Population aging, education and skill premium in international trade

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Abstract

This paper is primarily concerned with the impacts of population aging upon education and skill premium. To study population aging, we utilize an overlapping-generation model where survival into the old stage is uncertain and population aging is modelled via a higher survival rate. Featuring the household, education and production sectors, our model is analysed under both autarky and two-country trade environments. Under autarky, we find that population aging encourages educational efforts and decreases skill premium. In contrast, under the two-country trade environment, population aging in one country encourages domestic education while discourages education of the other country. Moreover, in our classical trade framework without skill biased technical change, we find that population aging increases skill premium in all countries. Our study provides a new explanation for the observed skill premium rises over recent decades.

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1 Introduction

Population aging has become one of the most important demographic problems facing many countries in the world. Due to improvements in heath and medication systems, the death rates have decreased significantly since last century. In most of developed and many developing countries, the distribution of the population is shifted toward older ages (Weil, 2006).

The potential impacts of population aging upon the economy have attracted extensive analysis. In particular, numerous studies have investigated the impacts of population aging upon human capital accumulation via education.[1] The general consensus is that population aging increases overall educational efforts. Intuitively, people do education during young ages, and an increased longevity let them reap more benefits from education, which increases the incentives for education.

While current literature analyzes population aging and education in great details, they are restricted to either closed economy or small open economy models.[2] Due to these model restrictions, potential interactions between population aging and international trade are neglected. For example, Heijdra and Romp (2009) uses a small open economy to study how domestic population aging affects domestic education. In such analysis, the impacts of domestic population aging upon education in other countries cannot be analyzed.

Realizing the current model limitations, our study gives a first attempt to analyze population aging and education in an international trade framework under general equilibrium. We find important results that are neglected so far in current literature. In a two-country trade equilibrium, while domestic population aging encourages domestic education, it discourages education in the other country. In other words, population aging results in more domestic people but less foreign people doing education. We call this the *education stealing effect*, which to our best knowledge, is not discovered in current analysis on population aging and education.

In addition to population aging and education, another focus of this paper is skill premium, which has experienced significant rises in recent decades. Since late 1960s, there has been a remarkable increase in skill premium, in nearly all developed and many developing countries. Autor, Katz and Kearney (2008) plots the US full-time weekly wage distribution from 1963 to 2003 and they find the gap in pay between college and high school educated workers have risen consistently since 1979. Similar rises in skill premium since 1970s are found in the UK and other OECD countries as well (Acemoglu and Autor, 2010; Gosling et al., 2000; Atkinson, 2007). In addition, wage premium has also risen in many developing countries in recent decades (Parro, 2013; Burstein et al., 2013, Anderson, 2005; Goldberg and Pavcnik, 2007).

The rise in skill premium, representing a form of wage inequality, triggers numerous studies trying to find its underlying reason. Current literature investigates rises in skill

[1] Chapter 2.5 of Chen (2014) presents a literature review on the relationship between population aging, economic growth and education.

[2] Autarkic analysis of population aging and education can be found in Ludwig and Vogel (2010), de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Boucekkine et al. (200s) among others. Heidra and Romp (2009) studies aging and education in a small open economy framework.

premium mainly from two perspectives, technical change and international trade. Studies from the viewpoint of technical change is triggered by the concurrence between skill premium rises and remarkable technological progress in recent decades. According to the technical change explanation, technological progress such as advancements in information and communication technologies, favors the skilled labor more than the unskilled labor and this increases the skill premium (Acemoglu, 2003; Acemoglu and Autor, 2011). The technical change that favors skilled labor is called skill biased technical change. Closely related to skill biased technical change is a theory called capital skill complementarity. According to this theory, since technology is embodied in capital goods (Parro, 2013), technical change will decrease the relative price of capital goods, and because skilled labor, compared to the unskilled, are more complementary to capital goods, the declined price of capital goods will increase the relative demand for skilled labor, leading to increases in skill premium (Griliches, 1969; Autor, Kazt and Krueger, 1998; Koren and Csillag, 2011). In a more general sense, capital skill complementarity and skill biased technical change can be unified. According to Acemoglu (2003), skilled-biased technical change can be defined as "any change in technology that increases the aggregate demand for skills." Therefore, the capital skill complementarity argument can be treated as a form of skill biased technical change.

Besides skill biased technical change, many studies investigate rises in skill premium from an international trade point of view. In the classical Heckscher-Ohlin framework, the Stolper-Samuelson theorem predicts that international trade should decrease skill premium in countries abundant in unskilled labor, mostly developing countries. This is in sharp contrast with the empirical evidence, where skill premium increases in both developed and developing countries. Given the fact that classical trade models cannot explain the worldwide skill premium rises, many studies try to introduce skill biased technical change into international trade. International trade could lower the relative price of skill intensive goods, which triggers skill biased technical change, leading to higher skill premium (Acemoglu, 2003; Burstein and Vogel, 2010; Parro, 2013).

According to the above literature on skill premium, it seems that skill biased technical change is a necessary factor in raising skill premium, while if put into a longer historical viewpoint, it might not be true. Van Zanden (2009) analyzes the long-term development of the skill premium in western Europe since 1300. He studies the relationship between skill premium and estimates of population for England (1300-1800) and Italy (1326-1800) and finds that sill premium tends to increase in periods of population growth. Skill biased technical change is only a recent phenomenon (Acemoglu, 2009), which cannot explain the skill premium rises before 1800. Given the positive relationship between population growth and skill premium in Van Zanden's data sample, skill premium could be related to demographics.

Even if ignoring the strong relationship between population aging and skill premium, and only looking at the skilled biased technical change explanation, population aging could also play an important role. In Acemoglu and Autor (2011), they argue that demographic trends can be the trigger for the increase in skilled labor, which in turn raises skill premium. According to their arguments, demographic trends rather than skill biased technical change, could be the underlying reason for skill premium. To our best knowledge, this argument has not been formally analyzed in current literature.

Noticing the potentially important relationship between population aging and skill premium, our paper investigates whether population aging itself, without skill biased technical change, can induce increases in skill premium. Using a classical two-country two-good trade model, we study how population aging affects skill premium under both autarky and trade equilibrium. We find that while population aging decreases skill premium under autarky, it will increase skill premium in both countries under trade equilibrium. With international trade, population aging changes the production pattern such that skilled labor become scarcer, which raises their relative wage. Moreover, our model does not feature skill biased technical change, which suggests that the underlying reason for skill premium rise could be population aging, rather than skill biased technical change.

Our paper has two contributions. To our best knowledge, our study is the first to investigate how population aging and education interact in an international trade framework under general equilibrium. We find population aging encourages education under autarky because the payoff to education is higher with an increased longevity, consistent with current literature. However, under international trade equilibrium, population aging only encourages domestic education and it discourages education in the other country. While the current literature only looks at how population aging affects domestic education, our study highlights the significant implications of population aging for education across other countries.

The second contribution of our paper lies in finding that population aging, instead of skill biased technical change, could be the underlying reason for skill premium rises. While the literature emphasizes the role of skill biased technical change in raising skill premium, our model works with a form of technical change that does not bias any factor. With only population aging and classical international trade framework, we find that population aging increases skill premium, by varying production patterns across countries. Our finding highlights the important role of population aging in raising skill premium.

The rest of the paper is organized as follows. Section 2 presents the model in autarky and describes the problems in the household, education and production sectors. Section 3 solves the optimization problems of all sectors and derives the autarky equilibrium. In addition, the impacts of population aging under autarky equilibrium are analyzed via comparative statistic analysis. After the autarky analysis, section 4 extends the autarky model to a two-country trade model, derives the international trade equilibrium and studies the impacts of population aging under trade equilibrium. After obtaining the results of population aging under autarky and trade equilibrium, section 5 provides a detailed discussion. In particular, we discuss how population aging affects education and skill premium, and other economic variables such as growth rate of technology and relative good price. In the end, section 6 concludes the paper with a summary of the main results and some future research ideas.

2 Autarky model

Our autarky model is based on Eicher (1996). Eicher (1996) uses a two-period overlapping-generation (OLG) model, including household, education and production sectors, to study human capital accumulation, education and endogenous economic growth. Our model differs from Eicher (1996) in two aspects. First, in Eicher's model, survival from the young into the old stage is certain for all people. In order to study population aging, we introduce survival uncertainty and population aging is modelled via a higher survival rate. Second, Eicher's model deals with closed economy, while we analyze the model in both closed economy and two-country two-good trade environment.[3] Using this model, we study the impacts of population aging upon education and skill premium under both autarky and trade equilibrium. In this section, we give a complete description of the model under autarky, and its extension to a two-country trade framework is in section 4.

2.1 Household sector

In an OLG model, each agent lives for at most two periods, young and old. During each period the number of new born agents is assumed to be constant and normalized to one. Survival from the young into the old age stage is uncertain, with survival rate equal to s where $0 \leq s \leq 1$. Following the literature such as Ludwig and Vogel (2010) and Irmen (2009), population aging is modelled via an increase in the survival rate s. During each period, the number young agents is one, and the number of old agents is s, hence a higher survival rate s implies there are more old agents per unit of young, which is a key feature of population aging.

During each period, young agents are born as unskilled labor, and they can choose to stay unskilled or go to school. If a young agent decides to stay unskilled, in the young stage, he works as an unskilled labor and gets wage. He then decides how much to consume for young and how much to save for old. If he survives into the old stage, he retires and consumes. His old age consumption comes from two aspects. First, he gets his previous saving plus interest revenues. Second, a fraction $(1-s)$ of the total unskilled labor does not survive into the old stage, and the savings they left are assumed to be evenly distributed among the survived unskilled.

Following Blanchard (1985), we assume there exists a perfect annuity market against survival risks of unskilled labor, and the savings of those unskilled labor who fail to survive are evenly distributed to those unskilled labor who indeed survive. If during t each young unskilled saves s_t, since only a fraction s of the young unskilled can survive, each survived unskilled labor gets $\frac{(1+r_{t+1})}{s}s_t$ when old.[4]

The problem of an unskilled labor during t, as described above, is expressed by the following

$$\max_{c_{1t}^{UH}, c_{1t}^{UL}, c_{2t+1}^{UH}, c_{2t+1}^{UL}} U_t^U \equiv \left(logc_{1t}^{UH} + \alpha \cdot logc_{1t}^{UL}\right) + \delta \cdot s \cdot \left(logc_{2t+1}^{UH} + \alpha \cdot logc_{2t+1}^{UL}\right); \quad (1)$$

[3] Another paper of Eicher (1999), extends the model of Eicher (1996) into a trade framework. Eicher (1999) deals with a small open economy framework. In contrast, we work with a two-country two-good trade model under general equilibrium.

[4] Denoting the total number of unskilled labor during t as x_t, then since each unskilled labor saves s_t, total saving during t is equal to $x_t \cdot s_t$. This amount of saving, plus interest revenue, becomes $(1+r_{t+1}) \cdot x_t \cdot s_t$ during period $t+1$, where r_{t+1} is the pure market interest rate from t to $t+1$. During period $t+1$, the number of survived unskilled labor is equal to $s \cdot x_t$ and total saving $(1+r_{t+1}) \cdot x_t \cdot s_t$ is distributed evenly among the, so each agent gets the among $\frac{(1+r_{t+1}) \cdot x_t \cdot s_t}{s \cdot x_t} = \frac{1+r_{t+1}}{s} s_t$.

$$
\begin{aligned}
s.t. \quad p_{Ht} \cdot c_{1t}^{UH} + p_{Lt} \cdot c_{1t}^{UL} + s_t &= w_t^U; & (2)\\
p_{Ht+1} \cdot c_{2t+1}^{UH} + p_{Lt+1} \cdot c_{2t+1}^{UL} &= \frac{1+r_{t+1}}{s} s_t. & (3)
\end{aligned}
$$

In this specification, c_{1t}^{UH} is young unskilled's consumption of good Y_H during t, and c_{1t}^{UL} is young unskilled's consumption of good Y_L during t, similarly c_{2t+1}^{UH} is old unskilled's consumption of good Y_H during $t+1$, and c_{2t+1}^{UL} is old unskilled's consumption of good Y_L during $t+1$. p_{Ht} and p_{Lt} are prices of good Y_H and Y_L during t, similarly for p_{Ht+1} and p_{Lt+1}. s_t is the saving during t, w_t^U the wage rate during t, r_{t+1} the pure market interest rate from t to $t+1$ and s the survival rate. $\delta \in (0,1)$ is for inter-temporal discounting.

After describing the problem for unskilled labor, we come to the problem of schooling agents (students). Young agents can choose to stay unskilled or go to school for education. If they go to school (called *students*), they cannot work when young and must borrow to finance their young consumption and the tuition fee. If students survival into old, they become skilled labor and choose to work in the education sector or the production sector. Skilled labor working in the education sector are called *teachers* while those working in the production sector are called *engineers*. A skilled labor gets payoff in the old stage, pays back his young-age borrowing and consumes the amount left. The problem of a student is expressed by the following

$$
\max_{c_{1t}^{SH}, c_{1t}^{SL}, c_{2t+1}^{SH}, c_{2t+1}^{SL}} U_t^S \equiv \left(logc_{1t}^{SH} + \alpha \cdot logc_{1t}^{SL}\right) + \delta \cdot s \cdot \left(logc_{2t+1}^{SH} + \alpha \cdot logc_{2t+1}^{SL}\right); \qquad (4)
$$

$$
\begin{aligned}
s.t. \quad p_{Ht} \cdot c_{1t}^{SH} + p_{Lt} \cdot c_{1t}^{SL} + z_t &= b_t; & (5)\\
p_{Ht+1} \cdot c_{2t+1}^{SH} + p_{Lt+1} \cdot c_{2t+1}^{SL} &= w_{t+1}^S - \frac{1+r_{t+1}}{s} b_t. & (6)
\end{aligned}
$$

In the above specification, z_t is the tuition fee for each student, b_t is the borrowing when young, and w_{t+1}^S is the wage rate for a skilled labor when old (teachers and engineers must have the same wage under equilibrium and this common wage is denoted by w_{t+1}^S). All students must borrow in order to consume and pay the tuition fee when young, but some students do not survive into the old, and their borrowing must be repaid by some other people. We assume there exists a perfect annuity market against survival risks of students, and the total borrowing of students during t are evenly paid by those skilled labor surviving into the old age, which explains why $\frac{1+r_{t+1}}{s} b_t$ appears in equation (6).

In our model, a young agent can freely choose to stay unskilled or go to school. When deciding the career path, an agent compares lifetime welfare of two career paths, and choose the one yielding higher lifetime welfare. In equilibrium where there are both unskilled labor and schooling agents during each period, these two career paths must yield the same lifetime welfare.[5]

[5] The fact that there are both unskilled and skilled labor during equilibrium will be clear when we solve for the equilibrium, in appendix 8.1 and 8.3.

During young, the unskilled must save for old age consumption. At the same time, students must borrow to pay the tuition fee and finance their young consumption. To regulate the saving and borrowing during each period, we assume there exists a bond market. In equilibrium, within each period total saving of the unskilled is equal to total borrowing of the students.

2.2 Education sector

The education sector plays a very important role in our model, because new technology vintages are invented in the education sector. In other words, technological progress occurs in the education sector. The education sector in this model serves two purposes. First, in this sector students are trained and they become skilled labor after surviving into the old stage. Second, in the education process, new technology vintages are invented and this leads to technological progress. The education sector can be interpreted as the R&D sector.

During period t, teachers work with students to generate new technology vintage, which is denoted v_{t+1}. The created vintage v_{t+1} can be used for production during the next period $t+1$. Same as in Eicher (1996; 1999), the technology progress is modelled as

$$v_{t+1} - v_t = \mu \cdot v_t \cdot min\left(\gamma \cdot P_t, S_t\right), \qquad \mu > 0, \gamma > 1, \tag{7}$$

where P_t is the number of teachers and S_t is the number of students. Equation (7) implies that students and teachers are perfect complements with a fixed ratio γ. From equation (7) we can get the growth rate of technology vintages expressed as

$$\frac{v_{t+1} - v_t}{v_t} = \mu \cdot min\left(\gamma \cdot P_t, S_t\right). \tag{8}$$

As implied by equation (8), if the numbers of teachers and students both increase, the growth rate of technology vintages also increases. Intuitively, with more teachers and students, we can think of the economy having more researchers and this should generate faster technological progress. In the autarky equilibrium presented in section 3, the numbers of students and teachers are constant over time, and technology is growing at a constant rate.

2.3 Production sector

Following Eicher (1999), there are two goods for final consumption, Y_H and Y_L. Goods Y_H and Y_L differ in skilled-labor intensity and technology vintage complexity. During t, v_t is the relatively advanced technology vintage while v_{t-1} is the relatively old technology, from the previous period. The production of good Y_H requires skilled labor (engineers) H_t, unskilled labor L_t^H, and advanced technology v_t. The production good Y_L requires only unskilled labor L_t^L and relatively old technology v_{t-1}.

Production functions are expressed as

$$\begin{aligned} Y_{Ht} &= v_t \cdot (H_t)^\rho \cdot \left(L_t^H\right)^{1-\rho}, \qquad 0 < \rho < 1, && (9) \\ Y_{Lt} &= v_{t-1} \cdot \theta \cdot L_t^L, \qquad \theta > 0. && (10) \end{aligned}$$

In this specification, H_t is the number of engineers, L_t^H the number of unskilled labor working in Y_H sector, and L_t^L is the number of unskilled labor working in Y_L sector. During each period, the production of Y_H requires the advanced technology, while the production of Y_L only requires technology from the previous period. Therefore, Y_H is called high-tech good and Y_L is called low-tech good. The production of Y_H needs skilled labor and unskilled labor, while the production of Y_L only needs unskilled labor, hence the high-tech good Y_H is skilled labor intensive and the low-tech good Y_L is unskilled labor intensive.

In our model, labor allocation during each period implies the following

$$
\begin{aligned}
1 &= L_t + S_t. && (11)\\
L_t &= L_t^H + L_t^L, && (12)\\
s \cdot S_{t-1} &= P_t + H_t. && (13)
\end{aligned}
$$

The total number of newborn during each period is normalized to one in our model and this is represented by equation (11). Equation (12) captures the idea that unskilled labor are employed in the high-tech and low-tech good sectors. In addition, survived students from previous period (only a fraction of s will survive) become teachers and engineers this period and this is represented by equation (13).

2.4 A brief discussion of the autarky model

In this model, new born agents first choose their career paths, staying unskilled or going to school. Upon the career choice, they choose their consumption plans to maximize expected lifetime welfare, taking income, tuition fee, and good prices as given. If they succeed in surviving into the old age (only a fraction s can survive into old), unskilled labor retire, while skilled labor choose either to be a teacher or engineer. Teachers work in the education sector, educate the students, and create more advanced technology vintages. Engineers work in the high-tech good production sector, with some unskilled labor, using the advanced technology vintage. Other young unskilled labor work in the low-tech sector, with the relatively old technology vintage.[6]

We study the model under general equilibrium, where all input prices, wages and output prices are endogenous. The equilibrium numbers of unskilled labor, students, teachers and engineers are also endogenous. This implies that the growth rate of technology vintages, hence the rate of technological progress is endogenous. The most important exogenous parameter is the survival rate. A higher survival rate means more people will survive to old, and there are more older people per young, which is population aging.

[6]In our two-period OLG model, skilled labor work during old and they do not have a retirement period. In contrast, the unskilled labor retire when old. This causes asymmetry between skilled and unskilled. In appendix 8.7, we extend the two-period OLG model into three-period and skilled labor also have a retirement period. We analyse the impacts of population aging upon education and we find having a three-period OLG model does not qualitatively change our two-period results. Due to model simplicity, we use the two-period OLG in our model.

Till now we have described the structure of the model in autarky. In the next section, we solve for the autarky equilibrium and investigate the impacts of population aging on variables such as student number, relative wage of skilled to unskilled so on.

3 Autarky equilibrium and impacts of population aging under autarky

In this section, we solve for the autarky equilibrium. Moreover, the impacts of population aging upon the following variables are studied, which are equilibrium number of students S, equilibrium relative wage $\frac{w^S}{w^U}$, equilibrium relative price of two consumption goods $\frac{p_L}{p_H}$ and equilibrium growth rate of technology vintage $\left(\frac{v_{t+1}-v_t}{v_t}\right)$.[7]

The equilibrium number of student is a measure of overall educational efforts, where more students in equilibrium meaning a higher educational effort. The relative wage of skilled to unskilled labor gives the skill premium, which is a key focus of our study. Since our model autarky model will be extended to international trade model in next section, we need to investigate the relative price of the low-tech to high-tech goods. Cross-country differences in good prices could induce international trade. In addition, since technological progress is the engine for modern economic growth, we study how population aging affects the growth rate of technology.

In order to solve the autarky model, we need to look at five aspects, including household utility maximization, education sector equilibrium, production sector profit maximization, no career arbitrage between unskilled and skilled labor and bond market clearing. The solution process is tedious and to emphasize the main results, we choose to put the derivation process of the autarky equilibrium in appendix 8.1. The autarky equilibrium is summarized by the proposition below.

Proposition 1 *In autarky equilibrium, the student number, relative wage of skilled to unskilled labor, relative price of low-tech to high-tech good, and growth rate of technology vintage, are all constant through time. They are expressed as below*

$$S = \frac{\frac{\delta s^2}{1+\delta s^2} + \frac{\rho}{(1-\rho+\alpha)(1-\gamma\cdot s)}}{1 + \frac{\rho}{(1-\rho+\alpha)(1-\gamma\cdot s)}}, \tag{14}$$

$$\frac{w^S}{w^U} = \frac{\rho\gamma}{(1-\rho+\alpha)(\gamma\cdot s-1)}\frac{1-S}{S}, \tag{15}$$

$$\frac{p_L}{p_H} = \left(\frac{1-\rho}{\theta}\left(\frac{1-\rho-\alpha}{1-\rho}\right)^{\rho}\left(s-\frac{1}{\gamma}\right)^{\rho}\right)(1+\mu S)\left(\frac{S}{1-S}\right)^{\rho}, \tag{16}$$

$$\left(\frac{v_{t+1}-v_t}{v_t}\right) = \mu S. \tag{17}$$

[7] We use $\left(\frac{v_{t+1}-v_t}{v_t}\right)$ to denote the equilibrium growth rate of technology vintages. The expression $\frac{v_{t+1}-v_t}{v_t}$ is used to denote growth rate of technology vintages during a general period t.

In Proposition 1, all variables of interests are solved in reduced forms and expressed as functions of survival rate s. Our paper is concerned with population aging. Population aging is defined via an increase in s, so the results in Proposition 1 allow us to study how population aging affects the key economic variables in equilibrium. Using Proposition 1, we analyze and discuss the impacts of population aging below. To study the impacts of population aging, comparative statistic analysis are conducted, with respect to the survival rate s.

As shown in appendix 8.2, the equilibrium student number S expressed in (14) is increasing in s. With S increasing in s, it can be seen from equations (15) to (17) that the relative wage of skilled labor versus unskilled labor is decreasing in s, while the relative price of Y_L versus Y_H and the growth rate of technology vintage are increasing in s. These results are summarized in the following proposition.

Proposition 2 *Under autarky equilibrium expressed in Proposition 1, population aging defined as an increase in the survival rate s will increase the student number (S), relative price of low-tech to high-tech good ($\frac{p_L}{p_H}$) and growth rate of technology vintage (μS), while decrease the relative wage of skilled to unskilled labor ($\frac{w^S}{w^U}$).*

Proposition 2 shows that population aging defined as a higher survival rate encourages educational efforts. With a higher survival rate, students are more likely to survive to old. In our model, since students get no income when young and only get payoff during the old age, a higher survival probability increases their expected reward from education. Therefore, their incentives to do education are higher.

As the number of students increases, the number of unskilled labor will decrease, and this decreases the relative scarcity of skilled labor, which in turn decreases the relative wage of skilled labor. In the two production sectors, low-tech good Y_L is unskilled labor intensive while high-tech good Y_H requires more of skilled labor. When the relative wage of skilled labor decreases, the relative input price of Y_L to Y_H increases, and this translates into the higher relative output price of Y_L. Lastly, since student number is increasing in the survival rate and technology vintage growth is a positive by-product of the education sector, the growth rate is positively related with student number. With the student number increasing with population aging, so is the growth rate of technology. More discussion about the impacts of population aging will be done in section 5, when we discuss the relationship among population aging, education and skill premium under international trade.

4 Population aging under international trade

In this section, the autarky model in section 2 is extended to an international trade framework with two countries, Home and Foreign. This extension allows us to study the impacts of population aging upon education and skill premium under international trade in general equilibrium. With international trade, we can investigate how population aging in country affects education not only in this country itself, but also in other countries. Moreover, using a classical two-country two-good trade model, we will analyze whether population aging, via changing the relative supply of skilled labor and production pattern across countries, can explain the skill premium rises observed over recent decades.

4.1 Trade model description

We consider a world economy where there are only two countries, named Home and Foreign. In both Home and Foreign, there are household, education and production sectors. The structure of and the problem facing each sector is the same as in autarky. In Home, the household, education and production sectors problems are fully represented by equations (1) to (13). In Foreign, the household, education and production sectors are represented by equations (1) to (13) with all variables added a superscript *.

The survival rates in Home and Foreign are denoted by s and s^* respectively. In order to focus on population aging, Home and Foreign are assumed to be identical in every aspect except for the survival rates. The purpose of the international trade model is to study the impacts of population aging, and therefore, countries' differences in other aspects are assumed away. We assume that $s > s^*$, hence during each period the fraction of older agents is higher in Home than in Foreign. Empirically, due to better health and medication systems, population aging are more severe in developed countries compared to developing countries, so Home can be interpreted as more developed than Foreign.

Imagine the world with Home and Foreign, and initially there is no trade between them and Home and Foreign reach their respective autarky equilibrium. Our results in autarky equilibrium, Propositions 1 and 2 apply to Home and Foreign (with Foreign variables all denoted with *). Now we allow Home and Foreign open to free international trade in goods Y_H and Y_L. If there are cross-country price differences in Y_H or Y_L, trade could occur. For simplicity, we assume trade is free, with no tariffs, no government regulations and no transportation costs.

According to Propositions 1 and 2 and the assumption that $s > s^*$, the educational efforts are higher in Home and there are more students in Home relative to Foreign. This gives Home a technology advantage over Foreign and the relative price of the high-tech good is lower in Home. When Home and Foreign are open to free international trade, Home exports the high-tech good and Foreign exports the low-tech good. The occurrence of international trade could change the relative good price in both countries, which in turn affects the factor prices and educational efforts.

The trade model is analyzed under general equilibrium, with all input prices, wages, and output prices determined endogenously. In trade equilibrium, the total world consumption of Y_L (Y_H) is equal to the total world production of Y_L (Y_H) and this determines the equilibrium price of Y_L (Y_H). Output prices in turn determine the wage rate in each country. Taking output prices and wage rates as given, agents make consumption plans and career decisions, which determine equilibrium student numbers and the rates of technological progress in each country.

With international trade in goods embodying different technologies, there could exist opportunities for technology spillover. In our model, Home is technologically more advanced than Foreign. Home exports the high-tech good which embodies advanced technology, and this raises the possibility of technology spillover from Home to Foreign. Below, we describe how technology spillover happens.

Technology spillover

In order to obtain a tractable trade equilibrium, we impose some simplifying assumptions on how the technology vintages are determined in Foreign. When Home and Foreign trade, Home exports the high-tech Y_H to Foreign. During period t, technology vintage v_t is embodied in Y_H and exported to Foreign. We assume that during t, Foreign can fully study the technology v_t in the Foreign education sector. The Foreign education sector can work either with its own technology v^*_{Ht}, or technology from Home v_t. The research outcome of Foreign education sector during t are used for production during $t+1$. Moreover, since Home technology v_t is fully studied in Foreign during t, we assume Foreign can use v_t for production during $t+1$. The key assumption here is that Foreign can fully study v_t during t and v_t can be used for production in Foreign during $t+1$. Because Foreign can study everything about v_t, we say Foreign has complete knowledge over v_t. In appendix 8.6, we show the importance of assuming complete knowledge in our model.

During t, because $v_t > v^*_{Ht}$, the Foreign education sector will study and work on v_t, instead of v^*_{Ht}. Then during $t+1$, Foreign can use technology vintage $v^*_{Ht+1} = (1+\mu S^*_t) \cdot v_t$ for production of Y_H. During $t+1$, Foreign will use v_t to produce Y_L. There is no technology spillover from Foreign to Home, since Home has more advanced technology and has no benefit from studying Foreign's technology. The following table illustrates the technology spillover process, by showing the available technology vintages for production and R&D in Home and Foreign, during two consecutive periods t and $t+1$.

Table 1. Home and Foreign technology vintages during two consecutive periods

		Y_H sector	Y_L sector	Technology worked on in R&D
At t	Home	v_t	v_{t-1}	v_t
	Foreign	$(1+\mu S^*_{t-1}) \cdot v_{t-1}$	v_{t-1}	v_t
At $(t+1)$	Home	v_{t+1}	v_t	v_{t+1}
	Foreign	$(1+\mu S^*_t) \cdot v_t$	v_t	v_{t+1}

Till now we have finished describing the international trade model. In the analysis below, we solve for the trade equilibrium and analyze the impacts of population. We focus on the following endogenous variables, student number, relative wage, relative good price and relative scarcity of skilled to unskilled labor. Due to the trade model complexity, the trade equilibrium does not give reduced form solutions for these endogenous variables. However, comparative statistic analysis can be conducted, with respect to the survival rates in both countries. In this sense, the impacts of population aging upon the above variables can be studied. In the next section, we solve for the trade equilibrium and investigate the impacts of population aging.

4.2 Impacts of population aging in international trade equilibrium

In order to emphasize the main results of our model, we choose to put the details of deriving the international trade equilibrium to appendix 8.3. Here, we simply make use of four important equations, derived in appendix 8.3. These four equations fully describe our key variables of interests under trade and they are

$$\frac{w^S}{w^U} = \frac{\rho}{1-\rho}\cdot\left(\frac{H}{L^H}\right)^{-1}; \tag{18}$$

$$\frac{p_L}{p_H} = \frac{1-\rho}{\theta}\cdot(1+\mu S)\cdot\left(\frac{H}{L^H}\right)^{\rho}; \tag{19}$$

$$\frac{H}{L^H} = \frac{1+\alpha-\rho}{1-\rho}\cdot\frac{1}{2-S-S^*}\cdot\left((s\cdot S-\frac{S}{\gamma})+\left(\frac{1+\mu\cdot S^*}{1+\mu\cdot S}\right)^{1/\rho}\cdot(s^*S^*-\frac{S^*}{\gamma})\right) \tag{20}$$

$$1 = \frac{\left(s\cdot S-\frac{S}{\gamma}\right)+\left(\frac{1+\mu S^*}{1+\mu S}\right)^{1/\rho}\cdot\left(s^*\cdot S^*-\frac{S^*}{\gamma}\right)}{\left(s^*\cdot S^*-\frac{S^*}{\gamma}\right)+\left(\frac{1+\mu S}{1+\mu S^*}\right)^{1/\rho}\cdot\left(s\cdot S-\frac{S}{\gamma}\right)}\cdot\frac{\frac{\delta s^2}{(1+\delta s^2)S}-1}{\frac{\delta(s^*)^2}{(1+\delta(s^*)^2)S^*}-1}. \tag{21}$$

Equation (18) expresses the relative wage as a function of $\frac{H}{L^H}$, where $\frac{H}{L^H}$ is the ratio of skilled labor to unskilled labor in the high-tech good sector. When $\frac{H}{L^H}$ decreases, skilled labor becomes scarcer relative to unskilled labor, which drives up the relative wage of skilled labor.[8] Equation (19) expresses the relative good price as a function of the equilibrium student number and $\frac{H}{L^H}$. The relative good price is important for trade pattern between Home and Foreign.

It can be seen from equations (18) and (19) that the ratio $\frac{H}{L^H}$ plays an important role in determining the skill premium and relative good price. Therefore, we present equation (20) above, which expresses the ratio $\frac{H}{L^H}$ as a function of student numbers (S, S^*) and survival rates (s, s^*). Equation (21) shows the connection between student numbers and survival rates.

The most important exogenous variables are two survival rates, which measure population aging in Home and Foreign. Once we find how the survival rates affect the student numbers, using equation (21), we can analyze how population aging affects the ratio $\frac{H}{L^H}$ from equation (20). With these results in hand, equations (18) and (19) allow to investigate the impacts of population aging on skill premium and relative good price.[9] In appendix 8.4, the details of analyzing impacts of population aging upon education are presented and here we directly show the following proposition.

Proposition 3 *Under international trade between Home and Foreign, if population aging occurs in Home, namely s increases, the equilibrium student number in Home increases, while the equilibrium student number in Foreign decreases. On the other hand, if population aging occurs in Foreign, namely s^* increases, the equilibrium student number in*

[8]In our model, skilled labor work in the education sector or high-tech good sector. Unskilled labor work in the low-tech or high-tech good sector. Therefore, only the high-tech good sector employs both skilled and unskilled labor. The ratio of skilled to unskilled in the high-tech good sector gives a measure for the relative scarcity of skilled labor.

[9]Equations (18) to (21) apply to Home, and they also apply to Foreign, with all Home variables and Foreign variables interchaning each other (s interchanges with s^*, S with S^*, H with H^* and L^H with L^{H*}). For expositional simplicity, we only present the four equations for Home.

Home decreases, while the equilibrium student number in Foreign increases. Formally, we have

$$\frac{\partial S}{\partial s} > 0, \frac{\partial S^*}{\partial s} < 0, \frac{\partial S}{\partial s^*} < 0 \text{ and } \frac{\partial S^*}{\partial s^*} > 0.$$

According to the above proposition, population aging in any country leads to more student in this country itself, but less students in the other country. In other words, population aging encourages domestic education, while it discourages foreign education. This can be interpreted as if population aging 'steals' education from other country to domestic country and we call this education stealing effect. To the best of our knowledge, this is never mentioned in the current literature. We will discuss its significance later in section 5.

With the impacts of population aging upon educational efforts in hand, we can come to investigate how skilled to unskilled ratio $\frac{H}{L^H}$, skill premium $\frac{w^S}{w^U}$ and relative good price $\frac{p_L}{p_H}$ are affected by population aging. Leaving technical details in appendix 8.5, the following proposition summarizes the impacts of population aging.

Proposition 4 *Under international trade between Home and Foreign, in equilibrium, if population aging occurs in either Home or Foreign, namely if s or s^* increases, then in both Home and Foreign, the ratio of skilled to unskilled labor in high-tech sector decreases, the relative wage of skilled to unskilled labor increases, and the relative price of low-tech to high-tech good decreases. Formally, we have*

1. $\frac{\partial H/L^H}{\partial s} < 0$, $\frac{\partial w^S/w^U}{\partial s} > 0$, $\frac{\partial H^*/L^{H*}}{\partial s} < 0$, $\frac{\partial w^{S*}/w^{U*}}{\partial s} > 0$, *and* $\frac{\partial p_L/p_H}{\partial s} < 0$*;*
2. $\frac{\partial H/L^H}{\partial s^*} < 0$, $\frac{\partial w^S/w^U}{\partial s^*} > 0$, $\frac{\partial H^*/L^{H*}}{\partial s^*} < 0$, $\frac{\partial w^{S*}/w^{U*}}{\partial s^*} > 0$, *and* $\frac{\partial p_L/p_H}{\partial s^*} < 0$.[10]

With population aging in Home or Foreign, the ratio of skilled to unskilled labor in the high tech good sector decreases in both Home and Foreign, though the underlying reason is different. More importantly, population aging in either Home or Foreign leads to an increase in the skill premium. Propositions 3 and 4 summarize the most important results in trade equilibrium, and we discuss their significance in the next section.

5 Discussions for impacts of population aging

The impacts of population aging under autarky are presented in Proposition 2, while those under trade are presented in Propositions 3 and 4. In this section, we discuss the economic significance of population aging. In doing so, we focus on the impacts of population aging upon education and skill premium. Once we understand these two, the impacts of population aging on other economic variables (such as growth rate of technology vintages and relative good price) are much more straightforward.

[10] Under free trade, the prices of Y_H and Y_L are equal across Home and Foreign, hence superscript $*$ is not added to p_L or p_H.

5.1 Population aging and education

We begin discussing the significance of population aging from its impacts upon education. Education has received much attention in economic studies mainly due to its important role in economic growth. The engine of modern economic growth lies in technological progress, and human capital plays a key role in this process. Human capital is accumulated mainly by education, hence population aging's impacts upon education will be reflected in technological progress.

As mentioned in the introduction, numerous studies have investigated the relationship between population aging and education, but they are restricted to either closed economy or small open economy environments, and the impacts of population aging in one country upon education in other countries are neglected. From a technical point of view, our model is the first to combine population aging, education, technological progress and international trade together, and analyze them under general equilibrium. More importantly, from an economic point of view, our model provides the first investigation on the cross-country impacts of population aging upon education.

In order to fully understand population aging and education, we compare our results in autarky and trade equilibrium, expressed by Propositions 2 and 3. In autarky, population aging encourages education, while under international trade, population aging encourages domestic and discourages foreign education. During autarky, when the survival rate increases, students are more likely to survive and get payoff during the old stage. A higher survival probability increases their payoffs of doing education, leading to higher education incentives. This intuition is similar to the general arguments in current literature mentioned in introduction, where they reach the general consensus that population aging increases educational efforts by raising the expected payoff to education.

Our key difference from current literature and one of our main contributions lies in our finding that, under trade equilibrium, while population aging encourages domestic educational efforts, it will do the opposite in other countries. With population aging in Home, student number rises in Home but declines in Foreign. This is like Home 'steals' some students from Foreign, which we call education stealing effect.

In trade equilibrium, if Home survival rate increases, the expected future payoff of being a student in Home increases, which results in higher educational efforts at Home. Foreign country exports the low-tech good Y_L due to its poorer technology and a higher relative price of Y_H, and if there are more students in Home due to population aging, the initial cross-country differences in technology and good prices are reinforced and Foreign finds it beneficial to produce even more Y_L to export. Because the production of low-tech good Y_L only requires unskilled labor, more production of Y_L calls for more unskilled labor and this results in less students in Foreign. As a result, in trade equilibrium, a higher survival rate in Home discourages education in Foreign. On the other hand, if the survival rate in Foreign goes up (s^* increases), the incentive to do education in Foreign goes up. More students in Foreign let the Foreign technology grow faster and this will decrease the cross-country differences in technology and relative price, which in turn lowers Home's benefit to produce and export $Y_{H.}$ As a result, Home produces less high-tech good, and Home demands less skilled and more unskilled labor, resulting in lower educational efforts in Home.

The key mechanism of our education stealing effect lies in the change in production pattern after population aging. Under international trade, with population aging, the relative good price changes and this results in production pattern change. It is the change in production pattern that affects the relative demand of skilled and unskilled labor, which in turn leads to changes in educational efforts. The current literature focuses on either closed economy, where the cross-country effects of population aging are totally excluded, or small open economy, where the rest of world is assumed to be unaffected by domestic economic behavior. Only by analyzing population aging in a two-country trade model under general equilibrium, are we able to uncover this education stealing effect.

5.2 Population aging and skill premium

In this subsection, we discuss how population aging, international trade and skill premium are related. The skill premium rises in recent decades have received much economic analysis and the current literature focuses on skill biased technical change. In contrast with current literature, our model does not feature skill biased technical change and we focus on population aging in affecting skill premium. In addition, we find that international trade plays a key role in the connection between population aging and skill premium.

According to Propositions 2 and 4, population aging decreases skill premium in autarky but increases that under international trade equilibrium. The key underlying reason explaining these different results lies in how population aging affects the relative scarcity of skilled labor. Intuitively, if population aging makes skilled labor scarcer relative to unskilled labor, skill premium should increase.

Under autarky, after s increases, overall educational efforts increase and there are less unskilled labor in Home. More skilled labor and less unskilled labor in the autarkic economy decreases the relative scarcity of skilled labor, leading to a decrease of the skill premium. Under trade, after s goes up in Home, there are more skilled labor in Home. However, this does not decrease the relative scarcity of skilled labor at Home, since now Home shifts to produce more of high-tech good Y_H and less of low-tech good Y_L. In fact, a higher s makes Home specialize so much in Y_H that it *increases* the relative scarcity of skilled labor.[11] This can be seen from Proposition 4, where the ratio of skilled labor to unskilled labor in the Y_H sector, $\frac{H}{L^H}$, is decreasing in s, hence skilled labor becomes scarcer after population aging. Due to a higher scarcity of skilled labor, the relative wage of skilled labor (skill premium) increases. In Foreign, after s goes up, there are more unskilled labor and less skilled labor, which makes the skilled labor scarcer and increases the skill premium in Foreign.[12] A similar logic applies to the case where population aging occurs in Foreign, namely if s^* goes up.

In closed economy, population aging decreases skill premium. In contrast, with international trade, population aging in any country will increase skill premium for all countries in the world. As discussed in the introduction, skill premium has increased significantly during recent decades, in both developed and developing countries. The autarky effect of

[11] In this model, the production of low-tech good Y_L does not need skilled labor at all, so Y_H has extreme skilled-labor intensity.

[12] A formal analysis of skilled labor scarcity can be obtained from Proposition 4, where the ratio of skilled labor to unskilled labor in the Foreign Y_H sector, $\frac{H^*}{L^{H*}}$, is decreasing in s.

aging upon skill premium is inconsistent with empirical evidence, but it is not surprising. The current literature considers skill biased technological change as necessary in raising skill premium, while technological change is not skill biased in our model.[13]

From the distinct impacts of population aging upon skill premium under autarky versus trade, we can see that international trade plays a key role in the relationship between population aging and skill premium. In the literature, some recent studies (such as Parro, 2013; Burstein and Vogel, 2010; Loren and Csillag, 2011) combine international trade with skill biased technical change, and they find since international trade can induce skill biased technical change via changing relative good prices, skill premium rises. In our model, without skill biased technical change, the impact of trade upon skill premium is not to induce skill biased technical change, but rather, via changing production patterns and factor scarcity. In this sense, we have shown that the standard two-country two-good trade model can explain the increase in skill premium to a much better extend than currently thought.

5.3 Other economic impacts of population

Having analyzed the relationship between population aging, education and skill premium above, we now discuss other impacts of population aging on the economy. In particular, we talk about how population aging affects the relative good price and the rate of technological progress.

From Propositions 2 and 4, with population aging, the relative price of low-tech good $\frac{p_L}{p_H}$ increases in autarky but decreases under trade. Prices of Y_L and Y_H depend on input prices, which are the wages of skilled and unskilled labor. Y_L is unskilled labor intensive, hence a higher relative wage of skilled labor decreases the relative price of Y_L. This, together with the impacts of population aging upon skill premium, explains why population aging affects relative good price differently under autarky and trade equilibrium. Empirically, the argument that trade should increase the relative price of skill intensive goods is inconsistent with observed evidence, where the relative price of skill intensive goods stays constant or declining (Acemoglu, 2003). According to our model results, the relative price of skill intensive goods does decrease during international trade with population aging, fitting well with the empirical evidence.

In modern economy, the engine of growth lies in technological progress. In our model, technological progress is through the development of new technology vintages in the education (R&D) sector. Technology vintage development is a positive by-product of the education sector, hence if there are more students doing education, the growth rate of technology vintages is higher. In autarky, population aging increases the rate of technological progress due to higher educational efforts. During international trade, due to technology spillover, the situation is more complex.

Under international trade, if Home survival rate s increases, there are more students in Home and Home technological progress is faster (growth rate is equal to μS from equation (8)). Due to educational stealing, there are less students in Foreign and Foreign

[13] Technical change is not skill biased in our model. This can be seen from equations (22) and (22) in appendix 8.3. Dividing the two equations gives skill premium, which does not relate with technology vintage (v_t) but only depends on factor ratio H/L^H.

technological progress *tend to be* slower. However, due to technological spillover, Foreign could also enjoy a faster technological progress. According to our technology spillover assumption in subsection 4.1, during any period, technology producing the low-tech good is the same in Home and Foreign, which implies the technological progress rate in the low-tech good sector is equal to μS in both Home and Foreign, and it is increasing with Home population aging. In Foreign high-tech good sector, even if the decrease in S^* tends to lower its technological growth rate, but because Foreign's R&D is based on Home technology vintage (v_t during t), a faster growth in v_t can offset the negative impacts from a lowered S^*.

If population aging occurs in Foreign so s^* increases, Foreign steals education from Home, leading to a decrease in growth rate in Home. If this happens, even if Foreign has more students, technological progress might be slower in Foreign. The growth rate in the Foreign low-tech sector is equal to μS, so it is decreasing with Foreign population aging. Foreign R&D for the technology in high-tech sector is based on v_t from Home, which is growing at a slower rate and this negative impacts can be so large that the growth rate in Foreign high-tech sector is also slower.

Combing the above discussion, population aging in Home tends to benefit all countries while that in Foreign tends to do the opposite. Empirically, due to better health and medicine systems, population aging is more severe in developed and advanced countries, which means population aging occurring in Home is a more likely outcome. Therefore, with population aging, the worldwide technological progress grows at a faster speed.

6 Conclusion

Population aging, characterized by a remarkable increase in the fraction of old agents, has become the most important demographic problem to many countries. Using an overlapping-generation framework where population aging is modelled via a higher survival rate from the young to the old, our study analyzes the impacts of population aging upon educational efforts and skill premium, in both autarky and two-country trade equilibrium.

Population aging encourages domestic educational efforts. After population aging via a higher survival rate, the expected payoff from education is higher, and this leads to a higher incentive to do education. Moreover, in the two-country trade equilibrium, population aging in any country discourages educational efforts in the other country. In this sense, population aging 'steals' education from the other country, which we call the *education stealing effect*.

Moreover, we find some important results contributing to the skill premium literature. In autarky, after population aging, there are more skilled labor compared to unskilled labor, and the increase in the relative supply of skilled labor decreases skill premium. However, we get the opposite result in trade equilibrium. In trade equilibrium, population aging in any country will increase skill premium of both countries, via production reallocation. After population aging in any country, there are more skilled labor and this country will produce more of high-tech (skilled labor intensive) goods and less of low-tech (unskilled labor intensive) goods, and this leads to an increase in the relative demand of skilled labor, hence raising skill premium. In the other country, there are less skilled labor (because of education stealing effect) and the relative supply of unskilled labor will rise, which leads to a decline of the relative wage of unskilled labor.

There are several possibilities for future research. First, in this model we have used Cobb-Douglas forms in utility functions and production functions of two goods, for tractability. One possible generalization is to use constant elasticity of substitution function forms. This can allow for a general degree of substitution elasticity between good consumptions in utility and input employments in production. Second, in this model, population aging is via an increase in the survival probability. In a broader sense, population aging is a combination of a higher survival rate and a reduction in fertility. Population growth rate can be introduced into the model, and population aging is associated with a lower population growth rate. Third, physical capital can be added to the production functions, and this allows for a form of technology-capital complementarity. If capital substitutes more for unskilled labor than for skilled labor, then one conjecture is that technical change should raise skill premium under autarky as well as trade equilibrium. These extension possibilities complicate the current model much and we leave them for future research.

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8 Appendix

8.1 Derivation of the autarky equilibrium in Proposition 1

In this subsection, we present the details solving our autarky model in section 2. Our model consists of the household, education, production sectors. In addition, young agents are free to choose their career paths, and there will be career arbitrage opportunities in equilibrium. Moreover, since students borrow and unskilled labor save, there will be a bond market regulating all the borrowing and savings. In the analysis below, we first solve the autarky model from five separate aspects, which are household utility maximization, education sector equilibrium, production sector profit maximization, no career arbitrage and bond market clearing. Then we combine results in the five aspects to solve for the autarky equilibrium.

Household utility maximization

We begin the household sector described in subsection 2.1. Unskilled labor need to solve the problem expressed by equations (1) to (3), and their consumption decisions are expressed as

$$
\begin{aligned}
c_{1t}^{UH} &= \frac{1}{(1+\alpha)(1+\delta s)} \frac{w_t^U}{p_{Ht}}, && \text{(A1)}\\
c_{1t}^{UL} &= \frac{\alpha}{(1+\alpha)(1+\delta s)} \frac{w_t^U}{p_{Lt}}, && \text{(A2)}\\
c_{2t+1}^{UH} &= \frac{\delta(1+r_{t+1})}{(1+\alpha)(1+\delta s)} \frac{w_t^U}{p_{Ht+1}}, && \text{(A3)}\\
c_{2t+1}^{UL} &= \frac{\alpha\delta(1+r_{t+1})}{(1+\alpha)(1+\delta s)} \frac{w_t^U}{p_{Lt+1}}, && \text{(A4)}
\end{aligned}
$$

where schooling agents need to solve the problem expressed by equations (4) to (6), and their consumption decisions are expressed as

$$
\begin{aligned}
c_{1t}^{SH} &= \frac{s}{(1+\alpha)(1+\delta s)(1+r_{t+1})} \left(w_{t+1}^S - \frac{1+r_{t+1}}{s} z_t \right) \frac{1}{p_{Ht}}, && \text{(A5)}\\
c_{1t}^{SL} &= \frac{\alpha s}{(1+\alpha)(1+\delta s)(1+r_{t+1})} \left(w_{t+1}^S - \frac{1+r_{t+1}}{s} z_t \right) \frac{1}{p_{Lt}}, && \text{(A6)}\\
c_{2t+1}^{SH} &= \frac{\delta s}{(1+\alpha)(1+\delta s)} \left(w_{t+1}^S - \frac{1+r_{t+1}}{s} z_t \right) \frac{1}{p_{Ht+1}}, && \text{(A7)}\\
c_{2t+1}^{SL} &= \frac{\alpha\delta s}{(1+\alpha)(1+\delta s)} \left(w_{t+1}^S - \frac{1+r_{t+1}}{s} z_t \right) \frac{1}{p_{Lt+1}}. && \text{(A8)}
\end{aligned}
$$

Education sector equilibrium

As mentioned earlier in subsection 2.2, students and teachers are perfect complements with a fixed ratio γ. In equilibrium, the number of students, S_t, and the number of teachers, P_t, satisfy

$$\gamma \cdot P_t = S_t, \tag{A9}$$

and using equation (A9), the technology evolution equation (7) becomes

$$v_{t+1} - v_t = \mu v_t S_t. \tag{A10}$$

During any period t under equilibrium, teachers and engineers have the same wage, and this common wage is denoted w_t^S. It is assumed that teachers' wages are fully funded by students via tuition fees. Given the student-teacher ratio $\gamma > 1$, the tuition fee of each student is given by

$$z_t = \frac{w_t^S}{\gamma}. \tag{A11}$$

Production sector profit maximization

Under competitive equilibrium, taking the Y_H price and inputs prices as given, the Y_H sector solves the problem expressed by[14]

$$\max_{H_t, L_t^H} \quad p_{Ht} \cdot v_t \cdot (H_t)^\rho \left(L_t^H\right)^{1-\rho} - w_t^S H_t - w_t^U L_t^H, \tag{A12}$$

and the above problem yields the wage rates for skilled and unskilled labor via the inverse demand functions as

$$w_t^S = p_{Ht} \cdot v_t \cdot \rho \left(\frac{H_t}{L_t^H}\right)^{\rho-1}, \tag{A13}$$

$$w_t^U = p_{Ht} \cdot v_t \cdot (1-\rho) \left(\frac{H_t}{L_t^H}\right)^{\rho}. \tag{A14}$$

Similar to the Y_H, the Y_L sector solves the problem expressed by

$$\max_{L_t^L} \quad p_{Lt} \cdot v_{t-1} \cdot \theta \cdot L_t^L - w_t^U L_t^L, \tag{A15}$$

and the above problem yields the wage rate for unskilled labor as

$$w_t^U = p_{Lt} \cdot v_{t-1} \cdot \theta. \tag{A16}$$

In equilibrium, unskilled labor must have the same wage working in the Y_H or Y_L sector, hence in equation (22) and (A16) the same notation w_t^U is used for the common wage rate.

No career arbitrage

In this model, since a young agent chooses between unskilled labor and students, career arbitrage implies these two career choices must yield the same lifetime welfare and this is called no career arbitrage condition. According to Eicher (1996, 1999), since agents share identical utility functions, and face identical inter-temporal rates of transformation, unskilled labor and students must have the same total expenditure, both during young and old. In order to obtain the total expenditure when young and old, the savings of

[14]Due to constant returns to scale in the production function and pricing taking behavior, the Y_H sector has zero profit hence there is no need to specify the ownership of Y_H sector. The same applies for Y_L sector.

unskilled and borrowings of students are solved first, which then are used to solve for total expenditures.

Substituting consumption choices (22) and (22) into budget constraint (2), the saving of unskilled labor during young is

$$s_t = \frac{\delta s}{1+\delta s} w_t^U, \tag{A17}$$

and substituting consumption choices (22) and (22) into budget constraint (5), the borrowing of students during young is

$$b_t = \frac{s}{(1+\delta s)(1+r_{t+1})} \left(w_{t+1}^S + \delta(1+r_{t+1}) z_t \right). \tag{A18}$$

Using the saving function of unskilled labor and borrowing function of students, the total expenditures of unskilled labor and students during young and old are denoted by

$$\begin{aligned} y_{1t}^U &\equiv w_t^U - s_t, && \text{(A19)} \\ y_{2t+1}^U &\equiv \frac{1+r_{t+1}}{s} s_t, && \text{(A20)} \\ y_{1t}^S &\equiv b_t - z_t, && \text{(A21)} \\ y_{2t+1}^S &\equiv w_{t+1}^S - \frac{1+r_{t+1}}{s} b_t, && \text{(A22)} \end{aligned}$$

and since unskilled labor and students have the same total expenditure, both during young and old, the following must hold

$$\begin{aligned} y_{1t}^U &= y_{1t}^S, && \text{(A23)} \\ y_{2t+1}^U &= y_{2t+1}^S. && \text{(A24)} \end{aligned}$$

Equations (22) and (22), together with the total expenditure expressions (A17)-(22), yield the relationship between wages of skilled labor and unskilled labor as

$$w_t^U = \frac{s}{1+r_{t+1}} w_{t+1}^S - z_t. \tag{A25}$$

Bond market clearing

In this model, unskilled labor save when young for old consumption and students borrow when young to finance their education and young consumption. Borrowings of students and savings of unskilled labor are regulated by a bond market. During each period, bond market clears so the total borrowing is equal to total saving, namely

$$L_t \cdot s_t = S_t \cdot b_t. \tag{A26}$$

Substituting saving expression (A17) and borrowing expression (A18) into the above condition, the bond market clearing interest rate can be expressed as a function of the number of students and unskilled labor:

$$1 + r_{t+1} = \frac{w_{t+1}^S}{\delta s \left(\frac{L_t}{S_t} \cdot w_t^U - z_t \right)}. \tag{A27}$$

Till now, the five conditions (household utility maximization, education sector equilibrium, production profit maximization optimization, no career arbitrage and bond market clearing) are solved and represented by equations from (22) to (A27). In the next subsection, these equations will be used to solve for the autarky equilibrium.

Solving for the autarky equilibrium

The variables of particular interest are the equilibrium student size S, relative wage of skilled to unskilled $\frac{w^s}{w^U}$, relative price of low-tech to high-tech good $\frac{p_L}{p_H}$ and the growth rate of technology vintages $\frac{v_{t+1}-v_t}{v_t}$. Combining no career arbitrage condition (A25) and bond market clearing condition (A27) together, and using equations (11) and (A9), the numbers of students and teachers can be expressed as functions of the relative wage by

$$S_t = \frac{\delta s^2}{1+\delta s^2} \frac{1}{\frac{w_t^S}{\gamma w_t^U} + 1}, \tag{A28}$$

$$P_t = \frac{\delta s^2}{1+\delta s^2} \frac{1}{\frac{w_t^S}{w_t^U} + \gamma}, \tag{A29}$$

and substituting equation (22) into equilibrium interest rate equation (A27), the interest rate can be expressed in terms of wage rates as

$$1 + r_{t+1} = \frac{1}{s} \frac{w_{t+1}^S}{w_t^U + \frac{w_t^S}{\gamma}}. \tag{A30}$$

From equations (22) and (22), the relative price of good Y_L to good Y_H is expressed as

$$\frac{p_{Lt}}{p_{Ht}} = \frac{\alpha c_{1t}^{UH}}{c_{1t}^{UL}}, \tag{A31}$$

and from equations (22) and (22) applying to an agent born at $t-1$, the relative price of good Y_L to good Y_H is expressed as

$$\frac{p_{LT}}{p_{Ht}} = \frac{\alpha c_{2t}^{UH}}{c_{2t}^{UL}}. \tag{A32}$$

Combining the above two equations, a basic algebraic property[15] implies

$$\frac{p_{Lt}}{p_{Ht}} = \alpha \cdot \frac{1 \cdot c_{1t}^{UH} + s \cdot c_{2t}^{UH}}{1 \cdot c_{1t}^{UL} + s \cdot c_{2t}^{UL}}. \tag{A33}$$

[15] If $\frac{w}{x} = \frac{y}{z}$, then $\frac{w}{x} = \frac{w+ay}{x+az}$ for any constant $a \neq -\frac{x}{z}$.

Due to equations (22) and (22) and since unskilled labor and students share the same utility structure, if they are born at the same period, their consumption plans are the same, namely $c_{1t}^{UH} = c_{1t}^{SH}$, $c_{1t}^{UL} = c_{1t}^{SL}$, $c_{2t+1}^{UH} = c_{2t+1}^{SH}$ and $c_{2t+1}^{UL} = c_{2t+1}^{SL}$. In period t, the number of new born agents is unity, and the number of old agents (survived from period $t-1$) is s, hence $1 \cdot c_{1t}^{UH} + s \cdot c_{2t}^{UH}$ is the total consumption of good Y_H during t and $1 \cdot c_{1t}^{UL} + s \cdot c_{2t}^{UL}$ is the total assumption of good Y_L during t.

In autarky, total consumption of each good is equal to domestic production, namely, $1 \cdot c_{1t}^{UH} + s \cdot c_{2t}^{UH} = Y_{Ht}$ and $1 \cdot c_{1t}^{UL} + s \cdot c_{2t}^{UL} = Y_{Lt}$. The above equation (A33) can be expressed as

$$\frac{p_{Lt}}{p_{Ht}} = \frac{\alpha Y_{Ht}}{Y_{Lt}}. \tag{A34}$$

Using the production functions for Y_{Ht} and Y_{Lt}, (9) and (10), yields

$$\frac{Y_{Ht}}{Y_{Lt}} = \frac{v_t}{v_{t-1}} \frac{1}{\theta} \frac{L_t^H}{L_t^L} \left(\frac{H_t}{L_t^H} \right)^\rho. \tag{A35}$$

Substituting (A35) into (A34) yields

$$\frac{p_{Lt}}{p_{Ht}} = \frac{v_t}{v_{t-1}} \frac{\alpha}{\theta} \frac{L_t^H}{L_t^L} \left(\frac{H_t}{L_t^H} \right)^\rho. \tag{A36}$$

From technology evolution equation (A10) applied to period from $t-1$ to t, technology vintage growth is expressed as

$$\frac{v_t}{v_{t-1}} = 1 + \mu S_{t-1}. \tag{A37}$$

Market clearing for the unskilled implied the unskilled must achieve the same wage in Y_H and Y_L sector. Combining (22) and (A16) yields another expression for the relative price as

$$\frac{p_{Lt}}{p_{Ht}} = \frac{v_t}{v_{t-1}} \frac{1-\rho}{\theta} \left(\frac{H_t}{L_t^H} \right)^\rho. \tag{A38}$$

Combining (A36) and (A38) yields

$$\frac{L_t^H}{L_t^L} = \frac{1-\rho}{\alpha}. \tag{A39}$$

Since during each period the number of new born agents is unity, $L_t^H + L_t^L = L_t = 1 - S_t$. L_t^H and L_t^L can be expressed as the following

$$L_t^H = \frac{1-\rho}{1-\rho+\alpha} L_t = \frac{1-\rho}{1-\rho+\alpha}(1-S_t), \tag{A40}$$

$$L_t^L = \frac{\alpha}{1-\rho+\alpha} L_t = \frac{\alpha}{1-\rho+\alpha}(1-S_t). \tag{A41}$$

From the teacher-student number relation (A9), and equation (13), H_t can expressed as

$$H_t = s \cdot S_{t-1} - P_t = s \cdot S_{t-1} - \frac{S_t}{\gamma}. \tag{A42}$$

Substituting equations (A39) to (A42) into the relative price formula (A38) yields the relative price as a function of student number as

$$\frac{p_{Lt}}{p_{Ht}} = (1 + \mu S_{t-1}) \frac{1-\rho}{\theta} \left(\frac{s \cdot S_{t-1} - \frac{S_t}{\gamma}}{\frac{1-\rho}{1-\rho+\alpha}(1 - S_t)} \right)^{\rho} \tag{A43}$$

$$= \frac{(1 + \mu S_{t-1})(1-\rho)}{\theta} \left(\frac{(1-\rho+\alpha)\left(s \cdot S_{t-1} - \frac{S_t}{\gamma}\right)}{(1-\rho)(1 - S_t)} \right)^{\rho}. \tag{A44}$$

Combining equations (22) and (22), the relative wage rate can be expressed as a function of the student number as

$$\frac{w_t^S}{w_t^U} = \frac{\rho(1 - S_t)}{(1-\rho+\alpha)\left(s \cdot S_{t-1} - \frac{S_t}{\gamma}\right)}. \tag{A45}$$

As shown in Eicher (1996) page 136, in equilibrium, the number of student is constant through time hence $S_{t-1} = S_t \equiv S$. All variables without time subscripts denote the equilibrium values of the associated variables. Substituting equation (A45) into (22), the equilibrium student number can be solved. Substituting the equilibrium student number into equation (A45) gives the equilibrium relative wage, into equation (A44) gives the relative price of low-tech good to high-tech good, into (A37) gives the equilibrium growth rate of technology vintage. The results are summarized in Proposition 1 in section 3.

8.2 Proof of $\frac{\partial S}{\partial s} > 0$ in Proposition 2

This subsection shows the proof for the result $\frac{\partial S}{\partial s} > 0$ in Proposition 2, where population aging will increase the student number in autarky equilibrium. The autarky equilibrium student number is expressed by equation (14), reproduced here as

$$S = \frac{\frac{\delta s^2}{1+\delta s^2} + \frac{\rho}{(1-\rho+\alpha)(1-\gamma \cdot s)}}{1 + \frac{\rho}{(1-\rho+\alpha)(1-\gamma \cdot s)}}.$$

The above equation can be expressed as

$$S = \frac{g(s) + h(s)}{1 + h(s)} \tag{14'}$$

where

$$g(s) \equiv \frac{\delta s^2}{1 + \delta s^2} = \frac{1}{1 + \frac{1}{\delta s^2}},$$

$$h(s) \equiv \frac{\rho}{(1-\rho+\alpha)(1-\gamma \cdot s)}.$$

We note that $0 < g(s) < 1$ and since the denominators of $g(s)$ is decreasing in s, $g'(s) > 0$. Moreover, the denominators of $h(s)$ is decreasing in s so $h'(s) > 0$. Differentiating equation (14') with respect to s yields

$$\frac{\partial S}{\partial s} = \frac{g'(s)(1+h(s)) + (1-g(s))h'(s)}{(1+h(s))^2}.$$

Using the facts that $0 < g(s) < 1$, $g'(s) > 0$ and $h'(s) > 0$, we get the result that $\frac{\partial S}{\partial s} > 0$, as in Proposition 2.

8.3 Solving the trade model

This subsection solves the trade model described in section 4. The basic model structure, expressed by equations (1) to (11), is still the same both in Home and Foreign, with all Foreign variables denoted by *. Accordingly, equations (22) to (A33) remain valid, for each country.

The procedures to solve for the trade equilibrium are exactly the same as the autarky equilibrium in subsection, and the only difference is in the market clearing condition for the consumption goods. Starting from equation (A33), which still applies to Home, the Foreign counterpart of equation (A33) is

$$\frac{p_{Lt}^*}{p_{Ht}^*} = \alpha \cdot \frac{1 \cdot c_{1t}^{UH*} + s^* \cdot c_{2t}^{UH*}}{1 \cdot c_{1t}^{UL*} + s^* \cdot c_{2t}^{UL*}}. \tag{A46}$$

With free trade and no transportation costs, and assume partial specialization in both country, it follows that

$$\begin{aligned} p_{Ht} &= p_{Ht}^*, \quad &\text{(A47)} \\ p_{Lt} &= p_{Lt}^*. \quad &\text{(A48)} \end{aligned}$$

Combining equations (A33), (A46)-(22) gives

$$\frac{p_{Lt}}{p_{Ht}} = \alpha \cdot \frac{1 \cdot c_{1t}^{UH} + s \cdot c_{2t}^{UH} + 1 \cdot c_{1t}^{UH*} + s^* \cdot c_{2t}^{UH*}}{1 \cdot c_{1t}^{UL} + s \cdot c_{2t}^{UL} + 1 \cdot c_{1t}^{UL*} + s^* \cdot c_{2t}^{UL*}}. \tag{A49}$$

With international trade in good Y_H and Y_L, total domestic consumption of each good is no longer equal to total domestic production. Instead, total world consumption is equal to total world production, and it follows that

$$\begin{aligned} 1 \cdot c_{1t}^{UH} + s \cdot c_{2t}^{UH} + 1 \cdot c_{1t}^{UH*} + s^* \cdot c_{2t}^{UH*} &= Y_{Ht} + Y_{Ht}^*, \quad &\text{(A50)} \\ 1 \cdot c_{1t}^{UL} + s \cdot c_{2t}^{UL} + 1 \cdot c_{1t}^{UL*} + s^* \cdot c_{2t}^{UL*} &= Y_{Lt} + Y_{Lt}^*. \quad &\text{(A51)} \end{aligned}$$

Combining equations (A49)-(22) yields

$$\frac{p_{Lt}}{p_{Ht}} = \alpha \cdot \frac{Y_{Ht} + Y_{Ht}^*}{Y_{Lt} + Y_{Lt}^*}, \tag{A52}$$

and this equation is the international trade counterpart of equation (A34). Solving the trade model, in a manner similar to the autarky model (steps from equations (A34) to (A45)), gives

$$\frac{p_{Lt}}{p_{Ht}} = \alpha \frac{v_t \cdot (H_t)^\rho \cdot (L_t^H)^{1-\rho} + v_{Ht}^* \cdot (H_t^*)^\rho \cdot (L_t^{H*})^{1-\rho}}{v_{t-1} \cdot \theta \cdot (L_t^L) + v_{Lt}^* \cdot \theta \cdot (L_t^{L*})} = \frac{v_t}{v_{t-1}} \frac{1-\rho}{\theta} \left(\frac{H_t}{L_t^H}\right)^\rho, \tag{A53}$$

$$\frac{v_t}{v_{t-1}} \frac{1-\rho}{\theta} \left(\frac{H_t}{L_t^H}\right)^\rho = \frac{v_{Ht}^*}{v_{Lt}^*} \frac{1-\rho}{\theta} \left(\frac{H_t^*}{L_t^{H*}}\right)^\rho, \tag{A54}$$

$$L_t^H + L_t^L = 1 - S_t, \tag{A55}$$

$$L_t^{H*} + L_t^{L*} = 1 - S_t^*, \tag{A56}$$

$$H_t = s \cdot S_{t-1} - \frac{S_t}{\gamma}, \tag{A57}$$

$$H_t^* = s^* \cdot S_{t-1}^* - \frac{S_t^*}{\gamma}, \tag{A58}$$

where $v_{Ht}^*(v_{Lt}^*)$ denotes the technology vintage used to produce $Y_H(Y_L)$ during t in Foreign.

The first equality of (A53) is obtained by putting production functions for Y_H and Y_L (equations (9), (10) and their Foreign counterparts) into equation (A52). The second equality of (A53) is obtained by combining (22) and (A16). Combining (22), (A16), their Foreign counterparts, and (22) and (22), gives equation (A54). (A55) is obtained by (11) and (12), with (A56) as its Foreign counterpart. (A57) is rewriting (A42), with (A58) as its Foreign counterpart.

The first step is to solve for six variables, L_t^H, L_t^L, L_t^{H*}, L_t^{L*}, H_t and H_t^* as functions of S_t, S_{t-1}, S_t^* and S_{t-1}^*. The second step is to solve the equilibrium values of S_t, S_{t-1}, S_t^* and S_{t-1}^*.

Using technology spillover assumption in subsection 4.1, it follows that $v_{Lt}^* = v_{t-1}$ and $v_{Ht}^* = (1 + \mu S_t^*) \cdot v_{t-1}$, then using these expressions, equations (A53) and (A54) become

$$L_t^H + L_t^{H*} = \frac{1-\rho}{\alpha}(L_t^L + L_t^{L*}), \tag{A59}$$

$$(1 + \mu \cdot S_t) \left(\frac{H_t}{L_t^H}\right)^\rho = (1 + \mu \cdot S_t^*) \left(\frac{H_t^*}{L_t^{H*}}\right)^\rho. \tag{A60}$$

Using equations (A55)-(22), L_t^H, L_t^L, L_t^{H*}, L_t^{L*}, H_t and H_t^* can be solved as functions of S_t, S_{t-1}, S_t^* and S_{t-1}^*. For simplicity, only L_t^H and L_t^{H*} (expressions for other four variables are not used below) are presented below, as functions of S_t, S_{t-1}, S_t^* and S_{t-1}^*, as

$$L_t^H = \frac{1-\rho}{1+\alpha-\rho}(2 - S_t - S_t^*) \left(1 + \left(\frac{1+\mu S_t^*}{1+\mu S_t}\right)^{1/\rho} \cdot \frac{s^* \cdot S_{t-1}^* - S_t^*/\gamma}{s \cdot S_{t-1} - S_t/\gamma}\right)^{-1}, \tag{A61}$$

$$L_t^{H*} = \frac{1-\rho}{1+\alpha-\rho}(2 - S_t - S_t^*) \frac{\left(\frac{1+\mu S_t^*}{1+\mu S_t}\right)^{1/\rho} \cdot \frac{s^* \cdot S_{t-1}^* - S_t^*/\gamma}{s \cdot S_{t-1} - S_t/\gamma}}{1 + \left(\frac{1+\mu S_t^*}{1+\mu S_t}\right)^{1/\rho} \cdot \frac{s^* \cdot S_{t-1}^* - S_t^*/\gamma}{s \cdot S_{t-1} - S_t/\gamma}}. \tag{A62}$$

Combining (A61) and (A62) gives

$$\frac{H_t}{L_t^H} = \frac{1+\alpha-\rho}{1-\rho} \cdot \frac{1}{2-S_t-S_t^*} \cdot \left((s \cdot S_{t-1} - \frac{S_t}{\gamma}) + \left(\frac{1+\mu \cdot S_t^*}{1+\mu \cdot S_t} \right)^{1/\rho} \cdot (s^* S_{t-1}^* - \frac{S_t^*}{\gamma}) \right). \tag{A63}$$

Using (22) and (22), the relative wage can be expressed as

$$\frac{w_t^S}{w_t^U} = \frac{\rho}{1-\rho} \cdot \frac{L_t^H}{H_t}, \tag{A64}$$

where evaluating (A64) at the steady state gives equation (18). Putting equation (A64) into equation (22) gives

$$\frac{1+\delta s^2}{\delta s^2} \cdot S_t = \frac{\gamma}{\frac{\rho}{1-\rho} \cdot \frac{L_t^H}{H_t} + \gamma}. \tag{A65}$$

Combining (A63) and (A65) gives

$$\frac{(1+\alpha-\rho)\gamma}{(2-S_t-S_t^*)\rho} \left(\left(s \cdot S_{t-1} - \frac{S_t}{\gamma} \right) + \left(\frac{1+\mu S_t^*}{1+\mu S_t} \right)^{1/\rho} \left(s^* \cdot S_{t-1}^* - \frac{S_t^*}{\gamma} \right) \right) \left(\frac{\delta s^2}{(1+\delta s^2) S_t} - 1 \right) = 1. \tag{A66}$$

Repeating the above process for the Foreign, the Foreign counterpart of equation (A66) can be expressed as

$$\frac{(1+\alpha-\rho)\gamma}{(2-S_t-S_t^*)\rho} \left(\left(s^* \cdot S_{t-1}^* - \frac{S_t^*}{\gamma} \right) + \left(\frac{1+\mu S_t}{1+\mu S_t^*} \right)^{1/\rho} \left(s \cdot S_{t-1} - \frac{S_t}{\gamma} \right) \right) \left(\frac{\delta (s^*)^2}{(1+\delta (s^*)^2) S_t^*} - 1 \right) = 1. \tag{A67}$$

Similar to autarky equilibrium, in trade equilibrium, $S_t = S_{t-1} \equiv S$ and $S_t^* = S_{t-1}^* \equiv S^*$, and (A66) and (A67) become

$$\frac{(1+\alpha-\rho)\gamma}{(2-S-S^*)\rho} \left(\left(s \cdot S - \frac{S}{\gamma} \right) + \left(\frac{1+\mu S^*}{1+\mu S} \right)^{1/\rho} \cdot \left(s^* \cdot S^* - \frac{S^*}{\gamma} \right) \right) \left(\frac{\delta s^2}{(1+\delta s^2) S} - 1 \right) = 1, \tag{A68}$$

$$\frac{(1+\alpha-\rho)\gamma}{(2-S-S^*)\rho} \left(\left(s^* \cdot S^* - \frac{S^*}{\gamma} \right) + \left(\frac{1+\mu S}{1+\mu S^*} \right)^{1/\rho} \cdot \left(s \cdot S - \frac{S}{\gamma} \right) \right) \left(\frac{\delta (s^*)^2}{(1+\delta (s^*)^2) S^*} - 1 \right) = 1. \tag{A69}$$

The above two equations determine the equilibrium student numbers, S and S^* as functions of survival rates s and s^* and other exogenous variables. From equation (A53), the equilibrium relative price in Home can be expressed as

$$\frac{p_L}{p_H} = \frac{1-\rho}{\theta} \cdot (1+\mu S) \cdot \left(\frac{H}{L^H} \right)^{\rho},$$

which gives equation (19) , and $\frac{H}{L^H}$ is obtained from (A64) with all variables evaluated at the equilibrium values as

$$\frac{H}{L^H} = \frac{1+\alpha-\rho}{1-\rho} \cdot \frac{1}{2-S-S^*} \cdot \left((s \cdot S - \frac{S}{\gamma}) + \left(\frac{1+\mu \cdot S^*}{1+\mu \cdot S} \right)^{1/\rho} \cdot (s^* S^* - \frac{S^*}{\gamma}) \right),$$

which gives equation (20). In addition, dividing equations (A68) and (A69) gives (21) in subsection 4.2.

8.4 Solving the signs of $\frac{\partial S}{\partial s}$, $\frac{\partial S}{\partial s^*}$, $\frac{\partial S^*}{\partial s}$, $\frac{\partial S^*}{\partial s^*}$ in Proposition 3

Equation (21) is reproduced below

$$\underbrace{\frac{\left(s \cdot S - \frac{S}{\gamma}\right) + \left(\frac{1+\mu S^*}{1+\mu S}\right)^{1/\rho} \cdot \left(s^* \cdot S^* - \frac{S^*}{\gamma}\right)}{\left(s^* \cdot S^* - \frac{S^*}{\gamma}\right) + \left(\frac{1+\mu S}{1+\mu S^*}\right)^{1/\rho} \cdot \left(s \cdot S - \frac{S}{\gamma}\right)}}_{\text{Part I}} \cdot \underbrace{\frac{\frac{\delta s^2}{(1+\delta s^2)S} - 1}{\frac{\delta (s^*)^2}{(1+\delta (s^*)^2)S^*} - 1}}_{\text{Part II}} = 1.$$

We can rewrite Part I of equation (21) as

$$\text{Part I of (21)} = \frac{1+BA}{A+B^{-1}} = \frac{1+BA}{1+BA} \cdot B = B \equiv \left(\frac{1+\mu S^*}{1+\mu S}\right)^{1/\rho}$$

where $A \equiv \frac{s^* \cdot S^* - \frac{S^*}{\gamma}}{s \cdot S - \frac{S}{\gamma}}$ and we find that

- $\frac{\partial log(\text{Part I of (21)})}{\partial s} = \frac{\partial log(\text{Part I of (21)})}{\partial s^*} = 0$,
- $\frac{\partial log(\text{Part I of (21)})}{\partial S} < 0$,
- $\frac{\partial log(\text{Part I of (21)})}{\partial S^*} > 0$.

Now Part II of (21) is analyzed. s and S only enter the numerator, while s^* and S^* only enter the denominator of part II of (21). We note that the numerator of part II of (21) is increasing in s and decreasing in S, while the denominator of part II of (21) is increasing in s^* and decreasing in S^*. Therefore, the following results hold

- $\frac{\partial(\text{Part II of (21)})}{\partial s} > 0$;
- $\frac{\partial(\text{Part II of (21)})}{\partial s^*} < 0$;
- $\frac{\partial(\text{Part II of (21)})}{\partial S} < 0$;
- $\frac{\partial(\text{Part II of (21)})}{\partial S^*} > 0$.

Combing the above results, comparative statistic analysis can be conducted. The purpose of comparative statistic analysis is to find whether S and S^* are increasing or decreasing in s and s^*. The usual way to do comparative statistic analysis is to find the signs for all the derivatives, but here the expressions are messy so a simpler method is used. Namely, by looking at how part I and part II of (21) change with s and s^*, how S and S^* change with s and s^* can be determined and this is done in the next paragraph.

If s increases, part II of (21) increases while part I is unchanged. For (21) to hold again, either S increases, or S^* decreases. Similarly, if s^* increases, part II of (21) decreases, while part I is unchanged. For (21) to hold again, either S decreases, or S^* increases. To sum up, S is increasing in s, decreasing in s^*, while S^* is decreasing in s and increasing in s^*. This finishes the proof for Proposition 3.

8.5 Derivations of the comparative statistics results in Proposition 4

From equation (18), w^S/w^U is negatively related with H/L^H. From equation (19), p_L/p_H is positively related with S and H/L^H. We need to find how the ratio H/L^H is affected by survival rates. Combining equations (A68) and (20) gives the following

$$\frac{(1-\rho)\gamma}{\rho} \cdot \frac{H}{L^H} \cdot \left(\frac{\delta s^2}{(1+\delta s^2)S} - 1 \right) = 1.$$

If s^* increases, due to Proposition 2, S decreases, and for the above equation to hold, $\frac{H}{L^H}$ must decrease. This in turns implies that w^S/w^U increases and p_L/p_H decreases.

In the Foreign country, the analysis is similar to Home. Using the Foreign counterparts of equations (18), (19), (20) and equation (A69), we can find how H^*/L^{H*}, w^{S*}/w^{U*} and p_L/p_H change after an increase in s. After s increases, H^*/L^{H*} decreases, which implies w^{S*}/w^{U*} increases and p_L/p_H decreases.

8.6 The necessity of technology spillover during each period with complete knowledge

Section 4 works with a special form of technology spillover process. During period t, Home country (aging and advanced country) exports the high good Y_H which embodies the technology vintage v_t. It is assumed that v_t can be fully studied by Foreign during t, then during $t-1$, v_t becomes available in Foreign to produce Y_L. The assumption that v_t can be fully studied by Foreign can be called complete knowledge. The analysis here shows that the assumption of complete knowledge is necessary to have any equilibrium in which the fractions of students in both countries are constant through time.

The analysis starts from equation (A53). Note that in any equilibrium where the fractions of students are constant through time in both countries, the following variables are also constant through time: S_t, H_t, L_t^H, L_t^L, S_t^*, H_t^*, L_t^{H*}, and L_t^{L*}. Time subscripts are dropped to denote equilibrium values. During equilibrium, the technology vintage evolution in Home becomes

$$v_t = (1 + \mu S) \cdot v_{t-1}.$$

During t, Home will export Y_H to Foreign and technology vintage v_t is embodied in Y_H. During technology spillover, v_t can be partially or fully studied in Foreign and used in the next period t. Section 4 assumes that Foreign has complete knowledge about v_t so v_t can be completely studied by Foreign. Now this model works with a general case, where Foreign may not have complete knowledge about v_t so that only part of v_t can be studied. During t, there are three technology vintages in Foreign, which are

1. v_t: embodied in the imported Y_H from Home;

2. v_{Ht}^*: the relative advanced technology vintage during t in Foreign, used to produce Y_H;

3. v_{Lt}^*: the relative old technology vintage during t in Foreign, used to produce Y_L.

During the next period $t+1$, there are two technology vintages in Foreign used to produce goods. The first is v^*_{Ht+1}, relatively advanced, used to produce Y_H. The second is v^*_{Lt+1}, relatively old, used to produce Y_L. v^*_{Ht+1} is obtained via R&D during t, while v^*_{Lt+1} is carried from the available technology vintages during t. During t, both v_t and v^*_{Ht} are more advanced than v^*_{Lt}, and this implies: (1) R&D during t is conducted on v_t and v^*_{Ht} and the outcome is v^*_{t+1}; (2) some combination of v_t and v^*_{Ht} is carried onto $t+1$ and the outcome is v^*_{Lt+1}. A formal representation of the above idea is the following

$$
\begin{aligned}
v^*_{Ht+1} &= (1+\mu S^*)(\phi \cdot v_t + \cdot(1-\phi)\cdot v^*_{Ht}), \quad 0 \le \phi \le 1; && \text{(I)}\\
v^*_{Lt+1} &= \pi \cdot v_t + (1-\pi)\cdot v^*_{Ht}, \qquad 0 \le \pi \le 1. && \text{(II)}
\end{aligned}
$$

Equation (22) implies that, during t, the Foreign is doing R&D on v_t and v^*_{Ht}, with relative weights equal to ϕ and $(1-\phi)$. Similarly, as implied by equation (22), some combination of v_t and v^*_{Ht} is carried onto $t+1$ to produce Y_L and the relative weights are π and $(1-\pi)$.

Using equations (22) and (22), the $(t+1)$ equilibrium version of equation (A53) becomes

$$
\begin{aligned}
&\alpha\frac{v_{t+1}\cdot H^\rho \cdot (L^H)^{1-\rho} + (1+\mu S^*)(\phi\cdot v_t + (1-\phi)\cdot v^*_{Ht})\cdot (H^*)^\rho \cdot (L^{H*})^{1-\rho}}{v_t\cdot\theta\cdot L^L + (\pi\cdot v_t + (1-\pi)\cdot v^*_{Ht})\cdot\theta\cdot L^{L*}}\\
&= (1+\mu S)\frac{1-\rho}{\theta}\left(\frac{H}{L^H}\right)^\rho, \quad \text{for all } t \text{ during equilibrium.}
\end{aligned}
$$

Simplifying the notations for the time-invariant equilibrium variables, the above equation can be represented by the following

$$
\frac{\text{constant} + \text{constant}\cdot\left(\phi + (1-\phi)\frac{v^*_{Ht}}{v_t}\right)}{\text{constant} + \text{constant}\cdot\left(\pi + (1-\pi)\frac{v^*_{Ht}}{v_t}\right)} = \text{constant},
$$
$$
\text{holding for all } t \text{ during equilibrium.}
$$

In order to have the above equation to hold for all t during equilibrium, the following must be true

$$
\begin{aligned}
\phi &= \pi = 1;\\
\text{or}\quad \frac{v^*_{Ht}}{v_t} &= \text{constant}, \quad \text{for all } t \text{ during equilibrium.}
\end{aligned}
$$

Next, the ratio $\frac{v^*_{Ht}}{v_t}$ is solved using (22). The periods t, $t-1$ and $t-2$ versions of equation (22) are

$$
\begin{aligned}
v^*_{Ht} &= (1+\mu S^*)(\phi\cdot v_{t-1} + \cdot(1-\phi)\cdot v^*_{Ht-1}),\\
v^*_{Ht-1} &= (1+\mu S^*)(\phi\cdot v_{t-2} + \cdot(1-\phi)\cdot v^*_{Ht-2}),\\
v^*_{Ht-2} &= (1+\mu S^*)(\phi\cdot v_{t-3} + \cdot(1-\phi)\cdot v^*_{Ht-3});
\end{aligned}
$$

and the above system implies

$$\begin{aligned}v^*_{Ht} =&(1+\mu S^*)\phi v_{t-1} + (1+\mu S^*)^2\phi(1-\phi)v_{t-2} \\ &+ (1+\mu S^*)^3\phi(1-\phi)^2 v_{t-3} + (1-\phi)^3(1+\mu S^*)^3 v^*_{Ht-3}.\end{aligned}$$

Denote the first period where trade equilibrium occurs by T, and extend the above equation to T, yields

$$\begin{aligned}v^*_{Ht} =&(1+\mu S^*)\phi v_{t-1} + (1+\mu S^*)^2\phi(1-\phi)v_{t-2} + \cdots + (1+\mu S^*)^{t-T}\phi(1-\phi)^{t-T-1}v_T \\ &+ [(1-\phi)(1+\mu S^*)]^{t-T}\, v^*_{HT}.\end{aligned}$$

Dividing the above expression by $v_t = (1+\mu S)^{t-T}v_T$, the $\frac{v^*_{Ht}}{v_t}$ ratio can be solved as

$$\begin{aligned}\frac{v^*_{Ht}}{v_t} =& \underbrace{\frac{1+\mu S^*}{1+\mu S}\phi + \left(\frac{1+\mu S^*}{1+\mu S}\right)^2\phi(1-\phi) + \cdots + \left(\frac{1+\mu S^*}{1+\mu S}\right)^{t-T}\phi(1-\phi)^{t-T-1}}_{\text{geometric sequence with common factor } = \frac{1+\mu S^*}{1+\mu S}(1-\phi)} \\ &+ \left[(1-\phi)\left(\frac{1+\mu S^*}{1+\mu S}\right)\right]^{t-T}\frac{v^*_{HT}}{v_T}.\end{aligned}$$

For the above expression to be constant through time, two conditions must be satisfied, which are (1) the geometric sequence in the first line above has common factor equal to 0, which requires $\phi = 1$; and (2) the second line above is equal to 0, which also requires $\phi = 1$. If $\phi = 1$, equation (22) becomes

$$v^*_{Ht+1} = (1+\mu S^*)\cdot v_t.$$

The above equation implies that in Foreign during t, the education sector can fully study the most advanced technology vintage of Home, namely v_t, and improve upon on that. In other words, in the R&D process, Foreign must has complete knowledge about the technology vintage embedded in the imported goods from Home during each period.

8.7 Three period extension

In this model, unskilled labor work during young and retire when old. In contrast, schooling agents do education when young and work (either as teachers or engineers) when old, and they do not have a retirement stage. This causes some asymmetry between the unskilled and skilled labor, in terms of the retirement stage.

One solution is to extend the two-period model into three periods: young, middle age and old. New born young agents are unskilled, and freely choose to stay unskilled, or do education. If he remains unskilled, when young, he borrows, consumes, and does nothing else (he does not work, nor saves); when middle age, he works as an unskilled labor, gets wage rate, pays back the borrowing (with interest), consumes, and saves for his retirement; when old, he retires and consumes his borrowing (plus any interest). Compared to the two-period model, in the three-period model, young unskilled labor do not work and do not have income. This is in order to keep symmetry with the schooling agents, who do not work and have no income when young.

If a young agent chooses to go to school, when young, he borrows, pays tuition fee for education and consumes; when middle age, he works in the education sector (teacher) or high-tech good production sector (engineer), get his wage, pays back the borrowing (with interest), consumes and saves for his retirement; when old, he retires and consumes his borrowing (plus any interest).

One immediate advantage of this three-period OLG model is on the bond market clearing issue. In the two-period OLG model, borrowing of young students must be financed by savings of young unskilled labor. In three-period OLG, borrowing of young students can be financed by savings of (i) middle aged skilled labor, (ii) young unskilled labor or/and (iii) middle aged unskilled. This allows for more flexibility on the bond marking clearing condition.

In this three period extension, there are two survival issues, namely survival from young to middle age, and survival from middle age to old. We call the survival rate from middle age to old as the adult survival rate, where the survival rate from young to middle age as the child survival rate. In the analysis below, we first allow for only one survival uncertainty, the survival from middle to old. Then, we allow both survival uncertainties.

8.7.1 Only one survival rate (adult survival rate)

Survival from young into the middle age is certain and survival from middle age into the old has a probability s. s is the adult survival rate. Population aging is modeled via an increase in s. After s increases, during each period, there are more old agents and the fraction of old agents increases, which is a key feature of population aging. In the three-period model, the survival rate plays the same role as that in the two-period model.

A representative unskilled labor solves the lifetime utility maximization problem, expressed by

$$\begin{aligned}\max_{c_{1t}^{UH},c_{1t}^{UL},c_{2t+1}^{UH},c_{2t+1}^{UL},c_{3t+2}^{UH},c_{3t+2}^{UL}} U_t^U \equiv & \left(logc_{1t}^{UH} + \alpha \cdot logc_{1t}^{UL}\right) \\ & +\delta \cdot s \cdot \left(logc_{2t+1}^{UH} + \alpha \cdot logc_{2t+1}^{UL}\right) \\ & +\delta^2 \cdot s \cdot \left(logc_{3t+2}^{UH} + \alpha \cdot logc_{3t+2}^{UL}\right);\end{aligned}$$

subject to

$$\begin{aligned} p_{Ht} \cdot c_{1t}^{UH} + p_{Lt} \cdot c_{1t}^{UL} + s_t &= b_t^U, \\ p_{Ht+1} \cdot c_{2t+1}^{UH} + p_{Lt+1} \cdot c_{2t+1}^{UL} + (1+r_{t+1})b_t^U + s_{t+1}^U &= w_{t+1}^U, \\ p_{Ht+2} \cdot c_{3t+2}^{UH} + p_{Lt+2} \cdot c_{3t+2}^{UL} &= \frac{1+r_{t+2}}{s} s_{t+1}^U. \end{aligned}$$

In this specification, $c_{1t}^{UH}, c_{1t}^{UL}, c_{2t+1}^{UH}, c_{2t+1}^{UL}, c_{3t+2}^{UL}, c_{3t+2}^{UL}$ are skilled labor's consumptions of goods Y_H and Y_L during three life periods. b_t^U is the borrowing of (young) unskilled labor during t, and s_{t+1}^{U} is the saving of (middle age) unskilled labor during $t+1$. Perfect annuity markets for adult survival risks are assumed to exist, same as in the two-period model, therefore s is in the denominator in the above third budget constraint.

In addition to unskilled labor, a representative schooling agent (student) solves the lifetime utility maximization problem, expressed by

$$\begin{aligned}
\max_{c_{1t}^{SH}, c_{1t}^{SL}, c_{2t+1}^{SH}, c_{2t+1}^{SL}, c_{3t+2}^{SH}, c_{3t+2}^{SL}} U_t^S \equiv & \left(logc_{1t}^{SH} + \alpha \cdot logc_{1t}^{SL}\right) \\
& +\delta \cdot s \cdot \left(logc_{2t+1}^{SH} + \alpha \cdot logc_{2t+1}^{SL}\right) \\
& +\delta^2 \cdot s \cdot \left(logc_{3t+2}^{SH} + \alpha \cdot logc_{3t+2}^{SL}\right);
\end{aligned}$$

subject to

$$\begin{aligned}
p_{Ht} \cdot c_{1t}^{SH} + p_{Lt} \cdot c_{1t}^{SL} + z_t &= b_t^S, \\
p_{Ht+1} \cdot c_{2t+1}^{SH} + p_{Lt+1} \cdot c_{2t+1}^{SL} + (1+r_{t+1})b_t^S + s_{t+1}^S &= w_{t+1}^S, \\
p_{Ht+2} \cdot c_{3t+2}^{SH} + p_{Lt+2} \cdot c_{3t+2}^{SL} &= \frac{1+r_{t+2}}{s} s_{t+1}^S.
\end{aligned}$$

In this specification, $c_{1t}^{SH}, c_{1t}^{SL}, c_{2t+1}^{SH}, c_{2t+1}^{SL}, c_{3t+2}^{SH}, c_{3t+2}^{SL}$ are this skilled labor's consumptions of goods Y_H and Y_L during three life periods. z_t is the tuition fee at t, b_t^S is the borrowing of (young) skilled labor during t, and s_{t+1}^S is the saving of (middle age) skilled labor during $t+1$. Perfect annuity markets for adult survival risks are assumed to exist, same as in the two-period model, therefore s is in the denominator in the above third budget constraint.

The production and education sectors are the same as in the two-period model, which means equations (7) to (10) still apply. The labor market clearing conditions (12) and (13) still apply. Since all young agents survive into middle age with certainty, equation (11) is modified as

$$L_{t+1} = s_1 \cdot (1 - S_t).$$

where $(1 - S_t)$ is total unskilled labor during t, and a fraction s of them survive and become the unskilled labor workforce during $t+1$.

The steps to solve this three-period model are the same as in the two-period model and they are omitted here. We focus on equilibrium, where the size of students is constant through time. Denoting this equilibrium student size by S, the equation determining S is

$$\frac{1-\rho+\alpha}{\alpha}\frac{S}{1-S} = (1 - \frac{1}{\gamma\delta^2 s})(1 - \frac{1}{\gamma}).$$

The left hand side of the above equation is increasing in S, and its right hand side is increasing in s. Therefore, as s increases, for the above equation to hold, S must increase. In other words, if the adult survival rate increases, the equilibrium student size increases and the educational efforts are higher. This result is the same as the two-period OLG model. However, the intuition is different here and discussed below.

In the two-period OLG model, schooling agents get wage from the second stage of life. Since their survival into the second stage is uncertain, they face the risk of dying before getting payments. After the survival rate increases, they have a higher probability to get their payoff. As a result, their incentives to do education are higher. In the current three-period OLG model, this intuition does not apply, since schooling agents get payoff in the middle age and they will survive into the middle age with certainty, hence a higher survival rate does not change their probability of getting payoff.

In the three-period model, the reason for the education efforts to rise with population aging lies in the bond market clearing condition. After the survival rate increases, both unskilled labor and skilled labor have a higher probability to consume in the retirement stage, and this will raise their savings during middle age (they save in the middle age in order to consume after retiring). The bond market clearing condition requires that total saving is equal to total borrowing, and if the total savings increase, so will the total borrowing, borrowing from students to do education. An increased total borrowing from students will result in more young agents choosing to go to school hence the overall educational efforts are higher after population aging.

Even if the channel through which population aging increases education efforts is different in two-period and three-period OLG models, the general result that population aging encourages education efforts is the same. Therefore, allowing for three periods do not qualitatively change the model results. For simplicity, two-period OLG is used in our paper.

8.7.2 Two (child and adult) survival rate

In the above analysis, the survival from young to middle age is certain. Here, the survival risk from young to middle age is introduced. The survival rate from young to middle age is denoted s_1, and that from middle age to old is denoted s_2. s_1 can be interpreted as child survival rate, and s_2 is adult survival rate. With two survival rates, the above model is modified as below.

A representative unskilled labor solves the lifetime utility maximization problem, expressed by

$$\begin{aligned}\max_{c_{1t}^{UH},c_{1t}^{UL},c_{2t+1}^{UH},c_{2t+1}^{UL},c_{3t+2}^{UL},c_{3t+2}^{UL}} U_t^U \quad \equiv \quad & \left(logc_{1t}^{UH} + \alpha \cdot logc_{1t}^{UL}\right) \\ & +\delta \cdot s_1 \cdot \left(logc_{2t+1}^{UH} + \alpha \cdot logc_{2t+1}^{UL}\right) \\ & +\delta^2 \cdot s_1 \cdot s_2 \cdot \left(logc_{3t+2}^{UH} + \alpha \cdot logc_{3t+2}^{UL}\right);\end{aligned}$$

subject to

$$\begin{aligned}p_{Ht} \cdot c_{1t}^{UH} + p_{Lt} \cdot c_{1t}^{UL} + s_t &= b_t^U, \\ p_{Ht+1} \cdot c_{2t+1}^{UH} + p_{Lt+1} \cdot c_{2t+1}^{UL} + \frac{1 + r_{t+1}}{s_1} b_t^U + s_{t+1}^U &= w_{t+1}^U, \\ p_{Ht+2} \cdot c_{3t+2}^{UH} + p_{Lt+2} \cdot c_{3t+2}^{UL} &= \frac{1 + r_{t+2}}{s_2} s_{t+1}^U.\end{aligned}$$

In addition, a representative schooling agent (student) solves the lifetime utility maximization problem, expressed by

$$\begin{aligned}\max_{c_{1t}^{SH},c_{1t}^{SL},c_{2t+1}^{SH},c_{2t+1}^{SL},c_{3t+2}^{SH},c_{3t+2}^{SL}} U_t^S &\equiv \left(logc_{1t}^{SH}+\alpha\cdot logc_{1t}^{SL}\right)\\ &+\delta\cdot s_1\cdot\left(logc_{2t+1}^{SH}+\alpha\cdot logc_{2t+1}^{SL}\right)\\ &+\delta^2\cdot s_1\cdot s_2\cdot\left(logc_{3t+2}^{SH}+\alpha\cdot logc_{3t+2}^{SL}\right);\end{aligned}$$

subject to

$$\begin{aligned}p_{Ht}\cdot c_{1t}^{SH}+p_{Lt}\cdot c_{1t}^{SL}+z_t &= b_t^S,\\ p_{Ht+1}\cdot c_{2t+1}^{SH}+p_{Lt+1}\cdot c_{2t+1}^{SL}+\frac{1+r_{t+1}}{s_1}b_t^S+s_{t+1}^S &= w_{t+1}^S,\\ p_{Ht+2}\cdot c_{3t+2}^{SH}+p_{Lt+2}\cdot c_{3t+2}^{SL} &= \frac{1+r_{t+2}}{s_2}s_{t+1}^S.\end{aligned}$$

The production and education sectors are the same as in the two-period model, which means equations (7) to (10) still apply. The labor market clearing conditions (12) and (13) still apply. Since all young agents survive into middle age with certainty, equation (11) is modified as

$$L_{t+1}=s_1\cdot(1-S_t)$$

Solving the above model, the equation determining the equilibrium student size S is

$$\frac{1-\rho+\alpha}{\alpha}\frac{S}{1-S}=\left(1-\frac{1}{\gamma\delta^2 s_1^2 s_2}\right)\frac{s_1}{s_1-1/\gamma}\equiv f(s_1,s_2).$$

The left hand side of the above equation is increasing in S. $f(s_1,s_2)$ is increasing in s_2. Therefore, if s_2 increases and s_1 is unchanged, for the above equation to hold again, S must increase. In other words, the equilibrium student size is increasing in the adult survival rate s_2.

In general, $f(s_1,s_2)$ is ambiguous in s_1. To see this, take logarithmic of $f(s_1,s_2)$, and differentiate it with respect to s_1, yielding

$$\frac{\partial\log f(s_1,s_2)}{\partial s_1}=\frac{2\gamma\delta^2 s_2 s_1^3+3\delta^2 s_2 s_1^2-2s_1+1/\gamma}{s_1(s_1-1/\gamma)(1-\gamma\delta^2 s_2 s_1^2)}.$$

If $1/\gamma<\delta^2 s_2 s_1^2$, then $s_1>1/\gamma$ and

$$\frac{\partial\log f(s_1,s_2)}{\partial s_1}=\frac{3\delta^2 s_2 s_1^2+1/\gamma+2s_1(\gamma\delta^2 s_2 s_1^2-1)}{s_1(s_1-1/\gamma)(1-\gamma\delta^2 s_2 s_1^2)}<0.$$

Since $f(s_1,s_2)$ is decreasing in s_1, S is decreasing in s_1, the child survival rate.

If $1/\gamma>\delta^2 s_2 s_1^2$, the sign of the numerator of $\frac{\partial\log f(s_1,s_2)}{\partial s_1}$ is ambiguous, and the sign of $\frac{\partial\log f(s_1,s_2)}{\partial s_1}$ is ambiguous. In this case, the impact of an increasing s_1 upon S is ambiguous.

After the child survival rate increases, both unskilled labor and schooling agents are more likely to survive to the middle age. Schooling agents get payoff in the middle age, so a higher child survival rate increases the education incentive. However, in contrast to the original model where unskilled labor get payoff when young, in this model the unskilled labor also get payoff in the middle age, hence a higher child survival rate also increases the incentive to stay unskilled. With these two conflicting effects, the net effect of child survival rate upon educational efforts is ambiguous.