Immigration, Offshoring, Tasks and Technology Adoption: Implications for Natives Wage Structure*

Ehsan Vallizadeh†

IAB and Maastricht University

July 13, 2015
Preliminary

Abstract

This paper addresses two main concerns regarding the effects of globalization: offshoring and immigration. A theoretical model is developed that is characterized by firm heterogeneity due to technology adoption, monopolistic competition, skill heterogeneity, and task allocation. Changes in natives wage structure induced by immigration and offshoring can be decomposed into three key channels: i) a market size effect, ii) an productivity effect; iii) a technology adoption effect. The analysis reveals two novel insights. First, easier degree of technology adoption mitigates the market size effects, while the productivity effect remains. Second, both immigration of medium-skilled workers and offshoring of medium skill-intensive jobs lead to a polarizing wage effect for native workers whenever the task shares assigned to immigrants or levels of offshoring rate are high.

Keywords Offshoring · Immigration · Skills · Task Assignment · Technology Adoption

JEL F16 · F22 · F66 · O33

*I am grateful for valuable comments and discussions to Joan Muysken and Thomas Ziesemer. I would also like to thank seminar participants at TASK III - the 3rd International BIBB/IAB/ZEW conference in Nuremberg.

†Institute for Employment Research (IAB), Research Department B1, Weddigenstr. 20 - 22, 90478 Nürnberg, Germany. ehsan.vallizadeh@iab.de, T: +49 911 179 78 36.

1 Introduction

Immigration and the relocation of jobs abroad by multinationals, i.e. offshoring of domestic jobs, are often blamed for adverse labor market outcomes for native workers. Yet, the empirical findings on the impact of these two aspects of globalization of labor services on wages and employment of native workers are ambiguous. The literature provides several explanations for this. On the one hand, there is a *labor supply effect*, indicating the direct displacement effect due to increasing labor market competition. On the other hand, there are several indirect channels which may offset the direct adverse effect.

Recent studies have emphasized one salient feature associated with a cost-efficiency enhancing effect of due to reallocation of jobs (tasks) from natives to immigrants and to offshore workers (cf. Grossman and Rossi-Hansberg, 2008; Ottaviano et al., 2013). This refers to the so-called *productivity effect*. The intuition is that native workers have a comparative advantage in performing some job tasks, e.g. complex and communication intensive tasks, which are usually complement to those tasks performed by immigrants, e.g. manual intensive tasks (cf. Peri and Sparber, 2009; Peri, 2012), as well as to those prone to offshoring, e.g. routine and easily codifiable tasks (cf. Grossman and Rossi-Hansberg, 2008). Thus, both types of labor supply shock lead to a more efficient allocations of skills to tasks. Another mechanism proposed in the literature that can explain the positive productivity effect of immigrant-induced skill supply shock is through directed technological changes (cf. Acemoglu, 2002) or the adoption of production techniques (cf. Beaudry et al., 2010). The idea is that immigration-induced increase in some production factors lead firms to adopt production technologies that are more efficient and intensive in the use of that factor (cf. Dustmann and Glitz, 2012; Gauthier-Loiselle and Hunt, 2008; González and Ortega, 2011; Lewis, 2003).[1] Thus, the productivity gains are associated with task specialization by natives and the choice of appropriate techniques by firms.

Thus, understanding the mechanisms that explain the magnitude between these various channels and their distributional implications from a general equilibrium perspective is the objective of this paper. As a first attempt, in Vallizadeh (2015), I have developed a theoretical model to analyze the distributional effect of immigration-induced changes in the skill mix by accounting for endogenous technology adoption behavior. The core analytic result reveals the importance of the degree of technology adoption which counteracts the direct market size effect, associated with higher labor market competition. More precisely, it is shown that as the degree of adoption across different technology modes becomes very easy, the technology adoption effect balances the labor supply effect, leaving the domestic relative wage structure insensitive to immigration shocks. Moreover, the direction of these two channels depends significantly on the interaction between consumer preferences (i.e. goods demand elasticity) and market power of monopolists (i.e. the elasticity of substitution between intermediates). While this novel result is able to reconciles the empirical findings focusing on spatial variation of skills and workers, it omits, however, to address recent empirical observations, emphasizing the importance of the internal adjustment process of reorganization of work. This accentuate the need for an alternative paradigm that permits, for example, to account for the differential specialization pattern between immigrants and natives, or between jobs produced at home and abroad.

Thus, this paper provides an integrated theoretical framework that allows to jointly investigate the distributional impact of immigration and offshoring and to analyze the adjustment mechanisms and their under-

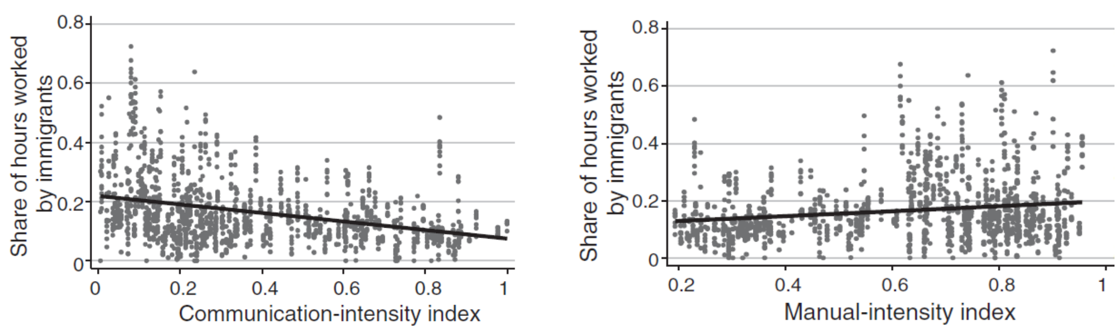
[1] In a recent study Ghosh et al. (2014) examine the impact of changes in the H-1B cap on the performance of US firms. They find that relaxing the cap on the H-1B visas would lead to gains in average labor productivity, firm size, and profits for firms that contact heavily R&D.

lying determinants. In doing so, I augment the framework in Vallizadeh (2015) with two important respects. First, I allow for differential task-specialization patterns between immigrants and natives, where task allocation is determined endogenously according to comparative advantages. Second, offshoring opportunities are introduced, where domestic firms can gain in efficiency regarding production costs by relocating and producing a fraction of domestic jobs abroad. In doing so, I develop a model that accounts for a rich set stylized facts. More precisely, I build on Vallizadeh (2015), which provides a tractable general equilibrium framework featuring various important stylized facts including endogenous technology adoption, skill and firm heterogeneity, and monopolistic competition, and integrate in that model the task-based approach as in Grossman and Rossi-Hansberg (2008) and offshoring of intermediates as in Acemoglu et al. (2012).[2]

I develop these extensions in two steps. First and in line with the empirical observations, I impose that within each job-skill group tasks are ordered according to their communication, interpersonal intensity, so that immigrants specialize in low index tasks, while natives perform the higher indexed tasks. In fact, Ottaviano et al. (2013) show for the U.S. that immigrants are proportionately more in jobs with high content of manual tasks, and are underrepresented in occupations characterized by low intensity of communication and complexity content of tasks compared to natives, see Figure 1. As elaborated below, immigration-induced labor supply shock exhibits now a dynamic characteristic, captured by an endogenous specialization of natives in more communication intensive tasks. In the second step, offshoring is introduced in the form of relocation of jobs (intermediate goods), similar to models of international fragmentation of the production process along the value added chain, or trade in intermediate inputs (cf. Feenstra and Hanson, 1999; Jones and Kierzkowski, 2001; Kohler, 2004; Rodríguez-Clare, 2010).

Figure 1: Immigrants and task specialization

(a) Immigrants and communication intensity (b) Immigrants and manual intensity



Notes: Communication intensity measures the ability regarding written and oral expression and understanding; manual intensity measures the ability regarding dexterity, strength, and coordination. The slope of the regression line (standard error) in Panel (a): -0.14 (0.01), in Panel (b): 0.087 (0.01). Source: Ottaviano et al. (2013).

The important implication of the model is that due to endogenous changes in the mass of monopolists profit-maximizing incentives generate externalities in the form of increasing returns to scale. More interestingly, both immigration and offshoring generate a *direct* market size effect, raising the mass of firms employing those factors most intensively, and an *indirect* scale effect, captured by an expansion of the range

[2]The task-based approach is akin to the one used in Groizard et al. (2014) in the context of offshoring, and allows for imperfect substitutability between tasks over a continuum unit interval. Thus, it generalizes the original specification of perfect complementarity in Grossman and Rossi-Hansberg (2008).

of final goods due to the endogenous technology adoption of complementing production modes. However, this indirect market size effect tends to reduce the direct scale effect by increasing the relative price of goods produced under the competing technology modes.[3]

The key feature of this approach is that it reveals two convenient structural properties. First, changes in natives wage structure induced by immigration and offshoring can be decomposed into three key channels: (i) a technology adoption effect, (ii) a market size effect, and (iii) a productivity effect. The first two channels are akin to those derived in Vallizadeh (2015), and as discussed there they counteract each other as a result of direct and indirect scale effects caused by market externalities. Moreover the relative magnitude between these two forces depend on the degree of adoption between different technology modes, while the direction of each of these effects is determined by the relative strength between goods demand elasticity and the market power of monopolists. Thus, the third channel is the new economics channel, which remains whenever technology adoption becomes approximately very easy. This highlights the crucial departure from models which rely on the static and implicit assumption of immigration (cf. Vallizadeh, 2015). Second, the relative magnitude between productivity and labor supply effects depends crucially on the initial levels of the task share performed by immigrants and of offshoring rate. At initial low levels, the productivity effect becomes unambiguously the dominating force.

The comparative statics analysis uncovers several novel predictions regarding the impact of immigration on natives wage structure. In the empirically more relevant case where the market power of monopolists (i.e. the elasticity of substitution between intermediates) is larger than the goods demand elasticity, low-skill (high-skill) immigration induces a wage effect similar to *unskill-biased* (*skill-biased*) technological change, associated with a monotonic decline (increase) in the skill premium across the skill groups, while medium-skill immigration induces a hump-shaped wage effect, i.e. an increase in the medium-skilled wage relative to both low- and high-skilled wages, whenever the level of tasks assigned to immigrants is relatively low. The opposite pattern is obtained for at low levels of tasks assigned to immigrants.

Moreover, the results of the impact of offshoring on natives wage structure reveal two key implications. First, offshoring of medium skill-intensive jobs generate similar wage effects to medium-skilled immigration. Particularly, offshoring-induced changes in natives relative wages is characterized by a U-shaped relationship, i.e. a relative decline in medium-skilled wages relative to low- and high-skilled wages, whenever the level of offshoring rate is high, while the converse is true at low levels of the offshoring rate. However the underlying determinants differ to medium-skilled immigration. The key feature in the case of offshoring is that the productivity effect is more pronounce when the wage gap between the source and host countries of offshoring is large (i.e. little offshoring), while in the case of medium-skilled immigration, productivity gains are obtained due to infra-marginal cost-savings, i.e. a decline in the factor price (wages) of the whole range of tasks performed initially by immigrants. In addition, the extent of offshoring-induced productivity effect is also importantly affected by the degree of technology adoption. Intuitively, a high degree of technology adoption rate tends to reduce the magnitude of both direct and indirect market size effects. This is the crucial departure to the results discussed in Acemoglu et al. (2012), who use a similar model to analyze the effects of offshoring on technical change and skill premium.

Although this paper does not account for other important factors like endogenous skill supply, labor

[3]Notice that this feature of direct and indirect scale effects discussed in this paper has a similar characteristics to the internal and external externalities discussed by Ethier (1982), but differs substantially in terms of the context and its determinants.

market frictions, or institutional factors, the results discussed above highlight the rich pattern of interacting adjustment mechanisms and are consistent with the broad empirical observations of various points in time registering changes in the wage structure in many advanced over the last decades.

The rest of the paper is organized as follows. In the next section, the related literature is reviewed. The basic setup of the theoretical framework together with the elaborating discussion of task-based approach are presented in section 3. The characteristics of the equilibrium are illustrated in section 4, followed by the comparative static analysis of the impact of immigration on natives wage structure in section 5. The extension of the model with respect to offshoring medium skill-intensive jobs and the analysis of its distributional effect for native workers are elaborated in section 6. Finally, concluding remarks are provided in section 7.

2 Related literature

There exists a large body of studies that have sought to analyze the labor market impacts of immigration and offshoring. This paper contributes to different strands in the literature. First, it is a contribution to a growing but still small literature addressing jointly the income an employment effects of immigration and offshoring (cf. Olney, 2012; Ottaviano et al., 2013; Rodríguez-Clare, 2010). The main point of departure to these studies is the endogenous adoption behavior between different production techniques. For example, guided by the theoretical task-based approach à la Grossman and Rossi-Hansberg (2008), Olney (2012) and Ottaviano et al. (2013) analyze empirically the direct and indirect effects of offshoring and immigration for the United States. Imposing implicitly perfect substitutability between immigrants and natives, Olney (2012) finds opposite effects where immigration hurts low-skilled native workers through higher competition, while offshoring induces a positive demand effect for the domestic low-skilled labor, driven by the indirect efficiency improvements in production process. Contrary, using the general task-assignment model of Acemoglu and Autor (2011), Ottaviano et al. (2013) allow immigrants and natives to specialize in different tasks, and find that immigration and offshoring both generate efficiency effects for native workers.[4]

Second, this paper is also contributing to the large literature on the theoretical determinants of offshoring- and technology-induced changes in the wage structure (cf. Acemoglu and Autor, 2011; Acemoglu et al., 2012; Acemoglu and Zilibotti, 2001; Egger et al., 2013; Grossman and Rossi-Hansberg, 2008).[5] Closely related to this paper is the study by Acemoglu et al. (2012), who investigate the impact of offshoring on directed technological changes and skill premium. The results in this paper share similar features to their analytical findings. For instance, they show that offshoring induces an efficiency effect, and its magnitude depends on the level of the offshoring rate. The offshoring-induced changes in the wage structure are the results of endogenous technological changes, where the direction of this effect is crucially affected by the interaction between goods demand elasticity and the elasticity of substitution between intermediates. However, the structure and the underlying determinants of the mechanism differ in three main respects from their approach. First, they consider only a dichotomous skill distribution and thus miss to address recent observations regarding

[4] Rodríguez-Clare (2010) examines the offshoring-induced interaction between productivity and term-of-trade effects in a two-country model. He shows that higher fragmentation shifts workers to the research sector. This effect leads, in the short-run, to a decline in the real wages due a dominating negative terms-of-trade effect for the offshoring country, while, in the long run, wages rise due to a productivity gain following higher stock of ideas per worker. Moreover, immigration induces a net gain effect only for immigrants, since they earn now higher wages in the offshoring country, while offshoring generates benefits for firms in offshoring countries due to lower unit costs.

[5] Other recent contributions studying the wage effects of offshoring are Baldwin and Robert-Nicoud (2007); Costinot and Vogel (2010).

polarizing wage rends. Second, this paper allows for firm heterogeneity, and thus accounts for important implications of technology adoption effects. A crucial feature that is missing in their model. Third, in their model the determinant of offshoring-induced endogenous technological changes is explained a biased price effect induced by a direct and a complementarity market size effect, where the latter is associated a the extent of low values of elasticity of substitution between intermediates. However, in this paper endogenous changes in technology margins driven by technology adoption behavior induces an indirect scale effect interacting with the direct market size effect induces a biased goods price (and thus factor price) effect favoring the competition technology modes.

It is also worth mentioning the recent study by Egger et al. (2013), who analyze in a two-country model the welfare and distributional impacts of offshoring by highlighting the importance of firm heterogeneity à la Melitz (2003). They show that offshoring generates a non-monotonic income distribution effect in the share of offshoring firms. The intuition is that at initial high levels of variable offshoring costs, the share of offshoring firms is low, and a marginal decline in offshoring cost induces an unfavorable shift of domestic labor from more to low productive firms, inducing a deteriorating welfare effect (in terms of income per capita). However, this result highlights the efficiency gains at the intensive margin, thus contrast the determinants of offshoring in this paper as highlighted by the fact that arbitrage from offshoring is highest at low levels of offshoring rate, associated to high wage gaps between domestic and foreign countries.

Finally, this paper is also related to the growing literature on the role of task content of jobs and the determinants of wage polarization. A number of these studies has emphasized the key role of information and communication technology (ICT) behind the recent wage polarization (cf. Autor and Dorn, 2013; Autor et al., 2006; Michaels et al., 2014). Another set of studies provide a competing explanation by emphasizing the role of offshoring domestic jobs as the key determinant behind this trend (cf. Goos and Manning, 2007; Goos et al., 2014; Vallizadeh et al., 2015). Particularly, the recent study by Vallizadeh et al. (2015) investigate the income and employment impact of offshoring using a generalized task-based approach of Acemoglu and Autor (2011) and find similar distributional effects of offshoring to those of immigration of various skill groups discussed in this paper.

3 The Model

In the next section, I commence with the description of the basic structure of the model in Vallizadeh (2015), while leaving formal details implicit for the stake of convenience. However, I will make them explicit when highlighting the differences. Then, the following sections elaborate the details of the extension with respect to task allocation between immigrants and natives and to offshoring opportunities.

3.1 Basic environment

The economy is described by a continuum of final goods $y(z)$ combined over the unit interval according to the following CES production function to produce a final consumption aggregate over

$$Y = \left[\int_0^{z_L} y_L(z)^{\frac{\sigma-1}{\sigma}} \mathrm{d}z + \int_{z_L}^{z_H} y_M(z)^{\frac{\sigma-1}{\sigma}} \mathrm{d}z + \int_{z_H}^{1} y_H(z)^{\frac{\sigma-1}{\sigma}} \mathrm{d}z\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where final goods are indexed $z \in [0,1]$ in increasing order of production technology productivity, and $\sigma \geq 1$ denotes the elasticity of substitution between final good varieties.

Final goods market

The final goods market is perfectly competitive and each final good z can be produced in three different technology modes: low- $(\varphi_L(z))$, medium- $(\varphi_M(z))$, and high-quality $(\varphi_H(z))$ techniques. It follows that each technology mode is used to produce a distinct range of final goods

$$y(z) = \begin{cases} \varphi_L(z)X_L & \text{for all} \quad z \in [0, z_L], \\ \varphi_M(z)X_M & \text{for all} \quad z \in (z_L, z_H), \\ \varphi_H(z)X_H & \text{for all} \quad z \in [z_H, 1]. \end{cases} \tag{2}$$

where X_k is a composite of intermediate goods, for $k = \{L, M, H\}$. The respective price schedule is given by

$$p(z) = \begin{cases} P_L\varphi_L(z)^{-1} & \text{for all} \quad z \in [0, z_L], \\ P_M\varphi_M(z)^{-1} & \text{for all} \quad z \in (z_L, z_H), \\ P_H\varphi_H(z)^{-1} & \text{for all} \quad z \in [z_H, 1], \end{cases} \tag{3}$$

where P_k denotes the effective price index of final goods, i.e. the price per productive technology k, which will be defined below.

The choice of technology mode is based on the Ricardian type of comparative advantages, where final good firms self-select endogenously into the most cost-efficient production technique. As discussed in Vallizadeh (2015), the equilibrium is characterized by two endogenous final goods margins, z_L and z_H, where final good firms are indifferent between using low- and medium-quality techniques and between medium- and high-quality techniques, respectively. These conditions are given by

$$\frac{P_L}{P_M} = \Lambda_L(z_L), \tag{4}$$
$$\frac{P_M}{P_H} = \Lambda_H(z_H), \tag{5}$$

where $\Lambda_L(z_L) \equiv \frac{\varphi_L(z_L)}{\varphi_M(z_L)}$ and $\Lambda_H(z_H) \equiv \frac{\varphi_M(z_H)}{\varphi_H(z_H)}$, and both are continuous and decreasing in their arguments, see Vallizadeh (2015) for formal details and discussion.

Then, by means of profit maximization subject to Eq. (1), the demand for any final good variety z is given by

$$y_k(z) = Yp_k(z)^{-\sigma}, \text{ for } k = \{L, M, H\} \tag{6}$$

where the price of aggregate consumption good is set to unity, i.e. $P_Y = 1$.

Intermediate goods market

In contrast to the final goods market, the intermediate goods market is characterized by monopolistic competition, where each intermediates, in turn, is manufactured by factor labor, denoted by $\tilde{L}_k(j)$ for variety j. Moreover, each intermediate variety requires a fixed cost (measured in terms of the numeraire good), where the size depends on the adopted technology mode, i.e. $f_L < f_M < f_H$. The production technology of the composite good X_k is represented by a CES aggregator over a continuum of intermediate varieties, indexed by j:

$$X_k = \left(\int_0^{A_k} x_k(j)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{7}$$

where $x(j)$ denotes the amount of intermediate variety $j \in [0, A_k]$, $\epsilon \geq 1$ captures the elasticity of substitution between varieties, and A_k measures the mass of monopolist in technology mode k.[6]

By means of cost minimization subject to Eq. (7), the demand for intermediate good j is given by:

$$p_k(j) = \zeta_k X_k^{\frac{1}{\epsilon}} x_k(j)^{-\frac{1}{\epsilon}}, \; \forall \, k = \{L, M, H\}, \tag{8}$$

where $p_k(j)$ is the price of intermediate variety j produced with technology mode k. Combining (8) with (7) yields the marginal cost of composite intermediate goods

$$\zeta_k = \left(\int_0^{A_k} p_k(j)^{1-\epsilon} \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}. \tag{9}$$

Under the constant elastic demand function (8) of degree $\epsilon > 1$, the profit maximizing monopolist sets the price equal to a markup $\epsilon/(\epsilon - 1)$ over the respective marginal cost, i.e.

$$p_k(j) = \frac{\epsilon}{\epsilon - 1} \tilde{w}_k, \tag{10}$$

where $\tilde{w}_k$ denotes the marginal cost of k-type factor labor, defined below.

3.2 Task allocation between immigrants and natives

As discussed above, recent empirical findings have highlighted the differential comparative advantages in job task performance between immigrants and natives, where immigrants tend to perform on average low communicative type of tasks and natives tend to specialize in communication- and complex-intensive tasks (Ottaviano et al., 2013; Peri, 2012; Peri and Sparber, 2009). This has been put forth as the key rationale behind the imperfect substitutability between immigrants and natives, inducing a productivity effect for native workers associated with the efficient pattern of task specialization. I build on Grossman and Rossi-Hansberg (2008) to design a task assignment model characterized by a continuum of tasks over the unit interval. While natives in each skill group k are homogeneous in terms of their communication abilities, the task productivity among immigrants is more heterogeneous, leading to specialization of immigrants in low communicative tasks.

[6] For a discussion on the degree of external increasing returns to scales and the relationship between composite intermediate good X_k and mass of monopolists A_k, see Vallizadeh (2015).

More precisely, the factor labor $\tilde{L}_k$ utilized by each intermediate variety $x_k(j)$, is denoted by composite of a continuum of tasks, indexed $i \in [0,1]$ in increasing order of communication, inter-personal intensity. If a task i is assigned to immigrants ($d = F$), its unit input requirement is $\gamma_k(i) \geq 1$, where $\gamma_k(i)$ is continuously increasing in i. The marginal cost associated with tasked performed by immigrants is denoted by w_k^F. The unit input requirement using natives ($d = D$) is 1, with the marginal cost w_k^D. Following the recent empirical studies highlighting the increasing importance of interactive, communication-intensive tasks in workplaces in the modern economy (cf. Borghans et al., 2008, 2013; Spitz-Oener, 2006; Stasz, 1997), I assume that the compensation of communication-intensive tasks is higher relative to non-communication tasks, i.e. $w_k^D > w_k^F$ for all $k = \{L, M, H\}$. An intermediate-goods producer $j \in [0, A_k]$ will assign tasks to immigrants up to I_k, where $I_k \in (0,1)$, so that the composite labor input is produced according to the following CES technology:

$$\tilde{L}_k(j) = \left(\int_0^{I_k} \left(\frac{l_k^F(i)}{\gamma_k(i)} \right)^{\frac{\alpha-1}{\alpha}} \mathrm{d}i + \int_{I_k}^1 \left(l_k^D(i) \right)^{\frac{\alpha-1}{\alpha}} \mathrm{d}i \right)^{\frac{\alpha}{\alpha-1}}, \tag{11}$$

where $\alpha \geq 1$ denotes the elasticity of substitution between tasks. The following lemma establishes the optimal conditions regarding the fraction of tasks performed by immigrants and natives

Lemma 1. *Let L_k^D and L_k^F denote the amount of native and immigrant workers, respectively, which are employed by differentiated-good producer $j \in [0, A_k]$, for $k = \{L, M, H\}$. Then, it follows*

$$\tilde{L}_k(j) \quad = \quad \left(\vartheta_k(I_k)^{\frac{1}{\alpha}} \left(L_k^F\right)^{\frac{\alpha-1}{\alpha}} + [1 - I_k]^{\frac{1}{\alpha}} \left(L_k^D\right)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}}, \tag{12}$$

where $\vartheta_k(I_k) \equiv \left(\int_0^{I_k} \gamma_k(i)^{1-\alpha} \mathrm{d}i \right)$, and the optimality condition for the marginal task I_k is denoted by

$$w_k^D = w_k^F \gamma_k(I_k). \tag{13}$$

Proof. See Appendix A.1.

Having defined the structure of the economy and the optimality conditions, I proceed now with the discussion of the characteristics of the equilibrium followed by the comparative statics.

4 Characteristics of equilibrium

The characteristics of the equilibrium are similar to Vallizadeh (2015), except for the labor market clearing conditions and related variables, which require now the market clearing for both immigrants and natives in every skill group. In a standard symmetric equilibrium, the varieties of the differentiated good X_k employ the same amount of factor labor $\tilde{L}_k(j) = \tilde{L}_k$, are sold at the same price $p_k(j) = p_k$ and are produced in equal amount $x_k(j) = x_k$, for all $k = \{L, M, H\}$.

Free entry: Free entry ensures that each intermediate-good firm makes zero profits so that the quantity for each variety is defined by

$$x_k = (\epsilon - 1) f_k, \quad \forall \quad k = \{L, M, H\}. \tag{14}$$

Marginal costs of intermediate goods and price index: Next, using the monopolistic pricing behavior, Eq. (10), the marginal cost of the differentiated good ζ_k, defined in Eq. (9), simplifies to

$$\zeta_k = A_k^{\frac{1}{1-\epsilon}} p_k = \frac{\epsilon}{\epsilon - 1} A_k^{\frac{1}{1-\epsilon}} \tilde{w}_k, \;\; \forall\, k = \{L, M, H\}, \tag{15}$$

where the margin cost is defined by $\tilde{w}_k = \left(\vartheta_k(I_k)\left(w_k^F\right)^{1-\alpha} + [1 - I_k]\left(w_k^D\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$ (see Appendix A.1 for a formal discussion).

The perfectly competitive nature of the final-goods market requires that any final good producer makes zero profits, i.e. $p_k(z)y_k(z) = \zeta_k X_k$, for all $k = \{L, M, H\}$. Utilizing Eqs. (2) and (3), it follows that the price index of technology k must equal the marginal cost, i.e.

$$P_k = \zeta_k. \tag{16}$$

Resource constraint: Let the total amount of labor used in each set of technology $[0, A_k]$ be denoted by n_k^d, for $d = \{F, D\}$, and the total resource constraint (i.e. over the range of final goods produced with technology mode k) by

$$\int_{z \in \mathcal{S}_k} n_k^d \mathrm{d}z \;\; = \;\; N_k^d,$$

where $\mathcal{S}_L = \{0, z_L\}$, $\mathcal{S}_M = \{z_L, z_H\}$, and $\mathcal{S}_H = \{z_H, 1\}$. Then, by symmetry it follows

$$L_L^F = \frac{n_L^F}{A_L} = \frac{N_L^F}{z_L A_L}, \qquad L_L^D = \frac{n_L^D}{A_L} = \frac{N_L^D}{z_L A_L}, \tag{17}$$

$$L_M^F = \frac{n_M^F}{A_M} = \frac{N_M^F}{(z_H - z_L)A_M}, \qquad L_M^D = \frac{n_M^D}{A_M} = \frac{N_M^D}{(z_H - z_L)A_M}, \tag{18}$$

$$L_H^F = \frac{n_H^F}{A_H} = \frac{N_H^F}{(1 - z_H)A_H}, \qquad L_H^D = \frac{n_H^D}{A_H} = \frac{N_H^D}{(1 - z_H)A_H}. \tag{19}$$

Thus, the first equalities in Eqs. (17), (19), and (18) denote the resource usage by the intermediate-goods firms, while the second equalities reflect the overall utilization of factor labor by each final-goods producer in the economy so that N_L^d, N_M^d, and N_H^d are the total (exogenously given) endowments of labor.

Composite intermediate output: Next, combining the labor market conditions (17), (18), and (19), with Eq. (12) yields the equilibrium value of intermediate output

$$x_L = \frac{\tilde{N}_L}{z_L A_L}, \quad x_M = \frac{\tilde{N}_M}{(z_H - z_L)A_M}, \quad x_H = \frac{\tilde{N}_H}{(1 - z_H)A_H}, \tag{20}$$

where $\tilde{N}_k \equiv \left(\vartheta_k(I_k)^{\frac{1}{\alpha}}\left(N_k^F\right)^{\frac{\alpha-1}{\alpha}} + [1 - I_k]^{\frac{1}{\alpha}}\left(N_k^D\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}$, for all $k = \{L, M, H\}$.

Substituting the results from (20) into the composite intermediate goods function (7) for the respective technology mode, we get

$$X_L = A_L^{\frac{1}{\epsilon-1}} \frac{\tilde{N}_L}{z_L}, \quad X_M = A_M^{\frac{1}{\epsilon-1}} \frac{\tilde{N}_M}{z_H - z_L}, \quad X_H = A_H^{\frac{1}{\epsilon-1}} \frac{\tilde{N}_H}{1 - z_H}. \tag{21}$$

Mass of monopolists: Combining the budget constraint for any final good producer z,

$$p(z)y(z) \quad = \quad w_k^F \int_0^{A_k} L_k^F \mathrm{d}j + w_k^D \int_0^{A_k} L_k^D \mathrm{d}j,$$

with Eqs. (2), (3), (7), (14) and the resource constraints (17) (18), and (19), then the mass of monopolists in each technology mode k can be pinned down to the skill endowment, fixed costs, the range of final goods, and the market power of monopolists (see Appendix A.2 for details of the formal derivation).

$$A_L = \frac{\tilde{N}_L}{\epsilon f_L z_L}, \quad A_M = \frac{\tilde{N}_M}{\epsilon f_M (z_H - z_L)}, \quad A_H = \frac{\tilde{N}_H}{\epsilon f_H (1 - z_H)}. \tag{22}$$

Labor composite and marginal costs: Contrary to Vallizadeh (2015), there exists within-skill group heterogeneity due to differential task productivity between immigrants and natives. The convenient block recursive structure of the model allows to solve first for the equilibrium task margin I_k and simplify the labor composite and the marginal cost of factor labor as expressions of the task margin as well as of natives endowments and wages, respectively. In doing so, utilize Lemma 1 and labor market clearing conditions (18) to rewrite the composite factor labor and marginal cost of every intermediate-good producer using technology mode k. Given the equilibrium task margin $I_k > 0$, it follows (see Appendix A.3)

$$\tilde{N}_k \quad = \quad \Theta_k(I_k)^{-\alpha} \frac{N_k^D}{1 - I_k}, \tag{23}$$

$$\tilde{w}_k \quad = \quad \Theta_k(I_k) w_k^D, \tag{24}$$

where the generalized measure of marginal cost of tasks is denoted by

$$\Theta_k(I_k) \quad \equiv \quad \left[\vartheta_k(I_k)\gamma_k(I_k)^{\alpha-1} + 1 - I_k\right]^{\frac{1}{1-\alpha}} < 1. \tag{25}$$

These results indicate the key feature of the task-based approach characterizing the productivity gains due to task specialization patterns between immigrants and natives. The following lemma summarizes this result.

Lemma 2. *For $I_k > 0$, an increase in k-skill endowment of immigrants raises the task margin, i.e.*

$$\frac{\mathrm{d} \ln I_k}{\mathrm{d} \ln N_k^F} > 0,$$

which in turn induces a cost-efficiency enhancing effect, associated with lower marginal costs, i.e.

$$\frac{\mathrm{d} \ln \Theta_k(I_k)}{\mathrm{d} \ln I_k} < 0,$$

and an increase in the composite labor, i.e.

$$\frac{\mathrm{d} \ln \tilde{N}_k}{\mathrm{d} \ln I_k} = -\frac{\mathrm{d} \ln \left[\Theta_k(I_k)^{\alpha}(1 - I_k)\right]}{\mathrm{d} \ln I_k} > 0$$

Proof. See Appendix A.4.

Thus, the results in Lemma 2 denote the crucial departure from models that account only implicitly for immigrants (or rely basically on the assumption of perfect substitutability between immigrants and natives). Immigration is now associated with an endogenous response by native workers, which has been put forth as the salient feature regarding the efficiency improvement, the so-called productivity effect. This effect is captured by $\Theta_k(I_k)$ in Eq. (25).

Final goods: Utilizing Eqs. (2) and (21), the production function of final goods can be then rewritten as

$$y(z) = \begin{cases} \varphi_L(z)X_L = \varphi_L(z)A_L^{\frac{1}{\epsilon-1}}\frac{\tilde{N}_L}{z_L}, & \text{for all} \quad z \in [0, z_L], \\ \varphi_M(z)X_M = \varphi_M(z)A_M^{\frac{1}{\epsilon-1}}\frac{\tilde{N}_M}{z_H - z_L}, & \text{for all} \quad z \in (z_L, z_H), \\ \varphi_H(z)X_H = \varphi_H(z)A_H^{\frac{1}{\epsilon-1}}\frac{\tilde{N}_H}{1-z_H}, & \text{for all} \quad z \in [z_H, 1], \end{cases} \tag{26}$$

Aggregate output by technology modes: Finally, we derive the equilibrium aggregate levels of output and expenditure for each technology mode k. Let the total cost using technology k be

$$P_k Y_k = \int_{z \in \mathcal{S}_k} p(z) y(z) \mathrm{d}z,$$

where Y_k denotes the aggregate level of output of final goods using technology mode k. Now, utilizing Eqs. (3) and (26) in the previously derived equation and manipulating slightly, yields the aggregate level of output per technology mode k (see Appendix A.5 for the formal derivation)

$$Y_L = A_L^{\frac{1}{\epsilon-1}}\tilde{N}_L, \quad Y_M = A_M^{\frac{1}{\epsilon-1}}\tilde{N}_M, \quad Y_H = A_H^{\frac{1}{\epsilon-1}}\tilde{N}_H, \tag{27}$$

Relative expenditure on technology modes: As shown in Vallizadeh (2015), it follows from Eq. (6) that the expenditure on final goods must equal at the equilibrium technology margins, z_L and z_H. Utilizing Eqs. (21), (22), and (23), then we obtain the relative aggregate demand for final goods produced with technology mode $k = M$

$$\frac{P_L}{P_M} = \left(\frac{f_L}{f_M}\right)^{\frac{1}{(\epsilon-1)}} \left(\frac{\mathcal{M}(I_M)}{\mathcal{M}(I_L)}\right)^{\frac{\epsilon}{(1-\epsilon)}} \left(\frac{N_M^D}{N_L^D}\right)^{\frac{\epsilon}{(\epsilon-1)}} \left(\frac{z_L}{z_H - z_L}\right)^{\frac{\epsilon}{(\epsilon-1)}}, \tag{28}$$

$$\frac{P_M}{P_H} = \left(\frac{f_M}{f_H}\right)^{\frac{1}{(\epsilon-1)}} \left(\frac{\mathcal{M}_H(I_H)}{\mathcal{M}_M(I_M)}\right)^{\frac{\epsilon}{(1-\epsilon)}} \left(\frac{N_H^D}{N_M^D}\right)^{\frac{\epsilon}{(\epsilon-1)}} \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{\epsilon}{(\epsilon-1)}}. \tag{29}$$

where $\mathcal{M}_k(I_k) \equiv \Theta_k(I_k)^{\alpha}(1 - I_k)$, for all $k = \{L, M, H\}$ measures the impact of migration associated with endogenous changes in the task margin, and is decreasing in I_k, implying that the labor composite is increasing in I_k, i.e. $\frac{\mathrm{d}\ln\tilde{N}_k}{\mathrm{d}\ln I_k} = -\frac{\mathrm{d}\ln\mathcal{M}_k(I_k)}{\mathrm{d}\ln I_k} > 0$ (see again Appendix A.4). It is worth mentioning the crucial differences to the case of implicit immigration discussed in Vallizadeh (2015). Evaluating the functional properties of $\mathcal{M}_k(I_k)$, it can be shown that $\lim_{I_k \to 0}\frac{\mathrm{d}\ln\tilde{N}_k}{\mathrm{d}\ln I_k} = -\frac{\mathrm{d}\ln\mathcal{M}_k(I_k)}{\mathrm{d}\ln I_k} = 0$ and $\lim_{I_k \to 1}\frac{\mathrm{d}\ln\tilde{N}_k}{\mathrm{d}\ln I_k} = -\frac{\mathrm{d}\ln\mathcal{M}_k(I_k)}{\mathrm{d}\ln I_k} = \infty$. This indicates the productivity effect of immigration: an increase in I_k induces an efficiency-enhancing reallocation of tasks between natives and immigrants.

To close the model, combine (28) and (29) with the efficient allocation of technology modes, Eqs. (4) and (5), respectively. Now taking logs, manipulating and rearranging slightly, we obtain two implicit functions $\mathcal{F}_L(\cdot)$ and $\mathcal{F}_H(\cdot)$ that define the equilibrium technology margins z_L and z_H as functions of parameters and variables of the model.

$$\begin{aligned}\mathcal{F}_L(z_L, z_H, f_L, f_M, I_L, I_M) \quad &\equiv \quad \frac{1}{\epsilon}\ln\left(\frac{f_L}{f_M}\right) + \ln\left(\frac{\mathcal{M}_M(I_M)}{\mathcal{M}_L(I_L)}\right) + \ln\left(\frac{N_M^D}{N_L^D}\right) - \frac{(\epsilon-1)}{\epsilon}\ln\Lambda_L(z_L) \\ &\quad + \ln\left(\frac{z_L}{z_H - z_L}\right) = 0, \qquad (30)\\ \mathcal{F}_H(z_L, z_H, f_H, f_M, I_H, I_M) \quad &\equiv \quad \frac{1}{\epsilon}\ln\left(\frac{f_M}{f_H}\right) + \ln\left(\frac{\mathcal{M}_H(I_H)}{\mathcal{M}_M(I_M)}\right) + \ln\left(\frac{N_H^D}{N_M^D}\right) - \frac{(\epsilon-1)}{\epsilon}\ln\Lambda_H(z_H) \\ &\quad + \ln\left(\frac{z_H - z_L}{1 - z_H}\right) = 0. \qquad (31)\end{aligned}$$

This implicit system of equations can be used to compute basic comparative statics of changes in factor endowment and fixed market entry costs and their implications for technology adoption.

5 Comparative Statics

5.1 Immigration and technology adoption

Recall again that immigration is associated with a shift in the task margin I_k. For the computation of the comparative statics it is therefore sufficient to consider changes in the task margin. This implies that all the results derived here are akin to those derived in Vallizadeh (2015), except that now the relationship between the technology margins, z_L and z_H, and the task margin I_k is affected by the initial level of task margin. To get a better idea, take the total differentiation of the implicit Eqs. (30) and (31) with respect to I_M to obtain

$$\begin{pmatrix}\frac{\partial\mathcal{F}_L}{\partial z_L} & \frac{\partial\mathcal{F}_L}{\partial z_H}\\ \frac{\partial\mathcal{F}_H}{\partial z_L} & \frac{\partial\mathcal{F}_H}{\partial z_H}\end{pmatrix}\begin{pmatrix}\mathrm{d}\ln z_L\\ \mathrm{d}\ln z_H\end{pmatrix} = \begin{pmatrix}-\frac{\partial\mathcal{F}_L}{\partial I_M}\\ -\frac{\partial\mathcal{F}_H}{\partial I_M}\end{pmatrix}\times \mathrm{d}\ln I_M,$$

$$\Leftrightarrow \begin{pmatrix}\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L + \frac{z_H}{z_H - z_L} & -\frac{z_H}{z_H-z_L}\\ -\frac{z_L}{z_H-z_L} & \frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_H z_H + \frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}\end{pmatrix}\begin{pmatrix}\mathrm{d}\ln z_L\\ \mathrm{d}\ln z_H\end{pmatrix} = \begin{pmatrix}-\Psi_M\\ \Psi_M\end{pmatrix}\times \mathrm{d}\ln I_M,$$

where $\tilde{\varepsilon}_k \equiv -\frac{\partial\ln\Lambda_k(z_k)}{\partial\ln z_k} > 0$, for $k = \{L, H\}$ denotes the elasticity (in absolute value) of relative efficiency schedules of technology modes k at the equilibrium margin z_k and $\Psi_M \equiv -\frac{\mathrm{d}\ln\mathcal{M}_M(I_M)}{\mathrm{d}\ln I_M} > 0$. The inspection of the matrix verifies that the Jacobi has a positive sign, i.e. $\mathcal{D}_J = \frac{\partial\mathcal{F}_L}{\partial z_L}\frac{\partial\mathcal{F}_H}{\partial z_H} - \frac{\partial\mathcal{F}_L}{\partial z_H}\frac{\partial\mathcal{F}_H}{\partial z_L} > 0$, and thus the solution to the above 2×2 system is unique. Then, by Cramer's rule, we obtain

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_M} = \frac{\mathrm{d}\ln\mathcal{M}_M}{\mathrm{d}\ln I_M}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln\mathcal{M}_M} = -\Psi_M\frac{\frac{z_H}{1-z_H} + \frac{(\epsilon-1)}{\epsilon}\tilde{\varepsilon}_H z_H}{\mathcal{D}_J} < 0, \qquad (32)$$

and

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_M} = \frac{\mathrm{d}\ln\mathcal{M}_M}{\mathrm{d}\ln I_M}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln\mathcal{M}_M} = \Psi_M\frac{1 + \frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L}{\mathcal{D}_J} > 0. \qquad (33)$$

The results of the comparative statics in Eqs. (32) and (33) highlight the convenient feature of the model.

It allows to decompose immigration-induced changes in the technology margins, z_L and z_H, into changes due to internal task reallocation between immigrants and natives, captured by Ψ_k, and changes in the technology margins due to endogenous technology adoption. The following proposition summarizes the main results.

Proposition 1. *An exogenous increase in the skill endowment of immigrants and in the fixed costs will affect the technology margins z_L and z_H in the following way:*

- *Immigration of high-skill workers and H-fixed cost*

$$\left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln I_H}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln I_H}\right\} < 0, \quad \left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln f_H}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln f_H}\right\} > 0,$$

- *Immigration of medium-skilled workers, easier offshoring, and M-fixed cost*

$$\left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln f_M}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln I_M}\right\} > 0, \left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln I_M}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln f_M}\right\} < 0,$$

- *Immigration of low-skilled workers and L-fixed cost*

$$\left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln I_L}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln I_L}\right\} > 0, \quad \left\{\frac{\mathrm{d}z_L}{\mathrm{d}\ln f_L}, \frac{\mathrm{d}z_H}{\mathrm{d}\ln f_L}\right\} < 0.$$

Proof. See Appendix A.6 and the discussion in Vallizadeh (2015).

Equipped with comparative statics for technology adoption effect, the next section computes the comparative statics of different immigration scenarios and changes in natives wage structure.

5.2 Immigration and natives wage structure

In equilibrium, the optimal demand condition for intermediate goods, Eq. (8), and the monopolistic price-setting condition, Eq. (10), imply

$$\frac{\epsilon}{\epsilon-1}\tilde{w}_k = \quad p_k \quad = \zeta_k X_k^{1/\epsilon} x_k^{-1/\epsilon}, \qquad \forall\, k = \{L, M, H\}. \tag{34}$$

Utilizing the equilibrium conditions (16), (20), (21), (22), (24), (27), into Eq. (34) and taking the ration between low- and medium-skilled and between medium- and high-skilled wages we obtain (see Appendix A.7 for a full formal discussion):

$$\frac{w_L^D}{w_M^D} = \left(\frac{z_L}{z_H - z_L}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \left(\frac{\mathcal{M}_L(I_L)}{\mathcal{M}_M(I_M)}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}} \left(\frac{\Theta_L(I_L)}{\Theta_M(I_M)}\right)^{-1} \mathcal{K}_{LM} \tag{35}$$

$$\frac{w_M^D}{w_H^D} = \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \left(\frac{\mathcal{M}_M(I_M)}{\mathcal{M}_H(I_H)}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}} \left(\frac{\Theta_M(I_M)}{\Theta_H(I_H)}\right)^{-1} \mathcal{K}_{MH} \tag{36}$$

where $\mathcal{K}_{LM} \equiv \left(\frac{N_L^D}{N_M^D}\right)^{\frac{(\sigma-\epsilon)}{(\epsilon-1)\sigma}} \left(\frac{f_L}{f_M}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}} \Lambda_L(z, z')$ and $\mathcal{K}_{MH} \equiv \left(\frac{N_M^D}{N_H^D}\right)^{\frac{(\sigma-\epsilon)}{(\epsilon-1)\sigma}} \left(\frac{f_M}{f_H}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}} \Lambda_H(z', z'')$ summarize the other exogenous variables.

An inspection of Eqs. (35) and (36) allows to decompose the impact of immigration on natives wage structure into the following key channels: (i) a technology adoption effect, captured by the terms $\left(\frac{z_L}{z_H - z_L}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}$ and $\left(\frac{z_H - z_L}{1-z_H}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}$, (ii) a market size effect , captured by $\left(\frac{\mathcal{M}_L(I_L)}{\mathcal{M}_M(I_M)}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}}$ and $\left(\frac{\mathcal{M}_M(I_M)}{\mathcal{M}_H(I_H)}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}}$, and (iii) a productivity effect, captured by $(\Theta_L(I_L)/\Theta_M(I_M))^{-1}$ and $(\Theta_M(I_M)/\Theta_H(I_H))^{-1}$.

As highlighted by Vallizadeh (2015), the technology adoption effect is interacting with the market size effect due to the the direct and indirect market size effects and the price effect induced by the market externalities mentioned earlier. The direction of these effects depends on the final goods demand elasticity, σ, and the market power of monopolists, ϵ.

Let the technology adoption effects be denoted by $\tilde{\Gamma}_{I_k} \equiv \frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_k}$ and $\tilde{\Delta}_{I_k} \equiv \frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_k}$. As shown in Appendix A.8, the convenient structure of the model allows to decompose the technology effect into the following terms: $\tilde{\Gamma}_{I_k} = \Psi_k\Gamma_k$ and $\tilde{\Delta}_{I_k} = \Psi_k\Delta_k$, indicating a scalar product between changes in labor composite due to internal task reallocation, $\Psi_k \equiv -\frac{\mathrm{d}\ln\mathcal{M}_k}{\mathrm{d}\ln I_k} = -\frac{\mathrm{d}\ln[\Theta_k(I_k)^{\alpha}(1-I_k)]}{\mathrm{d}\ln I_k} > 0$, and immigration-induced changes in the relative range of final goods due to endogenous changes in the technology margins, i.e. $\Gamma_k = \frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln\mathcal{M}_k}$ and $\Delta_k = \frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln\mathcal{M}_k}$. It worth noticing that the terms Γ_k and Δ_k are akin to those computed in Vallizadeh (2015), and as discussed there changes in the neighborhood of the technology margins z_L and z_H, depend on the degree of technology adoption, captured by the elasticity parameters $\tilde{\varepsilon}_L \equiv -\frac{\mathrm{d}\ln\Lambda_L(z_L)}{\mathrm{d}\ln z_L} > 0$ and $\tilde{\varepsilon}_H \equiv -\frac{\mathrm{d}\ln\Lambda_H(z_H)}{\mathrm{d}\ln z_H} > 0$, so that it follows that $\{|\Gamma_k|, |\Delta_k|\} \in (0,1)$, for $0 < \{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} < \infty$.

The rest of this section computes the impact of various skill immigration scenarios on natives wage structure and elaborates the determinants of and the interaction between the different channels, while the formal discussions are relegated to Appendix A.9. For ease of notation, let the relative wage of medium-skilled natives be $\omega^D_{LM} \equiv (w^D_L/w^D_M)$ and $\omega^D_{MH} \equiv (w^D_M/w^D_H)$.

Low-skill immigration

Now taking logs in Eqs. (35) and (36) and differentiating totally with respect to changes in the task margin I_L, and manipulating yields

$$\frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\ln I_L} = \left(1-\alpha(1-\Gamma_L)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\tilde{\Theta}_L(I_L) - (1-\Gamma_L)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\frac{I_L}{1-I_L} \tag{37}$$

$$\frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\ln I_L} = \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\tilde{\Theta}_L(I_L) + \frac{I_L}{1-I_L}\right)\Delta_L, \tag{38}$$

where $\tilde{\Theta}_L(I_L) \equiv -\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L} > 0$ denotes the productivity effect, and by Proposition 1 $\Gamma_L > 0$ and $\Delta_L < 0$.

Medium-skill immigration

Following the same steps, the impact of high-skill immigration can be computed, i.e. taking logs in Eqs. (35) and (36) and differentiating totally with respect to changes in the task margin I_H, associated with high-skill immigration, yields

$$\frac{\mathrm{d}\ln\omega_{LM}^{D}}{\mathrm{d}\ln I_M} = -\left(1-\alpha(1+\Gamma_M)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\tilde{\Theta}_M(I_M)+(1+\Gamma_M)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\frac{I_M}{1-I_M} \tag{39}$$

$$\frac{\mathrm{d}\ln\omega_{MH}^{D}}{\mathrm{d}\ln I_M} = \left(1-\alpha(1-\Delta_M)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\tilde{\Theta}_M(I_M)-(1-\Delta_M)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\frac{I_M}{1-I_M}, \tag{40}$$

where $\tilde{\Theta}_M(I_M)\equiv-\frac{\mathrm{d}\ln\Theta_M(I_M)}{\mathrm{d}\ln I_M}>0$ denotes the productivity effect, by Proposition 1, we get that $\Gamma_M<0$ and $\Delta_M>0$.

High-skill immigration

Following the same steps, the impact of high-skill immigration can be computed, i.e. taking logs in Eqs. (35) and (36) and differentiating totally with respect to changes in the task margin I_H yields

$$\frac{\mathrm{d}\ln\omega_{LM}^{D}}{\mathrm{d}\ln I_H} = \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\tilde{\Theta}_H(I_H)+\frac{I_H}{1-I_H}\right)\Gamma_H \tag{41}$$

$$\frac{\mathrm{d}\ln\omega_{MH}^{D}}{\mathrm{d}\ln I_H} = -\left(1-\alpha(1+\Delta_H)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\tilde{\Theta}_H(I_H)+(1+\Delta_H)\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\frac{I_H}{1-I_H}, \tag{42}$$

where $\tilde{\Theta}_H(I_H)\equiv-\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H}>0$ denotes the productivity effect, and by Proposition 1 $\Gamma_H>0$ and $\Delta_H<0$.

The examination of Eqs (37)–(42) reveals the following new insights. First, allowing for internal adjustment of work organization, i.e. endogenous task specialization between immigrants and natives, immigration is associated with a productivity effect, $\tilde{\Theta}_k$, and a labor supply effect, $\frac{I_k}{1-I_k}$. Moreover, the relative strength between these two forces exhibits now a dynamic nature, varying with the initial level of task margin. As elaborated below, this novel feature of the model provides a rationale behind the ambiguity of the impact of immigration on natives wage structure found often in the empirical literature.

In addition, the direction of each channel is importantly shaped by the degree of technology adoption between the different production modes, Γ_k and Δ_k, as well as by the relative strength between the final-goods demand elasticity (σ) and the market power of monopolists (ϵ). However, it worth highlighting that the magnitude of the productivity effect depends additionally on the level of the elasticity of substitution between tasks, α. The next proposition summarizes the first key result regarding the impact of immigration on natives relative wage structure.

Proposition 2. *Whenever $\sigma>\epsilon$ and for a sufficient positive finite degree of technology adoption, i.e. $0<\{\tilde{\varepsilon}_L,\tilde{\varepsilon}_H\}<\infty$, then changes in natives wage structure are characterized as follows:*

- ***Low-skill immigration** induces a wage effect similar to an "unskill-bias" technological change, i.e. an relative increase in wages between low- and medium-skilled workers and between medium- and high-skilled workers;*
- ***Medium-skill immigration** generates a hump shaped relationship, i.e. medium-skill wages rise relative to both low- and high-skill wages;*
- ***High-skill immigration** has a similar wage effect compared to a "skill-bias" technological change, i.e. a monotonic increase in relative wages across the skill groups.*

Proof. See the discussion in the text.

From Eqs. (37), (39), (40), and (42) is readily seen that whenever the goods demand elasticity is larger than the market power of monopolists, the direction of productivity and labor supply effects become independent from the extent of the elasticity of substitution between tasks (α) and the initial value of the task margin. The intuition behind this result turns on the *market size effect* and the *price effect* associated with the market externalities mentioned above. To illustrate these price and market size effects, consider the low-skill immigration scenario. First, the internal reallocation of tasks between low-skilled immigrants and natives enhances the cost-efficiency of firms hiring them, raising their profits and in turn the mass of those monopolists, denoting the *direct market size effect*.

As a consequence the comparative advantages of technology mode L increases, leading to an increase in the relative price index of goods produced with technology modes M and H. This in turn induces final goods firms to shift to the production techniques complementing low-skilled workers, leading to an expansion of threshold margin z_L, indicating the *indirect market size effect*. However, this effect raises the relative price of goods M and H thus induces an increase in the mass of monopolists in those markets, i.e. a rise in A_M and A_H, while counteracting the direct market size effect for technology mode L, i.e. a reduction in A_L (see Eq. (22)). This effect reflects the *price effect* inducing a biased shift in favor of M- and H-complement technologies. Since these effects work in opposite direction, the relative strength is ambiguous. Thus, for $\sigma > \epsilon$, the goods demand effect will be the dominating force, indicating the relative favorable shift in the demand for low-skill-intensive final goods. The empirical literature provide a range of values for these elasticities. As discussed in Acemoglu et al. (2012), the range of values for the elasticity of substitution across intermediates are usually estimated above 3, while between skill groups are in the range $[1.5, 2]$. This implies that the empirically plausible case is whenever $\sigma < \epsilon$, which I elaborate next.

Recalling Eqs. (37)–(42), the direction of immigration-induced changes in natives wage structure depends, on the one hand, the relative strength between the productivity effect and the labor supply effect. On the other hand, it depends on the degree of the elasticity of substitution between tasks and the degree of technology adoption. Thus, it follows that for certain degree of task substitutability/complementarity the immigration-induced efficiency enhancing effect will have a beneficial effect for native workers.

Lemma 3 (Tasks Complementarity)**.** *Given $\sigma < \epsilon$, i.e. the goods demand elasticity is lower than the elasticity of substitution between intermediates, it follows that tasks performed by immigrants are complement to those performed by natives if and only if $\alpha < \min\{\tilde{\alpha}_L, \tilde{\alpha}_H\}$, where $\tilde{\alpha}_L \equiv \frac{(\epsilon-1)\sigma}{(\epsilon-\sigma)(1+\Delta_H)}$, $\tilde{\alpha}_H \equiv \frac{(\epsilon-1)\sigma}{(\epsilon-\sigma)(1+\Gamma_L)}$, and $\min\{\tilde{\alpha}_L, \tilde{\alpha}_H\} = \max\{\tilde{\varepsilon}_L z_L(1-z_L), \tilde{\varepsilon}_H z_H(1-z_H)\}$.*

Proof. See Appendix A.10.

Lemma 3 characterizes two key properties. First, the efficiency gains accrued to natives arise up to a certain level of imperfect substitutability across the tasks. The empirical studies provide estimations that indicate the complementarity between tasks performed by immigrants and by natives. For example, Peri and Sparber (2009) estimate the value of the elasticity of substitution between manual and communicative, complex tasks in the range $[0.63, 1.43]$. Now, for empirically plausible values it can be shown that the upper boundary in Lemma 3 lies comfortably outside of this range. As discussed in Acemoglu et al. (2012), the estimated values of σ lie in the range $[1.5, 2]$, and that of ϵ are usually higher than 3.

Second, the extent of task complementarity/substitutability depends also on the degree of technology adoption. Intuitively, easier adoption across various production modes will mitigate the direct market size effect of immigration, and thus leaving the wage structure independent from the elasticity α. More precisely, as elaborated in Vallizadeh (2015), for the asymptotic case $\{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} \to 0$, i.e. when the adoption between the different technology modes in the neighborhood of the margins, z_L and z_H, becomes extremely easy, $\Gamma_L = -\Gamma_M = -\Delta_H = \Delta_M = 1$, and $-\Delta_L = \Gamma_H = 0$. This implies that the market size and the price effects will be asymptotically balanced. Utilizing these observations into Eqs. (37)–(42), the second key result can be summarized as follows

Proposition 3. *Whenever the adoption between different technology modes becomes extremely easy, changes in natives wage structure are asymptotically characterized by the immigration-induced productivity effect due to an efficiency enhancing internal task reallocation, such that*

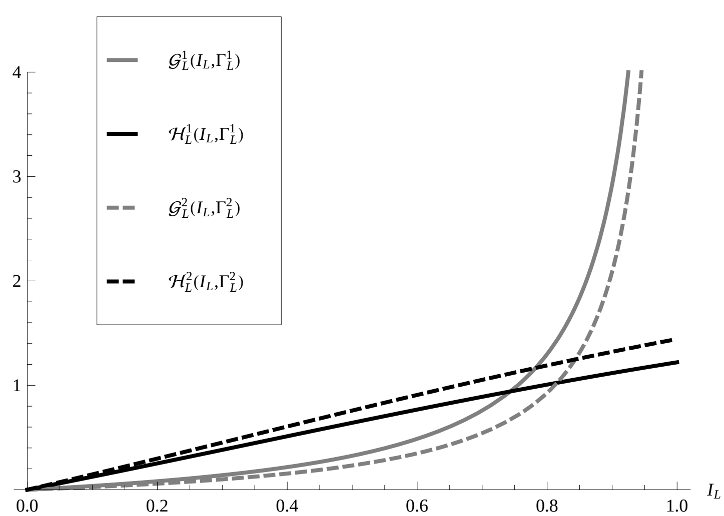
$$\frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\ln I_L} \approx \tilde{\Theta}_L(I_L), \quad \frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\ln I_H} \approx \tilde{\Theta}_H(I_H), \quad \frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\ln I_M} = \frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\ln I_M} \approx \tilde{\Theta}_M(I_M), \quad \textit{while} \quad \frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\ln I_L} = \frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\ln I_H} \approx 0.$$

Proof. See the discussion in the text.

Proposition 3 highlights the important role of task-reallocation and technology adoption mechanisms. While the latter is mitigating the magnitude of initial direct effects associated with the market size and price effects, the former shows the novel mechanism which becomes the remaining force. Thus, these results support the recent empirical evidence on immigration-induced efficiency gains (cf. Ottaviano et al., 2013), but they reconcile partially the predictions of empirical findings focusing primary on aggregate effects of endogenous technology adoption effects and long-run balancing wage effects of immigration-induced changes in the skill mix (cf. Beaudry et al., 2010).

However, in the case when the degree of technology adoption is positive but finite, i.e. $0 < \{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} < \infty$ immigration-induced changes in natives wage structure depend crucially on the initial level of the task margin. This in turn determines the relative magnitude between the productivity and labor supply effects. To ob-

Figure 2: Productivity and labor supply effects



Notes: The figure shows the relationship between productivity effect and labor supply effect, for the case $\Gamma^1_L = 0.4$ (solid line) and $\Gamma^2_L = 0.7$ (dashed line), see in the text for the remaining parameter values.

tain a better idea, recall Eq. (37) and let the productivity effect be captured by $\mathcal{H}_L(I_L) \equiv (1-\alpha\delta(1-\Gamma_L))\,\tilde{\Theta}_L(I_L)$ and the labor supply effect by $\mathcal{G}_L(I_L) \equiv \delta(1-\Gamma_L)\frac{I_L}{1-I_L}$, where $\delta \equiv \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}$. It can be straightforwardly shown that for $I_L \to 0$, $\mathcal{H}_L = \mathcal{G}_L \to 0$, while as $I_L \to 1$, $\mathcal{G}_L \to \infty$ and $\mathcal{H}_L \to (1-\alpha\delta(1-\Gamma_L))\,\mu_L$, where $\mu_L = \frac{\mathrm{d}\ln\gamma_L(I_L)}{\mathrm{d}\ln I_L}$ denotes the elasticity of task-comparative advantage schedule of immigrants. Moreover, given that $\gamma_L(i)$ is continuous and monotonically increasing over the unit interval, there must exist a single crossing between $\mathcal{H}_L$ and $\mathcal{G}_L$, defining a threshold margin at which both effects are balanced. Figure 2 depicts this relationship for the case of a log-linear functional form of $\gamma_L(i) = \exp[\mu_L i]$, for empirically plausible values of $\epsilon = 3.33$ and $\sigma = 1.6$ (cf. Acemoglu et al., 2012) and $\alpha = 1.2$, which lies within the estimated range of values in Peri and Sparber (2009). The same relationship can be also derived from Eqs. (39) and (40) for a medium-skilled task margin, and from Eq. (42) for the high-skilled task margin. Thus, the next key result can be summarized as follows.

Proposition 4. *Given $\epsilon > \sigma$ and a sufficient degree of task complementarity, defined by Lemma 3, it follows that whenever the task margin is higher than a threshold $\check{I}_k \in (0,1)$, the immigration-induced productivity effect is dominated by the labor supply effect, so that changes in natives wage structure induced by*

- ***Low-skill immigration** are like a "skill-bias" technological change, i.e. a relative decline in low- and medium-skilled wages compared to medium- and high-skilled wages, respectively.*
- ***Medium-skill immigration** are characterized by polarizing wage effect, i.e. a relative decline in medium-skilled wages compared to low- and high-skilled wages.*
- ***High-skill immigration** are like a "unskill-bias" technological change, i.e. a relative increase in low- and medium-skilled wages compared to medium- and high-skilled wages, respectively.*

Proof. See Appendix A.11 and the discussion in the text.

Proposition 4 highlights the third key feature of the model, associated with a non-monotonic relationship between immigration and the relative magnitude between productivity and labor supply effects. Moreover, the impact of immigration on natives wage structure contrasts now those derived in Proposition 2. Particularly, at high initial level of task margin, marginal increase in immigration induces a labor-saving technological effect for similarly skilled native workers, lowering their wages compared to other skill groups. However,

Table 1: Immigration and natives wage structure, if $\sigma < \epsilon$

	Low-skill		Medium-skill		High-skill	
	$I_L < \check{I}_L$	$I_L > \check{I}_L$	$I_M < \check{I}_M$	$I_M > \check{I}_M$	$I_H < \check{I}_H$	$I_H > \check{I}_H$
$\mathrm{d}\ln\omega^D_{LM}$	$+$	$-$	$-$	$+$	$+$	$+$
$\mathrm{d}\ln\omega^D_{MH}$	$-$	$-$	$+$	$-$	$-$	$+$

it is worth mentioning the case when the initial level of task margin is lower than the threshold margin $\check{I}_k$, implying that the the productivity effect is dominating the labor supply effect. In this case, both low-skill and high-skill immigration scenarios induce a polarization of natives wage structure, while medium-skill immigration scenario induce an inverse U-shaped wage effect. Table 1 summarizes these results.

Moreover, these results highlight the importance of both macor- and microecnomic structure of the model, i.e. technology adoption at the industry level and within-firm reorganization of work, which are crucial to gain insights behind the determinants of and the interaction between various channels.

6 Offshoring of intermediate inputs

This section extends the framework with respect to the possibility of offshoring for domestic firms. As discussed above, the empirical evidences has indicated that along the value added chain jobs concentrated in the middle range of the skill distribution, characterized by high intensity of routine content of tasks, are mostly prone to offshoring. This has been put forth as the potential source of wage polarization due to offshoring (cf. Acemoglu and Autor, 2011). Thus, I suppose that only intermediate goods utilized under technology mode $k = M$ are affect by offshoring.

In doing so, I follow the exogenous offshoring approach, discussed in Acemoglu et al. (2012), where offshoring occurs in the form of relocations of intermediate inputs, similar to models of fragmentation along the value added chain (cf. Jones and Kierzkowski, 2001). As pointed out by Acemoglu et al. (2012), this approach allows to capture changes in offshoring at the extensive margin. Yet, the key features of offshoring associated with the efficiency improvement due to lower offshoring costs and the labor supply effect, capturing the increased competition between offshore and domestic workers are preserved.[7] In the rest of this section, I describe the basic settings as well as the equilibrium outcomes of offshoring, followed by the comparative static analysis.

It is assumed that domestic firms can allocate and produce an exogenously given fraction κ of all intermediate goods x_M over the interval $[0, A_M]$ abroad. However, due to implicit trade costs, cost-savings from reallocation of domestic production are limited so that $0 < \kappa < \tilde{\kappa} < 1$, where at the threshold $\tilde{\kappa}$, there is no arbitrage from offshoring, i.e. wages between the source and host countries of offshoring are equalized. It follows that in equilibrium a measure κA_M of monopolists produce abroad, while the remaining measure $(1-\kappa)A_M$ of monopolists produce at home. In what follows the variables characterizing the offshoring equilibrium are denoted by the *hat*-symbol "$\hat{}$".

Imposing labor market clearing for both domestic medium-skilled and offshore workers and recalling the symmetry assumption, then the equilibrium output values of firms producing at home and abroad are characterized, respectively, by

$$x_M = \frac{\tilde{N}_M}{(1-\kappa)A_M(z_H - z_L)}, \quad x_O = \frac{N_O}{\kappa A_M(z_H - z_L)}, \tag{43}$$

where N_O denotes the total exogenously given endowments of offshore workers. Next, similar to the pricing-behavior of domestic monopolists, Eq. (10), offshoring firms charge a mark-up over the marginal cost, i.e. $p_O = \frac{\epsilon}{\epsilon-1}w_O$, where p_O and w_O denote the price and wage of offshoring goods and workers, respectively. The optimal demand condition for intermediate goods, Eq. (8), implies now the following demand conditions

[7] Alternatively, one could introduce offshoring following the task-based approach à la Grossman and Rossi-Hansberg (2008), which allows to account for changes at both intensive and extensive margins. See for example, Ottaviano et al. (2013) who introduce offshoring as an intermediate range of tasks between the range of tasks performed by immigrants and natives over the unit interval. However, this requires more structure and thus would unnecessary complicate the analysis.

for offshore and domestically-produced intermediates

$$p_O = \hat{\zeta}_M \left(\frac{\hat{X}_M}{x_O}\right)^{\frac{1}{\epsilon}}, \tag{44}$$

$$p_M = \hat{\zeta}_M \left(\frac{\hat{X}_M}{x_M}\right)^{\frac{1}{\epsilon}}. \tag{45}$$

Then, from Eqs. (44) and (45) and the markup-pricing behavior, it follows that the wage gap between offshore and domestic workers is given by

$$\frac{w_O}{\tilde{w}_M} = \frac{p_O}{p_M} = \left(\frac{x_O}{x_M}\right)^{-\frac{1}{\epsilon}} = \left(\frac{N_O}{\tilde{N}_M}\frac{1-\kappa}{\kappa}\right)^{-\frac{1}{\epsilon}}, \tag{46}$$

where from Eq. (46) it is readily evident that $\tilde{w}_M > w_O$ for $\kappa < \tilde{\kappa} \equiv \frac{N_O}{N_O+\tilde{N}_M}$. Moreover, in line with the recent empirical evidence, immigration of medium-skilled workers, i.e. $\frac{\mathrm{d}\ln \tilde{N}_M}{\mathrm{d}\ln I_M} > 0$, reduces the arbitrage from offshoring, indicating that offshoring and immigration are substitutes (cf. Ottaviano et al., 2013). However, since for $I_M \to 1$, $\frac{\mathrm{d}\ln \tilde{N}_M}{\mathrm{d}\ln I_M} \to \infty$ and thus $\tilde{\kappa} \to 0$, it is imposed that the task margin I_M is sufficiently below unity so that positive values of offshoring exits, i.e. $0 < \kappa < \tilde{\kappa}$.

Next, substituting (43) into the composite intermediate goods functions (7) for $k = M$ yields

$$\hat{X}_M = A_M^{\frac{1}{\epsilon-1}} \frac{\hat{N}_M}{z_H - z_L}, \tag{47}$$

where

$$\hat{N}_M = \left(\kappa^{\frac{1}{\epsilon}} N_O^{\frac{\epsilon-1}{\epsilon}} + (1-\kappa)^{\frac{1}{\epsilon}} \tilde{N}_M^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{48}$$

denotes the weighted average, captured by the offshoring rate κ, of domestic medium-skilled and offshore workers. Eq. (48) reveals now an interesting feature. For given number of varieties, the production is increasing in the extent of offshoring:

$$\frac{\mathrm{d}\hat{N}_M}{\mathrm{d}\kappa} = \frac{1}{\epsilon-1}\hat{N}_M^{1/\epsilon}\left[\left(\frac{N_O}{\kappa}\right)^{\frac{\epsilon-1}{\epsilon}} - \left(\frac{\tilde{N}_M}{1-\kappa}\right)^{\frac{\epsilon-1}{\epsilon}}\right] > 0,$$

where $\lim_{\kappa\to 0} \mathrm{d}\hat{N}_M/\mathrm{d}\kappa = \infty$ and $\lim_{\kappa\to\tilde{\kappa}} \mathrm{d}\hat{N}_M/\mathrm{d}\kappa = 0$. This is referred to as the offshoring-induced *efficiency effect*, associated with an efficient reallocation of production towards countries where wages are lower (Acemoglu et al., 2012). Intuitively, at low initial values of offshoring rate, the arbitrage from offshoring is very high, due to the high offshore-domestic wage gap.

Similarly, by means of cost minimization, the equilibrium cost index of composite intermediate goods is now given by[8]

$$\hat{\zeta}_M = A_M^{\frac{1}{1-\epsilon}} \left(\kappa p_O^{1-\epsilon} + (1-\kappa) p_M^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{49}$$

[8]More precisely, the optimization problem of a final-good producer is characterized by $\int_0^{\kappa A_M} p_O(j)x_O(j)\mathrm{d} + \int_0^{(1-\kappa)A_M} p_M(j)x_M(j)\mathrm{d}$ subject to $\hat{X}_M = \left(\int_0^{\kappa A_M} x_O(j)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d} + \int_0^{(1-\kappa)A_M} x_M(j)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}\right)^{\frac{\epsilon}{\epsilon-1}}$.

Now, the zero profit condition for each final good producer $z \in (z_L, z_H)$ implies

$$p_M(z)y_M(z) = \frac{\tilde{w}_M \tilde{N}_M + w_O N_O}{z_H - z_L}$$

Following the same steps discussed above, the mass of monopolists is now defined by (see Appendix A.12)

$$\hat{A}_M = \frac{\hat{N}_M}{f_M \epsilon (z_H - z_L)} \tag{50}$$

Equipped with these equilibrium conditions, the following section elaborates the comparative statics of offshoring regarding technology adoption and changes in the natives wage structure.

6.1 Offshoring and technology adoption

First notice that the relative expenditure conditions defined by Eqs. (28) and (29) has to be adjusted for offshoring. Thus, utilizing Eqs. (47) and (50) in Y_M and inserting the results in $P_L Y_L / z_L = P_M Y_M / (z_H - z_L)$ and $P_M Y_M / (z_H - z_L) = P_H Y_H / (1 - z_H)$ and solving for the relative price indices yields

$$\frac{P_L}{P_M} = \left(\frac{f_L}{f_M}\right)^{\frac{1}{(\epsilon-1)}} \left(\frac{\hat{N}_M}{\tilde{N}_L}\right)^{\frac{\epsilon}{(\epsilon-1)}} \left(\frac{z_L}{z_H - z_L}\right)^{\frac{\epsilon}{(\epsilon-1)}}, \tag{28'}$$

$$\frac{P_M}{P_H} = \left(\frac{f_M}{f_H}\right)^{\frac{1}{(\epsilon-1)}} \left(\frac{\tilde{N}_H}{\hat{N}_M}\right)^{\frac{\epsilon}{(\epsilon-1)}} \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{\epsilon}{(\epsilon-1)}}. \tag{29'}$$

Combining Eqs. (28') and (29') with Eqs. (4) and (5) yields two implicit functions adjusted for offshoring

$$\hat{\mathcal{F}}_L(z_L, z_H, f_L, f_M, I_L, I_M, \kappa) \equiv \frac{1}{\epsilon} \ln\left(\frac{f_L}{f_M}\right) + \ln\left(\frac{\hat{N}_M}{\tilde{N}_L}\right) - \frac{(\epsilon-1)}{\epsilon} \ln \Lambda_L(z_L) + \ln\left(\frac{z_L}{z_H - z_L}\right) = 0, \tag{30'}$$

$$\hat{\mathcal{F}}_H(z_L, z_H, f_H, f_M, I_H, I_M, \kappa) \equiv \frac{1}{\epsilon} \ln\left(\frac{f_M}{f_H}\right) + \ln\left(\frac{\tilde{N}_H}{\hat{N}_M}\right) - \frac{(\epsilon-1)}{\epsilon} \ln \Lambda_H(z_H) + \ln\left(\frac{z_H - z_L}{1 - z_H}\right) = 0. \tag{31'}$$

from which the comparative statics for easier offshoring ($\mathrm{d}\kappa > 0$) can be conducted. Now, let $\Phi_\kappa \equiv \frac{\mathrm{d} \ln \hat{N}_M}{\mathrm{d}\kappa}$, taking the total differentiation of the adjusted 2×2 system, Eqs. (30') and (31'), with respect to offshoring rate κ yields

$$\begin{pmatrix} \frac{\partial \mathcal{F}_L}{\partial z_L} & \frac{\partial \mathcal{F}_L}{\partial z_H} \\ \frac{\partial \mathcal{F}_H}{\partial z_L} & \frac{\partial \mathcal{F}_H}{\partial z_H} \end{pmatrix} \begin{pmatrix} \mathrm{d} \ln z_L \\ \mathrm{d} \ln z_H \end{pmatrix} = \begin{pmatrix} -\frac{\partial \mathcal{F}_L}{\partial \kappa} \\ -\frac{\partial \mathcal{F}_H}{\partial \kappa} \end{pmatrix} \times \mathrm{d}\kappa,$$

$$\Leftrightarrow \begin{pmatrix} \frac{\epsilon-1}{\epsilon} \tilde{\varepsilon}_L z_L + \frac{z_H}{z_H - z_L} & -\frac{z_H}{z_H - z_L} \\ -\frac{z_L}{z_H - z_L} & \frac{\epsilon-1}{\epsilon} \tilde{\varepsilon}_H z_H + \frac{z_H(1-z_L)}{(1-z_H)(z_H - z_L)} \end{pmatrix} \begin{pmatrix} \mathrm{d} \ln z_L \\ \mathrm{d} \ln z_H \end{pmatrix} = \begin{pmatrix} -\Phi_\kappa \\ \Phi_\kappa \end{pmatrix} \times \mathrm{d}\kappa,$$

Then, by Cramer's rule, we obtain

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\kappa} = \frac{\mathrm{d}\ln \hat{N}_M}{\mathrm{d}\kappa}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln \hat{N}_M} = -\Phi_\kappa \frac{\frac{z_H}{1-z_H} + \frac{(\epsilon-1)}{\epsilon}\tilde{\varepsilon}_H z_H}{\mathcal{D}_J} < 0, \tag{53}$$

and

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\kappa} = \frac{\mathrm{d}\ln \hat{N}_M}{\mathrm{d}\kappa}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln \hat{N}_M} = \Phi_\kappa \frac{1 + \frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L}{\mathcal{D}_J} > 0. \tag{54}$$

Eqs. (53) and (54) reveal again the convenient structure of the model, allowing to decompose changes in the technology margins into offshoring-induced efficiency effect (Φ_κ) and changes at the extensive margins due to technology adoption behavior. The next proposition summarizes the key result.

Proposition 5. *An exogenous increase in the offshoring rate ($\mathrm{d}\kappa > 0$) induces an efficiency effect for products produced under the complementing technology mode $k = M$, and thus raising the comparative advantages of adopting this technology mode. Consequently the economy becomes proportionately more specialized in technology mode M.*

Proof. See the discussion in the text.

Thus, Propositions 1 and 5 show that both immigration of medium-skilled workers and offshoring of intermediate goods induce a cost-efficiency improvement for firms and industries employing these factors most intensively. This raises the comparative advantages of production techniques complementing these two factors. Consequently, final goods that have been produced under the technology modes L and H are now produced by technology mode M.

6.2 Offshoring and natives wage structure

As discussed earlier, the relative wage structure can derived from the equilibrium condition (34). Thus, following the same steps and utilizing the offshoring adjusted equilibrium conditions (43), (45), (47), (48) together with previously derived equilibrium variables (16), (20), (21), (22), (24), (27), into Eq. (34) and taking ratios with respect to medium-skilled wages, yields

$$\omega^D_{LM} = \left(\frac{z_L}{z_H - z_L}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \left(\frac{\tilde{N}_L}{\hat{N}_M}\right)^{-\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \hat{N}_M^{-\frac{1}{\epsilon}} \left(\frac{\tilde{N}_M}{1-\kappa}\right)^{\frac{1}{\epsilon}} \left(\frac{\Theta_M}{\Theta_L}\right) \mathcal{K}_{LM} \tag{55}$$

$$\omega^D_{MH} = \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \left(\frac{\hat{N}_M}{\tilde{N}_H}\right)^{-\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}} \hat{N}_M^{\frac{1}{\epsilon}} \left(\frac{\tilde{N}_M}{1-\kappa}\right)^{-\frac{1}{\epsilon}} \left(\frac{\Theta_H}{\Theta_M}\right) \mathcal{K}_{MH} \tag{56}$$

Again Eqs. (55) and (56) allow to decompose the effect of κ into (i) a market size effect, $\hat{N}_M^{-\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}$, (ii) an efficiency effect, $\hat{N}_M^{\frac{1}{\epsilon}}$, (iii) a technology adoption effect, $\left(\frac{z_L}{z_H-z_L}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}$ and $\left(\frac{z_H-z_L}{1-z_H}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}$, and (iv) a labor supply effect, $\left(\frac{\tilde{N}_M}{1-\kappa}\right)^{\frac{1}{\epsilon}}$. This latter channel is the result of the assumption that offshoring accounts only the benefits at the extensive margins, and thus neglecting the beneficial effects of a reduction of offshoring costs on all intermediate inputs (intensive margins).

I now discuss these channels in more details. First, as mentioned above, the relative strength between the labor supply effect and the efficiency effect depend on the extent of offshoring rate. That is, for $\kappa \to 0$, $\Phi_\kappa \to \infty$, the efficiency effect will dominate the labor supply effect. Intuitively, at low values of offshoring rate the wage gap and thus the arbitrage from offshoring will be very large for domestic firms (Acemoglu et al., 2012). However, notice that this result differs from the non-monotonic relationship discussed in Egger et al. (2013), where gains from offshoring arise only at high levels of offshoring rate, as the efficiency effect at the intensive margin becomes large.

Second, as discussed in the case of medium-skill immigration, the direct market size effect are interacting with the indirect, i.e. the endogenous technology adoption, scale effects, where the relative magnitude between these two effects depends again on the extent of technology adoption in the neighborhood of the margins z_L and z_H, captured by the elasticity parameters $\tilde{\varepsilon}_L$ and $\tilde{\varepsilon}_H$. This is the crucial departure from Acemoglu et al. (2012), where this latter effect is omitted. To see this more formally, take logs in Eqs. (55) and (56) differentiate totally with respect to κ and make use of the comparative static results (53) and (54) to obtain

$$\frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\kappa} = \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}(1+\Gamma_M)\Phi_\kappa - \frac{1}{\epsilon}\left(\Phi_\kappa - \frac{1}{1-\kappa}\right) \tag{57}$$

$$\frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\kappa} = -\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}(1-\Delta_M)\Phi_\kappa + \frac{1}{\epsilon}\left(\Phi_\kappa - \frac{1}{1-\kappa}\right) \tag{58}$$

In Eqs. (57) and (58), the first terms describe the interaction between the technology adoption effect and the direct market size effect, while the second terms denote the interaction between the labor supply effect and the efficiency effect. It is readily seen that the direction of the first term depends again on the relative magnitude between the final-goods demand elasticity (σ) and the market power of monopolists (ϵ). Thus, whenever $\epsilon > \sigma$, i.e. the elasticity of substitution between intermediates is larger than the goods demand elasticity, the direction of the first terms is biased towards the low- and high-skilled workers in the sense that offshoring is associated with a low- and high-skill labor-augmenting effect.

The strength of the first terms in Eqs. (57) and (58) depends, o the one hand, on the degree of technology adoption, denoted by Γ_M and Δ_M. On the other hand, it depends on the level of offshoring rate κ. Thus, when either $\kappa \to \tilde{\kappa}$, or $\{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} \to 0$, the first terms vanish. The remaining term is then the labor supply effect (recall that $\lim_{\kappa\to\tilde{\kappa}} \Phi_\kappa = 0$). However, for $\kappa < \tilde{\kappa}$ and a sufficient positive and finite degree of technology adoption, an inspection of Eqs. (57) and (58) reveals that the relative magnitude between the efficiency effect and the scale effect is determined by the extent of technology adoption. The next proposition summarizes this result.

Proposition 6. *Given a positive and finite of the degree of technology adoption, i.e.* $\{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} > 0$*, an exogenous increase in offshoring rate* κ *induces an efficiency effect which is counteracting the scale effect whenever* $\epsilon > \sigma$*. The relative magnitude between these two effects depends on the degree of technology adoption. There exists a threshold degree of technology adoption* $\chi \equiv \frac{\epsilon^2+\sigma-2\sigma\epsilon}{\epsilon(\epsilon-\sigma)}$*, which determines a lower boundary for changes in the relative range of final goods. Whenever* $\min\{|\Gamma_M|, \Delta_M\} > \chi$*, the efficiency effect will be the dominating force. Moreover, for initial low values of offshoring rate the efficiency effect will also dominate the labor supply effect.*

Proof. See Appendix A.13.

The results in Proposition 6 highlight again the importance of technology adoption behavior and the rich pattern of interaction between various forces. Intuitively, the easier the technology adoption is in the neighborhood of the margins z_L and z_H, the stronger will be changes in the relative range of final goods, i.e. high values of $|\Gamma_M|$ and Δ_M. Thus, whenever the market power of monopolists is larger than the final-goods demand elasticity ($\epsilon > \sigma$), technology adoption effect will mitigate the extent of the market size effect. Now equipped with these results, the next proposition summarizes the main results regarding the impact of offshoring on natives relative wage structure.

Proposition 7. *Given the results in Proposition 6, changes in natives relative wage structure due to an exogenous increase in offshoring rate ($\mathrm{d}\kappa > 0$) are characterized by a*

(i) ***Medium-skill-augmenting wage effect****, associated with a relative increase of medium-skilled wages compared to both low- and high-skilled wages – a hump shaped relationship, whenever the offshoring rate is initially low.*

(ii) ***Polarizing wage effect****, associated with a relative decline of medium-skilled wages compared to both low- and high-skilled wages – a U-shaped relationship, whenever the offshoring rate is initially high.*

Proof. See the discussion in the text.

Comparing the results in Proposition 7 with those of the case of medium-skill immigration discussed in Proposition 2 and Proposition 4 it is readily seen that both types of labor shock have a similar impact on natives relative wage structure, but the adjustment mechanism is different. Medium-skill immigration leads to an internal reorganization of work (i.e. task reallocation), where natives react endogenously by specializing in communicative tasks, while offshoring of medium skill-intensive jobs (intermediates) affects both immigrants and natives proportionately, leaving task intensity unaffected. Although the latter is by construction, the results highlight that the direction of distributional impact of both medium-skill immigration and offshoring for native workers depends crucially on their initial share in the economy.

It is also worth noticing that the results in Propositions 2, 4, and 7 differ substantially from those derived in Olney (2012) and Ottaviano et al. (2013), who investigate jointly the impact of offshoring and immigration. The key arguments are that implicit assumption of perfect substitutability between immigrants and natives, e.g. as in Olney (2012), miss to capture important aspects of potential efficiency improvements highlighted by the recent empirical evidence, and that accounting only for partial equilibrium effects, e.g. as in Ottaviano et al. (2013), neglect important general equilibrium implications.

7 Conclusion

Immigration and offshoring of domestic jobs are two phenomenons that have considerably shaped the public debate regarding their potential labor market effects in many advanced countries. Thus, understanding the determinants of the impact of offshoring and immigration is one of the key objectives of both policy makers and researchers. This paper has analyzed jointly the impact of these two phenomena of globalized labor services on natives wage structure and how their effects are interlinked. In doing so, I develop an integrated theoretical framework that features a richer structure of the economy consisting of firm heterogeneity, endogenous technology adoption, monopolistic competition, and skill heterogeneity. Moreover, the systematic framework allows to account for various potential adjustment channels of the receiving economy to offshoring and immigration shocks, and to investigate the determinants of and the interaction between these different forces from a general equilibrium perspective.

The theoretical analysis provides several novel insights. First, immigration and offshoring induce three main channels: (i) a market size effect, (ii) a technology adoption effect, and (iii) a productivity effect. More importantly, the relative magnitude between the first two effects is crucially determined by the degree of technology adoption. In addition, the direction of each of these two effects is importantly determined by the relative magnitude between consumer preferences (i.e. final-goods demand elasticity) and the market power of monopolists (i.e. the elasticity of substitution between intermediate goods). An interesting result of the analysis reveals that whenever the degree of technology adoption between different production modes becomes very easy, the market size and technology adoption effects are asymptotically balanced. Thus, the remaining channel is the productivity effect, associated with the cost-efficiency enhancing. Third, the magnitude of the productivity effect is stronger at low initial levels of task shares assigned to immigrants and of offshoring rate. The intuition is that at higher levels the labor market competition among domestic workers, associated with the direct displacement effect due to task reallocation in the case of immigration, or job relocation in the case of offshoring, becomes dominating. These results highlight the rich pattern of interaction between different forces. Nevertheless, the tractability of the framework permits to derive clear conditions regarding when some forces dominate others, and thus enabling to make clear predictions regarding the impacts on natives wage structure.

The comparative statics reveal the following predictions regarding immigration-induced changes in the relative wage structure of native workers. Whenever consumer preferences react more elastic relative to the market power of monopolists, low- and high-skilled immigration induce wage effects similar to "unskill-" and "skill-biased" technological changes, respectively, while medium-skilled immigration generates a hum-shaped relationship. The rationale behind these results is twofold. On the one hand by the profit-maximizing incentives, leading to endogenous increase in the mass of those firms employing most intensively this type of workers – indicating the direct market size effect. On the other hand, immigration raises the comparative advantages of complementing technology modes and thus by endogenous technology adoption behavior this leads to an expansion in the range of final goods produced that type of production techniques – indicating the indirect market size effect. However, this latter effect induces in turn a relative increase in the price of goods produced with the competing technology modes, and thus by the standard Stolper-Samuelson theorem raises the relative wages of those skill groups – indicating the price effect. However, with strong elastic consumer preferences, the demand shifts in favor of goods and workers that were hit by the immigration shock, and thus overcompensating the initial adverse effects.

More interestingly, in the empirically plausible case, i.e. when the market power of monopolists is larger than the goods demand elasticity, the predictions of distributional effects of immigration depends crucially on the level of task shares assigned to immigrants. Thus, at low levels of task shares, low- and high-skilled immigration both generate a polarizing wage effect, while medium-skilled immigration induces again a hump-shaped wage effect. This is due to strong productivity effects at low initial task shares, and the spill-over effects through the scale and price externalities mentioned above. In the case of offshoring, changes in the wage structure of native workers is similar to those induced by medium-skilled immigration. However, an interesting result is that even in the very long-run when technology adoption becomes fully realized, the remaining channel is the productivity effect induced by gains due to efficient reallocation of tasks and jobs.

References

Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4):781–809.

Acemoglu, D. and Autor, D. (2011). Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings. In Ashenfelter, O. and Card, D., editors, *Handbook of Labor Economics*, volume 4, Part B, pages 1043–1171. Elsevier.

Acemoglu, D., Gancia, G., and Zilibotti, F. (2012). Offshoring and direct technical change. *NBER, Working Paper 18595*.

Acemoglu, D. and Zilibotti, F. (2001). Productivity differences. *Quarterly Journal of Economics*, 116:563–606.

Autor, D. H. and Dorn, D. (2013). The growth of low skill service jobs and the polarization of the U.S. labor market. *American Economic Review*, 103(5):1553–1597.

Autor, D. H., Katz, L. F., and Kearney, M. S. (2006). The Polarization of the U.S. Labor Market. *American Economic Review*, 96(2):189–194.

Baldwin, R. and Robert-Nicoud, F. (2007). Offshoring: General equilibrium effects on wages, production and trade. *National Bureau of Economic Research, No. w12991*.

Beaudry, P., Doms, M., and Lewis, E. (2010). Should the personal computer be considered a technological revolution? evidence from us metropolitan areas. *Journal of Political Economy*, 118(5):988–1036.

Borghans, L., ter Weel, B., and Weinberg, B. A. (2008). Interpersonal styles and labor market outcomes. *The Journal of Human Resources*, XLIII(4).

Borghans, L., ter Weel, B., and Weinberg, B. A. (2013). People skills and the labor-market outcomes of underrepresented groups. *Industrial and Labor Relations Review*, xxx(x).

Costinot, A. and Vogel, J. (2010). Matching and Inequality in the World Economy. *Journal of Political Economy*, 118(4):747–786.

Dustmann, C. and Glitz, A. (2012). How do industries and firms respond to changes in local labor supply? IZA Discussion Paper No.6257.

Egger, H., Kreickemeier, U., and Wrona, J. (2013). Offshoring domestic jobs. mimeo.

Ethier, W. (1982). National and International Returns to Scale in the Modern Theory of Iinternational Trade. *The American Economic Review*, 72(3):389–405.

Feenstra, R. C. and Hanson, G. H. (1999). The impact of outsourcing and high-technology capital on wages: Estimates for the united states, 1979–1990. *The Quarterly Journal of Economics*, 86(2):240–245.

Gauthier-Loiselle, M. and Hunt, J. (2008). How much does immigration boost innovation? *NBER Working Paper No. 14312*.

Ghosh, A., Mayda, A. M., and Ortega, F. (2014). The impact of skilled foreign workers on firms: An investigation of publicly traded us firms. *IZA DP No. 8684*.

González, L. and Ortega, F. (2011). How do very open economies adjust to large immigration flows? Evidence from Spanish regions. *Labour Economics*, 18:57–70.

Goos, M. and Manning, A. (2007). Lousy and Lovely Jobs: The Rising Polarization of Work in Britain. *Review of Economic Studies*, 89:118–133.

Goos, M., Manning, A., and Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review*, 104:2509–2526.

Groizard, J. L., Ranjan, P., and Rodriguez-Lopez, A. (2014). Offshoring and jobs: The myriad channels of influence. *European Economic Review*, 72:221–239.

Grossman, G. M. and Rossi-Hansberg, E. (2008). Trading Task: A Simple Theory of Offshoring. *American Economic Review*, 98:1978–1997.

Jones, R. W. and Kierzkowski, H. (2001). A framework for fragmentation. In Arndt, S. and Kierzkowski, H., editors, *Fragmentation: New Production Patterns in the World Economy*, pages 17–45. Oxford University Press, New York.

Kohler, W. (2004). Aspects of international fragmentation. *Review of International Economics*, 12:793–816.

Lewis, E. G. (2003). Local open economies within the U.S.: How do industries respond to immigration. Federal Reserve Bank of Philadelphia Working Paper No. 04-03.

Melitz, M. (2003). The Impact of Trade on Intra-Idustry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71:1695–1725.

Michaels, G., Natraj, A., and Van Reenen, J. (2014). Has ICT Polarized Skill Demand? Evidence from Eleven Countries over Twenty-Five Years. *The Review of Economics and Statistics*, 96(1):60–77.

Olney, W. W. (2012). Offshoring, immigration, and the native wage distribution. *Canadian Journal of Economics*, 45:830–856.

Ottaviano, G. I. P., Peri, G., and Wright, G. C. (2013). Immigration, Offshoring and American Jobs. *American Economic Review*, 103(5):1925–59.

Peri, G. (2012). The effect of immigration on productivity: Evidence from us states. *Review of Economics and Statistics*, 94(1):348–358.

Peri, G. and Sparber, C. (2009). Task specialization, immigration, and wages. *American Economic Journal: Applied Economics*, 1(3):135–169.

Rodríguez-Clare, A. (2010). Offshoring in a Ricardian World. *American Economic Journal: Macroeconomics*, 2(2):227–258.

Spitz-Oener, A. (2006). Technical change, job tasks, and rising educational demands: Looking outside the wage structure. *Journal of Labour Economics*, 24:235–270.

Stasz, C. (1997). Do employers need the skills they want? Evidence from technical work. *Journal of Education and Work*, 10(3):205–223.

Vallizadeh, E. (2015). Immigration, technology adoption and wage inequality. mimeo.

Vallizadeh, E., Muysken, J., and Ziesemer, T. (2015). Offshoring of Medium-skill Jobs, Polarization, and Productivity Effect: Implications for Wages and Low-skill Unemployment. IAB-Discussion Papers, 7/2015.

Mathematical Appendix

A

A.1 Proof of Lemma 1

The proof of the optimal task assignment between immigrants and natives can be conducted in two steps. First, I derive the optimal amount of composite factor labor L_k for a given I_k, and then derive the optimal task margin I_k. If L_k^D and L_k^F are the amounts of native and immigrant workers employed by any intermediate firm $j \in [0, A_k]$, it must satisfy

$$\int_0^{I_k} l_k^F(i)\mathrm{d}i = L_k^F, \tag{A.1}$$

$$\int_{I_k}^{1} l_k^D(i)\mathrm{d}i = L_k^D. \tag{A.2}$$

The optimal amount of workers across the respective range of tasks is obtained by minimizing the cost

$$\min_{l_k^F(i), l_k^D(i)} \int_0^{I_k} w_k^F l_k^F(i)\mathrm{d}i + \int_{I_k}^1 w_k^D l_k^D(i)\mathrm{d}i \quad \text{s.t.} \quad (11) \tag{A.3}$$

The first-order conditions yield

$$w_k^D = \tilde{w}_k \left(\frac{x_k}{l_k^D(i)}\right)^{\frac{1}{\alpha}}, \tag{A.4}$$

$$w_k^F = \tilde{w}_k \left(\frac{x_k}{l_k^F(i)}\right)^{\frac{1}{\alpha}} (\gamma_k(i))^{\frac{1-\alpha}{\alpha}}, \tag{A.5}$$

where $\tilde{w}_k$ denotes the Lagrangian multiplier. It follows from the optimality conditions (A.4) and (A.5)

$$l_k^D(i) = l_k^D(i'), \quad \text{for} \quad i, i' \in (I_k, 1] \tag{A.6}$$

$$l_k^F(i) = l_k^F(i') \left(\frac{\gamma_k(i)}{\gamma_k(i')}\right)^{1-\alpha}, \quad \text{for} \quad i, i' \in [0, I_k] \tag{A.7}$$

Thus, for the domestic labor it follows from equations (A.2) and (A.6) that

$$l_k^D(i) = \frac{L_k^D}{1 - I_k} \tag{A.8}$$

For the allocation of offshore workers, note that from Eq. (A.7), it must hold

$$l_k^F(i) = l_k^F(I_k) \left(\frac{\gamma_k(i)}{\gamma_k(I_k)}\right)^{1-\alpha}, \quad \text{for all} \ i \in [0, I_k] \tag{A.9}$$

Plugging Eq. (A.9) into Eq. (A.1), one can solve for $l_k^F(I_k)$ and substituting this expression back into Eq. (A.9) yields

$$l_k^F(i) = \left(\frac{\gamma_k(i)^{1-\alpha}}{\int_0^{I_k} \gamma_k(i)^{1-\alpha}\mathrm{d}i}\right) L_k^F \tag{A.10}$$

Thus, substituting Eqs. (A.8) and (A.10) into Eq. (11) yields

$$x_k = \tilde{L}_k = \left(\left[\int_0^{I_k} \gamma_k(i)^{1-\alpha} \mathrm{d}i \right]^{\frac{1}{\alpha}} \left(L_k^F\right)^{\frac{\alpha-1}{\alpha}} + [1 - I_k]^{\frac{1}{\alpha}} \left(L_k^D\right)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}}. \tag{A.11}$$

Now to prove that the optimal fraction of offshoring I_k is cost efficient, substitute the first-order conditions (A.4) and (A.5) for $l_k^D(i)$ and $l_k^F(i)$, respectively, into Eq. (11) and manipulate to obtain

$$\tilde{w}_k = \left(\left(\int_0^{I_k} \gamma_k(i)^{1-\alpha} \mathrm{d}i \right) \left(w_k^F\right)^{1-\alpha} + [1 - I_k] \left(w_k^D\right)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \tag{A.12}$$

Thus, Eq. (A.12) denotes the marginal cost of any differentiated-good producer $j \in [0, A_k]$ employing immigrant and native workers. Now, the optimal choice of marginal task I_k is obtained by minimizing $\tilde{w}_k$ with respect to I_k:

$$\frac{\mathrm{d}\tilde{w}_k}{\mathrm{d}I_k} = \frac{1}{1-\alpha} \tilde{w}_k^{\alpha} \left(\gamma_k(I_k)^{1-\alpha} \left(w_k^F\right)^{1-\alpha} - \left(w_k^D\right)^{1-\alpha} \right) = 0.$$

Thus, we get that $\frac{\mathrm{d}\tilde{w}_k}{\mathrm{d}I_k} = 0$ if and only if

$$\gamma_k(I_k) w_k^F = w_k^D,$$

so that an intermediate-good producer allocates tasks to immigrants if and only if $\gamma_k(i) w_k^F \leq w_k^D$. ■

A.2 Derivation of equilibrium mass of monopolists A_k

The derivation of the mass of monopolists in every technology mode can be derived as follows. First, notice that the perfectly competitive nature of the final goods market requires zero profits, i.e.

$$p_k(z) y_k(z) = \zeta_k X_k, \;\; \forall \;\; k = \{L, M, H\}$$

Using Eqs. (2) and (3), it follows that the price index of technology mode k has to be equal to its marginal cost

$$P_k = \zeta_k, \tag{A.13}$$

a result that has to hold in any perfectly competitive market. Now, utilizing the equilibrium value of ζ_k from Eq. (15), the equilibrium price index can be expressed as

$$P_k = \zeta_k = \frac{\epsilon}{\epsilon - 1} A_k^{\frac{1}{1-\epsilon}} \tilde{w}_k. \tag{A.14}$$

From the total budget constraint for any final good producer z using technology mode k requires

$$p_k(z) y_k(z) = w_k^F n_k^F + w_k^D n_k^D = \frac{1}{\mathcal{S}_k} \left(w_k^F N_k^F + w_k^D N_k^D \right),$$

where the second equality follows from Eq. (23).

Utilize Eqs. (2) and (3) in the left hand side of the previous equation and manipulate to obtain

$$P_k A_k^{\frac{\epsilon}{\epsilon-1}} x_k = \frac{w_k^D N_k^D}{\mathcal{S}_k}\left(1+\frac{w_k^F}{w_k^D}\frac{N_k^F}{N_k^D}\right).$$

Next, use the FE condition (14), and Eq. (A.14) to substitute for x_M and P_M in the previous equation, respectively, so that after some manipulation we obtain

$$A_k \epsilon f_k \tilde{w}_k = \frac{w_k^D N_k^D}{\mathcal{S}_k}\left(1+\frac{w_k^F}{w_k^D}\frac{N_k^F}{N_k^D}\right).$$

Next, substitute Eq. (A.18) for w_k^F/w_k^D in the right hand side and after simple manipulation we obtain

$$\begin{aligned} A_k \epsilon f_k \tilde{w}_k &= \frac{w_k^D N_k^D}{\mathcal{S}_k}\left(1+\left(\frac{\vartheta_k(I_k)}{(1-I_k)}\right)^{1/\alpha}\left(\frac{N_k^F}{N_k^D}\right)^{-\frac{1}{\alpha}}\frac{N_k^F}{N_k^D}\right) \\ A_k \epsilon f_k \tilde{w}_k &= \frac{1}{\mathcal{S}_k}\frac{w_k^D\left(N_k^D\right)^{\frac{1}{\alpha}}}{(1-I_k)^{1/\alpha}}\left((1-I_k)^{1/\alpha}\left(N_k^D\right)^{\frac{\alpha-1}{\alpha}}+\vartheta_k(I_k)^{\frac{1}{\alpha}}\left(N_k^F\right)^{\frac{\alpha-1}{\alpha}}\right) \end{aligned} \tag{A.15}$$

Recall the first order condition (A.5) and utilize Eqs. (A.8), (A.10), and (A.11) to obtain

$$\begin{aligned} w_k^D\left(l_k^D\right)^{1/\alpha} &= \tilde{w}_k x_k^{1/\alpha} \\ w_k^D\left(\frac{L_k}{1-I_k}\right)^{1/\alpha} &= \tilde{w}_k \tilde{L}_k^{1/\alpha} \end{aligned}$$

where recall

$$\begin{aligned} \tilde{L}_k &= \left(\vartheta_k(I_k)\left(L_k^F\right)^{\frac{\alpha-1}{\alpha}}+(1-I_k)^{1/\alpha}\left(L_k^D\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}} \\ \vartheta_k(I_k) &\equiv \left(\int_0^{I_k}\gamma_k(i)^{1-\alpha}\mathrm{d}i\right)^{1/\alpha} \end{aligned}$$

Now, substituting the total resource constraint condition (18) for L_M and L_O, we obtain

$$w_k\left(\frac{N_k^D}{1-I_k}\right)^{1/\alpha} = \tilde{w}_k \tilde{N}_k^{1/\alpha}. \tag{A.16}$$

Then, substituting (A.16) for $w_k^D\left(\frac{N_k^D}{1-I_k}\right)^{1/\alpha}$ in (A.15) and rearranging slightly yields

$$A_k = \frac{\tilde{N}_k}{\epsilon f_k \mathcal{S}_k} \tag{A.17}$$

A.3 Derivation of equilibrium labor composite and marginal costs

From (A.4) and (A.5) the inverse relative demand labor demand between offshore and medium-skilled workers is defined by

$$\frac{w_k^F}{w_k^D} = \left(\frac{l_k^F(i)}{l_k^D(i)}\right)^{-\frac{1}{\alpha}}\left[\gamma_k(i)\right]^{\frac{1-\alpha}{\alpha}},$$

and using Eqs. (A.8) and (A.10) yields

$$\frac{w_k^F}{w_k^D} = \left(\frac{L_k^F}{L_k^D}\right)^{-\frac{1}{\alpha}} \frac{\left[\int_0^{I_k} \gamma_k(i)^{1-\alpha} \mathrm{d}i\right]^{\frac{1}{\alpha}}}{(1-I_k)^{1/\alpha}}.$$

Finally, utilizing the overall labor market clearing condition derived in Eqs. (17), (18), and (19) the previous equation can be written as

$$\frac{w_k^F}{w_k^D} = \left(\frac{N_k^F}{N_k^D}\right)^{-\frac{1}{\alpha}} \left(\frac{\vartheta_k(I_k)}{(1-I)}\right)^{1/\alpha}, \tag{A.18}$$

where $\vartheta_k(I_k) \equiv \left(\int_0^I \gamma_k(i)^{1-\alpha}\mathrm{d}i\right)$. Then, combining (A.18) with the cost-efficient condition task allocation, Eq. (13), yields

$$\frac{1}{\gamma_k(I_k)} = \left(\frac{N_k^F}{N_k^D}\right)^{-\frac{1}{\alpha}} \left(\frac{\vartheta_k(I_k)}{(1-I)}\right)^{1/\alpha} \tag{A.19}$$

Next, solving the previous derived equation for N_k^F and combining the result with the labor market clearing conditions , Eqs. (17)–(19), to insert into Eq. (A.11), yields

$$x_k = \frac{\tilde{N}_k}{\mathcal{S}_k A_k} \tag{A.20}$$

$$\begin{aligned} \tilde{N}_k &= \left(\left[\int_0^{I_k} \gamma_k(i)^{1-\alpha}\mathrm{d}i\right]^{\frac{1}{\alpha}} \left(N_k^F\right)^{\frac{\alpha-1}{\alpha}} + (1-I_k)^{\frac{1}{\alpha}} \left(N_k^D\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}} \\ &= \left(\vartheta_k(I_k)^{\frac{1}{\alpha}} \left(N_k^F\right)^{\frac{\alpha-1}{\alpha}} + (1-I_k)^{\frac{1}{\alpha}} \left(N_k^D\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}. \end{aligned} \tag{A.21}$$

Now, using Eq. (A.19) into the previous equation and manipulating slightly yields

$$\tilde{N}_k = \left(\vartheta_k(I_k)\gamma_k(I_k)^{\alpha-1} + (1-I_k)\right)^{\frac{\alpha}{\alpha-1}} \frac{N_k^D}{1-I_k} = \mathcal{M}_k(\Theta_k, I_k) N_k^D, \tag{A.22}$$

and from Eqs. (13) and (A.12) we obtain

$$\tilde{w}_k = \left(\vartheta_k(I_k)\gamma_k(I_k)^{\alpha-1} + 1 - I_k\right)^{\frac{1}{1-\alpha}} w_k^D = \Theta_k(I_k) w_k^D, \tag{A.23}$$

where $\Theta_k(I_k)$ denotes the generalized measure of marginal cost of task production.

A.4 Proof of Lemma 2

Notice, first, that Eq. (A.19) implies that task margin I_k depends only on the endowments of immigrant and native workers. Rearranging Eq. (A.19) and taking logs, yields $\ln\left(\frac{N_k^F}{N_k^D}\right) = \alpha \ln \gamma_k(I_k) + \ln \vartheta_k(I_k) - \ln(1-I_k)$ Now, recalling the property of $\gamma_k'(i) > 0$ and differentiating this expression with respect to N_k^F we get

$$\frac{\mathrm{d}\ln I_k}{\mathrm{d}\ln N_k^F} = \left(\alpha\mu_{\gamma_k} + \frac{\gamma_k(I_k)^{1-\alpha} I_k}{\vartheta_k(I_k)} + \frac{I_k}{1-I_k}\right)^{-1} > 0. \tag{A.24}$$

From Eq. (A.23), we can compute the impact of changes in teh task margin on the marginal cost of monopolists. Differentiating (A.23) w.r.t. I_k yields

$$\frac{\mathrm{d}\ln\Theta_k(I_k)}{\mathrm{d}\ln I_k} = -\Theta_k(I_k)^{\alpha-1}\vartheta_k(I_k)\gamma_k(I_k)^{\alpha-1}\mu_{\gamma_k} < 0, \tag{A.25}$$

where $\mu_{\gamma_k} \equiv \frac{\gamma_k'(I_k)I_k}{\gamma_k(I_k)} > 0$ denotes the elasticity of comparative advantage schedule of immigrants with respect to task margin I_k. Moreover, utilizing the outcome in Eq. (A.25), it follows from Eqs. (A.22) and (A.23) that, for $I_k > 0$, the labor composite is increasing and the marginal cost of intermediate-good firm is decreasing in task margin I_k, i.e.

$$\frac{\mathrm{d}\ln\tilde{N}_k}{\mathrm{d}\ln I_k} = -\frac{\mathrm{d}\ln\mathcal{M}_k}{\mathrm{d}\ln I_k} = -\alpha\frac{\mathrm{d}\ln\Theta_k(I_k)}{\mathrm{d}\ln I_k} + \frac{I_k}{1-I_k} > 0 \tag{A.26}$$

$$\frac{\mathrm{d}\ln\tilde{w}_k}{\mathrm{d}\ln I_k} = \frac{\mathrm{d}\ln\Theta_k(I_k)}{\mathrm{d}\ln I_k} < 0 \tag{A.27}$$

■

A.5 Derivation of aggregate sectoral output

The derivation of the aggregate sectoral output is illustrated for the low-tech goods. The aggregation over the range of all final goods using the other two types of technology can be derived similarly. Define the total revenue using low-quality technology as

$$P_L Y_L = \int_0^{z_L} p(z)y(z)\mathrm{d}z.$$

Utilizing Eqs. (2) and (3) yields

$$\begin{aligned} P_L Y_L &= \int_0^{z_L} p(z)\varphi_L(z)X_L\mathrm{d}z, \\ &= \int_0^{z_L} P_L X_L\mathrm{d}z, \\ &= P_L z_L X_L. \end{aligned}$$

Then, by equilibrium condition (21), it follows

$$Y_L = A_L^{\frac{1}{\epsilon-1}}\tilde{N}_L.$$

Following the same steps, we obtain

$$\begin{aligned} Y_M &= A_M^{\frac{1}{\epsilon-1}}\tilde{N}_M, \\ Y_H &= A_H^{\frac{1}{\epsilon-1}}\tilde{N}_H. \end{aligned}$$

A.6 Proof of Proposition 1

The computation of the comparative statics for the other exogenous changes is similar to the one conducted for changes in the task margin I_M, associated with medium-skill immigration, recall Eq. (A.24). Notice that by Eq. (A.19) task margin I_k depends only on the endowments of skill type k. Recall the definition $\Psi_k = -\frac{\mathrm{d}\ln \mathcal{M}_k(I_k)}{\mathrm{d}\ln I_k} > 0$ and differentiate the implicit 2×2 system, Eqs. (30) and (31) with respect to changes in the task margins and the fixed costs, yields:

- Low-skill immigration and L-fixed costs:

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_L} = \frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln \mathcal{M}_L}\frac{\mathrm{d}\ln \mathcal{M}_L}{\mathrm{d}\ln I_L} = \Psi_L \frac{\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_H z_H + \frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}}{\mathcal{D}_J} > 0, \tag{A.28}$$

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln f_L} = -\frac{1}{\Psi_L}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_L} < 0 \tag{A.29}$$

and

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_L} = \frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln \mathcal{M}_L}\frac{\mathrm{d}\ln \mathcal{M}_L}{\mathrm{d}\ln I_L} = \Psi_L \frac{\frac{z_L}{(z_H-z_L)}}{\mathcal{D}_J} > 0. \tag{A.30}$$

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln f_L} = -\frac{1}{\Psi_L}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_L} < 0 \tag{A.31}$$

- High-skill immigration and H-fixed costs:

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_H} = \frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln \mathcal{M}_H}\frac{\mathrm{d}\ln \mathcal{M}_H}{\mathrm{d}\ln I_H} = -\Psi_H \frac{\frac{z_H}{(z_H-z_L)}}{\mathcal{D}_J} < 0, \tag{A.32}$$

$$\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln f_H} = -\frac{1}{\Psi_H}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_H} > 0 \tag{A.33}$$

and

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_H} = \frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln \mathcal{M}_H}\frac{\mathrm{d}\ln \mathcal{M}_H}{\mathrm{d}\ln I_H} = -\Psi_H \frac{\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L + \frac{z_H}{(z_H-z_L)}}{\mathcal{D}_J} < 0. \tag{A.34}$$

$$\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln f_H} = -\frac{1}{\Psi_H}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_H} > 0 \tag{A.35}$$

where the Jacobi is given by

$$\mathcal{D}_J = \left(\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L + \frac{1}{z_L}\right)\left(\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_H z_H + \frac{1-z_L}{(1-z_H)(z_H-z_L)}\right) + \frac{1}{z_H-z_L}\left(\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_H z_H + \frac{1}{1-z_H}\right) > 0 \tag{A.36}$$

Moreover, from (A.28) and (A.30) it can be readily verified that

$$\frac{\mathrm{d}\ln z_H - \mathrm{d}\ln z_L}{\mathrm{d}\ln I_L} = -\frac{\Psi_L}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_H z_H + \frac{1}{(1-z_H)}\right) < 0 \tag{A.37}$$

and similarly from (A.32) and (A.34)

$$\frac{\mathrm{d}\ln z_H - \mathrm{d}\ln z_L}{\mathrm{d}\ln I_H} = -\frac{\Psi_H}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\tilde{\varepsilon}_L z_L\right) < 0 \tag{A.38}$$

Since $\Psi_k(I_k)$ depends on the initial level of the task margin, it follows from Eq. (A.26) that for any $k = \{L, M, H\}$

$$\begin{aligned}\lim_{I_k \to 0} \Psi_k = 0 \quad &\Rightarrow \quad \lim_{I_k \to 0} \left| \frac{\mathrm{d} \ln z_L}{\mathrm{d} \ln I_k} \right| = \left| \frac{\mathrm{d} \ln z_H}{\mathrm{d} \ln I_k} \right| = 0 \\ \lim_{I_k \to 1} \Psi_k = \infty \quad &\Rightarrow \quad \lim_{I_k \to 1} \left| \frac{\mathrm{d} \ln z_L}{\mathrm{d} \ln I_k} \right| = \left| \frac{\mathrm{d} \ln z_H}{\mathrm{d} \ln I_k} \right| = \infty\end{aligned} \tag{A.39}$$

■

A.7 Derivation of relative wages

To derive the relative wages structure for native workers, we proceed as follows. Recall the first order condition defining the final goods demand, Eq. (6). Now, multiplying both sides by $p(z)$ and noticing that from Eqs. (3), (26), and (27) it follows $p_L(z)y_L(z) = P_LY_L/z_L$ and $p_M(z)y_M(z) = P_MY_M/(z_H - z_L)$, and $p_H(z)y_H(z) = P_HY_H/(1 - z_H)$, we get

$$\begin{aligned}\frac{P_LY_L}{z_L} &= YP_L^{1-\sigma}\varphi_L(z)^{\sigma-1}, \quad \forall\, z \in \{0, z_L\} \\ \frac{P_MY_M}{z_H - z_L} &= YP_M^{1-\sigma}\varphi_M(z')^{\sigma-1}, \quad \forall\, z' \in \{z_L, z_H\} \\ \frac{P_HY_H}{1 - z_H} &= YP_H^{1-\sigma}\varphi_H(z'')^{\sigma-1}, \quad \forall\, z'' \in \{z_H, 1\}\end{aligned}$$

Rearranging and taking the ration with respect to $k = M$, we get the relative aggregate demand for final goods

$$\begin{aligned}\frac{P_L}{P_M} &= \left(\frac{z_L}{z_H - z_L}\right)^{\frac{1}{\sigma}} \left(\frac{Y_L}{Y_M}\right)^{-\frac{1}{\sigma}} \Lambda_L(z, z'), \\ \frac{P_M}{P_H} &= \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{1}{\sigma}} \left(\frac{Y_M}{Y_H}\right)^{-\frac{1}{\sigma}} \Lambda_H(z', z''),\end{aligned}$$

where $\Lambda_L(z, z') \equiv \left(\frac{\varphi_L(z')}{\varphi_M(z'')}\right)^{\sigma-1}$ and $\Lambda_H(z', z'') \equiv \left(\frac{\varphi_M(z')}{\varphi_H(z'')}\right)^{\sigma-1}$.[9] Next, use the equilibrium outcomes (27) to substitute for Y_k to obtain

$$\frac{P_L}{P_M} = \left(\frac{z_L}{z_H - z_L}\right)^{\frac{1}{\sigma}} \left[\frac{\tilde{N}_L}{\tilde{N}_M} \left(\frac{A_L}{A_M}\right)^{\frac{1}{\epsilon-1}}\right]^{-\frac{1}{\sigma}} \Lambda_L(z, z'), \tag{A.40}$$

$$\frac{P_M}{P_H} = \left(\frac{z_H - z_L}{1 - z_H}\right)^{\frac{1}{\sigma}} \left[\frac{\tilde{N}_M}{\tilde{N}_H} \left(\frac{A_M}{A_H}\right)^{\frac{1}{\epsilon-1}}\right]^{-\frac{1}{\sigma}} \Lambda_H(z', z''), \tag{A.41}$$

Next, use the equilibrium condition (34) and take the ratio with respect to technology mode $k = M$ to

[9] Notice that the relative final goods demand at the margins, z_L and z_H, is undefined since producers are indifferent regarding the adoption of different production technology modes.

obtain

$$\frac{\tilde{w}_L}{\tilde{w}_M} = \frac{\zeta_L}{\zeta_M}\left(\frac{X_L}{X_M}\right)^{1/\epsilon}\left(\frac{x_L}{x_M}\right)^{-1/\epsilon}$$
$$\frac{\tilde{w}_M}{\tilde{w}_H} = \frac{\zeta_M}{\zeta_H}\left(\frac{X_M}{X_H}\right)^{1/\epsilon}\left(\frac{x_M}{x_H}\right)^{-1/\epsilon}$$

Now, recalling the symmetry assumption and using Eq. (16) to substitute for ζ_k we get

$$\frac{\tilde{w}_L}{\tilde{w}_M} = \frac{P_L}{P_M}\left(\frac{A_L}{A_M}\right)^{1/(\epsilon-1)}$$
$$\frac{\tilde{w}_M}{\tilde{w}_H} = \frac{P_M}{P_H}\left(\frac{A_M}{A_H}\right)^{1/(\epsilon-1)}$$

Next, utilize Eqs. (A.40) and (A.41) in the previous equations to substitute for P_L/P_M and P_M/P_H, respectively.

$$\frac{\tilde{w}_L}{\tilde{w}_M} = \left(\frac{z_L}{z_H-z_L}\right)^{\frac{1}{\sigma}}\left[\frac{\tilde{N}_L}{\tilde{N}_M}\left(\frac{A_L}{A_M}\right)^{\frac{1}{\epsilon-1}}\right]^{-\frac{1}{\sigma}}\left(\frac{A_L}{A_M}\right)^{1/(\epsilon-1)}\Lambda_L(z,z')$$
$$\frac{\tilde{w}_M}{\tilde{w}_H} = \left(\frac{z_H-z_L}{1-z_H}\right)^{\frac{1}{\sigma}}\left[\frac{\tilde{N}_M}{\tilde{N}_H}\left(\frac{A_M}{A_H}\right)^{\frac{1}{\epsilon-1}}\right]^{-\frac{1}{\sigma}}\left(\frac{A_M}{A_H}\right)^{1/(\epsilon-1)}\Lambda_H(z',z'')$$

Now, using the equilibrium expression for A_k, Eq. (22) and manipulating slightly we obtain

$$\frac{\tilde{w}_L}{\tilde{w}_M} = \left(\frac{z_L}{z_H-z_L}\right)^{\frac{1}{\sigma}}\left(\frac{z_L}{z_H-z_L}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}}\left(\frac{\tilde{N}_L}{\tilde{N}_M}\right)^{\frac{\sigma-\epsilon}{(\epsilon-1)\sigma}}\left(\frac{f_L}{f_M}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}}\Lambda_L(z,z')$$
$$\frac{\tilde{w}_M}{\tilde{w}_H} = \left(\frac{z_H-z_L}{1-z_H}\right)^{\frac{1}{\sigma}}\left(\frac{z_H-z_L}{1-z_H}\right)^{\frac{-(\sigma-1)}{(\epsilon-1)\sigma}}\left(\frac{\tilde{N}_M}{\tilde{N}_H}\right)^{\frac{\sigma-\epsilon}{(\epsilon-1)\sigma}}\left(\frac{f_M}{f_H}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}}\Lambda_H(z',z'')$$

Finally using the Eqs. (23) and (24) to substitute for $\tilde{N}_k$ and $\tilde{w}_k$, respectively, then we can rewrite the previously derived equations in terms of relative wages of native workers, i.e.

$$\frac{w_L^D}{w_M^D} = \left(\frac{z_L}{z_H-z_L}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}\left(\frac{\Theta_L(I_L)}{\Theta_M(I_M)}\right)^{\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1}\left(\frac{1-I_L}{1-I_M}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}}\left(\frac{N_L^D}{N_M^D}\right)^{\frac{(\sigma-\epsilon)}{(\epsilon-1)\sigma}}\left(\frac{f_L}{f_M}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}}\Lambda_L(z,z')$$
$$\frac{w_M^D}{w_H^D} = \left(\frac{z_H-z_L}{1-z_H}\right)^{\frac{\epsilon-\sigma}{(\epsilon-1)\sigma}}\left(\frac{\Theta_M(I_M)}{\Theta_H(I_H)}\right)^{\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1}\left(\frac{1-I_M}{1-I_H}\right)^{\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}}\left(\frac{N_M^D}{N_H^D}\right)^{\frac{(\sigma-\epsilon)}{(\epsilon-1)\sigma}}\left(\frac{f_M}{f_H}\right)^{-\frac{\sigma-1}{(\epsilon-1)\sigma}}\Lambda_H(z',z'')$$

A.8 Derivation of immigration-induced changes in relative range of final goods

Low-skill immigration

To obtain explicit solution for the terms $\tilde{\Gamma}_L \equiv \frac{\mathrm{d}\ln[z_L/(z_H-z_L)]}{\mathrm{d}\ln I_L} = \frac{\mathrm{d}\ln\mathcal{M}_L}{\mathrm{d}\ln I_L}\frac{\mathrm{d}\ln[z_L/(z_H-z_L)]}{\mathrm{d}\ln\mathcal{M}_L}$ and $\tilde{\Delta}_{I_L} \equiv \frac{\mathrm{d}\ln[(z_H-z_L)/(1-z_H)]}{\mathrm{d}\ln I_L} = \frac{\mathrm{d}\ln\mathcal{M}_L}{\mathrm{d}\ln I_L}\frac{\mathrm{d}\ln[(z_H-z_L)/(1-z_H)]}{\mathrm{d}\ln\mathcal{M}_L}$, we can utilize the results of the comparative statics in Eqs. (A.28) and (A.30) to obtain the following terms

$$\begin{aligned}\tilde{\Gamma}_L &= \frac{z_H}{z_H - z_L}\left(\frac{\mathrm{d}\ln z_L - \mathrm{d}\ln z_H}{\mathrm{d}\ln I_L}\right) \\ &= \Psi_L \Gamma_L > 0,\end{aligned} \tag{A.42}$$

where $\Gamma_L = \frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_H z_H z_H}{(z_H - z_L)} + \frac{z_H}{(1-z_H)(z_H-z_L)}\right) > 0$, and

$$\begin{aligned}\tilde{\Delta}_{I_L} &= -\left(\frac{z_L}{z_H - z_L}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_L} - \frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_L}\right) \\ &= \Psi_L \Delta_L < 0,\end{aligned} \tag{A.43}$$

where $\Delta_L = -\frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_H z_H z_L}{(z_H-z_L)}\right) < 0$. It is immediately evident that the size of Γ_L and Δ_L depends, on the one hand, on the degrees of $\tilde{\varepsilon}_L$ and $\tilde{\varepsilon}_H$. On the other hand, it depends on the initial level of task margin I_L, captured by Ψ_L. As shown in Vallizadeh (2015, Appendix B.3), it follows that $0 < \Gamma_L < 1$ and $0 < |\Delta_L| < 1$ for $0 < \{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} < \infty$.

Moreover, both the labor supply effect, $I_L/(1-I_L)$, and the productivity effect, $\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L}$, depend also on the initial level of the task margin. Thus, taking the limits and utilizing Eq. (A.25), we get that

$$\lim_{I_L\to 0}\frac{I_L}{1-I_L} = 0, \quad \text{and} \quad \lim_{I_L\to 1}\frac{I_L}{1-I_L} = \infty$$
$$\lim_{I_L\to 0}\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L} = 0, \quad \text{and} \quad \lim_{I_L\to 1}\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L} = -\mu_{\gamma_L}$$

Medium-skill immigration

Following the same steps, we can solve for an explicit solution of the terms $\tilde{\Gamma}_{I_M} \equiv \frac{\mathrm{d}\ln[z_L/(z_H-z_L)]}{\mathrm{d}\ln I_M}$ and $\tilde{\Delta}_{I_M} \equiv \frac{\mathrm{d}\ln[(z_H-z_L)/(1-z_H)]}{\mathrm{d}\ln I_M}$, we can utilize the results of the comparative statics derived in the main text, Eqs. (32) and (33), to obtain the following terms

$$\begin{aligned}\tilde{\Gamma}_M &= \frac{z_H}{z_H - z_L}\left(\frac{\mathrm{d}\ln z_L - \mathrm{d}\ln z_H}{\mathrm{d}\ln I_M}\right) \\ &= \Psi_M \Gamma_M < 0\end{aligned} \tag{A.44}$$

where $\Gamma_M = -\frac{1}{\mathcal{D}_J}\left(\frac{z_H}{(z_H-z_L)(1-z_H)} + \frac{\epsilon-1}{\epsilon}(\tilde{\varepsilon}_L z_L + \tilde{\varepsilon}_H z_H)\frac{z_H}{(z_H-z_L)}\right) < 0$, and

$$\begin{aligned}\tilde{\Delta}_M &= \left(\frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_M} - \frac{z_L}{z_H-z_L}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_M}\right) \\ &= \Psi_M \Delta_M > 0\end{aligned} \tag{A.45}$$

where $\Delta_M = \frac{1}{\mathcal{D}_J}\left(\frac{z_H}{(1-z_H)(z_H-z_L)} + \frac{\epsilon-1}{\epsilon}\left(\frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}\tilde{\varepsilon}_L z_L + \frac{z_L}{z_H-z_L}\tilde{\varepsilon}_H z_H\right)\right) > 0$ Now, utilizing the results in (A.39) for $k = H$ and given $0 < \{\tilde{\varepsilon}_L, \tilde{\varepsilon}_M\} < \infty$, we get that

$$\lim_{I_M\to 0}\Psi_M = 0 \quad \Rightarrow \quad \lim_{I_M\to 0}\tilde{\Delta}_M = -\tilde{\Gamma}_M = \infty$$
$$\lim_{I_M\to 1}\Psi_M = \infty \quad \Rightarrow \quad \lim_{I_M\to 1}\tilde{\Delta}_M = \tilde{\Gamma}_M = 0$$

Moreover, from Eqs. (A.44) and (A.45), we get that

$$\begin{aligned} \Delta_M &\lesseqgtr |\Gamma_M| \\ \frac{\tilde{\varepsilon}_L}{1-z_H} &\lesseqgtr \frac{\tilde{\varepsilon}_H}{z_H} \end{aligned}$$

implying that the magnitude of changes in the technology margin z_L relative to $(1-z_H)$ depends on the size of the elasticities at the respective margins. That is, if $\frac{\tilde{\varepsilon}_L}{1-z_H} < \frac{\tilde{\varepsilon}_H}{z_H}$ the adoption between L and M technology modes in the neighborhood of z_L is easier relative to that between M and H technology modes in the neighborhood of z_H.

High-skill immigration

Following the same steps, we can solve for an explicit solution of the terms $\tilde{\Gamma}_H \equiv \frac{\mathrm{d}\ln[z_L/(z_H-z_L)]}{\mathrm{d}\ln I_H}$ and $\tilde{\Delta}_{I_H} \equiv \frac{\mathrm{d}\ln[(z_H-z_L)/(1-z_H)]}{\mathrm{d}\ln I_H}$, we can utilize the results of the comparative statics in Eqs. (A.32) and (A.34) to obtain the following terms

$$\begin{aligned} \tilde{\Gamma}_H &= \frac{z_H}{z_H-z_L}\left(\frac{\mathrm{d}\ln z_L - \mathrm{d}\ln z_H}{\mathrm{d}\ln I_H}\right) \\ &= \Psi_H \Gamma_H > 0 \end{aligned} \tag{A.46}$$

where $\Gamma_H = \frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_L}{(z_H-z_L)z_L}\right) > 0$, and

$$\begin{aligned} \tilde{\Delta}_{I_H} &= \left(\frac{z_H(1-z_L)}{(1-z_H)(z_H-z_L)}\frac{\mathrm{d}\ln z_H}{\mathrm{d}\ln I_H} - \frac{z_L}{z_H-z_L}\frac{\mathrm{d}\ln z_L}{\mathrm{d}\ln I_H}\right) \\ &= \Psi_H \Delta_H < 0 \end{aligned} \tag{A.47}$$

where $\Delta_H = -\frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_L z_L(1-z_L)}{(1-z_H)(z_H-z_L)} + \frac{z_H}{(z_H-z_L)(1-z_H)}\right) < 0$.

Similarly, utilizing Eq. (A.25) it follows that both the labor supply effect, $I_H/(1-I_H)$, and the productivity effect, $\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H}$ converge to the following limits

$$\begin{aligned} \lim_{I_H\to 0}\frac{I_H}{1-I_H} &= 0, \quad \text{and} \quad \lim_{I_H\to 1}\frac{I_H}{1-I_H} = \infty \\ \lim_{I_H\to 0}\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H} &= 0, \quad \text{and} \quad \lim_{I_H\to 1}\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H} = -\mu_{\gamma_H} \end{aligned}$$

■

A.9 Impact of immigration on natives wage structure

Low-skill immigration and natives wage structure

To compute the impact of low-skill immigration on the wage structure of native workers, recall the Eqs. (35) and (36), take logs and differentiate with respect to task margin I_L to obtain

$$\begin{aligned}\frac{\mathrm{d}\ln(w_L^D/w_M^D)}{\mathrm{d}\ln I_L} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_L}\right)+\left(\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1\right)\left(\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L}\right)-\left(\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\frac{I_L}{1-I_L}\\ \frac{\mathrm{d}\ln(w_M^D/w_H^D)}{\mathrm{d}\ln I_L} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_L}\right)\end{aligned}$$

Utilizing now Eqs. (A.42) and (A.43) and making use of the definition of the terms $\Psi_L\equiv-\frac{\mathrm{d}\ln[\Omega_L(I_L)^{\alpha}(1-I_L)]}{\mathrm{d}\ln I_L}$ and $\tilde{\Theta}_L(I_L)\equiv-\frac{\mathrm{d}\ln\Theta_L(I_L)}{\mathrm{d}\ln I_L}$ we get

$$\begin{aligned}\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_L}\right) &= \Psi_L\Gamma_L=\left(\alpha\tilde{\Theta}_L(I_L)+\frac{I_L}{1-I_L}\right)\Gamma_L>0\\ \left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_L}\right) &= \Psi_L\Delta_L=\left(\alpha\tilde{\Theta}_L(I_L)+\frac{I_L}{1-I_L}\right)\Delta_L<0\end{aligned}$$

Now utilizing these results in the previously derived equations and rearranging yield Eqs. (37) and (38) derived in the text.

High-skill immigration and natives wage structure

Following the same steps we can compute the distributional impact of high-skill immigration for native workers. More precisely, take logs in Eqs. (35) and (36) and differentiate with respect to task margin I_H to obtain

$$\begin{aligned}\frac{\mathrm{d}\ln(w_L^D/w_M^D)}{\mathrm{d}\ln I_H} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_H}\right)\\ \frac{\mathrm{d}\ln(w_M^D/w_H^D)}{\mathrm{d}\ln I_L} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_H}\right)-\left(\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1\right)\left(\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H}\right)+\left(\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\frac{I_H}{1-I_H}\end{aligned}$$

Now utilize Eqs. (A.46) and (A.47) and making use of the definition of the terms $\Psi_H\equiv-\frac{\mathrm{d}\ln[\Omega_H(I_H)^{\alpha}(1-I_H)]}{\mathrm{d}\ln I_H}$ and $\tilde{\Theta}_H(I_H)\equiv-\frac{\mathrm{d}\ln\Theta_H(I_H)}{\mathrm{d}\ln I_H}$ to obtain

$$\begin{aligned}\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_H}\right) &= \Psi_H\Gamma_H=\left(\alpha\tilde{\Theta}_H(I_H)+\frac{I_H}{1-I_H}\right)\Gamma_H>0\\ \left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_H}\right) &= \Psi_H\Delta_H=\left(\alpha\tilde{\Theta}_H(I_H)+\frac{I_H}{1-I_H}\right)\Delta_H<0\end{aligned}$$

Substituting these results in the previously derived equations and rearranging yield Eqs. (41) and (42) derived in the text.

Medium-skill immigration and natives wage structure

To obtain the distributional impact of medium-skill immigration for native workers, take logs in Eqs. (35) and (36) and differentiate with respect to task margin I_M to obtain

$$\begin{aligned}
\frac{\mathrm{d}\ln(w_L^D/w_M^D)}{\mathrm{d}\ln I_M} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_M}\right)-\left(\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1\right)\left(\frac{\mathrm{d}\ln\Theta_M(I_M)}{\mathrm{d}\ln I_M}\right)+\left(\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\frac{I_M}{1-I_M}\\
\frac{\mathrm{d}\ln(w_M^D/w_H^D)}{\mathrm{d}\ln I_L} &= \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}\left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_M}\right)+\left(\frac{\alpha(\epsilon-\sigma)}{(\epsilon-1)\sigma}-1\right)\left(\frac{\mathrm{d}\ln\Theta_M(I_M)}{\mathrm{d}\ln I_M}\right)-\left(\frac{(\epsilon-\sigma)}{(\epsilon-1)\sigma}\right)\frac{I_M}{1-I_M}
\end{aligned}$$

Now utilize Eqs. (A.44) and (A.45) and making use of the definition of the terms $\Psi_M\equiv-\frac{\mathrm{d}\ln[\Omega_M(I_M)^\alpha(1-I_M)]}{\mathrm{d}\ln I_M}$ and $\tilde{\Theta}_M(I_M)\equiv-\frac{\mathrm{d}\ln\Theta_M(I_M)}{\mathrm{d}\ln I_M}$ to obtain

$$\begin{aligned}
\left(\frac{\mathrm{d}\ln(z_L/(z_H-z_L))}{\mathrm{d}\ln I_M}\right) &= \Psi_M\Gamma_M=\left(\alpha\tilde{\Theta}_M(I_M)+\frac{I_M}{1-I_M}\right)\Gamma_M<0\\
\left(\frac{\mathrm{d}\ln((z_H-z_L)/(1-z_H))}{\mathrm{d}\ln I_M}\right) &= \Psi_M\Delta_M=\left(\alpha\tilde{\Theta}_M(I_M)+\frac{I_M}{1-I_M}\right)\Delta_M>0
\end{aligned}$$

Substituting these results in the previously derived equations and rearranging yield Eqs. (39) and (40) derived in the text.

A.10 Proof of Lemma 3

Recall the thresholds of the elasticity of substitution between tasks,

$$\begin{aligned}
\tilde{\alpha}_L &\equiv \frac{(\epsilon-1)\sigma}{(\epsilon-\sigma)(1-\Gamma_L)},\\
\tilde{\alpha}_H &\equiv \frac{(\epsilon-1)\sigma}{(\epsilon-\sigma)(1+\Delta_H)},
\end{aligned}$$

Thus, for the relative strength between the two thresholds we obtain

$$\begin{aligned}
\tilde{\alpha}_L &\lesseqgtr \tilde{\alpha}_H\\
\Gamma_L &\lesseqgtr -\Delta_H
\end{aligned}$$

Now substituting the definitions of Γ_L and Δ_H from Eqs. (A.42) and (A.47), respectively, and manipulating yields

$$\begin{aligned}
\frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_H z_H z_H}{(z_H-z_L)}+\frac{z_H}{(1-z_H)(z_H-z_L)}\right) &\lesseqgtr \frac{1}{\mathcal{D}_J}\left(\frac{\epsilon-1}{\epsilon}\frac{\tilde{\varepsilon}_L z_L(1-z_L)}{(1-z_H)(z_H-z_L)}+\frac{z_H}{(z_H-z_L)(1-z_H)}\right)\\
\tilde{\varepsilon}_H z_H(1-z_H) &\lesseqgtr \tilde{\varepsilon}_L z_L(1-z_L) \qquad \text{(A.48)}
\end{aligned}$$

■

A.11 Proof of Proposition 4

The proof of the existent of threshold margins $\check{I}_L$ and $\check{I}_H$ can be straightforwardly verified as follows. Given $0 < \{\tilde{\varepsilon}_L, \tilde{\varepsilon}_H\} < \infty$ and the functional properties of $\gamma_k(i)$, it follows from Eqs. (37) and (42), respectively,

$$\begin{aligned} \frac{\mathrm{d}\ln \omega_{LM}}{\mathrm{d}\ln I_L} &= \mathcal{H}_L(\check{I}_L) - \mathcal{G}_L(\check{I}_L) = 0, \qquad (A.49)\\ \frac{\mathrm{d}\ln \omega_{MH}}{\mathrm{d}\ln I_H} &= \mathcal{H}_H(\check{I}_H) - \mathcal{G}_H(\check{I}_H) = 0, \qquad (A.50)\end{aligned}$$

so that

$$\mathcal{H}_k(I_k) \lesseqgtr \mathcal{G}_k(I_k), \quad \text{for} \quad I_k \gtreqless \check{I}_k,\ k = \{L, H\}.$$

The proof of the existent of threshold margin $\check{I}_M$ needs further elaboration. From Eqs. (39) and (40) two threshold task margins are obtained at which $\mathrm{d}\ln\omega^D_{LM} = 0$ and $\mathrm{d}\ln\omega^D_{MH} = 0$. Define these two thresholds, respectively, by the following implicit functions

$$\mathcal{I}_{1,M} \equiv (1 - \alpha\delta(1 + \Gamma_M))\,\tilde{\theta}_M(\check{I}_{1,M}) - \delta(1 + \Gamma_M)\frac{\check{I}_{1,M}}{1 - \check{I}_{1,M}} = 0, \qquad (A.51)$$

$$\mathcal{I}_{2,M} \equiv (1 - \alpha\delta(1 - \Delta_M))\,\tilde{\theta}_M(\check{I}_{2,M}) - \delta(1 - \Delta_M)\frac{\check{I}_{2,M}}{1 - \check{I}_{2,M}} = 0. \qquad (A.52)$$

Thus, it is readily seen from Eqs. (A.51) and (A.52) that if $-\Gamma_M = \Delta_M$, we get that $\check{I}_{1,M} = \check{I}_{2,M} = \check{I}_M$. Moreover, using the definitions of Γ_M and Δ_M from Eqs. (A.44) and (A.45), respectively, we get that

$$\begin{aligned} -\Gamma_M &\lesseqgtr \Delta_M \\ \frac{1}{\mathcal{D}_J}\left(\frac{z_H}{(z_H - z_L)(1 - z_H)} + \frac{\epsilon - 1}{\epsilon}(\tilde{\varepsilon}_L + \tilde{\varepsilon}_H)\frac{z_H}{(z_H - z_L)}\right) &\lesseqgtr \frac{1}{\mathcal{D}_J}\left(\frac{z_H}{(1 - z_H)(z_H - z_L)} + \frac{\epsilon - 1}{\epsilon}\left(\frac{z_H(1 - z_L)}{(1 - z_H)(z_H - z_L)}\tilde{\varepsilon}_L + \frac{z_L}{z_H - z_L}\tilde{\varepsilon}_H\right)\right),\end{aligned}$$

which after some manipulation yields

$$\tilde{\varepsilon}_H(1 - z_H) \lesseqgtr \tilde{\varepsilon}_L z_L. \qquad (A.53)$$

This in turn implies that

$$\check{I}_{1,M} \lesseqgtr \check{I}_{2,M}. \qquad (A.54)$$

To verify (A.54), it is sufficient to show that either from (A.51), $\frac{\partial \mathcal{I}_{1,M}}{\partial \Gamma_M} < 0$, or from (A.52), $\frac{\partial \mathcal{I}_{2,M}}{\partial \Delta_M} > 0$. Thus, differentiating Eq. (A.52) with respect to Δ_M, we obtain

$$\frac{\mathrm{d}\check{I}_{2,M}}{\mathrm{d}\Delta_M} = \frac{\alpha\delta\tilde{\Theta}_M(\check{I}_{2,M}) + \delta\check{I}_{2,M}/(1 - \check{I}_{2,M})}{\delta(1 - \check{\Gamma}_M)\frac{1}{(1 - \check{I}_{1,M})^2} - (1 - \alpha\delta(1 - \check{\Gamma}_M)\frac{\partial\tilde{\Theta}_M(\check{I}_{2,M})}{\partial\check{I}_{2,M}}} > 0$$

where by the functional properties discussed above the denominator is always positive. Thus, for an asymmetric degree of technology adoption in the neighborhood of the margins z_L and z_H, there exist two task margins, implying that the productivity (labor supply) effect dominates unambiguously the labor supply (productivity) effect when ever the task margin is lower (higher) than the minimum (maximum) of the two thresholds, i.e. $I_M < \min\{\check{I}_{1,M}, \check{I}_{2,M}\}$ ($I_M > \max\{\check{I}_{1,M}, \check{I}_{2,M}\}$). For all $\min\{\check{I}_{1,M}, \check{I}_{2,M}\} \leq I_M \leq \max\{\check{I}_{1,M}, \check{I}_{2,M}\}$, the relationship between the two forces becomes ambiguous.

Moreover, comparing Eqs. (A.48) and (A.53) it is readily seen that the condition $\min\{\tilde{\alpha}_L, \tilde{\alpha}_H\}$ defines the binding thresholds for elasticity of substitution between tasks. ■

A.12 Derivation of $\hat{A}_M$

Recall the zero profit condition

$$p_M(z)y_M(z) = \frac{\tilde{w}_M \tilde{N}_M + w_O N_O}{z_H - z_L}$$

Utilizing Eq. (3) in the right hand side and manipulating the left hand side slightly yields

$$P_M \hat{X}_M = \frac{\tilde{w}_M \tilde{N}_M}{z_H - z_L}\left[1 + \frac{w_O}{\tilde{w}_M}\frac{N_O}{\tilde{N}_M}\right]$$

Now perfect competition in the final goods market requires $P_M = \hat{\zeta}_M$ and by symmetry it follows that $\hat{X}_M = A_M^{\frac{\epsilon}{\epsilon-1}}\left(\kappa x_O^{\frac{\epsilon-1}{\epsilon}} + (1-\kappa)x_M^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$. Since the extent of offshoring is exogenously given, the FE condition (14) applies to both type of firms producing intermediates at home and abroad. Substituting these observations together with Eq. (49) and the markup-pricing in the left hand side of the previously derived equation yields

$$\epsilon A_M \hat{w}_M f_M = \frac{\tilde{w}_M \tilde{N}_M}{z_H - z_L}\left[1 + \frac{w_O}{\tilde{w}_M}\frac{N_O}{\tilde{N}_M}\right]$$

where $\hat{w}_M$ denotes the marginal cost index of $\hat{N}_M$ and is defined by

$$\hat{w}_M = \left[\kappa w_O^{1-\epsilon} + (1-\kappa)\tilde{w}_M^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Next substitute Eq. (46) for $w_O/\tilde{w}_M$ in the left hand side and manipulate to obtain

$$\hat{\zeta}_M A_M^{\frac{\epsilon}{\epsilon-1}}(\epsilon-1)f_M = \frac{\tilde{w}_M \tilde{N}_M^{1/\epsilon}}{(z_H - z_L)(1-\kappa)^{1/\epsilon}}\left[\kappa^{\frac{1}{\epsilon}} N_O^{\frac{\epsilon-1}{\epsilon}} + (-\kappa)^{\frac{1}{\epsilon}}\tilde{N}_M^{\frac{\epsilon-1}{\epsilon}}\right] \tag{A.55}$$

Now from the optimization problem

$$\min_{\tilde{N}_M, N_O} \tilde{w}_M \tilde{N}_M + w_O N_O \text{ s.t. } \hat{N}_M = \left(\kappa^{\frac{1}{\epsilon}} N_O^{\frac{\epsilon-1}{\epsilon}} + (-\kappa)^{\frac{1}{\epsilon}}\tilde{N}_M^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

the first order conditions imply

$$\tilde{w}_M = \hat{w}_M \left(\frac{\hat{N}_M}{\tilde{N}_M}\right)^{\frac{1}{\epsilon}}(1-\kappa)^{1/\epsilon} \tag{A.56}$$

$$w_O = \hat{w}_M \left(\frac{\hat{N}_M}{N_O}\right)^{\frac{1}{\epsilon}}\kappa^{1/\epsilon}, \tag{A.57}$$

Utilizing the first-order condition (A.56) in the left hand side of (A.55), manipulating further and solving with respect to $\hat{A}_M$ yields the solution derived in the text.

A.13 Proof of Proposition 6

Recall Eqs. (57) and (58) and rearrange slightly to obtain

$$\frac{\mathrm{d}\ln\omega^D_{LM}}{\mathrm{d}\kappa} = -\left(\frac{1}{\epsilon} - \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}(1+\Gamma_M)\right)\Phi_\kappa + \frac{1}{\epsilon}\frac{1}{1-\kappa}$$
$$\frac{\mathrm{d}\ln\omega^D_{MH}}{\mathrm{d}\kappa} = \left(\frac{1}{\epsilon} - \frac{\epsilon-\sigma}{(\epsilon-1)\sigma}(1-\Delta_M)\right)\Phi_\kappa - \frac{1}{\epsilon}\frac{1}{1-\kappa}$$

Now from the first terms on the right hand side, one obtains a threshold defined by

$$\chi \equiv \frac{\epsilon^2+\sigma-2\sigma\epsilon}{\epsilon(\epsilon-\sigma)},$$

which sets a lower boundary for changes in the relative range of final goods. Thus, it follows that whenever

$$\min\{|\Gamma_M|, \Delta_M\} > \chi,$$

the efficiency effect will be dominating. As defined by condition (A.53) in Appendix A.11 it follows that

$$\min\{|\Gamma_M|, \Delta_M\} = \min\{\tilde{\varepsilon}_H(1-z_H), \tilde{\varepsilon}_L z_L\}.$$

Moreover, since $\lim_{\kappa\to 0}\Phi_\kappa = \infty$, the efficiency effect will also dominate the labor supply effect (the second term on the right hand side) at low initial values of offshoring rate. ■