
Katharina Erhardt¶

June 21, 2016

Abstract

This paper reassesses the home market effect in the light of firm level heterogeneity. Industries with high barriers to trade, less differentiated products and high productivity dispersion are more likely to concentrate in larger markets. These results stand in contrast to predictions derived in models without selection into exporting. In the presence of economies of scale firms tend to locate in markets where export selection is less tough. These predictions are confirmed in an empirical investigation based on 34 OECD countries and 118 sectors.

Keywords: Home Market Effect, Heterogeneous Firms, Monopolistic Competition, Economic Geography

JEL Classification: F12, R12

1 Introduction

Countries tend to export those goods for which they have relatively large domestic markets. This concept was formally introduced by Krugman (1980) and has become known as the home market effect. The home market effect

*Acknowledgements: I am grateful to Peter Egger, Sergey Nigai, Benjamin Jung, Wilhelm Kohler, Matthieu Crozet, Simon Hänni as well as participants at the SMYE 2014 in Vienna and the Brown Bag Seminar at ETH Zürich for discussions and suggestions.

¶Affiliation: ETH Zürich. Address: ETH Zürich, KOF, Leonhardstrasse 21, 8092 Zürich, Switzerland. E-mail: erhardt@kof.ethz.ch.
arises in trade models featuring monopolistic competition, increasing returns to scale and transport costs. Increasing returns to scale induce firms to locate their production in one place while the larger market allows for minimizing transport costs. Since the larger country has a location advantage vis-à-vis the smaller country this advantage is offset by a higher wage rate in Home (Krugman (1980)). In a multi-sector environment the home market effect leads to a concentration of the differentiated product industry in the larger market (Helpman and Krugman (1985)). Allowing for a continuum of differentiated product sectors Hanson and Xiang (2004) generalize this result by showing that the home market effect is more pronounced in industries with high barriers to trade and more differentiated products.

Up to the present, the notion that differentiated product industries tend to concentrate in the larger market is established in the trade and economic geography literature.\(^1\) This alleged fact is even used as a discriminatory criterion to identify sectors subject to increasing returns to scale in the empirical literature (Davis and Weinstein (1996, 1999)).

However, all studies confirming the home market effect to be stronger in those industries that feature a higher degree of product differentiation are based on homogeneous firms models that do not allow for export selection. This is surprising given the fact that there is vast evidence on export selection\(^2\) and that this strand of literature – sometimes referred to as ‘new new trade theory’ – experienced a rapid growth over the last decade.\(^3\)

In this paper, I extend the standard model of monopolistic competition with heterogeneous firms introduced by Melitz (2003) to a multi-sector framework in order to identify industry characteristics that affect the strength of the home market effect across industries in a setting of export selection. In line

\(^2\)Helpman, Melitz and Yeaple (2004) show that exporters are more productive than non-exporters and Eaton, Kortum and Kramarz (2011) find vast evidence for selection into exporting using French firm-level data.
with previous results I confirm that industries with high transport costs tend to be subject to a stronger home market effect. However, I find that allowing for selection into exporting causes less differentiated industries to concentrate in the larger market, contradicting previous studies on home market effects. Equally, industries with a more dispersed productivity distribution tend to locate in the larger market. The coexistence of export selection and increasing returns introduces a new channel that makes firm choosing a location that provides easier access to foreign markets. As the share of exporters is higher in the smaller country, there is a tendency to locate in the smaller country. The interplay of the elasticity of substitution and the productivity dispersion in an industry determines how strong the selection effect is, reversing the role of the elasticity of substitution in this setting as opposed to a homogeneous firm setting.

The predictions derived in this theoretical model are strongly confirmed in an empirical investigation based on a sample of 34 OECD countries and 118 manufacturing sectors over the period 1998-2007. Both the empirical and theoretical part of this paper confirm a negative relationship between the productivity shape parameter – the trade elasticity – and the strength of the home market effect. This result is in line with other empirical studies on home market effects that have found a negative relationship between the presence of home market effects and the trade elasticity (Compare Hanson and Xiang (2004) and Crozet and Trionfetti (2008)). The difference between these studies and the present paper lies in the structural interpretation of the result. As Chaney (2008) pointed out the productivity shape parameter is the structural equivalent of the trade elasticity in a Melitz (2003) model with Pareto productivity distribution, while it relates to the elasticity of substitution in a homogeneous firm setting. In contrast to previous empirical studies, I measure the elasticity of substitution in an industry by the ratio between revenues and profits using balance sheet data on 306,226 European firms. Consistent with the theoretical prediction, I find the elasticity of substitution to be positively related to the presence of a home market effect.

To my knowledge, only few papers have examined the role of home market
effects in a heterogeneous firm setting. Felbermayr and Jung (2012) find a
home market effect to arise in a single-sector environment with export selec-
tion even without an outside sector. In their model, the advantage of the
larger market leads to both higher wages and a higher market entry proba-

4
blity. Okubo (2009) embeds the Melitz (2003) model into a Ricardian model of
c omparative advantage with a continuum of sectors à la Dornbusch, Fischer
and Samuelson (1977). Allowing for asymmetries in country size and minimum
productivity across countries, he finds that a home market effect à la Krugman
(1980) leads to wage rates that are proportional to the market size. Further-
more, he identifies a home market effect à la Helpman and Krugman (1985)
which is reflected in a more than proportional share of exporters. However,
in his setting, Okubo (2009) is not able to compare the strength of the home
market effect across sectors. Similarly, Nguyen (2012) identifies a home mar-
ket effect in a setting with one differentiated product sector and one outside
good sector, but is not able to draw any conclusions regarding the strength of
the home market effect across different differentiated product sectors. While
being analytically more tractable, the assumption of an outside sector with-
out transport costs is rather unrealistic and shuts down the wage channel, an
important driver of the results in this paper. Clearly, the home market effect
is a relative concept. In a setting with several industries, the presence of a
home market effect in a certain sector depends on the characteristics of the
other sectors in the economy. Here lies the main contribution of this paper: In
a setting with several industries, all of them being subject to a home market
effect when observed in isolation, I identify which sector characteristics are
those that foster a home market effect in an equilibrium with a continuum of
sectors.

The remainder of the paper is organized as follows. The next section
describes the model and investigates the characteristics of the equilibrium.
In section 3 the home market effect is defined and industry characteristics are
identified that induce a home market effect. Section 4 provides an empirical

4Note that the definition of a home market effect by Felbermayr and Jung (2012) is based
on the relative number of firms.
assessment of the predictions derived in section 3. Finally, section 5 concludes. Technical details can be found in the appendix.

2 The Model

In this section I develop a continuum of industries framework in which each industry is characterized by a Melitz (2003) type monopolistic competition with heterogeneous firms. There are two asymmetric countries, Home and Foreign, indexed by $i \in \{H, F\}$, that differ only in size with Home being the larger country. Each country has one factor of production, labor $L_i$, that is supplied inelastically and is paid a wage rate $w_i$. Without loss of generality I normalize $L_F \equiv 1$ and $w_F \equiv 1$. Since Home is defined as the large country it is endowed with more labor than Foreign, $L_H > 1$. Aggregate income $R_i$ is given by $R_i = w_i L_i$.

In each country there exists the same continuum of industries $z \in Z$. Along the continuum, industries vary with respect to their elasticity of substitution $\sigma(z)$, their iceberg-type transport cost $\tau(z)$ and their productivity dispersion $\beta(z)$. The implications of these parameters are explained in detail in what follows. For the moment, it is important to note that in each country the same industries exist that are subject to the same exogenous characteristics. This assumption is quite intuitive if equally developed countries are considered. Firms find the same industry specific preconditions in each country. Hence, the only force driving differences between countries in this model is the asymmetry in market size. In this setup the market size effects on trade and firm allocation can be isolated.

2.1 Demand

The preferences of a representative consumer are of Cobb-Douglas type across a continuum of industries $z \in Z$ with each industry being composed of horizontally differentiated goods, respectively. The exogenous consumption share of each

\[^5\]In the following, industry and sector are used interchangeably.
industry is denoted by $\alpha(z) < 1$. Over all sectors consumption shares sum up to unity: \( \int_0^1 \alpha(z)dz = 1 \). In every single sector \( z \) the market structure is assumed to be Dixit-Stiglitz monopolistic competition. Preferences across varieties are of constant elasticity of substitution form (CES) $\sigma(z) > 1$:

\[
U_i = \int_{z \in Z} \left[ \alpha(z) \frac{\sigma(z)}{\sigma(z) - 1} \ln \left( \int_{\omega_z \in \Omega_{z,i}} q(\omega_z) \frac{\sigma(z) - 1}{\sigma(z)} d\omega_z \right) \right] dz. \tag{1}
\]

$\Omega_{z,i}$ denotes the set of varieties available in a sector $z$ in country $i$. A specific variety within this set is indexed by lowercase letters $\omega_z$. Even though each country accommodates all industries the set of varieties available to consumers varies across countries: Consumers can choose between all domestic varieties and those foreign varieties produced by exporters. Varieties produced by non-exporters are only available to the respective local residents.

Maximizing utility (1) subject to the budget constraint

\[
R_i = \int_{z \in Z} \left[ \int_{\omega_z \in \Omega_z} q(\omega_z)p(\omega_z)d\omega_z \right] dz, \tag{2}
\]

the demand for a single variety $\omega_z$ is given by

\[
q(\omega_z) = \frac{\alpha(z)R_i}{P_z^{1-\sigma(z)}} p(\omega_z)^{-\sigma(z)} \tag{3}
\]

and depends negatively on the price of the variety, $p(\omega_z)$, positively on the income spent on industry $z$, $\alpha(z)R_i$, and the aggregate price in industry $z$, $P_z$, with the aggregate price being composed of all single varieties’ prices in sector $z$:

\[
P_z = \left( \int_{\omega_z \in \Omega_z} p(\omega_z)^{1-\sigma(z)}d\omega_z \right)^{-\frac{1}{1-\sigma(z)}}. \tag{4}
\]
2.2 Production

In each sector there is a continuum of firms with each firm producing a differentiated variety. Firms face constant marginal costs depending on their productivity $\varphi$ and fixed market access costs $f_{ij}$. These fixed costs are assumed to be symmetric for both countries, $f_{ii} = f_{jj} = f^D$ and $f_{ij} = f_{ji} = f^X$ and higher for serving the export market, $f^X > f^D$. That way, I introduce a selection channel such that only the most productive firms will be able to serve the export market profitably. More productive firms are characterized by lower labor input requirements for a given output. Hence, they can produce at lower cost and cover their fixed costs more easily. The presence of fixed costs leads to production under increasing returns to scale: average costs are decreasing in output. Moreover, iceberg-type transport costs $\tau_{ij}(z) = \tau_{ji}(z) = \tau(z) > 1$ constitute a barrier to trade since $\tau$ varieties have to be shipped for one variety to arrive in the destination country. I assume only cross-border trade to be subject to this kind of transport cost, $\tau_{ii}(z) = \tau_{jj}(z) = 1$. Given the wage rate $w_i$, production costs for serving a market $i$ are:

$$C_{ij}(\varphi(z)) = w_i \left( f_{ij} + \tau_{ij}(z) q(\varphi(z)) \varphi(z) \right).$$

(5)

Every firm acts as a monopolist for its differentiated variety. Thus, in order to maximize profits, firms set marginal cost equal to marginal revenue leading to a constant mark-up over marginal cost:

$$p_{ij}(\varphi(z)) = \frac{\tau_{ij}(z) w_i}{\varphi(z)} \frac{\sigma(z)}{\sigma(z) - 1},$$

(6)

where the mark-up indicates the degree of monopoly power in an industry: The more differentiated products are (the lower the elasticity of substitution) the higher is the mark-up firms can charge since consumers are ready to bear higher prices in this industry. Sales revenues $r_{ij}(z) = p_{ij}(z) q_{ij}(z)$ are used to

---

6 Export specific variables are denoted by X, variables specific to the domestic market are denoted by D.
obtain profits:

$$\pi_{ij}(\varphi(z)) = \frac{\alpha(z)R_i P_j(z)^{\sigma(z)-1}}{\sigma(z)} \left( \frac{\tau_{ij}(z)w_i}{\varphi(z)} \frac{\sigma(z)}{\sigma(z) - 1} \right)^{1-\sigma(z)} - w_i f_{ij}. \quad (7)$$

Hence, more productive firms (higher $\varphi$) can charge lower prices and thereby sell more quantities while obtaining higher profits. $\alpha(z)R_i$ indicates the market size of a sector $z$ in country $i$. Clearly, firm profits rise with the market size. If the aggregate price in the destination market, $P_j(z)$, rises the firm benefits in terms of higher profits as it can offer its good at relatively lower prices.

### 2.3 Equilibrium

As in the standard Melitz (2003) model, there exists an unbounded number $M^e$ of identical prospective entrants into each sector $z$. Productivities $\varphi$ are drawn from a common distribution $g(\varphi)$ after having paid a fixed entry cost in terms of labor, $f^e w_i > 0$. These fixed costs can be considered as being a unique initial investment such as research and development (R&D) with the random productivity draw being the uncertain outcome of research activities. For ease of tractability and due to broad general empirical support I assume the distribution to be Pareto with probability density function (p.d.f)

$$g(\varphi) = \beta(z) \varphi^{\beta(z)-1} \quad (8)$$

and the respective cumulative distribution function (c.d.f.)

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^{\beta(z)}, \quad (9)$$

where $\beta(z) > \sigma(z) - 1$ is a shape parameter that determines the dispersion of productivities: with rising $\beta(z)$ the productivity dispersion declines. Hence, for a high value of $\beta(z)$, productivities are concentrated at very low values. On the contrary, for a low $\beta(z)$, the inter-quartile range of $\varphi$ rises. The minimum value productivities can take is given by $b$ which is assumed to be equal across
sectors and countries for the sake of tractability in this paper.\footnote{For a discussion of the role of asymmetric minimum values, see Okubo (2009) and Felbermayr and Jung (2012). Demidova (2008) presents a model where firms draw from different productivity distributions.}

After incurring sunk investment costs $f$, firms learn about their productivity and decide to enter the market if production is profitable, i.e., if profits are nonnegative, \(\pi_{ij}(\varphi_{ij}(z)) \geq 0\): fixed costs of production must be at least covered by operating profits. Since fixed costs of exporting are higher than those of serving the domestic market, only the most productive firms become exporters.

\[
\frac{\alpha(z)R_jP_j(z)^{\sigma(z)-1}}{\sigma(z)} \left( \frac{\tau_{ij}(z)w_i}{\varphi_{ij}^*(z)} \frac{\sigma(z)}{\sigma(z) - 1} \right)^{1-\sigma(z)} = w_i f_{ij}. \tag{10}
\]

The above condition is referred to as the \textit{zero cutoff profit condition} as it determines the cutoff-level \(\varphi_{ij}^*(z)\) that is the minimum productivity level for which profits are nonnegative if selling to country \(j, j \in \{H, F\}\). In each industry \(z\) there exist four zero cutoff profit conditions corresponding to four cutoff-levels, \(\varphi_{HH}^*(z), \varphi_{HF}^*(z), \varphi_{FF}^*(z), \varphi_{FH}^*(z)\). In the following, parameters are chosen such that \textit{conventional sorting} holds. This is the case as long as cutoffs for serving the respective domestic market are lower than cutoffs for serving the export market, \(\varphi_{ij}^*(z) > \varphi_{ii}^*(z)\), \(i \in \{H, F\}, j \in \{H, F\}\).

Ex-ante, the probability of drawing a productivity level above the respective cutoff level \(\varphi_{ij}^*(z)\) is given by \((1-G(\varphi_{ij}^*(z)))\). The distribution of productivities \(g(\varphi)\) conditional on successful market entry is thus

\[
\mu(\varphi) = \begin{cases} 
g(\varphi) \quad & \text{if } \varphi \geq \varphi^* \\
1-G(\varphi) \quad & \text{if } \varphi < \varphi^*. 
\end{cases} \tag{11}
\]

As long as the assumption of \textit{conventional sorting} holds, only a fraction of active firms is exporting. Using (11) the share of exporting firms among all active firms in country \(i\) can be expressed in relative terms of respective entry probabilities, \(m_{ij}(z) = \frac{(1-G(\varphi_{ii}^*(z)))}{(1-G(\varphi_{ii}^*(z)))}\). Using the conditional distribution of
productivities, $\mu(\varphi)$, expected profits for an individual firm are:

$$\pi_i(\varphi(z)) = \sum_{j \in \{H,F\}} \int_{\varphi_{ij}(z)}^{\infty} \pi_{ij}(\varphi(z)) \mu(\varphi) d\varphi.$$  \hspace{1cm} (12)

The only reason to incur the sunk innovation cost is the prospect of positive expected profits covering these costs. Consequently, a second set of equilibrium conditions states that expected profits conditional on successful innovation equal the initial investment, $f^e$. Using the zero cutoff profit conditions expected profits can be rewritten and stated as the free entry condition:

$$f^e = (1 - G(\varphi^*_i(z))) (\theta(z) - 1) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij},$$  \hspace{1cm} (13)

where $\theta(z) \equiv \frac{\beta(z)}{1 - \sigma(z) + \sigma(z)} > 0$. Given the constraints on $\beta(z)$ and $\sigma(z)$, $\theta(z)$ is a positive constant specific to a sector $z$. For each country, Home and Foreign, there exists one free entry condition.

The aggregate shipment = aggregate expenditure condition ensures market clearing in every sector of Home and Foreign:

$$\theta(z) \sigma(z)(m_{ij}(z) + \eta) = \frac{\alpha(z) w_i L_i}{M_i(z) w_i + M_j(z) m_{ji}(z) w_j} + \frac{\alpha(z) w_j L_j m_{ij}(z) \eta}{M_j(z) w_j + M_i(z) m_{ij}(z) w_i \eta}$$  \hspace{1cm} (14)

with $\eta = \frac{L^X}{T^D}$ being a measure of relative market access cost. Finally, the balanced trade condition ensures market clearing at the aggregate level across all sectors:

$$0 = \int_0^1 \alpha(z) \left[ \frac{R_H}{\frac{\beta(z) \sigma(z)}{w^\eta(z)} - \tau(z) \beta(z) \eta^{\sigma(z) - 1} - 1} - \frac{\frac{\beta(z) \sigma(z)}{w^\eta(z)} - 1}{\tau(z) \beta(z) \eta^{\sigma(z) - 1} - 1 - \frac{\beta(z) \sigma(z)}{w^\eta(z)}} \right] dz.$$  \hspace{1cm} (15)

$^8 \beta(z) > \sigma(z) - 1$ and $\sigma(z) > 1$. 

10
Using (15) the relative wage rate can be determined. Single industries may have a trade surplus or deficit. The economy as a whole, however, will always feature balanced trade. For every sector $z$, the remaining endogenous variables $L_H(z), L_F(z), M_H(z), M_F(z), \varphi_{HH}^*(z), \varphi_{HF}^*(z), \varphi_{FF}^*(z), \varphi_{FH}^*(z)$ can be solved for using (15), (14), (13) and (10).\footnote{See the appendix for details.}

The share of exporters can be expressed in terms of the relative wage rate $w$:

\[
m_{HF}(z) = \frac{(w^{\sigma(z)} \tau(z)^{\sigma(z)-1} \eta^{-\beta(z) \sigma(z)-1})}{(1 - \eta (w^{\sigma(z)} \tau(z)^{\sigma(z)-1} \eta^{-\beta(z) \sigma(z)-1}))} \tag{16}
\]

\[
m_{FH}(z) = \frac{(w^{-\sigma(z)} \tau(z)^{\sigma(z)-1} \eta^{-\beta(z) \sigma(z)-1})}{(1 - \eta (w^{-\sigma(z)} \tau(z)^{\sigma(z)-1} \eta^{-\beta(z) \sigma(z)-1}))} \tag{17}
\]

The relative market access cost $\eta$ and the iceberg-type transport costs $\tau(z)$ influence the share of exporters in Home and Foreign in the same way as these barriers to trade are assumed to be symmetric across countries. The share of exporters is higher for that country that is subject to lower wages.

The free entry condition can now uniquely determine the entry cutoffs for each country using (16) and (17):

\[
(\varphi_{FF}^*(z))^{\beta(z)} = b^{\beta(z)} (\theta(z) - 1) \left( \frac{f_d + m_{FH}(z) f^x}{f_e} \right) \tag{18}
\]

\[
(\varphi_{HH}^*(z))^{\beta(z)} = b^{\beta(z)} (\theta(z) - 1) \left( \frac{f_d + m_{HF}(z) f^x}{f_e} \right), \tag{19}
\]

which in turn allows to solve for the export cutoffs using the relative wage.
from (15):

\[
\begin{align*}
\varphi^*_F(z) &= \varphi^*_H(z) w^{\frac{\sigma(z)}{\sigma(z)-1}} T(z) \frac{1}{\sigma(z)-1} \\
\varphi^*_H(z) &= \varphi^*_F(z) w^{\frac{\sigma(z)}{\sigma(z)-1}} T(z) \frac{1}{\sigma(z)-1}.
\end{align*}
\]

(20)

(21)

2.4 Characteristics of the Equilibrium

**Proposition 1.** A unique solution for the balanced trade condition (15) can be obtained only for a relative wage rate greater than one, \( w > 1 \). Hence wages are higher in the large country.

![Figure 1](image-url)

Figure 1: The trade balance condition for a sector \( z \) depicted together with the range of asymptotes.

*Proof.* Recall the balanced trade condition (15): an equilibrium is obtained if and only if the integral on the right hand side becomes 0. Hence, even if
there might be imbalances on the sectoral level trade must be balanced across sectors. The trade balance of a single sector from Home is given by the term in brackets of (15) and denoted by $T(w, z)$:

$$T(w, z) = \frac{R_H}{w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}} \tau(z)^{\beta(z)} \eta^{\frac{\beta(z)}{\sigma(z)-1}} - 1} - \frac{w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}}}{\tau(z)^{\beta(z)} \eta^{\frac{\beta(z)}{\sigma(z)-1}} - w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}}}$$  

(22)

Clearly, $T(w, z)$ is always decreasing with $w$, $T'(w, z) < 0$. Moreover, it can be shown that without a wage differential, $w = 1$, the sectoral trade balance would be positive in any case. Hence, in order to achieve a sectoral balance of trade, the relative wage rate has to be larger than 1. Each sectoral trade balance enters the balanced trade condition as its share of total consumption, $\alpha(z) > 0$. Since $T(w, z)$ is positive in all sectors for a wage rate smaller than or equal to 1, $w \leq 1$, an integral summing up over all sectoral trade balances can never be 0. Hence, in order to meet the balanced trade condition, the relative wage must be higher in Home. Further investigation of $T(w, z)$ shows that it is not properly defined for $w = \tau(z)^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}}$, and for $w = \tau(z)^{\frac{1-\sigma(z)}{\sigma(z)}} \eta^{\frac{1-\beta(z)-\sigma(z)+1}{\sigma(z)}}$. Thus, in order to obtain an equilibrium the wage rate must lie between $\tau(z)^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}} \eta^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}} < w < \tau(z)^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}} \eta^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}}$. Since the first term is smaller than 1 in any case we can conclude that - taking into account all sectors $z$ - the wage rate must lie in the range of:

$$1 < w < \min \left[ \tau(z)^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}} \eta^{\frac{\beta(z)-\sigma(z)+1}{\sigma(z)}} \right],$$

(23)

where the set on the right hand-side captures all ill-defined ranges of $w$ across all sectors. Figure 1 plots $T(w, z)$ for a specific sector $z$ in order to visualize the discussion above. Instead of a single asymptote a range of asymptotes that restrict the relative wage is depicted.\(^{10}\)

Proposition 1 shows that the wage rate must be higher in the large coun-

\(^{10}\)An exact analytical derivation of the results can be found in the appendix.
try in order to establish an open economy equilibrium. As already laid out by Krugman (1980) and the subsequent home market effect literature, wage rates are one of the main adjustment channels to account for country size asymmetries.\textsuperscript{11} In the presence of transport costs the larger market in Home provides a possibility to save these costs for a large part of demand. This advantage is counteracted by the rise in wage rates which ensures full employment in Home as well as in Foreign. By offering lower wage rates Foreign establishes an own location advantage.

While the wage rate is determined across all sectors, all other endogenous variables are sector specific. The entry and export probabilities in different industries are thus highly dependent on how these industries are able to cope with wage differentials. Relative entry cutoffs for some sector $z$ can be rewritten as:

$$\frac{(\varphi_{FF}^*(z))^{\beta(z)}}{(\varphi_{HH}^*(z))^{\beta(z)}} = 1 - w \frac{-\beta(z)\eta^{\beta(z)}}{\beta(z)\eta^{\beta(z)} - 1} \Gamma(z) = 1 - w \frac{-\beta(z)\eta^{\beta(z)}}{\beta(z)\eta^{\beta(z)} - 1} \Gamma(z),$$

(24)

where $\Gamma(z) = \tau(z)^{-\beta(z)}\eta^{-\beta(z)}\eta^{\beta(z)}$, $0 < \Gamma(z) < 1$ is a measure of freeness of trade.\textsuperscript{12}

Equation (24) shows that the entry cutoff in Home is lower than the entry cutoff in Foreign as long as the relative wage rate is greater than one. Hence, the probability of successful innovation is higher in Home than in Foreign, $(1 - G(\varphi_{HH}^*(z))) > (1 - G(\varphi_{FF}^*(z)))$ and $\varphi_{FF}^*(z) > \varphi_{HH}^*(z)$. Due to the access to a larger market even less productive firms can recover their fixed costs and enter the market in Home. This result is in line with former studies on home market effects, in particular with Felbermayr and Jung (2012) who find trade liberalization to be enhancing relative probability of successful entry in a one-sector model.


\textsuperscript{12}It is increasing as iceberg-transport costs and/or relative market access costs $\eta$ are decreasing. For $\Gamma(z)$ near 1, almost no trade barriers exist. $\Gamma(z)$ near 0, in contrast, indicates prohibitive barriers to trade.
Relative export cutoffs can be stated in a similar fashion:

\[ \frac{\varphi_{HF}(z)}{\varphi_{FH}(z)} = w^{\phi_0(z)} \left( \frac{\varphi_{FF}(z)}{\varphi_{HH}(z)} \right)^{1/(\sigma(z) - 1)}. \]  

(25)

Since the relative entry cutoff of Foreign to Home is greater than one the export cutoff in Home is higher than the respective cutoff in Foreign. Using the conventional sorting assumption, the equilibrium thresholds in the economy are ranked as follows:

\[ \varphi_{HH}(z) < \varphi_{FF}(z) < \varphi_{FH}(z) < \varphi_{HF}(z) \]  

(26)

**Proposition 2.** As long as conventional sorting holds and Home is larger than Foreign, the entry thresholds in Home are lower than in Foreign. The export probability, however, is higher in Foreign.

*Proof.* In the text. \(\square\)

Country size asymmetries are not only balanced by the wage adjustment channel described in Proposition 1 but also by different average productivities in countries that differ in size. The overall productivity level in Home is lower than in Foreign since the entry probability is higher in the larger market. However, the average productivity of exporters is higher in Home than in Foreign. Home exporters are put at a disadvantage when competing in Foreign, where firms can produce at lower costs. Foreign high productivity firms, however, can easily bear transport costs as their competitors in Home produce at higher labor costs. Hence, of all active firms in a country there is a larger share of exporters in Foreign than in Home, \(m_{HF}(z) < m_{FH}(z)\).

**Proposition 3.** As \(\beta\) approximates its lower bound, \(\beta(z) \to \sigma(z) - 1\), the above model nests the homogeneous firm model by Hanson and Xiang (2004).

*Proof.* See appendix.\(^{13}\)

\(^{13}\)Compare also the results found in Burstein and Vogel (2011).
With $\beta(z)$ approximating its lower bound, $\sigma(z) - 1$, the balanced trade condition of the present model and the one by Hanson and Xiang (2004) are identical and so are the prevailing wage rates. Though there is still heterogeneity at the firm level the pattern of trade is identical to that found by Hanson and Xiang (2004). How can this convergence be interpreted? There is an inverse relationship between the shape parameter of the Pareto distribution $\beta(z)$ and the dispersion of productivity. Firms are more concentrated in the case of low productivity dispersion. Reaching its lower bound, $\beta(z) \to \sigma(z) - 1$, productivity dispersion is maximized. In the Krugman (1980) case of homogeneous firms there do exist differentiated varieties but firms are identical regarding their productivity. Therefore, all firms are exporters. No selection occurs as it does in the Melitz (2003) case where only the most productive firms export. By approximating the lower bound, the selection channel - and thus one of the distinctive features of the Melitz (2003) model vis-à-vis the Krugman (1980) model - is turned off: $\eta^{\frac{\beta(z)}{\sigma(z) - 1}} \to 1$. All firms that serve the domestic market serve the export market as well. Selection of the more productive firms into the export market was driven by the fact that fixed market access costs were higher for the exporting market. For a sufficiently high productivity dispersion, $\beta(z) = \sigma(z) - 1$, this barrier to export becomes obsolete. The least productive firm to enter the domestic market will be the cutoff firm for the exporting market as well.

3 The Home Market Effect in a Continuum of Melitz Sectors

The sector specific home market effect $H(z)$ is defined by the share of world production being produced in Home’s sector $z$ adjusted for the share in world demand stemming from Home’s demand for that particular industry:

$$H(z) = \frac{1}{1 + R_H} \left[ \frac{R_H}{w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \Gamma(z)^{-1} - 1} - \frac{w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}}}{\Gamma(z)^{-1} - w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \Gamma(z)^{-1}} \right]$$  \hspace{1cm} (27)
Definition 1. A sector $z$ is subject to a **Home Market Effect** if Home’s value of production in sector $z$ relative to world production in $z$ exceeds Home’s relative demand for that sector.$^{14}$ Technically, this can be expressed as: $H(z) > 0$.

Rearranging (27) one can show that there exists a home market effect in sector $z$ if and only if:

$$R_H \left[ \frac{\Gamma(z)^{-1}}{\frac{\eta(z)\sigma(z)}{w}} - 1 \right] > \Gamma(z)^{-1}w^{\frac{\beta(z)}{\sigma(z)}} - 1$$ \hspace{1cm} (28)

The intuition for a home market effect to arise is strongly connected to the presence of increasing returns to scale and transport costs. In fact, both characteristics are a necessary condition for a home market effect to exist. Economies of scale ensure that concentrating production in one place is beneficial in order to maximize profits - producing in the large country allows for savings in transport costs. Inequality (28) illustrates the cost-benefit analysis underlying a firm’s location decision. When deciding where to locate firms have to bear several aspects in mind. In Foreign they benefit from lower relative wage rates. A higher wage differential unambiguously increases the advantage of locating production in Foreign. However, these firms have to bear transport costs in order to access the larger market. The impact of transport costs increases as the size of the larger market, $R_H$, rises. Hence, locating in Home is the more attractive the larger the local market is because transport costs can be saved. Clearly, the trade-off between costs and benefits depends crucially on industry characteristics: productivity dispersion, elasticity of substitution and freeness of trade.

$^{14}$Compare Helpman and Krugman (1985) and Hanson and Xiang (2004) who define a home market effect conceptually equivalent and Crozet and Trionfetti (2008) and Behrens, Lamorgese, Ottaviano and Tabuchi (2009) who define a home market effect in the same way as in definition 1.
3.1 The Home Market Effect Across Sectors

Clearly, an analytical solution for $H(z)$ depends on the exact distributions of $\alpha(z), \sigma(z), \beta(z)$ and $\Gamma(z)$. However, even without any assumptions about these distributions, one can compare the values of $H(z)$ between two industries, $z_0$ and $z_1$, and obtain a measure of concentration of industry $z_0$ relative to industry $z_1$. Pairwise comparisons of two industries that differ in one of the exogenous characteristics - elasticity of substitution, freeness of trade or productivity dispersion - allow for identification of those industry characteristics that foster a home market effect.

**Proposition 4.** If an industry $z_0$ exhibits a Home Market Effect $H(z_0) > 0$, then so will all industries $z_1$, $H(z_1) > 0$, that are characterized by lower freeness of trade $\Gamma(z_1) \leq \Gamma(z_0)$, higher elasticity of substitution $\sigma(z_1) \geq \sigma(z_0)$ and higher productivity dispersion $\beta(z_1) \leq \beta(z_0)$.

**Proof.** From (28) follows that a Home Market Effect exists if and only if:

$$R_H > w^\frac{2\beta(z_0)\sigma(z_0)}{\sigma(z)-1} + \Gamma(z)w^\frac{\beta(z_0)\sigma(z_0)}{\sigma(z)-1}(R_H - 1)$$

Noticing that $w^\frac{\sigma(z_0)\beta(z)}{\sigma(z)-1} > 1$, $\Gamma \in (0, 1)$ and $R_H = wL_H > 1$, it is obvious that the right-hand side is increasing in the freeness of trade $\Gamma(z)$, decreasing in the elasticity of substitution $\sigma(z)$ and increasing in the shape parameter $\beta(z)$. 

Proposition 4 states that industries with high barriers to trade, less differentiated products and high productivity dispersion tend to concentrate in larger markets. However, Proposition 4 does not make any predictions about the strength of the home market effects.

**Proposition 5.** Consider two countries, Home and Foreign, both accommodating a continuum of Melitz-type industries and Home being employed with a higher labor force.

(a) Given the freeness of trade and the productivity dispersion in two industries $z_0, z_1$, $\Gamma(z_1) = \Gamma(z_0), \beta(z_1) = \beta(z_0)$, the home market effect is higher for industries that feature a higher elasticity of substitution: $H(z_1) > H(z_0)$ if
\(\sigma(z_1) > \sigma(z_0).\)

(b) Given the freeness of trade and the elasticity of substitution in two industries \(z_0, z_1, \Gamma(z_1) = \Gamma(z_0), \sigma(z_1) = \sigma(z_0),\) the home market effect is higher for industries that feature a higher productivity dispersion: \(H(z_1) > H(z_0)\) if \(\beta(z_1) < \beta(z_0).\)

(c) Given the productivity dispersion and the elasticity of substitution in two industries \(z_0, z_1, \beta(z_1) = \beta(z_0), \sigma(z_1) = \sigma(z_0),\) the home market effect is higher for industries that feature a lower freeness of trade: \(H(z_1) > H(z_0)\) if \(\Gamma(z_1) < \Gamma(z_0),\)

provided freeness of trade is not too low: \(1 - \frac{R_H \left( \Gamma(z) \sigma(z)^{-1} - \frac{\beta(z) \sigma(z)}{w} \right)^2}{\left( \frac{\beta(z) \sigma(z)}{w} \right)^2} > 0.\)

Proof. See Appendix.

Proposition 5 states that for two industries with the same elasticity of substitution and the same productivity dispersion an industry with higher trade barriers will exhibit a stronger home market effect as long as trade barriers are not too high. Industries with high transport costs are especially interested in saving these costs which can be achieved by relocating to the larger market. However, for very high transport costs - hence, low freeness of trade - \(H(z)\) might be decreasing in \(\Gamma(z).\) Goods in sectors with very high transport costs are barely traded at all. While any industry \(z_1\) with higher transport costs \(\Gamma(z_1) < \Gamma(z_0)\) exhibits a home market effect if \(H(z_0) > 0\) (see Proposition 4), industries with intermediate levels of transport costs tend to have a stronger home market effect.\(^{15}\)

Regarding the elasticity of substitution, Proposition 5 contradicts the relationship typically found in a homogeneous firm setting such as Helpman and Krugman (1985) or Hanson and Xiang (2004). Hanson and Xiang (2004), for instance, find that the home market effect is stronger in industries with a lower elasticity of substitution. In the two-sector case, Helpman and Krugman \(\footnote{15}{\text{This illustrates the difference between Proposition 4 and 5. Compare also Hanson and Xiang (2004).}}\)
(1985) show that a home market effect occurs in the differentiated product sector. The impact of product differentiation on home market effects operates via the impact of the wage differential on the location decision. Recall that firms considering to locate in the larger market face a trade-off between saving transport costs and being subject to higher production costs. The elasticity of substitution governs the capacity to deal with high production costs by being a measure of price sensitivity.

Clearly, the same channel works in a heterogeneous firm setting. However, in this case there is a second channel to be borne in mind: selection. The occurrence of a home market effect is connected to the presence of economies of scale. In order to exploit economies of scale best, it is important to be able to export in the first place, leading to a location force towards that market where selection is less tough, which is the smaller market. Comparing two industries $z_0$ and $z_1$, the tendency to locate in the larger market, meaning a higher home market effect, is hence higher for that industry that is less restrictive in terms of export selection. The export share of the larger market is given by:

\[
m_{HF}(z) = \eta^{-1} \Gamma(z) w^{-\beta(z)\sigma(z)} - \eta^{-1} \Gamma(z)^2 \left(1 - \Gamma(z) w^{-\beta(z)\sigma(z)}\right)^{-1}.
\]

Comparing two industries, the export share is higher in that industry that features a higher elasticity of substitution leading to a stronger home market effect in that industry. The selection channel overcompensates the price sensitivity channel and reverses the effects of product differentiation for the home market effect.

Since the impact of selection depends on the interplay between productivity dispersion and the elasticity of substitution, the intuition for the role of productivity dispersion in determining a home market effect is the same as for the elasticity of substitution: selection is tougher in sectors with ceteris paribus lower productivity dispersion leading to a less pronounced home market effect in these sectors.
3.2 Robustness

A natural question arising in this context concerns the robustness of the results. First, the predictions of Proposition 5 contradict results previously found in the literature that have been confirmed in various empirical analyses. But in fact, the empirical tests confirming previous predictions are in line with the results found in this paper. Most studies evaluating the relationship between the elasticity of substitution and the home market effect approximate the elasticity of substitution using estimates of import demand elasticities.\(^{16}\) While the import demand elasticity corresponds to \((1 - \sigma(z))\) in a homogeneous firm framework it corresponds to \(-\beta(z)\) in a heterogeneous firms framework. Proposition 5 predicts a negative relationship for the Pareto shape parameter \(\beta(z)\) which is in line with the empirical results found in earlier studies. The main difference lies in the structural interpretation of the trade elasticity.

Second, the model is stylized and operates under certain assumptions. The interesting question, however, is if these assumptions are reasonable and if the basic channels driving the results can arise in a wider range of models. Of course, assuming productivities to be distributed Pareto is core to the predictions derived in Proposition 5 when it comes to the interpretation of the parameters.\(^{17}\) Still, a selection mechanism such as revealed in this stylized model is at work in any model of heterogeneous firm with export selection independent of the true productivity distribution.\(^{18}\) This implies that in any model featuring economies of scale there will be a tendency to locate in that market that is subject to less restrictive export selection since firms benefit from the access to a larger market. Also, the theoretical model used in this paper does not rely on an outside sector that ties down wages. Taking into account the wage effect makes this model more general and allows for interesting pair-wise comparisons across industries.\(^{19}\) By choosing the exogenous

\(^{16}\)See e.g. Hanson and Xiang (2004) and Crozet and Trionfetti (2008).

\(^{17}\)There is strong empirical support for the assumption of Pareto-distributed productivities. See e.g. Luttmer (2007).

\(^{18}\)Compare Melitz (2003) who does not specify an exact productivity distribution.

\(^{19}\)E.g. Nguyen (2012) shuts down the wage channel - notably an important driver of the results - by assuming an homogeneous goods outside sector. Any comparisons across
industry characteristics of the model accordingly one can easily nest different models and sectors.

Eventually, the external validity of the results is an empirical question. Therefore, the predictions of the theoretical model are tested in an empirical exercise in the subsequent section.

4 Empirical Implementation

The theoretical model gives clear empirical predictions regarding the size of the home market effect across different kinds of industries. In choosing the empirical specification I take the model seriously and select all variables to be close to their theoretical counterpart. Also, the set of control variables is solely guided by the theoretical model in the baseline specification. Even if I do include further controls in the robustness section, I emphasize that none of the results found in that section are to be interpreted causally.

4.1 Variables and Data

The home market effect as defined in equation (27) relates the share of production in a sector to the share of demand for that sector in a given country. A country $i$’s production share in sector $z$ at time $t$ is simply

$$ prodshare_{i,zt} = \frac{prod_{i,zt}}{\sum_{k=1}^{N} prod_{k,zt}}, $$

where $N$ denotes the number of countries and $prod_{i,zt}$ the value of production in country $i$’s sector $z$ in year $t$. To construct a valid measure of demand the two-country setting has to be extended to a multi-country setting where potential demand for producers in country $i$ originates from different source industries are in fact comparisons of different kinds of differentiated product industries to the homogenous good sector. Modelling only a single differentiated product industry, a comparisons across industry characteristics for two differentiated goods sectors is not possible.
countries \(j\) and is weighted by bilateral market accessibility \(\phi_{ij,zt}\). Following Head and Ries (2001) and Crozet and Trionfetti (2008) I construct \(\phi_{ij,zt}\) using bilateral trade flows \(tflow_{ij,zt}\) between two countries \(i\) and \(j\) in sector \(z\) at time \(t\) normalized by the respective internal trade flows to get rid off any endogenous determinants of trade such as wages and price indices:

\[
\phi_{ij,zt} = \sqrt{\frac{tflow_{ij,zt}tflow_{ji,zt}}{tflow_{ii,zt}tflow_{jj,zt}}}. \tag{31}
\]

Clearly, this formulation assumes market accessibility to be symmetric for two countries \(i\) and \(j\) in sector \(z\) at time \(t\). With this measure at hand, the potential demand for goods from sector \(z\) in country \(i\) at time \(t\), \(dem_{i,zt}\), can be constructed using the sectoral expenditures across countries \(j \in N\) at time \(t\), \(exp_{j,zt}\), weighted by \(\phi_{ij,zt}\):

\[
dem_{i,zt} = \sum_{j} \phi_{ij,zt}exp_{j,zt}. \tag{32}
\]

Analogously to the production share, the demand share potentially addressed to firms in country \(i\) in sector \(z\) at time \(t\) is given by:

\[
demshare_{i,zt} = \frac{dem_{i,zt}}{\sum_{k=1}^{N} dem_{k,zt}}. \tag{33}
\]

In order to construct these variables I use data on bilateral trade flows at the 4-digit ISIC Rev.3.1 level from UN Comtrade’s International Trade Statistics Database and production data at the same level of aggregation from UNIDO’s Indstat database. Data on sectoral expenditure in country \(i\) in a given year \(t\) is proxied by \(exp_{i,zt} = prod_{i,zt} + \sum_{k \neq i}^{N} tflow_{ki,zt} - \sum_{k \neq i}^{N} tflow_{ik,zt}\) and sectoral internal flows in country \(i\) in a given year \(t\) by \(tflow_{ii,zt} = prod_{i,zt} - \)

---

20 This formulation of the demand share closely corresponds to the one used by Crozet and Trionfetti (2008). While I allow for a multi-country setting in determining the actual demand for a good, I do not account for third country effects in determining the production pattern. See Behrens et al. (2009) for an intensive discussion of home market effects in a multi-country setting.
The theoretical model assumes countries to be identical except for their size. Therefore, I restrict the sample for the main analysis to 34 OECD countries over the years 1998-2007.\footnote{These countries are Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, South Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.}

**Productivity Dispersion**

In a Melitz (2003)-type model with Pareto productivity distribution the productivity shape parameter $\beta(z)$ corresponds to the import demand elasticity of an industry $z$.\footnote{Compare Chaney (2008) for a discussion of the differences in trade elasticities in a homogeneous versus a heterogeneous firm setting. See also Arkolakis, Costinot and Rodriguez-Clare (2012) for an excellent overview.} To obtain a measure of $\beta(z)$ I use import demand elasticities provided by Kee, Nicita and Olarreaga (2008) at the 6-digit HS96 level for 149 countries. Consistent with their theoretical model, I aggregate these elasticities to 4-digit ISIC Rev.3.1 level using import shares and GDP shares provided by the authors as well as industry correspondence tables from WITS. Finally, I aggregate these data across importers using an unweighted as well as an importshare-weighted mean.

**Elasticity of Substitution**

In order to obtain a valid measure for $\sigma(z)$ at the sector level I make use of the relationship between operating profits and operating revenues implied by the theoretical model. The elasticity of substitution $\sigma(z)$ corresponds exactly to the ratio of operating revenues to operating profits.\footnote{Operating profits are defined as the differential between operating revenue and operating expenses: $\bar{\pi}_i(\varphi(z)) = p_{ii}(\varphi(z))q_{ii}(\varphi(z)) + p_{ij}(\varphi(z))q_{ij}(\varphi(z)) - w_iq_{ii}(\varphi(z)) - \tau_iw_iq_{ij}(\varphi(z)) = (r_{ii}(\varphi(z)) + r_{ij}(\varphi(z)))/\sigma(z)$.} Using balance sheet data provided by Bureau van Dijk’s Amadeus data base I calculate this ratio for 306,226 firms.\footnote{These data comprise firms from 39 countries covering the years 2007 - 2013} I proxy sector-level $\sigma(z)$ by taking the median of that ratio for every 4-digit industry category. Figure 2 plots these values across
Figure 2: The distribution of the elasticity of substitution $\sigma(z)$ across industries.

industries and confirms that their magnitude is in a reasonable range. The lowest value of 1.82 is obtained for the sector ‘publishing of music’, a rather differentiated sector while ‘manufacture of prepared animal feeds’ seems to be a less differentiated industry yielding the maximum value of 17.61.

**Freeness of Trade**

Bilateral market accessibility as measured by $\phi_{ij,zt}$ is generally seen as a good proxy for freeness of trade between two countries $i$ and $j$. I construct a measure of freeness of trade at the sectoral level, $\Gamma(z)$, by taking the mean of $\phi_{ij,zt}$ across countries and time.

The final sample consists of 34 countries over a period of 10 years (1998-2007) comprising 118 manufacturing sectors.

---

26The sectors considered are all ISIC Rev.3.1 4-digit manufacturing sectors except for sector 3330 (manufacture of watches and clocks), 3710 and 3720 (recycling of metal and non-metal waste, respectively) for which data on the productivity dispersion is unavailable.
Table 1: Theoretical predictions about the sign of the coefficient of the interaction term across industries characteristics.

<table>
<thead>
<tr>
<th>Industry characteristic</th>
<th>$X_z$</th>
<th>Theoretical Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution</td>
<td>$\sigma(z)$</td>
<td>$\alpha_2 &gt; 0$</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>$\beta(z)$</td>
<td>$\alpha_2 &lt; 0$</td>
</tr>
<tr>
<td>Freeness of trade</td>
<td>$\Gamma(z)$</td>
<td>$\alpha_2 &lt; 0$</td>
</tr>
</tbody>
</table>

4.2 Empirical Analysis and Results

A home market effect exists if the production share in a sector exceeds the demand share for the same sector. This relationship can be easily tested by estimating the following regression:

$$\text{prodshare}_{i,zt} = \alpha_1 \cdot \text{demshare}_{i,zt} + u_{i,zt}. \tag{34}$$

The coefficient $\alpha_1$ is expected to be larger than unity if a home market effect exists. In order to test the empirical predictions regarding the size of the home market effect across different sectors, I interact the industry characteristics with the demand share while conditioning on the remaining characteristics.

$$\text{prodshare}_{i,zt} = \alpha_1 \cdot \text{demshare}_{i,zt} + \alpha_2 \cdot X_z \times \text{demshare}_{i,zt} + \alpha_3 \cdot \text{controls}_z + u_{i,zt}, \tag{35}$$

where $X_z$ represents one out of the three industry characteristics. Table 1 summarizes the predictions derived in Proposition 5. The baseline results are based on a pooled OLS regression and presented in Table 2. Column (1) reports the results for the basic home market effect regression equation (34). The coefficient $\alpha_1$ is significantly greater than 1 at the 1% level indicating that, averaging across all sectors, a home market effect can be identified. The coefficient is smaller than the one obtained by Crozet and Trionfetti (2008) which can be attributed to the more aggregated sector definition in their sample.

However, the main interest in this empirical analysis lies how this effect varies with industry characteristics. Columns (2)-(4) include interaction terms for the elasticity of substitution, the productivity shape parameter and the
freeness of trade, respectively. Column (2) shows clear evidence for a positive relationship between the elasticity of substitution and the home market effect. This confirms the theoretical prediction that less differentiated industries are rather exhibiting a home market effect. Equally in line with the predictions of the previous section, $\beta(z)$ as an inverse measure of productivity dispersion is negatively associated with the strength of a home market effect according to column (3). The coefficient of interest in column (4) bears the expected negative sign but is insignificant at the 10% level. Recalling Proposition 5, the negative relationship between freeness of trade and the size of the home market effect is unambiguous for higher levels of freeness of trade only. Therefore, I rerun the regression restricting the sample to those sectors that have a freeness of trade that is at least as high as the 25th, the 50th and the 75 percentile of the distribution in column (5)-(7). As expected the coefficient on the interaction term becomes significant in all specifications and increases in absolute value as the sample is gradually reduced to sectors that are more open to trade. This confirms not only the theoretically implied negative relationship between the home market effect and freeness of trade but suggest also that only industries with a certain amount of trade freeness are subject to this relationship. Overall, the results in Table 2 strongly confirm the predictions derived from the theoretical model: The strength of the home market effect increases in the elasticity of substitution and in the productivity dispersion (that is inversely related to $\beta(z)$) as well as the freeness of trade of a sector. In all cases, the inclusion of other sectoral controls as guided by theory seems to be statistically relevant. Note that the size of the coefficients on the interaction terms lacks any interpretation beyond its sign without further assumption on the distribution of industry characteristics across sectors.

4.3 Robustness

I conduct various robustness checks to confirm the validity of the results. Adding the respective baseline effect of the industry characteristic of interest does not change the results as can be seen in columns (1)-(3) of Table 3.
Table 2: OLS estimates: Evaluation of the home market across industry characteristics.

<table>
<thead>
<tr>
<th>Dependent variable: prodshare_{i,zt}</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>demshare_{i,zt}</td>
<td>1.06748***</td>
<td>1.00354***</td>
<td>1.14472***</td>
<td>1.10402***</td>
<td>1.18262***</td>
<td>1.27622***</td>
<td>1.30267***</td>
</tr>
<tr>
<td></td>
<td>(0.00739)</td>
<td>(0.02110)</td>
<td>(0.01969)</td>
<td>(0.00917)</td>
<td>(0.01643)</td>
<td>(0.03292)</td>
<td>(0.08438)</td>
</tr>
<tr>
<td>σ(z) × demshare_{i,zt}</td>
<td>0.00984***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00209)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β(z) × demshare_{i,zt}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.02093***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.00679)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Γ(z) × demshare_{i,zt}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.32306</td>
<td>-1.49115***</td>
<td>-2.79265***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.43359)</td>
<td>(0.52848)</td>
<td>(0.70935)</td>
</tr>
<tr>
<td>signa(z)</td>
<td>-0.00033***</td>
<td>-0.00019***</td>
<td>-0.00027***</td>
<td>-0.00026**</td>
<td>0.00012</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00004)</td>
<td>(0.00005)</td>
<td>(0.00008)</td>
<td>(0.00012)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β(z)</td>
<td>-0.00131***</td>
<td>-0.00082***</td>
<td>-0.00113***</td>
<td>-0.00146***</td>
<td>-0.00217**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00015)</td>
<td>(0.00016)</td>
<td>(0.00032)</td>
<td>(0.00051)</td>
<td>(0.00089)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Γ(z)</td>
<td>-0.02539***</td>
<td>-0.01718*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00920)</td>
<td>(0.01018)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>27538</td>
<td>27538</td>
<td>27538</td>
<td>27538</td>
<td>20710</td>
<td>13631</td>
<td>6946</td>
</tr>
<tr>
<td>Sample</td>
<td>Full</td>
<td>Full</td>
<td>Full</td>
<td>Full</td>
<td>Γ(z) &gt;</td>
<td>Γ(z) &gt;</td>
<td>Γ(z) &gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25th percentile</td>
<td>50th percentile</td>
<td>75th percentile</td>
</tr>
<tr>
<td>R²</td>
<td>0.8521</td>
<td>0.8539</td>
<td>0.8537</td>
<td>0.8536</td>
<td>0.8205</td>
<td>0.7759</td>
<td>0.7058</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01
Including the baseline effect seems to be statistically significant for columns (1) and (3) only. Also, in these cases, the coefficients on the interaction term increase in absolute size indicating an even stronger relationship between the industry characteristics on the strength of the home market effect. Note that in column (3) the sample is restricted to have a freeness of trade of at least the 25th percentile’s value.

Columns (4)-(6) of Table 3 report the result when including year fixed effects. Even though most year fixed effects are statistically significant, the coefficients of the relevant interaction terms are barely changed.\textsuperscript{27}

The theoretical predictions are derived under the assumption that the countries considered share common preferences and distributions of parameters. For this reason, I decided for the subset of OECD countries in the main analysis. Qualitatively, the results are not sensitive to this decision as can be seen in columns (1)-(3) of Table 4 where 113 countries are included.

Clearly, the choice of variables for the industry characteristics is key to the analysis. To show that this choice is not driving the results I provide alternative measures for the elasticity of substitution and the productivity shape parameter in columns (4)-(6) of Table 4. In column (4) I report results using a measure of the elasticity of substitution based on the mean of the operating-revenues-to-operating-profits ratio for firms that report non-negative profits.\textsuperscript{28}

Alternative measures for the productivity dispersion are obtained using a weighted mean across importers to obtain the sector-level productivity shape parameter $\beta(z)$ in column (5) and taking Caliendo and Parro (2015)’s measure of productivity dispersion in column (6). Since their measure varies at the 2-digit sector level only, the size of the coefficient can not be meaningfully com-

\textsuperscript{27}Note that including country fixed effects is not recommended in this setting because country size is strongly correlated with the demand share - being part of the interaction that is the main focus of the empirical setting. Therefore, it is unclear how the results would have to be interpreted in that setting.

\textsuperscript{28}Note that I exclude sector 3691 (manufacture of jewellery) in that specification since the alternative measure of elasticity of substitution is an extreme outlier in this case (30.19 as compared to 7.33 in the baseline specification driven by few extreme observations in the balance sheet data).
pared to the baseline coefficient. However, the sign of the coefficient confirms the expected direction for the relationship between productivity dispersion and the size of the home market effect.

This confirms that the correlations found in the empirical investigation are not only in line with all theoretical predictions of the previous section but also robust to various specifications and sample compositions. All in all, this provides very strong evidence for the theoretically identified relationship between certain industry characteristics and the strength of home market effects.

5 Conclusion

This paper reassesses the home market effect under firm level heterogeneity. The model used for this purpose introduces a continuum of Melitz-type sectors into a world of two countries that differ in size. Sector characteristics such as product differentiation, productivity dispersion and transport costs are symmetric across countries and vary along the continuum. In an equilibrium industries can be identified that tend to locate in the large country rather than the small one. In the presence of increasing returns to scale, firms prefer to serve both markets. Consequently, the home market effect is the stronger the higher the probability to become an exporter to the small country. Since this probability is related to the selection forces within a sector, the role of industry characteristics on selection is crucial for the derivation of home market effects. Previous research on the home market effect ignores these effects by neglecting firm heterogeneity. It can be shown that the selection effect depends on elasticity of substitution and productivity dispersion within a sector. As long as selection occurs the effect of elasticity of substitution on selection outweighs its direct effect on location decisions and reverses previous results.

In the proposed model, I can identify a home market effect in two dimensions. On the one hand, the large country will always be subject to higher wage rates. On the other hand, the relative size of industries differs across countries resulting from country size asymmetries. The analysis of the equilibrium reveals that sectors with high barriers to trade, low product differentiation and
Table 3: Robustness tests: Including baseline effects and year fixed effects.

<table>
<thead>
<tr>
<th>Dependent variable: prodshare_{i,zt}</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>demshare_{i,zt}</td>
<td>0.97493***</td>
<td>1.14215***</td>
<td>1.19832***</td>
<td>1.00478***</td>
<td>1.14550***</td>
<td>1.18280***</td>
</tr>
<tr>
<td>(0.02224)</td>
<td>(0.02080)</td>
<td>(0.01946)</td>
<td>(0.02117)</td>
<td>(0.01974)</td>
<td>(0.01643)</td>
<td></td>
</tr>
<tr>
<td>σ(\tilde{z}) × demshare_{i,zt}</td>
<td>0.01323***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.00231)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β(\tilde{z}) × demshare_{i,zt}</td>
<td>-0.01970***</td>
<td></td>
<td></td>
<td></td>
<td>-0.02079***</td>
<td></td>
</tr>
<tr>
<td>(0.00751)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.00679)</td>
<td></td>
</tr>
<tr>
<td>Γ(\tilde{z}) × demshare_{i,zt}</td>
<td></td>
<td>-1.94853***</td>
<td></td>
<td></td>
<td></td>
<td>-1.48751***</td>
</tr>
<tr>
<td>(0.64040)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.52930)</td>
<td></td>
</tr>
<tr>
<td>σ(\tilde{z})</td>
<td>-0.00045***</td>
<td>-0.00029***</td>
<td>-0.00047***</td>
<td>-0.00020***</td>
<td>-0.00021**</td>
<td></td>
</tr>
<tr>
<td>(0.00006)</td>
<td>(0.00006)</td>
<td>(0.00009)</td>
<td>(0.00005)</td>
<td>(0.00009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β(\tilde{z})</td>
<td>0.00002</td>
<td>-0.00020</td>
<td>-0.00124***</td>
<td>-0.00084***</td>
<td>-0.00102***</td>
<td></td>
</tr>
<tr>
<td>(0.00017)</td>
<td>(0.00018)</td>
<td>(0.00032)</td>
<td>(0.00017)</td>
<td>(0.00033)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Γ(\tilde{z})</td>
<td>-0.00083</td>
<td>-0.01721*</td>
<td>0.06264***</td>
<td>-0.01691</td>
<td>-0.01371</td>
<td></td>
</tr>
<tr>
<td>(0.01060)</td>
<td>(0.01018)</td>
<td>(0.01905)</td>
<td>(0.01036)</td>
<td>(0.01051)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixed effect</th>
<th>No</th>
<th>No</th>
<th>No</th>
<th>Year</th>
<th>Year</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>27538</td>
<td>27538</td>
<td>20710</td>
<td>27538</td>
<td>27538</td>
<td>20710</td>
</tr>
<tr>
<td>Sample</td>
<td>Full</td>
<td>Full</td>
<td>Γ(\tilde{z}) &gt; 25th percentile</td>
<td>Full</td>
<td>Full</td>
<td>Γ(\tilde{z}) &gt; 25th percentile</td>
</tr>
<tr>
<td>R²</td>
<td>0.8542</td>
<td>0.8537</td>
<td>0.8207</td>
<td>0.8540</td>
<td>0.8538</td>
<td>0.8205</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01
Table 4: Robustness tests: Using the full set of countries and alternative industry characteristics.

<table>
<thead>
<tr>
<th>Dependent variable: prodshare&lt;sub&gt;i,zt&lt;/sub&gt;</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>demshare&lt;sub&gt;i,zt&lt;/sub&gt;</td>
<td>1.02062***</td>
<td>1.14306***</td>
<td>1.19049***</td>
<td>1.03877***</td>
<td>1.12726***</td>
<td>1.10243***</td>
</tr>
<tr>
<td></td>
<td>(0.02090)</td>
<td>(0.01979)</td>
<td>(0.02825)</td>
<td>(0.01647)</td>
<td>(0.01758)</td>
<td>(0.01010)</td>
</tr>
<tr>
<td>(\sigma(z) \times \text{demshare}_{i,zt})</td>
<td>0.00841***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00203)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta(z) \times \text{demshare}_{i,zt})</td>
<td>-0.01834***</td>
<td>-0.01392**</td>
<td>-0.00099*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00670)</td>
<td>(0.00582)</td>
<td>(0.00053)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Gamma(z) \times \text{demshare}_{i,zt})</td>
<td>-2.53530*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.51064)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma(z))</td>
<td>-0.00018***</td>
<td>-0.00019***</td>
<td>-0.00034***</td>
<td>-0.00036***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.00006)</td>
<td>(0.00004)</td>
<td>(0.00004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta(z))</td>
<td>-0.00068***</td>
<td>-0.00054**</td>
<td>-0.00154***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00007)</td>
<td>(0.00027)</td>
<td>(0.00015)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Gamma(z))</td>
<td>-0.03587***</td>
<td>-0.02391**</td>
<td>-0.01651*</td>
<td>-0.01506</td>
<td>-0.01253</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01025)</td>
<td>(0.01122)</td>
<td>(0.00913)</td>
<td>(0.01032)</td>
<td>(0.01040)</td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>60606</td>
<td>60606</td>
<td>30167</td>
<td>27291</td>
<td>27538</td>
<td>27538</td>
</tr>
<tr>
<td>Sample</td>
<td>Full</td>
<td>Full</td>
<td>(\Gamma(z)) &gt; 25th percentile</td>
<td>Full</td>
<td>Full</td>
<td>Full</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.8428</td>
<td>0.8427</td>
<td>0.7712</td>
<td>0.8553</td>
<td>0.8536</td>
<td>0.8535</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

* p < 0.1, ** p < 0.05, *** p < 0.01
high productivity dispersion are more likely to locate in the large market.

An empirical assessment of these predictions based on a set of 34 OECD countries and 118 sectors strongly confirms the theoretically identified relationship between the different industry characteristics and the strength of the home market effect.
References


A Appendix

A.1 Equilibrium

Rewrite the price index

\[
P_i(z) = \left( \int_{\omega_z \in \Omega_z} p(\omega_z)^{1-\sigma(z)} d\omega_z \right)^{\frac{1}{\sigma(z)}}
\]

\[
P_i(z)^{1-\sigma(z)} = \sum_{j \in \{H,F\}} \left( \frac{\sigma(z)}{\sigma(z)-1} \tau_{ji}(z) w_j \right)^{1-\sigma(z)} M_j(z) \frac{m_{ji}(z)}{1 - g(\varphi_{ji}^*(z))} \int_{\varphi_{ji}^*(z)}^\infty (\varphi(z))^\sigma(z)-1 \beta(z) b_\beta(z) \varphi^{-\beta(z)-1} d\varphi
\]

\[
P_i(z)^{1-\sigma(z)} = \sum_{j \in \{H,F\}} \left( \frac{\sigma(z)}{\sigma(z)-1} \tau_{ji}(z) w_j \right)^{1-\sigma(z)} M_j(z) \frac{m_{ji}(z)}{b_\beta(z) (\varphi_{ji}^*)^{-\beta(z)}} \int_{\varphi_{ji}^*(z)}^\infty (\varphi(z))^{-\beta(z)+\sigma(z)-2} \beta(z) b_\beta(z) d\varphi
\]

\[
P_i(z)^{1-\sigma(z)} = \sum_{j \in \{H,F\}} \left( \frac{\sigma(z)}{\sigma(z)-1} \tau_{ji}(z) w_j \right)^{1-\sigma(z)} M_j(z) \frac{m_{ji}(z)}{b_\beta(z) (\varphi_{ji}^*)^{-\beta(z)}} \left[ -\frac{\beta(z)}{\sigma(z)-\beta(z)-1} b_\beta(z) (\varphi_{ji}^*(z))^{\sigma(z)-\beta(z)-1} \right]_{\varphi_{ji}^*(z)}
\]

\[
P_i(z)^{1-\sigma(z)} = \theta(z) \sum_{j \in \{H,F\}} \left( \frac{\sigma(z)}{\sigma(z)-1} \tau_{ji}(z) w_j \right)^{1-\sigma(z)} M_j(z)m_{ji}(z) (\varphi_{ji}^*(z))^{\sigma(z)-1}
\]

where \(\theta(z) \equiv \frac{\beta(z)}{1-\sigma(z)+\beta(z)} > 0\) and use this to derive equilibrium conditions:

A.1.1 Zero Profit Cutoff Condition

The Zero Profit Condition (ZPC) states that the cutoff firm (denoted by an asterisk) makes zero profits, hence:

\[
\pi_{ij}(\varphi_{ij}^*(z)) = 0
\]

\[
\iff \varphi_{ij}^*(z)^{\sigma(z)-1} = w_i^{\sigma(z)} f_{ij} \frac{\sigma(z)}{\alpha(z) R_j P_j(z)^{\sigma(z)-1}} \left( \frac{\tau_{ij}(z)}{\sigma(z)-1} \right)^{\sigma(z)-1}
\]

\[
\iff \varphi_{ij}^*(z) = w_i^{\sigma(z)-1} f_{ij}^{\frac{1}{\sigma(z)-1}} \left( \frac{\sigma(z)}{\alpha(z) R_j P_j(z)^{\sigma(z)-1}} \right)^{\frac{1}{\sigma(z)-1}} \left( \frac{\tau_{ij}(z)}{\sigma(z)-1} \right)
\]

A.1.2 Free Entry Condition

Expected profits are given by:

\[
\bar{\pi}_i(\varphi(z)) = \sum_{j \in \{H,F\}} \int_{\varphi_{ij}^*(z)}^\infty \frac{\pi_{ij}(\varphi(z)) g(\varphi(z))}{1 - G(\varphi_{ii}^*(z))} d\varphi
\]
As \( \lim_{\varphi \to \infty} (g(\varphi)) = 0 \), using \( \theta(z) \equiv \frac{\beta(z)}{1-\sigma(z)+\beta(z)} \) and plugging in \( \varphi^*_{ij}(z)^{\sigma(z)-1} \) from ZPC:

\[
\pi_i(\varphi(z)) = \sum_{j \in \{H,F\}} \frac{m_{ij}(z)}{1 - G(\varphi^*_{ij}(z))} \left[ \int_{\varphi^*_{ij}(z)}^{\infty} \pi_{ij}(\varphi(z)) g(\varphi(z)) d\varphi \right]
\]

\[
\pi_i(\varphi(z)) = w_i (\theta(z) - 1) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij}
\]

The **Free Entry Condition (FE)** states that fixed entry costs \( f^e \) are equal to ex-ante expected profits, i.e. discounted by the probability of successful entry \( (1 - G(\varphi^*_{ii}(z))) \equiv p_i^{\text{in}}(z) \):

\[
p_i^{\text{in}}(z) w_i (\theta(z) - 1) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij} = f^e w_i
\]

\[
\iff f^e = p_i^{\text{in}}(z) (\theta(z) - 1) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij}
\]

Average revenues are thus given by:

\[
\overline{r}_i = \sum_{j \in \{H,F\}} \int_{\varphi^*_{ij}(z)}^{\infty} \frac{r_{ij}(\varphi(z)) g(\varphi(z))}{1 - G(\varphi^*_{ij}(z))} d\varphi
\]

\[
= \sum_{j \in \{H,F\}} \int_{\varphi^*_{ij}(z)}^{\infty} \frac{p_{ij}(\varphi(z)) q_{ij}(\varphi(z)) g(\varphi(z))}{1 - G(\varphi^*_{ii}(z))} d\varphi
\]

\[
= \sum_{j \in \{H,F\}} \int_{\varphi^*_{ij}(z)}^{\infty} \frac{\alpha(z) R_i}{P_{j}(1-\sigma(z))} \left( \frac{\tau_{ij}(z) w_i}{\varphi(z)^{\sigma(z)-1}} \right)^{1-\sigma(z)} \beta(z) \beta(z)^{-\beta(z)-1} d\varphi
\]

\[
= \sum_{j \in \{H,F\}} \int_{\varphi^*_{ij}(z)}^{\infty} \frac{\alpha(z) R_i}{P_{j}(1-\sigma(z))} \left( \frac{\tau_{ij}(z) w_i}{\varphi(z)^{\sigma(z)-1}} \right)^{1-\sigma(z)} \beta(z) \beta(z)^{-\beta(z)-1} \left( \frac{\varphi^*_{ij}(z)^{\sigma(z)-1}}{1 - G(\varphi^*_{ij}(z))} \right) d\varphi
\]

Plug in \( \varphi^*_{ij}(z)^{\sigma(z)-1} \) from ZPC:

\[
\overline{r}_i = w_i \theta(z) \sigma(z) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij}
\]
The labor employed in a single sector \( z \) can be derived such that:

\[
L_i(z) = \frac{M_i(z)}{p_i(z)} f^e + M_i(z) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij} + M_i(z) \sum_{j \in \{H,F\}} \int_{\varphi_i(z)}^{\infty} \frac{\tau_{ij}(z) q_{ij}(z) g(\varphi(z))}{1 - G(\varphi_{ij}(z))} d\varphi
\]

Using the free entry condition \( p_i(z) (\theta(z) - 1) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij} = f^e \) and the cutoff productivity from the ZPC:

\[
L_i(z) = M_i(z) \theta(z) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij} + M_i(z) \sum_{j \in \{H,F\}} m_{ij}(z) \theta(z) \left( f_{ij} \varphi(z) \left( \frac{\varphi(z)}{\sigma(z)} - 1 \right)^{-1} \right) \\
= M_i(z) \theta(z) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij} + M_i(z) \sum_{j \in \{H,F\}} m_{ij}(z) \theta(z) f_{ij} (\sigma(z) - 1) \\
= M_i(z) \theta(z) \sigma(z) \sum_{j \in \{H,F\}} m_{ij}(z) f_{ij}
\]

A.1.3 Aggregate Shipment = Aggregate Expenditure

Define \( f_{ii} = f_{jj} = f^d \) and \( f_{ij} = f_{ji} = f^e \) and \( f^e \equiv \eta > 1 \). In the following, the index \( z \) is dropped where convenient since derivations are identical for all sectors \( z \). The share of the expenditure on sector \( z \) that is spent on domestic and foreign goods respectively is given by:

\[
\alpha R_i = M_i \int_{\varphi_i(z)}^{\infty} \frac{p_{ii}(\varphi) c_{ii}(\varphi) g(\varphi) d\varphi}{1 - G(\varphi_{ii})} + M_j m_{ji} \int_{\varphi_{ji}}^{\infty} \frac{p_{ji}(\varphi) c_{ji}(\varphi) g(\varphi) d\varphi}{1 - G(\varphi_{ji})} \\
= M_i \int_{\varphi_i(z)}^{\infty} \frac{(w_i^\sigma)^{1-\sigma} \alpha R_i \beta \varphi^{\sigma-2} d\varphi}{1 - G(\varphi_{ii})} + M_j m_{ji} \int_{\varphi_{ji}}^{\infty} \frac{\tau_{ij}(w_j^\sigma)^{1-\sigma} \alpha R_i \beta \varphi^{\sigma-2} d\varphi}{1 - G(\varphi_{ji})} \\
= M_i w_i \theta f^d \sigma + M_j m_{ji} \theta w_j f^e \sigma
\]

Analogue:

\[
\alpha R_j = M_j w_j \theta f^d \sigma + M_i m_{ij} \theta w_i f^e \sigma
\]

Share of domestic spending on domestic goods:

\[
\frac{M_i w_i \theta f^d \sigma}{M_i w_i \theta f^d \sigma + M_j m_{ji} \theta w_j f^e \sigma} = \frac{M_i w_i f^d}{M_i w_i f^d + M_j m_{ji} w_j f^e} = \frac{M_i w_i}{M_i w_i + M_j m_{ji} w_j \eta}
\]

Share of domestic spending on foreign goods:

\[
\frac{M_j m_{ji} w_j \eta}{M_i w_i + M_j m_{ji} w_j \eta}
\]
Share of foreign spending on domestic goods:

\[
\frac{M_i m_{ij} w_i \eta}{M_j w_j + M_i m_{ij} w_i \eta}
\]

Share of foreign spending on foreign goods:

\[
\frac{M_j w_j}{M_j w_j + M_i m_{ij} w_i \eta}
\]

Aggregate shipment = Aggregate expenditure:

\[
\alpha R_i = M_i r_i = \frac{M_i w_i f^d}{M_i w_i f^d + M_j m_{ji} w_j f^x} + \alpha w_j L_j \frac{M_i m_{ij} w_i f^x}{M_j w_j f^d + M_i m_{ij} w_i f^x}
\]

\[\Leftrightarrow \quad \bar{r}_i = \alpha w_i L_i \frac{w_i f^d}{M_i w_i f^d + M_j m_{ji} w_j f^x} + \alpha w_j L_j \frac{m_{ij} w_i f^x}{M_j w_j f^d + M_i m_{ij} w_i f^x}
\]

\[\Leftrightarrow \quad w_i \theta \sigma (m_{ij} f^x + f^d) = \alpha w_i L_i \frac{w_i f^d}{M_i w_i f^d + M_j m_{ji} w_j f^x} + \alpha w_j L_j \frac{m_{ij} w_i f^x}{M_j w_j f^d + M_i m_{ij} w_i f^x}
\]

\[\Leftrightarrow \quad \theta \sigma (m_{ij} f^x + f^d) = \alpha w_i L_i \frac{f^d}{M_i w_i f^d + M_j m_{ji} w_j f^x} + \alpha w_j L_j \frac{m_{ij} f^x}{M_j w_j f^d + M_i m_{ij} w_i f^x}
\]

Analogue:

\[\theta \sigma (m_{ji} f^x + f^d) = \alpha w_j L_j \frac{f^d}{M_i w_i f^d + M_j m_{ij} w_i f^x} + \alpha w_i L_i \frac{m_{ij} f^x}{M_i w_i f^d + M_j m_{ji} w_j f^x}
\]

A.1.4 Solving for the Number of Active Firms

Normalize \( w_j \equiv 1, L_j \equiv 1 \), hence \( R_i = w_i L_i = w L = R \) and write down two conditions:

I)

\[\theta \sigma (m_{ij} f^x + f^d) = \alpha R \frac{f^d}{M_i w f^d + M_j m_{ji} w f^x} + \alpha m_{ij} f^x
\]

II)

\[\theta \sigma (m_{ji} f^x + f^d) = \alpha M_i w f^d + M_j m_{ij} w f^x
\]

Since

\[\frac{\alpha R}{\alpha \theta} = M_i w_i \theta f^d \sigma + M_j m_{ji} \theta w_j f^x \sigma
\]

\[\Leftrightarrow \quad M_j = \left( \frac{\alpha R - M_i w f^d}{m_{ji} f^x} \right)
\]
Hence:

\[ \frac{\alpha}{\sigma^2} R f^d - f^d M_i w f^d = \frac{\alpha}{\sigma^2} m_{ji} f^x - M_i m_{ij} w f^x m_{ji} f^x \]

\[ \iff M_i = \frac{\alpha}{\sigma^2 w} \frac{R f^d - m_{ji} f^x}{(f^x)^2 - m_{ij} m_{ji} (f^x)^2} \]

Hence, \( M_j \) is given by:

\[ M_j = \left( \frac{\alpha}{\sigma^2} R - M_i w f^d \right) \]

\[ = \frac{\alpha}{\sigma^2} R \frac{w f^d}{m_{ji} f^x} \left( \frac{\alpha}{\sigma^2} \left( \frac{f^x m_{ji} - R f^d}{w (m_{ij} m_{ji} (f^x)^2 - (f^d)^2)} \right) \right) \]

\[ = \frac{\alpha}{\sigma^2} \left( \frac{R m_{ij} m_{ji} (f^x)^2 - R f^d + f^d R f^d - f^d f^x m_{ji}}{m_{ji} f^x (m_{ij} m_{ji} (f^x)^2 - (f^d)^2)} \right) \]

\[ = \frac{\alpha}{\sigma^2} \left( \frac{f^d - R m_{ij} f^x}{(f^d)^2 - m_{ij} m_{ji} (f^x)^2} \right) \]

### A.1.5 Solving for the Export Shares

Assuming conventional sorting holds, the free entry condition can be rewritten in relative form:

\[ \frac{p^i_n}{\sum_{j \in \{H,F\}} m_{ij} f_{ij}} \frac{\sum_{j \in \{H,F\}} m_{ji} f_{ji}} {p^i_n} = 1 \]

\[ \frac{(1 - G(\varphi_{ii}^*) (f^d + m_{ij} f^x))}{(1 - G(\varphi_{jj}^*) (f^d + m_{ji} f^x))} = 1 \]

\[ \iff \left( \frac{\varphi_{ii}^*}{\varphi_{jj}^*} \right)^{-\beta} = \frac{f^d + m_{ji} f^x}{f^d + m_{ij} f^x} \]

Since

\[ m_{ji} = \frac{1 - G(\varphi_{ji}^*)}{1 - G(\varphi_{jj}^*)} \]

\[ = \frac{1 - G(\varphi_{ji}^*)}{1 - G(\varphi_{ii}^*)} \frac{(1 - G(\varphi_{ii}^*))}{1 - G(\varphi_{jj}^*)} \]

\[ = \left( \frac{\varphi_{ji}^*}{\varphi_{ii}^*} \right)^{-\beta} \left( \frac{\varphi_{ii}^*}{\varphi_{jj}^*} \right)^{-\beta} \]

Analogue:

\[ m_{ij} = \left( \frac{\varphi_{ij}^*}{\varphi_{jj}^*} \right)^{-\beta} \left( \frac{\varphi_{ii}^*}{\varphi_{jj}^*} \right)^{\beta} \]
The ZPC in relative form yields:

\[
\frac{\varphi_{ij}^{\sigma-1}}{\varphi_{jj}^{\sigma-1}} = w^{\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d}
\]

and

\[
\frac{\varphi_{ji}^{\sigma-1}}{\varphi_{ii}^{\sigma-1}} = w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d}
\]

Hence,

\[
(\varphi_{ji}^{\sigma-1})^{-\beta} = \left( w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta}
\]

\[
\iff m_{ji} = \left( w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta} \left( f^d + m_{ji} f^x \right)
\]

\[
f^d m_{ji} + m_{ij} f^x m_{ji} = \left( w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta} \left( f^d + m_{ji} f^x \right)
\]

\[
m_{ij} = \frac{\left( w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta} \left( f^d + m_{ji} f^x \right) - f^d m_{ji}}{f^x m_{ji}}
\]

\[
m_{ij} = \left( w^{-\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta} \left( f^d \frac{1}{f^x m_{ji}} + 1 \right) - \frac{f^d}{f^x}
\]

Analogue:

\[
m_{ji} = \left( w^{\sigma} r^{\sigma-1} f^x \frac{f^d}{f^d} \right)^{-\beta} \left( f^d \frac{1}{f^x m_{ij}} + 1 \right) - \frac{f^d}{f^x}
\]

Plugging in \(m_{ji}\) in \(m_{ij}\) the quadratic equation can be solved:

\[
m_{ij} = \frac{(1 + m_{ij} \eta) \left( w^{\sigma} r^{\sigma-1} \eta \right)^{-\beta} \left( \eta (w^{-\sigma} r^{\sigma-1} \eta)^{-\beta} - 1 \right) + m_{ij}}{\left( \eta (1 + m_{ij} \eta) (w^{\sigma} r^{\sigma-1} \eta)^{-\beta} - \eta m_{ij} \right)}
\]

\[
\iff 0 = \frac{(1 + m_{ij} \eta) \left( w^{\sigma} r^{\sigma-1} \eta \right)^{-\beta} \left( \eta (w^{-\sigma} r^{\sigma-1} \eta)^{-\beta} - 1 \right) + m_{ij}}{\left( \eta (1 + m_{ij} \eta) (w^{\sigma} r^{\sigma-1} \eta)^{-\beta} - \eta m_{ij} \right)}
\]
\[(m_{ij})^2 \eta \left( \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1 \right) \]

\[+ m_{ij} \left( \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1 \right) \]

\[-m_{ij} \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \left( \eta \left( w^{-\sigma} \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1 \right) \]

\[- \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \left( \eta \left( w^{-\sigma} \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1 \right) \]

Two solutions can be obtained:

\[m_{ij1} = \left( \frac{\left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \left( 1 - \eta \left( w^{-\sigma} \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \right)} {\left( 1 - \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \right)} \right) > 0 \]

\[m_{ij2} = -\frac{1}{\eta} < 0. \]

where only \( m_{ij1} \) is a reasonable solution. Hence \( m_{ji} \) is given by:

\[ m_{ji} = \frac{\left( \frac{1}{m_{ij}} + \eta \right) \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1} {\eta \left( \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} - 1 \right) + \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}}} \]

\[= \frac{\eta}{\left( 1 - \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \right) \left( 1 - \eta \left( w^{-\sigma} \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}} \right)} \]

\[= \left( \frac{1 - \eta \left( w^\sigma \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}}} {1 - \eta \left( w^{-\sigma} \tau^{\sigma-1} \eta \right)^{\frac{\sigma^\beta}{\sigma-1}}} \right) \left( \frac{w^{-\sigma} \tau^{\sigma-1} \eta^{\frac{\sigma^\beta}{\sigma-1}}} {w^\sigma \tau^{\sigma-1} \eta^{\frac{\sigma^\beta}{\sigma-1}}} \right) \]
A.1.6 Balanced Trade Condition

The balanced trade condition can be derived noting that aggregate income is aggregate expenditure. Plug-in for aggregate income on the left hand-side: \( wL = R \)

\[
R_i = \int_0^1 M_i(z)\pi_i(z)dz = \int_0^1 \frac{\alpha(z)}{v_i^2(z)\theta(z)} \left( \frac{f^x m_{ji}(z) - R_i f_i^d}{m_{ij}(z)m_{ji}(z) (f^x)^2 - (f_i^d)^2} \right) w_i\theta(z)\sigma(z) \sum_{j\in\{H,F\}} m_{ij}(z)f_{ij}dz = \\
R_i = \int_0^1 \alpha(z) \left( \frac{f^x m_{ji}(z) - R_i f_i^d}{m_{ij}(z)m_{ji}(z) (f^x)^2 - (f_i^d)^2} \right) (f^d + m_{ij}(z)f^x)dz = \\
R_i = \int_0^1 \alpha(z) \left( \frac{f^d f^x m_{ji}(z) - R_i f_i^d f^d + f^x m_{ji}(z)m_{ij}(z) f^x - R_i f_i^d m_{ij}(z) f^x}{m_{ij}(z)m_{ji}(z) (f^x)^2 - (f_i^d)^2} \right) dz = \\
R_i = \int_0^1 \alpha(z) \left( \frac{\eta m_{ji}(z) - R_i + m_{ij}(z)m_{ij}(z) (\eta)^2 - R_i m_{ij}(z) \eta}{m_{ij}(z)m_{ji}(z) (\eta)^2 - 1} \right) dz = \\
R_i = \int_0^1 \alpha(z) \left( \frac{(\eta m_{ji}(z) - R_i) (1 + m_{ij}(z) \eta)}{m_{ij}(z)m_{ji}(z) (\eta)^2 - 1} \right) dz
\]

Plugging in for the export shares and noting that \( R_i = R = \int_0^1 \alpha(z)Rdz \):

\[
0 = \int_0^1 \alpha(z) \left[ \frac{R}{\beta(z)\sigma(z)} - \frac{\beta(z)\sigma(z)}{\sigma(z)w^{2\sigma(z)-1}} \right] dz
\]

A.2 Characteristics of the Equilibrium

A.2.1 Proof of Proposition 1

It is sufficient to conduct the proof for a single sector \( z \), as \( \alpha(z) \) is greater than 0 and does not change any of the results. The integrand of the balanced trade condition is denoted by \( T(w, z) \)

\[
T(w, z) = \left[ \frac{R}{\beta(z)\sigma(z)} w^{\sigma(z)-1} \tau(z)^{\beta(z)} \eta^{\beta(z)-1} - 1 \right] \tau(z)^{\beta(z)} \eta^{\beta(z)-1} - w^{\sigma(z)-1} \]

The derivative of \( T(w, z) \) is given by:

\[
T'(w, z) = \left[ -R \left( \frac{\beta(z)\sigma(z)}{\sigma(z)-1} w^{\sigma(z)-1} \tau(z)^{\beta(z)} \eta^{\beta(z)-1} - 1 \right)^2 \right] \tau(z)^{\beta(z)} \eta^{\beta(z)-1} - w^{\sigma(z)-1} \]

VIII
One can show that

$$T'(w, z) < 0$$

as long as:

$$-R \left( w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - 1 \right)^2 < \left( \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \right)^2$$

which is always true for $R > 0$. Hence, $T(w, z)$ is always falling in $w$. Examining $T(w, z)$ further shows that:

$$T(1, z) = \left[ \frac{R - 1}{\tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1}} \right] > 0,$$

as $R > 1$. Hence, a wage rate $w$ for which $T(w, z) = 0$ must be greater than one since $T(w, z)$ is always falling in $w$. Furthermore, $T(w, z)$ is ill defined where the denominator is 0:

$$T(w, z) = \int_0^1 \alpha(z) \left[ \frac{R \left( w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - 1 \right) - \left( w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} \right)}{\left( \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \right)} \right] dz$$

Hence, for either:

$$\tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} = w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} < 1$$

or:

$$\tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} = w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} > 1$$

It can be concluded that $T(w, z)$ has a solution for $1 < w < \tau(z)^{\frac{\sigma(z) - 1}{\sigma(z)}} \eta^{\frac{\beta(z) - \sigma(z)}{\beta(z)\sigma(z)}} \leftrightarrow \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} < w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}}, T(w, z) > 0$:

$$T(w, z) = \int_0^1 \alpha(z) \left[ \frac{R \left( w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - 1 \right) - \left( w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} \right)}{\left( \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \right)} \right] dz$$

Because the denominator is negative:

$$\left( \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \right) \left( \tau(z)^{\beta(z)\eta^{\beta(z)\sigma(z) - 1} - 1} - w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \right) < 0$$
And the numerator is negativ:

\[
R \left( w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}} - 1 \right) - \left( w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}} - 1 \right) = Rw^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \left( \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}} - w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \right) < 0
\]

As

\[
-w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \left( \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}} - w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \right) < 0
\]

and

\[
Rw^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \left( \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}} - w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \right) < 0
\]

Hence, \( T(w > \tau(z))^{\frac{\sigma(z)-1}{\sigma(z)} \eta^{\frac{\beta(z)-\sigma(z)+1}{\beta(z)\sigma(z)}}} > 0 \).

It is known that \( T(0) = 0 \) by recalling that \( R = wL \). Hence, as \( T(w, z) \) is always falling in \( w \), for \( w < \tau(z) \frac{1-\sigma(z)}{\sigma(z)} \eta^{\frac{\beta(z)-\sigma(z)+1}{\beta(z)\sigma(z)}} \), \( T(w, z) < 0 \). Summarizing these results across all sectors it can be concluded that:

\[
\max \left[ \tau(z)^{\frac{1-\sigma(z)}{\sigma(z)} \eta^{\frac{\beta(z)-\sigma(z)+1}{\beta(z)\sigma(z)}}} \right] < 1 < w < \min \left[ \tau(z)^{\frac{\sigma(z)-1}{\sigma(z)} \eta^{\frac{\beta(z)-\sigma(z)+1}{\beta(z)\sigma(z)}}} \right].
\]

Using the freeness of trade measure, \( \tau(z)^{\frac{\beta(z)}{\sigma(z)-1}+1} = \Gamma(z) \), this can be rewritten:

\[
0 < \Gamma(z) < 1 < w^{\frac{\beta(z)e(z)}{\sigma(z)-1}} < \Gamma(z)^{-1}
\]

\[
\iff 0 < \Gamma(z)w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} < 1
\]

\[
\iff 0 < \Gamma(z)w^{\frac{\beta(z)e(z)}{\sigma(z)-1}} < 1
\]

Rewriting \( m_{ij}(z) \):

\[
m_{HF}(z) = \left( \eta^{-1}\Gamma(z)w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \left( 1 - \Gamma(z)w^{\frac{\beta(z)e(z)}{\sigma(z)-1}} \right) \right) > 0
\]

\[
m_{FH}(z) = \left( \eta^{-1}\Gamma(z)w^{\frac{\beta(z)e(z)}{\sigma(z)-1}} \left( 1 - \Gamma(z)w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \right) \right) > 0
\]

where

\[
0 < \left( 1 - \Gamma(z)w^{\frac{\beta(z)e(z)}{\sigma(z)-1}} \right) < \left( 1 - \Gamma(z)w^{\frac{-\beta(z)e(z)}{\sigma(z)-1}} \right)
\]

In order to make sure that in every sector every country has at least some active firms, conditions on
the number of active firms have to be fulfilled.

\[ M_H(z) = \frac{\alpha(z)}{\sigma(z) \theta(z)} \frac{Rf^d - m_{FH}(z)f^x}{w((f^d)^2 - m_{HF}(z)m_{FH}(z)(f^x)^2)} \]

\[ M_F(z) = \frac{\alpha(z)}{\sigma(z) \theta(z)} \frac{f^d - Rm_{HF}(z)f^x}{((f^d)^2 - m_{HF}(z)m_{FH}(z)(f^x)^2)} \]

Since:

\[ \left( (f^d)^2 - m_{HF}(z)m_{FH}(z)(f^x)^2 \right) \]

\[ = (1 - m_{HF}(z)m_{FH}(z)\eta^2) \]

\[ = 1 - \Gamma(z)^2 > 0, \]

the following restriction on aggregate income must be met:

\[ \frac{m_{FH}(z)}{\eta} < R < \frac{1}{\eta m_{HF}(z)} \]

As \( m_{FH}(z) < 1 \) and \( \eta > 1 \) the left-hand side is always met since \( R > 1 \).

A.2.2 Proof of Proposition 2:

Rewriting the equilibrium entry cutoffs in relative terms using the free entry condition yields:

\[ \left( \frac{\varphi_{FF}^*(z)}{\varphi_{HH}^*(z)} \right)^{\beta(z)} = \frac{1 + m_{FH}(z)\eta}{1 + m_{HF}(z)\eta} \]

\[ \times \left[ 1 - \Gamma(z)w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \left( 1 - \Gamma(z)w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \right) \right] \]

\[ \times \left[ 1 - \Gamma(z)w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}} \left( 1 - \Gamma(z)w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}} \right) \right] \]

\[ = \frac{[1 - \Gamma(z)^2](1 - \Gamma(z)w^{\frac{-\beta(z)\sigma(z)}{\sigma(z) - 1}})}{[1 - \Gamma(z)^2](1 - \Gamma(z)w^{\frac{\beta(z)\sigma(z)}{\sigma(z) - 1}})} \]
From Proposition 1:

\[
0 < 1 - \Gamma(z)w^{\beta(z)\sigma(z)} < 1 - \Gamma(z)w^{\beta(z)\sigma(z) - 1}
\]

Hence,

\[
\left( \frac{\varphi_{FF}(z)^{\beta(z)}}{\varphi_{HH}(z)^{\beta(z)}} \right) > 1.
\]

Furthermore, using the ZPC in relative terms:

\[
\frac{\varphi_{HF}(z)^{\sigma(z) - 1}}{\varphi_{HF}(z)^{\sigma(z) - 1}} = w^{\sigma(z)\tau(z)^{\sigma(z) - 1}\eta}
\]

and

\[
\frac{\varphi_{FH}(z)^{\sigma(z) - 1}}{\varphi_{HH}(z)^{\sigma(z) - 1}} = w^{-\sigma(z)\tau(z)^{\sigma(z) - 1}\eta}
\]

the relative export cutoffs in terms of the entry cutoffs can be stated:

As \(\frac{\varphi_{FF}(z)}{\varphi_{HH}(z)} > 1\), \(w > 1\) and \(\frac{2\sigma(z)}{\sigma(z) - 1} > 1\)

\[
\frac{\varphi_{HF}(z)}{\varphi_{FH}(z)} > 1
\]

Thus, as long as conventional sorting holds:

\[
\varphi_{HH}(z) < \varphi_{FF}(z) < \varphi_{FH}(z) < \varphi_{HF}(z)
\]

Equally,

\[
m_{FH}(z) = 1 - G(\varphi_{FH}(z)) = \left( \frac{\varphi_{FH}(z)}{\varphi_{HF}(z)} \right)^{-\beta(z)} > m_{HF}(z) = \left( \frac{\varphi_{HF}(z)}{\varphi_{HH}(z)} \right)^{-\beta(z)}
\]

**A.2.3 Proof of Proposition 3**

The balanced trade condition approximates the Hanson and Xiang case of a continuum of Helpman Krugman sectors, as soon as \(\beta(z) \to \sigma(z) - 1\)

\[
0 = \int_{0}^{1} \alpha(z) \left[ \frac{R}{w^{\sigma(z)\tau(z)^{\sigma(z) - 1} - 1} - \frac{w^{\sigma(z)}}{\tau(z)^{\sigma(z) - 1} - w^{\sigma(z)}}} \right] dz
\]
A.3 Derivation of the Home Market Effect $H(z)$

The home market effect $H(z)$ is defined by the share of Home’s production in world production in industry $z$ minus the share of Home’s demand for that particular sector:

$$H(z) = \frac{M_H(z)\bar{r}_H(z)}{M_F(z)\bar{r}_F(z) + M_H(z)\bar{r}_H(z)} - \frac{\alpha(z)R_H}{\alpha(z) + \alpha(z)R_H}$$
$$= \frac{(R_H - m_{HF}(z)\eta)(1 + m_{HF}(z)\eta)}{(1 - R_Hm_{HF}(z)\eta)(1 + m_{HF}(z)\eta) + (R_H - m_{HF}(z)\eta)(1 + m_{HF}(z)\eta)} - \frac{R_H}{1 + R_H}$$
$$= \frac{R_H + \Gamma(z)^2 - \Gamma(z)w^{\sigma(z)-1}}{(1 + R_H)[1 + \Gamma(z)^2 - \Gamma(z)(w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}} + w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}})]} - \frac{R_H}{1 + R_H}$$
$$= \frac{1}{1 + R_H} \left[ \frac{R_H}{w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}} \Gamma(z)^{-1} - 1} - \frac{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}}{\Gamma(z)^{-1} - w^{\frac{\beta(z)\sigma(z)}{\sigma(z)-1}}} \right]$$

A.3.1 Sketch of Proof of Proposition 5

Elasticity of Substitution
Comparing two sectors $z_0$ and $z_1$ with $\beta(z_0) = \beta(z_1)$, $\Gamma(z_0) = \Gamma(z_1)$ and $\sigma(z_0) < \sigma(z_1)$, clearly $\frac{\sigma(z_1)}{\sigma(z_1)-1} < \frac{\sigma(z_0)}{\sigma(z_0)-1}$. Since $w > 1$, $\beta(z_0) = \beta(z_1) > 0$, $w^{\frac{\beta(z_0)\sigma(z_0)}{\sigma(z_0)-1}} < w^{\frac{\beta(z_0)\sigma(z_0)}{\sigma(z_0)-1}}$. Hence, unambiguously, $H(z_1) > H(z_0)$.

Productivity Dispersion
Comparing two sectors $z_0$ and $z_1$ with $\beta(z_0) > \beta(z_1)$, $\Gamma(z_0) = \Gamma(z_1)$ and $\sigma(z_0) = \sigma(z_1)$, clearly $\frac{\sigma(z_1)}{\sigma(z_1)-1} < \frac{\sigma(z_0)}{\sigma(z_0)-1}$. Since $w > 1$, $\sigma(z_0) = \sigma(z_1) > 0$, $w^{\frac{\beta(z_0)\sigma(z_0)}{\sigma(z_0)-1}} < w^{\frac{\beta(z_0)\sigma(z_0)}{\sigma(z_0)-1}}$. Hence, unambiguously, $H(z_1) > H(z_0)$.

Freeness of Trade
Comparing two sectors $z_0$ and $z_1$ with $\beta(z_0) = \beta(z_1)$, $\Gamma(z_0) > \Gamma(z_1)$ and $\sigma(z_0) = \sigma(z_1)$. Define $\Gamma(z_0) - \Gamma(z_1) = h$. Since $\Gamma(z_0) > \Gamma(z_1)$, $h > 0$. Clearly, $H(z_0) - H(z_1) = H(\Gamma(z_0)) - H(\Gamma(z_1)) < 0$, if $H(\Gamma(z_0)) - H(\Gamma(z_1)) = \frac{H(\Gamma(z_1)+h) - H(\Gamma(z_1))}{\Gamma(z_1)+h - \Gamma(z_1)} < 0$. Without loss of generality let $h \to 0$. Hence,

$$\lim_{h \to 0} \frac{H(\Gamma(z_1)+h) - H(\Gamma(z_1))}{\Gamma(z_1)+h - \Gamma(z_1)} = \frac{\partial H(z_1)}{\partial z_1} = \frac{R_Hw^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}} \Gamma(z_1)}{w^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}} \Gamma(z_1)^{-1} - 1} - \frac{w^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}} \Gamma(z_1)^{-2}}{\Gamma(z_1)^{-1} - w^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}}} < 0$$

$$\rightarrow 1 - \frac{R_H \left( \Gamma(z_1)^{-1} - w^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}} \right)^2}{\left( w^{\frac{\beta(z_1)\sigma(z_1)}{\sigma(z_1)-1}} \Gamma(z_1)^{-1} - 1 \right)^2} > 0$$
B Conventional Sorting

Conventional sorting holds as long as:

\[ \varphi_{FF}^*(z) < \varphi_{FH}^*(z) \]

\[ \left[ b^\beta(z) (\theta(z) - 1) \left( \frac{f^d + m_{FH}(z)f^x}{f^e} \right)^{1/\sigma(z)} \right] < \varphi_{HH}^*(z) w^{-\sigma(z)} \tau(z) \eta^{1/\sigma(z)-1} \]

\[ \frac{(1 + m_{FH}(z))}{(1 + m_{HF}(z))} < w^{-\sigma(z)} \tau(z) \beta(z) \eta^{\beta(z)} - \sigma(z) \]

\[ \left[ 1 - \eta \left( w^{\sigma(z)} \tau(z) \sigma(z) - 1 \right)^{\frac{-\beta(z)}{\sigma(z)-1}} \right] \left( 1 - \left( \eta \left( \tau(z) \sigma(z) - 1 \right)^{\frac{-\beta(z)}{\sigma(z)-1}} \right)^2 \right) < \frac{1}{\beta(z) \eta^{\beta(z)}} \]

\[ w^{\frac{\beta(z) \sigma(z)}{\sigma(z)-1}} \Gamma(z) \eta^{-1} \left[ 1 - w^{\frac{-\beta(z) \sigma(z)}{\sigma(z)-1}} \Gamma(z) \right] < 1 - w^{\frac{\beta(z) \sigma(z)}{\sigma(z)-1}} \Gamma(z) \]

\[ w^{\frac{\beta(z) \sigma(z)}{\sigma(z)-1}} \left( \Gamma(z) \eta^{-1} + \Gamma(z) \right) < 1 + \Gamma(z)^2 \eta^{-1} \]

\[ w^{\frac{\beta(z) \sigma(z)}{\sigma(z)-1}} < \frac{\eta \Gamma(z)^{-1} + \Gamma(z)}{1 + \eta} \]

From the balanced trade condition it is known that:

\[ w^{\frac{\beta(z) \sigma(z)}{\sigma(z)-1}} < \Gamma(z)^{-1} \]

Moreover:

\[ \frac{(\eta \Gamma(z)^{-1} + \Gamma(z))}{1 + \eta} < \Gamma(z)^{-1} \]

\[ (\eta \Gamma(z)^{-1} + \Gamma(z)) < \Gamma(z)^{-1} [1 + \eta] \]

\[ \Gamma(z) < \Gamma(z)^{-1} \]

Hence, as long as conventional sorting holds, the relative wage rate is always properly defined.