Hierarchy of Trade and Experience as an Exporter

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Abstract

Trade models à la Melitz predict a hierarchy of trade. However, Eaton et al. (2011) shows that firms do not enter markets according to a hierarchy. The present article aims to reassess the empirical and theoretical tenets of this hierarchy of trade. First, we reveal that the fact that firms obey a hierarchy of trade strongly depends on their experience as exporters. Firms export to different markets when they are young exporters, while, conditional on survival and over time, they tend to export to the same ordered set of countries. In so doing, we reconcile Melitz’s prediction with Eaton et al. (2011): the conclusion of an absence of hierarchy made by Eaton et al. (2011) is valid in the short run, whereas firms come closer to Melitz’s prediction the longer they have been exporters. Thus, we argue that what we observe in the data is not a drawback but a snapshot of the convergence toward the Melitz’s prediction. Second, we develop a unified model that explains the evidence outlined above. We offer a new class of export choice models in which firms always export to different sets of markets in the short run, whereas the strategies of firms tend to stick more to a hierarchy over time. We demonstrate that this result is due to a baseline competition effect that is present in any model with monopolistic competition. In particular, we identify a new firms’ self-selection effect based on a trade-off between accessibility and competition.

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1 Introduction

Over the last decade, the Melitz (2003) model has become the cornerstone in international trade. The overwhelming popularity of this model stems from its ability to reproduce a well-established empirical fact: the self-selection of firms into export markets (see, among others, Bernard and Jensen (1999), Bernard et al. (2007), Mayer and Ottaviano (2008), Berthou and Fontagné (2008) and Manova and Zhang (2009)). This means that, with foreign countries ranked by their attractiveness, firms with high productivity serve more difficult markets than firms with low productivity. Melitz (2003) explains this fact using the existence of increasing costs that prevent firms with insufficient productivity from entering difficult markets. Another key feature of this theoretical framework is that it predicts a hierarchy of trade: if a firm exports to the kth most attractive market, then it necessarily export to the k – 1th most attractive country as well. However, in a recent work, Eaton et al. (2011) show that this benchmark model is not fully consistent with the data. It is true that the average productivity of firms is shown to be increasing with the costs to enter the destination market. However, these authors assert that firms do not enter markets according to a hierarchy. In other words, firm-level data display a large amount of heterogeneity in terms of export destinations.

The present article aims to reassess the empirical and theoretical tenets of this hierarchy of trade. In particular, we attempt to answer the following question: are Melitz’s prediction and Eaton et al.’s (2011) empirical findings irreconcilable?

Empirically, we considerably reduce the conflict between Eaton et al.’s (2011) empirical findings and Melitz’s prediction by revealing new striking findings regarding how firms enter foreign markets.

To do so, we exploit French customs data (i.e., firm-level data) over the period 1994-2008. We restrict our empirical analysis to the study of how French manufacturing firms enter different foreign markets. In line with Eaton et al. (2011), we find, on average, little support for the existence of a hierarchy of trade: only 11.51 % of the total number of firms follows a hierarchy of trade. However, we underscore that this overall lack of hierarchy encompasses significant heterogeneity. Namely, we emphasize the strong age dependence in the firms’ strategies: the fact that firms obey a hierarchy of trade strongly depends on their experience as exporters. For instance, the proportion of firms ordered in terms of destinations is 6.76 % for exporters with one year of experience and 16.53 % for firms with 14 years of experience as exporters. More important, we build an indicator that measures the deviation in the French firms’ behavior from the Melitz prediction. The range of the latter is [0%, 100%]. It takes the value 0 % when the firms’ behavior exactly follows the
Melitz prediction. In that case, there is no deviation from the benchmark. Likewise, it takes the value 100% when firms’ behavior completely contradicts the Melitz model. We again find that the deviation dramatically decreases with experience as an exporter. For firms with one year of experience as exporters, the observed deviation is 81.18%, whereas the latter is 46.14% for firms with more than 14 years of experience as exporters.

These new stylized findings may be meaningful from a theoretical point of view. Our empirical regularities suggest that the absence of a hierarchy of trade is a short-run phenomenon. Firms export to heterogeneous foreign countries when they are young. But, with time, for those who survive, they export more and more to the same hierarchy of trade. As a result, given that the Melitz (2003) model makes predictions concerning long-run economies, it does not account for these short-run characteristics. Thus, we argue that what we observe in the data is not a failure of Melitz (2003). On the contrary, we reckon that there is an unambiguous convergence toward the Melitz (2003) prediction.

We then provide a theoretical explanation of these features of the data by showing that natural competition forces present in any trade model are sufficient to generate the findings highlighted previously. In particular, we identify a new self-selection effect: firms’ self-selection into export markets now results from a trade-off between accessibility and competition.

To this end, we construct a qualitative model of export choice in which a fixed mass of homogeneous firms play the following two-stage game. They choose a unique destination to export to, and once the destination is set, they monopolistically compete with other exporters for this destination. This destination choice process finds empirical support. Notably, it has been documented that a substantial share of new exporters enters just one destination. For example, Albornoz et al. (2012) report that, among new Argentine manufacturing exporters between 2002 and 2007, 79% of them serve a unique foreign market, whereas only 6% of them enter more than three countries.1 Furthermore, the fact that the mass of firms is fixed implies that the free entry condition does not apply, and therefore, firms make profits. In that context, we study the behavior of the economy in the short run and show that firms face a trade-off. This trade-off is summarized by the fact that, naturally, profits decrease in relation to the density of competitors (i.e., fear of competition) and in relation to the distance to the destination due to transport costs. Hence, firms have an incentive to export to the most attractive economies to avoid paying high transport costs. However, they anticipate that these destinations are precisely those to which a high number of competitors will export. As a consequence, they are encouraged to export to less attractive countries to escape competition. Based on this mechanism, equilibrium is a situation in which a non-degenerated distribution of firms with respect to

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1See Appendix B.1.
their destination choices that balances the two opposite forces summarized above. Thus, the aforementioned baseline trade-off is the key feature of our model because it generates a non-hierarchy of trade. Last, using comparative statics, we demonstrate that productive firms serve more distant countries and have higher mark-ups. This means that, even if firms do not adhere to a hierarchy of trade; on average, firms with high productivity serve less attractive markets than firms with low productivity, which is consistent with the data. Motivated by empirical evidence, we add periods to our baseline model. In our dynamic framework, each firm chooses one destination per period (i.e., sequential exporting). This hypothesis is empirically motivated by Heid et al. (2010) and Albornoz et al. (2012), who focus on the dynamics of exports. Using Chinese and Argentinean data, the latter shows that the export choices of firms feature a sequential entry into foreign markets. Unlike what is implied in long-run models with free entry, firms do not choose all of their destinations simultaneously, but rather one at a time. Adding time periods to our static game has two implications. On the one hand, we demonstrate that our model tends to yield the same predictions as Melitz (2003). Over time, firms with the same level of productivity tend to export to the same basket of destinations (i.e., have similar strategies in terms of destinations). This feature of the model is a result of our escape-competition effect (i.e., our new self-selection effect). Therefore, our argument is that what we observe in the data is not a drawback but a snapshot of the convergence toward the Melitz (2003) prediction. On the other hand, because firms are engaged in sequential exporting, the competition becomes tougher. This process indicates that, at a certain time, it is unprofitable for a firm to continue entering new destinations. As a result, a unique period can be identified where the game stops, and profits have been driven to zero. In that case, the economy is in the long run, and we show that there are mixed strategies at equilibrium. Thus, the model never converges to an exact hierarchy in trade and so provides an intuitive rationale for the data.

These results are robust according to many various dimensions. To be solvable and to maintain consistent results, only three reasonable characteristics are required: profits must be continuous and decreasing with respect to distance (which captures attractiveness) and the degree of competition. Thus, we argue that any trade model encompasses this baseline competition effect and can potentially reproduce the key features of the data. We also extend our baseline export choice model to cope with simultaneous exporting, local firms and entry and exit of firms and heterogeneous firms (in terms of marginal cost). All of these alternatives lead to the same results as the basic model.

On top of its simplicity and robustness, our qualitative model has the appeal of deriving new interesting predictions concerning mark-ups, distance, quality, etc. We show that the mark-up over the marginal cost increases with distance. Using comparative statics, we demonstrate that firms that produce high-quality goods make more profits, have higher
mark-ups and integrate into more distant markets than firms that specialize in low-quality goods. In so doing, we provide a theoretical understanding of the recent empirical findings emphasized by Martin and Mayneris (2013) and Fontagné and Hatte (2013). We finally assert that our new self-selection mechanism (i.e., the presence of the escape-competition effect) induces non-obvious implications of trade liberalization in terms of welfare. When trade liberalization is asymmetric (i.e., preference trade agreements), the model displays trade externalities. We underscore that trade liberalization in any given region matters for more than that region because it prompts firms to export to that region and to desert others (i.e., a negative third country effect emerges as an outcome). Melitz and Ottaviano (2008) find that preferential trade liberalization causes a welfare loss in the third country in long run. In our model, there is a loss in both the short and long run.

In the recent literature, two articles (Eaton et al. (2011) and Chaney (2014)) address the issue that we examine. In both papers, firms with identical marginal costs can serve different sets of foreign markets. The common feature of these two models is that they introduce a second dimension of heterogeneity between firms. In the Eaton et al. (2011) model, the second dimension of heterogeneity that is added is market- and firm-specific heterogeneity in entry costs and idiosyncratic demand shocks. The model adds these factors by incorporating the Arkolakis (2010) formulation of market access. This extra element explains why two identically productive firms make different choices regarding their decision to enter a given market: only the firm with the highest market-specific component of demand and/or the lowest market-specific component of fixed costs enters the country. Chaney (2014) also integrates a second dimension of heterogeneity between firms. He argues that firms can meet trading partners in two ways. On the one hand, they can meet trading partners by direct search, which is modeled as a geographically biased random search, while on the other hand, once a firm has acquired trading contacts in foreign locations, it can develop a new network from these locations: firms differ in their ability to develop a network of consumers in a given market. This new ingredient triggers the heterogeneity of export choices across firms with the same level of productivity: firms export to those markets where they are able to develop their network. In contrast to these two contributions, we propose a new rationale for the absence of a hierarchy of trade without the need to add another heterogeneous dimension between firms. Trade models have centered around the value of fixed costs in replicating firms’ self-selection into export markets. By contrast, in our export choice model, firms’ self-selection relies on the trade-off emphasized previously between accessibility and competition. In that sense, our paper is also related to the literature of pure wage (or price) dispersion. Burdett and Judd (1983) demonstrate that price dispersion can constitute a stable equilibrium even when firms and consumers are identical. Likewise, Burdett and Mortensen (1998) show that wage dispersion can emerge
among homogeneous firms and workers. In both cases, firms reach the same profit level despite paying different wages or setting unequal prices. As in our article, this state of the world is explained by a trade-off from the point of view of the firms that encapsulates a competition effect.

The remainder of the paper is organized as follows. Section 2 introduces the empirical evidence. Section 3 presents the body of the export choice model. Different extensions are discussed in Section 4. Section 5 provides the conclusions.

2 Stylized Facts

This section reveals new stylized facts regarding how manufacturing firms enter foreign markets using French customs data over the period 1994-2008. Importantly, the new empirical regularities that we note reconcile Melitz (2003) with Eaton et al. (2011). Ultimately, all of these empirical findings motivate the use of a new type of export choice model based on homogenous firms and that features firms’ dynamics. We present this model in Sections 3 and 4.

2.1 Hierarchy of Trade

We begin our empirical examination by briefly reproducing Eaton et al.’s (2011) analysis. This is performed in two steps.

First, we determine the eight most popular destinations to which French manufacturing firms export. Table 1 displays these destinations for 2008 as well as the associated number of exporters and the proportion that these firms represent of the overall number of French exporters. Table 1 shows that the top eight destinations are fairly stable over the time.\(^2\) The obtained ranking is almost identical that found by Eaton et al. (2011) for the period 1986-1992. For instance, Belgium is the most popular country, hosting 21,643 French exporters, which accounts for 39.25 % of the total number of exports. For comparison purposes, Eaton et al. (2011) obtain 17,699 and 52 %. The only difference with Eaton et al. (2011) is that Spain reaches third place in our sample.

Second, we describe the strategy of French firms in entering these eight different markets. In particular, we focus on the hierarchy of countries that firms follow. Previously, we stated that firms obey a hierarchy of trade if they exhibit the following behavior. If a firm exports to the \(k\)th most attractive market, then it necessarily export to the \((k - 1)\)th most attractive

\(^2\)Note that this ranking could be easily explained by various factors, including, but not limited to, the size of the market, the geographic distance from France and the share of cultural aspects, such as the use of the same language.
Table 1: French Firms Exporting to the Eight Most Popular Destinations

<table>
<thead>
<tr>
<th>Destinations</th>
<th>Nb. of French Exporters</th>
<th>Prop. of French Exporters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (BE)</td>
<td>21,643</td>
<td>39.25 %</td>
</tr>
<tr>
<td>Germany (DE)</td>
<td>13,010</td>
<td>23.59 %</td>
</tr>
<tr>
<td>Spain (ES)</td>
<td>10,768</td>
<td>19.52 %</td>
</tr>
<tr>
<td>Switzerland (CH)</td>
<td>10,628</td>
<td>19.27 %</td>
</tr>
<tr>
<td>Italy (IT)</td>
<td>10,021</td>
<td>18.17 %</td>
</tr>
<tr>
<td>United Kingdom (UK)</td>
<td>9,795</td>
<td>17.76 %</td>
</tr>
<tr>
<td>Netherlands (NL)</td>
<td>8,134</td>
<td>14.75 %</td>
</tr>
<tr>
<td>United States (US)</td>
<td>7,814</td>
<td>14.17 %</td>
</tr>
<tr>
<td>Any destination</td>
<td>55,141</td>
<td>100 %</td>
</tr>
</tbody>
</table>

Table 2 reproduces and confirms the results of Eaton et al. (2011): French exporters, at the first approximation, do not follow a hierarchy of trade. Only 11.51 % (6,352/55,141) of the total number of firms confirm a hierarchy. The economic literature replicates this non-hierarchy by adding exogenous heterogeneity dimensions. Eaton et al. (2011) suppose market- and firm-specific heterogeneity in entry costs and demand, while Chaney (2014) assumes that firms differ in their ability to develop a network of consumers in a given market. In our model, we highlight that the natural competition forces present in any standard trade model are strong enough to generate heterogeneity in terms of export destination.
2.2 Hierarchy of Trade and Experience as an Exporter

We now draw a link between experience as an exporter and the fact that firms obey a hierarchy of trade.

The preliminary issue that we address is measuring whether heterogeneity in terms of experience matters for export destination choice. To do so, we use the specificity of our data. Using SIREN codes, we are able to characterize the dynamics of export destinations over time for each firm. We define new exporters at time $t$ as firms that export at $t$ but not at $t - 1$. We also define experience as an exporter as the time spent since the first year of exporting. We then build 4 cohorts of firms: $1^{st}$ year as exporters, $5^{th}$ year as exporters, $10^{th}$ year as exporters and more than 14 years as exporters. To clarify, the cohort $10^{th}$ year as exporters pools the firms that were $1^{st}$ year exporters in 1995 and are still surviving and exporting 10 years thereafter. Table 3 displays the number of exporters according to the top eight destinations and the previously defined age groups. It is striking that destination choice is age-dependent. Surprisingly, for firms with 1 year of experience, the most popular destination is Switzerland, then Belgium and then the United States. By contrast, firms that experience a long period of exporting share the same ranking as that shown in Table 1, except for the Italy, which is in the $4^{th}$ position instead of the $5^{th}$. This temporal discrepancy strongly suggests that experience as an exporter impacts a firm’s preference in the choice of markets to enter. Presumably, this result also means that age should affect the hierarchy of trade followed by French exporters.

To investigate the connection between export experience and hierarchy of trade more directly, we carry out two other simple but convincing analyses.

In the first, we report in Figure 1 the proportion of French exporters that obey a hierarchy of trade as their experience increases.
hierarchy according to experience.\textsuperscript{3} The results in Figure 1 are clear. We find that a pronounced age-dependent convergence is at work. The global proportion of firms ordered in terms of destinations goes from 6.76\% for one-year exporters to 16.53\% for more than 14-years exporters, which indicates that two firms are more likely to have similar strategies at time $t$ than at time $t - 1$. Our extrapolation is the following. The absence of a hierarchy of trade is a short-run effect, and, in the long run, firms should export to similar sets of destinations. Given that the Melitz (2003) model derives predictions concerning long-run economies, we assert that what we observe in the data is not necessarily a failure of the Melitz (2003) model. On the contrary, we argue that there is undoubtedly a convergence toward the Melitz (2003) prediction. One can think of many reasons why such a phenomenon emerges. In our theoretical model, we find such a partial convergence. We emphasize the crucial role of sequential exporting and the intensification of competition over time in featuring this state of the world.

Concerning the second exercise, we measure the importance of the deviations observed in Figure 1 from the benchmark and how these deviations evolve with experience as an exporter. Obviously, a "small deviation" would not contradict the theory as much as a "large deviation". For this purpose, we identify firms that export to the United States in 2008. We then count the number of "destination errors" of these firms. We define a destination error as a firm not serving one of the seven other most popular countries. For

\textsuperscript{3}15\textsuperscript{+} captures more than 14-years exporters.
example, a firm that exports to all of these countries except Switzerland makes one error, whereas a firm that exports only to the United States makes 7 errors.\textsuperscript{4} We then re-do the same exercise for each export string (i.e., for firms that export to the Netherlands but not to the United States, etc). Under this view, the deviation from the Melitz prediction can be captured by the following statistic $\text{error} \times 100 \in [0\%, 100\%]$ where $\text{error}$ is the (observed) average number of destination errors of the firms and $\text{maxerror}$ is the average number of destination errors of the firms that completely contradict the Melitz model.\textsuperscript{5} As a consequence, this indicator takes the value 0 \% when the firms’ behavior exactly follows the Melitz prediction. In that case, there is no deviation from the benchmark. Likewise, it takes the value 100 \% when firms’ behavior completely contradicts the Melitz model. Figure 2 displays the average number of destination errors and the deviation from Melitz by experience as an exporter.\textsuperscript{6} The results are remarkable. The deviation strongly decreases over time. For example, firms that had 1 year of experience as exporters in 2008 make, on average, 3.24 errors, which represents a deviation of 81.18 \% from the Melitz prediction, whereas firms that had more than 14 years of experience as exporters only make 1.84 errors,\textsuperscript{4} One can certainly argue that not exporting to Belgium (or Germany) represents a larger deviation from the predicted hierarchy that not exporting to Switzerland (or Italy). Therefore, firms’ behaviors would come closer to the predictions of Melitz not only because they make fewer errors over time but also because they make fewer errors in the top destinations (Belgium or Germany) than in the bottom (the Netherlands and the USA). Integrating this dimension does not change our results.\textsuperscript{5} $\text{maxerror}$ is defined as $\text{maxerror} = \sum_{i \in I} \text{maxerror}_i = \frac{7+6+5+4+3+2+1}{8} = 4$ with $I = \{US, NL, IT, CH, ES, DE\}$.\textsuperscript{6} See Appendix B.2 for more details.
which represents a deviation of 46.14 % from the benchmark. This confirms the fact that
firms come closer to the Melitz hierarchy the longer they have been exporters.

3 Baseline Model

Motivated by the empirical regularities established in Section 2, we offer a new export
choice model that explains:

1. Why firms do not obey a hierarchy of trade when they are young exporters.

2. Why firms that experience a longer spell of exporting are more likely to be ordered
in terms of destination choices (i.e., why firms converge toward Melitz’s prediction
over time).

3.1 Environment

Let us consider a world composed of a home country and a continuum of foreign countries
distributed on a Hotelling line $X = [0, X]$. Foreign countries are ranked with respect to their
distance $x \in X$ to the home country. There are two goods in this world: one homogeneous
good produced under perfect competition in all countries and one horizontally differentiated
good that is only produced by domestic firms. We assume that there are no local firms
in foreign countries. See Section 4 for a proof that adding local firms does not change the
nature of our results.

3.1.1 Supply Side

In the home country, there is a unit mass of (ex ante) homogeneous and risk-neutral
firms. All of the firms produce a variety of the horizontally differentiated good at the
same marginal cost $c > 0$ and at the same fixed cost $f > 0$. These firms also have
the possibility of exporting to a single foreign country (see Albornoz et al. (2012) and
Appendix B.1 for a justification). To export to $x$, the firms must incur a transport cost
$\tau x > 0$ for each unit of exported good knowing that, once the destination is selected, they
will be engaged in a monopolistic competition only with other domestic firms that export
to the same destination.$^7$ Note that the model can be derived by assuming that each firm
chooses a number $1 < n < \infty$ of destinations (i.e., simultaneous exporting).$^8$ Moreover,
we, respectively denote $q_i(x)$ and $Q(x)$ as the quantities sold in country $x$ for variety $i$ and

$^7$ also captures the notion of multilateral trade resistance. Likewise, $f$ also captures the notion of fixed
export cost.

$^8$More globally, see Section 4 for a discussion of the robustness of our framework.
for all varieties available in country $x$ (i.e., the total quantity exported to country $x$) so that:

$$Q(x) = \int_{I_x} q_i(x) \, di$$

(1)

with $I_x = \mu(x)$ being the set of available varieties in $x$ or, equivalently, the density of firms exporting to $x$.

### 3.1.2 Demand Side

In each foreign country, the population size is normalized to one, and the demand side is summarized by a representative consumer with homogeneous preferences. The upper-tier utility function of this consumer is quasi-linear:

$$U(z, q(x)) = z + U(q(x))$$

(2)

with $z$ being the consumption of the Hicksian composite good produced under perfect competition and used as the numeraire. We define $q(x)$ as the vector of consumption of varieties of the horizontally differentiated good such that $q(x) = (q_i(x))_{i=0}^{I_x}$. The lower-tier utility of consuming the differentiated good is given by a quadratic utility function à la Melitz and Ottaviano (2008):

$$U(q(x)) = \int_{I_x} a q_i(x) - \frac{b}{2} q_i(x)^2 \, di - \frac{\gamma}{2} \left[ \int_{I_x} q_i(x) \, di \right]^2$$

(3)

Parameter $a$ represents the consumer’s intrinsic evaluation of the differentiated good. It also stands as a measure of the quality of the good (see Di Comite et al. (2012)). It is the same for all varieties, which implies that firms sell a good that is only horizontally differentiated. Parameter $b$ reflects substitutability between varieties. It is a constant and is equal for each variety, which means that no variety is more or less substitutable than another. Parameter $\gamma$ captures the demand linkage between varieties. A higher $\gamma$ indicates that varieties are less differentiated, and the marginal utility of consuming a unit of variety $i$ decreases more rapidly with the consumption of any variety $j \neq i$. We suppose that $a$, $b$, $\gamma$ are positive and the same in all countries. As a result, the demand function addressed to each single firm will be strictly the same in each country. Finally, the budget constraint of the representative consumer is:

$$z + \int_{I_x} p_i(x) q_i(x) \, di \leq y$$

(4)

with $y$ as the exogenous income of consumers. In this environment, the marginal utility of income can be normalized to one (see Spence (1976) and Neary (2009) for further explanations), and each firm faces the same downward-sloping demand function in each $x$:

$$p_i(x) = a - bq_i(x) - \gamma Q(x)$$

(5)
Firms take $Q(x)$ as given: they do not anticipate their impact on aggregates. In this sense, there is no strategic interaction, and firms monopolistically compete with other firms exporting to the same destination.

3.2 Static Setting

Within this simple framework, domestic firms play the following two-step game:

1. They choose a single country to export to.

2. They (monopolistically) compete in quantity in country $x$ with all firms that export to this location.

Therefore, the entire program of a firm is given by:

$$\max_{x,q_i(x)} \pi(x,q_i(x)) = \max_{x,q_i(x)} \left\{ p_i(x)q_i(x) - (c + \tau x)q_i(x) - f \right\}$$

with $\pi$ as the profit of firms. The model is solved by backward induction. Stage 2 determines an equilibrium quantity sold by firms in location $x$ denoted by $q^*(x)$. Stage 1 pins down an equilibrium distribution of firms in destination $x$ or, comparably, an equilibrium number of varieties exported to foreign country $x$ denoted by $\mu^*(x) = I^*_x$.

3.2.1 Stage 2

Given that a firm exports to $x$, Stage 2 solves the following program:

$$\max_{q_i(x)} \left\{ [a - bq_i(x) - \gamma Q(x)]q_i(x) - (c + \tau x)q_i(x) - f \right\}$$

It yields the following best response function:

$$q^*_i(x) = \frac{a - c - \tau x - \gamma Q(x)}{2b}$$

As all firms are homogeneous in terms of their marginal cost, in addition to the fact that best response functions are linear in $Q(x)$, all firms that export to the same destination choose the same quantity. Let this quantity be $q(x)$. Hence, the total quantity sold in each country is simply $Q(x) = \mu(x)q(x)$, and the equilibrium quantity produced by a firm exporting in $x$ becomes:

$$q^*(x) = \frac{a - c - \tau x}{2b + \gamma \mu(x)}$$

With this quantity, the equilibrium profit made by a firm that exports to $x$ is simply:

$$\pi(x,q^*(x)) = bq^*(x)^2 - f$$

Because the equilibrium profit is strictly increasing in quantities, it is a decreasing function of both distance (at a given transport cost level) and the mass of firms exporting to $x$. 

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3.2.2 Stage 1

For a given equilibrium in stage 2, firms choose a destination so that:

$$\max_x \pi(x,q^*(x)) = \max_x \left\{ b \left( \frac{a-c-\tau x}{2b+\gamma \mu(x)} \right)^2 - f \right\}$$  \hspace{1cm} (11)

Equation (11) emphasizes that firms face a trade-off. The trade-off is captured by the decreasing relationships between profits $\pi$ and both $x$, the distance, and $\mu(x)$, the number of competitors. Thus, it represents the desire to export to the most attractive countries and the desire to escape competition. The intuition is easy to grasp. As transport costs increase with distance, firms should export to attractive countries. However, they also anticipate that more competitors will export to these countries. Therefore, to escape competition, they have an incentive to export to less attractive countries. This program also shows that firms select a destination for their exports in accordance with their preferences and the strategies of others, summarized by the density of competitors. In that particular case, the suitable equilibrium takes the following form:\textsuperscript{9}

**Definition 1** A firm’s distribution $\mu^* \in \mathcal{M}(\mathcal{X})$ is an equilibrium if, and only if:\textsuperscript{10}

$$\begin{cases}
\pi(x,q^*(x)) = \pi(x,\mu^*(x)) \leq \pi^* \quad \text{for almost every } x \in \mathcal{X} \\
\pi(x,q^*(x)) = \pi(x,\mu^*(x)) = \pi^* \quad \text{for almost every } x \in \mathcal{X} \text{ such that } \mu^*(x) > 0
\end{cases}$$  \hspace{1cm} (12)

An equilibrium is a situation in which each firm receives the same total profit wherever it exports to because in such a configuration, unilateral deviations of strategies are impossible. Using (11) and Definition 1, we obtain the following proposition:

**Proposition 1** A unique firm’s distribution $\mu^*$ exists such that: $\forall x \in [0, \hat{x}^*]$  

$$\mu^*(x) = \frac{a-c-\tau x}{\gamma \Gamma^*} - \frac{2b}{\gamma}$$  \hspace{1cm} (13)

with $\hat{x}^*$ the threshold country above which no firms export:

$$\hat{x}^* = \frac{a-c-2b\Gamma^*}{\tau}$$  \hspace{1cm} (14)

with $\Gamma^*$ a constant equal to:

$$\Gamma^* = \frac{4(a-c)b + 2\gamma \tau - \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{8b^2}$$  \hspace{1cm} (15)

and with $\pi^*$ the equilibrium profit given by:

$$\pi^* = b [\Gamma^*]^2 - f$$  \hspace{1cm} (16)

\textsuperscript{9}See Cardaliaguet (2012) for further explanations regarding the form of the equilibrium.

\textsuperscript{10}$\mathcal{M}(\mathcal{X})$ is the set of absolutely continuous Borel probability measures on $\mathcal{X}$ (absolutely continuous with respect to the Lebesgue measure).
For convenience of analysis, we also plot two distributions in Figure 2. The dashed line represents the initial equilibrium $\mu^*$, which is analytically defined in (13)-(16). The solid line depicts the same distribution but with a change in parameters. This new distribution is denoted by $\tilde{\mu}$.

The first striking result, sketched in Proposition 1, is that the firm’s distribution is never degenerated. This result implies that homogeneous firms do export differently in the short run (i.e., they do not follow a hierarchy of trade in the short run). As outlined previously, this feature is due to a baseline competition effect. Firms want to export to the most attractive countries to avoid paying high transport costs. However, at the same time, because they have the same profit function, they anticipate that these markets will be more coveted by their competitors. The competition will be tougher and will prompt some firms to deviate by exporting to less attractive destinations.

Unlike the results in Proposition 1, which seem to be cumbersome, the effect of each parameter on the firm’s distribution is unambiguous (see Proposition 2).\footnote{Hereafter, we focus on the impact of $c$, $a$ and $f$. The other parameters, including $\gamma$, $b$ and $\tau$, have a standard interpretation (see Figure 2).}

**Proposition 2** The effect of an increase in each parameter on $\mu^*$ is displayed in Figure 2.
One can intuitively deduce the influence of a firm’s productivity on the destination choice. We see that the slope of \( \mu^*(x) \) with respect to \( x \) is:\(^{12}\)

\[
\frac{\partial \mu^*(x)}{\partial x} = -\frac{\tau}{\gamma \Gamma^*} < 0 \tag{17}
\]

This slope is steeper for a higher \( c \) because the equilibrium profit \( \pi^* \) is a decreasing function of \( c \). Thus, for a higher productivity (i.e., lower \( c \)), the equilibrium profit is higher, which in turn implies a more slight influence of distance on trade. Moreover, plugging (15) into (14) gives:

\[
\hat{x}^* = \frac{-2\gamma \tau + \sqrt{16(a - c)b\gamma \tau + 4\gamma^2 \tau^2}}{4b\tau} \tag{18}
\]

The threshold country \( \hat{x}^* \) is itself a decreasing function of \( c \). Consequently, for a lower \( c \), the distribution \( \mu^* \) is flatter. This feature suggests that, on average, more productive firms export further, which is consistent with the data (see, for example, Bernard and Jensen (1999)). Another interesting result lies in parameter \( a \). An increase in \( a \) has exactly the same impact as a decrease in \( c \). Because this parameter can be interpreted as a measure of quality, our model predicts that firms producing high quality goods generate more profits and integrate into more distant markets than firms producing low-quality goods. This theoretical result goes along with recent empirical findings highlighted by Martin and Mayneris (2013) and Fontagné and Hatte (2013). Last, \( f \), the fixed cost, does not determine firms’ location decision, it only reduces firms’ profit.\(^{13}\) Firms’ self-selection only results from the trade-off emphasized previously between accessibility and competition. However, as in the basic trade theory, \( f \) prevents firms with insufficient productivity from exporting; only firms with \( c < \hat{c} \) export with \( \hat{c} \) being the productivity threshold.\(^{14}\)

Using (10), (15) and (16), it also results that:

\[
q^*(x) = \frac{4(a - c)b + 2\gamma \tau - \sqrt{16(a - c)b\gamma \tau + 4\gamma^2 \tau^2}}{8b^2} = \Gamma^* \tag{19}
\]

The equilibrium quantity sold by firms (i.e., the intensive margin) does not hinge on distance anymore, which reflects, once again, the fact that at equilibrium, the distance effect and the competition toughness compensate for each other. This result also means that our model can be viewed as a model of the extensive margin only. In so doing, we are in line with (among others) Chaney (2014). There are three reasons for leaving this outcome unchanged. First, in our empirical examination, our purpose has been to focus on the extensive margin of trade; we have deliberately omitted the study of the intensive margin.

\(^{12}\)Note that (17) shows that the extensive margin is decreasing with distance, which is consistent with the findings of Bernard et al. (2007).

\(^{13}\)Recall that \( f \) includes fixed export costs.

\(^{14}\)We obtain: \( \pi^* = 0 \Leftrightarrow 4(a - \hat{c})b + 2\gamma \tau - \sqrt{16(a - \hat{c})b\gamma \tau + 4\gamma^2 \tau^2} = 8b^2 \sqrt{f} \).
Second, we want to keep the setup simple. Embedding heterogeneity in the intensive margin of trade in the existing model can be cumbersome and may rule out the possibility of obtaining analytical and interpretable results. We leave this extension for future research. Third, this approach permits us to obtain a new, interesting prediction concerning firms’ mark-up. Note that the price faced by firm $i$ in each country $x$ can be expressed as a function of the extensive margin $\mu^*(x)$:

$$p^*_i(x) = a - [b + \gamma \mu^*(x)] \Gamma^*$$

(20)

Because $\mu^*(x)$ is, as mentioned above, decreasing with distance, $\frac{p^*_i(x) - c}{c}$, the endogenous mark-up over the marginal cost is increasing with distance!

### 3.3 Dynamic Setting

By choosing one destination at a time, homogeneous firms are forced to have different strategies to escape competition, which is what we highlighted in the previous section by presenting a single-period version of the model (i.e., in the short run). This section aims to present the same model but with several periods to stress the positive link between experience as exporters and the fact that firms follow a hierarchy of trade.

To achieve this goal, we add periods to our basic model so that, at every $t$, firm $i$ faces the following downward-sloping demand function in each $x$:

$$p_{i,t}(x) = a - bq_{i,t}(x) - \gamma Q_t(x)$$

(21)

where $Q_t(x)$ is the number of firms that export to $x$ at time $t$. Furthermore, as firms are myopic (i.e., they make decisions only according to their current utilities), the model resolution is unchanged. Stage 2 pins down an equilibrium quantity sold by firms in location $x$ at time $t$ denoted by $q^*_t(x)$:

$$q^*_t(x) = \frac{a - c - \tau x - \gamma Q_t(x)}{2b}$$

(22)

and Stage 1 determines an equilibrium density of firms at time $t$ denoted by $\mu^*_t(x)$.

#### 3.3.1 Sequential Exporting

In what follows, each firm is now allowed to choose a supplementary destination at each period as soon as profits are positive.\footnote{One could consider the same framework using $1 < n < \infty$ destinations or a certain entry/exit rate of firms. One could also assume that only a proportion $\psi$ of firms are allowed to choose a supplementary destination. In all cases, the nature of the results is the same, although these extensions make the formulas much more complex.} This approach naturally implies that, at each
period $t > 1$, all firms select an additional destination. Indeed, one can demonstrate that it is always optimal for a firm to enter a new market at $t + 1$. One can guess that the total profits at $t$ are $t\pi^*$. If a firm did not enter at $t$, then its profits would be $(t - 1)\pi^*$. This approach also implies that firms do not have incentives to exit markets to which they are already exporting. Therefore, they continue to export to the destinations chosen at $t - k$ with $k \in \{1, t - 1\}$.

Within this environment, the quantity sold in each $x$ by each firm at $t$ is given by (22) with:

$$ Q_t(x) = \begin{cases} q_t^*(x)\mu_t(x) & \text{for } t = 1 \\ q_t^*(x) \left[\mu_t(x) + \sum_{k=1}^{t-1} \mu_k^*(x)\right] & \text{if } t > 1 \end{cases} $$

(23)

where $\sum_{k=1}^{t-1} \mu_k^*(x)$ stands for the number of firms already exporting to destination $x$ in $t$, so the instantaneous profit made by a firm that exports to $x$ at $t$ becomes:

$$ \pi_t(x, q^*_t(x)) = bq^*_t(x)^2 - f $$

(24)

and the total profit made by a firm at $t$ is: $\Pi_t = t(bq^*_t(x)^2 - f)$. Therefore:

**Definition 2** A distribution of firms $\mu_t^* \in M(\mathcal{X})$ is an equilibrium if, and only if:

$$ \begin{cases} \pi_t(x, q_t^*(x)) = \pi_t(x, \mu_t^*(x)) \leq \pi_t^* & \text{for almost every } x \in \mathcal{X} \\ \pi_t(x, q_t^*(x)) = \pi_t(x, \mu_t^*(x)) = \pi_t^* & \text{for almost every } x \in \mathcal{X} \text{ such that } \mu_t^*(x) > 0 \end{cases} $$

(25)

Using (23)-(24) and Definition 2, we obtain:

**Proposition 3** A unique distribution $\mu_t^*$ exists such that: $x \in [0, \tilde{x}_t^*]$

$$ \mu_t^*(x) = \begin{cases} \frac{a - c - \tau x}{\gamma \Gamma_t^*} - \frac{2b}{\gamma} & \text{for } t = 1 \\ \frac{a - c - \tau x}{\gamma \Gamma_t^*} & \text{for } t > 1 \end{cases} $$

(26)

with $\tilde{x}_t^*$ the threshold country above which no firms export at time $t$:

$$ \tilde{x}_t^* = \begin{cases} \frac{a - c - 2b \Gamma_t^*}{\tau} & \text{for } t = 1 \\ \frac{a - c}{\tau} & \text{for } t > 1 \end{cases} $$

(27)

with $\Gamma_t^*$ a constant that equals:

$$ \Gamma_t^* = \begin{cases} \frac{4(a-c) + 2b - 4\sqrt{(a-c)b + 4\gamma^2 + 4\gamma^2}}{8b^2} & \text{for } t = 1 \\ \frac{(a-c)^2}{2\tau \gamma} & \text{for } t > 1 \end{cases} $$

(28)

and $\pi_t^*$, the instantaneous equilibrium profit at time $t$, is given by:

$$ \pi_t^* = b \left[ \frac{(a - c)^2 \Gamma_t^*}{(a - c)^2 + 2 \gamma (t - 1) \Gamma_t^*} \right]^2 - f $$

(29)
We also plot two distributions in Figure 3. The dotted line represents the initial density distributions at time \( t = 1 \). We label this distribution \( \mu_1^* \). The solid line depicts the same distribution but at time \( t > 1 \). The latter is denoted by \( \mu_t^* \). Coupling (26)-(29) and Figure 3, several issues emerge.

First, we obviously find the same expression as that found in Section 3.2 for the spatial distribution of firms at \( t = 1 \). At time \( t > 1 \), we obtain a flatter distribution: \( \mu_1^*(0) > \mu_t^*(0) \), \( \partial \mu_1^*(x)/\partial x < \partial \mu_t^*(x)/\partial x \) and \( \tilde{x}_1^* < \tilde{x}_t^* \). More surprising, the obtained distribution becomes constant over time: the threshold country \( \tilde{x}_t^* \) increases between the first and the second period, and then it stays the same beginning in the second period. The phenomenon is the same for the trade gradient (see Figure 3) due to the absence of entries and exits of firms. Indeed, in Section 4, we will demonstrate that when considering the same setting but with an exogenous rate of entry of new exporting firms and allowing for firms' destruction, this characteristic is no longer valid. Until the appearance of these differences, the predictions made in Section 3.2 are preserved in this dynamic setting. Notably, the effects of the parameters are the same in the short run and in the long run: the long-run firms' distribution is steeper when \( c \) increases and \( a \) decreases.

Second, adding periods is strictly equivalent to allowing a supplementary unit mass of firms to choose a unique destination. As a consequence, competition is increasingly more difficult with time, and profits decrease over time: \( \pi_{t+1}^* < \pi_t^* \) \( \forall t \). Because firms produce at the same fixed cost \( f \), this decrease in profits implies that there exists a unique period \( t^* < \infty \) that leads to zero profits: \( \Pi_t^* = 0 \). This result means that when \( t = t^* \), the game stops, as the economy is in the long run and all firms export to \( t^* \) destinations. Using (28)-(29), we find:

\[
t^* = 1 + \frac{(a - c)^2}{2\tau\gamma} \left( \sqrt{\frac{b}{f}} - \frac{8b^2}{4(a - c)b + 2\gamma\tau - \sqrt{16(a - c)b\gamma\tau + 4\gamma^2\tau^2}} \right)
\]  

(30)

The above equation seems to be cumbersome. However, it is easy to show with basic algebra that \( t^* \) is an increasing function of \( a \) and a decreasing function of \( c \) and \( f \). Therefore, our simple model predicts that more productive firms and firms producing high-quality goods export to more destinations in the long run. Finally, note that, as before, \( f \), the fixed cost, does not affect firms' location decisions. However, it controls the speed of convergence because it determines the period where the game stops.

Third, a corollary to the Melitz (2003) model in such a framework would be that all firms with a given level of productivity should export to all countries between 0 and \( \frac{a - c}{\tau} \). This result is not exactly the case in our model. At any \( t > 1 \), the threshold country \( \tilde{x}_t^* \) is the same, and all firms select a supplementary destination. Also note that \( \mu_t(x) \) can be viewed as the probability for a firm to export to \( x \) at \( t \). Because \( \mu_t(x) \) decreases

\[\text{More precisely, } \exists t^* \text{ such that: } t^* = \lceil \tilde{t} \rceil \text{ and } \Pi_{\tilde{t}}^* = 0. \]
with $x$, it implies that firms are more likely to enter new destinations near $x = 0$ than near $x = X$. Therefore, two firms observed at $t$ are more likely to have similar baskets of destinations than if they were observed at $t' < t$. Hence, a prediction of our model is that firms tend to have more similar strategies. Nevertheless, as stated previously, because $\pi^*_t$ is decreasing over time, there exists a unique period $t^*$ that leads to zero profits and, in the long run, firms export to a finite number of $t^*$ destinations. As a consequence, the model never exactly converges toward the Melitz prediction. It is still true that firms tend to have more similar baskets of destinations over time, but at $t^*$ there still exists dispersion. Finally, because more productive firms and firms producing high-quality goods export to more destinations in the long run, these firms are more likely to follow a hierarchy of trade than less productive firms and firms producing high-quality goods.

### 3.4 Welfare Analysis

In what follows, we study the effects of trade liberalization in terms of welfare. Because there is a single exporting country in our setting, trade liberalization is associated with the case where $\tau$ falls.\(^{17}\) It is also worth noting that, with no local firms, welfare is only consumers’ surplus.

We then show that gains from trade liberalization are ambiguous in the short run (and

\(^{17}\)Recall that $\tau$ includes the motion of multilateral trade resistance.
therefore in the long run). We also find that, when the latter is asymmetric, the region that does not undergo trade liberalization always experiences welfare losses. These two results occur because a decrease in transport costs affects firms’ location decisions, which endogenously changes the degree of competition across countries. Such firms’ reallocations generate gains and losses in terms of welfare. In sum, we prove that the previous trade-off between attractiveness and competition plays a substantial role in determining the welfare extent of trade liberalization.

This shift in the pattern of entry and the welfare loss in "the third country" have already been isolated, for example, by Melitz and Ottaviano (2008). However, our approach differs from that framework in two dimensions. On the one hand, we obtain these findings in a very different model by adopting an export choice model with symmetric firms. On the other hand, Melitz and Ottaviano (2008) demonstrate that countries always gain from trade liberalization in the short run because of the standard "pro-competition effect". In our model, there is a loss in both the short and long run.

3.4.1 Symmetric Trade Liberalization

In every destination \( x \), welfare is:

\[
W^*(x) = \frac{b}{2} \mu^*(x) q^*(x)^2 + \frac{\gamma}{2} [\mu^*(x) q^*(x)]^2
\]

(31)

Two comments are necessary. First, the less attractive the country is, the lower the consumer surplus is (i.e., \( \frac{\partial W(x)}{\partial x} < 0 \)). This result is partially driven by the fact that there are not local firms in every country. However, even with a uniform distribution of local firms, the surplus would be lower in less attractive markets because fewer firms would export to these countries, and each exporting firm would sell lower quantities.

Second, at the extreme, when trade liberalization is perfect, and assuming negligible transport costs (i.e., \( \tau = 0 \)), the firms’ distribution is uniform on \([0, X]\), and therefore, the consumer surplus is strictly the same in every country. This result indicates that trade liberalization tends to equalize the consumer surplus across countries. However, it does not mean that trade liberalization has the same impact in each country (for a better understanding, see Figure 4, where the solid line depicts a distribution with lower transport costs than the dashed line). In fact, trade liberalization is harmful for the most attractive countries and beneficial for the least attractive countries. For remote countries, trade liberalization unambiguously leads to higher welfare because it increases both the varieties and the quantity sold of those varieties. By contrast, for close economies, the results are more ambiguous: trade liberalization improves \( q^* \), the intensive margin, but it also decreases \( \mu^*(x) \), the extensive margin. This result occurs because a decrease in trade costs causes a shift of firms according to their export destination. The equilibrium distribution \( \mu^* \) becomes flatter, which means that some firms desert close countries for
remote countries. This non-monotonicity of the impact of trade liberalization also makes
the evolution of the global welfare uncertain. By integration, we obtain:

\[ W^* = \int_{\text{Supp}(\mu^*)} W^*(x) = \frac{bq^*}{2} + \frac{(a - c - 2bq^*)^3}{6\tau\gamma} \] 

with \( W^* \) as the global welfare of our fictive world. One can see that, due to firms’ reallocation, the effect of a decrease in \( \tau \) is completely ambiguous!

### 3.4.2 Asymmetric Trade Liberalization and Trade Externalities

Previously, we established that a decrease in trade costs has a non-monotonous impact on welfare due to firms’ reallocation in the short run. This reallocation mechanism is also interesting if the trade liberalization is asymmetric, in that it concerns only a single region. To analyze such a situation, let us marginally modify our setting by assuming that firms can now export either to \( x \geq 0 \) or to \( x < 0 \) such that \( x \in [-X, X] \). Let us also assume that exporters must incur transport costs \( \tau \) if they export to \( x > 0 \) and transport costs \( \phi \) if they export to \( x < 0 \): they pay \( \tau x \) on each unit sold when they export to any \( x > 0 \) and \( \phi |x| \) when they export to any \( x < 0 \). Using the same methodology as mentioned above, we
find the following distribution: $\forall x \in [\bar{x}^*, \tilde{x}^*]$

$$
\mu^*(x) = \begin{cases} 
\frac{a-c+\phi x}{\gamma \sqrt{\frac{a-c}{b}}} - \frac{2b}{\gamma} & \text{if } x < 0 \\
\frac{a-c}{\gamma \sqrt{\frac{a-c}{b}}} - \frac{2b}{\gamma} & \text{if } x = 0 \\
\frac{a-c-2x}{\gamma \sqrt{\frac{a-c}{b}}} - \frac{2b}{\gamma} & \text{if } x > 0 
\end{cases}
$$

(33)

Threshold countries $\tilde{x}^*_+$ and $\tilde{x}^*_-$ are given by:\18

$$
\tilde{x}^*_+ = \frac{a - c - 2b}{\phi}\sqrt{\frac{\pi^* + f}{b}}
$$

(34)

and

$$
\tilde{x}^*_- = -\frac{a - c - 2b}{\tau}\sqrt{\frac{\pi^* + f}{b}}
$$

(35)

There also exists a unique solution of profits $\pi^*$ given by:

$$
\pi^* = b \left[ \frac{4(a - c)b + 2\gamma \Phi - \sqrt{16(a - c)b\gamma\Phi + 4\gamma^2\Phi^2}}{8b^2} \right]^2 - f
$$

(36)

with $\Phi = \frac{\phi \tau}{\phi + \tau}$. The firms’ profit depends on both $\tau$ and $\phi$. This feature has implications for the concern of asymmetric trade liberalization. Let us consider that only $\tau$ falls and $\phi$ remains the same. To achieve our goal, in Figure 5, we plot $\mu^*$ with different $\tau$. The dashed line represents symmetric transport costs ($\phi = \tau$), while the solid line depicts a case with a lower $\tau$. Obviously, it flattens $\mu^*$ for positive values of $x$. However, even if $\phi$ remains the same, it also changes the firms’ distribution for negative values of $x$ because a decrease in $\tau$ increases the equilibrium profit, which decreases the absolute value of $\tilde{x}^*_-$.

As a result, firms proportionally desert the countries in the negative region. In terms of welfare, there are two opposite impacts. The positive part of the distribution support undergoes the same welfare evolution as in Section 3.4.1, while for the negative part, it unambiguously decreases welfare.\19 Thus, in this model, due to the reallocation of destinations by firms, trade liberalization in any given region does not only matter for that region; it prompts firms to export to that given region and to desert others. Therefore, this model displays trade externalities. Based on this feature, we can call for an international coordination of trade policy.

\18We reasonably have $\tilde{x}^*_+ > |\tilde{x}^*_-| \forall \phi > \tau$. If $\phi = \tau$, then the distribution is symmetric with respect to 0.

\19Note that the global welfare in the region not experiencing trade liberalization is $W^*_- = \frac{b(a-c-2bq^*)^2q^*}{4\gamma^2} + \frac{(a-c-2bq^*)^3}{6\phi\gamma}$. Then, it is simple to show that $\frac{\partial W^*_-}{\partial \tau} > 0$ as $\frac{\partial q^*}{\partial \tau} < 0$.
4 Extensions and Robustness

In Section 3, we highlighted the critical role of an escape competition effect in explaining why firms do not follow a hierarchy of trade in the short run and why experience as an exporter is positively correlated with the likelihood of following a hierarchy of trade in the long run. To derive these key results, we have pinned down a novel export choice model based on several assumptions, including quasi-linear preferences, linear transport costs, and unique exporting country. In what follows, we generalize our model to the case of unspecified preferences, simultaneous exporting and local firms. We also extend our framework to manage the entry and exit of firms and heterogeneous firms. We obtain similar results, which demonstrates the robustness of our findings.

4.1 General Preferences and Hierarchy of trade

In the baseline model, we have constructed a simple model based on linear transport costs and quadratic utility functions. In the trade literature, it is well established that the forms of transport costs (i.e., iceberg transport costs) and consumers’ preferences play a critical role in generating different predictions. Therefore, the natural question to address is determining whether the simplifying assumptions that we use matter to the non-existence of a hierarchy of trade. To this end, we consider our baseline model, but we leave transport costs and preferences unspecified. Using Cardaliaguet (2012), the following proposition is made:
Proposition 4 If $\pi$ is continuous,

$$\frac{\partial \pi(x, \mu(x))}{\partial x} < 0$$  \hspace{1cm} (37)

for every $x$ in $X$, and

$$\frac{\partial \pi(x, \mu(x))}{\partial \mu(x)} < 0$$  \hspace{1cm} (38)

then $\mu^*$ is not degenerated and

$$\frac{\partial \mu^*(x)}{\partial x} < 0$$  \hspace{1cm} (39)

To maintain consistent results, only three reasonable characteristics are required: profits must be continuous and decreasing with respect to distance and the degree of competition. These conditions are very natural and verified by any model with downward-sloping demand functions and no perfect complementary goods. Thus, we argue that any trade model encompasses this baseline competition and can reproduce the key features of the data. This approach also reveals that the analytical model presented in Section 3 can be considered to be an illustration. In addition, the intuition behind this proposition is the same as that of Section 3. The decreasing relationships between the utility $\pi$ and both $x$ the strategy and $\mu(x)$ the indicator of how difficult the competition is in $x$ induce the following trade-off for firms. As (37) holds, they have an incentive to choose $x = 0$ as the best strategy when maximizing their utility. However, because all of the players share the same utility function, they also guess that this strategy has the highest number of competitors. Because they fear competition (see assumption (38)), they are encouraged to play differently (i.e., no hierarchy of trade). Also note that, based on this mechanism, we again find that an equilibrium is a situation in which each agent receives the same equilibrium payoff $\pi^*$, whatever its action, because both parts of the trade-off identified above compensate for each other, that is, $\mu^*$ decreases with respect to $x$ to balance the two opposite forces summarized in (37) and (38).

4.2 Simultaneous Exporting

Thus far, we have supposed that firms select a unique export destination. In a recent work, Albornoz et al. (2012) document that the most productive firms are engaged in simultaneous exporting. Our model is flexible enough to feature this characteristic. To capture this phenomenon, let us marginally assume that now each firm chooses $1 < n < \infty$ different destinations. Following Proposition 1, firms export to a vector $x \in \mathcal{X}^n$ of foreign countries such that:

$$\pi(x, \mu(x)) = b \left( \frac{a-c-\tau|x|}{2b+\gamma \mu(x)} \right)^2 - f$$  \hspace{1cm} (40)

and the equilibrium in stage 1 becomes:

$$\mu^*(x) = \frac{a-c-\tau|x|}{\gamma \Gamma^*} - \frac{2b}{\gamma}$$  \hspace{1cm} (41)
with
\[ \Gamma^* = \frac{4(a - c)b + 2\gamma\tau n - \sqrt{16(a - c)b\gamma\tau n + 4\gamma^2\tau^2n^2}}{8b^2} \] (42)
and \( \pi^* = b[\Gamma^*]^2 - f \). The obtained findings are an order of magnitude equivalent to what was found in Section 3.2. However, differences lie in terms of vectors and the fact that parameter enters the equilibrium profit function. Thus, we reasonably conclude that this small difference does not change the results (no hierarchy of trade and a partial convergence toward Melitz’s prediction).

### 4.3 Local Firms

In the present model, there are no domestic competitors in foreign markets and thus no foreign exporters. It is as if the home country is a monopoly supplier of the differentiated goods of the world. Hence, the sole determinant of competition in a foreign country is summarized only by the number of firms selling there. We relax this assumption in two steps. First, we study the case of exogenous local firms. Second, we focus on the inclusion of endogenous local firms. In both cases, our results derived in Section 3 are remarkably robust.

#### 4.3.1 Exogenous Local Firms

Here, we introduce exogenous local firms to the existing model. For that purpose, let \( \tilde{\mu}(x) \) be the exogenous density of local firms in country \( x \). These firms produce at the same marginal cost \( c \) and at the same fixed cost \( f > 0 \). We also denote \( \tilde{q}(x) \) as the quantity sold by a local firm. In this new environment, we obtain:

\[ \mu^*(x) = \frac{2b(a - c - \tau x) - \tau\gamma\tilde{\mu}(x)x}{2\gamma b \sqrt{\frac{\pi^* + f}{b}}} - \frac{2b}{\gamma} - \tilde{\mu}(x) \] (43)

with \( \pi^* \) solving the following equation:

\[ \int_0^{\tilde{x}^*} \frac{2b(a - c - \tau x) - \tau\gamma\tilde{\mu}(x)x}{2\gamma b \sqrt{\frac{\pi^* + f}{b}}} - \frac{2b}{\gamma} - \tilde{\mu}(x)dx = 1 \] (44)

where \( \tilde{x}^* \) is determined as:

\[ \frac{2b(a - c - \tau\tilde{x}^*) - \tau\gamma\tilde{\mu}(\tilde{x}^*)\tilde{x}^*}{2\gamma b \sqrt{\frac{\pi^* + f}{b}}} = \tilde{\mu}(\tilde{x}^*) + \frac{2b}{\gamma} \] (45)

A general solution for the spatial distribution of firms is obtained, but the solution is fairly complicated because considering an unspecified form for \( \tilde{\mu} \) adds too many degrees of freedom to the model. To gain a sense of the effect of adding exogenous local firms, it is useful to take
the particular case where local firms are evenly distributed across destinations \( \mu(x) = \psi \) with \( \psi \) a constant. In this configuration, introducing local firms increases transport costs but decreases equilibrium profits because \( \psi \) implies tougher competition, which reduces the incentive to export to less attractive countries. However, firms escape competition by exporting to less attractive countries: the firms’ distribution is steeper. The other results stemming from Section 3 are unchanged in nature.

4.3.2 Endogenous Local Firms

Presumably, allowing for endogenous firms in each location could be very cumbersome. Rather, we explore the case in which \( \mathcal{X} = [0,1] \) and a second exporting country is located at the other end of the "long narrow" world economy. The distribution of firms on the left side is labeled \( \mu_0 \) whereas the distribution of firms on the right side is denoted by \( \mu_1 \). We also consider the case with completely symmetric "regions": firms are identical in terms of fixed cost, marginal cost, transportation cost, etc. Under this setup, we obtain:

**Proposition 5** An equilibrium \((\mu_0, \mu_1)\) is as shown in Figure 6.a.

The striking result is that, even with a second exporting country, we find again the same equilibrium of Section 3.2. The striking result is that, even with a second exporting country, we again find the same equilibrium of Section 3.2. A configuration in which the two exporting countries serve the same markets cannot constitute an equilibrium (see Figure 6.b). The crux of the argument is as follows. If the two exporting countries entered the same destinations, the firms in these locations would incur both higher transport costs and more competitors, which would break the trade-off between accessibility and difficulty of competition that, in turn, rules out the possibility of obtaining a spatial firm distribution that equalizes all profits. To conclude, Figure 6.c depicts the situation where the first exporting country is populated by more productive firms: \( c_0 < c_1 \). Figure 6.c clearly shows that integrating heterogeneity does not affect the nature of the aforementioned findings.

4.4 Entry and Exit of Firms with Sequential Exporting

In Section 3.3, the convergence toward the Melitz prediction results from the fact that firms are engaged in sequential exporting. Hereafter, at each period \( t > 1 \), we assume that a proportion \( \rho \) of exporters present at \( t - 1 \) in \( x \) disappears, and a mass \( \rho \) of new firms enter and choose a destination.\(^{20}\) Firms that survive keep producing and have the possibility of choosing an additional country to serve. In that new case, the quantity sold in each \( x \) by

\(^{20}\)Several extensions could be considered. For example, one could assume that the proportion of exporters present at \( t - 1 \) in \( x \) that disappears is destination and time specific (i.e., \( \rho_l(x) \)).
each firm at $t$ equals (22), where now:

$$Q_t(x) = \begin{cases} q_t^*(x) \mu_t(x) & \text{for } t = 1 \\ q_t^*(x) \left[ \mu_t(x) + \sum_{k=1}^{t-1} (1 - \rho)^{t-k} \mu_k^*(x) \right] & \text{if } t > 1 \end{cases}$$  \hspace{1cm} (46)$$

where $\sum_{k=1}^{t-1} (1 - \rho)^{t-k} \mu_k^*(x)$ represents the number of exporters still surviving in $t$ at destination $x$. Moreover, the equilibrium profit made by a firm that exports to $x$ at $t$ remains the same $\pi_t(x, q_t^*(x)) = bq_t^*(x)^2 - f$, but now, due to entries and exits, the firms differ according to their total profit $\Pi_t = k\pi_t(x, q_t^*(x)), \forall k \in \{1, t\}$. Thus, following Section 3,
we obtain: $x \in [0, \bar{x}_t^*]

\mu_t^*(x) = \begin{cases} 
\frac{a-c-\tau x}{\tau \Gamma_t^*} - \frac{2b}{\gamma} & \text{for } t = 1 \\
\frac{a-c-\tau x}{\tau \Gamma_t^*} - \frac{2bp}{\gamma} & \text{for } t > 1 
\end{cases} \quad (47)

with $\bar{x}_t^*$ as the threshold country above which no firms exports at time $t$:

$$\bar{x}_t^* = \begin{cases} 
a - c - \frac{2b}{\gamma} \tau \Gamma_t^* & \text{for } t = 1 \\
a - c - \frac{2bp}{\gamma} \tau \Gamma_t^* & \text{for } t > 1 
\end{cases} \quad (48)$$

with $\Gamma_t^*$, a constant equal to:

$$\Gamma_t^* = \begin{cases} 
\frac{4(a-c)b + 2\gamma - \sqrt{16(a-c)b\gamma + 4\gamma^2 \tau^2}}{8b^2} & \text{for } t = 1 \\
\frac{4(a-c)b\rho + 2\gamma \tau m_t - \sqrt{16(a-c)b\rho\gamma \tau m_t + 4\gamma^2 \tau^2 m_t^2}}{8b^2 \rho^2} & \text{for } t > 1 
\end{cases} \quad (49)$$

and $\pi_t^* = b \left[ \frac{\Gamma_t^* \sqrt{\tau \Gamma_t^* + (1-\rho)\Gamma_t^*}}{\sqrt{\tau \Gamma_t^* + f + \frac{1}{b}}} \right]^2 - f$.\footnote{$m_t$ is the mass of firms that enter a new destination at $t > 1$.} Up to parameters $\rho$ and $m_t$ in the firms’ distribution $\mu_t^*$ and in the constant $\Gamma_t^*$, the equilibrium distribution at time $t > 1$ is identical to that at time $t = 1$. Reasonably, these small differences do not invalidate the results found in the case of sequential exporting without entry and exit. The obtained findings are an order of magnitude equivalent to what was highlighted in Section 3.3. Namely, over time, homogeneous firms tend to have increasingly similar baskets of destinations (i.e., $\mu_t$ remains a decreasing function of $x$). However, as the data suggest, we show that this convergence is incomplete in the sense that there still exists dispersion in the long run. This feature of the model is explained by the entry of new firms and the fact that incumbent firms are engaged in sequential exporting. This intensifies the competition between foreign destinations (i.e., $\pi_t^*$ decreases with time because $m_t$ increases with $t$) and, at a given time period, this becomes unprofitable for a firm to continue entering the game (i.e., $\exists t^*$ such that $\Pi_t^* = 0$, $\forall k$). This extension only departs from our benchmark model according to one dimension: $\mu_t^*$ is no longer constant for $t > 1$. On the contrary, the distribution of new destinations is steeper over time: $\mu_{t+1}^*(0) > \mu_t^*(0), \frac{\partial \mu_{t+1}^*(x)}{\partial x} < \frac{\partial \mu_t^*(x)}{\partial x}$ and $x_{t+1}^* > x_t^*$. This is because $m_t$ is an increasing function of $t$.

### 4.5 Heterogeneous Firms

The objective of the present article is to explain how homogeneous firms export over time, which is why we build a model with symmetric firms. However, it is reasonable to want to know how the model works when firms are heterogeneous in terms of marginal cost. To this end, we consider the same setting as in Section 3 but with different populations of
firms. This new framework is based on a specific Nash equilibrium. Unfortunately, this particular form of equilibrium is new in the trade literature. Consequently, we highlight some important definitions and results concerning our peculiar Nash equilibrium before presenting the case with heterogeneous firms.

4.5.1 Theoretical Preliminaries

Hereafter, we describe a population game in which players are engaged with intra- and inter-group local interactions, which means that the players’ utility functions rely on their strategy and the densities of the other players choosing this strategy. We then analyze the main characteristics (i.e., existence and uniqueness) of a Nash equilibrium in the pure strategy of such a game.

Note that the present game can be interpreted as a population game à la Sandholm (2001), in which interactions are local, and the set of actions is not assumed to be finite. It can also be viewed as an extension of the framework of Cardaliaguet (2012) to several populations of players or a particular case of the mean field game developed by Cirant (2014). Finally, it is also related to large crowding games (see Milchtaich (2000)).

Environment Let $\mathcal{X}$ be a compact subset of $\mathbb{R}$ and $\mathcal{M}(\mathcal{X})$ be the set of absolutely continuous Borel probability measures on $\mathcal{X}$ (absolutely continuous with respect to the Lebesgue measure) denoted by $\mu$ and having a density also denoted by $\mu$ in $C^0(\mathcal{X})$, with $C^0(\mathcal{X})$ as the set of continuous functions. $\mathcal{M}(\mathcal{X})$ is endowed with the Kantorovich-Rubinstein distance:

$$d(\mu, \tilde{\mu}) = \sup \left\{ \int_{\mathcal{X}} g(x) d(\mu - \tilde{\mu})(x) : g \in C^{0,1}(\mathcal{X}), Lip(g) \leq 1 \right\}$$  \hspace{1cm} (50)$$

where $C^{0,1}(\mathcal{X})$ is the set of continuous and differentiable functions on $\mathcal{X}$ and with $Lip(g)$ as the minimal Lipschitz constant for $g$. These assumptions imply that $d$ metricizes the weak-* convergence on $\mathcal{M}(\mathcal{X})$, and $\mathcal{M}(\mathcal{X})$ is compact for $d$. Subsequently, we consider a continuum of players sharing a common set of strategies and that are distributed in two different group indexes $k \in \mathcal{K} = \{1, 2\}$.\footnote{A generalization for $k \in \mathbb{N}_+^*$ is possible without any difficulty.} In a given population $k$, players are homogenous and have to choose a single strategy from $\mathcal{X}$ the common set of strategies.\footnote{A generalization for $n \in \mathbb{N}_+^*$ actions is possible without any difficulty.} To do so, they maximize a population-specific utility function $A_k : \mathcal{X} \times (0, \infty)^2 \rightarrow \mathbb{R}$. The latter depends on $x$, their strategy, and $(\mu_1(x), \mu_2(x))$, the densities of players (in each population) choosing the same strategy. It is worth noting that the effects of densities on utilities may be positive or negative and may also differ according to the populations.
Definition A (pure-strategy) Nash equilibrium, in the sense that all agents play best response, is given by the following definition:

**Definition 3** A vector \((\mu_1^*, \mu_2^*) \in \mathcal{M}(\mathcal{X})^2\) is a (pure-strategy) Nash equilibrium if, and only if:

\[
\int_{\mathcal{X}} A_k(x, \mu_1^*(x), \mu_2^*(x)) \, d\mu_k(x) = \sup_{\mu \in \mathcal{M}(\mathcal{X})} \int_{\mathcal{X}} A_k(x, \mu_1(x), \mu_2(x)) \, d\mu(x)
\]

Existence and Uniqueness Here, we question the existence and uniqueness of a Nash equilibrium of the form (59). In particular, we get:

**Proposition 6** Suppose that \(A_1\) and \(A_2\) are continuous, then there exists at least one vector \((\mu_1^*, \mu_2^*) \in \mathcal{M}(\mathcal{X})^2\) satisfying Definition 3.

We also find that:

**Proposition 7** Suppose that \(A_1\) and \(A_2\) are differentiable and

\[
A(x, \mu_1, \mu_2) + A(x, \mu_1, \mu_2)^T
\]

is negative definite

with

\[
A(x, \mu_1, \mu_2) = \begin{pmatrix} a_{11}(x) & a_{12}(x) \\ a_{21}(x) & a_{22}(x) \end{pmatrix} = \begin{pmatrix} \frac{\partial A_1(x, \mu_1(x), \mu_2(x))}{\partial \mu_1(x)} & \frac{\partial A_1(x, \mu_1(x), \mu_2(x))}{\partial \mu_2(x)} \\ \frac{\partial A_2(x, \mu_1(x), \mu_2(x))}{\partial \mu_1(x)} & \frac{\partial A_2(x, \mu_1(x), \mu_2(x))}{\partial \mu_2(x)} \end{pmatrix}
\]

and \(A(x, \mu_1, \mu_2)^T\) the transpose of the matrix \(A(x, \mu_1, \mu_2)\), then there is at most a vector \((\mu_1^*, \mu_2^*) \in \mathcal{M}(\mathcal{X})^2\) satisfying Definition 3.

This proposition simply states that uniqueness only depends on the relative importance of the local interactions. To clarify, note that (52) collapses to:

\[
2a_{11}(x) < 0
\]

and

\[
4a_{11}(x)a_{22}(x) - [a_{12}(x) + a_{21}(x)]^2 > 0
\]

This sheds light on the key driving forces governing multiplicity. Specifically, one can notice that multiple equilibria emerge as an outcome if:

1. \(a_{11}(x) = a_{12}(x) = a_{21}(x) = a_{22}(x)\).
2. \(a_{11}(x), a_{12}(x), a_{21}(x), a_{22}(x) > 0\).
3. \(a_{11}(x) = a_{12}(x), a_{21}(x) = a_{22}(x)\) and \(a_{11}(x) \neq a_{21}(x)\) or \(a_{11}(x) = a_{21}(x), a_{12}(x) = a_{22}(x)\) and \(a_{11}(x) \neq a_{12}(x)\).

The ideas behind these conditions are clear. Multiple equilibria arise if, when the local interactions equally affect utilities, the local interactions positively affect payoff functions or the local interactions symmetrically affect utilities. Thus, our conjecture is that an equilibrium is unique if individuals face (i) negative and (ii) asymmetric local interactions.
4.5.2 Model

In this section, we address heterogeneous firms to ascertain whether our predictions are still valid. Thus, we assume two different productivities in the model: \(c_1\) and \(c_2\), with \(c_1 < c_2\). This indicates that firms with different levels of productivity compete with each other. We also suppose that there is a mass one of type \(1\) and a mass \(m > 1\) of type \(2\).\(^{24}\) In that case, we obtain:

\[
q_1^*(x) = \frac{2b(a - c_1 - \tau x) + \gamma \mu_2(x)(c_2 - c_1)}{2b \{2b + \gamma [\mu_1(x) + \mu_2(x)]\}} \tag{56}
\]

\[
q_2^*(x) = \frac{2b(a - c_2 - \tau x) + \gamma \mu_1(x)(c_1 - c_2)}{2b \{2b + \gamma [\mu_1(x) + \mu_2(x)]\}} \tag{57}
\]

and

\[
\pi_1(x) = bq_1^*(x)^2 - f \tag{58}
\]

\[
\pi_2(x) = bq_2^*(x)^2 - f \tag{59}
\]

The main difference from our baseline model is that firms are now engaged with intra- and inter-group local interactions, which implies that not only the mass of firms matters for the magnitude of competitive pressure but also the mean productivity of those firms. Using Proposition 7, there exists a unique equilibrium if, and only if:

\[
4 \frac{\partial \pi_1(x)}{\partial \mu_1(x)} \frac{\partial \pi_2(x)}{\partial \mu_2(x)} - \left[\frac{\partial \pi_1(x)}{\partial \mu_2(x)} + \frac{\partial \pi_2(x)}{\partial \mu_1(x)}\right]^2 > 0 \tag{60}
\]

Because of the non-linearity of \(\pi_1\) and \(\pi_2\), this condition appears to comply with a very specific parametrization, which means that uniqueness does not hold when firms are heterogeneous. This approach also prevents the possibility of providing a robust comparative statics analysis.

In the preceding section, we presented an export choice model populated by symmetric firms. To provide some insights into the effect of heterogeneity, we conducted a detailed comparative statics concerning the parameter \(c\) that captures the marginal cost of firms. We have demonstrated that firms with a lower marginal cost serve more distant markets, on average, than firms with a higher marginal cost. Subsequently, the natural exercise is to verify whether such a configuration can be generated in the case of two populations of firms. Using equations (56)-(59), we obtain the last proposition of the article:

**Proposition 8** If

\[
c_1 + b\Gamma_1 < \frac{c_1 + c_2}{2} < a + b \left[\frac{c_1 \Gamma_1 + (a - c_2)\Gamma_2}{c_2 - c_1}\right] \tag{61}
\]

then firms with a lower marginal cost serve more distant markets (on average) than firms with a higher marginal cost: \(\mu_2^*(0) > \mu_1^*(0)\), \(\frac{\partial \mu_2^*(x)}{\partial x} < \frac{\partial \mu_1^*(x)}{\partial x}\) and \(x_2^* < x_1^*\).

\(^{24}\)\(m\) is assumed to be greater than one, which is in line with evidence that shows that less productive firms are more numerous.
Thus, even if the equilibrium is multiple, the prediction made in Section 3 remains true under (61). The latter condition determines an upper and lower band for the average productivity of firms. However, because $\Gamma_1$ and $\Gamma_2$ are two endogenous variables (not necessarily unique), the analysis of (61) is fairly complicated and does not shed light on the connection between the parameters and the overall configuration, which reinforces our earlier argument that our export choice model is more tractable when considering symmetric firms.

5 Conclusions

The present paper tackles the issue of hierarchy of trade. Using recent French data, we first document that the fact that French exporters follow a hierarchy of trade depends on their experience as exporters. This means that they strongly export to different markets when they are young exporters, while, conditional on survival and over time, they tend to export to the same ordered set of countries. In so doing, we reconcile Melitz (2003) with Eaton et al. (2011). Second, we offer a new model of export choice that provides a rationale of this new stylized fact. Based on a simple competition effect that is present in any model of monopolistic competition, we show how two identical firms can enter different markets. Our economic argument is that trade can be considered as a way to escape competition. In a multi-period setting, our model displays the dynamics emphasized by the empirical part, namely, that firms tend to have more similar strategies as they become more experienced exporters. However, though the model converges toward the Melitz prediction, it never reaches it fully. Thus, we argue that what we observe in the data is not a failure of Melitz (2003) but a convergence toward his prediction.

Several extensions can be considered. In particular, we see, among others, three possible refinements. First, we have called for an additional extension: the inclusion of heterogeneity in the intensive margin of trade. Second, our article is qualitative in nature. As a consequence, future research should be directed toward quantitatively matching the empirical regularities that we have highlighted. Third, we derive new predictions concerning the relationship between mark-up and distance and gains from trade that may provide a direction for empirical works. For example, it would be very interesting to empirically confirm that mark-ups increase with distance. We leave all of these issues for future studies.
References


A Proofs

Proof 1 \( \pi \) is continuous and \( \frac{\partial \pi(x, q^*(x))}{\partial \pi(x)} < 0 \). From Cardaliaguet (2012), this implies that a unique \( \mu^* \) exists. Then, we determine \( \mu^*(x) \) a density of firms such that:

\[
\begin{cases}
\pi(x, q^*(x)) = b \left( \frac{a-c-\tau x}{2b+\gamma \mu^*(x)} \right)^2 - f = \pi^* \in \mathbb{R}_+^* \quad \forall x \in \mathcal{X} \\
\int_{\text{Supp}(\mu^*)} \mu^*(x) dx = \int_0^{x^*} \mu^*(x) dx = 1 \\
\mu^*(x) \geq 0 \quad \forall x \in \mathcal{X}
\end{cases}
\]

With the first condition of the system, we immediately find:

\[
\mu^*(x) = \frac{a-c-\tau x}{\gamma \sqrt{\frac{\pi+I}{b}}} - \frac{2b}{\gamma}
\]

Note that the trade gradient is \( t g^* = \frac{\partial \pi^*(x)}{\partial x} = -\frac{\tau}{\gamma \sqrt{\frac{\pi+I}{b}}} \) and \( \mu^*(0) = \frac{a-c-2b\sqrt{\frac{\pi+I}{b}}}{\gamma \sqrt{\frac{\pi+I}{b}}} \). Using the third condition of the system, the threshold country \( \bar{x}^* \) is determined as \( \mu^*(\bar{x}^*) = 0 \), that is,

\[
\bar{x}^* = \frac{a-c-2b\sqrt{\frac{\pi+I}{b}}}{\tau}
\]

If \( 0 < \bar{x}^* < X \) then integrating \( \mu^* \) on its endogenous support \([0, \bar{x}^*]\) yields the following quadratic polynomial

\[
4b^2\Gamma^2 - [4(a-c)b + 2\gamma \tau] \Gamma^* + (a-c)^2 = 0 \quad \text{with} \quad \Gamma^* = \sqrt{\frac{\pi+I}{b}}. \]

The discriminant of this quadratic polynomial is \( \Delta = [4(a-c)b + 2\gamma \tau]^2 - 16(a-c)^2b^2 = 16(a-c)b\gamma \tau + 4\gamma^2 \tau^2 > 0 \). Because \( \Delta > 0 \), it admits two solutions \( \text{Sol}_1 = \frac{4(a-c)b + 2\gamma \tau - \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{8b^2} > 0 \) because \( 16(a-c)^2b^2 > 0 \) and \( \text{Sol}_2 = \frac{4(a-c)b + 2\gamma \tau + \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{8b^2} > 0 \) so that: \( \text{Sol}_1 < \text{Sol}_2 \).

Using \( \text{Sol}_1 \), we obtain \( \mu^*(0) = -\frac{2\gamma \tau + \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{4b\gamma \text{Sol}_1} < 0 \) and \( \bar{x}^* = -\frac{2\gamma \tau + \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{4b\tau} \). Note that: \( \mu^*(0) > 0 \) and \( \bar{x}^* > 0 \) because \( 16(a-c)b\gamma \tau > 0 \). However, plugging \( \text{Sol}_2 \) in \( \mu^*(0) \) and \( \bar{x}^* \) leads to \( \mu^*(0) = -\frac{2\gamma \tau - \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{4b\gamma \text{Sol}_2} < 0 \) and \( \bar{x}^* = -\frac{2\gamma \tau - \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{4b\tau} < 0 \). This implies that the unique solution for \( \Gamma^* \) is:

\[
\Gamma^* = \frac{4(a-c)b + 2\gamma \tau - \sqrt{16(a-c)b\gamma \tau + 4\gamma^2 \tau^2}}{8b^2}
\]

and \( \pi^* = b \left| \Gamma^* \right|^2 - f \).\(^{25}\) If \( \bar{x}^* \geq X \) then the support of \( \mu^* \) is \([0, X]\) and integrating \( \mu^* \) on the new support gives \( \Gamma^* = \frac{2(a-c)X - X^2}{2(\gamma + 2bX)} \).\(^{26}\)

Proof 2 Previously, note that using equations (13)-(15) to determine the effect of each parameter on the equilibrium distribution can be cumbersome. In what follows, we perform

\(^{25}\)We assume that \( \pi^* > 0 \).

\(^{26}\)Both cases exhibit similar results. Therefore, for the clarity of the exposition, we only display results for \( \bar{x}^* < X \) in the article.
a simpler analysis. From Proof 1, we know that the equilibrium profit is given by the following expression \((a - c - 2b\Gamma)^2 = 2\gamma\tau\Gamma\). Let \(g_1\) be a continuous function on \(\mathbb{R}_+\) defined as \(g_1(\Gamma) = (a - c - 2b\Gamma)^2\) and observe that \(g_1(0) = (a - c)^2 > 0\), \(\lim_{\Gamma \to +\infty} g_1(\Gamma) = +\infty\) and \(\frac{\partial g_1(\Gamma)}{\partial \Gamma} = -4b(a - c - 2b\Gamma)\). Let \(g_2\) be a continuous function on \(\mathbb{R}_+\) defined as \(g_2(\Gamma) = 2\gamma\tau\Gamma\) and note that \(g_2(0) = 0\), \(\lim_{\Gamma \to +\infty} g_2(\Gamma) = +\infty\) and \(\frac{\partial g_2(\Gamma)}{\partial \Gamma} > 0\). For the clarity of the exposition, we plot \(g_1\) (blue line), \(g_2\) (red line) and the locus of the equilibrium profit \(\Gamma^*\) on a graph (see Figure 7). Afterward, we compute the derivative of these two functions with respect to each parameter to determine the effect on \(\Gamma^*\) and \(\pi^*\). Then, we compute the derivative of the trade gradient \(tg^*\) with respect to each parameter. Since \(\int_{\text{Supp}(\mu^*)} \mu^*(x)dx = 1\), when the slope of the trade gradient decreases (respectively increases), the threshold country \(\tilde{x}^*\) increases (respectively decreases) and the density of firms in \(x = 0\) decreases (respectively increases).\(^{27}\)

- **Effect of parameter \(a\):** \(\frac{\partial g_1(0)}{\partial a} > 0\) (because \(a - c > 0\)), \(\frac{\partial^2 g_1(\Gamma)}{\partial a \partial a} < 0\), \(\frac{\partial g_2(\Gamma)}{\partial a} = 0\) and \(\frac{\partial^2 g_2(\Gamma)}{\partial a \partial a} = 0\). This leads to: \(\frac{\partial \Gamma^*}{\partial a} > 0\) and so \(\frac{\partial \pi^*}{\partial a} > 0\), \(\frac{\partial g^*}{\partial a} < 0\), \(\frac{\partial \gamma^*}{\partial a} < 0\) and \(\frac{\partial x^*}{\partial a} > 0\) (see Figure 8.a and Figure 2).

- **Effect of parameter \(c\):** \(\frac{\partial g_1(0)}{\partial c} < 0\) (because \(a - c > 0\)), \(\frac{\partial^2 g_1(\Gamma)}{\partial c \partial c} > 0\), \(\frac{\partial g_2(\Gamma)}{\partial c} = 0\) and \(\frac{\partial^2 g_2(\Gamma)}{\partial c \partial c} = 0\). This leads to: \(\frac{\partial \Gamma^*}{\partial c} < 0\) and so \(\frac{\partial \pi^*}{\partial c} < 0\), \(\frac{\partial g^*}{\partial c} > 0\), \(\frac{\partial \gamma^*}{\partial c} > 0\) and \(\frac{\partial x^*}{\partial c} < 0\) (see Figure 8.b and Figure 2).

- **Effect of parameter \(f\):** trivial. \(\diamond\)

**Proof 3** Using (22)-(25), we have \(\mu^*_t(x) = \frac{a - c - \tau x}{\gamma t} - \frac{2b}{\tau} - \sum_{k=1}^{t-1} \mu^*_k(x)\) if \(t > 1\). By iteration, we obtain:

\[
\mu^*_t(x) = \frac{a - c - \tau x}{\gamma \Gamma^*_t}
\]

\(\forall t > 1\) and with \(\Gamma^*_t = \sqrt{\frac{c + f}{b} + \frac{x_t - f}{b} + \sqrt{\frac{x_t - f}{b} + \frac{x_t - f}{b} + f}}\). Following Proof 1, if \(t = 1\) then \(\pi^*_1 = b\Gamma^*_{1} - f > 0\) and \(\Gamma^*_1 = \frac{4(a - c)b + 2\gamma - \sqrt{16(a - c)b + 4a + 4\gamma^2} + 4b^2}{8b^2}\). In the case of \(t > 1\), the threshold country \(\tilde{x}^*_t\) is determined as \(\mu^*_t(\tilde{x}^*_t) = 0\), that is,

\[
\tilde{x}^*_t = \frac{a - c}{\tau}
\]

Assuming \(\tilde{x}^*_t < X\) \(\forall t\) and then integrating \(\mu^*_t\) on its endogenous support \([0, \tilde{x}^*_t]\) yields the following:

\[
\Gamma^*_t = \frac{(a - c)^2}{2\tau \gamma}
\]

\(^{27}\)We perform the case of \(a, c\) and \(f\). It is easy to use the same method for \(\gamma, b\) and \(\tau\).
This also implies that \( \sqrt{\frac{\pi_t^*+f}{b}} = \frac{(a-c)^2\sqrt{\pi_{t-1}^*+f}}{2\tau \gamma} \). By iteration, we finally find \( \sqrt{\frac{\pi_t^*+f}{b}} = \)

\[
\frac{(a-c)^2\Gamma^*_1}{2\tau \gamma} + (t-1)\Gamma^*_1
\]

that is,

\[
\pi_t^* = b \left[ \frac{(a-c)^2\Gamma^*_1}{(a-c)^2 + 2\tau \gamma(t-1)\Gamma^*_1} \right]^2 - f
\]

Proof 5 We have:

\[ q^*_0(x) = \frac{2b(a - c - \tau x) + (1 - 2x)\gamma \tau \mu_1(x)}{2b[2b + \gamma \mu_0(x) + \gamma \mu_1(x)]} \]

and

\[ q^*_1(x) = \frac{2b[a - c - \tau(1 - x)] - (1 - 2x)\gamma \tau \mu_0(x)}{2b[2b + \gamma \mu_0(x) + \gamma \mu_1(x)]} \]

An equilibrium depict in Figure 6.b exists if and only if:

\[ \frac{a - c - \tau x}{2b + \gamma \mu_0(x)} = \frac{2b(a - c - \tau x) + (1 - 2x)\gamma \tau \mu_1(x)}{2b[2b + \gamma \mu_0(x) + \gamma \mu_1(x)]} \]

and

\[ \frac{a - c - \tau(1 - x)}{2b + \gamma \mu_1(x)} = \frac{2b[a - c - \tau(1 - x)] - (1 - 2x)\gamma \tau \mu_0(x)}{2b[2b + \gamma \mu_0(x) + \gamma \mu_1(x)]} \]

which is impossible since \( \mu_0, \mu_1 > 0 \). Thus, the only equilibrium that arises is the one displayed in Figure 6.a with

\[ q^*_0(x) = \frac{a - c - \tau x}{2b + \gamma \mu_0(x)} \]

and

\[ q^*_1(x) = \frac{a - c - \tau(1 - x)}{2b + \gamma \mu_1(x)} \]

Following Proof 1, we naturally obtain: \( \mu^*_0(x) = \mu^*(x) \) and \( \mu^*_1(x) = \mu^*(1 - x) \).

For Proofs 6 and 7, we follow and adapt Cardaliaguet (2012) and Cirant (2014).

Proof 6 In what follows, let us assume that \( Y = \mathcal{M}(X) \) and let \( \Gamma : Y \times Y \to 2^Y \times 2^Y \) be defined by: \( \Gamma_1(\mu_1, \mu_2) = \arg \max_{\mu_1 \in Y} \int_X A_1(x, \mu_1(x), \mu_2(x))d\mu^*_1(x) \) and by symmetry \( \Gamma_2(\mu_1, \mu_2) = \arg \max_{\mu_2 \in Y} \int_X A_2(x, \mu_1(x), \mu_2(x))d\mu^*_2(x) \). \( \Gamma \) is upper-semicontinuous multi-application with convex compact values. Furthermore, \( Y \times Y \) is a convex compact set of a locally convex Hausdroff space. This implies that, by Ky Fan fixed point theorem, \( \Gamma \) admits a fixed point, that is to say, \( \exists (\mu^*_1, \mu^*_2) \in Y \times Y \) such that for \( k \in \{1, 2\}, \int_X A_k(x, \mu^*_1(x), \mu^*_2(x))d\mu^*_k(x) = \sup_{\mu \in Y} \int_X A_k(x, \mu_1(x), \mu_2(x))d\mu(x) \). \( \diamond \)

Proof 7 The proof is performed in two steps.

We prove that if \( \int_X \sum_{k=1}^2 A_k(x, \mu_1^*(x), \mu_2^*(x)) - A_k(x, \tilde{\mu}_1(x), \tilde{\mu}_2(x))d(\mu_k^* - \tilde{\mu}_k)(x) < 0 \) for all \( \mu^*_1, \mu^*_2, \tilde{\mu}_1, \tilde{\mu}_2 \in \mathcal{M}(X) \), then, there is at most a vector \( (\mu^*_1, \mu^*_2) \in \mathcal{M}(X)^2 \) satisfying Definition 3. Assume that \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \) satisfy the above condition. From Definition 3, it comes that \( \int_X A_1(x, \mu^*_1(x), \mu^*_2(x))d\mu^*_1(x) \geq \int_X A_1(x, \tilde{\mu}_1(x), \tilde{\mu}_2(x))d\tilde{\mu}_1(x) \) and (by symmetry) \( \int_X A_1(x, \tilde{\mu}_1(x), \tilde{\mu}_2(x))d\tilde{\mu}_1(x) \geq \int_X A_1(x, \mu^*_1(x), \mu^*_2(x))d\mu^*_1(x) \). By subtracting both
equations above, we obtain \( \int_X A_1(x, \mu_1(x), \mu_2(x)) - A_1(x, \bar{\mu}_1(x), \bar{\mu}_2(x)) d(\mu_1 - \bar{\mu}_1)(x) \geq 0 \).

By the same token, we get \( \int_X A_2(x, \mu_1(x), \mu_2(x)) - A_2(x, \bar{\mu}_1(x), \bar{\mu}_2(x)) d(\mu_2 - \bar{\mu}_2)(x) \geq 0 \).

This implies that \( \mu_1^* = \bar{\mu}_1 \) and \( \mu_2^* = \bar{\mu}_2 \).

Second, using Lagrange Theorem and assuming that \( A_1 \) and \( A_2 \) are differentiable, it appears that a sufficient condition for uniqueness is that \( A(x, \mu_1, \mu_2) + A(x, \mu_1, \mu_2)^T \) is negative definite with:

\[
A(x, \mu_1, \mu_2) = \begin{pmatrix}
    a_{11}(x) & a_{12}(x) \\
    a_{21}(x) & a_{22}(x)
\end{pmatrix}
= \begin{pmatrix}
    \partial A_1(x, \mu_1(x), \mu_2(x)) / \partial \mu_1(x) & \partial A_1(x, \mu_1(x), \mu_2(x)) / \partial \mu_2(x) \\
    \partial A_2(x, \mu_1(x), \mu_2(x)) / \partial \mu_1(x) & \partial A_2(x, \mu_1(x), \mu_2(x)) / \partial \mu_2(x)
\end{pmatrix}
\]

and with \( A(x, \mu_1, \mu_2)^T \) being the transpose of the matrix \( A(x, \mu_1, \mu_2) \).

\textbf{Proof 8} Coupling (56)-(59) and after simple computations, we end up with:

\[
\begin{align*}
A\mu_1^*(x) &= a - \frac{c_2 - c_1}{\gamma_1^*} x + \frac{(c_2 - c_1)(a - c_2 - \tau x)}{2b^* \Gamma_2^*} - \frac{c_2 - c_1}{\gamma_1^*} - \frac{a - c_2 - \tau x}{\gamma_1^*} \\
A\mu_2^*(x) &= a - \frac{c_2 - c_1}{\gamma_1^*} x - \frac{(c_2 - c_1)(a - c_2 - \tau x)}{2b^* \Gamma_2^*} + \frac{c_2 - c_1}{\gamma_1^*} - \frac{a - c_2 - \tau x}{\gamma_1^*}
\end{align*}
\]

where \( A = c_2 - c_1 - 2b(\Gamma_1^* - \Gamma_2^*) \).

Our goal is to determine the conditions under which firms with a lower marginal cost serve more distance markets (on average) than firms with a higher marginal cost, that is, \( \mu_2^*(0) > \mu_1^*(0) \), \( \frac{\partial \mu_2^*(x)}{\partial x} < \frac{\partial \mu_1^*(x)}{\partial x} \) and \( x_2^* < x_1^* \). Due to linearity of \( \frac{\partial \mu_2^*(x)}{\partial x} \) and \( \frac{\partial \mu_1^*(x)}{\partial x} \), it is sufficient to prove \( \mu_2^*(0) > \mu_1^*(0) > 0 \) and \( 0 < x_2^* < x_1^* \).

Evaluating the first equation of the system at \( x = 0 \), we easily find that \( \mu_1^*(0) > 0 \) if \( c_2 - c_1 - 2b(\Gamma_1^* - \Gamma_2^*) > 0 \iff \ A > 0 \). Similarly, evaluating \( \mu_2^* \) at \( x = 0 \), we derive that \( \mu_2^*(0) > \mu_1^*(0) \) if \( 2b[(a - c_1)\Gamma_1^* - (a - c_2)\Gamma_2^*] - (c_1 - c_2)(2a - c_1 - c_2) > 0 \). We now turn to the condition concerning thresholds countries. In what follows, we assume that \( A > 0 \).

Using the second equation of the system and after simple algebra, the threshold country for \( c_2 \)-type firms is \( \tilde{x}_2^* = \frac{a - c_1}{\tau} - \frac{2b(a - c_1)\Gamma_2^*}{2b(a - c_2)\Gamma_2 - (c_2 - c_1)} \). Likewise, we find \( \tilde{x}_1^* = \frac{a - c - 2b\Gamma_1^*}{\tau} \) for \( c_1 \)-type firms. Then, we derive the following conditions \( \tilde{x}_2^* > 0 \) if \( 1 > \frac{2b\Gamma_1^*}{2b(a - c_2)\Gamma_2^* - (c_2 - c_1)} \) and \( \tilde{x}_1^* > \tilde{x}_2^* \) if \( \Gamma_1^* > \frac{(a - c_1)\Gamma_2^*}{2b(a - c_2)\Gamma_2^* - (c_2 - c_1)} \).

Combining all the conditions, we easily find:

\[
c_1 + b\Gamma_1 < \frac{c_1 + c_2}{2} < a + b\left[ \frac{c_1\Gamma_1 + (a - c_2)\Gamma_2}{c_2 - c_1} \right]
\]

\( \Box \)

\section*{B Stylized Facts (cont’d)}

\subsection*{B.1 New Exporters and Number of Destinations}

Tables 4 and 5 report the proportion of French exporters according to the number of destinations they serve. For instance, Table 4 shows that among new French manufacturing

\footnote{We assume that \( \Gamma_1, \Gamma_2^* > 0 \)}
exporters in 1995, 82.03 % of them enter a unique foreign market, whereas only 7.50 % of them enter more than three countries. Tables 4 and 5 confirms the results of Albornoz et al. (2012): a substantial share of new exporters enters just one destination.

<table>
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<tr>
<td>1</td>
<td>82.03 %</td>
<td>81.66 %</td>
<td>80.80 %</td>
<td>81.55 %</td>
<td>81.15 %</td>
<td>81.64 %</td>
<td>82.16 %</td>
</tr>
<tr>
<td>2</td>
<td>10.47 %</td>
<td>10.73 %</td>
<td>11.22 %</td>
<td>10.41 %</td>
<td>11.42 %</td>
<td>10.43 %</td>
<td>10.70 %</td>
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<tr>
<td>3+</td>
<td>7.50 %</td>
<td>7.61 %</td>
<td>7.97 %</td>
<td>8.03 %</td>
<td>7.43 %</td>
<td>7.93 %</td>
<td>7.14 %</td>
</tr>
</tbody>
</table>

Table 4: New Exporters and Number of Destinations: 1995-2001

<table>
<thead>
<tr>
<th>Nb.</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
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<tr>
<td>1</td>
<td>83.32 %</td>
<td>82.62 %</td>
<td>82.13 %</td>
<td>82.23 %</td>
<td>83.02 %</td>
<td>82.68 %</td>
<td>81.59 %</td>
</tr>
<tr>
<td>2</td>
<td>9.59 %</td>
<td>10.04 %</td>
<td>10.55 %</td>
<td>10.36 %</td>
<td>9.93 %</td>
<td>10.24 %</td>
<td>11.01 %</td>
</tr>
<tr>
<td>3+</td>
<td>7.08 %</td>
<td>7.34 %</td>
<td>7.31 %</td>
<td>7.41 %</td>
<td>7.03 %</td>
<td>7.07 %</td>
<td>7.04 %</td>
</tr>
</tbody>
</table>

Table 5: New Exporters and Number of Destinations: 2002-2008

### B.2 Destination Errors, Deviation from Melitz and Experience as an Exporter

In this section, we give the results for the average number of destination errors and the deviation from Melitz by experience as an exporter and export string. For example, Figure 10 shows that firms that had one-year of experience as exporters to the US in 2008 make, on average, 5.25 errors, which represents a deviation of 75.06 % from the Melitz prediction, whereas firms that had more than 14 years of experience as exporters only make 1.39 errors, which represents a deviation of 19.97 % from the benchmark. Figures 10-16 confirm the results of Section 2.2: firms that experience a longer spell of exporting are more likely to be ordered in terms of destination choices. However, the results are less pronounced for destinations in the top destinations (Belgium or Germany) than in the bottom (the Netherlands and the USA).
Figure 10: Destination Errors, Deviation from Melitz and Experience as an Exporter: US

Figure 11: Destination Errors, Deviation from Melitz and Experience as an Exporter: Netherlands
Figure 12: Destination Errors, Deviation from Melitz and Experience as an Exporter: United Kingdom

Figure 13: Destination Errors, Deviation from Melitz and Experience as an Exporter: Italy
Figure 14: Destination Errors, Deviation from Melitz and Experience as an Exporter: Switzerland

Figure 15: Destination Errors, Deviation from Melitz and Experience as an Exporter: Spain
Figure 16: Deviation from Melitz and Experience as an Exporter: Germany