\omega + \omega^*)(1 - \omega \omega^*)Z/(2\omega \omega^*) \) and \( I_W = (\omega + \omega^*)(1 + \omega \omega^*)Z/(2\omega \omega^*) \), the share locus (20) and the investment locus \( (n_H = n_L) \) are written in implicit form respectively as follows:

\[
\Omega = \frac{\omega^*}{\omega} - \frac{\delta - \varphi + (1 - \delta \varphi)\omega \omega^* + (1 - \delta)(1 + \varphi)(b - b^*\omega \omega^*)\alpha \rho}{\delta - \varphi + (1 - \delta \varphi)\omega \omega^* + (1 - \delta)(1 + \varphi)(b^* - b \omega \omega^*)\alpha \rho},
\]

\[
\Phi = \frac{(\sigma - 1)\gamma \omega k}{\sigma(\rho + \lambda)} - \frac{(1 - \alpha \rho)(1 - \omega \omega^*)\omega k}{\alpha(1 + \omega \omega^*)(\rho + \lambda)} - \frac{2(1 - \alpha \rho)\omega \omega^*}{\alpha(1 + \omega \omega^*)l_F} + \frac{\sigma - \alpha}{\alpha \sigma l_F} + \frac{\lambda}{l_F}.
\]
We use these expressions to obtain the following partial derivatives:

\[
\begin{align*}
\frac{\partial \Omega}{\partial \omega} &= \frac{\omega^*}{\omega^2} + \frac{(1 - \alpha \rho)(\omega - \omega^*)^2 \omega^*}{\alpha \rho (b - b^*)(1 + \omega \omega^*)^2 \omega^2} \geq 0, \\
\frac{\partial \Omega}{\partial \omega^*} &= \frac{1}{\omega} + \frac{(1 - \alpha \rho)(\omega - \omega^*)^2}{\alpha \rho (b - b^*)(1 + \omega \omega^*)^2 \omega^2} > 0, \\
\frac{\partial \Omega}{\partial \delta} &= -\frac{(1 - \delta)(1 + \varphi)(\omega + \omega^*)(1 + \omega \omega^*) \alpha \rho}{(\delta - \varphi + (1 - \delta \varphi) \omega \omega^* + (1 - \delta)(1 + \varphi)(b - b^*) \omega \omega^*) \alpha \rho} \omega \geq 0, \\
\frac{\partial \Omega}{\partial \varphi} &= -\frac{(1 - \delta)(1 + \varphi)^2 (b - b^*) \omega^2}{\alpha \rho (1 - \delta)(1 + \varphi)^2 (b - b^*) \omega^2} < 0, \\
\frac{\partial \Omega}{\partial b} &= -\frac{(1 - \delta \varphi)^2 (1 + \varphi)^2 (b - b^*) (1 + \omega \omega^*) \omega^2}{\alpha \rho (1 - \delta \varphi)^2 (1 + \varphi)^2 (b - b^*) (1 + \omega \omega^*) \omega^2} > 0, \\
\frac{\partial \Phi}{\partial \omega} &= \frac{n \omega^*}{(\omega + \omega^*) \omega} + \frac{2(1 - \alpha \rho)(\omega \omega^* - \rho - \lambda) \omega^*}{\alpha (\rho + \lambda)(1 + \omega \omega^*)^2 l_F} > 0, \\
\frac{\partial \Phi}{\partial \omega^*} &= \frac{n \omega}{(\omega + \omega^*) \omega} + \frac{2(1 - \alpha \rho)(\omega \omega^* - \rho - \lambda) \omega}{\alpha (\rho + \lambda)(1 + \omega \omega^*)^2 l_F} > 0, \\
\frac{\partial \Phi}{\partial \delta} &= \frac{n}{1 + \delta} > 0.
\end{align*}
\]

The slope of the share locus is \((d\omega^*/d\omega)|_{\Omega=0} = -(\partial \Omega/\partial \omega)/(\partial \Omega/\partial \omega^*)\), and is positive or negative depending on the sign of \(\partial \Omega/\partial \omega\). The slope of the investment locus is \((d\omega^*/d\omega)|_{\Phi=0} = -(\partial \Phi/\partial \omega)/(\partial \Phi/\partial \omega^*) < 0\). We consider the stability of the long-run equilibrium described by \(n_H = n_L = n\) and \(\Omega = \Phi = 0\) using a Taylor expansion of (21) and (22) to obtain the following determinant for the Jacobian matrix \(J_1\) of the linearized system:

\[
|J_1| = \frac{(1 + \omega \omega^*)(\rho + \lambda)l_F}{2\omega^*} \left( \frac{\partial \Phi}{\partial \omega^*} \right) \left( \frac{d\omega^*}{d\omega} \bigg|_{\Phi=0} - \frac{d\omega^*}{d\omega} \bigg|_{\Omega=0} \right).
\]

The relative wage rate \((\omega)\) is a control variable and the level of market entry \((n)\) is a state variable. Thus, we require one positive and one negative eigenvalue for saddle-path stability. As such, we consider long-run equilibria that satisfy \(|J_1| < 0\), which is ensured for \(\partial \Phi/\partial \omega^* > 0\) and \((d\omega^*/d\omega)|_{\Omega=0} > (d\omega^*/d\omega)|_{\Phi=0}\), where \(\omega kl_F > \rho + \lambda\) is a sufficient condition for \(\partial \Phi/\partial \omega > 0\) and \(\partial \Phi/\partial \omega^* > 0\).
Appendix B: Proposition 1

The steady-state comparative statics associated with national labor allocations are

\[
\begin{align*}
\frac{d\omega}{db} &= -\frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega} \frac{\partial \Omega}{\partial b}, \\
\frac{d\omega}{d\varphi} &= -\frac{1}{|J_2|} \frac{\partial \Phi}{\partial \varphi}, \\
\frac{d\omega}{d\delta} &= -\frac{1}{|J_2|} \left( \frac{\partial \Phi}{\partial \omega} \frac{\partial \Omega}{\partial \delta} - \frac{\partial \Phi}{\partial \delta} \frac{\partial \omega}{\partial \varphi} \right),
\end{align*}
\]

\[
\begin{align*}
\frac{d\omega^*}{db} &= \frac{1}{|J_2|} \frac{\partial \Phi}{\partial \omega} \frac{\partial \Omega}{\partial b}, \\
\frac{d\omega^*}{d\varphi} &= \frac{1}{|J_2|} \frac{\partial \Phi}{\partial \varphi}, \\
\frac{d\omega^*}{d\delta} &= \frac{1}{|J_2|} \left( \frac{\partial \Phi}{\partial \omega} \frac{\partial \Omega}{\partial \delta} - \frac{\partial \Phi}{\partial \delta} \frac{\partial \omega}{\partial \varphi} \right),
\end{align*}
\]

where \(|J_2| = (\partial \Omega/\partial \omega) (\partial \Phi/\partial \omega^*) ((d \omega^*/d \omega)|_{\Phi=0} - (d \omega^*/d \omega)|_{\Omega=0}) < 0\) for \(|J_1| < 0\).

These comparative statics are used to derive Proposition 1. First, we define \(\varphi_B\) as the threshold value of the freeness of trade for which the denominator of \(\partial \Omega/\partial b\) equals zero. Then, we have \(\partial \Omega/\partial b < 0, \omega^* > \omega, s_X > 1/2,\) and \(s_I > 1/2\) for \(\varphi < \varphi_B,\) and \(\partial \Omega/\partial b > 0, \omega^* < \omega, s_X < 1/2,\) and \(s_I < 1/2\) for \(\varphi > \varphi_B.\) Next, considering the effects of an increase in \(\varphi,\) since \(\partial \Omega/\partial \varphi < 0,\) we have \(d\omega/d\varphi < 0\) and \(d\omega^*/d\varphi > 0\) for \(s_X \in (0,1).\) As such, when \(\varphi < \varphi_B,\) \(ds_X/d\varphi > 0\) until \(s_X = 1\) at \(\varphi = \varphi_X,\) where \(\varphi_X\) is the threshold value of the freeness of trade at which the investment locus and the \(\omega^*/\omega = 1/\delta\) line intersect in Figure 2. Similarly, \(ds_I/d\varphi > 0\) until \(s_I = 1\) at \(\varphi = \varphi_I,\) where \(\varphi_I\) is the threshold value of the freeness of trade at which the investment locus intersects the \(\omega^* = 1\) line. Alternatively, when \(\varphi > \varphi_B,\) \(s_X = 0\) until \(\varphi \geq \varphi_X^*,\) after which \(ds_X/d\varphi > 0,\) where \(\varphi_X^*\) is the threshold value of the freeness of trade at which the investment locus and the \(\omega^*/\omega = \delta\) line intersect. Likewise, \(s_I = 0\) until \(\varphi \geq \varphi_I^*,\) after which \(ds_I/d\varphi > 0,\) where \(\varphi_I^*\) is the threshold level of value of the freeness of trade at which the investment locus intersects the \(\omega = 1\) line.

The ranking of \(\varphi_X\) and \(\varphi_I\) depends on the degree of knowledge diffusion. Given that \(d\omega/d\delta|_{\Phi=0, \omega^*=0} = -(\partial \Phi/\partial \delta)/(\partial \Phi/\partial \omega) < 0,\) an increase in \(\delta\) shifts the investment locus downwards. Hence, as a rise in \(\delta\) rotates the \(\omega^*/\omega = 1/\delta\) line clockwise around the origin, we can define the threshold value \(\delta = \delta^*\) at which \(\varphi_X = \varphi_I.\) Thus, we have
\( \varphi_X > \varphi_I \) for \( \delta < \bar{\delta} \) and \( \varphi_X \leq \varphi_I \) for \( \delta \geq \bar{\delta} \). It can be similarly shown that \( \varphi_I^* > \varphi_X^* \) for \( \delta < \bar{\delta} \) and \( \varphi_I^* \leq \varphi_X^* \) for \( \delta \geq \bar{\delta} \).

**Appendix C: Propositions 2 and 3**

First, we find that \( b - s_X \geq 0 \) for \( \omega^*/\omega \leq (b + \delta b^*)/(b^* + \delta b) \), where \( 1/\delta > (b + \delta b^*)/(b^* + \delta b) > 1 \). In addition, evaluating (20) at \( \varphi = 0 \), we have

\[
\frac{\omega^*}{\omega} \bigg|_{\Omega=0; \ \varphi=0} = \frac{\delta + \omega^* + (1 - \delta)(b - b^* \omega^*)\alpha \rho}{\delta + \omega^* + (1 - \delta)(b^* - b^* \omega^*)\alpha \rho} < \frac{b + \delta b^*}{b^* + \delta b},
\]

given that \( 1 > \alpha \rho \). As \( d(\omega^*/\omega)|_{\Omega=0}/d\varphi > 0 \), there is a threshold value \( \varphi = \varphi_{XO} \in (0, \varphi_X) \) at which \( b = s_X \). Thus, \( b < s_X \) for \( \varphi \in (\varphi_{XO}, \varphi_B) \) and \( b > s_X \) for \( \varphi \not\in (\varphi_{XO}, \varphi_B) \). This proves Proposition 2.

Next, we set \( b = s_I \) to obtain \( b\omega(1 - \omega^2) = b^* \omega^*(1 - \omega^2) \). Given \( b > b^* \), for relative wage combinations that satisfy this condition, we have \( \omega^* > \omega \). In addition, this condition is strictly concave with a positive slope:

\[
\frac{d\omega^*}{d\omega} = \frac{\omega^2(1 + \omega^2)b^*}{\omega^2(1 + \omega^2)b} > 0, \quad \frac{d^2\omega^*}{d\omega^2} = -\frac{2b^* \omega^2}{b(1 + \omega^2)\omega^3} \left( 1 - \frac{(1 - \omega^2)(1 + \omega^2)^2}{(1 - \omega^2)(1 + \omega^2)^2} \right) < 0,
\]

where the second term in parentheses in \( d^2\omega^*/d\omega^2 \) is less than one for \( \omega^* > \omega \). Since the investment locus has a negative slope, it intersects \( b\omega(1 - \omega^2) = b^* \omega^*(1 - \omega^2) \) once at the threshold value \( \varphi = \varphi_{IO} \in (0, \varphi_I) \). Thus, we have \( b < s_I \) for \( \varphi \in (\varphi_{IO}, \varphi_B) \), and \( b > s_I \) for \( \varphi \not\in (\varphi_{IO}, \varphi_B) \). The existence of the range \( (\varphi_{IO}, \varphi_X) \) depends on the position of the \( \omega^*/\omega = 1/\delta \) line in Figure 2. As increases in \( \delta \) cause the \( \omega^*/\omega = 1/\delta \) line to rotate clockwise around the origin and the investment locus to shift downwards \( (d\omega/d\delta)|_{\varphi=0; \ \omega^*=0} = -(\partial \Phi/\partial \delta)/(\partial \Phi/\partial \omega) < 0 \), there is a threshold value \( \delta = \bar{\delta} \) at which \( \varphi_{IO} = \varphi_X \). Therefore, we have \( \varphi_{IO} < \varphi_X \) for \( \delta < \bar{\delta} \) and \( \varphi_{IO} \geq \varphi_X \) for \( \delta \geq \bar{\delta} \), as outlined in Proposition 3.

Finally, as \( s_X \) and \( s_I \) are increasing functions of \( \omega^*/\omega \), we consider the effects of
changes in $\varphi$ and $\delta$ on $s_X$ and $s_I$ using

$$
\frac{1}{\Theta_1} \frac{d(\omega^*/\omega)}{d\varphi} = (1 + \omega^*) \left( \frac{n}{\omega^*} + \Theta_2 \right),
$$

$$
\frac{1}{\Theta_1} \frac{d(\omega^*/\omega)}{d\delta} = -\frac{(1 - \varphi)n}{(1 - \delta)^2(1 + \varphi)\omega^*} + \frac{2(1 - \alpha\rho)n}{(1 + \delta)(1 + \omega^*)} - \frac{(1 - \varphi)\Theta_2}{(1 - \delta)^2(1 + \varphi)(\rho + \lambda)},
$$

where $\Theta_1 = -((\omega^* - \omega^*)^2/\omega^*)/(\alpha\rho(b - b^*)(1 + \omega^*)\omega^3|J_2|) > 0$ and $\Theta_2 = 4(1 - \alpha\rho)(\omega k l_F - \rho - \lambda)/((\rho + \lambda)(1 + \omega^*)^2a l_F) > 0$. Therefore, $dS_X/d\varphi < 0$ and $dS_I/d\varphi < 0$. In addition, a sufficient rise in $\delta$ shifts the $S_X$ and $S_I$ curves upwards in Figure 3 for $\varphi < \varphi_B$.

**Appendix D: Propositions 4 and 5**

First, the effects of changes in $\omega k$ on market entry and productivity growth are

$$
\frac{dn}{d(\omega k)} = -\frac{n(\rho + \lambda)}{l_F l_W k - \rho - \lambda},
$$

$$
\frac{dg}{d(\omega k)} = \frac{(\sigma - 1)\gamma l_F}{\sigma(\nu - \rho - \lambda)}.
$$

Then, the following are used with $|J_2| < 0$ to obtain Propositions 4 and 5:

$$
\frac{d(\omega k)}{db} = \frac{2(1 - \alpha\rho)(\omega k l_F - \rho - \lambda)\omega k(\omega^* - \omega)^2}{\alpha(\rho + \lambda)(b - b^*) l_F (1 + \omega^*)^2 \omega^2 |J_2|} < 0,
$$

$$
\frac{d(\omega k)}{d\varphi} = \frac{2(1 + \delta)(1 - \alpha\rho)(\omega k l_F - \rho - \lambda)\omega k(\omega^* - \omega)^3}{\alpha^2(\rho + \lambda)l_F (1 - \delta)(1 + \varphi)^2(b - b^*)(\omega + \omega^*)(1 + \omega^*)^2 \omega^2 |J_2|},
$$

$$
\frac{d(\omega k)}{d\delta} = -\frac{2(1 - \alpha\rho)(\omega k l_F - \rho - \lambda)\omega k}{\alpha(\rho + \lambda)(1 + \omega^*)^2(\omega + \omega^*) l_F |J_2|} \left( \frac{2\omega^*}{\omega} + \frac{(1 - \delta)^{-2}(1 - \varphi)(\omega^* - \omega)^3}{\alpha(1 + \varphi)(b - b^*)(1 + \omega^*)\omega^2} \right).
$$

**References**


Growth 6, 5-37.


