Wage Disparities and Internal Migration Patterns

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Abstract

I investigate how wage disparities between the rural and urban areas affect internal migration patterns. In particular, I attempt to clarify which internal migration pattern people choose between parental and family migration under different wage disparities.

Many studies have shown that income differences or wage disparities between rural and urban areas are one of the main reasons for people living in rural areas to migrate to urban areas. In this paper, I demonstrate how the choice of migration pattern is associated with wage disparities.

I first show that wage disparities have different effects on parental migration and family migration. In particular, wage disparities have negative as well as positive effects on human capital formation under parental migration and positive effects under family migration.

I then show that the choice of internal migration pattern depends on the degree of wage disparity and differences in migration costs. In particular, if wage disparities and cost differences are neither large nor small, then parental migration is more likely preferred to family migration with decreases in wage disparities, whereas family migration is more likely preferred to parental migration with increases in wage disparities. In this case, it is possible that wage disparities facilitate human capital formation.

My results suggest that not only do wage disparities cause internal migration but also affect the choice of the internal migration pattern and human capital formation.

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1. Introduction

In this paper, I deal with wage disparities between rural and urban areas and choice of internal migration pattern. In particular, I attempt to clarify how wage disparities affect the choice of internal migration pattern and human capital formation.

In China, the number of people who migrate internally has increased very rapidly despite the presence of various barriers.\(^1\) Chan (2008) estimates that rural migrant labourers numbered 79.8 million in 1998, 98.2 million in 2003, 102.6 million in 2004, 108.2 million in 2005 and 114.9 million in 2006. This growth is mainly due to growing income differences between the rural and urban areas (Zhang and Song, 2003) or wage disparities between them (Chan, 2008).\(^2\) According to the China Development Research Foundation (2013), the urban–rural income ratio has increased from 2.78 in 2000 to 3.23 in 2003 and 3.32 in 2007, although it has dropped consistently from 2010, decreasing to 3.13 in 2011.

Internal migration can be broadly divided into two types, i.e. parental migration and family migration (Shimada, 2014).

In general, not only school education but also home education is imperative in building human capital (Glomm and Kaganovich, 2003; Casarico and Sommacal, 2012). The quality and quantity

\(^1\) Obstacles to labour mobility include the household registration (\textit{hukou}) system, local protectionism in the destination, lack of a well-developed social safety net for rural migrants in the destination and discrimination against migrants (Zhang and Zou, 2012).

\(^2\) Reform of the household registration system might have also encouraged internal migration. However, Melander and Pelikanova (2013) and Chan (2010) argue that concrete proposals for such reform are scarce and the core of the dualistic structure supported by the system has not changed much.
of children’s education at school and home are different under parental migration and family migration. Accordingly, human capital will be formed differently across these two migration patterns.

Clearly, human capital determines workers’ earnings. This suggests that migration pattern can be an important factor in determining future income differences or wage disparities. Accordingly, we should pay sufficient attention to people’s choice of internal migration pattern when analysing the evolution of income differences or wage disparities.

As mentioned earlier, previous studies have revealed that people in rural areas move to urban areas due to the existence of income differences or wage disparities. In addition, there have been controversies on whether internal migration will help reduce income differences or wage disparities (Keidel, 2009; Lu, 2009).

However, previous studies have not placed sufficient emphasis on the effects of income differences or wage disparities on choice of internal migration pattern. Which pattern people choose given the income difference or the wage disparity has not been fully understood, despite the fact that migration patterns have significant implications for the future course of the economy via education and human capital.

Therefore, I attempt to identify which pattern of migration people in rural areas utilise under different wage disparities when moving to urban areas. In particular, I reveal how migration pattern choice is associated with wage disparities.

I obtain the following results: Wage disparities increase parent’s utility under both parental migration and family migration. However, such disparities lower as well as raise human capital under parental migration but raise human capital under family migration.

As for the choice of internal migration pattern, if wage disparities are sufficiently large, the optimal internal migration pattern is determined only by differences in migration cost. Otherwise,
migration pattern is determined by wage disparities as well as migration cost difference. In particular, if wage disparities are neither large nor small and the cost does not differ significantly, then parental migration is more likely to be selected with decreases in the wage disparity, whereas family migration is more likely with increases in the wage disparity.

My results suggest that wage disparities affect the choice of internal migration patterns and human capital formation in addition to the fact that they cause internal migration.

The remainder of this paper is organised as follows: Section 2 models parental migration and family migration and formulates the maximisation problem for each migration pattern. Section 3 solves the maximisation problem for parental migration and determines how migration duration, human capital and parent’s utility are associated with wage disparities. Section 4 solves the maximisation problem for family migration and determines how human capital and parent’s utility are associated with wage disparities. Section 5 compares parent’s utility between the two migration patterns and determines how wage disparities affect migration pattern choice. Section 5 provides concluding remarks.

2. Model

I assume an overlapping-generations economy. The economy is comprised of the rural and urban areas. The rural area is connected to the urban area through labour emigration. The rural and urban areas differ in that wages are higher in the urban area than in the rural area. It is implicitly assumed that physical capital is more abundant in the urban area than in the rural area. This causes the wage disparity between the two areas. In general, the wage disparity between the rural and urban areas refers to the difference in wages earned by rural people in the rural area and those earned by urban people in the urban area. This suggests that the wage disparity arises not only from the difference in physical capital in the two areas but also from the difference in human
capital of people in the two areas. Accordingly, if urban people’s human capital is larger than rural
people’s human capital, then the wage disparity in the general sense will be much larger than the
wage disparity caused only by the difference in physical capital. However, in this paper, I assume
that the wage disparity is generated by the difference in physical capital.

I focus on a representative household in the rural area consisting of a parent and a child. Each
agent is assumed to live for two periods, childhood and parenthood. As a child, he goes to school
and receives school education. He also receives parental care, i.e. home education while his parent
resides with him. Unlike previous studies, such as Vidal (1998) and Docquier et al. (2008), I
explicitly assume that human capital is formed not only by school education but also by home
education.3

Utilising human capital accumulated in childhood, an agent provides labour in parenthood. The
parent may migrate to the urban area alone or with a child and stay there for part or all of the
period to work. As he is altruistic towards the child, he derives utility not only from consumption
in the present period but also from human capital in the next period. Although the parent shares
consumption with the child, this is not because he is altruistic towards the child but to satisfy the
child’s survival needs.

2.1 Parental Migration

The parent resides in the urban area to work for either part or all of the parenthood period. The
migration duration in period $t$ is $0 \leq t' \leq 1$. The parent manipulates the migration duration to

\[ \text{migration duration in period } t \text{ is } 0 \leq t' \leq 1. \]

According to Jingzhong and Lu (2011), lack of parental disciplining due to parental absence has
negative effects on children’s school performance. Accordingly, not only does home education
have an independent role but it also complements school education.
maximise his utility.

The parent earns wages in period $t$ by $w_t$ in the rural area and $w_t^*$ in the urban area. Wages are the product of human capital measured in terms of efficiency units of labour and wages per efficiency. Wages per efficiency in the rural area are denoted as $w$, and those in the urban area are $w^*$. Due to the difference in the availability of physical capital, a disparity in wages per efficiency exists by $\alpha^* > 0$ between the rural and urban areas and $w^*$ is equal to $(1 + \alpha^*)w$. Larger values of $\alpha^*$ mean larger disparities in wages per efficiency. Henceforth, unless otherwise specified, I refer a disparity in wages per efficiency as a wage disparity.

Under these assumptions, parent’s income in period $t$ is 

$$
(1 - l_t^*)w_t + l_t^*w_t^* = (1 + \alpha^*l_t^*)wh_t,
$$

where $h_t$ is human capital in period $t$ measured in efficiency units of labour under parental migration.

The parent spends $1 - \theta_t$, where $0 < \theta_t < 1$, of his income on consumption and $\theta_t$ on child’s school education in period $t$. The parent manipulates this ratio to maximise his utility.

Measuring the amount of school education by money expended on it and denoting the amount of home education, $2 - l_t^*$, where 2 (two) is assumed to be the maximum amount of home education, human capital available in period $t + 1$, $h_{t+1}$, is built according to the following human capital formation equation under parental migration:

$$
h_{t+1} = \{\theta_t(1 + \alpha^*l_t^*)wh_t\}^{s}(2 - l_t^*)^p, \ s, p > 0, \ s + p < 1,
$$

where $s$, which equals $\partial \ln h_{t+1}/\partial \ln \{\theta_t(1 + \alpha^*l_t^*)wh_t\}$ (the elasticity of human capital with respect to school education), measures the effectiveness of school education and $p$ measures the effectiveness of home education.\(^4\) The human capital formation function exhibits decreasing

\(^4\) If the maximum amount of home education were 1 (one), which is equal to the length of the period, then given the amount of home education, the contribution of home education to human
returns to scale.

The human capital formation equation under parental migration is endowed with the property wherein human capital in period \( t \) affects human capital in period \( t + 1 \) through wages in period \( t \). Intergenerational externality is operative in the same manner as in Galor and Stark (1994).

The parent derives utility in period \( t \) from consumption in period \( t \) and the amount of human capital available to his child in period \( t + 1 \). Since the parent is altruistic towards the child, higher wages for his child, which do not matter to the parent directly, nevertheless also raise the parent’s utility. Thus, parent’s utility before deducting parental migration costs, \( u^{PM}_i \), is

\[
u^{PM}_i = \ln(1 - \theta_i)(1 + \alpha'^{PM}_i)wh_i + \varepsilon \ln \rho \{1 + \alpha'^{PM}_i \}wh_i \{1 - \alpha'^{PM}_i \}, \tag{1}
\]

where \( 0 < \varepsilon < 1 \) is the parent’s degree of altruism towards the child and \( 0 < \rho < 1 \) measures the subjective time rate of the discount. I represent the costs for parental migration, \( \varphi^{PM} > 0 \), by negative utility since they include non-pecuniary elements such as those arising from family disintegration as well as pecuniary ones. Costs are given exogenously.

The parent’s maximisation problem under parental migration is summarised as follows:

\[
\max_{\theta_i, \varepsilon} u^{PM}_i \quad \text{subject to} \quad 0 < \varepsilon < 1, \quad 0 < \theta_i < 1. \]

2.2 Family Migration

Under family migration, the parent and the child move to the urban area together, and thus the migration duration is always equal to one. However, they return to the rural area at the end of the period and go again to the urban area at the beginning of the next period since they are not permanent residents of the urban area.

capital formation (and also to parent’s utility defined by Equation 1), \( \ln(1 - \theta_i)^\rho \), would be smaller with higher effectiveness of home education, i.e. larger values of \( \rho \).
Accordingly, the parent provides labour in the urban area for all of the period, giving rise to an income of \((1 + \alpha^*)w\tilde{h}_t = w^*\tilde{h}_t\), where \(\tilde{h}_t\) is human capital in period \(t\) measured in efficiency units of labour under family migration. The child receives home education fully, i.e. by \(2\) (two).

Denoting the ratio of parent’s income spent on consumption by \(1 - \tilde{\theta}_t\), where \(0 < \tilde{\theta}_t < 1\), and the ratio spent on child’s school education by \(\tilde{\theta}_t\), then human capital in period \(t + 1\), denoted by \(\tilde{h}_{t+1}\), is built according to

\[
\tilde{h}_{t+1} = (\tilde{\theta}_t(1 + \alpha^*)w\tilde{h}_t)^{2^p}.
\]

The effectiveness of both school education and home education are assumed to be the same as those under parental migration. This is, of course, to simplify the analysis. In general we cannot determine whether or not the effectiveness of school education differs between parental and family migration, as this is the product of the school education’s quality and the school’s accessibility. Under family migration, the quality of school education is higher.\(^5\) However, migrant children’s accessibility to local schools in the urban area is strictly limited if the migrant is non-hukou.

I represent parent’s utility derived from consumption and human capital in a similar manner as in parental migration.

\[
\ln(1 - \tilde{\theta}_t)(1 + \alpha^*)w\tilde{h}_t + \varepsilon \ln \rho(\tilde{\theta}_t(1 + \alpha^*)w\tilde{h}_t)^{2^p}.
\]

To compare parent’s utility between the two migration patterns, I deduct the difference in costs between two migration patterns from above utility. Parent’s utility under family migration excluding the cost difference, \(u_{t}^{FM}\), is represented as follows:

\[
u_{t}^{FM} = \ln(1 - \tilde{\theta}_t)(1 + \alpha^*)w\tilde{h}_t + \varepsilon \ln \rho(\tilde{\theta}_t(1 + \alpha^*)w\tilde{h}_t)^{2^p} - \varpi,
\]

\(^5\) Wang and Li (2009) find a large discrepancy between urban and rural education quality and infer the causes.
where $\bar{c}$ is the difference of migration costs, i.e. $\bar{c}^{FM} - \bar{c}^{PM}$. As with the costs for parental migration, the costs for family migration, $\bar{c}^{FM} > 0$, are given exogenously and include both pecuniary and non-pecuniary elements. We cannot determine a priori the migration pattern that incurs larger costs. In other words, $\bar{c}$ can be negative, zero, or positive.

The parent's maximisation problem under family migration is summarised as follows:

$$\max_{\theta} u^{FM} \quad \text{subject to } 0 < \theta < 1.$$ 

3. Parental Migration under Different Wage Disparities

In this section, I solve the maximisation problem for parental migration and determine migration duration, human capital, and parent’s utility.

3.1 Migration Duration under Parental Migration

Differentiating Equation (1) with respect to $\theta$, the optimal ratio of parent’s income spent on the child’s school education, $\theta^*$, is determined as follows:

$$\theta^* = \frac{\alpha}{1 + \alpha^*}.$$  \hspace{1cm} (3)

Equation (3) suggests that as the parent cares more about the child or as school education is more effective, the parent spends a larger fraction of income on the child’s school education ($\partial \theta^*/\partial \alpha$, $\partial \theta^*/\partial s > 0$). The optimal ratio does not depend on the level of human capital and remains unchanged over time.

I differentiate Equation (1) with respect to $l^*$ and find

$$2\alpha^*(1 + \alpha) - \varphi - \alpha^*(1 + \alpha + \varphi)l^* = 0.$$  \hspace{1cm} <

When $l^* = 0$, the left-hand side of the above equation is positive if $\alpha^* > \varphi/2(1 + \alpha)$ and negative if $\alpha^* < \varphi/2(1 + \alpha)$. On the other hand, when $l^* = 1$, the left-hand side is positive if
$\alpha^* > \varepsilon p / (1 + \varepsilon s - \varepsilon p)$ and negative $\alpha^* < \varepsilon p / (1 + \varepsilon s - \varepsilon p)$. Clearly, $\varepsilon p / 2(1 + \varepsilon s) < \varepsilon p / (1 + \varepsilon s - \varepsilon p)$.

Moreover, the left-hand side decreases with $I^*_{t}$. These facts suggest that

\[
\frac{\partial u^*_{PM}}{\partial I^*_{t}} < 0 \text{ if } \alpha^* < \varepsilon p / 2(1 + \varepsilon s), \text{ hereafter called Case A},
\]

\[
\frac{\partial u^*_{PM}}{\partial I^*_{t}} > 0 \text{ if } \varepsilon p / 2(1 + \varepsilon s) \leq \alpha^* \leq \varepsilon p / (1 + \varepsilon s - \varepsilon p), \text{ hereafter called Case B},
\]

\[
\frac{\partial u^*_{PM}}{\partial I^*_{t}} > 0 \text{ if } \alpha^* > \varepsilon p / (1 + \varepsilon s - \varepsilon p), \text{ hereafter called Case C}.
\]

Therefore, the optimal migration duration, $[I^*_{t}]$, differs across different wage disparities.

\[
[I^*_{t}] = 0 \quad (\forall I^*_{t} \leq \varepsilon p / 2(1 + \alpha)),
\]

\[
[I^*_{t}] = \frac{2\alpha^* (1 + \varepsilon s) - \varepsilon p}{\alpha^* (1 + \varepsilon s + \varepsilon p)} \quad (\forall \varepsilon p / 2(1 + \alpha) \leq \alpha^* \leq \varepsilon p / (1 + \alpha - \varepsilon p)), \quad (4)
\]

\[
[I^*_{t}] = 1 \quad (\forall I^*_{t} \geq \varepsilon p / (1 + \alpha - \varepsilon p)).
\]

In Case $A$, that is, when the wage disparity is small, people do not migrate but rather stay in the rural area. The interior solution $[I^*_{t}]_{\varepsilon p / 2(1 + \alpha) \leq \alpha^* \leq \varepsilon p / (1 + \alpha - \varepsilon p)}$ increases with $\alpha^*$, starting from $[I^*_{t}]_{\alpha^* = \varepsilon p / 2(1 + \alpha)} = 0$ and ending with $[I^*_{t}]_{\alpha^* = \varepsilon p / (1 + \alpha - \varepsilon p)} = 1$. In Case $C$, that is, when the wage disparity is large, people stay in the urban area all the period. Accordingly, Equations (4) show that the migration duration increases with the wage disparity.

[Figure 1 around here]

From this we have the following proposition:

**Proposition 1** Under parental migration, people do not migrate if the wage disparity is small.

Otherwise, they migrate and its duration monotonically increases with the wage disparity.

By staying longer in the urban area, the parent earns a larger income, enabling more money to be spent on consumption and the child’s school education. However, home education decreases. If the wage disparity is small, the latter negative effects on home education and thereby on parent’s
utility are larger than the former positive effects, dissuading the parent from migration. As the wage disparity increases further, the positive effects come to outweigh the negative effects, thus motivating the parent to stay longer in the urban area.

3.2 Human Capital under Parental Migration

In Case A, that is, when the wage disparity is small, human capital in period $\tau + 1$, $h_{\tau+1}|_{a^* < q/(1+\alpha^*)}$, for given $h_\tau$, is

$$h_{\tau+1}|_{a^* < q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\}w_h \right]^\prime \alpha^p,$$

and its steady state, $h^*|_{a^* < q/(1+\alpha^*)}$, is

$$h^*|_{a^* < q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\}w \right]^{\gamma/(1-\gamma)} \alpha^{p/(1-\gamma)}.$$

The steady state is stable.

In Case B, that is, when the wage disparity is neither small nor large, human capital in period $\tau + 1$, $h_{\tau+1}|_{q/(2+\alpha^*) \leq a^* \leq q/(1+\alpha^*)}$, for given $h_\tau$, is

$$h_{\tau+1}|_{q/(2+\alpha^*) \leq a^* \leq q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\} \{1 + 2\alpha^*\} \alpha/(1 + \epsilon\alpha + q\alpha) \right] \{1 + 2\alpha^*\} q\alpha/(1 + \epsilon\alpha + q\alpha) \alpha^p,$$

and its steady state, $h^*|_{q/(2+\alpha^*) \leq a^* \leq q/(1+\alpha^*)}$, is

$$h^*|_{q/(2+\alpha^*) \leq a^* \leq q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\} \{1 + 2\alpha^*\} \alpha/(1 + \epsilon\alpha + q\alpha) \right]^{\gamma/(1-\gamma)} \{1 + 2\alpha^*\} q\alpha/(1 + \epsilon\alpha + q\alpha) \alpha^p.$$

The steady state is stable.

In Case C, that is, when the wage disparity is large, human capital in period $\tau + 1$, $h_{\tau+1}|_{a^* > q/(1+\alpha^*)}$, for given $h_\tau$, is

$$h_{\tau+1}|_{a^* > q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\} \{1 + \alpha^*\} w_h \right]^\prime,$$

and its steady state, $h^*|_{a^* > q/(1+\alpha^*)}$, is

$$h^*|_{a^* > q/(1+\alpha^*)} = \left[ \{\epsilon\alpha/(1+\alpha)\} \{1 + \alpha^*\} w \right]^{\gamma/(1-\gamma)}.$$

The steady state is stable.
If the wage disparity is smallest in Case B, i.e. $\alpha^* = \frac{\varepsilon p}{2(1 + \varepsilon s)}$, then from Equations (7) and (8), human capital in the short run $h_{t+1}|_{\alpha^* = \frac{\varepsilon p}{2(1 + \varepsilon s)}}$ is $[\{\varepsilon s/(1 + \varepsilon s)\} w_{t}^{\varepsilon}]^{2^p}$ and human capital in steady state $h^*|_{\alpha^* = \frac{\varepsilon p}{2(1 + \varepsilon s)}}$ is $[\{\varepsilon s/(1 + \varepsilon s)\} w]^{\nu(l-1)} 2^{p(l-1)}$. These are the same values as those in Case A (see Equations 5 and 6). Similarly, if the wage disparity is largest in Case B, i.e. $\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}$, then from Equations (7) and (8), human capital in the short run $h_{t+1}|_{\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}}$ is $[\{\varepsilon s/(1 + \varepsilon s)\} (1 + \varepsilon)/(1 + \varepsilon s + \varepsilon p)] w_{t}^{\varepsilon}$ and human capital in steady state $h^*|_{\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}}$ is $[\{\varepsilon s/(1 + \varepsilon s)\} (1 + \varepsilon)/(1 + \varepsilon s + \varepsilon p)] w^{\nu(l-1)}$. These are the same values as those obtained when $\alpha^*$ approaches to $\frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}$ in Equations (9) and (10). Moreover,

$$h_{t+1}\big|_{\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}} \quad \text{for the given } h, \quad h^*\big|_{\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}}$$

decrease with $\alpha^*$.

Therefore, human capital in both the short run ($h_{t+1}$ for the given $h$) and the steady state ($h^* = h^*$) remain unchanged or decrease with the wage disparity ($dh_{t+1}/d\alpha^*, \; dh^*/d\alpha^* \leq 0$) until $\alpha^* = \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}$.

However after $\alpha^*$ has exceeded $\frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}$, human capital in the short run and in steady state increases with the wage disparity, as clear from Equations (9) and (10).

[Figures 2a and 2b around here]

I summarise these results by the following proposition:

**Proposition 2** Under parental migration, initially, human capital monotonically decreases with the wage disparity. After the wage disparity has exceeded a certain level, human capital increases

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6 Signs of $dh_{t+1}/d\alpha^*$ and $dh^*/d\alpha^*$ in Case B are determined by the sign of $-(\frac{p}{2s} - \alpha^*)$ and by the fact that $\frac{p}{2s} > \varepsilon/(1 + \varepsilon s - \varepsilon p)$. In addition, in Case B, $\alpha^* \leq \frac{\varepsilon p}{(1 + \varepsilon s - \varepsilon p)}$. Accordingly, $\frac{p}{2s} - \alpha^* > \frac{p}{2s} - \varepsilon p/(1 + \varepsilon s - \varepsilon p) > 0$. This suggests that $dh_{t+1}/d\alpha^*$ and $dh^*/d\alpha^*$ are negative.
As the wage disparity increases, the migration duration increases. As the parent’s absence is longer, the negative effects on human capital formation via the smaller amount of home education outweigh the positive effects via the larger amount of school education. Accordingly, the wage disparity hinders human capital formation. However, beyond a certain level, even without home education, larger income due to larger wage disparities makes school education’s positive effects dominant. Accordingly, the wage disparity facilitates human capital formation.

### 3.3 Parent’s Utility under Parental Migration

In **Case A**, the parent’s utility in period \( t \), \( u_t^{PM}_{a^* < q/(1+\alpha)} \), for the given \( h_t \), is

\[
u_t^{PM} = \ln \left\{ \frac{1}{(1+\alpha)} \right\} w h_t + \varepsilon \ln \rho \left[ \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right] w h_t \right\} + \varepsilon \ln 2 + \varepsilon \ln \left( \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right) w h_t \right\} + \varepsilon \ln 2 + \varepsilon \ln \left( \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right) w h_t \right\}
\]

(11)

where \( v(h_t) = \ln \left\{ \frac{1}{(1+\alpha)} \right\} w h_t + \varepsilon \ln \rho \left[ \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right] w h_t \right\} + \varepsilon \ln 2 + \varepsilon \ln \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} w h_t \}. \)

It should be noted that in **Case A**, the parent does not migrate so no migration costs are incurred. However, parent’s utility under family migration (Equation 2) is defined as to include parental migration costs. Accordingly, for the comparison with family migration to make sense, I add the costs for parental migration to parent’s utility when no parental migration occurs.

In **Case B**, the parent’s utility in period \( t \), \( u_t^{PM}_{q/(1+\alpha) < \alpha^* < \alpha} \), for the given \( h_t \), is

\[
u_t^{PM} = \ln \left\{ \frac{1}{(1+\alpha)} \right\} w h_t + \varepsilon \ln \rho \left[ \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right] w h_t \right\} + \varepsilon \ln \rho \left[ \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} \right] w h_t \right\} + \varepsilon \ln 2 + \varepsilon \ln \frac{\alpha \left(1 + \alpha s\right)}{(1 + \alpha)} w h_t \right\}
\]

(12)

In **Case C**, parent’s utility in period \( t \), \( u_t^{PM}_{\alpha^* > q/(1+\alpha)} \), for the given \( h_t \), is

\[
u_t^{PM} = v(h_t) + (1 + \alpha) \ln (1 + \alpha^*)
\]

(13)

In Equation (12), \( u_t^{PM}_{\alpha^* = q/(1+\alpha)} \) = \( v(h_t) + (1 + \alpha) \ln \left( \frac{(1 + \alpha s) \alpha}{(1 + \alpha)} \right) \), and in Equation (13), \( \lim_{\alpha^* \to q/(1+\alpha)} u_t^{PM}_{\alpha^* > q/(1+\alpha)} = v(h_t) + (1 + \alpha) \ln \left( \frac{(1 + \alpha s) \alpha}{(1 + \alpha)} \right) \). Moreover, parent’s
utility increases with wage disparities in both Case B \( (du_{t}^{PM})_{\alpha' \leq \alpha < \alpha'} > 0 \) and 
Case C \( (du_{t}^{PM})_{\alpha > \alpha / (1 + \alpha - \alpha')} > 0 ). \)

This provides us with the following proposition:

**Proposition 3** Under parental migration, parent’s utility increases with the wage disparity if the parent decides to migrate.

As **Proposition 2** states, wage disparities make the parent stay longer in the urban area. By earning a larger income, the parent spends more money on consumption and child’s education. These positive effects dominate the negative effects arising from the smaller amount of home education. As a result, the parent’s utility rises.

To summarise the results in this section, it was found that under parental migration, a wage disparity makes the parent stay longer in the urban area and raises his utility. However, human capital decreases as well as increases as the wage disparity widens.

4. Family Migration under Different Wage Disparities

In this section, I solve the maximisation problem for family migration and determine human capital and parent’s utility.

4.1 Human Capital under Family Migration

I differentiate Equation (2) with respect to \( \tilde{\theta} \) to find 
\[
\tilde{\theta}_{t}^* = \frac{\alpha}{(1 + \alpha)}. \tag{14}
\]

\( ^{7} \frac{du_{t}^{PM}}{\alpha' \leq \alpha \leq \alpha'} \bigg/ d\alpha^* \) = \( \frac{1}{(1 + 2\alpha^*)\alpha^*} \) \( \{2(1 + \alpha)\alpha^* - \alpha\} \). \( \frac{du_{t}^{PM}}{\alpha > \alpha / (1 + \alpha - \alpha')} \bigg/ d\alpha^* \) = \( \frac{(1 + \alpha^*)}{(1 + \alpha)}. \)
The optimal ratio of parent’s income spent on the child’s school education is the same as under parental migration (see Equation 3).

Substituting Equation (14) into the human capital formation equation under family migration, human capital in period $t + 1$, $\tilde{h}_{t+1}$, for the given $\tilde{h}_t$, is

$$ \tilde{h}_{t+1} = \left[ \frac{\alpha \varepsilon}{(1 + \varepsilon)} \right] (1 + \alpha^*) w \tilde{h}_t]^{\varepsilon} 2^\varepsilon, \quad (15) $$

and its steady state, $\tilde{h}^*$, is

$$ \tilde{h}^* = \left[ \frac{\alpha \varepsilon}{(1 + \varepsilon)} \right] (1 + \alpha^*) w]^{\varepsilon/(1-\varepsilon)} 2^\varepsilon/(1-\varepsilon). \quad (16) $$

Equations (15) and (16) suggest that human capital in the short run ($\tilde{h}_{t+1}$ for the given $\tilde{h}_t$) and in steady state ($\tilde{h}_{t+1} = \tilde{h}_t \equiv \tilde{h}^*$) increase with the wage disparity ($\partial \tilde{h}_{t+1} / \partial \alpha^*$, $\partial \tilde{h}^* / \partial \alpha^* > 0$).

I summarise this property by the following proposition:

**Proposition 4** Under family migration, human capital monotonically increases with the wage disparity.

The larger wage disparity enables the parent to earn larger income, and the larger amount of money can be spent on child’s school education. Unlike under parental migration, the amount of home education remains unchanged under family migration. As a result, the wage disparity has only positive effects on human capital formation.

### 4.2 Parent’s Utility under Family Migration

By substituting Equation (14) into Equation (2), we have

$$ u^{FM}_t = v(h_t) + (1 + \varepsilon) \ln(1 + \alpha^*) + \varepsilon p \ln 2 - \varepsilon. \quad (17) $$

Equation (17) suggests that parent’s utility increases with the wage disparity.

I summarise this property as follows:
**Proposition 5** Under family migration, the parent’s utility monotonically increases with the wage disparity.

Under family migration, both consumption and human capital are larger with larger wage disparities. Accordingly, parent’s utility is also larger.

To summarise the results in this section, it was found that under family migration, a wage disparity increases both human capital and parent’s utility.

5. Choice of the Internal Migration Pattern under Different Wage Disparities

I am now in a position to examine how the choice of internal migration pattern is associated with wage disparities.

When the wage disparity is small, i.e. \( \alpha^* < \varphi / 2(1 + \varepsilon) \) as in Case A, the difference of parent’s utility is, from Equations (17) and (11)

\[
\begin{align*}
& u^{FM}_t \bigg|_{\alpha^* < \varphi / 2(1 + \varepsilon)} \quad u^{PM}_t \bigg|_{\alpha^* < \varphi / 2(1 + \varepsilon)} = (1 + \varepsilon^*) \ln(1 + \alpha^*) - \bar{c}^{FM}.
\end{align*}
\]

According to Equation (18), if the wage disparity is sufficiently small, i.e. \( \alpha^* \) is sufficiently near to 0 (zero), parental migration provides higher utility. Accordingly, the parent decides not to migrate in such a case.

If \( (1 + \varepsilon) \ln[1 + \{ \varphi \} / 2(1 + \varepsilon)] - \bar{c}^{FM} > 0 \), the difference in parent’s utility is positive for a value of \( \alpha^* \) sufficiently near to \( \varphi / 2(1 + \varepsilon) \). In this case, the parent chooses family migration for wage disparities larger than the particular level at which two migration patterns produce the same parent’s utility, and he chooses parental migration and decides not to migrate for wage disparities lower than that level.

[Figure 4a around here]

On the other hand, if \( (1 + \varepsilon) \ln[1 + \{ \varphi \} / 2(1 + \varepsilon)] - \bar{c}^{FM} < 0 \), the difference in parent’s utility...
is always negative. Accordingly, the parent always chooses parental migration and stays in the rural area.

Figure 4b around here

These results can be summed up by the following proposition:

**Proposition 6** When the wage disparity is small, the optimal internal migration pattern is determined by both migration costs and wage disparities. In particular, if family migration costs are sufficiently small, then for smaller wage disparities, parental migration is more likely preferred whereas for larger wage disparities, family migration is more likely preferred. Otherwise, the parent always chooses parental migration and stays in the rural area.

I move on to Case B, where the wage disparity is neither small nor large, i.e. \( \frac{\varepsilon \rho}{2(1 + \varepsilon)} \leq \alpha^* \leq \frac{\varepsilon \rho}{(1 + \varepsilon - \varepsilon \rho)} \).

When \( \alpha^* \) is smallest, i.e. \( \alpha^* \leq \frac{\varepsilon \rho}{2(1 + \varepsilon)} \),

\[
u_i^{FM} = v(h_i) + (1 + \varepsilon \alpha) \ln \{1 + \varepsilon \rho / 2(1 + \varepsilon)\} + \varepsilon \rho \ln 2 - \overline{c} \text{ and from } (1 + 2\alpha^*)(1 + \varepsilon \alpha) \big/ (1 + \varepsilon \alpha + \varepsilon \rho) = 1 \text{ and } (1 + 2\alpha^*)\varepsilon \rho / (1 + \varepsilon \alpha + \varepsilon \rho) \alpha^* = 2, \]

\[
u_i^{PM} = v(h_i) + \varepsilon \rho \ln 2.
\]

When \( \alpha^* \) is largest, i.e. \( \alpha^* \geq \frac{\varepsilon \rho}{(1 + \varepsilon - \varepsilon \rho)} \),

\[
(1 + 2\alpha^*) \big/ (1 + \varepsilon \alpha + \varepsilon \rho) = (1 + \varepsilon \rho) \big/ (1 + \varepsilon \alpha - \varepsilon \rho), \text{ and } (1 + 2\alpha^*)\varepsilon \rho / (1 + \varepsilon \alpha + \varepsilon \rho) \alpha^* = 1,
\]

\[
u_i^{FM} = v(h_i) + (1 + \varepsilon \alpha) \ln \{(1 + \varepsilon \alpha) \big/ (1 + \varepsilon \alpha + \varepsilon \rho)\} + \varepsilon \rho \ln 2 - \overline{c} \text{ and } \nu_i^{PM} = v(h_i) + (1 + \varepsilon \alpha) \ln \{(1 + \varepsilon \alpha) \big/ (1 + \varepsilon \alpha - \varepsilon \rho)\}.
\]

If the cost difference is larger than \( \varepsilon \rho \ln 2 \), then from Equation (19)

\[
\nu_i^{FM} \bigg|_{\alpha^* = \varepsilon \rho / (1 + \varepsilon \alpha - \varepsilon \rho)} - \nu_i^{FM} \bigg|_{\alpha^* = \varepsilon \rho / (1 + \varepsilon \alpha - \varepsilon \rho)} < (1 + \varepsilon \alpha) \ln \{1 + \varepsilon \rho / 2(1 + \varepsilon)\} - \varepsilon \rho \ln 2.
\]

Since the right-hand side of the above equation is negative, parental migration produces higher
parent’s utility at \( \alpha^* = \varphi p / (1 + \varphi) \). From Equation (20), parental migration produces higher
parent’s utility at \( \alpha^* = \varphi p / (1 + \varphi - \varphi) \) as well. Moreover, with increases in the wage disparity,
not only does the parent’s utility increase under both migration patterns but parent’s utility also
increases more under family migration than under parental migration, i.e.

\[
\frac{\partial u_i^{FM}}{\partial \alpha^*} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} > \frac{\partial u_i^{PM}}{\partial \alpha^*} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} > 0.
\]

These facts suggest that the curves for \( u_i^{FM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \) and \( u_i^{PM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \) do not intersect and that the
curve for \( u_i^{FM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \) lies above the curve for \( u_i^{PM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \). Therefore, parental migration always provides higher utility for the parent if \( \varphi > \varphi p \ln 2 \), i.e.

\[
u_i^{FM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} < \nu_i^{PM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \text{ if } \varphi > \varphi p \ln 2.
\]

[Figure 5a around here]

If the cost difference is smaller than \((1 + \varphi) \ln \{1 + \varphi p / (1 + \varphi)\}\), then from Equation (19),
family migration produces higher parent’s utility at \( \alpha^* = \varphi p / (1 + \varphi) \). From Equation (20),
family migration produces higher parent’s utility at \( \alpha^* = \varphi p / (1 + \varphi - \varphi) \) as well. These facts
suggest that the curve for \( u_i^{FM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \) lies above the curve for
\( u_i^{PM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \). Therefore, family migration always provides higher utility for the parent
if \( \varphi < (1 + \varphi) \ln \{1 + \varphi p / (1 + \varphi)\} \), i.e.

\[
u_i^{FM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} > \nu_i^{PM} \bigg|_{\varphi p / (1 + \alpha) \leq \varphi p / (1 + \alpha - \varphi)} \text{ if } \varphi < (1 + \varphi) \ln \{1 + \varphi p / (1 + \varphi)\}.
\]

[Figure 5b around here]

So far in Case B, I have found parental migration is chosen when the cost difference is
sufficiently large but family migration is chosen when the cost difference is sufficiently small.
Accordingly, in such cases, the choice of the migration pattern is determined by the cost
difference.

However, when the cost difference is not significant, i.e. \((1 + \varphi) \ln \{1 + \varphi p / (1 + \varphi)\} \leq \varphi \leq \varphi p \ln 2\),
parental migration and family migration can provide the same utility. In particular, the parent
derives the same utility at $\alpha^* = \varphi/p/(1+\alpha)$ if $\tilde{c} = (1+\alpha)\ln\{1 + \varphi/2(1+\alpha)\}$ (see Equation 19) and at $\alpha^* = \varphi/p/(1+\alpha - \varphi)$ if $\tilde{c} = \varphi \ln 2$ (see Equation 20). At these wage disparities, the parent is indifferent between the two migration patterns.

In addition, the parent derives the same utility at a certain wage disparity between $\varphi/p/(1+\alpha)$ and $\varphi/p/(1+\alpha - \varphi)$ if $(1+\alpha)\ln\{1 + \varphi/2(1+\alpha)\} < \tilde{c} < \varphi \ln 2$. In this case, we have

$$u_t^{FM}\bigg|_{\alpha^* = \varphi/p/(1+\alpha)} < u_t^{PM}\bigg|_{\alpha^* = \varphi/p/(1+\alpha - \varphi)} \quad \text{and} \quad u_t^{FM}\bigg|_{\alpha^* = \varphi/p/(1+\alpha - \varphi)} > u_t^{PM}\bigg|_{\alpha^* = \varphi/p/(1+\alpha)}$$

from Equations (19) and (20), and the curve for $u_t^{PM}\bigg|_{\varphi/p/(2(1+\alpha)) \leq \alpha^* \leq \varphi/p/(1+\alpha - \varphi)}$ and the curve $u_t^{FM}\bigg|_{\varphi/p/(2(1+\alpha)) \leq \alpha^* \leq \varphi/p/(1+\alpha - \varphi)}$ intersect only once. Accordingly, parental migration produces higher utility before the wage disparity reaches a certain level at which $u_t^{PM} = u_t^{FM}$, and beyond that level of wage disparity, family migration produces higher utility.

Therefore, in the case where the cost difference is not significant, the wage disparity determines the migration pattern choice. For smaller wage disparities, parental migration is more likely chosen, whereas for the larger wage disparity, family migration is more likely chosen.

Summarising the results yields the following proposition:

**Proposition 7** If the wage disparity is neither small nor large and the cost does not differ significantly between parental migration and family migration, then the optimal internal migration pattern is determined by the wage disparity. In particular, for smaller wage disparities, parental migration is more likely preferred whereas for larger wage disparities, family migration is more likely preferred.

This proposition suggests that when the migration cost difference is not significant, human capital likely increases with the wage disparity since family migration is more likely preferred with the wage disparity and, as derived, family migration raises human capital with the wage
disparity.

Now I look at Case C, the case wherein the wage disparity is large, i.e. $\alpha^* > \varphi/(1 + \alpha - \varphi)$. The difference in the parent’s utility is, from Equations (17) and (13)

$$u^*_i \left|_{\alpha^* > \varphi/(1 + \alpha - \varphi)} - u^*_i \left|_{\alpha^* > \varphi/(1 + \alpha - \varphi)} = \varphi \ln 2 - \bar{c}. \right.$$

Equation (21) suggests that the outcome of which internal migration pattern produces higher parent’s utility does not depend on the wage disparity. Instead, the choice of migration pattern is determined solely by the cost difference. In particular, if the cost does not differ or is larger for parental migration than for family migration, i.e. $\bar{c}^{FM} \geq \bar{c}^{PM}$, or if the cost for family migration is larger than the cost for parental migration but by less than $\varphi \ln 2$, i.e. $\varphi \ln 2 > \bar{c}^{FM} - \bar{c}^{PM} > 0$, then family migration is seen as the optimal pattern. On the other hand, if family migration incurs larger costs than parental migration by more than $\varphi \ln 2$, then parental migration is chosen.

I summarise this result by the following proposition:

**Proposition 8** If the wage disparity is large, then the internal migration pattern is determined solely by the migration cost. Internal migration with smaller costs is likely preferred as an optimal pattern.

From Propositions 2, 4, and 8, we find that regardless of the migration cost difference, the wage disparity always raises human capital when the wage disparity is large.

To summarise the results in this section, I found that the wage disparity as well as the cost difference determines the choice of migration pattern.

6. Concluding Remarks
In developing countries, large income differences or wage disparities between the rural and urban areas tend to motivate people in the rural areas to attempt to move to and work in urban areas. In China, even with various migration prevention measures in place, a growing number of people from rural areas have attempted to migrate to urban areas. In undertaking the migration process, people choose different internal migration patterns that bring about different levels of human capital.

I attempted to determine whether and how people’s choice of internal migration pattern is associated with wage disparities, and showed that the degree of wage disparity as well as migration cost differences affect the choice of migration pattern when the migration cost does not differ significantly across two patterns and the wage disparity is not so large.

I infer from my results that the present wage disparity might affect the future wage differences between the rural and urban areas. In particular, when the choice of the migration pattern is made solely by the wage disparity, people become likely to choose the family migration with increases in the wage disparity, and human capital increases with the wage disparity under the family migration. Since human capital is passed on to the future generations, people in the rural area will have larger human capital. Accordingly, even if the physical capital is not so abundant in the rural area as in the urban area, wages in the rural area will not be so significantly lower than those in the urban area, and future wage differences will be smaller.

In the analysis, I gave the wage disparity, in particular, the disparity in wages per efficiency exogenously. Given such a wage disparity, people choose a migration pattern. A certain migration pattern produces a certain level of human capital. However, since human capital affects wages, the wage disparity in the general sense will change and this might affect the choice of the migration pattern. If this is the case, the choice of migration pattern might change over time.
References


Lu, Y. (2012). “Education of Children Left Behind in Rural China.” *Journal of Marriage and


Figure 1
Migration Duration under Parental Migration

Figure 2a
Human Capital in the Short Run under Parental Migration
Figure 2b
Human Capital in Steady State under Parental Migration

Figure 3a
Human Capital in the Short Run under Family Migration
Figure 3b
Human Capital in Steady State under Family Migration

Figure 4a
Parent's Utility when $\alpha' < \frac{\sigma \rho}{2(1 + \varepsilon)}$ and $(1 + \varepsilon) \ln[1 + \{\sigma \rho/2(1 + \varepsilon)\}] - \varepsilon^M > 0$. 
Figure 4b
Parent's Utility when $\alpha' < \frac{\varphi}{2(1 + \varepsilon)}$ and $(1 + \varepsilon) \ln[1 + \{\frac{\varphi}{2(1 + \varepsilon)}\}] - \bar{c}^{FM} < 0$.

Figure 5a
Parent's Utility when $\frac{\varphi}{2(1 + \varepsilon)} \leq \alpha' \leq \frac{\varphi}{(1 + \varepsilon - \varphi)}$ and $\bar{c} > \varphi \ln 2$. 

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Figure 5b
Parent's Utility when \(\frac{\varphi p}{2(1 + \alpha)} \leq \alpha' \leq \frac{\varphi p}{1 + \alpha - \varphi} \) and \(\bar{c} < (1 + \alpha) \ln \left(1 + \frac{\varphi p}{2(1 + \alpha)}\right)\).

Figure 5c
Parent's Utility when \(\frac{\varphi p}{2(1 + \alpha)} \leq \alpha' \leq \frac{\varphi p}{1 + \alpha - \varphi} \) and \((1 + \alpha) \ln \left(1 + \frac{\varphi p}{2(1 + \alpha)}\right) < \bar{c} < \varphi p \ln 2\).
Figure 6a
Parent's Utility when $\alpha^*>\epsilon p/(1+\epsilon_s-\epsilon p)$ and $\epsilon p\ln 2 > \bar{\epsilon}$.

Figure 6b
Parent's Utility when $\alpha^*>\epsilon p/(1+\epsilon_s-\epsilon p)$ and $\bar{\epsilon} > \epsilon p\ln 2$. 