It’s the Competition – Heterogeneous Firms, Trade Liberalization and $R&D$ Investment∗

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Abstract

It is well known that trade liberalization leads to fiercer competition and that competition affects firms’ innovation. In recent trade models with heterogeneous firms, however, better access to foreign markets, rather than increased competition pressure, is most often the reason why firms increase their levels of innovation. This paper proposes a novel framework of process-$R&D$ investment with stochastic outcome based on a heterogeneous-firms model. The paper’s key result shows that if the $R&D$ returns differ according to the hazard rate stochastic dominance criterion, trade liberalization unambiguously increases firms’ $R&D$ spending. In this case, the fiercer competition that accompanies trade liberalization pushes firms to innovate more in order to remain competitive, implying a more fat-tailed productivity distribution of firms, improved industrial productivity and new welfare gains from trade.

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1 Introduction

It is a widely accepted fact that trade liberalization increases competition. The fiercer competition that accompanies trade liberalization, in turn, forces unproductive firms out of the market, and, at the same time, induces firms to innovate more in order to remain competitive.\(^1\) As a consequence, industrial productivity increases.\(^2\) From a theoretical point of view, recent static models with firm heterogeneity and endogenous innovation are able to explain most of these empirical regularities. However, the reason why firms innovate more in these models is typically caused by better access to foreign markets, whereas increased competition pressure that accompanies trade liberalization in principle discourages innovation. Neither of these models is therefore able to explain the link between greater competition and increased innovation.

The contribution of the paper is to propose a novel mechanism that is able to reproduce these empirical findings. The paper adds a pre-stage to Melitz (2003), in which firms can decide on the size of the investment to be made in process-R&I. The outcome of R&I is uncertain and depends on how much a firm invests in R&I. As usual in heterogeneous-firms models, trade liberalization leads to better foreign market access and at the same time increases the competition that firms face. The paper shows that these changes affect a firm’s R&I investment behavior if R&I returns differ according to the hazard rate stochastic dominance (HRSD) criterion. In this case, by investing more in R&I, a firm can reduce its likelihood of becoming one of the low-productivity firms which are forced to exit the market as competition increases through trade liberalization. As a result, a firm’s incentive to raise its R&D spending increases

\(^1\)See, for example, Iacovone et al. (2013), Bernard et al. (2006), Pavcnik (2002) for empirical evidence that trade liberalization leads to increased firm exits; and Bloom et al. (2015), Bernard et al. (2006), Bertschek (1995) for empirical evidence that trade liberalization increases the pressure on firms to innovate.

\(^2\)See, for example, Eslava et al. (2013), Lileeva and Trefler (2010), Bernard et al. (2006), and Pavcnik (2002) for empirical evidence that trade liberalization leads to increased industrial productivity via resource reallocation from low-productivity (exiting) firms towards high-productivity firms and via productivity growth within firms. Additionally, trade liberalization can affect the productivity of an industry, for example, through a better reallocation of resources within firms for the purpose of product diversification (Bernard et al., 2011), or through a reduction in resource misallocation (Eslava et al., 2013).
as trade becomes liberalized. The increased R&D investments made by (some) firms lead to an even fiercer level of competition than would exist if an endogenous change of R&D investments would not take place. This, in turn, forces all firms to invest more in R&D in order to remain competitive in the new market environment. This mechanism could in principle be made part of any heterogeneous-firms trade model, as I show, for example, for the linear demand specification of Melitz and Ottaviano (2008).

Moreover, the changes at the firm level strongly affect the long-run aggregate outcomes. In accordance with the above-mentioned empirical findings, trade liberalization increases the level of industrial productivity due to (i) a reallocation of resources from low-productivity (partly exiting) firms towards higher-productivity firms, and (ii) to firms’ increased R&D investments. In addition, the firms’ internal productivity-changes imply that the firm distribution after trade liberalization hazard rate stochastically dominates the distribution previous to trade liberalization. This means that the fraction of high-productivity firms is larger than it would be if firms did not increase their R&D spending in response to trade liberalization, implying a more fat-tailed productivity distribution of firms. Thus, the increased R&D investments offer an explanation for the empirically identified changes in the firm distribution of industries after opening up to trade (Okubo and Tomiura, 2014; Di Giovanni et al., 2011).

The increased R&D spending also leads to new aggregate welfare gains from trade. This may help to minimize the critique raised by Arkolakis et al. (2012). They show that the constant trade elasticity and the domestic trade share are enough to determine the welfare gains from trade in homogeneous-firms models and in heterogeneous-firms models if firms are distributed according to an untruncated Pareto distribution. They subsequently argue that heterogeneous-firms models cover no new gains from trade compared to models with homogeneous firms. By considering the particular case of Pareto-distributed firms, the model of this paper shows, however, that this critique no longer holds if firms can invest in R&D. Instead, in this case, the trade elasticity

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3If, however, uncertain R&D returns cannot be ordered according to the HRSRD criterion, it is generally not possible to determine how trade liberalization affects firms’ R&D decisions. Ambiguity occurs, for example, if R&D returns differ according to the commonly used first-order stochastic dominance criterion.
depends on the degree of trade openness even in the specific case of an untruncated Pareto distribution.\footnote{See also Melitz and Redding (2014) for another contribution where trade elasticity depends on the degree of trade openness.}

This paper contributes to the increasing literature on trade with heterogeneous firms and endogenous firm productivity. In particular, it focuses on the impact of trade liberalization with a long-term stationary equilibrium perspective. This means that firms can respond to trade liberalization by developing entirely new production techniques in an endogenous market structure, while the outcomes of such developments are highly uncertain (see, for example, Dasgupta and Stiglitz, 1980, for a related interpretation of the long run).

The model is therefore most closely associated with the work of Long et al. (2011). In their model with linear demand, firms can also choose how much to invest in process R&D before knowing their productivity. In contrast to the model of this paper, they exactly specify how R&D spending influences a firm’s expected profits by assuming that R&D investments affect a firm’s expected profits by a multiplicative factor. As a result, the firm’s innovation activity remains constant for any level of trade openness in case of free entry, since, with this R&D specification, the relative profitability between two investment levels remains constant.

Atkeson and Burstein (2010) study the impact of trade liberalization on firms’ innovation in a dynamic Melitz (2003) framework. In that model, firms can continuously decide on their innovation investment intensity. In the long run in most of their analyzed scenarios, however, firm exit and entry again offset firms’ incentives to change their investment intensity in response to trade liberalization. Moreover, they have to rely on numerical methods to derive most of the results, whilst the present model analyzes trade liberalization analytically. Taken together, in these models, the impact of trade liberalization is similar to conventional heterogeneous-firms models, in which firms have no investment choice.

Other related papers are Rubini (2014), Dhingra (2013), Bustos (2011), and Van-
The main difference in these models compared to mine is that firms make their innovation decision after the productivity draw, implying that the outcome of innovations is deterministic. The general result is that, in line with empirical evidence (Bustos, 2011; Lileeva and Trefler, 2010), high-productivity exporting firms increase their innovation investments in response to trade liberalization, whereas low-productivity non-exporting firms do not. This is the case, because these firms gain from improved foreign market access. However, since the innovation decision is based on certain outcomes and not, as in the model of this paper, on expected outcomes, fiercer competition generally reduces the incentive to innovate. Hence, these trade models are unable to explain the positive impact that competition has on firms’ innovation activities which is found in the data.

To the best of my knowledge, Desmet and Parente (2010) provide the only static general equilibrium model in which fiercer competition owing to trade liberalization causes firms to innovate more. However, the mechanism of their model differs substantially from the one I use in this paper, because it works by changing the price elasticity of demand. Moreover, since Desmet and Parente (2010) assume that the innovation outcomes are deterministic, in equilibrium all firms end up with the same productivity. As a result, their model is not able to explain changes in industrial productivity caused by resource reallocation among firms.

I structure the rest of the paper as follows. Section 2 introduces the model by first describing the R&D choice of firms and the equilibrium concept. Section 3 then analyzes how trade liberalization changes the R&D choice of firms, while Section 4 analyzes how the consideration of firm R&D investment affects industrial productivity and welfare. Section 5 discusses under which specific conditions R&D investments

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5 All these models build on Melitz (2003), except that of Dhingra (2013), who assumes a linear demand specification. Furthermore, in the model of Dhingra (2013) firms can, besides upgrading their production technology, also invest in product innovations, which is not possible in the other models mentioned above.

6 Ederington and McCalman (2008) or Yeaple (2005) also study how trade liberalization affects the adoption of new technologies by firms. However, in their models, sector heterogeneity is a result of firms that adopt different technologies, and not the other way around.

7 Recently, this mechanism has also been used in models of endogenous growth to show that a more competitive environment caused by trade liberalization pushes firms to increase their innovation activity. See, among others, Impullitti and Licandro (2015).
depend on trade openness in static heterogeneous-firms models. In Section 6, I use the Pareto distribution to give an example, and finally, Section 7 summarizes the main results.

## 2 The model

The main assumptions are similar to those of Melitz (2003) in the case of two symmetric countries. There is one significant difference: Firms can influence their productivity draw by means of R&D investment. Figure 1 compares the decision process of firms in the present model with that of Melitz (2003). As in Melitz (2003), the decision process of firms has two stages.

Starting at the second stage, the domestic profits of a firm with productivity \( z \) are, in the case of standard CES consumer preferences,

\[
\pi_d(z) = \frac{R}{\sigma} \left( P z^\sigma - 1 \right)^\sigma - f_d, \tag{1}
\]

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8The symmetry assumption ensures that both countries have the same aggregate variables, and that firms therefore behave similarly in both countries. Hence, in order to save space, the analysis is done from the perspective of one country. However, one should bear in mind that all results apply to both countries.
where $R$ denotes aggregate revenue, $P$ the aggregate price index, $\sigma$ the constant elasticity of substitution between two consumed varieties, and $f_d$ the fixed costs required to remain in the domestic market. The nominal wage rate does not appear, because it serves as the numéraire and equals one. Since there is a continuum of firms in the market, a single firm has no influence on $P$ and $R$. Only firms with a high enough productivity draw to earn non-negative profits remain in the market. Hence, $\pi_d(z_D) = 0$ implicitly defines the productivity at which a firm earns exactly zero profit in the domestic market. One can calculate $z_D$ by setting (1) equal to zero as

$$z_D = \left( \frac{f_d \sigma}{R} \right)^{\frac{1}{\sigma-1}} \frac{\sigma}{(\sigma - 1)P}. \quad (2)$$

In addition, a firm that exports has to pay fixed costs $f_x$ and variable trade costs of Iceberg-type $\tau \geq 1$ per unit shipped. The export profits of a firm with productivity $z$ are

$$\pi_x(z) = R \left( \frac{Pz}{\tau \sigma} - \frac{1}{\sigma - 1} \right) \left( \frac{Pz}{\tau \sigma} - \frac{1}{\sigma - 1} \right) - f_x. \quad (3)$$

By setting this equation equal to zero, $\pi_x(z_X) = 0$, and using (2) leads to the export cut-off

$$z_X = \frac{z_D}{\phi} \quad (4)$$

as a function of the domestic market cut-off and the measure of trade openness $\phi$, with $\phi = \frac{1}{(\frac{f_d}{f_x})^{\frac{1}{\sigma-1}}}$. Autarky would be $\tau = \infty \rightarrow \phi = 0$ and free trade would be $\tau = 1 \rightarrow \phi = \left( \frac{f_d}{f_x} \right)^{\frac{1}{\sigma-1}}$. As in the standard model of Melitz (2003), only the most productive firms export as long as $f_x \tau^{\sigma-1} > f_d$. Hence, $z_X > z_D$. In addition, due to additive fixed costs and market-independent sunk $R&D$ costs, all export firms also sell domestically.

In the first stage, a firm can decide whether to enter or not to enter the industry, and if it decides to enter, how much to invest in $R&D$. If it decides not to enter the industry,
its pay-off is zero. Similar to the heterogeneous-firms trade model of Long et al. (2011), if in the model of this paper an entrant decides to enter, it has to decide how much to invest in R&D. For simplicity, I constrain the choice to two investment levels, low $L$ and high $H$. In the following, the index $i \in \{L, H\}$ indicates the choice. This, however, is only for the purpose of simplified exposition. As I show in Appendix H, the results do not change if there is a continuum of R&D choices available to an entrant. A high level of investment requires higher sunk costs than a low level of investment, $F_H > F_L$. Furthermore, an entrant that invests in an $i$-level of R&D draws its productivity from a common distribution $G_i(z)$ with support $z \geq 0$. As I show later on in Section 5, the following assumption is crucial for the results of the model:

**Assumption 1.** A high level of R&D investment leads to productivity draws that hazard rate stochastically dominate those of a low level.¹⁰

The distribution function $G_H(z)$ hazard rate stochastically dominates the distribution function $G_L(z)$, $G_H(z) >_hr G_L(z)$, if for any possible productivity draw

$$\frac{g_H(z)}{1 - G_H(z)} < \frac{g_L(z)}{1 - G_L(z)}.$$  

(5)

The hazard rate stochastic dominance (HRSD) concept implies that the relative probability between a high and a low level of R&D investment of drawing a productivity above a specific productivity level increases when this specific productivity level increases;¹¹ $\frac{\partial}{\partial z} \frac{1 - G_H(z)}{1 - G_L(z)} = -\frac{g_H(z)(1 - G_L(z)) + g_L(z)(1 - G_H(z))}{(1 - G_L(z))^2} > 0$ due to $g_H(z)(1 - G_L(z)) < g_L(z)(1 - G_H(z))$. Furthermore, HRSD implies first-order stochastic dominance (FOSD), but not the other way around. In addition, equation (5) implies that an investment in a high level of R&D has a larger conditional and unconditional expectation value than an investment in a low level of R&D. As a result, the model’s R&D

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³See Section 5 for a discussion of the model of Long et al. (2011).

¹⁰This includes, besides the HRSD order by itself, all stochastic orders that imply HRSD, e.g., the likelihood ratio order or the mean residual life order (see Shaked and Shanthikumar, 2007). Pflüger and Russek (2013) and Demidova (2008) also use the concept of HRSD in heterogeneous-firms trade models à la Melitz (2003). However, neither of them assume endogenous R&D investments; instead, they assume that the productivity distributions of two countries differ exogenously according to this stochastic order criterion.

¹¹For mathematical details see, for example, Shaked and Shanthikumar (2007).
specification captures, in addition to these technical aspects, some essential characteristics of R&RD that are highlighted in the literature. R&RD has an uncertain outcome. However, if firms invest more in R&RD, this is likely to lead to better results (Doraszelski and Jaumandreu, 2013).

The expected firm value prior to the draw is, for both R&RD investments, equivalent to the expected profits from domestic and export sales minus the sunk investment costs. Hence, by using (2) in (1) and (4) together with (2) in (3) and taking each associated conditional expectation, the expected firm value is similar to Melitz (2003), besides the fact that it now depends on an entrant’s R&RD investment level $i$:

$$V_i(z_D, \phi) = j_i(z_D)f_d + j_i(z_X)f_x - F_i,$$  
(6)

with $j_i(z_s) = (1 - G_i(z_s))k_i(z_s)$. $1 - G_i(z_s)$ stands for the probability of entering the domestic market with $z_s = z_D$ (and, respectively, of entering the export market with $z_s = z_X = z_D/\phi$). In addition, $k_i(z_s) = \int_{z_s}^{\infty} \left( \frac{z}{z_s} \right)^{\sigma-1} \frac{g_i(z)}{1 - G_i(z_s)} \, dz - 1$, is the expected profits earned in the domestic and export market conditioned on a successful market entry, respectively.$^{12}$ Since $HRSD$ implies $j_H(z_s) > j_L(z_s)$ for any $z_s > 0$, the expected profits net of R&RD costs are always higher if an entrant decides to invest in a high level rather than a low level of R&RD, $V_H(z_D, \phi) + F_H > V_L(z_D, \phi) + F_L$.

By taking equation (6), I am able to state an entrant’s R&RD investment decision problem if it decides to enter the industry. In this case, an entrant seeks to maximize its expected firm value

$$V(z_D, \phi) = \max_i \{V_i(z_D, \phi)\},$$  
(7)

with respect to the R&RD investment level $i$. The decision depends on the degree of trade openness $\phi$, which is treated as exogenous in the model. Moreover, it depends on the expected general equilibrium value $z_D$. As usual in heterogeneous-firms trade models with monopolistic competition, a single firm is assumed to be too small to

$^{12}k_i(z_s)$ has to be finite. Following Melitz (2003), this is the case if the $(\sigma - 1)$th uncentered moment of $g_i(z)$ is finite.
have any influence on $z_D$. However, the entry and $R&D$ decision of all firms together determine $z_D$ endogenously. Changes in $z_D$ affect the expected profits of each $R&D$ investment according to:

$$\frac{\partial V_i(z_D, \phi)}{\partial z_D} = j'_i(z_D)f_d + j'_i(z_X)f_x\phi^{-1} < 0,$$

with $j'_i(z_s) = -\frac{1}{z_s} (\sigma - 1)(1 - G_i(z_s))(k_i(z_s) + 1) < 0$ (see Melitz, 2003) and $j'_H(z_s) < j'_L(z_s) < 0$ (for the proof, see Appendix B). As can be seen from (2), $z_D$ contains the aggregate price index. Any change in $z_D$ can therefore be interpreted as a change in the competitive environment of a firm. Since the higher the cut-off is, the lower a firm’s expected profits will be, I characterize a higher cut-off $z_D$ as a “tougher” market environment.

### 2.1 Determination of the equilibrium

Assume that there is an infinite number of identical entrants. Assume furthermore that all entrants simultaneously decide on their entry and $R&D$ decision. Moreover, as is standard in the literature of innovation and endogenous market structure, I only focus on symmetric Nash equilibria (e.g., Long et al., 2011; Desmet and Parente, 2010; Dasgupta and Stiglitz, 1980). An equilibrium must then satisfy the following definition:

**Definition 1.** The cut-off productivity level $z_D$ and the $R&D$ investment level $i$ determine an equilibrium with free entry if $V_i(z_D, \phi) \geq V_k(z_D, \phi)$, with $i, k \in \{L, H\}$ and $i \neq k$, and $V_i(z_D, \phi) = 0$ hold.

The first condition says that in any equilibrium no entrant wants to change its $R&D$ decision, taking as given the value of $z_D$, i.e., the entry and $R&D$ decision of the other entrants. The second condition says that in a free-entry equilibrium expected profits have to be equal to zero. One can easily show that such an equilibrium is always

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13To make the point, I neglect asymmetric characteristics of entrants. This assumption could be relaxed, for example, by assuming different $R&D$ costs among entrants.
characterized by a positive number of firms in the market.\textsuperscript{14} In the following:

Definition 2. Let $z_D^i$ be the cut-off for which an investment in the R&D level $i$ leads to zero expected profits: $V_i(z_D^i, \phi) = 0$.

Due to the assumption of identical entrants, it follows from the definition of an equilibrium that:

Lemma 1. Two pure equilibria are possible: Either all entrants find it optimal to invest in a low level of R&D, which is the case if $V_L(z_D^L, \phi) = 0$ and $V_H(z_D^L, \phi) \leq 0$ hold, or to invest in a high level of R&D, which is the case if $V_H(z_D^H, \phi) = 0$ and $V_L(z_D^H, \phi) \leq 0$ hold.

In the following, I focus only on pure equilibria.\textsuperscript{15} Since entrants only care about the competition intensity $z_D$, a pure equilibrium always exists, because $HRSD$ implies that $\partial V_H(z_D, \phi) / \partial z_D < \partial V_L(z_D, \phi) / \partial z_D < 0$ (for details, see Appendix C). Which pure equilibrium exists, however, depends on the expected return and cost of each R&D investment. Figure 2 illustrates both kinds of pure equilibria. In the left-hand (right-hand) graph in the figure, the only equilibrium is given by an investment in a low (high) level of R&D, because the investment costs of a high (low) level of R&D are too high relative to its expected returns.

The interesting question is whether an exogenous change in the market environment alters the equilibrium R&D investment level, and if this is the case, how it does this. The following analysis focuses on trade liberalization. In general, however, the framework may be used to study how a change in any parameter that affects an entrant’s value (6) alters the equilibrium R&D investment level.

\textsuperscript{14}In order to have a positive number of firms in the market in equilibrium, it is required that $V_i(z_D, \phi) = 0$. This is the case if, for at least one R&D investment level, $V_i(z_D, \phi) > 0$ holds at the lowest possible $z_D$, $z_D = 0$, and $V_i(z_D, \phi) < 0$ holds for $z_D \to \infty$. This is always fulfilled for finite R&D costs $F_i$, because, as shown by Melitz (2003), $j_i(z_D)$ decreases from infinity to zero on $z_D = (0, \infty)$.

\textsuperscript{15}If $V_H(z_D^H, \phi) = V_L(z_D^H, \phi) = 0$, an entrant is indifferent between both investment levels. Since $\partial V_H(z_D, \phi) / \partial z_D < \partial V_L(z_D, \phi) / \partial z_D < 0$, it follows from the definition of $z_D^i$ that $z_D^L = z_D^H$. As a result, in this case, according to Definition 1, in addition to the two pure equilibria that exist, an infinite number of mixed equilibria also exist.
Figure 2: Low R&D spending equilibrium (left). High R&D spending equilibrium (right).

3 Trade liberalization and R&D spending

Now I will analyze how lower variable trade costs \( \tau \), which is equivalent to a larger \( \phi \), affects the optimal R&D investment level. As in Melitz (2003) and other models with no dynamic adjustments, one can see the following results as a steady state comparison of different trade-openness levels. I proceed in two steps. First, I analyze an entrant’s R&D choice in dependency on trade openness by assuming that none of the other entrants will deviate from their R&D choice. Second, given the knowledge of the first step, I am able to determine an entrant’s equilibrium R&D investment strategy in dependency on trade openness.

To start with, suppose that an entrant assumes that all other firms were to invest in an \( i \)-level of R&D, and hence the general equilibrium value is \( z^*_D \). The R&D choice function of an entrant (7) in this case is

\[
V(z^*_D, \phi) = \max \left\{ V_i(z^*_D, \phi); V_k(z^*_D, \phi) \right\}, \text{ where } k \in \{L, H\} \text{ and } k \neq i.
\]  

The first argument represents an entrant’s expected firm value of an \( i \)-level of R&D investment. Its value is equal to zero due to free entry for any level of trade openness as long as all other firms were also going to invest in an \( i \)-level of R&D. Hence, by
using (6) and (7) the free entry condition is

\[ V_i(z^i_D, \phi) = j_i(z^i_D) f_d + j_i(z^i_X) f_x - F_i = 0. \]  \(10\)

Trade liberalization changes the free entry condition according to

\[ dV_i(z^i_D, \phi) = j'_i(z^i_D) f_d dz^i_D + j'_i(z^i_X) f_x \phi^{-1} dz^i_D - j'_i(z^i_X) f_x z^i_X \phi^{-1} d\phi. \]  \(11\)

This is the total differential of equation (10). If an economy becomes more open, competition will increase in the domestic market (term one) and in the export market (term two) and lead to lower expected profits (see equation 8); however, at the same time, expected export profits will increase (term three). The total change has to be equal to zero due to free entry. Hence, I can write equation (11) as

\[ \frac{dz^i_D}{d\phi} = \frac{j'_i(z^i_X) f_x z^i_X}{j'_i(z^i_D) f_d \phi + j'_i(z^i_X) f_x} > 0. \]  \(12\)

The last inequality is due to \(j'_H(z^i_s) < j'_L(z^i_s) < 0\) (for the proof, see Appendix B), better access to the foreign market leads to a stronger increase in expected profits if a firm invests in a high level of R&D, rather than a low level of R&D. At the same time, however, tougher competition leads to a stronger decrease in expected profits if a firm invests in a high level rather than a low level of R&D. Thus, the weight of each effect matters for the sign of (13). By reformulating equation (13) slightly differently, I obtain

\[ dV_k(z^i_D, \phi) = j'_k(z^i_D) f_d dz^i_D + j'_k(z^i_X) f_x \phi^{-1} dz^i_D - j'_k(z^i_X) f_x z^i_X \phi^{-1} d\phi. \]  \(13\)

Since \(j'_{H_L}(z^i_s) < j'_{L_H}(z^i_s) < 0\) (for the proof, see Appendix B), better access to the foreign market leads to a stronger increase in expected profits if a firm invests in a high level R&D. At the same time, however, tougher competition leads to a stronger decrease in expected profits if a firm invests in a high level rather than a low level of R&D. Thus, the weight of each effect matters for the sign of (13). By reformulating equation (13) slightly differently, I obtain

\[ dV_k(z^i_D, \phi) = j'_k(z^i_X) f_x \phi^{-1} \left( \left( \frac{j'_k(z^i_D) f_d}{j'_k(z^i_X) f_x} \phi + 1 \right) dz^i_D - z^i_X d\phi \right). \]  \(14\)
where the first term of the large brackets reveals the weight of the competition effect and the second term reveals the weight of better foreign market access. The sign of $dV_k(z_D^i, \phi)$ depends on the competition intensity $z_D^i$, which can, as previously shown, either be $z_H^D$ or $z_L^D$. First, assume the equilibrium consists of firms that have all invested in a low level of R&D, i.e., the indexes are $i = L$ and $k = H$. The concept of HRSD implies that $0 < \frac{j_H^L(z_L^D)}{j_H^L(z_X^L)} < \frac{j_H^L(z_H^D)}{j_H^L(z_X^H)}$, as long as $z_X^L > z_D^L$ (see Appendix A). This is the case, because the likelihoods between a high and a low level of R&D investment drawing a productivity above a certain level increase when the value of this level increases, i.e., $\partial(1 - G_H(z_L^L))/(1 - G_L(z_s^L))/\partial z_s^L > 0$. Thus, by raising the amount of R&D spending, an entrant can reduce its likelihood (relative to an entrant not raising its R&D spending) of becoming one of the low-productivity firms that are forced to exit the market as competition toughens. As a result, an entrant’s incentive to invest more in R&D increases as trade becomes liberalized, because the positive effect of better foreign market access ($d\phi > 0$) carries a stronger weight than the negative effect of fiercer competition. Formally, $dV_H(z_D^L, \phi) > 0$, since the term in the large bracket of (14) is negative and $j_H^L(\cdot) < 0$.

Now, suppose that an entrant assumes that the equilibrium consists of firms that have all invested in a high level of R&D, i.e., the indexes are $i = H$ and $k = L$. By applying the same steps as before, one can show that an entrant’s incentive to invest in a low level of R&D declines as trade becomes liberalized. Formally, $dV_L(z_D^H, \phi) < 0$, since the term in the large bracket of (14) is now positive and $j_L^H(\cdot) < 0$.

Taken together, if Assumption 1 holds (i.e., as long as the returns of two R&D investments differ according to the HRSD criterion), fiercer competition cannot offset

\[ \text{If one allows for } f_x \tau^{\sigma-1} < f_d, \text{ the sign of (14) would be reversed as soon as trade costs reach a level at which low-productivity firms find it optimal to export but not to serve the domestic market anymore; i.e., as soon as } z_X = z_D. \text{ In this (rather unrealistic) case, further trade liberalization would reduce the incentive to invest more in R&D. At the same time, however, competition would increase further (see equation 12). Hence, by considering this (rather unrealistic) case, the model can generate an inverted U-shaped relationship between competition and innovation as reported, for example, by Aghion et al. (2005).} \]

\[ \text{However, it is important to note that the results are not driven by changes in the relative profitability between the two expected firm values conditional on successful entry, since HRSD only implies that the conditional expectation values of two R&D investment levels differ. It does not, in general, imply a change in their relative value when the condition changes.} \]
(more than offset) the positive direct effect of lower trade costs. Formally:

**Lemma 2.**

\[
\begin{align*}
    dV_H(z^L_D, \phi) &> 0 \text{ and } dV_L(z^L_D, \phi) = 0 \quad \forall \quad z^L_X > z^L_D, \\
    dV_H(z^H_D, \phi) &> 0 \text{ and } dV_L(z^H_D, \phi) < 0 \quad \forall \quad z^H_X > z^H_D.
\end{align*}
\]  
(15)

**Proof.** See Appendix A.

Moreover, Lemma 2 implies the following:

**Lemma 3.** As long as the benefit to the cost ratio derived from a high level of R&D is sufficiently, but not excessively, different from that derived from a low level of R&D, there exists a level of trade openness \( \phi^* \), with \( \phi^* \in [0, 1] \), at which \( V_H(z^L_D, \phi^*) = V_L(z^H_D, \phi^*) \). At the same time, at \( \phi^* \) it follows that \( z^H_D = z^L_D \). Hence,

\[
V_H(z^L_D = z^H_D, \phi^*) = V_L(z^H_D = z^L_D, \phi^*) = 0
\]
(16)

due to free entry. Moreover, if \( \phi^* \) exists, it is unique.\(^{18}\)

**Proof.** See Appendix D.

The two conditions \( F_H > f_d j_H(j^{-1}_L(F_L/F_d)) \) and \( F_L > (f_d + f_x) j_L(j^{-1}_H(F_H/F_d + f_d)) \), with \( j^{-1}_i(\cdot) \) as the inverse function of \( j_i(\cdot) \), ensure the existence of \( \phi^* \).\(^{19}\) I show the derivations in Appendix D. The first condition states that an entrant finds it optimal under autarky \( (\phi = 0) \) to invest in a low level of R&D if all other firms were to invest in a high level of R&D. In addition, the second condition states that an entrant finds it optimal under free trade \( (\phi = 1) \) to invest in a high level of R&D if all other firms were to invest in a low level of R&D.

### 3.1 Equilibrium before and after trade liberalization

Now, I am able to determine the entrants’ equilibrium R&D choice, and thus the equilibrium as defined in Definition 1, in dependency on trade openness. Figure 3

---

\(^{18}\)The sunk investment costs can be adjusted such that an intersection \( \phi^* \) point always exists, because \( \frac{\partial V_i(z^*_D, \phi)}{\partial F_i} < 0 \) and \( \frac{\partial V_i(z^*_D, \phi)}{\partial F_i} > 0 \).

\(^{19}\)The inverse function exists, because \( j_i(\cdot) \) is continuous and monotonically decreasing in its argument (see 8), as long as \( z_D \geq 0 \). Moreover, \( z^L_D, z^H_D \geq 0 \), since \( z \geq 0 \).
Figure 3: \( R\&D \) choice in dependency on trade openness \( \phi \).

displays the results. From Lemmas 2 and 3 it follows that \( V_L(z_H^H, \phi) > V_H(z_H^H, \phi) = 0 \)
and \( V_L(z_L^L, \phi) = 0 > V_H(z_L^L, \phi) \) as long as \( \phi < \phi^* \). As a result:

**Proposition 1.** *If trade costs are sufficiently high, \( \phi < \phi^* \), and Assumption 1 holds, an investment in a low level of \( R\&D \) is the unique equilibrium.*

Moreover, from Lemma 3 it follows immediately that at \( \phi^* \) an entrant is indifferent between a low and a high level of \( R\&D \) investment. Hence, it is not possible to clearly determine the firms’ \( R\&D \) investment level at this particular level of trade openness. Nevertheless, it is possible to determine the general equilibrium value \( z_D \) as \( z_D = z_L^L = z_H^H \), because its value is at this particular level of trade openness independent from firms’ \( R\&D \) investments (for the proof, see Appendix D). Furthermore, for trade-openness levels beyond \( \phi^* \), it follows from Lemmas 2 and 3 that \( V_L(z_H^H, \phi) < V_H(z_H^H, \phi) = 0 \) and \( V_L(z_L^L, \phi) = 0 < V_H(z_L^L, \phi) \). Hence:

**Proposition 2.** *If trade costs are sufficiently low, \( \phi > \phi^* \), a high level of \( R\&D \) invest-
ment is the unique equilibrium as long as Assumption 1 is fulfilled.\textsuperscript{20}

To understand the results of Propositions 1 and 2, consider investors who finance the projects of entrants. In the absence of asymmetric information, full enforceability of contracts and uncorrelated projects, a profit-maximizing risk-neutral investor will only finance the projects that have the highest expected return on investment: \( \max \{ \frac{V_i + F_i}{F_i} \} \). This expression is equivalent to Tobin’s famous \( q \), the ratio of expected market value to replacement cost (Tobin, 1969). Consistent with the literature, it tends to unity in the model owing to free entry.\textsuperscript{21}

A high level of R&D investment leads for any level of trade openness to a higher expected productivity draw and thus to a higher expected market value compared to a low level of R&D investment. Thus, if an entrant has to decide between these two investments without taking investment costs into account, he will always invest in a high level of R&D. For low levels of trade liberalization, however, an investment in a high level of R&D is not profitable, because the investment costs are too high relative to the expected sales opportunities (which implies that the expected market value is too low). Hence, for low levels of trade openness, an investor will only finance entrants that decide to carry out \( L \)-projects. The incentive to innovate more and carry out \( H \)-projects increases with trade liberalization. Thus, an entrant can raise its expected overall profits above zero by investing in an \( H \)-project, once trade has been liberalized beyond a given level of trade openness \( \phi^* \). Other entrants will also take advantage of this possibility. As a result, the competition intensity will increase (equation 8). This leads to a decrease in the expected profits for both kinds of R&D investments, until investing in a high level of R&D leads to zero expected profit, \( V_H(z_H^D, \phi) = 0 \). However, the competition intensity is now too strong to have zero expected profits by investing in a low level of R&D: \( z_H^D > z_L^D \rightarrow V_L(z_H^D, \phi) < 0 \). Investors will anticipate

\textsuperscript{20}This argument still holds if trade liberalization takes place and incumbent firms are hit by an exogenous exit rate \( \delta \) as in Melitz (2003). For trade-openness levels beyond \( \phi^* \), an entrant finds it optimal to invest in a high level of R&D for all possible competition intensities \( z_D \), where \( z_D \in [z_L^D, z_H^D] \). In the beginning, all incumbent firms would have invested in a low level of R&D. However, these firms are hit gradually by the shock \( \delta \). All new entrants that fill the gap will invest in a high level of R&D, and the competition intensity increases from \( z_L^D \) to \( z_H^D \).

\textsuperscript{21}If Tobin’s \( q \) were larger than one, more firms would enter. This increases the competition intensity summarized by \( z_D \) until \( q = 1 \).
this change in the market structure and, subsequent to trade liberalization, will only finance entrants that invest in an \( H \)-project.\(^{22}\)

This means that, from the perspective of a single entrant, better access to a foreign market acts in the first place as a pull factor for innovations. The increased innovations of all firms together, however, toughens competition with the ultimate result that entrants are pushed to innovate more; otherwise their expected competitiveness would be too low, and, thus investors would not finance them anymore.

4 Trade liberalization and the aggregate level

The next step analyzes what impact the increased firm-level \( R\&D \) investments resulting from trade liberalization have on industrial productivity and on welfare. At a particular level of trade openness, industrial productivity is similar to Melitz (2003), formulated as

\[
\tilde{z}_i(z_D) = \left( \frac{\bar{z}_D + p_X^i \tau^{1-\sigma} \bar{z}_X}{1 + p_X^i} \right)^{\frac{1}{1-\sigma}},
\]

(17)

where the index \( t \) stands for total industrial productivity, \( p_X^i = \frac{1-G_i(z_X)}{G_i(z_D)} \) being the share of export firms to all firms, \( \bar{z}_D = \int_{z_D}^{\infty} z^{\sigma-1} \frac{g(z)}{1-G_i(z_D)} dz \) being the average productivity of all firms in the market, and \( \bar{z}_X = \int_{z_X}^{\infty} z^{\sigma-1} \frac{g(z)}{1-G_i(z_X)} dz \) being the average productivity of export firms. The productivity of an industry is a weighted average of the productivity used in producing the domestic sales, \( \bar{z}_D \), and the productivity used in producing the exports, \( \bar{z}_X \) (note that the export cut-off \( z_X \) is a function of the domestic market cut-off \( z_D \)). Trade costs \( \tau \) reduce the export productivity if it is measured “at the foreign consumers” by the factor \( \tau^{1-\sigma} \).

All firms find it optimal to invest in a high level of \( R\&D \) and draw from a better distribution compared to the distribution that was preferred before trade liberaliza-

\(^{22}\)Another perspective is to think of entrants as each being endowed with an amount of \( F_H \) of resources. This can be, for example, due to business angels. As long as trade is rarely liberalized, it would be optimal for an entrant to invest only \( F_L \) and keep the rest. After trade liberalization, however, it would be optimal to invest all resources \( F_H \).
Figure 4: **The impact of trade liberalization on the productivity distribution of all firms:**
(a) in a model à la Melitz (2003) without firm-level R&D investment; (b) in this model with firm-level R&D investment.

Figure 4 illustrates the difference between a model with and without endogenous R&D investment for the particular case of Pareto-distributed productivity (see Section 6). In a model with R&D investment, trade liberalization leads to fiercer competition compared to a model with no R&D investment, \( z_D^f - z_D^{f'} < z_D^H - z_D^{L'} \), where the prime indicates the value of the pre-liberalization competition. The inequality follows immediately from the free-entry adjustment, (8) and (10), and the level of competition at \( \phi^* \), (16). In addition, an increase in firms’ R&D spending leads to an increase in the productivity within firms.

In order to disentangle the impact of both effects on the industrial productivity, I first focus on the impact of fiercer competition. In doing so, I compare the industrial productivity for some \( \phi > \phi^* \) that only considers the fiercer competition resulting from the firms’ increased R&D investments, \( \tilde{z}_L(z_D^H) \), with the industrial productivity of a model with no R&D investment, \( \tilde{z}_L(z_D^L) \) (i.e., \( \tilde{z}_L(z_D^L) \) corresponds to that of Melitz, 2003). Afterwards, I compare the industrial productivity that only involves the greater number of high-productivity firms, but not the fiercer competition, \( \tilde{z}_H(z_D^L) \), with the industrial productivity of no R&D investment, \( \tilde{z}_L(z_D^L) \). Finally, I use these results to show the total effect of firms’ increased R&D spending on industrial productivity.

To start with, the existence of a larger number of high-productivity firms due to in-
creased R&D spending fosters competition. This requires a higher level of productivity for firms to survive in the domestic market and also in the foreign market compared to a model with no R&D investment, $z_D^L - z_D^{L'} < z_D^H - z_D^{L'}$ (effect 1a vs 1b in Figure 4) and $z_X^L - z_X^{L'} < z_X^H - z_X^{L'}$, respectively. The first inequality holds for the same reasons as before. The second inequality holds by additionally taking the linear relationship between $z_X$ and $z_D$ into account (i.e., $z_X \phi = z_D$). As a result, the average productivity used for domestic production and for exports is, at some level of trade openness $\phi > \phi^*$, higher compared to that of Melitz (2003), $\tilde{z}_D^L(z_D^H) > \tilde{z}_D^L(z_D^L)$ and $\tilde{z}_X^L(z_X^L) > \tilde{z}_X^L(z_X^P)$.\textsuperscript{23} The share of export firms among all firms, however, can be larger or smaller: $p_X^L(z_D^H) = (1-G_L(z_X^H))/(1-G_L(z_D^H)) \leq p_X^L(z_D^L) = (1-G_L(z_X^L))/(1-G_L(z_D^L))$.\textsuperscript{24} This in turn may induce more efficiency losses compared to a model with no R&D investment. However, as long as the industrial productivity is measured “at the firm gate” (i.e., without taking the Iceberg efficiency losses into account), it follows:\textsuperscript{25}

**Lemma 4.** Higher R&D spending of firms in response to trade liberalization implies fiercer competition. This fiercer competition always increases the level of the industrial productivity, as long as it is measured “at the firm gate”: $\tilde{z}_X^L(z_D^H) > \tilde{z}_X^L(z_D^L)$.

**Proof.** See Appendix E.

Next, I want to determine the effect of increased R&D investments without considering the fiercer competition. In the present model, the increased R&D investments of firms affect the tail of the productivity distribution (effect (2) in Figure 4). In equilibrium, there are a greater number of high-productivity firms in the market due to the properties of HRSD. As a result, the average productivity used for domestic production and that used for exports are larger, $\tilde{z}_D^H(z_D^L) > \tilde{z}_D^L(z_D^L)$ and

\textsuperscript{23}The inequalities follow from $\frac{\partial z_D^L(z_D^L)}{\partial z_D} = \frac{(z_D^L - z_D^{L'})}{1-G_L(z_D^L)} > 0$.

\textsuperscript{24}Too see this, note $\frac{\partial (1-G_L(z_X))}{\partial z_D} = \frac{-G_L(z_X)\phi^{-1}(1-G_L(z_X))}{(1-G_L(z_D))} \leq 0$, depending on $z_D$, $\phi$ and the functional form of $G_L(\cdot)$. Hence, $p_X^L(z_D^H) \leq p_X^L(z_D^L)$.

\textsuperscript{25}Similar to Melitz (2003), it is possible that, due to the reallocation of resources to high-productivity firms, the industrial productivity can decline if it is measured “at the consumers”. If, however, the productivity averages are compared with each other at some highly liberalized trade level, the efficiency losses will always be more than offset by the larger productivity averages “at the firm gate”, i.e., by $\tilde{z}_X^L(z_D^H) > \tilde{z}_X^L(z_D^L)$ and $\tilde{z}_D^L(z_D^H) > \tilde{z}_D^L(z_D^L)$.
\( \bar{z}_H(\tilde{z}_D^i) > \bar{z}_L(\tilde{z}_D^i) \), respectively.\(^{26}\) At the same time, the share of export firms \( \tilde{p}_X^H(\tilde{z}_D^i) = (1 - G_H(\tilde{z}_D^i))/(1 - G_H(\tilde{z}_X^i)) > \tilde{p}_X^L(\tilde{z}_D^i) = (1 - G_L(\tilde{z}_D^i))/(1 - G_L(\tilde{z}_X^i)) \) (see equation 21 in the Appendix) is also larger. This means that more weight is now on exports, inducing larger efficiency losses. Nevertheless:

**Lemma 5.** An increase in firms’ R&D investments in response to trade liberalization leads to an increase in the productivity within firms. This always boosts the industrial productivity if productivity is measured “at the firm gate”: \( \tilde{z}_t^H(\tilde{z}_D^i) > \tilde{z}_t^L(\tilde{z}_D^i) \).\(^{27}\)

*Proof.* See Appendix F.

The increased R&D investments of firms in response to trade liberalization affect industrial productivity by increasing the productivity within firms across the whole firm distribution. This implies fiercer competition compared to a model without R&D investments, and in turn leads to a reallocation of resources from unlucky (possibly exiting) innovators to successful innovators. Hence, Lemma 4 and Lemma 5 lead to the following Proposition:

**Proposition 3.** As long as the productivity is measured “at the firm gate”, competition and innovation contribute jointly to the higher level of industrial productivity after trade liberalization compared to a heterogeneous-firms model without R&D investment: \( \tilde{z}_t^H(\tilde{z}_D^i) > \tilde{z}_t^L(\tilde{z}_D^i) > \tilde{z}_t^L(\tilde{z}_D^i) \).

Since empirical work measures the industrial productivity “at the firm gate” (Bernard et al., 2011), the results stated in Proposition 3 are, in general, consistent with the empirical evidence that was mentioned in the Introduction. In addition, the productivity increase within firms implies that the distribution of firm productivity becomes more fat-tailed in the long run. In other words, an industry consists of a higher fraction

\(^{26}\) The inequality of the domestic and the export average productivity follows immediately from the property of conditional expectations under \( HRSD \) (see equation 22 in the Appendix).

\(^{27}\) Again, as noted in footnote 25, if the Iceberg efficiency losses are considered, firms’ increased R&D investments always lead also to a larger industrial productivity when it is measured at some highly liberalized level of trade openness.
of high-productivity firms after trade liberalization.\footnote{The fraction of high-productivity firms to all firms is \((1 - G_i(z_H))/(1 - G_i(z_D))\) for any arbitrary classification of high-productivity firms \(z_H > z_D\). If \(G_H(z) > \gamma h G_L(z)\), then \(\frac{1-G_H(z_H)}{1-G_H(z_D)} > \frac{1-G_L(z_H)}{1-G_L(z_D)}\) (see Appendix A).} Hence, firms’ increased R&D investments in response to trade liberalization may explain the positive relationship found in the data between trade openness and an industry’s right-tail of the firm distribution (Okubo and Tomiura, 2014; Di Giovanni et al., 2011).

Finally, following Melitz (2003), welfare can be written by using (2) as

\[
W_i \equiv w/P_i = \frac{\sigma - 1}{\sigma} \left( \frac{R}{f_d\sigma} \right)^{1/(\sigma - 1)} z_{D}^i. \tag{18}
\]

Since labor is the only factor of production and wages are normalized to one, aggregate income \(R\) is equal to the exogenously given number of workers. Hence, changes in the cut-off productivity \(z_D\) affect welfare proportionally in heterogeneous-firms models à la Melitz (2003). We know from the previous analysis that \(z_H^D > z_L^D\) for any level of trade openness as soon as firms find it optimal to invest in a high level of R&D, i.e., \(\phi > \phi^*\). Using this in (18) leads to \(W_H > W_L\). As a result:

**Proposition 4.** *The increased R&D investments of firms in response to trade liberalization lead to additional gains from trade compared to conventional heterogeneous-firms models.*

## 5 Robustness

The previous results have been derived by assuming CES demand and R&D returns that differ according to the HRSD criterion. One may ask how important these assumptions are for the results. In the following, I will give an answer to this question by investigating under which conditions firm R&D investment depends on the degree of trade openness in static heterogeneous-firms models when the outcome of R&D is uncertain.

Assume the expected returns of different R&D investment levels cannot be ordered according to the HRSD criterion. Without more information on the R&D returns,
it is then no longer possible to compare \( \frac{j_H'(z_D)}{j_H'(z_X)} \) and \( \frac{j_L'(z_D)}{j_L'(z_X)} \) with each other (for details, see Appendix A). Thus, no general conclusion can be made on how the R&D choice function (9) changes as trade becomes liberalized, because it is not possible to determine the sign of equation (14). This is, for example, the case if one assumes that R&D returns differ only according to the FOSD criterion.

By considering an explicit functional form that fulfills the FOSD but not the HRSD criterion, a firm’s optimal R&D investment level may even be independent of the degree of trade openness. This is, for example, the case in the heterogeneous-firms trade model of Long et al. (2011) with linear demand (see Appendix G.2 for mathematical details). In their model, the level of R&D spending affects a firm’s expected profits by a multiplicative constant:

\[
V(z_D, r) = \frac{h(r)}{4} \left( \int_{z_D}^{\infty} (1/z_D - 1/z)^2 g(z)dz + \int_{z_X}^{\infty} (1/z_X - 1/z)^2 g(z)dz \right) - F(r),
\]

where \( h(r) \) is a constant depending on the R&D investment level \( r \geq 0 \), with \( h(0) = 1 \) and \( h'(r) > 0 \); \( F(r) > 0 \) and \( F'(r) > 0 \). Better foreign market access raises an entrant’s incentive to invest more in R&D. However, due to their specific assumption on R&D returns, the increase is exactly offset by fiercer competition. Moreover, as shown in Appendix G.2, this offsetting effect still occurs if one considers the R&D specification of Long et al. (2011) together with a CES demand structure. In this case, \( \frac{j_H'(z_D)}{j_H'(z_X)} = \frac{j_L'(z_D)}{j_L'(z_X)} \). As a result, equation (14) is zero for both levels of investment \( k = L, H \), implying that R&D investments are independent of the degree of trade openness.

If, instead, R&D returns differ according to the HRSD criterion and firms face a linear demand function, then, too, as shown in Appendix G.1, the optimal R&D spending of firms increase as trade becomes liberalized. This means that whether trade liberalization has an impact on firms’ R&D investments or not will hinge strongly on the assumed R&D returns and not on the demand specification.

### 6 An example: The Pareto distribution

Since numerous trade models with firm heterogeneity assume Pareto-distributed firms, I use it as an example to illustrate how sensitive the results are with respect to the
**R&D specification.** Assume that a firm’s R&D choice determines the Pareto distribution from which the productivity is to be drawn. By considering a one-tailed Pareto distribution with positive support over \((z_0^i, \infty)\) and density

\[
g_i(z) = \frac{z_0^\theta_i}{z_0^\theta_i + 1},
\]

the R&D returns can differ due to differences in the minimum possible productivity draw \(z_0^i\) or due to differences in the shape parameter \(\theta_i\). Figure 4 on page 20 provides a graphical representation of a one-tailed Pareto distribution. Its distribution function is

\[
G_i(z) = 1 - \left(\frac{z_0^i}{z}\right)^\theta_i \quad \text{for} \quad z \geq z_0^i.
\]

By using the definition of HRSD (see equation 5), it is possible to show that distinct Pareto distributions according to the minimum support parameter \(z_0^i\) alone, with \(z_0^H > z_0^L\), do not fulfill the HRSD criterion. In this case, an investment in a high level of R&D implies that all productivity draws in the range from \(z_0^L\) to \(z_0^H\) are eliminated. However, both distributions are alike if conditioned on \(z \geq z_0^H\), since

\[
\frac{g_H(z)}{1-G_H(z)} = \frac{\theta_i}{z} \quad \text{and} \quad \frac{g_L(z)}{1-G_L(z)} = \frac{\theta_i}{z} \quad \forall z > z_0^H.
\]

As a result, they only differ according to the FOSD criterion, because

\[
G_L(z) = 1 - \left(\frac{z_0^L}{z}\right)^\theta_i > G_H(z) = 1 - \left(\frac{z_0^H}{z}\right)^\theta_i.
\]

In this case, an entrant’s R&D investment decision is independent of the level of trade openness. One can show this by using the explicit functional form of the distribution, (19), in (11) and (14). The optimal R&D investment level will either be a low level of R&D or a high level of R&D for any degree of trade openness, depending on the investment costs \(F_L\) and \(F_H\). Additionally, as I show in Appendix G.2, this R&D specification is a particular case of the one assumed by Long et al. (2011).

If, however, one considers distinct Pareto distributions according to the shape pa-

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29 Redding (2011) discusses why a large number of heterogeneous-firms trade models use the Pareto distribution.

30 It is required that \(z_D^i > z_0^H\). This, in turn, is fulfilled once an equilibrium as defined in Definition 1 exists; otherwise, one can show that an investment in a high (low) level of R&D leads to negative expected profits in a case where the cut-off value is determined by a low (high) level of R&D; i.e., no equilibrium would exist.

31 Pfüger and Suedekum (2013) show that governments will increase their R&D spending in response to trade liberalization in a model with Pareto-distributed productivity. In their model, an increase in R&D spending changes the technological potential of a country, summarized by the lowest possible productivity draw of firms. However, the investment level would not change if firms invest in R&D, instead of governments.
rameter $\theta_i$, a Pareto distribution with a lower shape parameter $\theta_H$ hazard rate stochastically dominates a Pareto distribution with a higher shape parameter $\theta_L$: $\frac{g_H(z)}{1-G_H(z)} = \frac{\theta_H}{z}$ and $\frac{g_L(z)}{1-G_L(z)} = \frac{\theta_L}{z} \to \frac{g_H(z)}{1-G_H(z)} < \frac{g_L(z)}{1-G_L(z)}$, since $\theta_L > \theta_H$. Hence, with this particular assumption on R&D returns, firms prefer, prior to trade liberalization, to draw their productivity from the Pareto distribution with the larger shape parameter, namely $\theta_L$, if the costs of a high level of R&D are sufficiently large compared to the costs of a low level of R&D. After trade has been liberalized, however, they prefer to draw from the Pareto distribution with the lower shape parameter $\theta_H$ if the costs of the two R&D investments do not differ substantially. Thus, distinct Pareto distributions with regard to the shape parameter offer one particular functional form for which the results of the model hold.\footnote{In an earlier version of this paper, I derive all results with Pareto-distributed firms. It is available upon request.}

This has an important implication. Current research discusses the additional welfare gains from trade that heterogeneous-firms models offer compared to homogeneous-firms models (Melitz and Redding, 2014; Arkolakis et al., 2012).\footnote{See also Melitz and Trefler (2012) for an overview of the welfare gains of trade in heterogeneous-firms models.} Arkolakis et al. (2012) argue that heterogeneous-firms models do not cover any additional gains from trade compared to models with homogeneous-firms when firms are distributed according to an untruncated Pareto distribution. In particular, they show that knowledge about how trade liberalization affects the domestic trade share $\lambda$, where $\lambda$ equals domestic sales divided by total sales, and about the constant trade elasticity $\epsilon$ is sufficient to determine in both types of models the overall welfare gains from a reduction in variable trade costs in terms of elasticities which is formulated as $d\ln(W) = d\ln(\lambda)/\epsilon$. In addition, they show that the heterogeneous-firms model’s trade elasticity is equal to the Pareto shape parameter multiplied by minus one, i.e., $\epsilon = -\theta$. Consider now, as proposed in this paper, that firms respond to trade liberalization by increasing their R&D spending. At a given level of trade openness, the model is similar to the heterogeneous-firms model of Arkolakis et al. (2012). However, the shape parameter $\theta$ is no longer constant. Instead, it depends negatively on the degree of trade openness, i.e., $\theta = \theta_L$ if $\phi < \phi^*$.
and $\theta = \theta_H$, with $\theta_H < \theta_L$, if $\phi > \phi^*$. This, in turn, implies that the welfare elasticity increases in the degree of trade openness even if one does not take changes in the trade share $\lambda$ into account.

Intuitively, the higher an economy’s level of liberalization is, the higher will be the level of firm $R&D$ investment and the lower the fraction of low-productivity firms. As a consequence, if further trade liberalization takes place, and consequently competition toughens once again, a lower fraction of firms would be forced to exit the market than would be the case if they had not increased their $R&D$ investments in response to earlier reductions in trade costs. As a result, the consideration of firms’ $R&D$ investments implies higher welfare gains from trade in terms of elasticities in heterogeneous-firms models compared to homogeneous-firms models even if firms are distributed according to an untruncated Pareto distribution.

7 Conclusion

The present paper extends the workhorse model of Melitz (2003) by introducing endogenous investments in process-$R&D$ with uncertain outcome, modeled as the choice made by entrants to draw their productivity from distinct distributions without making specific functional form assumptions. The findings demonstrate that whether trade liberalization has an effect or not on firms’ $R&D$ investments depends strongly on how uncertainty in $R&D$ outcomes is modeled.

If the $R&D$ returns differ according to the hazard rate stochastic dominance crite-

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34To see this, note that the fraction of high-productivity firms to all firms in the case of Pareto-distributed firms is $(1 - G(z_H))/(1 - G(z_D)) = (z_D/z_H)^{\theta} < 1$ for any arbitrary classification of high-productivity firms $z_H > z_D$. Since it is smaller than one, a lower shape parameter $\theta$ implies a larger fraction of high-productivity firms.

35This is the case, because as Melitz and Redding (2014) show, if firms are distributed according to an untruncated Pareto distribution, the expected domestic and export market productivity per entering firm will have a constant elasticity function with respect to the cut-off for each corresponding market, formally $\int_{z_D}^{\infty} z^{\sigma-1} g(z) dz = \int_{z_D}^{\infty} z^{\sigma-1} \theta z_0^{\theta} z^{\theta-1} dz = (\theta z_0^{\sigma-\theta})/(\theta - \sigma + 1) z_0^\theta$. Moreover, both functions have the same elasticity, which equals to $\theta - \sigma + 1$. Hence, when measured in terms of elasticities, the change in the expected export productivity caused by a change in the cut-off $z_X$ is exactly offset by a change in the expected domestic productivity caused by a change in the cut-off $z_D$. As a result, the higher welfare gains occur solely, because the elasticity of the firms’ survival probability $\partial(1 - G(z_D))/(\partial z_D)/(1 - G(z_D)) = \theta$ is a function of the firms’ $R&D$ investments.
rion, the model is able to reproduce important empirical findings on the impact of trade liberalization, mentioned in the Introduction. In this case, fiercer competition triggered by trade liberalization pushes firms to raise their $R&\!\!D$ spending. This increases the industrial productivity by simultaneously reallocating resources from low-productivity firms to high-productivity exporting firms and by increasing firms’ internal productivity. Moreover, the increased $R&\!\!D$ investments lead to a more fat-tailed distribution of firms and new aggregate welfare gains from trade. Additionally, as briefly outlined in Section 6, if one considers firms’ $R&\!\!D$ investments, heterogeneous-firms models imply new welfare gains from trade compared to homogeneous-firms models even under the assumption of Pareto-distributed firms. Future research could gain insightful results, for example with regard to recent trade agreements, by quantifying these gains in a calibration study.

Furthermore, by combining the results of this paper with those of the technology-adoption models (e.g., Bustos, 2011) helps to refine our understanding of trade liberalization and firm innovation: The deterministic innovation returns in technology-adoption models may reflect a short- and medium-term perspective, while the uncertain investment returns in the present model reflect a long-term perspective. Thus, one can conclude that in the short run, better access to foreign markets may act as a pull factor for high-productivity exporting firms to increase their innovation investments. In the long run, in contrast, the fiercer competition that accompanies trade liberalization may push all firms to innovate more. This implies that future empirical work should distinguish between time horizons when analyzing the impact of trade liberalization on firm innovation. In this respect, it would also be interesting to combine both frameworks in a calibration study.

Finally, although the paper focuses on trade liberalization, the framework is particularly well suited to analyze other changes in the market environment, which are highlighted, for example, in the literature on industrial organization. Such research could investigate, for example, changes in the degree of product substitutability or in market entry barriers. This may allow future research to better understand the mechanism by which changes in the market environment affect the reciprocal relationship
between firm R&D investment and competition, and the resulting adaptations in the structure of industries.

Appendix

A Proof of Lemma 2

Before proofing Lemma 2, let me first introduce some important properties of the HRSD criterion. Following Shaked and Shanthikumar (2007), the productivity distribution $G_H(z)$ hazard rate stochastically dominates the productivity distribution $G_L(z)$, $G_H(z) >_{hr} G_L(z)$, if for any given productivity level $z$

$$\frac{g_H(z)}{1 - G_H(z)} < \frac{g_L(z)}{1 - G_L(z)}. \quad (20)$$

In addition, HRSD is equivalent to

$$\frac{1 - G_H(z_X)}{1 - G_H(z_D)} > \frac{1 - G_L(z_X)}{1 - G_L(z_D)} \quad (21)$$

for all $z_X > z_D$, since $\frac{\partial}{\partial z_x} \frac{1 - G_H(z_x)}{1 - G_L(z_x)} > 0$. Furthermore, HRSD establishes the following ordering of conditional expectations:

$$E_H[y(z)|z > z_s] > E_L[y(z)|z > z_s], \quad (22)$$

for any increasing function $y(z)$. Moreover, HRSD implies FOSD ($G_H(z) >_{st} G_L(z)$) but not the other way around. Therefore, all properties that apply to the FOSD criterion also apply to the HRSD criterion. Especially the following properties of FOSD are important later on. It allows us to rank the distribution functions as

$$G_H(z) < G_L(z), \quad (23)$$
and the unconditional expectation values as
\[ E_H[y(z)] > E_L[y(z)]. \] (24)

Now to the model. Free entry implies
\[ j_i(z_i^D)f_d + j_i(z_i^X)f_x = F_i, \] (25)
with \( j_i(z_i^i) = (1 - G_i(z_i^j))k_i(z_i^j) \). The free entry condition changes with trade liberalization according to
\[ j'_i(z_i^D)f_d dz_i^D + j'_i(z_i^X)f_x \phi^{-1} dz_i^X - j'_i(z_i^X)f_x z_i^i \phi^{-1} d\phi = 0. \] (26)

If all firms were to invest in a low level of R&D \((i = L)\), equation (26) can be written as
\[ dz_L^D d\phi = j'_L(z_L^X)f_x z_L^X \frac{j'_L(z_L^D)}{j'_L(z_L^D) f_d \phi + j'_L(z_L^X)f_x}. \] (27)

The expected value of a high level of R&D increases as variable trade costs \(\tau\) decline if, and only if, \(dV_H(z_L^D, \phi) = j'_H(z_L^D)f_d dz_L^D + j'_H(z_L^X)f_x \phi^{-1} dz_L^X - j'_H(z_L^X)f_x z_L^X \phi^{-1} d\phi > 0\). By using (27), this can be written as \((j'_H(z_L^D)f_d + j'_H(z_L^X)f_x \phi^{-1})j'_L(z_L^X)f_x z_L^X d\phi < (j'_L(z_L^D)f_d \phi + j'_L(z_L^X)f_x)j'_H(z_L^X)f_x z_L^X \phi^{-1} d\phi\), which reduces to
\[ \frac{j'_H(z_L^X)}{j'_L(z_L^D)} = \frac{j'_L(z_L^X)}{j'_L(z_L^D)}. \] (28)

Since by assumption \(z_L^X > z_L^D\), the last inequality is fulfilled iff \(\frac{j'_H(z_L^X)}{j'_L(z_L^D)} = \frac{\int_{z_L^D}^{z_L^X} z^{\sigma-1} g_H(z) dz}{\int_{z_L^D}^{z_L^X} z^{\sigma-1} g_L(z) dz}\) increases in \(z_L^D\). (The expression used here for \(\frac{j'_H(z_L^X)}{j'_L(z_L^D)}\) can be derived by using \(j'_i(z_L^i) = -\frac{1}{z_L^i}(\sigma - 1)(1 - G_i(z_s^L))(k_i(z_L^L) + 1) < 0\) and \(k_i(z_s^L) = \int_{z_s^L}^{\infty} \left(\frac{z}{z_s^L}\right)^{\sigma-1} \frac{g_i(z)}{1-G_i(z_s^L)} dz - 1\).)
By using (29) in (30), it leads to changes according to (26), with firms having invested in a high level of investments differ according to the criterion, the incentive to invest in a low level of R&D declines as trade becomes liberalized if the free entry condition is defined by firms having invested in a high level of R&D. In this case, the free entry condition changes according to (26), with \( i = H \):

\[
\frac{dz^H}{d\phi} = \frac{j'_H(z^H_X)f_Xz^H_X}{j'_L(z^H_D)f_d + j'_L(z^H_X)f_X}. \tag{29}
\]

Lemma 2 requires that

\[
dV_L(z^H_D, \phi) = j'_L(z^H_D)f_d dz^H_D + j'_L(z^H_X)f_X \phi^{-1} dz^H_X - j'_L(z^H_X)f_X z^H_X \phi^{-1} d\phi < 0. \tag{30}
\]

By using (29) in (30), it leads to \( \frac{j'_H(z^H_X)}{j'_H(z^H_D)} > \frac{j'_L(z^H_X)}{j'_L(z^H_D)} \forall z^H_X > z^H_D \). As one can show by applying the same steps as before, this inequality is always fulfilled if the expected return of a high level of R&D investment hazard rate stochastically dominates that of a low level of R&D investment.

If, however, the R&D returns cannot be ordered according to the HRS criterion, it is no longer possible to make a general statement on the relationship between \( \frac{g_H(z_s)}{1 - G_L(z_s)} \) and \( \frac{g_H(z_s)}{1 - G_H(z_s)} \). As a result, it is no longer possible to unambiguously determine how \( \frac{j'_H(z_X)}{j'_H(z_D)} \) relates to \( \frac{j'_L(z_X)}{j'_L(z_D)} \), and, hence how the relative profitability between a high and a
low level of R&D investment changes in the degree of trade openness. The widely used FOSD criterion is one particular example for which such an ambiguity occurs.

B Expected firm value and cut-off changes

To proof that $\frac{\partial V_H}{\partial z_D}(z_D, \phi) < \frac{\partial V_L}{\partial z_D}(z_D, \phi) < 0$ is equivalent to show that $j'_H(z_s) < j'_L(z_s)$, with $j'_i(z_s) = -\frac{1}{z_s} (\sigma - 1)(1 - G_i(z_s))(k_i(z_s) + 1) < 0$. This, in turn, is equivalent to show that $(1 - G_H(z_s)) \int_{z_s}^{\infty} \frac{z_s^{-1}g_H(z)}{1-G_H(z_s)} dz > (1 - G_L(z_s)) \int_{z_s}^{\infty} \frac{z_s^{-1}g_L(z)}{1-G_L(z_s)} dz$, which is always the case, as long as the return of a high level of R&D hazard rate stochastically dominates that of a low level of R&D (due to equations 22 and 23).

C Proof that a pure equilibrium always exists

This paragraph proofs that there always exists a pure equilibrium satisfying Definition 1. To start with, there can be at most one $z$-value at which $V_H(z, \phi) = V_L(z, \phi)$ holds for a given level of trade openness $\phi$, because $\frac{\partial V_H}{\partial z}(z, \phi) < \frac{\partial V_L}{\partial z}(z, \phi) < 0$ (for proof, see Appendix B). We know from Definition 2 that $V_H(z_H^D, \phi) = 0$. In general, there are three cases possible: (a): $V_L(z_H^D, \phi) < 0$, (b): $V_L(z_H^D, \phi) = 0$ and (c): $V_L(z_H^D, \phi) > 0$. All three cases are illustrated in Figure 5. By applying Definition 1, if follows from the single crossing property that a high level of R&D is the only equilibrium in case (a). In case (b), too, a high level of R&D investment is an equilibrium according to Definition 1, because it is not optimal for an entrant to deviate given its belief that the competition intensity will be $z_H^D$. At the same time, and for the same reason, a low level of R&D determines also an equilibrium, because in this case Definition 2 and the single crossing property imply that $z_D = z_H^D = z_L^D$. Case (c) is just the opposite of case (a), because $\frac{\partial V_L}{\partial z_D}(z_D, \phi) < 0$ and $V_L(z_H^L, \phi) = 0$ imply that $z_H^L > z_H^D$, and, thus, $V_H(z_H^L, \phi) < 0$ due to $\frac{\partial V_H}{\partial z_D}(z_D, \phi) < 0$ and $V_H(z_H^L, \phi) = 0$. Hence, an investment in a low level of R&D is in this case the only pure equilibrium. In summary, in each

\footnote{This equality always exists for some value of $z_D$ as long as the R&D cost $F_H$ is finite. For details, see Footnote 14.}
possible case there exists at least one pure equilibrium.

Figure 5: Free-entry equilibrium of case (a), left figure, of case (b), center figure and of case (c), right figure.

D Existence of $\phi^*$

This paragraph shows under which parameter conditions $V_H(z_D^L, \phi)$ crosses $V_L(z_D^H, \phi)$ between $\phi = 0$ and $\phi^{\text{max}} = 1$, with $\phi \equiv \phi^*$ being the intersection point for which $V_H(z_D^L, \phi) = V_L(z_D^H, \phi)$ holds. To start with, $dV_H(z_D^L, \phi) > 0$ and $dV_L(z_D^H, \phi) < 0$ (see Lemma 2) imply that if an intersection point exists, it is unique. Moreover, at $\phi^*$ it follows that $z_D^L = z_D^H$. Proof: Assume $z_D^j > z_D^i$ at $\phi^*$. As a result: $V_i(z_D^j, \phi^*) > V_i(z_D^i, \phi^*) = 0$; $V_j(z_D^i, \phi^*) < V_j(z_D^j, \phi^*) = 0$ due to (a) $\partial V(z_D, \phi)/\partial z_D < 0$ and (b) the definition of $z_D^j$ (see Definition 2). Hence, in this case $V_i(z_D^j, \phi^*) \neq V_j(z_D^j, \phi^*)$. Instead, assume $z_D^i = z_D^j$. Then $V_i(z_D^j, \phi^*) = V_i(z_D^i, \phi^*) = V_j(z_D^i, \phi^*) = V_j(z_D^j, \phi^*) = 0$. Hence, $V_H(z_D^L, \phi) = V_L(z_D^H, \phi)$ implies that $z_D = z_D^L = z_D^H$. 

In the following, I will determine for which parameter values $\phi^*$ exists. Assume entrants find it under autarky optimal to invest in a low level of $R&D$. This is the case if $V_L(z_D, \phi = 0) \geq 0$ and $V_H(z_D, \phi = 0) < 0$. Due to free entry (10), $V_L(z_D^L, \phi = 0) = 0 \iff f_d j_L(z_D^L) - F_L = 0$. Solve it after $z_D^L$ leads to $z_D^L = j_L^{-1}(F_L/f_d)$, with $j_L^{-1}(\cdot)$ as the inverse function of $j_L(\cdot)$. Use this in equation (6) with $i = H$ leads to

$$V_H(z_D^L, \phi = 0) < 0 \iff f_d j_H(j_L^{-1}(F_L/f_d)) - F_H < 0. \quad (31)$$
If this inequality is fulfilled, entrants find it under autarky optimal to invest in a low level of \( R & D \). Note this implies that \( F_H > F_L \), because \( f_d \dot{j}_H(f_L/f_d) - F_L > 0 \) due to \( \dot{j}_H() > \dot{j}_L() \).

Moreover, assume that entrants find it at the maximal level of trade openness \( \phi^{\max} \), with \( \phi^* < \phi^{\max} \) and \( \phi^{\max} = 1/\tau \left( \frac{\mu}{f_x} \right)^{\frac{1}{\beta}} \leq 1 \), optimal to invest in a high level of \( R & D \). This is the case if \( V_L(z_D, \phi = \phi^{\max}) < 0 \) and \( V_H(z_D, \phi = \phi^{\max}) \geq 0 \) hold. For simplicity, assume \( \phi^{\max} = 1 \). This can be the case if either \( \tau < 1 \) or \( f_d = f_x \). Due to free entry \( V_H(z_D^H, \phi = \phi^{\max}) = 0 \leftrightarrow (f_d + f_x)j_H(z_D^H) - F_H = 0.37 \) Solve it after \( z_D^H \) leads to \( z_D^H = j_H^{-1}(F_H/(f_d + f_x)) \). Using this in equation (6) with \( i = L \) shows that \( V_L(z_D^H, \phi = \phi^{\max}) < 0 \) if

\[
(f_d + f_x)j_L(j_H^{-1}(F_H/(f_d + f_x))) - F_L < 0. \tag{32}
\]

If the parameter values fulfill this condition, entrants find it from a certain degree of trade openness onwards optimal to invest in a high level of \( R & D \).

E Proof of Lemma 4

This paragraph proofs \( \tilde{z}_i^L(z_D^H) > \tilde{z}_i^L(z_D^L) \) if aggregate productivity is measured “at the firm gate”. Following Melitz (2003), the aggregate productivity without taking the efficiency losses due to Iceberg trade costs into account can be written as

\[
\Phi_i^L(z_D^L) = h^{-1} \left( \frac{1}{R} \int_0^\infty h(z) r(z) g_i(z) dz \right),
\]

\(33\)

37 Otherwise, if \( \phi^{\max} < 1 \), \( V_H(z_D^H, \phi = \phi^{\max}) = 0 \leftrightarrow f_d j_H(z_D^H) + f_x j_H(z_D^H \phi^{-1}) - F_H = 0 \). In this case, it is no longer possible to solve explicitly for \( z_D^H \) and replace it in \( V_L(z_D^H, \phi) < 0 \). In such a case, the two conditions, \( f_d j_H(z_D^H) + f_x j_H(z_D^H \phi^{-1}) - F_H = 0 \) & \( f_d j_L(z_D^H) + f_x j_L(z_D^H \phi^{-1}) - F_L < 0 \), have to be simultaneously fulfilled.
where \( h(z) \) is any increasing function and \( r_i(z) \) are the combined export and domestic market revenues. The combined revenues are used as the weights:

\[
\begin{align*}
    r_i(z) &= \begin{cases} 
    0 & \forall z < z_D^i, \\
    r_d^i(z) = \left( \frac{z}{z_D^i} \right)^{\sigma - 1} \sigma f & \forall z_D^i \phi > z \geq z_D^i, \\
    (1 + r^{-1})r_d^i(z) & \forall z \geq z_D^i \phi^{-1}.
    \end{cases}
\end{align*}
\]

We know from Proposition (2) that \( z_D^H > z_D^L \). Hence, \( r_d^H(z) = (z/z_D^H)^{\sigma - 1} \sigma f < r_d^L(z) = (z/z_D^L)^{\sigma - 1} \sigma f \forall z \geq z_D^L \) and, therefore, \( r_H(z) < r_L(z) \forall z \geq z_D^L \) (see 34). This implies that \( \int_0^z r_H(y)h(y)g(y)dy \leq \int_0^z r_L(y)h(y)g(y)dy \forall z > 0 \) and with strict inequality for \( z \geq z_D^L \). Thus, the revenue distribution associated with the density \( \frac{r_H(z)g(z)}{R} \) first-order stochastically dominates the distribution associated with the density \( \frac{r_L(z)g(z)}{R} \). As a result, \( \tilde{z}_i^L(z_D^H) > \tilde{z}_i^L(z_D^L) \) if measured “at the firm gate”, since \( h(z) \) is an increasing function.\(^{38}\) □

**F Proof of Lemma 5**

It has been claimed that \( \tilde{z}_i^H(z_D^i) > \tilde{z}_i^L(z_D^i) \) if measured “at the firm gate”. To show this, I use again equation (33) as the aggregate productivity. By assumption, I compare both aggregates at the same cut-off value \( z_D^i \). Thus, both industrial productivity values are calculated with the same normalized weights \( r(z)/R \) (see 34). However, since \( \int_0^z r(y)h(y)g_D(y)dy \leq \int_0^z r(y)h(y)g_L(y)dy \forall z \) due to \( G_D(z) > hr G_L(z) \) (note HRSD implies FOSD), it follows that, as long as firms draw from the better productivity distribution, the resulting firm revenue distribution first-order stochastically dominates the revenue distribution associated with the worse productivity distribution. Since FOSD implies a larger mean, it follows \( \tilde{z}_i^H(z_D^i) > \tilde{z}_i^L(z_D^i) \), as long as the productivity

\(^{38}\)The function \( h(z) \) increases only always in \( z \) if the productivity is measured “at the firm gate”. Otherwise, a non-export firm with lower \( z \) “at the firm gate” may have a higher productivity “at the consumers” compared to a export firm, since the Iceberg trade costs induce an efficiency loss on the resources used to produce the exports. Thus, if productivity is measured “at the consumers”, \( \int_0^z r_H(y)h(y)g(y)dy \leq \int_0^z r_L(y)h(y)g(y)dy \forall z \) may no longer hold. As a result, \( \tilde{z}_i^L(z_D^H) > \tilde{z}_i^L(z_D^L) \) holds only always if the productivity is measured “at the firm gate”.

is measured “at the firm gate”.39

G Robustness

G.1 Linear demand specification

Assume a linear demand specification as in the heterogeneous-firms trade model of Melitz and Ottaviano (2008). In the following, I show that, as long as innovation returns differ according to the HRSD criterion, firms find it also in this case optimal to increase their R&D investments as trade becomes liberalized. To make the point, I set each country’s market size equal to one without loss of generality. Following Melitz and Ottaviano (2008), the free entry condition is

\[ V_i(z_D, \phi) = \int_{z_D}^{\infty} \left( \frac{1}{z_D - 1/z} \right)^2 g_i(z) dz + \int_{z_X}^{\infty} \left( \frac{1}{z_X - 1/z} \right)^2 g_i(z) dz - F_i = 0. \]

Totally differentiating leads to

\[ dV_i(z_D, \phi) = -2 \int_{z_D}^{\infty} \frac{1}{z_D - 1/z} z_D^2 g_i(z) dz d z_D - 2 \int_{z_X}^{\infty} \frac{1}{z_X - 1/z} \phi^{-1} z_X^2 g_i(z) dz d z_D + 2 \int_{z_X}^{\infty} \left( \frac{1}{z_X - 1/z} \right) (z_X \phi)^{-1} g_i(z) dz d \phi. \]

Which can be written as

\[ dV_i(\cdot) = -2 \int_{z_X}^{\infty} \left( \frac{1}{z_X - 1/z} \right) g_i(z) dz \left( \frac{\phi \int_{z_D}^{\infty} \left( \frac{1}{z_D - 1/z} \right) z_D^2 g_i(z) dz}{z_X^2 \int_{z_X}^{\infty} \left( \frac{1}{z_X - 1/z} \right) g_i(z) dz} + 1 \right) dz_D - z_X d \phi. \]

(35)

Following Shaked and Shanthikumar (2007), p. 23, HRSD between two random variables x and y, x > hr y, is equivalent to \( E(\beta(y)) > \frac{E(\beta(x))}{E(\alpha(x))} \) for all functions \( \beta(\cdot) \) and \( \alpha(\cdot) \) for which the expectation exists and \( \beta(\cdot) \) is positive and both, \( \alpha/\beta \) and \( \beta \), are increasing in their arguments. If we set \( \alpha \equiv (1/z_X - 1/z) \) and \( \beta \equiv (1/z_D - 1/z) \) it turns out that \( \beta \geq 0 \) and \( \beta \) is increasing in z.40 Moreover, \( \alpha/\beta = (z - z_X)/(zz_X/zD - z_X) \) and \( \frac{\partial \alpha/\beta}{\partial z} = z_X(zX/zD - 1) > 0 \) as long as \( z_X > z_D \). Hence, the term in large brackets of (35) is lower if a firm invests in a high than in a low level of R&D, as long as the R&D

39See footnote (38).

40Note that \( \beta \) is equal to zero for \( z < z_D \). This means that the expectation value of \( \beta(z) \) is given by \( E(\beta(z)) = \int_{0}^{z_D} 0 g_i(z) dz + \int_{zD}^{\infty} \beta(z) g_i(z) dz \), since \( g_i(z) \) has support over \( z \geq 0 \). Similar results occur for \( \alpha \). Hence, both functions, \( \alpha \) and \( \beta \), are increasing but not strictly increasing in z.
returns can be ranked according to the \(HRSD\) criterion. Assume all other firms have invested in a low level of \(R\&D\). In this case, \(dV_L(z_D^L, \phi) = 0\) requires that the term in large brackets of (35) is zero for an investment in a low level of \(R\&D\). This in turn implies that \(dV_H(z_D^H, \phi) > 0\). Same reasoning implies that \(dV_L(z_D^H, \phi) < 0\), because \(dV_H(z_D^H, \phi) = 0\). As a result, the relative profitability between a high and a low level of \(R\&D\) investment increases as trade becomes liberalized. If, however, for example, the investment returns differ only according to the \(FOSD\) criterion, it is in general no longer possible to compare the value of the large bracket of (35) for two different \(R\&D\) investment levels with each other. Hence, the results are similar as in the case of CES demand.

G.2 The model of Long et al. (2011)

Long et al. (2011) consider a heterogeneous-firms trade model with linear demand and firms’ \(R\&D\) investments. In their model, more \(R\&D\) spending increases a firm’s chance to become a high-productivity firm. They assume that without \(R\&D\) investment a firm draws its productivity from a distribution \(G(z)\) with support \([z_{\text{min}}, \infty]\) and \(z_{\text{min}} \geq 0\). If, however, a firm invests an amount of \(r > 0\) in \(R\&D\), the chance of obtaining a higher productivity is increased. In particular, they assume that the distribution from where an entrant draws its productivity in case of \(R\&D\) investment is \(H(z, r) = \min\{h(r)G(z); 1\}\), with \(h(0) = 1\) and \(h(r)' > 0\), and \(H(rz_{\text{min}}, r) = 0\) and \(H(\infty, r) = 1\). This \(R\&D\) specification leads to the following expected firm value: \(V(z_D, r) = \frac{h(r)}{4} \left(\int_{z_D}^\infty (1/z_D - 1/z)^2 g(z) dz + \int_{z_X}^\infty (1/z_X - 1/z)^2 g(z) dz\right) - F(r)\). As one can easily see, the relative profitability between two \(R\&D\) investment levels \(r\) is independent of the degree of trade openness: \(\frac{\frac{h(r_i)}{h(r_k)}\left(\int_{z_D}^\infty (1/z_D - 1/z)^2 g(z) dz + \int_{z_X}^\infty (1/z_X - 1/z)^2 g(z) dz\right)}{\frac{h(r_i)}{h(r_k)}\left(\int_{z_D}^\infty (1/z_D - 1/z)^2 g(z) dz + \int_{z_X}^\infty (1/z_X - 1/z)^2 g(z) dz\right)} = \frac{h(r_i)}{h(r_k)}\). As a result, trade liberalization does not affect the \(R\&D\) investment decision of firms.\(^{41}\)

Moreover, this requires that the expected returns of two distinct \(R\&D\) investment levels cannot be ordered according to the \(HRSD\) criterion; as otherwise it would

\(^{41}\)Free entry implies that the expected value of at least one \(R\&D\) choice remains unchanged. Hence, it is enough to show that the relative profitability between two \(R\&D\) investment levels changes as trade becomes liberalized.
contradict with the previous paragraph G.1. Indeed, one can show that the returns differ only according to the FOSD criterion, because, $h(r)g(z) > g(z)\forall r > 0$, but $\int_{z_D} h(r)g(z)dz = \frac{g(z)}{1-G(z_D)}$ as long as $z_D \geq rz_{min}$.

**Pareto distribution**  
The Pareto distribution, as used in Section 6, offers an example with an explicit functional form. Assume $R&D$ investments lead to productivity draws from distinct Pareto distributions according to their minimum support parameter $z_i$. By considering the specific case of Long et al. (2011), the draw associated with a high level of $R&D$, $x$, is just a linear transformation of the draw associated with a low level of $R&D$, $y$: $x(y) = ry$. Assume the density of $y$ is $g(y)$. By applying the change of variable theorem, the density associated with the random variable $x$ can in general be written as $h(x) = g(y(x)) \frac{dy}{dx}$. In particular, assume the density of $y$ is $g(y) = \frac{\theta}{(\theta + 1) (z_0^H / x)^\theta}$ with $h(r) = (\frac{\theta}{\theta + 1}) (z_0^H / x)^\theta$. Integrating leads to $H(x, r) = 1 - (z_0^H / x)^\theta$. This shows that distinct Pareto distributions with respect to the minimum possible draw are a particular case of Long et al. (2011).

**The model of Long et al. (2011) with CES demand**  
Assume the $R&D$ specification of the previous paragraph: $H(z, r) = \min \{h(r)G(z); 1\}$. By using this relationship in (6), one can show that an entrant’s expected value of investing in the $R&D$ level $r$ and facing CES demand is

$$V(z_D, r) = h(r) \left( \int_{z_D}^{\infty} \left( \frac{z}{z_D} \right)^{\sigma-1} g(z)dz - \int_{z_D}^{\infty} g(z)dz \right) + \int_{z_X}^{\infty} \left( \frac{z}{z_X} \right)^{\sigma-1} g(z)dz - \int_{z_X}^{\infty} g(z)dz \right) - F(r).$$

Since $h(r)$ can be factored out, the relative profitability between two $R&D$ investment levels is again independent of the degree of trade openness. Thus, similar to the case of a linear demand structure, $R&D$ investments do not depend on the degree of trade openness.
Continuous R&D choices

Assume an entrant can choose among a continuum of different R&D investment levels $r \in [r_{\min}, r_{\max}] \in \mathbb{R}^+$. Each R&D investment level $r$ is associated with a random variable $z(r)$, where $z(r)$ has a continuous probability density $g(z, r)$ with support over $z \in [0, \infty]$ and a continuous distribution function $G(z_s, r) = \int_{z_s}^{\infty} g(z, r)dz \leq 1$. If an entrant invests in the R&D level $r$, its expected profit is

$$V(z_D, \phi, r) = f_d \left( \int_{z_D}^{\infty} \left( \frac{z}{z_D} \right)^{\sigma - 1} g(z, r)dz - (1 - G(z_D, r)) \right) + f_x \left( \int_{z_X}^{\infty} \left( \frac{z}{z_X} \right)^{\sigma - 1} g(z, r)dz - (1 - G(z_X, r)) \right) - F(r).$$

(36)

Assume that if an entrant invests more in R&D, both, its expected profit net of sunk costs and its sunk costs are higher: $\int_{z_s}^{\infty} \left( \frac{z}{z_s} \right)^{\sigma - 1} g_r(z, r)dz > 0$ and $F'(r) > 0$. Moreover, an optimal R&D level has to satisfy the following first-order condition:

$$\frac{\partial V(z_D, \phi, r)}{\partial r} = \sum_s f_s \left( \int_{z_s}^{\infty} \left( \frac{z}{z_s} \right)^{\sigma - 1} g_r(z, r)dz + G_r(z_s, r) \right) - F'(r) = 0.$$

(37)

Uniqueness of an equilibrium  It is possible to show that if an increase in the R&D spending let the R&D costs increase more than the R&D benefits, $F''(r) > (V(z_D, r) + F(r))''$, and if the R&D costs additionally fulfill $F'(r) > 0$ and $F(r_{\min}) < \infty$, there exists a unique optimal R&D level $r^* \in [r_{\min}, r_{\max}]$ for a given level of trade openness. To start with, in any free-entry equilibrium satisfying Definition 1, equations (36) and (37) have to be equal to zero. This means that in equilibrium the highest possible expected firm value has to be equal to zero due to free entry. Since $\partial V(z_D, r)/\partial z_D < 0 \forall r$, such an equilibrium corresponds to the highest possible value of $z_D$. Hence, to show that there exists an interior equilibrium is equivalent to show that there exists an $r$, satisfying $\partial z_D(r)/\partial r = 0$. Moreover, this is a unique equilibrium if $\partial^2 z_D(r)/\partial^2 r < 0 \forall r$. Since $V(z_D, r) = 0$ implicitly determines $z_D$, we have

$^{42}$The expected profit net of sunk costs increases in $r$ if, and only if, $G$ is shifted in $r$ according to FOSD. This is the case if $G_r(r, z) < 0$. 

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to work with the total differential of $V(z_D, r) = 0$. The idea is illustrated in Figure 6. Totally differentiate the free entry condition by holding the level of trade openness fix leads to (note, in order to save space, I set $f_d = f_x$ without loss of generality)

$$dV(z_D, r) = \frac{\partial V(z_D, r)}{\partial r} dr + \frac{\partial V(z_D, r)}{\partial z_D} dz_D = 0,$$

which can be written as

$$dz_D = \frac{\int_{z_D}^{\infty} \left( \frac{z}{z_D} \right)^{\sigma-1} g_r(z, r) dz + G_r(z_D, r)}{\frac{\sigma-1}{z_D} \int_{z_D}^{\infty} \left( \frac{z}{z_D} \right)^{\sigma-1} g(z, r) dz + \frac{\sigma-1}{z_X} \int_{z_X}^{\infty} \left( \frac{z}{z_X} \right)^{\sigma-1} g(z, r) dz} dr.$$

The denominator is always positive. The numerator is also positive as long as $F'(r)$ is not considered. However, as long as $F''(r) > (V(z_D, r) + F(r))^\prime$, it follows from the first-order condition (37) that

$$\frac{dz_D}{dr} > 0 \text{ if } r < r^*; \frac{dz_D}{dr} = 0 \text{ if } r = r^*; \frac{dz_D}{dr} < 0 \text{ if } r > r^*.$$

As a result, if $R&D$ costs increase sufficiently, there exists a unique $R&D$ investment level $r^*$ that maximizes $z_D$.\footnote{If $R&D$ costs are generally high, the optimal level of $R&D$ may be the corner solution $r^* = r^{\min}$. If this is the case, there would still be positive entry as long as $F(r^{\min}) < \infty$, because, as shown by Melitz (2003), $J(z_D)$ decreases from infinity to zero on $z_D = (0, \infty)$. Thus, $V(z_D, r^{\min}) = 0$ always exists if $F(r^{\min}) < \infty$.}

Defining this particular $z_D$ as $z_D(r^*)$ and noting that $z_D(r^*) > z_D(r) \forall r \neq r^*$, it follows that $V(z_D(r^*), r) < 0 \forall r \neq r^*$ due to $V(z_D(r^*), r^*) = 0$ and $\partial V(z_D(r), r) / \partial z_D < 0$. This, in turn, proofs that there exists a unique optimal
R&D level for a given level of trade openness.

**Trade liberalization**  In the following, I show that the equilibrium R&D investment level increases in the level of trade openness. To start with, suppose all other entrants invest in $r_i$. In this case, free entry requires that the expected firm value (36) remains at zero for the investment level $r_i$ after a small decline in trade costs:

$$dV(z^i_D, \phi, r_i) = j'(z^i_X, r_i) f_x \phi^{-1} \left( \frac{j'(z^i_D, r_i) f_d}{j'(z^i_X, r_i) f_x} \phi + 1 \right) dz^i_D - z^i_X d\phi = 0,$$

with

$$j'(z^i_s, r_i) = -\frac{1}{\sigma} (\sigma - 1) \int_{z^i_D}^{\infty} g(z, r_i) dz < 0.$$  

Now assume, instead, that an entrant invests more in R&D, say $r_k$, with $r_k > r_i$. As long as the return of any two R&D investment levels stand in a relation of HRSD,

$$G(r_k) > hr G(r_i),$$

it follows that

$$0 < \frac{j'(z^i_D, r_k)}{j'(z^i_X, r_k)} < \frac{j'(z^i_D, r_i)}{j'(z^i_X, r_i)}$$

if $z_X > z_D$.\(^44\) As a result, $dV(z^i_D, \phi, r_k) > 0$ for all $r_k > r_i$. By the same reasoning, one can show that $dV(z^k_D, \phi, r_i) < 0$ for all $r_k > r_i$. Hence, by investing more in R&D, an entrant can increase its expected profits above zero as trade becomes liberalized. The R&D level $r_i$ can then no longer be an equilibrium, because all entrants increase their R&D spending. Competition toughens until a new equilibrium is reached for which $V(z_D, \phi, r_k) > V(z_D, \phi, r \neq r_k)$ and $V(z_D, \phi, r_k) = 0$ hold (note, the first inequality follows from equation (38) and $\partial V(z_D, \phi, r)/\partial z_D < 0$).

Together with the result that there exists only one equilibrium for a given level of trade openness (see the previous paragraph), it follows that the equilibrium R&D investment level of a firm continuously increases in the degree of trade openness.

If, however, the R&D returns cannot be ordered according to the HRSD criterion, it is in general no longer possible to unambiguously compare the term $\frac{j'(z^i_D, r_k)}{j'(z^i_X, r_k)}$ for different levels of R&D. In consequence, it is no longer possible to generally state how trade

\(^44\)Following Shaked and Shanthikumar (2007), pp. 24, in order that a continuum of R&D investment levels $r$ can be ordered according to the HRSD criterion, it is sufficient that $g(z, r)$ is TP\(_2\) (totally positive of order 2). This is the case if

$$\begin{vmatrix} g(z_1, r_i) & g(z_2, r_i) \\ g(z_1, r_k) & g(z_2, r_k) \end{vmatrix} > 0$$

for every $z_2 > z_1$ and $r_k > r_i$.\[40]
liberalization affects an entrant’s R&D investment decision.

References


