International Trade and Financial Integration under Technological Specialization and Uncertainty

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Abstract

This paper presents a general equilibrium framework for analyzing the interaction of international trade and financial markets. Trade and investment allow to exploit the productivity gains from specialized production and to diversify the inherent risk of such specialized production across countries. If financial markets are integrated trade responds more sensitive to trade costs and, vice versa, risky international investment are more attractive under integrated goods market. International integration and financial liberalization shift investment from technologies with a low productivity to specialized technologies which are more productive but risky, and they lead to a higher variance in the trade accounts. Distinguishability of location specific environments and an accurate measurement of their risks are the fundamental limits of specialization and gains from diversification. If limited knowledge, uncertainty or market imperfections lead to erroneous risk assessment in financial markets, in particular if they neglect correlated risk, then globalization of goods and financial market generate welfare shocks rather than diversification gains.

Keywords: Trade and finance, international specialization, international diversification, trade imbalances, globalization under uncertainty

JEL classification: F10, F21, F30

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1 Introduction

Since Adam Smith’s classical work on the “Wealth of Nations” it has become a doctrine of economic reasoning that specialization is the engine of economic growth; and David Ricardo made international specialization a cornerstone in demonstrating the gains from trade. At the same time, however, specialization can also be a source of risk. The neglect of these risks is one of the reasons why a considerable gap has emerged between the positive evaluation of globalization in academic research and its more critical perception by the general public. For instance, results from polls show that the majority of the interviewees have negative sentiments towards globalization. Thereby, the perception of the increasing risk involved in deeper economic integration plays a prominent role for explaining rising skepticism of the general public about the benefits of globalization (cf. Arpe et al., 2012).

The aim of this paper is to close the gap in the assessment of globalization and its risks inside and outside academic circles by analyzing international specialization in a general equilibrium framework with technological uncertainty. For this purpose, we enrich a simple model of international trade with a financial market that allocates the savings of risk-averse agents on investment projects, which differ in their risk attributes. Savings can be invested at home or abroad. Investing abroad leads to an export of resources that are used as an input in foreign production now; and it causes an import of goods that are manufactured with these resources in the future. Thus, globalization in our model captures both product and financial market integration.

The risk level of an investment project depends on the degree of specialization of the underlying technology. In each country there exists, in addition to a low-return robust technology, a specialized risky technology, which offers a high expected return. The robust technology delivers the same low return under all possible circumstances. Specialized technologies are sensitive to the circumstances under which production takes place. These circumstances are exogenous and subject to uncertainty at the time when investments in a particular technology are made. We summarize these circumstances under the notion of environment or state of nature. The higher the degree of specialization is, the stronger
is the dependence of a technology on the specific environment it is designed for. Higher
degrees of specialization increase the return of investment if the environmental conditions
support the technology, but they also reduce the probability that the required conditions
are realized. This captures in a simple way the fundamental trade-off between expected
return and risk that is prevalent in most economic transactions.

The environmental conditions differ across different locations (countries). This pro-
vides a reason for agents to include domestic and foreign projects in their investment
portfolio. With the cross-border investment they can diversify the risks of technological
specialization. The investment decision of consumers generates trade in two time periods.
In the investment period, there is an outflow of goods to provide the capital input for for-
ign production projects. In the consumption period, domestic investors are remunerated
if the foreign project was successful, leading to an inflow of goods. Hence, the incentives
of agents to diversify the risks of investment in the international financial market generate
exports and imports in the goods market, with the specific motive of trade being to ex-
loit the gains from technological specialization and to diversify its risks at the same time.
Taking stock, we set up a model with intertemporal trade of goods and a financial market
that offers instruments to allocate savings on state-contingent investments in domestic
and foreign markets. To keep the analysis tractable, we focus on the case of symmetric
countries.

A fundamental limit to specialization and diversification comes from the nature of
uncertainty and the limited knowledge about possible future events. Specialization to
different environments is only possible to the extent that the environments are clearly
distinguishable from each other; and diversification of risks by state-contingent invest-
ments is only possible for distinguishable states. Diamond (1967) pointed to this fact
in his seminal paper on asset markets and technological uncertainty. In our model, this
implies that countries are only distinguishable to the extent that they offer technologies,
which differ in the environments they are designed for. This distinguishability limits the
number of trading partners, which can contribute to the gains from specialization in our
setting.
Shedding light on the interaction of international product and financial markets is the main purpose of our analysis. Thereby, the interaction is two-way. Lower trade barriers increase the expected return on foreign investment and therefore trigger a reallocation of resources towards foreign investment projects. Moreover, since in addition to the local robust technology agents can also opt for state-contingent investments at home and abroad, a decline in trade costs induces agents to shift their portfolio from robust towards state-contingent investment projects. The reason is that lower trade costs expand the possibilities to diversify the risks of state-contingent technologies, rendering the investment into risky projects more attractive. The financial sector in our model has the crucial role of providing insurance opportunities against risks from technological specialization by offering state-contingent securities. Feasibility and attractiveness of these securities depend on financial transaction costs, which reflect productivity in the financial industry, imperfections in the financial market and the degree of international financial integration. Thus, trade costs and financial frictions jointly determine the possibilities of diversifying technological risk by international investment and trade.

Whereas financial markets provide the instruments to diversify the risk of investment and therefore make the gains from technological specialization accessible in a first place, they are also a source of imbalances of external accounts. As outlined above, improved possibilities of risk diversification induce agents to shift their portfolio towards risky investment and this increases trade imbalances. In the symmetric world considered in this paper, trade is balanced by construction in the investment period. However, in the consumption period trade flows are state-contingent. Countries whose risky technologies are productive under the realized conditions end up being exporters. Countries whose risky technologies are not productive will be importers – provided that at least one of the foreign technologies is successful. Hence, even though the condition of market clearing implies that trade accounts are balanced in terms of expectations, there are imbalances in the consumption period. These trade imbalances are the higher, the more integrated financial and product markets are.\textsuperscript{1}

\textsuperscript{1}In contrast, current accounts are balanced in both the investment and the consumption period. In the investment period, the symmetry of countries implies balanced current accounts due to balanced

4
The link between financial integration and trade imbalances is a novel feature of our model, which deserves further exploration. In Figure 1, we report evidence on this link for OECD countries and display for this country sample the development of international debt securities, international trade flows, and the variance of bilateral trade imbalances in current USD over the period of 1990-2011, combining information from the International Bank of Settlement (BIS) and the OECD STAN Database.²

Figure 1 provides strong empirical support for the main mechanisms of our model. First of all, it documents a clear positive correlation between the volume of international trade flows and the volume of international debt securities. According to our model, this is because making the gains from specialization and diversification accessible requires both financial transactions and goods trade. The correlation coefficient between international trade accounts. In the consumption period, the trade of goods is mirrored in international factor income flows, again establishing a balanced current account.

²The BIS provides information on debt but not equity securities. Furthermore, we report data on international debt securities instead of data on total debt securities, because the former is available for more countries over a longer time horizon and it puts international transactions into the focus.
trade and international debt securities is very high and amounts to 0.97. Second, the figure also shows a clear positive correlation between the volume of international debt securities and the variance of bilateral imbalances. According to our model, this pattern exists because economic integration expands the possibilities to diversify the risks of state-contingent technologies by making use of state-contingent instruments provided by the financial market. Better diversification possibilities induce agents to shift resources from robust towards state-contingent investment projects, thereby increasing the variance of output and trade imbalances in our setting. The correlation coefficient between international debt securities and the variance of trade imbalances amounts to 0.81 and is thus also very high.3

Since economic integration – either due to lower costs of international transactions for a given set of countries or an expansion of the set of countries with integrated financial and product markets – allows to diversify the risks involved in technological specialization, gains from globalization do exist in our model also from the perspective of a risk-averse representative agent. Still, one may ask if negative sentiments towards globalization have a rational economic basis. For an answer, we model the fundamental limits of uncertainty and distinguishability explicitly, by allowing for erroneous risk assessment. Accounting for neglected correlation between possible location-specific environments, we show that the integration of product and financial markets can lead to wrong expectations regarding the aptitude of international investment and trade to diversify the risk of state-contingent investment projects. In this case, globalization in the form of stronger specialization due to an expansion in the set of available state-contingent technologies does not only increase trade imbalances, but can also increase the variance in consumption, with negative consequences for the welfare of risk-averse agents.

The remainder of the paper is organized as follows. Section 2 relates our analysis to existing literature. Section 3 outlines the basic model. Section 4 characterizes the autarky equilibrium and Section 5 extends the model to cross-border investment and international trade. In Section 6, the international diversification of technological risks and its implica-

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3In Section 8, we present and discuss additional descriptive evidence that supports the mechanisms of our model.
tion for trade imbalances are examined. In Section 7, we consider two possible extensions of our model. On the one hand, we discuss expansions in the number of trading partners and available state-contingent technologies under fully integrated financial markets and free trade. On the other hand, we address the possible downsides of globalization due to uncertainty and erroneous risk assessment in financial markets. Section 8 provides further descriptive evidence for the mechanisms of our model and it presents a numerical exercise to quantify the impact of trade and financial liberalization on expected income and external imbalances. Section 9 concludes with a summary of the main results and a discussion of possible directions for future research.

2 Related literature

In modeling the relationship between technological specialization and risk, our analysis is closely related to Acemoglu and Zilibotti (1997), who analyze the diversification of risks in a model of growth in a closed economy. Falkinger (2014) additionally accounts for limitations following from the requirements that states must be distinguishable and their risk accurately measurable; yet he too does not address international diversification as a specific motive for trade that renders gains from specialization accessible to agents.

By emphasizing that the gains from specialization in open economies may come at the cost of additional risk, we build on a well-established literature that addresses the conditions under which the insights from traditional trade theory extend to settings with production uncertainty. In their pioneering work, Helpman and Razin (1978) point out that the insights from trade models with deterministic production remain valid under uncertainty if the higher variance of specialized production in open economies can be insured by international asset trade. In this literature, our paper is most closely related to Grossman and Razin’s (1984) analysis of international capital allocation under uncertainty. They consider a two-factor economy (with one or two production sectors) in a two-country world with integrated financial markets, in which both physical capital and goods are traded if the countries differ in their endowments or risk patterns. Risk is
modeled in the form of country-specific (multiplicative) productivity shocks. Helpman (1988) generalizes the analysis to an $n$-good, $m$-factor and $J$-country framework, keeping the same type of productivity shock.\(^4\) This early literature on trade and investment under uncertainty abstracts from trade impediments and barriers to international investment. This rules out the existence of a home bias in international investment, a stylized fact that is well documented and has received a lot of attention in the literature since the seminal work of French and Poterba (1991).\(^5\) Emphasizing the role of trade costs and financial barriers and studying their comparative static effects on capital allocation and external imbalances, we complement previous research on trade and investment in open economies.\(^6\)

Furthermore, studying the ability of international financial markets to diversify the risk of state-contingent investments relates our analysis to a strand of the macroeconomics and international finance literature that looks in particular on the role of financial market integration for output growth and consumption volatility (cf. Obstfeld, 1994; Tesar, 1995). Models in this literature build on the assumption that output levels of individual countries follow stochastic processes, which are not perfectly correlated. This implies that the combined output of two countries is less volatile than the individual output levels, so that acquiring mutual claims in output allows to smooth consumption compared to the autarky case.\(^7\) Yet, these studies do neither shed light on the interaction of trade costs and financial frictions as a key determinant of the international investment portfolio, nor do they analyze the consequences of the portfolio composition for external imbalances. Moreover, in our analysis the stochastics of the production process are explicitly modeled

\(^4\)More recently, the link between risk diversification and production specialization in open economies has also been addressed by empirical research. For instance, Asdrubali et al. (1996) and Kalemli-Ozcan et al. (2001, 2003) provide evidence on this link for the US and OECD countries. di Giovanni and Levchenko (2009) identify specialization according to the law of comparative advantage towards more volatile sectors as a major channel through which trade openness triggers higher volatility of output.

\(^5\)For a literature review, see Lewis (1999).

\(^6\)A few recent studies on uncertainty in open economies show that output is more volatile in developing than in developed countries (see Koren and Tenreyro, 2007, 2013; Krishna and Levchenko, 2013). Due to the symmetry assumption, this issue is beyond the scope of our analysis.

\(^7\)More recently, attention in this literature has shifted towards the international portfolio choice and its limits to smooth consumption in models that abstract from physical investment and production decisions (cf. Lewis and Liu, 2015; Callen et al., 2015). Azzimonti et al. (2014) point to the role of government debt for diversifying risk in international financial markets and lowering the volatility of consumption.
in a way, which accounts for the productivity risk of specialized technologies and therefore allows us to focus on the role of international technological specialization.

Finally, the increasing risk of workers to lose their jobs or experience wage cuts in the aftermath of globalization features prominently in the literature of trade and labor markets (cf. Davidson and Matusz, 2006; Egger and Kreickemeier, 2009; Helpman et al., 2010). This literature emphasizes the individual risk of sub-groups of agents, whereas our focus is on aggregate risk and macroeconomic imbalances. Gains from trade under uncertainty also play a role in the literature on multinational firms. For instance, Rowland and Tesar (2004) examine the diversification potential of multinationals, whereas Rob and Vettas (2003) and Conconi et al. (2013) point out that uncertainty interacts with the mode of market penetration.

3 Model

Let \( L = \{0, 1, ..., L\} \) be a set of locations (countries) in the world and let \( \Omega \subset \mathbb{R}_+^L \) be the set of possible global environments, where \( \omega = (\omega_1, ..., \omega_L) \in \Omega \) is a particular realization with \( \omega_l \) describing the environment realized in country \( l \).

Each country is endowed with capital stock \( K_l, \ l \in L \), that can be invested into production possibilities the performance of which is in general sensitive to environment. Investments are taking place under uncertainty, that is, they have to be made before the relevant environment is realized.\(^8\) For an example, we can interpret \( \omega \) as weather conditions and think of \( K \) as stock of corn that can be treated in different ways and used as variants of seeds, which are more or less specialized to weather conditions.

Production technologies are assumed to be linear. That is, if \( I_s \) units of capital are invested in technology \( s \) of country \( l \), then

\[
x_s(\omega) = a_s(\omega)I_s
\]  

\(^8\)This is in contrast to flexible input use ex post, as for instance considered by Caselli et al. (2014), who focus on the relationship between output volatility and the tradability of inputs in the spot market.
units of output are generated under environment $\omega$.

Each country hosts two types of technologies, $s \in \{0, 1\}$, which differ in their risk and productivity profile. Technology $s = 0$ is robust to environmental variation but has low productivity. Formally,

$$a_0(\omega) = a_0 \quad \text{for all } \omega \in \Omega.$$  

(2)

Technology $s = 1$ is highly productive under ideal conditions but fails to deliver output in non-ideal environments. Formally,

$$a_1(\omega) = \begin{cases} a_H & \text{if } \omega \in \Omega^+_l \\ 0 & \text{otherwise} \end{cases}$$  

(3)

where $\Omega^+_l = \{\omega \in \Omega \mid \omega_l \text{ is ideal for } s = 1\}$. Define

$$\pi_l \equiv \prob{\omega \in \Omega^+_l} \quad \text{and} \quad \bar{\mu} \equiv \prob{\omega \in \Omega^+_l \text{ for some } l \in L} \leq 1.$$

For a meaningful analysis, $\pi_l a_H > a_0$ is assumed throughout the paper. Technologies are specialized to locations. A high-productive technology for which ideal conditions exist in $l$ has no ideal conditions in $l'$. In other words, if the high-productive technology of $l$ works whenever it works in $l'$ and vice versa, then the two locations are considered as one region.

**Assumption 1.** For $l \neq l' \in L$, $\Omega^+_l \cap \Omega^+_l = \emptyset$ and $\pi \left( \bigcup_{l \in L} \Omega^+_l \right) = \sum_{l \in L} \pi_l = \bar{\mu}$.

This assumption expresses the idea of international diversification possibilities. It requires that the considered locations are complementary with respect to their risky technologies. How many complementary locations satisfy Assumption 1 depends not only on the environmental conditions but also on the degree of technological specialization. If technical progress allows to target high-productive technologies to very specific conditions, the set of locations satisfying Assumption 1 increases. The effects of rising technological specialization on the interaction between financial integration and trade are considered in
Section 7. For the moment, however, set \( L \) is fixed.

The extent to which the potential gains from specialization and diversification can be exploited depends on trade costs and financial instruments. While trade costs will be modeled as iceberg costs we use the investment model of Falkinger (2014) (which in turn is related to Acemoglu and Zilibotti, 1997) for modeling the role of financial intermediation. The next section illustrates this role for the autarky case.

4 Autarky

Endowment \( K \) can be invested in two types of assets provided by the financial sector: a fixed-interest asset with pay-off \( r \) and a state-contingent security that pays off only if ideal conditions are realized for production. Formally, the pay-off of the state-contingent security is

\[
R(\omega) = \begin{cases} 
R & \text{if } \omega \in \Omega^+ \\
0 & \text{otherwise.}
\end{cases}
\]

(Country index \( l \) is dropped in this section.)

The capital invested in the safe asset is used for financing investments in the robust technology \((s = 0)\) whereas the saving in the contingent security is used for investments in the risky technology \((s = 1)\). We assume that the financial sector absorbs transaction cost \( \tau \) per unit of risky investment. No such cost is imposed on robust investment. Hence, we have

\[
r = a_0, \quad R = a_H(1 - \tau).
\]

With these pay-offs a portfolio of safe investment, \( I_0 \), and risky investment, \( I_1 \), generates the following income stream:

\[
X(\omega) = \begin{cases} 
a_0I_0 + a_H(1 - \tau)I_1 & \text{if } \omega \in \Omega^+ \\
a_0I_0 & \text{otherwise.}
\end{cases}
\]  

Agents are assumed to have logarithmic preferences. Their portfolio choice is thus deter-
mined by the program

\[
\max_{I_0,I_1} \pi \log [rI_0 + RI_1] + (1 - \pi) \log [rI_0] \quad \text{s.t.} \quad I_0 + I_1 \leq K.
\]

For \( \pi R > r \), the solution of this program is\(^9\)

\[
\begin{align*}
I_0 &= \frac{(1 - \pi)}{1 - r/R} K, \\
I_1 &= \frac{\pi - r/R}{1 - r/R} K.
\end{align*}
\]

(5)

While \( \pi < 1 \) by assumption, an interior solution for \( I_1 \) requires in addition \( \pi a_H (1 - \tau) > a_0 \). That means, transaction costs in the securities market must not be too high. Formally, there exists a threshold

\[
\bar{\tau} \equiv 1 - \frac{a_0}{\pi a_H}
\]

(6)

so that \( I_1 = 0 \) if \( \tau \geq \bar{\tau} \) and \( I_1 > 0 \) otherwise.

Substituting (5) into (4), we obtain for \( \tau < \bar{\tau} \):

\[
X(\omega) = \begin{cases} 
\pi RK & \text{if } \omega \in \Omega^+, \\
\frac{1 - \pi}{1 - r/R} rK & \text{otherwise.}
\end{cases}
\]

(7)

In contrast, if \( \tau \geq \bar{\tau} \): \( X(\omega) = rK \) for all \( \omega \in \Omega \). Since \( \pi R > r \) by assumption, we have: \( \pi RK > rK > \frac{1 - \pi}{1 - r/R} rK \).

5 International integration

Assume that country \( H \) opens up to another country \( l \neq H \). In autarky the income streams generated by the optimal investment portfolio are given by (7). Thus, under the

\( \text{Substituting } I_1 = K - I_0 \) and differentiating with respect to \( I_0 \), we get the first-order condition

\[
\frac{\pi (R - r)}{rI_0 + RI_1} = \frac{1 - \pi}{I_0}
\]

which together with \( I_0 + I_1 = K \) gives (5).
assumption that $\tau < \bar{\tau}$, if $\omega \in \Omega^+_H - \Omega^+_l$ then $X_H(\omega) = \pi RK > X_l(\omega) = \frac{1-\pi}{1-\tau/R} rK$. (Note that $\tau < \bar{\tau}$ implies $\pi R > r$.) Vice versa, $X_l(\omega) > X_H(\omega)$ if $\omega \in \Omega^+_l - \Omega^+_H$. The countries have specialization advantages in different environments. No specialization advantage would exist for $\omega \in \Omega^+_l \cap \Omega^+_H$. Assumption 1 excludes such overlap. It reflects our focus on the diversification advantages under technological uncertainty. Such advantages require that the potential trading partners are sufficiently different in terms of the ideal environment for their high-productive technologies.

Diversification advantages can only be exploited by trade, if cross border investment in specialized risky technologies is feasible. This requires financial market integration. Then investors in country $H$ have, in addition to the state-contingent asset with pay-off $R_H(\omega)$, also access to foreign state-contingent assets with pay-offs $R_l(\omega)$, $l \neq H$. Suppose that the degree of financial integration allows cross-border investment from $H$ to $n(\leq L)$ different countries. We call $n$ the degree of financial market integration.

Investing resources abroad and consuming the pay-off from this investment requires that resources are shipped from home to foreign and final output is shipped from foreign to home. This may involve trade costs. Let $t$ be the total trade costs involved in these transactions, per unit of investment. Then, the net pay-off to one unit of capital invested in $l \neq H$ is given by

$$R_l^n(\omega) = \begin{cases} (1-t)R & \text{if } \omega \in \Omega^+_l, \\ 0 & \text{otherwise.} \end{cases}$$

In sum, under integrated financial markets of degree $n$, the representative agent in $H$ can choose between investment in the robust technology at home, $I_0$, the risky technology at home, $I_H$, and $n$ risky technologies abroad, $(I_l)_{l=1,\ldots,n}$. The optimal portfolio choice in $H$ changes thus to the program:

$$\max_{I_0, I_H, (I_l)_{l=1,\ldots,n}} EU = \pi_H \log X_H^+ + \sum_{l=1}^n \pi_l \log(X_l^+) + (1-\mu) \log(r I_0)$$
subject to

\[ I_0 + I_H + \sum_{l=1}^{n} I_l \leq K, \quad X_H^+ = rI_0 + RI_H, \quad X_l^+ = rI_0 + (1-t)RI_l \] (8)

with \( \mu \equiv \pi_H + \sum_{l=1}^{n} \pi_l \leq \bar{\mu} \).

In the appendix we provide a full characterization of the solution of this program. Two determinants are important for the portfolio structure: \( \rho \equiv R/r = a_H(1-\tau)/a_0 \) and \( \alpha \equiv 1 + n/(1-t) \). The first is the relative return of the risky compared to the robust technology in the absence of uncertainty and trade costs. The second is the number of trading partners to which a country is integrated, discounted by trade costs. Furthermore, since agents are risk-averse, the expected return of the state-contingent technology must exceed the return of the robust technology in order to render risky investment attractive in the first place. This requires \( \mu \rho > \alpha \).\(^{10}\) For \( \mu < 1 \) the conditions for positive investment in the risky technology in location \( l \in \{H, 1, ..., n\} \) are:\(^{11}\)

\[ \pi_H > \frac{1 - \mu}{\rho - \alpha} \quad \text{and} \quad \pi_l > \frac{1 - \mu}{(1-t)(\rho - \alpha)} \quad \text{for} \quad l \in \{1, ..., n\}. \] (9)

If the conditions for an interior equilibrium with diversification of investment over all trading partners are fulfilled, the optimal investment portfolio is given by

\[ I_0 = \frac{1 - \mu}{1 - \alpha/\rho} K, \]
\[ I_H = \left[ \frac{\pi_H - \frac{1 - \mu}{\rho - \alpha}}{K}, \right] \] (10)
\[ I_l = \left[ \frac{\pi_l - \frac{1 - \mu}{1-t \rho - \alpha}}{K}, \right] \]

\(^{10}\)For symmetric countries \((\mu = \pi(1+n))\) and zero trade costs, inequality \( \mu \rho > \alpha \) reduces to \( \pi \rho > 1 \), which is the condition for risky investment in the closed economy. Trade costs increase \( \alpha \) and therefore reduce the attractiveness of investment in foreign risky technologies.

\(^{11}\)If \( \mu = 1 \), risks can be perfectly diversified in the international market and all capital is invested into the risky technology.
\[ l = 1, ..., n. \] Total risky investment is
\[ K - I_0 = \frac{\mu - \alpha/\rho}{1 - \alpha/\rho} K. \] (11)

For the comparative-static effects of trade costs on the investment portfolio, note first that \( \alpha \) is increasing in \( t \). Thus, the condition \( \alpha < \mu \rho \) for positive cross-border investment becomes more likely if trade costs decline. As soon as trade costs are low enough to make investment in a foreign country profitable, any further decline in trade costs increases the volume of capital that \( H \) invests to the foreign country. According to Eq. (10) \( I_l \) is decreasing in \( t \) – both through a direct channel and indirectly through \( \alpha \). Finally, a comparison of \( I_H \) and \( I_l \) shows that, ceteris paribus, (for \( \pi_l = \pi_H \)), the risky domestic technology is used more heavily than a particular foreign risky technology because of the trade cost. Interestingly, through the \( \alpha \)-channel a decline in \( t \) increases not only foreign investment but investment into the risky domestic technology, \( I_H \), as well – provided the risky technology is used at all. The reason is that lower trade cost allow more diversification, which provides “insurance” against technological risks. The following proposition summarizes the impact of trade costs on the investment portfolio of households.

**Proposition 1.** A decline in trade costs \( t \) increases the amount of capital invested abroad. Investment into the domestic risky technology rises as well, since lower trade costs allow for better international risk diversification.

**Proof.** Analysis in the text. \( \square \)

According to Proposition 1 product market integration leads to a reallocation of capital towards more specialized technologies, implying a higher degree of international specialization. The interaction between the portfolio decision of agents and trade costs, \( t \), exists because international investment and consumption smoothing require the shipment of goods across borders. Financial development, as captured by a fall in \( \tau \), increases the return to risky technologies net of financial transaction costs, \( R = A_H(1 - \tau) \), relative to productivity \( r \) of the robust technology. Therefore, financial development has similar
effects as product market integration regarding the reallocation of capital from robust towards state-contingent investment projects and it causes an expansion of international trade. The existence and relevance of such effects are well documented by empirical research. For instance, Acharya et al. (2011) provide evidence that deregulation of bank branching regulations in the US have significantly lowered state-level output volatility due to capital reallocation. Michalski and Ors (2012) show for the US that financial deregulation which enables inter-state banking increases bilateral trade between the liberalizing states. Also the findings of Becker and Hoffmann (2006) that intra-state risk-sharing within the US is higher than international risk-sharing is evidence that financial frictions matter for global diversification possibilities.

The gains from international integration can be seen by calculating $X^+_H$ and $X^+_l$ under optimally diversified investments. Substituting $I_H$ and $I_l$ from Eq. (10) in Eq. (8), we obtain

$$X^+_H = \pi_H RK,$$

$$X^+_l = (1 - t)\pi_l RK, \quad l = 1, \ldots, n$$

$$X_0 \equiv rI_0 = \frac{1 - \mu}{\rho - \alpha} RK \ (< X^+_l).$$

Using this in EU, we see that adding a country $l = n + 1$ to the set of countries integrated with $H$, increases EU by the term $\pi_{l+1} \log(1 - t)\pi_{l+1} RK - \pi_{l+1} \log X_0$, which is positive since $(1 - t)\pi_{l+1} > \frac{1 - \mu}{\rho - \alpha}$ if $I_{l+1} > 0$. (The effect of $\pi_{l+1}$ which works through the adjustment of $I_H$, $I_l$ and $I_0$ to the rise of $\mu$ to $\mu + \pi_{l+1}$ can be neglected due to the envelope theorem.)

Probability $\mu$ measures the extent to which the international economy provides diversification opportunities by international investment and trade. The extent to which these opportunities are actually used, depends on $\tau$ as well as on $t$. The condition $\tau < \bar{\tau}$ is required for any risky investment, and trade costs must be sufficiently low to activate foreign investment. If these two conditions are fulfilled, then, according to Eq. (10), an increase in $\mu$ shifts the investment portfolio away from the robust domestic technology to the specialized risky technologies employed at home or abroad. Since $(1 - t)(\rho - \alpha)$ is declining in both $t$ and $\tau$, not only declining trade cost, but also declining cost of financial
intermediation enhance international integration.

Now, $\mu = \pi_H + \sum_{i=1}^{n} \pi_i$ depends on the one side on $\pi_H$ and the average $\pi_i$, that is on the size of ideal environments for the feasible risky technologies. On the other hand, it increases with $n$, that is the degree of international financial integration. To disentangle the two determinants of $\mu$, we focus in the further analysis on symmetric countries with $\pi_i = \pi, l = H, 1, ..., n$. In this case, we have $\mu = (1 + n)\pi$. Therefore, the necessary condition $\mu < 1$ for $I_0 < K$ requires:

$$\pi < \frac{1}{1 + n}.$$  

Moreover, the sufficient conditions $\alpha < \mu \rho$ for $I_0 < K$, and $\pi_H > \frac{1 - \mu}{\rho - \alpha}$ and $\pi_l > \frac{1 - \mu}{(1 - l)(\rho - \alpha)}$ for $I_H > 0$, $I_l > 0$, become:\

$$\tau < \tilde{\tau} \quad \text{(equivalent to } \pi \rho > 1) \quad \text{and} \quad t < \tilde{t} \equiv \frac{\pi \rho - 1}{\pi (\rho - 1)}. \quad (13)$$

An increase in $\pi$ means that in each country the high-productive technology works under a broader range of environmental conditions. Comparative static analysis with respect to $\pi$ therefore shows the effects of technical change towards more robust high-productive technologies. We discuss the effects of such technical change on international integration in Section 7. Here, we focus on the degree of international integration measured by $n$. Using $\alpha = 1 + n/(1 - t)$, $\pi_H = \pi_l = \pi$ and $\mu = (1 + n)\pi$ in Eq. (10), we have for $\pi \rho > 1$ and $\pi < 1/(1 + n)$:

$$I_0 = \frac{\rho [1 - (1 + n)\pi]}{\rho - \frac{n}{1 - t} - 1} K,$$

$$I_H = \frac{\pi (\rho - \frac{nt}{1 - t}) - 1}{\rho - \frac{n}{1 - t} - 1} K, \quad (10')$$

\[12\text{Note first that } \tau < \tilde{\tau} \text{ is equivalent to } \pi \rho > 1. \text{ (Use (6) and } \rho = \frac{\bar{a}}{\tau} = \frac{(1-\tau)\mu}{\alpha(1-\tau)}). \text{ Note next that, for } \pi_H = \pi_l = \pi, \pi_l > \frac{1 - \mu}{(1 - l)(\rho - \alpha)} \text{ is equivalent to } t < \tilde{t} \text{ and that } \pi > \frac{1 - \mu}{(1 - l)(\rho - \alpha)} \text{ implies } \pi_H > \frac{1 - \mu}{\rho - \alpha}. \text{ Finally, } t < \tilde{t} \text{ and } \pi \rho > 1 \text{ are sufficient for } \alpha < \mu \rho \text{ as the following argument shows: } t < \tilde{t} \text{ is equivalent to } \alpha < \tilde{\alpha} \equiv 1 + \frac{n}{(1 - t)(\rho - \alpha)} \text{. Thus } \alpha < \mu \rho \text{ if } \tilde{\alpha} < \mu \rho. \text{ The latter condition is equivalent to the inequality } (1 - \pi) + n\pi(\rho - 1) < (1 - \pi)(1 + n)\pi \rho \text{ which can be rewritten as } 1 - (1 + n)\pi < \pi \rho[1 - (1 + n)\pi] \text{ and reduces to } 1 < \pi \rho. \text{ (Recall that } \mu = (1 + n)\pi < 1.)\]
\[ I_l = \frac{\pi (\rho - 1) - \frac{1 - \pi}{1 - t}}{\rho - \frac{n}{1 - t} - 1} K \equiv I^*, \; l = 1, \ldots, n. \]

As a first conclusion we see that, for a given \( \pi \), investment \( I_0 \) in the domestic technology – which is robust towards environmental variations though poorly productive – is declining in \( n \). At the same time, investments in the high-productive risky technologies – both abroad and at home – are rising. The domestic effect is however less pronounced than the one on foreign investment. A second observation, following from differentiating \( I_l \) with respect to \( t \) and \( \tau \), is that declining trade costs and diminishing costs of financial intermediation are substitutes in the promotion of investment and trade with foreign countries. Investment in foreign, \( I_l \), is rising in \( \rho^n \equiv (1 - t)(\rho - 1) \) since \( \frac{1 - \pi}{\pi} > n \) because of \((1 + n)\pi < 1\). Moreover,

\[
\frac{\partial I_l}{\partial (1 - t)} = (\rho - 1) \frac{\partial I_l}{\partial \rho^n}, \quad \frac{\partial I_l}{\partial (1 - \tau)} = \frac{1 - t}{1 - \tau} \frac{\partial I_l}{\partial \rho^n}.
\tag{14}
\]

Thus, a decline of \( \tau \) – due to financial liberalization or cost-decreasing progress in the financial industry – can compensate for high trade costs (and vice versa) in promoting investment and trade with a foreign country. According to (14), ceteris paribus (at \( t = \tau \)), financial liberalization has a stronger effect on foreign investment than trade liberalization. Finally, investment in the risky domestic technology, \( I_H \), is also increasing if financing costs \( \tau \) or trade costs \( t \) decline. Increased possibilities to hedge the risk of the high-productivity technologies by foreign trade and investment induce more risky investment at home too. The following proposition summarizes the role of \( n \) and the interaction between trade costs and financial frictions for the structure of the investment portfolio.

**Proposition 2.** A higher degree of international specialization \( n \) raises investment into foreign risky technologies. Also domestic risky investment increases, though to a lesser extent. For a given \( n \), lower financial costs (\( \tau \)) and trade costs (\( t \)) complement each other

---

13 \( \partial I_0 / \partial n < 0 \) if \( 1 - \pi < \pi(\rho - 1)(1 - t) \), which is equivalent to \( t < \bar{t} \).

14 For the effect of \( n \) on \( I_H \), note that \( \partial I_H / \partial n > 0 \) is equivalent to the inequality \( \pi \left( \rho - \frac{n}{1 - \tau} \right) - 1 > \pi t \left( \rho - \frac{n}{1 - \tau} - 1 \right) \) which reduces to \( t < \bar{t} \left( \frac{n}{\pi(\rho - 1)} \right) \).

15 For \( \partial I_H / \partial \tau < 0 \) note that \( \partial I_H / \rho > 0 \); and \( \partial I_H / \partial t < 0 \) is equivalent to \((1 + n)\pi < 1\).
in promoting foreign investment and international trade and shifting portfolios towards risky technologies.

Proof. Analysis in the text.

6 International diversification of technological risks and trade imbalances

The diversification of technological risks analyzed in this paper involves international trade of goods and international flows of capital income. Suppose that agents live two periods and capital fully depreciates after one period. At the beginning of each period a new generation is born and endowed with $K$. What are the consequences of integration for the external accounts?

Consider a country $i$ that shares integrated goods and financial markets with $j \in \{1, \ldots, n\}$ foreign countries. Let $I_{i,t}^j$ be investment of country $i$ in country $j$ in period $t$. $I_{i,t}^F \equiv \sum_{j=1}^{n} I_{i,t}^j$ is then total outward investment of country $i$ in period $t$ and $I_{F,t}^i \equiv \sum_{j=1}^{n} I_{i,t}^j$ is the flow of foreign investment into the country. This gives us for the country’s net-export of investment goods:

$$TA_{i,t}^I = I_{i,t}^F - I_{F,t}^i.$$

The investment flows generate in period $t + 1$ the following external account of capital incomes:

$$Z_{i,t+1} = \begin{cases} 
-RI_{F,t}^i & \text{if } \omega \in \Omega_i^+ \\
RI_{i,t}^j & \text{if } \omega \in \Omega_j^+, j = 1, \ldots, n \\
0 & \text{otherwise.}
\end{cases}$$

The incomes are spent on consumption goods in the market in which they are generated – which is the country with a positive output shock from the technology specialized to
the country. Thus, the country’s net export of consumption goods in period \( t + 1 \) is

\[
TA_{t,t+1}^c = \begin{cases} 
RI_{F,t}^i & \text{if } \omega \in \Omega_i^+ \\
-RI_{i,t}^j & \text{if } \omega \in \Omega_j^+, j = 1, \ldots, n \\
0 & \text{otherwise.}
\end{cases}
\]

Collecting the different components within one period, we obtain for the trade account in period \( t \):

\[
TA_{t,t} = I_{i,t}^F - I_{i,F,t}^i + \begin{cases} 
RI_{F,t-1}^i & \text{if } \omega \in \Omega_i^+ \\
-RI_{i,t-1}^j & \text{if } \omega \in \Omega_j^+, j = 1, \ldots, n \\
0 & \text{otherwise,}
\end{cases}
\]

which is state-dependent and generically unbalanced. By comparison, the country’s current account is state-independent:

\[
CA_{t,t} = TA_{t,t} + Z_{t,t} = I_{i,t}^F - I_{F,t}^i.
\]

In each period, the net-foreign asset position is fully determined by the cross-border investment flows within the period.

For assessing the effect of international diversification on external accounts in the long-run, we focus on symmetric countries with \( \pi_i = \pi_j = \pi \) and \( I_{i,t}^j = I_{j,t}^i = I^* \). Then the current account is balanced in each period. The trade account, however, is given by the following random process:

\[
TA = \begin{cases} 
nRI^* & \text{with probability } \pi \\
-RI^* & \text{with probability } n\pi \\
0 & \text{otherwise.}
\end{cases}
\]

(15)
Calculating expectation and variance of this process, we have

\[
\begin{align*}
E[TA] &= 0 \\
\text{VAR}[TA] &= n\mu(RI^*)^2, \quad \mu = (1 + n)\pi. 
\end{align*}
\]  

(16)

Hence, diversification of technological risks by international trade and investment balances trade accounts on average but generates high bilateral imbalances. The variance of these imbalances rises with the degree of international integration, ceteris paribus, that is for any given level of cross-border investment flows \(I^*\). Recalling from the previous section that \(I^*\) is rising in \(n\), we conclude that the direct variance-increasing effect of \(n\) is reinforced by the positive effect of \(n\) on the volume of cross-border investments. A similar indirect effect on the imbalances of trade accounts emerges if, for a given number of trading partners, the investment flows rise due to declining financial intermediation or trade costs. Since \(R = (1 - \tau)a_H\), the variance-increasing effect of declining financing costs through \(I^*\) is reinforced by a direct effect through the rising return on securities. Declining trade costs have only the variance-increasing effect through adjustments in \(I^*\).

In sum, increased diversification of technological risks by cross-border investment is accompanied by more imbalanced trade accounts. We have:

Proposition 3. If trade costs or financial transaction costs decline, the variance of a country’s trade account rises. If the country is integrated with a larger number of trading partners who are specialized in different technologies, the variance of trade accounts is higher.

Proof. Analysis in the text.

7 Two model extensions

In this section, we present two extensions of our model. In the first extension, we analyze technological innovation that either expands the set of available technologies or makes existing technologies more robust, whereas in the second extension we study limitations of gains from trade due to erroneous risk assessment in the financial market.
7.1 Technological innovation

Cross-border investments allow to exploit productivity gains from technologies that are specialized to particular environments, without increasing risk. Although there is risk in the performance of specialized technologies, the risk is diversified by international investment and trade. Where are the limits of such gains?

As shown, one limit is that trade costs have to be small enough to make foreign investment profitable. Let us assume that trade costs have fallen to zero. Then the other limit is that for all countries with specialized technologies state-contingent securities must be offered in the international financial market at transaction costs $\tau < \bar{\tau}$. Let us eliminate this limit, too, by assuming $\tau = 0$. Then the degree of international integration $n$ coincides with $L$, the number of locations with high-productivity technologies specialized to the specific locational conditions. Under such perfect integration, we have $\mu = \bar{\mu}$, with $\bar{\mu} = (1 + L)\pi$ in the case of symmetric countries. This points to a further, more fundamental limit of global diversification of technological risk: Not all environments are covered by specialized technologies, that means $\bar{\mu} < 1$ in the presented framework.\(^{16}\)

Innovation may lead to technologies which work in locations that so far have not been covered by high-productivity methods adapted to their specific conditions. In our framework this aspect can be captured by extending the set of specialized technologies $L = \{0, 1, ..., L\}$ and raising thus the measure $\bar{\mu}$ of environments covered by technologies targeted to the different locations. Alternatively, the measure $\bar{\mu}$ would also increase if, for a given set of specialized technologies, the technologies become more robust, that is, if the probability $\pi$ that a risky technology successfully delivers, rises. To focus on the effects of technological innovation, we assume perfect integration with zero trade costs and no financial frictions.

\(^{16}\)Only for $\bar{\mu} = 1$ all risk could be fully diversified and each country would achieve the income $X = RK$, whatever events were realized in the world.
Applying Eq. (10’) and assuming \( \pi \rho > 1 \), we have then for the optimal portfolio\(^{17}\)

\[
\begin{align*}
I_0 &= \rho \frac{1 - (1 + L)\pi}{\rho - (1 + L)} K \\
I_l &= \frac{\pi \rho - 1}{\rho - (1 + L)} K \equiv I_1, \ l = H, 1, ..., L.
\end{align*}
\]

The income stream generated by this portfolio is

\[
X(\omega) = \begin{cases} 
\pi RK & \text{with probability } \bar{\mu} = (1 + L)\pi \\
\frac{1-(1+L)\pi}{\rho-(1+L)} RK & \text{otherwise.}
\end{cases}
\]

Hence, technological innovations leading to a larger set of specialized technologies or more robust specialized technologies reduce the share of resources invested in domestic robust technologies and diversifies a larger share across countries. In this way, international investment and trade generate fully insured high returns over a larger measure of circumstances.\(^{18}\)

**Proposition 4.** In a fully integrated world without financial frictions and zero trade costs, technological innovation – expanding the set of specialized technologies or making specialized technologies more robust – reduces investment in the robust technology and increases investment in the risky technology at home and abroad.

*Proof.* Analysis in the text. \(\square\)

The following section addresses possible downsides of this favorable message.

### 7.2 Globalization under uncertainty

Global trade and financial markets allow a riskless use of risky high-productivity technologies, specialized to location-specific conditions around the globe. Trade volumes may become very large relative to GDP and the variance of trade accounts increases. But,

\(^{17}\)Note that \( \pi \rho > 1 \) is equivalent to \( \frac{\rho}{\pi + L} \rho > 1 \) and implies \( \rho > 1 + L \).

\(^{18}\)This corresponds to the diversification effect analyzed by Acemoglu and Zilibotti (1997) in a growth context.
expected GDP of all countries and expected utility of a country’s representative agent rise.

The fundamental assumption underlying this optimistic outlook on the benefits of global finance and trade is: that different environments can be clearly distinguished to a sufficiently fine degree so that they do not overlap across locations (Assumption 1). The further analysis focuses on the consequences of neglected overlap. For this purpose we assume that actually there is overlap – contrary to beliefs.

Assumption 2. For \( L \), let \( \{ \Omega^+_l \subset \Omega | l \in L \} \) be a family of environments with pairwise\(^{19} \) overlap so that for any \( l \) there exists \( l' \) with

\[
\pi \left( \Omega^+_l \cap \Omega^+_{l'} \right) = \pi_\Phi < \frac{\pi}{2},
\]

whereas \( \pi \left( \Omega^+_l \cap \Omega^+_{l'} \right) = 0 \) for all \( \tilde{l} \neq l, l' \).

Suppose that financial markets and investors’ beliefs are based on Assumption 1. For the case of symmetric countries that means, portfolio decisions are made on the basis of \( \bar{\mu} = (1 + L)\pi \) uncorrelated international investment opportunities with high returns. Actually, however, the measure of events covered by high-productivity technologies, \( \Omega^+ \equiv \bigcup_{l \in L} \Omega^+_l \), is

\[
\pi \left( \Omega^+ \right) \equiv (1 + L)\pi - \frac{(1 + L)\pi_\Phi}{2} \equiv \mu^* < \bar{\mu}.
\]

(19)

Moreover, the income streams generated by investments in high-productivity technologies in different countries are correlated. This has two severe consequences.

First, the negative shock that a country experiences if only the robust technology works occurs more frequently than believed and it is deeper than it would be under a lower degree of international integration. To see this, recall from (18):

\[
X_l(\omega) = \begin{cases} 
\pi_{RK} \equiv X^+ & \text{if } \omega \in \Omega^+_l, l \in L, \\
\frac{1-(1+L)\pi}{\rho-(1+L)} \pi_{RK} \equiv X^- & \text{if } \omega \in \Omega - \bigcup_{l \in L} \Omega^+_l.
\end{cases}
\]

(20)

\(^{19}\)To avoid integer problems we assume that \( 1 + L \) is a multiple of 2.
Hence, the negative shock occurs with probability $1 - \mu^* = 1 - \bar{\mu} + (1 + L)\pi_\Phi/2$ rather than probability $1 - \bar{\mu}$ as believed. The unexpected frequency of shocks is rising with the range of specialized international investments, $L$, and the size of the neglected overlap, $\pi_\Phi$. Moreover, $X^-$ is the lower, the larger is $L$.

Second, international investment and trade no longer provide full insurance of technological risks even under conditions in which high-productivity technologies work. For $\omega \in \Omega^+$, a country’s income is given by the random variable $X(\omega) = \sum_{l \in L} X_l(\omega)$, with conditional expectation $E[X(\omega)|\Omega^+] = \sum_{l \in L} E[X_l(\omega)|\Omega^+]$, where $E[X_l(\omega)|\Omega^+] = (\pi/\mu^*)X^+$ and $X^+ \equiv \pi RK$, according to (20). Substituting $\pi = \bar{\mu}/(1 + L)$, we have

$$E[X(\omega)|\Omega^+] = \frac{\bar{\mu}}{\mu^*}X^+. \tag{21}$$

The conditional variance of $X(\omega)$ is, by definition,

$$VAR[X(\omega)|\Omega^+] = \sum_{l \in L} VAR[X_l(\omega)|\Omega^+] + \sum_{l \in L} \sum_{l' \in L} COV[X_l(\omega), X_{l'}(\omega)|\Omega^+], \tag{22}$$

which, as formally shown in the appendix, can be written in the form

$$VAR[X(\omega)|\Omega^+] = \frac{\pi_\Phi}{2\pi} \left(1 - \frac{\pi_\Phi}{\pi}\right) \left(\frac{\bar{\mu}}{\mu^*}X^+\right)^2. \tag{23}$$

We see that the variance within $\Omega^+$ – which is zero if overlap is neglected, i.e. if $\pi_\Phi = 0$ and $\mu^* = \bar{\mu}$ – is positive since $\pi_\Phi < \pi$ and increasing in $\pi_\Phi/\pi$ for $\pi_\Phi < \pi/2$. To see this, note that $\bar{\mu}/\mu^* = [1 - \pi_\Phi/(2\pi)]^{-1}$ increases in $\pi_\Phi/\pi$.

**Proposition 5.** Erroneous assessment of risky technologies by the financial market undermines the diversification gains from international investment and trade. In particular, it generates income and consumption shocks.

**Proof.** Analysis in the text and derivation details in the appendix. \qed
8 From theory to numbers

The purpose of this section is twofold. On the one hand, we provide further supportive evidence for the link between goods market integration, financial liberalization and external imbalances, by looking at different measures of trade and financial flows as well as at barriers of these flows. On the other hand, we conduct a numerical exercise, in which we quantify the effects of lower trade and financial barriers on welfare and external imbalances.

8.1 Supportive evidence

In Figure 2, we present two variations of Figure 1. In the left panel, we adjust import figures to correct for aggregate trade imbalances between the OECD and the rest of the world so that the expected trade imbalance is zero, as suggested by our theoretical model. Accordingly, an increase in the variance of trade flows refers to a mean-preserving spread. This modification leaves our results more or less unaffected. In the right panel of this figure, international goods trade and international debt securities are expressed at constant 2005 prices. Thereby, we employ GDP figures from the World Development Indicators (WDI) database of the World Bank to construct a GDP deflator, which we then use for the price correction. Considering real instead of nominal variables lowers the correlation coefficient between international trade flows and international debt securities from 0.97 to 0.86 and the correlation coefficient between international debt securities and the variance of trade imbalances from 0.81 to 0.71. However, the respective correlations remain high.

In the left panel of Figure 3, we distinguish between exports of capital goods (solid line) and trade flows in household consumption goods (dashed line). Furthermore, we use international flows of consumption goods to compute the variance of trade imbalances. All the relevant information is again available from the OECD STAN database. This exercise allows an even closer look at the mechanisms of our model. The model distinguishes between investment and consumption period and it points to a positive correlation be-
Notes: International debt securities (black), variance of trade imbalances (blue), trade flows (red). *Left panel* – Trade figures are corrected for aggregate imbalances of the OECD vis-à-vis the rest of the world. *Right panel* – All variables are measured at constant 2005 USD.

Figure 2: Corrected imbalances; Real instead of nominal variables

twixbetween both the volume of exports of investment goods and the volume of trade (imports plus exports) in consumption goods. This hypothesis is well in line with the evidence for OECD countries. Furthermore, the model also predicts a positive correlation of both of these trade flows with the variance of consumption goods trade imbalances and the volume of financial transactions. Again, this theoretical relationship is well supported by the data. In the right panel of Figure 3, we show to what extent the variance in net international income flows is correlated with the volume of international debt securities and the volume of international trade flows. Thereby, we compute international income flows as the difference between gross domestic product (GDP) and gross national income (GNI) for each country in our sample. Data on GDP and GNI is obtained from the WDI database. According to our model, trade imbalances are counterbalanced by corresponding imbalances in international income flows. This seems well supported by the strongly positive correlation between the three curves in this panel.

The left panel of Figure 4, reports evidence on the development of total debt securities. Whereas the coverage for this data is lower than the coverage for international debt securities, the main insight of a positive correlation between financial transactions on the one hand and the volume of international trade flows or the variance of international trade imbalances on the other hand remains unaffected, when considering the broader measure of debt securities. In the right panel of Figure 4, we use data on outward foreign investment from UNCTAD as a measure for international financial flows. Whereas the
outcome of this modification may look not promising at a first glance, the correlation between international financial flows and the variance of trade imbalances is in fact even higher than in the benchmark model – with the correlation coefficient increasing from 0.81 to 0.83.

In the left panel of Figure 5, we report evidence on two measures of financial liberalization (red lines). Thereby, the dashed line presents the liberalization index of Abiad et al. (2008) and covers different aspects of liberalizations in the financial sector. The solid line displays the financial openness index of Chinn and Ito (2008), which has been recently updated online. In line with our theoretical model, we find evidence for a positive correlation between financial liberalization and the variance of trade imbalances. The correlation
coefficient amounts to 0.67 when considering the liberalization index proposed by Abiad et al. (2008), whereas it is with a value of 0.33 lower when considering the liberalization index proposed by Chinn and Ito (2008). In the right panel, we report evidence on the link between trade costs (red line), the volume of international debt securities and the variance of trade imbalances. Thereby, we consider changes in average, trade-weighted tariff rates from the WDI database (solid line) as well as recent estimates from gravity models on trade barriers (dashed line), as reported by Milner and McGowan (2013). As expected from our model, there is a negative correlation between trade costs and the variance in trade imbalances.

![Figure 5: Measures of financial liberalization and trade costs](image)


Taking stock, the descriptive evidence displayed in Figures 2-5 provides strong empirical support for our main theoretical hypothesis of a positive link between financial transactions, international trade flows and the variance of external imbalances.

### 8.2 A quantitative exercise

In a next step, we use part of the data presented above to parametrize our model. Thereby, we collapse Belgium and Luxembourg to a single economy and therefore set the number of trading partners in the OECD equal to \( n = 32 \). Furthermore, since our insights regarding the link between financial integration and trade imbalances builds on a setting

\[20\] We are grateful to Chris Milner for providing country-level estimates of trade barriers.
with symmetric countries, we do not make use of the cross-country variation in the data, but instead associate all members with the OECD average. More specifically, we use average tariffs at the country level as a proxy for trade costs. This information is obtained from the WDI and has, in comparison to broader estimates of trade costs as those reported by Milner and McGowan (2013), the advantage of being available for all OECD countries for the requested time period.\footnote{Since the WDI presents tariffs as a percentage increase of prices, we transform the respective rates to make them consistent with the trade cost variable in our model: $t = t_{WDI} / (1 + t_{WDI})$.} We weight the country-specific trade costs by trade flows to obtain an OECD average of trade costs for each year. We then compute a simple average over the period 1991-2011 to obtain a single measure of trade costs for our experiment. This gives $t = 0.030$ and allows us to compute $\alpha = 1 + n / (1 - t) = 33.978$. At some instances, we require information on financial barriers. To construct this variable, we divide the liberalization indices reported in Abiad et al. (2008) by the maximum possible level of financial deregulation and compute a GDP-weighted average of the ratio for the OECD countries in our data-set. Subtracting the resulting expression from 1 and computing a simple average for the period 1991-2005, we set $\tau = 0.093$.

Lacking detailed information on the returns to investment and on the probability that a state-contingent investment delivers, we construct the respective parameter values by accounting for the linear relationship between the standard deviation of trade imbalances and the capital stock in our model. In view of Eqs. (10') and (16), we have

$$STD[\tilde{T}A] = \nu \tilde{K}, \quad \nu \equiv \sqrt{n} \mu \left( \frac{\pi (\rho - 1) - \frac{1 - \pi}{1 - \tau}}{\rho - \alpha} \right) R, \quad (24)$$

where $\tilde{T}A = \sum_{i=1}^{32} TA_i / 32$ and $\tilde{K} = \sum_{i=1}^{32} K_i / 32$ are associated with the OECD averages of trade imbalances and capital stocks of its member countries. We compute the standard deviation of trade imbalances in consumer goods for the period 1991-2011, using bilateral trade data from the OECD STAN database. Furthermore, we employ capital stock data as reported by Berlemann and Wesselhöft (2014). They use the perpetual inventory method to estimate capital stock data for a large sample of 103 countries for the period 1970-
We use capital stock information for the period 1990-2010 and regress $\text{STD}[^\text{TA}]$ in period $s$ on the average capital stock of OECD countries in period $s-1$, using OLS. This gives a parameter estimate of $\hat{\nu} = 0.006$ with a standard error of 0.0001. The $R^2$ of this simple exercise amounts to 0.99.

We set the return to the robust technology is constant and equal to 1.02. Accounting for $R = r\rho$, we can employ the $\nu$-estimate from above to compute theory-consistent values of the relative return to state-contingent investment $\rho$ for different values of $\mu = (1+n)\pi$. The second row of Table 1 gives the thus computed parameter estimates of $\rho$. Due to a constant value of $\nu$, $\rho$ decreases in $\mu$. The third row reports the gross return on state-contingent investment in a good state: $a_H = \rho/(1-\tau)$, whereas the fourth row presents the expected net return per unit of risky investment in the international market relative to the return of the robust technology: $\mu\rho/\alpha$. This must be larger than one to render risky investment attractive for the risk-averse agents in our model. There are two counteracting effects. On the one hand, $\mu\rho/\alpha$ increases in the probability that at least one state-contingent technology is successful, $\mu$. On the other hand, it decreases because, for a given $\nu$, an increase in $\mu$ is associated with a decline in $\rho$. We find that the second effect dominates. This establishes the intuitive result that the risk premium that has to be paid by risky investment projects decreases with the diversification opportunities of investors.

With these parameter estimates at hand, we can now quantify the impact of trade

Note that the capital stock data is measured in constant 2000 USD, whereas our trade data is measured in constant 2005 USD. This could in principle lead to biased results due to omitted price changes. Therefore, we have also conducted trade data at constant 2000 prices, using the respective GDP deflator. Since our results turn out to be robust to this modification, we stick to trade data at constant 2005 USD throughout this exercise. Furthermore as a robustness check, we have also conducted the numerical exercise with capital stock data from the Penn World Tables, which is available at constant 2005 prices in USD. Although the capital stock information differs slightly in the two datasets, the insights from our numerical exercise remain more or less unaffected when considering this alternative measure of capital stocks.

This parameter value can be motivated by information from the US Treasury about real long-term interest rates on Treasury Inflation-Protected Securities (TIPS). The simple average of these interest rates for the years 2000-2011 amounts to 2.25 percent. Since it is well established that the long-term interest rates were higher in the 1990s than in the first decade of this century, we have accounted for higher values of $r$ in robustness checks. We have also checked lower values of $r$. It turns out that variations in $r$ do not change the insights from our numerical analysis in a qualitative way. Therefore, we do not report the respective results from the robustness checks here, but provide them upon request.
Table 1: Estimates of $\rho$ and $\mu \rho/\alpha$ for different values of $\mu$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0.05</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>683.20</td>
<td>341.08</td>
<td>136.22</td>
<td>68.04</td>
<td>45.33</td>
<td>37.76</td>
<td>35.77</td>
</tr>
<tr>
<td>$a_H$</td>
<td>768.16</td>
<td>383.49</td>
<td>153.16</td>
<td>76.50</td>
<td>50.96</td>
<td>42.46</td>
<td>40.22</td>
</tr>
<tr>
<td>$\mu \rho/\alpha$</td>
<td>1.0054</td>
<td>1.0038</td>
<td>1.0023</td>
<td>1.0012</td>
<td>1.0005</td>
<td>1.0002</td>
<td>1.0001</td>
</tr>
</tbody>
</table>

and financial liberalization on expected income and external imbalances. Thereby, we conduct four different experiments. In the first two ones, we reduce trade cost parameter $t$ by 10 percent and by 1 STD of the observed variation of this parameter over the period 1991-2011, respectively. The latter amounts to a reduction of $t$ of almost 50 percent and therefore indicates a sizable trade integration event. In the last two experiments we consider reductions in the financial barriers $\tau$, again distinguishing a small liberalization event of a 10 percent reduction of $\tau$ and a large liberalization event, in which $\tau$ falls by 1 STD or about 50 percent of its initial value.

The effects of these four liberalization experiments on expected income are summarized in Table 2. There, we see that the income effects of both the small and the large liberalization events are tiny. The main reason for this is that OECD countries are open economies with liberal financial sectors to begin with. Due to this, even a significant reduction of trade and financial barriers of up to 50 percent does not exert a big income stimulus for OECD members. More importantly, we see from a comparison of the different columns in Table 2 that the income stimulus of the respective liberalization events are more pronounced for higher values of $\mu$. That means further liberalization within the OECD would generate significant gains only in interaction with higher robustness of risky high-productivity technologies.\textsuperscript{24}

The quantitative effects of trade and financial liberalization on trade imbalances are reported in Table 3. There, we see that the respective effects are in general large and increasing in $\mu$. Of course, the size of these effects have to be interpreted with care. Since our model attributes all trade to the diversification motive of financial investors, it prob-

\textsuperscript{24}Due to the assumption of a logarithmic utility function, the welfare consequences of trade and financial liberalization are less pronounced than the income effects.
Table 2: Income effects of trade and financial liberalization

<table>
<thead>
<tr>
<th>µ</th>
<th>0.05</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δt 10%</td>
<td>0.0008</td>
<td>0.0016</td>
<td>0.0045</td>
<td>0.0126</td>
<td>0.0349</td>
<td>0.0877</td>
<td>0.1459</td>
</tr>
<tr>
<td>-1 STD</td>
<td>0.0036</td>
<td>0.0072</td>
<td>0.0203</td>
<td>0.0575</td>
<td>0.1572</td>
<td>0.3847</td>
<td>0.6195</td>
</tr>
<tr>
<td>Δτ 10%</td>
<td>0.0032</td>
<td>0.0065</td>
<td>0.0182</td>
<td>0.0511</td>
<td>0.1391</td>
<td>0.3402</td>
<td>0.5495</td>
</tr>
<tr>
<td>-1 STD</td>
<td>0.0279</td>
<td>0.0568</td>
<td>0.1622</td>
<td>0.4538</td>
<td>1.1777</td>
<td>2.5539</td>
<td>3.6650</td>
</tr>
</tbody>
</table>

Notes: The numbers refer to welfare changes in percent.

ably overestimates the role of trade and financial liberalization on external imbalances. Contrasting the results from Tables 2 and 3, we can conclude that further integration of goods markets as well as further liberalization of financial markets in the OECD, while exerting only minor positive income effects, will significantly increase trade imbalances within the OECD.\textsuperscript{25}

Table 3: The impact of trade and financial liberalization on external imbalances

<table>
<thead>
<tr>
<th>µ</th>
<th>0.05</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δt 10%</td>
<td>12.34</td>
<td>13.11</td>
<td>13.87</td>
<td>14.12</td>
<td>13.73</td>
<td>12.25</td>
<td>10.34</td>
</tr>
<tr>
<td>-1 STD</td>
<td>47.16</td>
<td>50.10</td>
<td>52.88</td>
<td>53.57</td>
<td>51.31</td>
<td>44.15</td>
<td>35.66</td>
</tr>
<tr>
<td>Δτ 10%</td>
<td>42.53</td>
<td>45.18</td>
<td>47.85</td>
<td>48.84</td>
<td>47.26</td>
<td>41.18</td>
<td>33.69</td>
</tr>
<tr>
<td>-1 STD</td>
<td>221.09</td>
<td>234.35</td>
<td>246.09</td>
<td>245.20</td>
<td>222.46</td>
<td>168.55</td>
<td>120.13</td>
</tr>
</tbody>
</table>

Notes: The numbers refer to percentage changes in the STD of trade imbalances.

9 Concluding remarks

This paper has set up a model, in which exploitation of technological specialization by international investment and trade leads to diversification gains. These diversification gains materialize in the form of higher expected income and an insurance against the risks of high-productivity technologies. However, the gains are accompanied by a rising variance in the trade accounts. Since international technological specialization and diversification

\textsuperscript{25}It is worth noting that the effects of trade and financial liberalization on external imbalances do not monotonically increase in µ. In the case of an integration in goods markets, we can formally show that at µ = 1 the impact of a change in t on trade imbalances vanishes.
require financial markets as well as goods markets, trade cost reduction and reduction of the costs of financial transactions complement each other in promoting globalization gains from technological specialization.

In two extensions of our framework, we have elaborated on technological innovations and on erroneous assessment of the risks of international investment in specialized technologies. Thereby, we have shown that technological specialization of regions improves the diversification opportunities of risk-averse investors, which stimulates international investment in risky technologies at home and abroad and fosters international trade. Furthermore, since the crucial assumption for the positive effects of global risk sharing in our model is a reliable assessment of environments and the performance of technologies targeted to a specific environment, the distinguishability of environments is crucial for beneficial international investment. If there is a partial overlap of environments across the different locations and investors have erroneous beliefs about the probability of success of risky technologies, the income streams generated by cross-border investments are correlated. We show that this correlation leads to a misallocation of capital that counteracts the diversification gains of international investment.

Whereas the model is too stylized to provide guidance for a rigorous empirical analysis of the link between international investment, international trade and external imbalances, we provide descriptive evidence to support our arguments. Furthermore, in a numerical exercise, we collect parameter values from existing empirical work and estimate the return to risky technologies, employing data on trade imbalances and capital stocks for OECD countries. We then quantify the effects of trade and financial liberalization on income and external imbalances in our setting. This exercise shows that although welfare gains from further trade and financial integration are rather small in general, the consequences for trade imbalances can be substantial. Further liberalization within the OECD can generate sizable welfare gains only in interaction with higher robustness of risky state-contingent technologies.

Since it has been the purpose of this paper to elaborate on the fundamental link between international investment, international trade and external imbalances in an ana-
lytically tractable framework, we have imposed several simplifying assumptions that limit the suitability of our framework to explain all aspects that might be important in this respect. For instance, we have assumed that capital is the only factor of production. Hence, our model does not provide insights regarding the consequences of international investment for the income of immobile labor and its volatility. Furthermore, by focussing on the diversification motive of investors, we have abstracted from other reasons for goods trade, so that our numerical results may overestimate the impact of financial and trade liberalization on external imbalances. Expanding the analysis in both of these directions, though beyond the scope of this paper, is a worthwhile task for future research and may provide suitable guidance for empirical analysis.

References


A Theoretical appendix

Solution to the capital allocation problem in the open economy

The solution to the portfolio allocation problem in the open economy can be characterized as follows.

Lemma 1. Under Assumption 1, if $H$ shares integrated financial markets with $n$ different countries, in each of which $\tau < \bar{\tau}$, the following holds:

i) If $\mu = 1$ then $I_0 = 0$, $I_H = \pi_K K$ and $I_l = \pi_l K$. If $\mu < 1$ and $\alpha \geq \mu \rho$, then $I_0 = K$.

ii) If $\mu < 1$, $\alpha < \mu \rho$ and $\pi_H > \frac{1 - \mu}{\rho - \alpha}$, $\pi_l > \frac{1 - \mu}{(1-t)(\rho - \alpha)}$ for all $l = 1, \ldots, n$, then the optimal investment portfolio is given by Eq. (10).

iii) If $\pi_H < \frac{1 - \mu}{\rho - \alpha}$ or $\pi_l < \frac{1 - \mu}{(1-t)(\rho - \alpha)}$ for some $l \in \{1, \ldots, n\}$, then $I_l = 0$ for the respective $l \in \{H, 1, \ldots, n\}$.

Proof: The first-order conditions for $\max_{I_0, I_H, (I_l)_{l=1, \ldots, n}} EU$ s.t. (8) are:

\[(I_H) \quad \frac{\pi_H}{X_H} R = \lambda,\]
\[(I_l) \quad \frac{\pi_l}{X_l} (1-t) R = \lambda,\]
\[(I_0) \quad \left(\frac{\pi_H}{X_H} + \sum_{l=1}^{n} \frac{\pi_l}{X_l}\right) r + \frac{1-\mu}{I_0} = \lambda,\]  (A.1)

where $\lambda$ is the Lagrange multiplier for budget constraint $I_0 + I_H + \sum_{l=1}^{n} I_l \leq K$.

Obviously, condition $(I_0)$ only applies if $\mu < 1$. For $\mu = 1$, we have $I_0 = 0$ and thus, after using (8) in $(I_H)$ and $(I_l)$ of (A.1), $I_H = \pi_H K$ and $I_l = \pi_l K$. The further proof considers the case $\mu < 1$.

According to (8), $X_H^+ - X_l^+ = R[I_H - (1-t)I_l]$. Combining this with $(I_H)$ and $(I_l)$ from (A.1), we obtain

\[I_H = \frac{\pi_H - \pi_l (1-t)}{\lambda} + (1-t)I_l.\]  (A.2)

Summing over $l$, we have $nI_H = \frac{n\pi_H - (\mu - \pi_H)(1-t)}{\lambda} + (1-t)(K - I_0 - I_H)$, which reduces to:

\[I_H = \frac{\pi_H - \mu / \alpha(t, n)}{\lambda} + \frac{K - I_0}{\alpha(t, n)}\]  (A.3)

with $\alpha(t, n) \equiv 1 + \frac{n}{1-t}$.
Moreover, using in (A.1) the conditions for \((I_H)\) and \((I_L)\) in the condition for \((I_0)\), we obtain \(\frac{\lambda}{R} \left(1 + \frac{n}{1-t}\right) r + \frac{1-\mu}{I_0} = \lambda\), which gives us for \(\alpha < \rho \left(\equiv R/r\right)\) and \(\mu < 1:\)

\[
\lambda \left(1 - \frac{\alpha}{\rho}\right) = \frac{1-\mu}{I_0}
\]

(A.4)

where in \(\alpha\) the arguments \(t,n\) have been dropped. If \(\alpha \geq \rho\), then the left side of \((I_0)\) would be larger than \(\lambda\) so that \(I_0 = K\) if \(\alpha \geq \rho\). Actually, \(\alpha < \rho\) is only a necessary condition for \(I_0 < K\). Since \(I_0 \leq K\), the left side of \((I_0)\) would be larger than the right side if \(\frac{\alpha}{\rho} + \frac{1-\mu}{\lambda K} > 1\). Thus, \(I_0 = K\) if \(\frac{\alpha}{\rho} + \frac{1-\mu}{\lambda K} > 1\) for the \(\lambda\) determined below.

Substituting \(X_H^+\) from (8) into condition \((I_H)\) of (A.1), we have

\[
rI_0 + RI_H = \frac{\pi_H R}{\lambda}.
\]

(A.5)

The system (A.3)-(A.5) determines \(I_0, I_H\) and \(\lambda\). Then (A.2) gives us \(I_l\).

For solving the system (A.3)-(A.5), substitute first (A.3) and (A.4) into (A.5) to get for the case \(\alpha < \rho\):

\[
\frac{r(1-\mu)}{\lambda \left(1 - \frac{\alpha}{\rho}\right)} + R \frac{\pi_H - \mu/\alpha}{\lambda} + \frac{R(K - I_0)}{\alpha} = \frac{\pi_H R}{\lambda},
\]

which reduces to

\[
K - I_0 = \frac{1}{\lambda} \left(\mu - \frac{\alpha(1-\mu)}{\rho - \alpha}\right).
\]

(A.6)

Substituting then \(I_0\) from (A.4) into (A.6), we have \(K = \frac{\rho(1-\mu)}{\lambda(\rho - \alpha)} + \frac{1}{\lambda} \left(\mu - \frac{\alpha(1-\mu)}{\rho - \alpha}\right)\),

which reduces to

\[
K = \frac{1}{\lambda}.
\]

(A.7)

Using this in (A.4) and solving for \(I_0\), we have

\[
I_0 = \frac{1-\mu}{1 - \alpha/\rho} K
\]

(A.8)

and \(K - I_0 = K \left(1 - \frac{1-\mu}{1-\alpha/\rho}\right)\), which reduces to

\[
K - I_0 = \frac{\mu - \alpha/\rho}{1 - \alpha/\rho} K.
\]

(A.9)
The condition $\frac{\alpha}{\rho} + \frac{1-\mu}{\mu*} > 1$ for a corner solution $I_0 = K$ reduces to $\alpha \leq \mu \rho$. We have:
\[I_0 < K \text{ if and only if } \alpha < \mu \rho. \quad (A.10)\]

Substitution of (A.7) and (A.9) into (A.3) gives us $I_H = K \left( \pi_H - \frac{\mu}{\alpha} + \frac{1}{\alpha/\rho} \right)$, which reduces to
\[I_H = K \left[ \pi_H - (1 - \mu) \frac{1}{\rho - \alpha} \right]. \quad (A.11)\]

Using this and (A.7) in (A.2), we have finally
\[I_l = K \left( \pi_l - \frac{1 - \mu}{1 - t} \frac{1}{\rho - \alpha} \right). \quad (A.12)\]

It is worth noting that $I_H > 0$ if and only if $\pi_H > (1 - \mu) \frac{1}{\rho - \alpha}$ and $I_l > 0$ if and only if $\pi_l > \frac{1 - \mu}{1 - t} \frac{1}{\rho - \alpha}$. If none of these inequalities holds, then $\pi_H + \sum_{l=1}^{n} \pi_l \leq (1 - \mu) \frac{1}{\rho - \alpha} + n \frac{1 - \mu}{1 - t} \frac{1}{\rho - \alpha}$, which is equivalent to $\mu (\rho - \alpha) \leq (1 - \mu) \alpha$ and implies $\mu \rho \leq \alpha$, that is $I_0 = K$. If $\pi_l \leq \frac{1 - \mu}{1 - t} \frac{1}{\rho - \alpha}$ for some $l$, then the respective investment levels are zero and the proof for determining the non-negative high-productive investments has to proceed by deleting for all $l$ with $I_l = 0$ the condition $(I_l)$ in (A.1) and using $X_l^+ = r I_0$ in the condition $(I_0)$.  

### Derivation of Eq. (23)

Note first that
\[VAR \left[ X_l(\omega) | \Omega^+ \right] = \frac{\pi}{\mu^*} \left( X^+ - \frac{\pi}{\mu^*} X^+ \right)^2 + \frac{\mu^* - \pi}{\mu^*} \left( \frac{\pi}{\mu^*} X^+ \right)^2 \]
\[= \pi \left( \frac{X^+}{\mu^*} \right)^2 \left[ \mu^* \left( 1 - \frac{\pi}{\mu^*} \right)^2 + \pi \left( \frac{\mu^* - \pi}{\mu^*} \right)^2 \right], \quad (A.13)\]

where the square-bracketed term reduces to $(\mu^* - \pi)$, establishing
\[VAR \left[ X_l(\omega) | \Omega^+ \right] = \pi \left( \mu^* - \pi \right) \left( \frac{X^+}{\mu^*} \right)^2. \quad (A.14)\]

The sum of variances is then given by
\[\sum_{l \in L} VAR \left[ X_l(\omega) | \Omega^+ \right] = (1 + L) \pi \left( \mu^* - \pi \right) \left( \frac{X^+}{\mu^*} \right)^2 = \bar{\mu} \left( \mu^* - \pi \right) \left( \frac{X^+}{\mu^*} \right)^2. \quad (A.15)\]
To determine the covariance we use that, according to Assumption 2, there are \(1 + L\) overlapping pairs and \((1 + L)(L - 1)\) non-overlapping ones. For the first type of pairs we have

\[
X_l(\omega)X_l'(\omega) = \begin{cases} 
(X^+)^2 & \text{with probability } \pi_\Phi, \\
0 & \text{with probability } \mu^* - \pi_\Phi.
\end{cases}
\] (A.16)

Thus, \(E[X_l(\omega)X_l'(\omega)|\Omega^+] = (\pi_\Phi/\mu^*)(X^+)^2\). Moreover, \(E[X_l(\omega)|\Omega^+] \cdot E[X_l'(\omega)|\Omega^+] = (\pi X^+ / \mu^*)^2\) for all pairs. This gives us:

\[
COV[X_l(\omega), X_l'(\omega)|\Omega^+] = \begin{cases} 
(\mu^* \pi_\Phi - \pi^2) \left( \frac{X^+}{\mu^*} \right)^2 & \text{for } 1 + L \text{ pairs}, \\
-\pi^2 \left( \frac{X^+}{\mu^*} \right)^2 & \text{for } (1 + L)(L - 1) \text{ pairs}.
\end{cases}
\] (A.17)

The sum of the covariances is then given by

\[
\sum_{l \in L} \sum_{l' \in L} COV[X_l(\omega), X_l'(\omega)|\Omega^+] = (1 + L)\pi \left( \mu^* \frac{\pi_\Phi}{\pi} - \pi \right) \left( \frac{X^+}{\mu^*} \right)^2 - (1 + L)(L - 1)\pi^2 \left( \frac{X^+}{\mu^*} \right)^2,
\] (A.18)

which reduces to

\[
\sum_{l \in L} \sum_{l' \in L, l' \neq l} COV[X_l(\omega), X_l'(\omega)|\Omega^+] = \bar{\mu} \left[ \mu^* \frac{\pi_\Phi}{\pi} - \pi L \right] \left( \frac{X^+}{\mu^*} \right)^2.
\] (A.19)

Collecting terms we thus obtain

\[
VAR[X(\omega)|\Omega^+] = \bar{\mu} \left[ \mu^* \left( 1 + \frac{\pi_\Phi}{\pi} \right) - \bar{\mu} \right] \left( \frac{X^+}{\mu^*} \right)^2,
\] (A.20)

Using (19) and \(\bar{\mu} = (1 + L)\pi\), we have \(\mu^* = \bar{\mu} \left( 1 - \frac{\pi_\Phi}{2\pi} \right)\). Thus, the square-bracketed term reduces to \(\bar{\mu} \frac{\pi_\Phi}{2\pi} \left( 1 - \frac{\pi_\Phi}{\pi} \right)\). This establishes Eq. (23).