"Brain Grain, Brain Drain and Optimal Educational Policies: A General Conceptual Framework"

by

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Abstract

A general conceptual framework is proposed for understanding how heterogeneous abilities and levels of educational attainment, along with differences in the quality of educational systems and access conditions, critically co-determine individuals’ international educational choices and subsequent professional options, thereby impacting national economic welfare. Whether students will ultimately chose to be trained and/or work at home, or abroad, depends on an array of factors, including the quality and pricing of educational offerings, the openness and selectivity of universities, international salary differentials and foreign job market access conditions. Self-selection is a key element determining the balance between brain drain and brain gain effects, along with the relative efficacy of countries’ optimal educational policies.

Key Words: heterogeneous agents, human capital, brain gain, brain drain, international migration, educational policies, self-selection

JEL classification codes: F22, D82, I25, I28, J24

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PRELIMINARY VERSION (in process and incomplete): Comments are welcome.
Section I: Introduction

International human capital mobility has been at the heart of the brain drain literature, initiated in large part by Bhagwati and Hamada (1974), and subsequently elaborated to consider the counterbalancing effects of brain gain effects on economic welfare in source and recipient countries. In what Schiff (2006) has termed the “new brain drain literature”, Mountford (1997) and Stark, Helmenstein and Prskawetz (1997), have identified a potentially important source of brain gain, which is independent from return migration. Specifically, although migration can generate a loss of domestic talent, it can also prompt an upsurge in the overall educational level of a home country, as a result of higher propensities to invest in human capital. Attractive foreign labour market conditions offer heightened incentives for domestic workers to strive to attain higher qualification levels, whether or not they ultimately find jobs abroad, thereby fostering, ceteris paribus, increases in average productivity levels at home.

While certain existing approaches to modelling brain drain and brain gain effects entail macroeconomic frameworks with representative agents, as in Vidal (1998), many also consider microeconomic decisions at the level of individual agents, including choices regarding optimal investment levels in education. Stark, Helmenstein and Prskawetz (1997) have proposed a framework, which demonstrates how, given the opportunity to migrate, choices regarding educational attainment will determine an individual’s wage on the foreign labour market. In other modelling frameworks, as proposed by Stark, Helmenstein, and Prskawetz (1998), the potential migrant takes into account a probability of finding a job abroad, which is identical for all individuals, or, as in Stark (2004), constrained by a minimum threshold level of qualification. Mountford (1997) and Beine, Docquier, and Rapoport (2001, 2008)) propose models where an individual’s decision is of a binary form – whether to undertake education or not, while the probability of finding foreign employment is exogenous. This does not allow a role for differences in individuals’ characteristics, so that migrants are randomly selected. In contrast, Chiswick (1999) provides for self-selection by migrants, since, assuming two categories of individuals, the rate of return to migration is greater for those with high-ability, relative to lower-ability persons.
Nonetheless, the literature has principally focused on the links between incentives to invest in human capital at home and subsequent migration flows.

The evaluation of brain drain/brain gain effects is made in the literature by assessing the impact of migration on a variety of specific economic objectives, which, however, do not include an explicit social welfare per se. Notably, migration is shown to influence the growth rate of the home economy, as in Beine, Docquier, and Rapoport (2001), the average educational level, as highlighted by Stark et al. (1997, 1998) and Lien and Wang (2005), average productivity in Mountford (1997), as well as the wages of non-migrants in Stark (2004).

Although there is now a burgeoning number of empirical studies, assessing different dimensions of the potential impact of brain drain and gain, there remains a lack of consensus regarding the size of conjectured positive effects of migration upon levels of education, welfare and/or growth. Notably, Beine, Docquier, and Rapoport (2001, 2008) find that the proportion of migrants must be low for such effects to be apparent. According to Schiff (2006), preliminary studies by the World Bank show no positive impact, while Groizard and Llull (2006) indicate a similar finding.

A recent critique by Rosenzweig (2006), which faults existing approaches to the analysis of brain drain and gain in two crucial respects, is particularly germane for motivating the modelling framework proposed in the current research. First, he contends that the potential impact of the “‘risk’ of emigrating” for “domestically-educated tertiary educated person(s)” is de facto quite minimal. Second, Rosenzweig goes on to suggest that “the literature ignores the endogeneity of the emigration probability”, while arguing that, in fact, “the choice of the location of tertiary education significantly affects the probability that the person can emigrate.”

1 While the analytical framework proposed by Rosenzweig does not allow for differences in individual abilities, his empirical findings are consistent with a number of the modeling assumptions which are subsequently invoked here. Notably, he reports evidence that students are motivated by foreign studies in order to obtain employment in a host country and that quality differences in university systems also appear to trigger the decision to study abroad.
Critically, existing analytical research has paid relatively little attention to the question of whether distinctive brain drain and gain effects may arise, depending on the extent to which educational investments take place either in home and/or host countries. Nonetheless, the policy stakes of the international mobility of high-skilled workers are increasingly recognized as a source of substantial policy concern.²

This paper proposes an enlarged framework for analyzing the nexus between human capital formation and international migration. It extends the existing research of Franck and Owen (2009, 2010) to consider distinct categories of brain drain and brain gain effects, arising from the eventual decision to undertake further human capital formation, either at home, or abroad. Associated international welfare implications are explored in a two country, game-theoretic setting, where heterogeneous individuals face the option of eventually pursuing further studies, while choosing between the alternative university systems. These educational investment decisions are based, among other considerations, on the interrelation between differences in students’ innate abilities, quality and access costs of the educational systems, as well as subsequent employment prospects and anticipated wage earnings in both countries. Nonetheless, under certain conditions, certain individuals may opt to remain relatively less trained and, consequently, only be able to work in their country of origin at a lower wage rate. Accordingly, the analysis does not consider the additional complication of international migration by unskilled workers.

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The organization of this paper is the following. The basic modelling analysis, proposed in Section II, starts with a general formulation of the heterogeneous individuals’ ex ante choices, regarding whether to undertake additional human capital formation at home or abroad, or remain less skilled. An individual’s underlying ability determines known productivity gains from studying in either university system, along with expected probabilities of subsequently obtaining foreign market employment at higher wages. These anticipated gains depend on the hypothetically realizable gains in

² See, for example, Leipziger (2008) and Solimano (2008).
productivity, which are, in turn, a function of individuals’ abilities across the heterogeneous population, as well as the quality of the chosen national university system. A determination of the alternative evaluations of the net returns to additional educational investments also depends on the specific costs borne by students in each university system. However, the net returns from undertaking further university studies also need to be compared with the lower wage for lower skilled workers in the home country, since individuals can opt not to upgrade their skills, and then remain at a lower and uniform level of productivity. Unlike more educated workers, workers with less skills are understood to only have the option of working at home.

Section II then characterizes how alternative configurations of educational decisions are critically dependent on certain modelling parameters, including the heterogeneity of innate abilities, quality of university systems, educational costs, as well as employment prospects and anticipated wage earnings. Distinctive brain drain and brain gain depends on the size of the sub-populations of individuals who migrate permanently, as compared with those who return home with enhanced productivity, or upgrade their human capital by pursuing further studies at home. Comparative static results highlight how changes in key model parameters impact international educational decisions and associated migration flows.

The analysis in Section III then starts by characterizing the determinants of economic welfare in the two countries. A key issue is again how the interrelation between a diverse set of modelling parameters, educational decisions and subsequent employment prospects, generates alternative regimes with distinctive brain drain and brain gain flows across the population of heterogeneous individuals, thereby defining the welfare calculations. The welfare implications of non-cooperative and cooperative national policies are then investigated in an initial shorter-term scenario where the quality of the university systems in the two countries is taken as given. However, governments can potentially impact individuals’ decisions to undertake further studies, along with the flows of students between countries, through their policies with regard to tuition payments and, in the case of the
foreign country, the extent of labour market access. In the case of non-cooperative solution, the domestic (foreign) country seeks to maximize the gain in productivity resulting from a lowering of tuition fees in order to foster more human capital formation, but faces potential tradeoffs to the extent that brain drain dominates brain gain effects and due to the cost of public sector funding, when tuition fees fall short of the actual educational costs incurred by the universities. In the initial analysis presented here governments are assumed to have perfect information regarding students’ abilities. Yet, a potentially critical issue concerns how imperfect information can impact the determination of governments’ optimal policies. Accordingly, certain of the potential implications of alternative assumptions regarding the extent of a public authority’s knowledge of students’ underlying abilities are also identified, in line with the earlier work by Franck and Owen (2009, 2010).

A concluding section highlights the overall contribution of this research, while briefly summarizing key findings. The nature of the ongoing investigation of the proposed model is outlined, while certain directions for subsequent research are also suggested.

Section II: Basic Modeling Framework

A. General Overview

A two-country setting provides the basic framework for the subsequent game-theoretic analysis of the interrelation between international educational choices, migration flows and economic welfare at home and abroad. A central concern is how the international educational choices of a population of a heterogeneous population of individuals in a domestic country, in terms of their abilities, is impact by quality and cost differences in university systems, as well as subsequent employment prospects and conditions in both countries.

A distinctive feature of the proposed analysis is the demonstration of how the option to invest in further human capital formation, either at home, or abroad, can generate distinct sets of brain drain and brain gain effects. Economic welfare in the two countries is thereby shown to be critically
dependent on the quality of, and cost differences between, university systems, along with employment prospects and wage earnings in the domestic and foreign labor markets. More specifically, a representative individual, coming from a heterogeneous population of individuals in the domestic country, faces an ex ante educational choice as to whether to remain unskilled, or to upgrade his/her human capital by either undertaking further studies at home, or abroad in the foreign university system, which is assumed to be of higher quality. Of course, these educational decisions are also influenced by the portion of cost differentials between the two countries educational systems, which are borne by students. These, in turn, depend on countries’ educational pricing strategies, as well as eventual grant programs, aimed at promoting countries either country’s economic welfare non-cooperatively, or cooperatively.

B. Initial Elements of the Model

The point of departure for the more formal modeling is a characterization of the interrelation between the abilities of the heterogeneous individuals in the domestic country and attainable productivity levels, where the latter depend on potential educational decisions. The domestic country is understood, then, to comprise a set of individuals with a range of abilities, where the capability of the representative $k$th agent, is designated as $a_k$. These abilities are distributed across the population, such that $a_k \in [a_1, a_2]$, where $a_1$ and $a_2$ indicate, respectively, the most, and least, capable persons in this heterogeneous set. An educational production function is understood to characterize how abilities, along with differences in the quality of national educational systems, co-determine an attainable productivity level, $e_k$, for skilled individuals. However, individuals, who do not pursue further studies, remain relatively unskilled, and are assumed to remain at a lower productivity, $e_0$, which is the same regardless of innate abilities.

More formally, the educational production function, specified by $e_k = f(a_k, Z)$, is an increasing function of its arguments and the cross-derivative, $f_{12}$, is positive. Here, $Z$ represents the quality of a particular country’s
educational system, such that $Z \in \{z, z^*\}$. The latter symbols distinguishing the educational quality of, respectively, the domestic and foreign countries, where it will be assumed, in general, here that the foreign educational system is of higher quality, such that $z^* \geq z$. Consequently, a distinction can be made between the higher level of productivity realized by the $k$th individual, $e_k^*$, when studying abroad $e_k^* = f(a_k, z^*)= e_k^*(a_k)$, relative to the level attainable through studies at home, $e_k = f(a_k, z)= e_k(a_k)$. Furthermore, it is assumed that there is increased productivity gain for more capable individuals, when they are educated in a higher quality system, which in light of the foregoing discussion means that:

(H1a) $e_k$ and $e_k^*$ are both strictly increasing functions of $a_k$, while $e_k^* > e_k$ for all $k$; and

(H1b) $e_k^* - e_k$ is a non-decreasing function of $a_k$, again for all $k$.

In light of the assumed superior quality of the foreign university system, the hypothetical educational options of pursuing further studies, either at home, or abroad, translate for the representative $k$th individual into an unique combination of productivity values $(e_k, e_k^*)$. The overall set of attainable combinations of productivity levels can be represented for the heterogeneous population, as a whole, by a line segment in a graphical framework, where conceivable levels of domestic and foreign productivity are represented, respectively, on the horizontal and vertical axes. Such a line segment, which is referred to, here, as the talent-educational quality locus, represents the nexus of attainable productivity gains, determined by the interrelation between the distribution of individuals’ talents and the performance-enhancement generated by the quality of the two countries’ educational systems.

There are four major determinants of the characteristics of the talent-educational quality locus, and hence the position and shape of the associated line segment, which warrant further elaboration. First, the degree to which the foreign university system offers a superior opportunity to enhance certain individuals’ productivity levels is captured by the extent to which any part of
the locus diverges away from a bisecting straight line, emanating from the origin, in the space of hypothetical realizable productivity values \((e_k, e_k^*)\) demarcated by the horizontal and vertical axes. Second, a related remark is that higher degrees of convexity of the upper part of the locus corresponds to scenarios where a higher quality of the foreign educational system offers a potential higher enhancement of the most talented individuals, since they would experience relatively greater relative productivity gains, as compared to those which could be realized through foreign studies by less able persons. Such an issue of heightened quality differentiation, according to students’ abilities, will be termed here as the relative degree of “elitism” of the foreign educational system. Third, for any given productivity scale, the initial point of departure of the talent-educational quality locus, relative to the origin, reflects the relative quality of the domestic country’s pre-university educational system.

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**NB.** A principal focus of the subsequent comparative analysis is to establish specific propositions on the basis of changes in variables which impact the relative positions of the talent-educational quality locus and the curves demarcating the three hypothetical educational regimes. More specifically, a particular concern is how the balance between brain drain, grain and waste effects can depend on the hypothetical combinations of source and recipient countries.

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The initial position of the talent-educational quality locus is determined, in part, by the quality of pre-university educational system in the source country, where the latter can be regarded as a separate variable entering the educational production function. Thus, its endpoints are determined by the extreme values for individuals’ talents, \(a_1\) and \(a_2\), and the quality of the domestic primary educational systems, its length by the degree of elitism in different educational systems and its slope will be steeper than a 45 degree line to the extent the foreign university educational system is superior to that of the domestic country.
In the proposed model, whereas skilled workers have the possibility of migrating abroad, it is postulated that unskilled individuals only can work at home at a fixed wage rate, \( w_0 \). In each labor market, the wages of skilled workers are understood to be an increasing function of workers’ realized productivity levels, which, as noted, depend on both their abilities and educational choices. A necessary condition for the possibility of permanent international migration, driven by more favorable employment prospects abroad, is that, for a given level \( e \) of individual productivity, the foreign salary is greater than that in the home country.\(^3\) Accordingly, for any given level of productivity, it is assumed that the corresponding salary in the foreign country, \( w^* = w^*(e) \), is greater than that at home, \( w = w(e) \), for all productivity levels and individuals. Furthermore, it will be postulated that the difference between these levels widens as productivity levels increase. In view of a lower productivity level, \( e_0 \), the wages attainable by unskilled persons, denoted as \( w_0 = w(e_0) \), are always superior to those for skilled workers. In sum, the following holds:

(H2a) \( w \) and \( w^* \) are increasing functions of \( e \), such that \( w(e) < w^*(e) \) and \( w(e) > w_0 \) for all \( e > e_0 \),

(H2b) \( w^* - w \) increases with \( e \).

In order to facilitate the subsequent analysis of the critical interrelation between wages, productivity and educational choices, it is useful to introduce additional notation for the representative \( k \)th individual. More specifically, the wages earned on the home or foreign labor markets will differ depending on whether the individual is trained at home or abroad. The higher quality, foreign university system yields a greater productivity gain, which, in turn, yields relatively higher wages in the domestic and foreign job markets, denoted, respectively, as \( w(e_k^*) = w[e^*(a_k)] \) and \( w^*(e_k^*) = w^*[e^*(a_k)] \).

\(^3\) However, as subsequently elaborated, temporary migration can occur in order to undertake studies abroad even when there is no prospect of foreign employment. A necessary condition is that the expected additional salary gain at home, resulting from enhanced productivity because of more favorable university conditions abroad, more than offsets any greater educational costs.
Accordingly, the following inequalities summarize, then, the interrelation between wage earnings and the location of human capital formation: \( w(e_k) < w(e_k^*) \) and \( w^*(e_k) < w^*(e_k^*) \).

C. The Ex Ante Model of Human Capital Formation with Heterogeneous Individuals

The analysis now characterizes the interrelation between educational choices and both temporary and permanent migration flows by focusing on the individuals’ decisions of whether, or not, to invest in further human capital, either at home, or abroad. While this choice will be formulated for a representative \( k \)th person, it is essential to recognize that the specific choices can vary across the heterogeneous population as a function of differences in abilities. A variety of other factors can also critically impact the decision whether or/not to pursue further studies in one of the two countries’ university systems. These include the range of probabilities of gaining access to the foreign labor market, which as previously noted depend on the choice of university systems; as well as the interrelation between salary differentials and productivity levels for skilled workers in the two labor markets, as compared to the fixed domestic wage for unskilled workers. Furthermore, both increases in productivity and associated gains in salaries differ across the population of heterogeneous individuals, according to abilities. Finally, the *ex ante* human capital decisions are also potentially impacted by the tradeoff between expected higher financial returns from further education and the corresponding relative costs, either at home or abroad.

More specifically, a representative individual faces three conceivable choice options, designated as outcomes [0], [1] and [2]. These are, respectively, to: i. not undertake any further studies, ii. pursue further studies at home, or iii. undertake studies abroad. Those individuals, opting not to continue their studies, remain relatively unskilled, are unable to work abroad and face an exogenously given domestic wage equal to \( w_0 \). However, by undertaking further studies, a representative individual, \( k \), can upgrade his/her level of productivity. This permits access to the foreign labor market
with variable probabilities, depending on where the individual is educated and his/her ability. More specifically, if the individual were to study at home, or abroad, the corresponding probabilities of being hired in the foreign country are designated, respectively, as \( p(e_k) \) and \( p^*(e_k^*) \), where \( p(e_k) \leq p^*(e_k^*) \). There are two distinct rationales for assuming that a foreign education can lead to enhanced prospects of being employed abroad. First, this may be due to the assumption that the foreign university system is of a higher quality. Second, there may be informational, network and host country labor market policies, which generate more favorable labor market access for host country trained students, even when they are equally qualified relative to those trained abroad. In light of the foregoing discussion, the following additional assumption applies:

\[ (H3) \quad p \text{ and } p^* \text{ are increasing functions of both } e \text{ and } e^*, \text{ such that } \]
\[ p^*(e_k^*) > p(e_k^*) > p(e_k). \]

The decision to pursue further studies potentially depends on a weighing of the expected salary gains in relation to the additional costs of further studies at home or abroad, where the latter are denoted, respectively, as \( I \) and \( I^* \). Note that such costs may include not just tuition costs, but also living and other expenses. The net cost differential for undertaking further studies in the two systems, is designated as \( i \), such that \( i = I^* - I \). The expected salary earnings from studying in either country can be represented by the following two general functional forms:

\[ (1) \quad g(e) = p(e) w^*(e) + (1-p(e)) w(e), \text{ and } \]
\[ g^*(e^*) = p^*(e^*) w^*(e^*) + (1-p^*(e^*)) w(e^*). \]

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4 While the latter inequality is not immediately relevant to an individual’s choice, it reflects the assumptions that the productivity level generated by a foreign education is higher than for a domestic one and that the probability of foreign employment is an increasing function of productivity.

5 While the analysis here abstracts from the potential heterogeneity of costs across the population. Such differences could arise because of locational, and other, individual specific factors, as well as government educational grants, which as analyzed by Franck and Owen (2009), can be tailored to promote social welfare in a given country.
A series of technical propositions regarding the expected earnings functions $g(e)$ and $g^*(e^*)$ constitute the underpinnings for a subsequent analysis, characterizing how different modeling parameters determine the configuration of the three educational choice regimes – [0], [1] and [2]. In order to establish certain of these propositions, it is necessary to invoke the following two further assumptions:

(H4) $g(e)$ and $g^*(e^*)$ are convex functions of $e$ and $e^*$.

(H5) $p^*(e) - p(e)$ is a non-decreasing function of $e$.

Whereas an initial proposition follows directly from certain of the foregoing assumptions, the demonstrations of the subsequent lemmas are provided following their statement.

**Lemma 1**

Given assumptions (H1a), (H2a,b) and (H3), the expected earnings functions from further studies at home or abroad, $g$ and $g^*$, are increasing functions of the respective attainable productivity levels, $e$ and $e^*$, and, consequently of students’ abilities, $a_k$, for all $k$.

**Lemma 2**

Given assumptions (H1b) and (H4), the expected incremental wage return, $g(e^*) - g(e)$, from a foreign, instead of a domestic, education, when evaluated in terms of the domestic expected earnings function, $g$, is an increasing function of students’ abilities, $a_k$, for all $k$.

**Demonstration:**

The derivative of the expression for the difference in the expected wages is given by:
\[
\frac{d}{da} \left[ g(e^*) - g(e) \right] = \frac{dg}{de^*} \frac{de^*}{da} - \frac{dg}{de} \frac{de}{da}.
\]
In light of assumption (H1b), \(\frac{de^*}{da} > \frac{de}{da}\), and given (H4), \(\frac{dg}{de^*} > \frac{dg}{de}\), so that, it follows that \(\frac{d}{da} \left[ g(e^*) - g(e) \right] > 0\).

**Lemma 3**

In light of the set of assumptions (H1) through (H5), the expected incremental wage return, \(g^*(e^*) - g(e)\), from a foreign education, evaluated in terms of the foreign expected earnings function, as compared with a domestic education, evaluated in terms of the domestic earnings function, is an increasing function of students’ abilities, \(a_k\), for all \(k\).

**Demonstration:**

The difference between these two earnings functions, \(g^*(e^*) - g(e)\), can be equivalently expressed as: \(\left[ g^*(e^*) - g(e^*) \right] + \left[ g(e^*) - g(e) \right]\). In light of Lemma 2, the second term, \(g(e^*) - g(e)\), is an increasing function of individuals’ abilities, \(a_k\); whereas the first term, \(g^*(e^*) - g(e^*)\), is equal to \([p^*(e^*) - p(e^*)][w^*(e^*) - w(e^*)]\). Finally, in light of assumptions (H2) and (H4), \(g^*(e^*) - g(e^*)\) is a non-decreasing function of \(e^*\) and, consequently, of \(a_k\).

The proposed formulation here of the decision determining individuals’ eventual investments in human capital is more general than in existing models of the brain drain, since it allows for both heterogeneous individuals and agent specific, human capital arbitrage decisions between two educational systems. The critical choice of each individual is whether to pursue further studies and, if so, in which country; where the latter decision depends on the quality of the different educational systems and the relative prospects of access to the higher wage foreign labor market. Accordingly, three conditions characterize this educational decision for the representative \(k\)th individual.
First, there is an incentive to continue his/her education at home, rather than remain unskilled, if the following condition, labeled (C1):

\[(2a) \ p(e_k)w^*(e_k) + (1-p(e_k))w(e_k) - I > w_0,\]

If this condition holds, option [1] will be chosen in preference to [0], and it can be equivalently expressed as condition (C2):

\[(2b) \ g(e_k) > I + w_0.\]

Second, studies abroad will be preferred to remaining unskilled, i.e. option [2] dominates [1], if condition (C12) holds:

\[(3a) \ p^*(e_k^*)w^*(e_k^*) + (1-p^*(e_k^*))w(e_k^*) - I^* > w_0,\]

Or, alternatively,

\[(3b) \ g^*(e_k^*) > I^* + w_0.\]

Conditions (2) and (3) are necessary for an individual to choose to undertake further studies, either at home, or abroad. Nonetheless, to determine a student’s final educational choice, it is also essential to consider an additional arbitrage condition, which compares the relative net returns from studying in the two university systems. The additional sufficiency condition for option [2] to prevail over [1] is:

\[(4a) \ p^*(e_k^*)w^*(e_k^*) + (1-p^*(e_k^*))w(e_k^*) - i > p(e_k)w^*(e_k) + (1 - p(e_k))w(e_k)\]

More simply, the latter can be expressed as:

\[(4b) \ g^*(e_k^*) - g(e_k) > i.\]

Together, inequalities (3) and (4) constitute sufficient conditions for an individual to decide to study abroad. However, when the opposite inequality to (4) holds, along with condition (2), an individual will instead elect to study at home, rather than either studying abroad, or remaining unskilled. Hence, option [1] then prevails.
Corresponding to each of the foregoing inequalities, (C1), (C2) and
(C12), are equations consisting of equalities, which identify limiting values in a
plane of productivity levels \((e, e^* )\); thereby demarcating zones, such that each
of these conditions is met. More specifically, in the case of (C1) that equality
can be expressed as \(e = g^{-1}[I + w_0]\), which represents a vertical line, as
depicted in Figure 1. Analogously, the frontier, determining the set of
productivity values such that condition (C2) is satisfied consists of a horizontal
line, defined by \(e^* = g^{*-1}[I^* + w_0]\). Ceteris paribus, an increase in the
opportunity cost of studying in the domestic (foreign) university system leads
to a rightward (upward) shift in the corresponding boundary line and,
consequently, increases the threshold productivity level, \(e (e^*)\), necessary to
undertake such further studies. The determination of nature of the boundary
curve corresponding to Condition (C12) requires an evaluation of its slope, on
the basis of the formula:

\[
\frac{de^*}{de} = \frac{g'(e)}{g^*(e^*)} = \frac{g'(e)}{g'(e^*)} \frac{g'(e^*)}{g^*(e^*)}.
\]

Since \(g\) is a convex function of productivity, if \(e^* > e\), then \(g'(e^*) > g'(e)\), so that the first ratio of
derivatives in the right-hand expression is less than one. Furthermore, given
that \(g^*(e^*) - g(e^*)\) is a non-decreasing function of \(e^*\), \(g^{**}(e^*) > g'(e^*)\). Hence,
since all four derivatives are positive and both ratios are less than unity, it
follows that \(de^*/de < 1\), so that the boundary associated with (C12) has a
positive slope with a value less than one in the area of the plane where \(e^* > e\).

As depicted in Figure 1, a combination of two out of the three boundary
lines, along with the associated inequality conditions, permits an identification
of the three alternative, and mutually exclusive, choice regimes. Whereas in
zone (0) there is no further investment in human capital, zones (1) and (2)
correspond to a pursuit of further studies in, respectively, the home or foreign
university systems.

Figure 1

The Interrelation between Productivity Levels, Financial Opportunity Costs
of Further Studies and the Configuration of Human Capital Regimes
Lemmas 1, 2 and 3 have established that the functions \( g(e), g^*(e^*) \) and \( g^*(e^*) - g(e) \) are all increasing functions of individuals’ abilities, \( a_k \).
Furthermore, as already shown, the three conditions, (C1), (C2) and (C3) are each associated with equalities, which together identify the boundaries for each of the three regimes. Hence, each of these equations defines a unique hypothetical threshold value of abilities, such that the condition is satisfied (violated) above (below) such a critical value. Accordingly, these specific threshold levels, referred to as \( A_{01}, A_{02} \) and \( A_{12} \), are determined, respectively, by the following equations:

\[
\begin{align*}
(5a) \quad & g[e(A_{01})] = I + w_0 \\
(5b) \quad & g^*[e^*(A_{02})] = I^* + w_0 \\
(5c) \quad & g^*[e^*(A_{12})] - g[e(A_{12})] = i.
\end{align*}
\]

Crucially, by ascertaining the different conceivable orderings for the relative values of these three critical ability parameter values, it is possible to establish the position of all the possible talent-educational quality loci, in relation to the three zones, [0], [1] and [2].
Lemma 4

There are only two possible orderings for the three threshold levels of abilities, defined by equations (5). These are as follows: \( A_01 < A_02 < A_{12} \) and \( A_{12} < A_02 < A_{01} \).

Demonstration:

If it is initially supposed that \( A_01 < A_02 \), it can then be established that it is not possible to have \( A_{12} < A_{01} \). For values of individuals’ abilities lying in the interval between \( A_01 \) and \( A_02 \), so that \( a \in [A_01, A_02] \), it must be the case that \( g[e(a)] > I + w_0 \) and \( g^*[e^*(a)] < I^* + w_0 \). This in turn means that \( g^*[e^*(a)] - g[e(a)] < i \), so that the set of abilities is such that \( a < A_{12} \). This is contrary to the initial hypothesis that \( A_{12} \) could be less than \( A_{01} \). Furthermore, it is also impossible that \( A_01 < A_{12} < A_02 \). If the latter inequalities were to hold, then it would have to be the case that \( g[e(A_{12})] > I + w_0 \) and \( g^*[e^*(A_{12})] < I^* + w_0 \). However, by summing the latter inequalities to be valid, it follows that, \( g^*[e^*(A_{12})] - g[e(A_{12})] < i \); but this violates the definition of \( A_{12} \) in equation (5c), so that the initial supposition cannot be valid. Consequently, together the foregoing demonstrations establish that \( A_01 < A_02 < A_{12} \).

Under the hypothesis that \( A_02 < A_01 \), an analogous proof can be used to establish that the only feasible ordering of abilities is such that \( A_{12} < A_02 < A_{01} \).

The actual educational choices over a hypothetical population of heterogeneous individuals requires an examination of the interrelation between, on the one hand, the previously identified factors determining the positions of the boundary conditions for the three regimes; and, on the other hand, the specific distributions of students’ abilities, such that \( a_k \in [a_1, a_2] \), as well as the quality differences in the two countries educational systems - both prior to and during university studies. The combined effects of these factors translate into the determination of the position, length and slope of a line segment in \( (e_k, e_{k^*}) \) space, which may, or may not, straddle more than one of the zones, \([0]\), \([1]\) or \([2]\).
Notably, such a line segment represents the nexus of attainable productivity gains, determined by the interrelation between the distribution of individuals’ talents and the performance-enhancement generated by the quality of the two countries’ educational systems. For expository simplicity, this curve will be referred to, here, as the talent-educational quality locus. Its endpoints are determined by the extreme values for individuals’ talents, $a_1$ and $a_2$, and the quality of the domestic primary educational systems, its length by the degree of elitism in different educational systems and its slope will be steeper than a 45 degree line to the extent the foreign university educational system is superior to that of the domestic country.

The determinants of the relative position of the talent-educational quality locus, in relation to the human capital choice regimes, will now be examined in more detail.

The orderings established in Lemma 4, in conjunction with the relative positions of the regime boundary equations, represented by equations (5a), (5b) and (5c), uniquely determine the positions of all the conceivable talent-educational quality loci for a given population of individuals with heterogeneous abilities. The representative $k$th individual with ability $a_k$ will realize a productivity level of $e_k = e(a_k, z)$ if he/she undertakes further human capital investment at home, but will achieve a higher productivity, equaling $e_k^* = e^*(a_k, z^*)$, as a result of foreign studies. By way of simplification, the relation between the two educational production functions can be expressed in terms of a function $e_k^* = \mu(e_k)$, where in keeping with the assumptions (H1a,b), $\mu$ is an increasing function. For a given set of modeling parameter values, this functional form can be incorporated into the productivity plane $(e, e^*)$, depicted in Figure 1, as a specific curve having a slope greater than one. The endpoints of such a curve, corresponding to the extreme values of abilities $a_1$ and $a_2$, may, or may not, straddle more than one educational choice regime. Of course, when such a curve lies wholly within one of the three zones, all individuals choose the same human capital investment option. However, if such a talent-educational quality locus were to intersect at least one of the lines demarcating different regimes, the associated ordering for the
critical boundary values of A01, A02 and A12 must be consistent with the ranking established in Lemma 4.

In this model, two distinct scenarios characterize the interrelation between the regime boundary conditions and the talent-educational quality locus. These different configurations depend on whether A01 is either interior, or superior to A02. First, as shown in Figure 2a, when the relevant ranking is A01<A02<A12, one conceivable outcome applies for the case when these boundary values lie between the limits of the postulated uniform distribution of attributes, a1 and a2. Then, whereas less capable individuals, whose abilities fall in the interval $a_1 < a_k < A01$, will not invest further in human capital (zone [0]), students with abilities lying in the interval, $A01 < a_k < A12$, will pursue an university education at home. Finally, the most capable persons with abilities such that $A12 < a_k < a_2$, will opt to continue their studies abroad. Second, when the relevant ranking is A12<A02<A01, a feasible result is that depicted in Figure 2b. This case is critically different from the previous outcome, since at most two human capital formation regimes will be manifest. While less capable individuals with abilities such that $a_1 < a_k < A02$, do not invest in human capital, more talented persons, with abilities in the interval $A02 < a_k < a_2$, will only study abroad.6 Crucially, given the configuration of abilities, quality of the potential university educational systems and the potential financial opportunity costs entailed by continued studies at home and abroad, there is no scope for the domestic country to maintain a viable system of higher education. Finally, another scenario, which also entails only two, out of the three, choices, is illustrated in Figure 2c. In this instance, the talent-educational quality locus does not contain one of the threshold values for the boundary conditions over the spectrum of individuals’ abilities, $a_1$ through $a_2$. and productivity gains, along the. Specifically, since $a_1 < A01 < a_2 < A12$, less talented individuals with abilities such $a_1 < a_k < A01$ will not undertake further human capital investment, while more capable persons for whom $A01 < a_k < a_2$, will receive a further education at home, so that no studies are undertake abroad.

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6 Note that the value A12, corresponding to the point M12 in Figure 2b, lies in the interval of lower abilities, where $a_1 < a_k < A01$. 
Figure 2a

Scenario Where All Three Human Capital Regimes Are Chosen by Certain Subsets of Individuals Across Heterogeneous Population

To facilitate the illustration of certain choice outcomes in this, and certain of the subsequent, illustrations, the scales for the productivity levels are assumed to differ between the $e_k$ and $e_k^*$ axes. Consequently, the talent-education locus does not necessarily lie above a bisecting 45-degree line, as would otherwise be the case without such a distortion.
Figure 2b

Scenario Where Subsets of Heterogenous Individuals Choose Only Either to Remain Unskilled or to Study Abroad

\[ g^{-1}[l^* + w_0] \]

\[ g^{-1}[l + w_0] \]

\[ e_k \]

\[ e_2 \]

\[ e_k^* \]

\[ e_2^* \]
Figure 2c

Scenario Where Subsets of Heterogenous Individuals Choose Only Either to Remain Unskilled or to Study at Home
D. A More Detailed Analysis of a Linearized Version of the Ex Ante Model

For the foregoing general formulation of the model, the associated expressions, characterizing both the populations of individuals’ choices and the associated consequences for economic welfare in the two countries, are highly non-linear. Accordingly, simplifying linearity assumptions are invoked here, in order to make the analysis more tractable. Specifically, the achievable levels of productivity, for the representative \( k \)th individual, will again depend on whether he/she is educated at home, or abroad. These are given, respectively, by:

\[
(6a) \quad e_k = \lambda a_k + \beta \\
(6b) \quad e_k^* = \lambda^* a_k + \beta^*
\]

The assumption of a higher quality foreign educational system can then be represented by:

\[
(7) \quad e_k^* = \mu e_k,
\]
where $\mu > 1$. The extent to which $\mu$ deviates from 1 reflects, then the opportunity, in terms of a foregone productivity gain, for an individual studying abroad, rather than at home. Note, furthermore, that the applicable set of attainable productivity levels is truncated, since it is determined by the underlying set of individuals’ abilities, which have been assumed to have lower and upper bounds, such that $a_k \in [a_1, a_2]$.

Linear specifications for the interrelation between the wages of skilled workers and alternative values of levels of labor productivity, $e$, are for, respectively, the domestic and foreign countries given by:

(8b) $w_1(e) = \theta e$

(8b) $w^*(e) = \theta^* e$

Here, the foreign country is understood to offer higher wages, so that $\theta^* > \theta$, while the wage received by all unskilled workers is such that $w_0 = \theta e_0$. A final simplification of exogenous employment probabilities in the two countries is also made, in order to avoid certain of the multiplicative terms involving the individual productivity levels. Specifically, it is assumed that:

(9) $p(e_k^*) = p^*$, $p(e_k) = p < p^*$, $\forall k$

In light of the foregoing simplifications the expressions for the expected earnings from studying at home, or abroad, can be readily deduced, and are, shown as follows:

(10a) $g(e_k) = [p\theta + (1-p) \theta] e_k = \alpha e_k$

(10b) $g^*(e_k^*) = [p^*\theta^* + (1-p^*) \theta] e_k^* = \alpha^* e_k^*$

Since wages are higher in the foreign country, $\theta^* > \theta$, and the probability of being hired abroad is greater when educated abroad, $p^* > p$, it follows that $g^*(e_k^*) > g(e_k)$ for $\forall k$.

---

8 This expression abstracts, without loss of generality, from the inclusion of an additional constant term.
More specific, but analytically equivalent, expressions for (5a), (5b) and (5c) capture individuals’ educational choices in this linearized version of the model. Accordingly, the conditions under which individuals are willing to pursue further studies, either at home, or abroad, are given, respectively, by:

\[(9) \quad \alpha e_k > I + w_0\]

\[(10) \quad \alpha e_k^* > I^* + w_0 .\]

Consistent with expressions (5c), the additional condition determining whether the expected returns from being educated abroad are sufficiently large, to offset the higher educational costs of studying abroad is given by:

\[(11) \quad \alpha e_k^* - \alpha e_k > i\]

The foregoing three inequalities are represented in Figure 3, which is analytically equivalent to Figure 1, and identifies combinations of productivity levels \((e, e^*)\), corresponding to the three distinct educational regimes. In particular, (9) and (10) correspond, respectively, to vertical and horizontal lines, which demarcate those productivity levels, which are compatible with undertaking further studies, respectively, at home, or abroad. The additional condition, (11), determines whether it is more advantageous for individuals to study abroad, rather than at home. It reflects whether the difference in the expected earnings from being educated abroad, rather than at home, offset the additional costs of such further studies. This condition can be represented in the \((e, e^*)\) graphical framework by a portion of a line emanating from the vertical access, which has a slope less than one, since \(\alpha/\alpha^* < 1\). That line segment is truncated by conditions (9) and (10).
The interrelation between conceivable values of individuals’ productivity levels and threshold levels defining their educational choices are illustrated in Figure 3. When the attainable productivity values for individuals are relatively low, individuals will not pursue further studies, since the returns to educational investments in either country do not offset the associated cost. Such outcomes are reflected by the set of equilibria, represented by zone [0]. However, further studies are justified for productivity levels corresponding to regimes [1] and [2], which again represent, respectively, the decisions to study at home, or abroad. As specified by condition (11), When studies abroad generate relatively greater increases in productivity, than those at home, net of the difference in educational costs, condition (11) is satisfied so that studies in the foreign university system are preferred, corresponding to zone [2].

**Figure 3**

Characterization of Boundary Conditions and Alternative Human Capital Regimes for the Linearized Version of the Model
The feasible set of alternative talent-educational quality frontiers can now be incorporated into Figure 3, so as to identify different configurations of the conceivable educational choices across the heterogeneous population. For each individual, with an ability \( a_k \) comprise between the limits of the distribution of abilities \( a_1 \) and \( a_2 \), there is an associated unique combination of productivity levels, corresponding to a point on the line \( e^* = \mu \cdot e \). These values lie between two extreme values, which are labeled here as M1 and M2. Across the continuum of abilities, \( a_k \), there are associated productivity values, \( e_k \), which lie on the regime boundary conditions, specified by (9), (10) and (11). These are denoted as follows:

\[(12a) \quad \varepsilon_0 = \frac{I + w_0}{\alpha} ; \quad (12b) \quad \varepsilon_2 = \frac{I^* + w_0}{\alpha^* \mu} ; \quad \text{and} \quad (12c) \quad \varepsilon_1 = \frac{I^* - I}{\alpha^* \mu - \alpha} \]

It is straightforward to establish the following interrelation between these threshold values, which is \( \varepsilon_2 = \lambda \varepsilon_0 + (1-\lambda)\varepsilon_1 \), where \( \lambda = \frac{\alpha}{\alpha^* \mu} \). Again, there are only two conceivable orderings for these threshold values. These are: \( \varepsilon_0 < \varepsilon_2 < \varepsilon_1 \) or \( \varepsilon_1 < \varepsilon_2 < \varepsilon_0 \). In the first instance, illustrated in Figure 4a, when the extreme values of productivity levels attainable from further studies in the domestic country, \( e_1 \) and \( e_2 \), encompass all three boundary values, so that \( e_1 < \varepsilon_0 < \varepsilon_2 < \varepsilon_1 < e_2 \), those individuals with abilities such that \( e_k < \varepsilon_0 \) will remain unskilled, while those for whom attainable productivity levels are \( \varepsilon_0 < e_k < \varepsilon_1 \), while the sub-population with abilities such that \( \varepsilon_1 < e_k \), will undertake studies abroad.

*****IN PROCESS*****

**Figure 4a**

Case Where All Three Educational Regimes Are Chosen by Specific Sub-populations of Heterogeneous Individuals such that \( \varepsilon_0 < \varepsilon_2 < \varepsilon_1 \)
\( e_k^* = \mu e \)

\[
\begin{align*}
(\varepsilon_0 + w_0)/\alpha & = (1 + w_0)/\alpha \\
\varepsilon_1 & = i(\alpha \mu - \alpha) \\
\end{align*}
\]
**Figure 4b**

Case Where Only Two Educational Regimes Are Chosen – Either No Studies or Abroadlations of Heterogeneous Individuals such that $\varepsilon_1 < \varepsilon_2 < \varepsilon_0$

**Figure 4c**

Case Where All Three Educational Regimes Are Chosen by Specific Sub-populations of Heterogeneous Individuals such that $\varepsilon_0 < \varepsilon_2 < \varepsilon_1$
Alternative ways of demarcating how the different educational choice regimes potentially depend on certain key modeling, parameter values are depicted in Figures 5 and 6ab. The first of these shows how the configuration of the regimes is related to the threshold levels, $\varepsilon_0$ and $\varepsilon_1$, which are defined by equations .. and.. As a reminder, the first of these two critical values delimits the boundary between zones 0 and 1, thereby determining for any individual productivity level $e_k$, whether a representative individual with ability $a_k$ will remain unskilled or study at home. The second of these critical values determines whether such an agent would choose between zones 1 and 2, which corresponds to the decision of whether to study at home or abroad. .... The graphical representation of the alternative regimes in Figure 5 can, alternatively, be transformed into an equivalent framework presented in Figures 6ab, which show how the educational choice regimes depend on the values of educational opportunity costs, including the educational costs which are borne by students in the two countries, I and I*. Not surprisingly, there exist threshold levels for both countries’ fees, such that beyond those levels it will no longer be profitable for a representative individual to pursue further studies, as represented by the set of equilibria corresponding to zone 0. However, when one of the country’s level of educational costs is relatively high, while the other’s is relatively low, that will favor the choice of the lower-cost educational system.
Figure 5

Representation of Alternative Educational Regimes in Terms of Critical Values ($\varepsilon_0$ and $\varepsilon_1$)
Figure 6a

Representation of Alternative Educational Regimes in Terms of the Opportunity Costs of Continuing Studies in the Two Countries \((I + w_0, I^* + \frac{w_0}{\mu})\)
The subsequent game-theoretic framework will initially focus on the interrelation between each country’s choice of the level of educational fees can potentially impact individuals’ human capital decisions under alternative assumptions of non-cooperative and cooperative solutions.
Section III: An Evaluation of Educational Policies in a Linearized Modeling Framework

*****IN PROCESS****

General Research Objective and Problematic

Provide a general framework for understanding how the specificity of how national economic and educational conditions can determine the balance between brain gain and brain drain, along with associated welfare effects.

Comparative Static Analysis

A. Linearized Version of the Model

Salaires - both at home and abroad - are assumed to be linear functions of individuals’ levels of productivity, such that:

1. \( w^*(e) = \theta^*e \)
2. \( w(e) = \theta e. \)

Since the general modeling framework hypothesizes that domestic students, who have undertaken more advanced levels of education, can earn higher salaries abroad, it follows that \( \theta^* > \theta \). However, lesser educated individuals only have the option of working at home at a fixed rate of \( w_0 \), which is lower than that for workers who pursued further training.

While studying abroad offers the twin advantages of access to a better quality university system and heightened access to the foreign job market, it is more costly to do so. With \( z \) and \( z^* \) corresponding to measures of educational quality at home and abroad, it follows that the ratio of these quality indices \( \mu = z^*/z, \) where \( \mu \geq 1 \). More favorable access to the foreign job market translates in the current modeling framework to the assumption that \( p^* \geq p, \) where \( p^* \) and \( p \) correspond then, respectively, to the probability of being hired abroad when training is, respectively, at home or abroad. These probabilities are taken to be exogenous, while not depending on either abilities, or attained productivity levels. The latter specification is essential in order to keep the analysis tractable. The
positive price differential borne by each student, when studying abroad, rather than at home, is given by \( i = I^* - I \), where \( I \) and \( I^* \) correspond, respectively, to the overall costs of tuition, living and other expenses paid per students in the two countries. Hence, the overall opportunity costs of deciding to study at home, or abroad, can be denoted by \( J \) and \( J^* \), where \( J = I + w_0 \) and \( J^* = I^* + w_0 \). Nonetheless, the actual share of the overall costs paid by individual students in most educational systems falls short of the true cost of their studies, where, most often, higher educational quality requires more costly investments per student. Hence, the cost per student generated by studying in the two system can be specified by the following two expressions:

2. \( I = \gamma v z = \sigma z \)
   \( I^* = \gamma^* v^* z^* = \sigma^* z^* \).

Accordingly, the cost incurred by each student represents only a fraction, either \( \gamma \), or \( \gamma^* \), of the actual gross costs incurred at the level of the institutions (public or private), which assume that financial burden in the two countries. These total costs per student are proportional to the educational quality in either system, where the variables \( v \) and \( v^* \) translate indicators of quality into units of cost, while the coefficients, \( \sigma \) and \( \sigma^* \), reflect the interrelation between educational quality and the costs actually incurred by students in either country.

Again, for simplicity, the heterogeneous ability, \( a_k \), for a representative \( k \)th student is assumed to be uniformly distributed over an interval \([a_1, a_2]\). By undertaking further studies in one of the two educational systems, it is possible for an individual to attain a level of productivity \( e_k \) or \( e_k^* \). Linear relations are also understood to characterize these transformation processes, which depend on the quality of the domestic or foreign educational systems, \( z \) or \( z^* \), such that:

3. \( e_k = z a_k \)
   \( e_k^* = z^* a_k \)

Hence, when further education is pursued in the domestic country the attainable levels of productivity over the heterogeneous populations of individuals lie in a range defined by \( e_k \in [e_1, e_2] \), while an analogous either overlapping or higher range of productivity levels applies in the case of further studies in the foreign
country, which can also be defined for the representative individual by $e_k^* = \mu e_k$. However, individuals who do not engage in further studies are assumed to only realize a relatively unskilled level of productivity, denoted as $e_0$, which is lower than attainable by the least capable individual in the home country, so that $e_0 < e_1$.

A set of three inter-temporal arbitrage conditions characterize the decision of whether to undertake further studies and, if so, whether at home, or abroad. These comprise an initial condition determining whether the expected wage earnings minus educational costs borne by the student (i.e. net returns) are greater, depending on whether foreign, or domestic, studies are undertaken, as specified below:

4a. $p^* w^*(e^*) + (1-p^*) w(e^*) - I^* > p w^*(e) + (1-p) w(e) - I$

Two additional conditions, indicating whether, or not, studies should be undertaken in either country, are as follows:

4b. $p^* w^*(e^*) + (1-p^*) w(e^*) - I^* > w_0$

4c. $p w^*(e) + (1-p) w(e) - I > w_0$

In light of the linear specification for wages in 1. and after introducing a simplifying notation, by defining $\alpha^* = p^* \theta^* + (1-p^*) \theta$ and $\alpha = p \theta^* + (1-p) \theta$ (so that, in general, $\alpha^* \geq \alpha$ and $p = p^* \Rightarrow \alpha = \alpha^*$), the arbitrage conditions 4abc. can be expressed as:

5a. $\alpha^* e^* - \alpha e > I^* - I$

$(\alpha^* \mu - \alpha) e > I^* - I$

5b. $\alpha^* e^* > I^* + w_0$

5c. $\alpha e > I + w_0$

The General Problematic:

A first exercise in comparative statistic analysis examines how the formulation of optimal educational policy in the home country, aimed at favoring brain gain (BG) and minimizing brain drain (BD). The principal domestic policy
instruments are the price of tuition and other costs borne by students when embarking in domestic studies, I, the educational quality offer by the home country, z, as well as the price for a student studying abroad, since a government can opt to provide fellowships for study abroad, even though it does not control the tuition policies and other costs in the foreign country. This analysis is undertaken for a specific paradigm (regime), where students are deciding between whether to study at home, or abroad. The analysis also considers implications of comparative state changes in the educational quality, z*, of the foreign educational system.

The focus of the analysis is on two alternative educational scenarios. First, a modeling framework is considered where the options are either to study only in the national system, or to also have access to educational opportunities abroad. Such a scenario can be viewed as entailing a benchmark case of national educational autarchy, as compared with international educational mobility. Promoting international university opportunities can be conceived of as a national and/or international policy objective, as in the initiatives designed under the Erasmus program of international student exchanges in the EU. Notably, a particular focus is on the potentially beneficial effects of access to a higher quality system abroad, although potentially at a higher cost.

Second, a scenario is also considered where there is initially no national higher-level university option, as opposed to one in which students can pursue their studies in either country’s educational systems, but with potentially different costs and quality offerings. This latter paradigm corresponds, for example, to the

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9 Franck and Owen (2015) have studied in a somewhat different heterogeneous-agent framework, where probabilities of employment, rather than wages, depend on individuals’ attainable productivity levels, conditions under which the effects of grants for studying abroad will improve a domestic country’s economic welfare.
evolution of educational prospects in a country like Luxembourg, as well as certain developing countries. Notably, it is only recently that Luxembourg founded a national university, which was a costly undertaking that such a prosperous country could afford. Intuitively, the welfare implications of using resources for investing in a country’s own system are likely to depend on both what educational quality is targeted and on both the overall national cost and on the tuition fees charged to students. Indeed, there are potentially offsetting effects. On the one hand, national educational systems can offer conditions under which there is a wider access to education for much of a country’s population, while dampening pressures for brain drain, given that there are often geographically specific synergies between the choice of educational location and the spatial distribution of subsequent job prospects. Yet, considerable financial investments may be initially to set up such a system, in the absence of an accumulation of physical and knowledge assets. Hence, there may be conditions under which it is advantageous to “piggy-back” on existing educational offerings internationally, given that the feasible prospects for developing inclusive, high-quality domestic educational opportunities, particularly in the short run, may be rather restricted.

The first of the two alternative scenarios offers then a setting for assessing the educational policy options, which arise, either when there is only a university in one country – either domestic, or foreign – thereby providing a basis from the perspective of a domestic country to calculate the net benefit differential associated with an expanded set of geographical locations in which an individual can study, along with a facilitated access to a foreign labor market.

It is supposed then that students initially only have the option of attending a home university – either since the direct and indirect costs of studying are excessively hard, or due to restricted access to the foreign university system. The analysis will
assess whether, or not, under what conditions it is advantageous for individuals to undertake studies abroad.

1.1 As an initial point of reference, it will be hypothesized that there is not exist a local university at home, so that individuals have only the option of studying abroad, or not pursuing further studies. For such a scenario the analysis will examine whether it is advantageous to create an university at home (under the assumption that the quality of that educational offering will remain inferior to that abroad).

The following three situations, characterizing the employment prospects in the foreign market, will receive particular emphasis:

  a) a relatively open employment market abroad, regardless of educational background, i.e. both p and p* high;
  b) a selectively open job market abroad, depending on educational background: p* high and p positively related to the quality of the home educational system;
  c) a relatively closed employment market abroad, regardless of educational background: p and p* low.

Nonetheless, since the probabilities of employment in the two job markets are taken here to be exogenous, a focus on a certain number of key scenarios, depending on specific combinations of the parameters p and p*, can offer insights regarding the nature of the principal properties of the model. In particular the following polar cases will be examined: p = 0, p = p*, p*=1, while also considering eventual changes in the relative quality of the university system:

p and p* high, z low (so µ high): case a)
p and \( p^* \) high, \( z \) high (so \( \mu \) low): case a) or b)

\( p^* \) high and \( p \) low, \( z \) high: case b)

\( p \) and \( p^* \) low: case c)

**Polar case 1**: \( p^* = 1 \), regardless of value of \( p \) (so that the “distance”, reflecting the extent to which employment prospects depend on educational location choices, \( d, = p^* - p \), does not matter)

In this instance all foreign-trained individuals are part of brain drain, so that brain gain is precluded. Hence, the apparent intuition is that the pursuit of more advanced education will be welfare decreasing, although the extent of such a loss will depend on what segment of the population decides to study abroad.

**Polar case 2**: \( p^* = p \) (distance, \( d, = 0 \))

Here, educational choice does not affect employment prospects abroad, so that being educated at home is likely to be more advantageous, provided that it is less costly. Nonetheless, a simultaneous symmetric increase in both of the employment prospect parameters, \( p \) and \( p^* \), creates a heightened incentive, per se, to become more educated, thereby generating associated positive welfare effects for the home country (provided that the two parameters are not both equal to one). Yet, a limitation of the analysis for this case is that the employment do not depend on productivity levels of the candidates, where an extended modeling framework could allow for an elitist effect, such that studying in a relatively higher quality system translates into more favorable job access conditions.
Section 1: Comparative static characterization of a setting in which subsets of students attend both domestic and foreign universities.

Whereas the productivity levels for the sub-population of students who opt to undertake their studies at home lie in the interval,

6a. \( e_k \in \left[ \frac{j}{\alpha}, \frac{i}{\alpha^* - \alpha} \right], \)

those who study abroad are defined by:

6b. \( e_k \in \left[ \frac{i}{\alpha^* - \alpha}, e_2 \right] \)

Recalling the simplified notation, \( \alpha^* = p^* \theta^* + (1-p^*) \theta \) and \( \alpha = p \theta^* + (1-p) \theta \), the expression \( \Delta = e_2 - e_1 = z(a_2 - a_1) \) can also be defined. It follows that the overall number of students undertaking advanced studies, along with their further division between those doing so at home or abroad, can be further divided (as shares of the total population \( N \)) into the following sub-populations, including an identification of the associated brain drain and brain gain effects:

7. Total number engaged in higher education - \( NE = (e_2 - \frac{j}{\alpha}) \frac{1}{\Delta} \)

Locally-educated number of students - \( NE_1 = \left( \frac{i}{\alpha^* - \alpha} - \frac{j}{\alpha} \right) \frac{1}{\Delta} \)

Brain gain for domestic students - \( BG_1 = (1 - p) \left( \frac{i}{\alpha^* - \alpha} - \frac{j}{\alpha} \right) \frac{1}{\Delta} \)

Brain drain for domestic students - \( BD_1 = p \left( \frac{i}{\alpha^* - \alpha} - \frac{j}{\alpha} \right) \frac{1}{\Delta} \)

Foreign-educated number of students - \( NE_2 = \left( e_2 - \frac{i}{\alpha^* - \alpha} \right) \frac{1}{\Delta} \)

Brain gain for foreign educated students –

\( BG_2 = (1 - p^*) \left( \frac{e_2 - i}{\alpha^* - \alpha} \right) \frac{1}{\Delta} \)

Brain drain for foreign educated students - \( BD_2 = p^* \left( \frac{e_2 - i}{\alpha^* - \alpha} \right) \frac{1}{\Delta} \).
The foregoing sub-populations permit an assessment of the total brain drain total, BD ( = BD1 + BD2), as well as the value of the brain gain, BG ( = BG1 + BG2). While there is an associated opportunity cost for the home country, generated by the total level of brain drain, it is the level of brain grain which defines the overall level of total productivity locally. The corresponding evaluation of the gross brain gain translated into units of productivity, which corresponds to an overly favorable assessment of the domestic value added since it does not include either the opportunity cost of foregoing unskilled production or the cost of undertaking more advanced studies, is given by:

8. \[ BGB = \bar{e}_1(1 - p)NE1 + \mu \bar{e}_2(1 - p^*)NE2 \]

In this expression, the average levels of individual productivity are defined by:

\[ \bar{e}_1 = \frac{1}{2} \left( \frac{i}{\alpha^* \mu - \alpha} + \frac{j}{\alpha} \right) \quad \text{and} \quad \bar{e}_2 = \frac{1}{2} \left( e_2 + \frac{i}{\alpha^* \mu - \alpha} \right). \]

A net expression for brain gain, which deducts the amount of production, which would have been realized by individuals had they remained unskilled corresponds then to:

9. \[ BGN = BGB - e_0 NE \]

Of course, the overall effect on the economic welfare of the domestic economy, in this cost-benefit perspective, amounts to:

10. \[ W = BGN - SCE, \]

where SCE represents the social cost of education. This amount comprises whatever grants are offered by the national authorities to partially, or entirely, offset the cost of studies abroad, as well as that part of the domestic higher-level education which are not covered by the student tuition payments.
An Analysis of the Influence of Educational Policies: Changes in Tuition Fees and Educational Quality in Both Countries

Six alternative policy scenarios are examined here for a framework where the probabilities of employment, p and p*, are held constant. These involve the following policy initiatives for the domestic and foreign countries:

1) Domestic country
   _ Variation in I with z constant
   _ Variation in z with I constant
   _ Variation in z with a proportional change in I

2) Foreign country
   _ Variation in I* with z* constant
   _ Variation in z* with I* constant
   _ Variation in z* with a proportional variation in I*

In light of the foregoing analysis, the general expressions for assessing brain gain and brain drain are given, respectively by:

\[
11a. \quad BG = (1-p) \, NE_1 + (1-p^*) \, NE_2 = (1-p^*) \, NE + (p^*-p) \, NE_1
\]

\[
11b. \quad BD = p \, NE_1 + p^* \, NE_2 = p^* \, NE - (p^*-p) \, NE_1 = p \, NE + (p^*-p) \, NE_2
\]

The values of productivity which demarcate the threshold between the two sub-populations of individuals, who study at home relative to those who go abroad, amounts to:

\[
12a. \quad R = \frac{I^*-I}{\alpha^*\mu - \alpha \Delta} = \frac{I^*-I}{\alpha^*z^* - \alpha z \Delta}
\]

For a specific setting in which there is a proportional relation between the tuition fees and the quality of an university education, that value for R becomes:

\[
12b. \quad R = \frac{\sigma^*z^*-\sigma z}{\alpha^*z^* - \alpha z \Delta} = \frac{\sigma^*\mu - \sigma z}{\alpha^*\mu - \alpha \Delta}
\]

Analogously, the corresponding threshold values separating the sub-populations of individuals who remain unskilled from those who pursue further studies at home is specified by:
13a. \[ R_0 = \frac{J}{\alpha \Delta} = \frac{J}{\alpha z \Delta} \]

13b. \[ R_0^* = \frac{J^*}{\alpha^* \Delta^*} \]

In light of the foregoing expressions, the overall the effects on numbers of students, linked then to an assessment of the impact on brain gain (once associated consequences for productivities are incorporated), along with the components distinguishing between the contributions related to numbers of locally and foreign educated individuals (equivalent to the expressions derived in equation 7), can be expressed in terms of these threshold values, as follows:

14a. \[ NE = \frac{e_2}{\Delta} - R_0 \]

14b. \[ NE_1 = R - R_0 \]

14c. \[ NE_2 = \frac{e_2}{\Delta} - R \]

In the envisaged scenarios, the expression \( \frac{e_2}{\Delta} \) is constant, so that variations in the different forms of brain gain can be initially analyzed on the basis of the two critical threshold values demarcating the three sub-populations of less skilled, locally educated and foreign trained individuals. More specifically, the overall brain gain effect in terms of numbers of students, NE, is inversely related to \( R_0 \). Ceteris paribus, as that threshold level is lowered (increased) the pool of individuals becoming educated at home (hence overall total) is increased, thereby heightening the potential for brain gain. Analogously reason suggests that the threshold level \( R \) is inversely to the foreign-educated brain gain effect in terms of numbers, \( NE_2 \), whereas the domestic-educated brain gain (again in terms of numbers), \( NE_1 \), increases as the interval \( R - R_0 \) expands.

The impact of changes in the properties of these threshold values \( R \) and \( R_0 \) can, in turn, be analyzed in relation to changes in the tuition fees, \( I \) and \( I^* \), as well as the quality of the two countries’ systems, \( z \) and \( z^* \).

Assuming \( z \) and \( z^* \) constant, it can be shown that:
i. when I diminishes, R increases, R_0 diminishes, and the interval R-R_0 is augmented;

ii. when I* diminishes, R diminishes, R_0 is constant, while the interval R-R_0 decreases.

The analysis presupposes that a positive brain locally-educated brain gain effect, NE1>0, which means that R > R_0 =>

\[
\frac{I^* - I}{\alpha^* z^* - \alpha z} > \frac{I + w_0}{\alpha z}
\]

and

\[
I^* - I > (I + w_0) \frac{\alpha^* z^* - \alpha z}{\alpha z} > I \frac{\alpha^* z^* - \alpha z}{\alpha z}
\]

Hence, \( \frac{I^*}{I} > \frac{\alpha^* z^*}{\alpha z} \)

If \( I^* = \sigma^* z^* \) and \( I = \sigma z \), it can be deduced that \( \frac{\sigma^*}{\sigma} > \frac{\alpha^*}{\alpha} \).

\( \frac{z}{\Delta} \) is constant; \( \frac{\sigma^*}{\sigma} > \frac{\alpha^*}{\alpha} = \Rightarrow \frac{\partial R}{\partial \mu} < 0 = \Rightarrow \frac{\partial R}{\partial z} > 0 \), \( \frac{\partial R}{\partial z^*} < 0 \)

while, \( \frac{\partial R_0}{\partial z} < 0 \)

Thus, R is an increasing fonction of the quality of domestic higher education system, z, and a decreasing fonction of the quality of foreign education, z*. R_0 is a decreasing fonction of z. As a consequence, the inverval R – R_0 is an increasing fonction of z, while being a decreasing fonction of z*.

The associated implications for comparative static changes in the overall brain gain effects in terms of numbers, NE, along with its sub-components, NE1 and NE2, for these different cases under considerations are summarized in Table 1.

The analysis is now extended to consider the quality-weighted effects on brain gain, BG, and brain drain, BD, which incorporate the variation in productivity levels across the population with heterogeneous abilities.

**The Evolution of BG and BD**
An analysis of the implications of changes in educational policies can now be presented in summary form, as follows:

a) BG

\[ BG = (1 - p) (R - R_0) + (1 - p^*) \left( \frac{e_z}{\Delta} - R \right) = (p^* - p) R - (1 - p) R_0 + (1 - p^*) \frac{e_z}{\Delta} \]

_For z and z* constant, if I increases, R diminishes, R_0 increases, BG diminishes; if I* increases, R increases, and BG increases._

_For z* constant, if z increases, R also increases, while R_0 diminishes and BG increases._

_For z constant, if z* increases, R diminishes, and BG diminishes._

b) BD

\[ BD = p (R - R_0) + p^* \left( \frac{e_z}{\Delta} - R \right) = - (p^* - p) R - p R_0 + p^* \frac{e_z}{\Delta} \]

_For z and z* constant,_

a) when I varies, \[ \Delta \frac{\partial BD}{\partial I} = \frac{p^* - p}{\alpha^* \mu - \alpha} - \frac{p}{\alpha} \]

so that \[ \frac{\partial BD}{\partial I} > 0 \leftrightarrow \frac{p^* - p}{p} > \frac{\alpha^* \mu - \alpha}{\alpha} \leftrightarrow \mu < \frac{\alpha^* p^*}{\alpha^* p} \]

It can be noted that for the specific case where domestically-educated individuals are foreclosed from the foreign job, so that \( p = 0 \), \( \frac{\partial BD}{\partial I} > 0 \); whereas when \( p = p^* \), \( \frac{\partial BD}{\partial I} < 0 \).

b) when I* diminishes, R diminishes, and BD increases.

_ For z constant, if z* increases, R diminishes, and BD increases._

_ For a constant level of foreign education, z*, if the quality of domestic education, z, increases, R increases, R_0 diminishes, and the overall brain drain effect, BD, has_
an associated maximum value. More specifically, for the case when $I$ is constant, this maximum is realized for a value specified by:

$$\mu = \frac{\alpha}{\alpha^*} \left[ 1 + \sqrt{\frac{p^* - p_i}{p_i}} J \right]$$

For the case where the probabilities of foreign education are not impacted by the locational choice of where to study, so $p = p^*$, $\alpha^* = \alpha$, so that this maximum corresponds to $\mu = 1$. This means that the BD effect is always increasing when the quality of foreign education is higher than that offered domestically, i.e. for $z \leq z^*$. In contrast, when the probability that locally-educated individuals will obtain a job abroad is low (i.e. $p$ close to zero), the value of $\mu$ is high, so that the effect on brain drain, BD, is almost always a decreasing function of the quality of education at home, $z$. 
**Table 1**

<table>
<thead>
<tr>
<th>Condition</th>
<th>( I ), ( z ) cst</th>
<th>( I ) cst, ( z )</th>
<th>( I = \sigma z ) cst</th>
<th>( I^* ), ( z^* ) cst</th>
<th>( I^* = \sigma z^* )</th>
<th>( NE )</th>
<th>( NE_1 )</th>
<th>( NE_2 )</th>
<th>( BD )</th>
<th>( BG )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu &lt; \frac{1 - p}{1 - p^*} )</td>
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<tr>
<td>( \mu &gt; \frac{1 - p}{1 - p^*} )</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<td>BGB</td>
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</table>
In light of the foregoing analysis a series of principal propositions, relating to the overall effect of specific educational policies on brain drain and brain gain, can be established. The first of these concerns the effects of varying tuition fees on brain drain and brain gain, as also summarized in Table 1.

**Proposition 1: The Effect of Changes in Tuition Fees on Brain Gain Effects When Assessed in Terms of Flows of Students**

For a constant quality of education, \( z \), the impact of decreasing home tuition fees, \( I \), is to increase brain gain, \( NE \), when assessed in terms of actual numbers of students, even though it is has an ambiguous effect on a productivity-weighted measure of brain drain.

Nonetheless, it is crucial to point out that the foregoing results depend on whether, or not, there are compensatory changes in the quality of education in the home country, resulting from decreased (increased) tuition fees. A related remark concerning Proposition 1 is that it is due to two distinct price effects, which i. encourage more students to become educated, while ii. certain students opt for domestic studies rather than pursuing more expensive studies abroad (price substitution effect). The higher propensity to become educated unambiguously favors brain gain, but also generates more brain drain, where the latter effect is heightened, ceteris paribus, when \( dp > 0 \). An associated implication is that since it is assumed that \( p < p^* \), there is more brain gain from educating individuals at home. Nonetheless, since \( z < z^* \) there are potentially offsetting dimensions of this second price effect, given that there is an opportunity cost of training students at home, in a lower quality educational environment.
Proposition 2: Effects of Changes in Tuition Fees on a Productivity-weighted Evaluation of Brain Gain, BG

For a constant quality of education, \( z \), there exists a threshold value \( \mu \) ("hat") which determines the directional impact of decreasing home tuition fees, \( I \), on the value of brain gain, when evaluated in productivity terms. Such a threshold level of \( \mu \) depends on the relative probabilities of brain drain when studying abroad, or at home.

Whereas for values lying below \( \mu \) ("hat") a decrease in the tuition fee, \( I \), always generates an increase in the productivity-weighted value of brain gain, BGB, above such a critical value \( \mu \) ("hat") there is a maximal value of such brain gain. Hence, decreases in domestic tuition fees will initially increase brain gain, but then decrease such domestic welfare enhancing effects.

The effects of decreasing \( I \) on brain drain, again evaluated in productivity terms, is ambiguous.

(****FURTHER WRITE-UP IN PROCESS****)

Section IV: Conclusion

****IN PROCESS****

References


Washington.


