Asymmetric Monotone Comparative Statics for the Industry Compositions

Anders Rosenstand Laugesen*

Preliminary working paper: July 27, 2015

Abstract

We let heterogeneous firms face decisions on a number of complementary activities (including offshoring and exporting) in a monopolistically-competitive industry with free entry. Only the most productive firms export and are directly exposed to the demand conditions in each of the two asymmetric countries we analyse. Within this model of international trade, we derive sufficient conditions for monotone comparative statics (MCS) for the industry composition. This phenomenon is defined as first-order stochastic dominance shifts in the equilibrium distributions of all activities across active firms. It is found that MCS for the industry composition is a likely outcome in a country which unilaterally liberalises international trade or experiences a decline in the costs of other complementary activities. In the nonliberalising country, the industry-level implications are exactly opposite. Firm-level complementarities manifest themselves very clearly at the industry level in the two countries. As a key application, we show how our results provide new and strong testable predictions for the recent offshoring model by Antràs, Fort, and Tintelnot (2014).

Keywords: Complementary Activities; Firm Heterogeneity; Offshoring; Exporting; Asymmetries

JEL Classifications: D21; F12; F61; L11

*Aarhus University, Fuglesangs Allé 4, 8210 Aarhus V, Denmark. Email: alaugesen@econ.au.dk. Phone: +45-30253599. I thank Peter Bache, Kristian Behrens, Peter Neary, Allan Sørensen, and seminar participants in Saint Petersburg and Venice for helpful comments.

1 Introduction

In this paper, we analyse the responses of industries to exogenous industry-wide shocks in a two-country heterogeneous-firms model of international trade with monopolistic competition à la Melitz (2003), Melitz and Redding (2014), and not at least Antràs et al. (2014) (henceforth AFT). The most productive firms export and are thus exposed to the demand levels in each of the two asymmetric countries. A demand level comprises an inverse measure of the level of competition in the industry.[1] Within this model of international trade, we derive sufficient conditions for monotone comparative statics (MCS) for the industry composition. This phenomenon is defined as first-order stochastic dominance shifts in the equilibrium distributions of all activities across active firms. An activity refers to any variable at the discretion of the firm. It is found that MCS for the industry composition is a very likely outcome in a country which unilaterally liberalises international trade or experiences a decline in the costs of other complementary activities. In the nonliberalising country, the industry-level implications are exactly reverse implying asymmetric MCS for the industry compositions in the two countries. Importantly, these clear industry-level results hold while things are much less clear at the firm level of analysis.

To give you a flavour of the meaning and significance of these results, let us be specific. Hence, let us turn attention to the recent offshoring model of AFT which can be nested within our model below under standard assumptions.[2] In an important and natural extension of their baseline framework to nonprohibitive costs of trading final goods across countries, AFT analyse the problem of heterogeneous firms determining the optimal extensive margin offshoring and exporting strategies in a multi-country model. The extensive margin offshoring and exporting decisions are complementary under a particular parametrisation. These complementarities receive empirical support in the empirical analysis in AFT. Recall that the present paper focuses on the case of complementary activities and two asymmetric countries. In this case, we show that an unilateral trade liberalisation implies MCS for the industry composition in the AFT model implying that the fractions of offshorers (i.e., importers of intermediate inputs) and exporters increase in the liberalising country if e.g. productivity is distributed Pareto while competition (mea-

[1] Under CES preferences, the demand levels are functions of total industry spending and the price index of the industry. They are thus demand shifters.

[2] Section 2 covers the AFT model in greater detail.

sured by the inverse demand level) increases in the liberalising country. This latter condition is easily checked in a practical context like the AFT model and is very likely to be satisfied as argued below. Moreover, under exactly the same circumstances, the fractions of offshorers and exporters decrease in the nonliberalising country. Consequently, the industry-level implications for the liberalising and nonliberalising countries are precisely opposite.

Why are these industry-level results interesting? First of all, the comparative statics in the AFT model treat the levels of competition in the various countries as exogenous during comparative statics such as trade liberalisations which is not particularly appealing under an assumption of free entry. Our results show how strong and testable industry-level results can be obtained when one allows for endogenous levels of competition. Interestingly, our results hold for a class of productivity distributions that contains the Pareto distribution, namely the class of productivity distributions where log-productivity is distributed with nonincreasing hazard rate.[3] Further, these monotone industry-level results hold even though the responses at the firm level are nonmonotone and in line with the empirical evidence in AFT. Second, we reveal how one can obtain clear industry-level results during comparative statics without having to solve a full general-equilibrium model. The presented technique works even when shocks and countries are asymmetric, as is most often the case, and when the model setup is flexible enough to encompass a long list of prominent trade models, cf. below. The apparent complexity is handled by applying the monotonicity theorems of Topkis (1978) and Milgrom and Shannon (1994).

A large strand of literature within the field of international trade has recently exploited various kinds of firm-level complementarities. This strand of literature contains key contributions such as Melitz (2003), Antràs and Helpman (2004), Helpman et al. (2004), Melitz and Ottaviano (2008), Arkolakis (2010), Helpman and Itskhoki (2010), Helpman et al. (2010), Arkolakis and Muendler (2011), Bernard et al. (2011), Bustos (2011), Davis and Harrigan (2011), Caliendo and Rossi-Hansberg (2012), Amiti and Davis (2012), Arkolakis et al. (2012), Mayer et al. (2014), Melitz and Redding (2014), and AFT. We strongly relate to this literature by simply assuming that the activities faced by firms are complementary with: (i) each other; (ii) firm productivity;

[3]However, our results do not necessarily hold when productivity is distributed Frechet or log-normally as log-productivity is distributed with increasing hazard rate in these cases. When log-productivity is distributed with nonincreasing hazard rate, we are assured a sufficient mass of exiting firms in the liberalising country and this does the trick.

(iii) the demand level; (iv) the foreign demand level(s); and (v) the exogenous industry-wide parameters that we vary during the comparative statics. Our focus lies on the industry-level implications of these common complementarities. Mrazova and Neary (2013) emphasise the role of supermodularity (complementarity) in shaping the sorting pattern of firms in a given equilibrium. Our approach here differs by not only focusing on a given equilibrium but rather conducting comparative statics across equilibria. The present paper should also be perceived as an extension of Bache and Laugesen (2014a) to the more plausible case of asymmetric countries and asymmetric shocks which cannot be analysed in the model of Bache and Laugesen (2014a) when firms make decision concerning their export activities. The cost of allowing for country and shock asymmetry in the present paper is that the propositions are conditional on the endogenous changes in the levels of competition in the two countries. Thankfully, these endogenous changes are very often easily determined as we argue.

2 Extensive Margins of Offshoring and Exporting

AFT have recently developed a multi-country sourcing model in which monopolistically-competitive heterogeneous firms decide upon their offshoring (and potentially also their exporting) strategies. Offshoring occurs when intermediate inputs are produced abroad. As in e.g. Amiti and Davis (2012), the gains from offshoring in AFT consist of a variable costs reduction which comes at the cost of higher fixed costs. The pros and cons of exporting are similar to those in Melitz (2003). In the present section, which presents some new comparative statics based on the AFT model, we will let the costs of exporting final goods be nonprohibitive, but still not negligible, implying that some but not all firms will export their final goods. This is done in an attempt to bridge the two literatures on offshoring and exporting and because industry-level comparative statics for the AFT model with prohibitive costs of exporting are easily obtained using the original methodology in Bache and Laugesen (2014a).[4]

[4]The case of J potentially asymmetric countries in the AFT model can be analysed within the setup of Bache and Laugesen (2014a) given that firms cannot export their final products. Allowing for exporting complicates the analysis because exporting firms get exposed to different demand levels, A_k (see below), which are affected by the comparative statics. Hence, the initial focus on a two-country model below.

In Section 3.5 of AFT and the underlying appendix, AFT express the problem of determining the optimal extensive margin offshoring and exporting strategies of a firm from country 1 with productivity θ as

$$\max_{I_{1j}^M \in \{0,1\}_{j=1}^2,\; I_{1k}^X \in \{0,1\}_{k=1}^2} \left(\gamma \sum_{j=1}^{2} I_{1j}^M T_j (\tau_{1j} w_j)^{-\psi} \right)^{(\sigma-1)/\psi} \sum_{k=1}^{2} I_{1k}^X (\tau_{1k}^X)^{1-\sigma} A_k \theta - w_1 \sum_{j=1}^{2} I_{1j}^M f_{1j} - w_1 \sum_{k=1}^{2} I_{1k}^X f_{1k}^X, \quad (1)$$

where the notation will now be briefly explained. The choice variables I_{1j}^M and I_{1k}^X are indicator dummies for importing intermediates (M) from country j (i.e., offshoring to country j) and exporting final goods (X) to country k, respectively. Note that our exposition here implies a slight abuse of terminology given that a country 1 firm can "offshore" production to country 1 and "export" to country 1. Hence, j and k are country indices. For the present purpose $\gamma > 0$ can be perceived as a parameter even though γ is in fact a function of model parameters that are kept constant during comparative statics. T_j denotes the technology level in country j. τ_{1j} and τ_{1k}^X denote the variable iceberg trade costs of, respectively, offshoring and exporting to country j and k from the view point of a country 1 firm. w_j denotes the wage rate in country j which is determined through an assumption of free trade in an outside numeraire good produced under perfect competition. σ and ψ are positive parameters. Let us henceforth assume that $(\sigma - 1) > \psi > 0$ such that the extensive margin offshoring and exporting decisions are complementary as in Amiti and Davis (2012) and Bache and Laugesen (2014b). The results of the empirical analysis in AFT are in line with this assumption. The intuition for the complementarity between offshoring and exporting is described in AFT. f_{1j} and f_{1k}^X denote the positive fixed costs of offshoring some production of intermediate inputs to country j and exporting final goods to country k, respectively.

Notice that in contrast to AFT who allow for J asymmetric countries, we will initially constrain ourselves to just 2 countries namely the home country 1 and the foreign country 2. This choice is partly made to strengthen the intuition for our results below, and partly made because the results are stronger in the case of just two countries. However, we will also briefly touch upon the case with J asymmetric countries. Let $\beta_1 = (-\tau_{12}, -\tau_{12}^X, -f_{12}, -f_{12}^X)$ be the vector of parameters that we vary (weakly increase) during the comparative statics.[5] Hence, β_1 contains minus the fixed and minus the variable costs of trading final goods and intermediate inputs across countries from the view point of country 1. Note that an increase in β_1 amounts to an asymmetric and unilateral trade liberalisation where e.g. an increase in $-\tau_{12}$ can be seen as an input trade liberalisation in country 1. An increase in $-\tau_{12}^X$ can be seen as a final-good import trade liberalisation in country 2. Following most of the heterogeneous-firms trade literature, let us henceforth assume that trade cost parameters are such that the least productive active firms do neither export final goods to foreign countries nor import intermediate inputs from foreign countries. This is in line with the empirical findings in Kohler and Smolka (2011) and Bernard et al. (2012) and implies that $x_1 \equiv (I_{11}^M, I_{12}^M, I_{11}^X, I_{12}^X) = (1, 0, 1, 0)$ for the least productive active firms. Finally, assume an equilibrium with free entry (which determines the two demand levels A_1 and A_2) where firms realise their productivity, θ, upon paying the entry cost f_e but before choosing their extensive margin offshoring and exporting strategies.

Under this set of assumptions, it is shown in the remainder of the paper that trade liberalisation through an increase in β_1 implies that the fractions of offshorers and exporters increase in country 1 if e.g. productivity is distributed Pareto (or log-productivity is distributed with nonincreasing hazard rate) while A_1 is nonincreasing and A_2 is nondecreasing. These required adjustments amount to an increase in competition in country 1 and a decrease in competition in country 2. Under exactly the same circumstances and a constant β_2 (the vector of minus the trade cost parameters in country 2), it follows from the analysis below that the fractions of offshorers and exporters decrease in country 2. Hence, we experience asymmetric monotone comparative statics (MCS) for the industry compositions in the two countries.[6] These strong industry-level results for the two countries will also hold when the total fixed cost function in (1) is increasing in the extensive margin offshoring and exporting decisions and submodular in these decisions instead of additively separable and thus modular. This is because the complemen-

[5]One could also include the parameter T_2 in β_1. This observation implies that the industry-level implications (MCS for the industry composition) of increasing T_2 are qualitatively similar to the explained implications of increasing β_1.

[6]It also follows from the analysis in Section 3 that exactly the same industry-level implications would appear for the two countries if country 2 opens up for horisontal FDI (modelled along the lines of Helpman et al., 2014). Such a shock can be modelled trough an increase in the below choice set, S. More information is available upon request.

tarity between offshoring and exporting is maintained under this alternative assumption. Note that the needed movements in A_1 and A_2 can be checked for in a practical context like the present. See for instance the exercise in Section 6.1 in AFT which confirms the needed demand-level adjustments under an increase in $-\tau_{12}$ or T_2 The related exercise regarding liberalisation of final-goods trade in Section 2.4 of Demidova and Rodriguez-Clare (2013) also confirms the needed demand-level adjustments. It is also shown below that a nonincrease in A_1 implies a nondecrease in A_2 and vice versa when β_2 is held constant. This observation, which follows from the two free-entry conditions, eases the requirements for asymmetric MCS for the industry compositions. Moreover, we show that an increase in β_1 implies MCS for the industry composition (increasing fractions of offshorers and exporters) in country 1 if this country can be characterised as a small open economy with positive production of differentiated goods and β_2 is held constant. This is because A_1 is certainly nonincreasing in this case while A_2 is unaffected by the developments in the small open economy named country 1.

Before we end our coverage of the AFT model, let us mention two additional points. First, qualitatively similar comparative static results for the industry composition in country 1 can be found within a multi-country model with J asymmetric countries if the induced changes (by an increase in β_1) in all foreign demand levels are nonnegative while the induced change in A_1 is nonpositive.[7] Second, while the above results make it clear that the comparative statics are monotone at the industry level of analysis in country 1, things are much less clear at the firm level of analysis in country 1. In fact, it follows from the below Appendix B that the comparative statics at the firm level in country 1 are nonmonotone and entirely in line with the empirical evidence in AFT.[8,9] Hence, the monotone industry-level results do not arise based on an aggregation of monotone firm-level results. All the above claims

[7]This follows from a straightforward generalisation of the decomposition in the below equation (8) to $J-1$ total foreign indirect effects which will share the same sign (all nonpositive).

[8]In their conclusion, AFT write: *"A distinctive characteristic of our framework is that a sectoral import competition shock that does not simultaneously increase export opportunities may still lead to intraindustry reallocation effects by which firms sourcing from these shocked countries may expand, while those not sourcing from those countries shrink".*

[9]Note, however, that in contrast to the baseline AFT model with prohibitive costs of exporting, nonprohibitive costs of exporting introduce an extra driver of intraindustry reallocations under import competition shocks namely a nonnegative foreign indirect effect in Appendix B which tends to make the larger exporting firms expand.

are proved below.

3 Model

After paying a sunk entry cost of f_e units of labour, atomistic firms enter an industry characterised by monopolistic competition. Upon entry, a firm realises its productivity level, $\theta \in [\theta_0, \infty)$ where $\theta_0 \geq 0$. Individual firms are fully characterised by their productivity level, θ, which is a realisation of the continuous random variable Θ with c.d.f. $F(\Theta)$. We let $F(\Theta)$ be strictly increasing on its entire support on which it is C^1. Firms with strictly higher θ are assumed to be able to earn strictly higher profits. Profits are assumed to be continuous in θ. Labour is the sole factor of production. We focus on a two-country model where the wage rates (w and w_f) are determined through a freely-traded homogeneous good produced under perfect competition in both countries.

After realising its productivity level, a firm has to choose whether to start producing or to exit the industry. If a firm chooses to produce, it has to make a decision, $x = (x_1, \ldots, x_n)$, where x_i denotes the chosen level of activity i. An activity refers to any variable at the discretion of the firm. The level of an activity can be either discrete or continuous. We let $x \in X$ where $X \subseteq \mathbb{R}^n$ is the set of all conceivable, but not necessarily available, decisions. The set X is assumed to be a lattice which, loosely speaking, means that undertaking a higher level of any activity may require, but importantly, cannot prevent undertaking a higher level of another activity. Restricting attention to lattices will allow complementarities between the n activities in x to take effect. The profitability of the decisions in X is influenced by a vector of exogenous industry-wide parameters, $\beta \in B$, with $B \subseteq \mathbb{R}^m$. Further, the actual choice set of all firms is restricted to a set of available decisions, $S \subseteq X$, with S being a sublattice of X. Our comparative statics will focus on changes in (β, S) which determines the attractiveness (all else equal) and availability of activities in the home country, but not in foreign. Similar assumptions to those above (and those below) hold in foreign where parameters will be denoted $f_{ef}, \theta_{0f}, \beta_f, S_f$, and where the distribution of productivities is denoted by $F_f(\Theta)$. Foreign variables and parameters thus appear with the subscript f. Domestic firms interact with foreign firms through, for instance, exporting, which is one of the n activities in x, or horisontal FDI as in Helpman et al. (2004). The exposition below will mainly focus on the home country.

3.1 Profits, Complementarities, and the Optimal Decision

Firm profits depend on a common and endogenous aggregate statistic which captures the (inverse) level of competition in the industry. We will refer to this variable, $A \in \mathbb{R}_+$, as the (domestic) demand level and let optimal firm profits be strictly increasing in A. In general, firm profits also depend on the foreign demand level, $A_f \in \mathbb{R}_+$. Optimal firm profits are weakly increasing in this variable and strictly increasing for exporters. In line with monopolistic competition among atomistic firms, individual firms perceive A and A_f as exogenous. Later, in equilibrium, A and A_f are determined through two free-entry conditions. Under CES demand, one can think of A and A_f as the two demand shifters.

Profits, π, of a firm with productivity θ depend on the decision, x, the demand level, A, the foreign demand level, A_f, and the industry parameters, β. We make the following assumption.

Assumption 1. *A, A_f, and θ only enter the profit function through the products, $A\theta$ and $A_f\theta$, and profits are additively separable in these two products when profits depend on A_f.*

Assumption 1 is often satisfied in models of heterogeneous firms with segmented markets such as Melitz (2003), Melitz and Redding (2014), and AFT. Formally,

$$\pi = \pi(x; A\theta, A_f\theta, \beta), \tag{2}$$

where the semicolon separates choice variables from arguments that are perceived as exogenous by the firms. The following assumption summarises the four key complementarities in our model.

Assumption 2. *For all $(A\theta, A\theta_f, \beta)$, the profit function, $\pi(x; A\theta, A_f\theta, \beta)$, is supermodular in x on X and exhibits increasing differences in $(x, A\theta)$, $(x, A_f\theta)$, and (x, β) on $X \times \mathbb{R}_+$, $X \times \mathbb{R}_+$, and $X \times B$, respectively.*

Supermodularity in x implies that the n activities are complementary.[10] An increase in the choice set, S, can e.g. capture the advent of a new

[10] An extensive body of recent research within international trade relies heavily on complementary activities. Parts of this literature are surveyed in Section 9 in Melitz and Redding (2014). For illustrative examples, see for instance Bustos (2011) and Amiti and Davis (2012).

marketing technique that becomes available to all firms. The implied effects on the levels of competition are captured by the endogenous changes in A and A_f. The assumptions of increasing differences imply that productivity, the demand level, the foreign demand level, and the elements of β are all complementary to the n activities.[11] Proper ordering of activity levels and parameters is crucial for profits to satisfy Assumption 2. Note that profits in (2) do not have to represent a certain payoff to firms. In case of uncertainty after a firm has realised its productivity level and made its decision, (2) could be interpreted as expected profits; see e.g. Athey and Schmutzler (1995).

Faced with the profit function (2), a firm makes its optimal decision, x^*, under the constraint that $x \in S$, while taking θ, A, A_f, and β as given. Formally we have that

$$x^*(A\theta, A_f\theta, \beta, S) = \arg\max_{x \in S} \pi(x; A\theta, A_f\theta, \beta).$$

Lemma 1. *The optimal decision, $x^*(A\theta, A_f\theta, \beta, S)$, is nondecreasing in $(A\theta, A_f\theta, \beta, S)$.*

Lemma 1 follows readily from Theorem 1 in Appendix A and is simply the manifestation of the four key complementarities in Assumption 2. Importantly, these comparative statics are partial in nature since the endogeneity of A and A_f is ignored which will prove to be important. The profits obtained under the optimal decision are defined as

$$\pi^*(A\theta, A_f\theta, \beta, S) \equiv \max_{x \in S} \pi(x; A\theta, A_f\theta, \beta).$$

3.2 Entry

Firm profits upon entry are bounded below by zero because the firm exits the industry and forfeits the sunk cost of entry when optimal firm profits, π^*, happen to be negative. Expected profits upon entry are thus given by

$$\Pi(A, A_f, \beta, S) \equiv \int \max\{0, \pi^*(A\theta, A_f\theta, \beta, S)\} \, \mathrm{d}F(\theta),$$

and are assumed to be finite. It is well known that this may require some restrictions on the distribution of productivities; see Melitz (2003). We assume unrestricted entry and an unbounded pool of potential entrants. In

[11] Note that β only contains those parameters that comply with Assumption 2.

equilibrium, the expected profits upon entry must therefore be equal to the cost of entry,

$$\Pi(A, A_f, \beta, S) = wf_e. \tag{3}$$

In foreign, we have that

$$\Pi_f(A_f, A, \beta_f, S_f) \equiv \int \max\{0, \pi_f^*(A_f\theta, A\theta, \beta_f, S_f)\} \, \mathrm{d}F_f(\theta) = w_f f_{ef}. \tag{4}$$

Equations (3) and (4) jointly determine the demand levels A and A_f as functions of e.g. (β, S) and their foreign counterparts. For discussion of existence and uniqueness in a concrete context similar to our model, see AFT.

3.3 Industry Composition

We denote the c.d.f. of the equilibrium distribution of activity i across active firms by $H_i(x_i; \beta, S)$, $i = 1, ..., n$. To characterise these distributions, consider the cross-section of firms in a given equilibrium where (A, A_f, β, S) is given. Since firms with strictly higher θ are able to earn strictly higher profits, the self-selection or sorting of firms into being active or exiting obeys the rule that all firms with productivities above a certain threshold are active and all firms with productivities below exit. Denoting this threshold by θ_a and assuming that the marginal active firms do not export, we have that

$$\theta_a(A, \beta, S) \equiv \inf\{\theta : \pi^*(A\theta, A_f\theta, \beta, S) > 0\}. \tag{5}$$

It is important to note that θ_a is not directly affected by A_f because the marginal active firms do not export. This simplifies the analysis in Section 4. We focus on the case with endogenous entry and exit by assuming that the lowest productivity firms are not able to produce profitably. That is, $\theta_a(A, \beta, S) > \theta_0$. The underlying reason could e.g. be the presence of some fixed costs of production or the presence of a choke price as in Melitz and Ottaviano (2008). The next step is to characterise the sorting of active firms into the activities based on productivity. By Lemma 1, the complementarities of our model imply that, in a given equilibrium, higher productivity firms choose weakly higher levels of all activities. Let θ_i be the lowest level of productivity at which a firm undertakes at least level x_i of activity i. Bounding this threshold from below by θ_a, it is given by

$$\theta_i(x_i; A, A_f, \beta, S) \equiv \max\{\theta_a, \inf\{\theta : x_i^*(A\theta, A_f\theta, \beta, S) \geq x_i\}\}. \tag{6}$$

On the basis of the above sorting pattern, we now characterise the equilibrium distributions of activities and the industry composition. In the following, we focus on a particular level, x_i, of activity i, which could be any level of any of the n activities. Applying the law of large numbers, let $s_a \equiv 1 - F(\theta_a)$ be the share of firms that are active and let $s_i \equiv 1 - F(\theta_i)$ denote the share of firms undertaking at least level x_i of activity i. Note that $s_a \geq s_i$. Using these shares, the c.d.f. of the equilibrium distribution of each activity i can be expressed as

$$H_i(x_i; \beta, S) = 1 - \frac{s_i(x_i; A, A_f, \beta, S)}{s_a(A, \beta, S)}. \tag{7}$$

The industry composition refers jointly to these n distributions.

4 Comparative Statics for the Industry Composition

We now investigate the equilibrium response of the industry composition to increases in the industry-wide parameters (β, S). Consistent with Lemma 1, both increases in β and increases in S provide firms with an incentive to increase their levels of all activities, all else equal. Increases in β do so by increasing the attractiveness of undertaking higher levels of the activities, while increases in S do so by shifting upwards the choice set of available decisions.[12] Importantly, these incentives can be brought about in two distinct ways. By increasing β, the attractiveness of higher levels of activities is increased both if profits associated with higher levels increase and if profits associated with lower levels decrease. Analogously, S is shifted upwards both if higher levels of activities become available and if lower levels become unavailable. While these two types of increases in β and S are not mutually exclusive, their distinct effects on firm profits, expected profits upon entry, and the demand levels, A and A_f, are crucial for the comparative statics. This is because, by Lemma 1, the optimal decision, x^*, is nondecreasing in $(A\theta, A_f\theta)$. Appendix B takes a closer look at these effects at the firm level. Apart from these effects, selection effects through changes in θ_a, and hence entry and exit, are central for our industry-level analysis. These arise since

[12] S can e.g. increase by allowing higher levels of existing activities. This obviously includes the case of allowing levels higher than zero of a given activity, i.e., introducing new complementary activities.

some marginal firms may either leave the industry or become active producers as a result of the change in (β, S). Our notion of monotone comparative statics (MCS) for the industry composition is formalised as follows.

Definition 1. *The industry composition exhibits MCS when increases in (β, S) induce first-order stochastic dominance (FSD) shifts in the equilibrium distributions of all activities. That is, $H_i(x_i; \beta, S)$ is nonincreasing for all levels, x_i, of all activities, $i = 1, \ldots, n$.*

MCS for the industry composition thus mean that the equilibrium distributions of the n activities unambiguously shift towards higher values such that the share of active firms which undertake at least any (positive) level of any activity is nondecreasing. Consequently, the average level of any activity increases. An example of MCS for the industry composition could be increases in the fractions of active firms that export and offshore production in the AFT model. Given the assumption of endogenous exit $(\theta_a > \theta_0)$, the equilibrium distributions, H_i, are both affected by the levels of the activities undertaken by firms conditional on being active (level effect; see Appendix B) and by the endogenous selection of which firms are active (selection effect; see below). The level effect is represented by the effects on s_i and the selection effect is represented by the effects on s_a.

4.1 Sufficient Conditions for MCS for the Industry Composition

To derive sufficient conditions for MCS for the industry composition, we decompose the change in H_i. Denote by ΔH_i the change in H_i induced by an increase in (β, S) from (β', S') to (β'', S'') where either β or S could remain unchanged. This change can be decomposed as follows.

$$\Delta H_i = \underbrace{\frac{s_i(x_i; A', A'_f, \beta', S')}{s_a(A', \beta', S')} - \frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A', \beta'', S'')}}_{\text{Total direct effect}} + \underbrace{\frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A', \beta'', S'')} - \frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'', \beta'', S'')}}_{\text{Total domestic indirect effect}} + \underbrace{\frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'', \beta'', S'')} - \frac{s_i(x_i; A'', A''_f, \beta'', S'')}{s_a(A'', \beta'', S'')}}_{\text{Total foreign indirect effect}}. \tag{8}$$

A' and A'_f relate to (β', S') (and (β'_f, S'_f)) and A'' and A''_f relate to (β'', S'') (and (β''_f, S''_f)). Notice that a change in (β_f, S_f) could capture a foreign response to the exogenous changes in the home country. In order to derive sufficient conditions for MCS for the industry composition, we start by considering the total direct effect on H_i in (8). Since an increase in (β, S) tends to increase the levels of the activities chosen by individual firms given A and A_f, c.f. Appendix B, it tends to increase the share of firms that undertake at least a given level of activity i, s_i.[13] This works in favour of MCS for the industry composition. Next, we need the direct effect on the share of active firms, s_a, to be nonpositive. Intuitively, the marginal active firms have low productivities and therefore undertake relatively low levels of the activities conditional on being active. An increase in the share of active firms therefore works against MCS for the industry composition. To ensure that the total direct effect of an increase in (β, S) works in the right direction, i.e., towards a nonpositive ΔH, we impose the following assumption which applies henceforth.

Assumption 3. *The direct effect of an increase in* (β, S) *on* $s_a(A, \beta, S)$ *is nonpositive.*

Assumption 3 implies that the direct effect (given A and A_f) of increases in (β, S) on the profits of the least productive active firms must be nonpositive such that θ_a is nondecreasing in (β, S) given A and A_f. On the one hand,

[13]To see this formally, note that, by Lemma 1, x^* is nondecreasing in $(A\theta, A_f\theta, \beta, S)$. Thus, it follows from (6) that θ_i is nonincreasing in (β, S) given A and A_f. Therefore, $s_i = 1 - F(\theta_i)$ is nondecreasing in (β, S) given A and A_f.

Assumption 3 is clearly satisfied when the direct effect of an increase in (β, S) on the profits of all firms is nonpositive. On the other hand, Assumption 3 is satisfied when the direct effect of an increase in (β, S) on the profits of the least productive active firms is zero while the effect on the profits of all other firms is nonnegative and positive for some.[14] This situation is very often seen in models of international trade building on Melitz (2003) since the least productive active firms are very often not directly affected by the comparative statics considered (mainly trade liberalisations) because of selection into trading activities. Hence, while Assumption 3 indeed refers to an endogenous object namely the profits of the marginal active firms, Assumption 3 is straightforward to check under typical comparative statics exercises in models of international trade given selection into trading activities. Note that, under Assumption 3, the total direct effect in (8) is nonpositive. The analysis below makes sure that the two remaining indirect effects in (8) are also nonpositive.

Next, consider the total foreign indirect effect in (8) which works through the endogenous change in s_i induced by the change in A_f and which is nonpositive if and only if $\Delta A_f \equiv A''_f - A'_f$ induced by the increase in (β, S) is nonnegative.[15] Note that the total foreign indirect effect is straightforward to sign because of the assumption that the marginal active firms do not export. This implies that s_a is independent of A_f.

Finally, consider the total domestic indirect effect on H_i in (8) which operates through the change in the domestic demand level, A. An increase in A tends to make all firms weakly increase their levels of activity i. This makes the effect on s_i nonnegative which works in favour of MCS for the industry composition.[16] At the same time, an increase in A will allow some previously inactive low-productivity firms to produce profitably wherefore θ_a decreases, all else equal. This follows from (5). The effect on s_a is therefore also nonnegative which works against MCS for the industry composition. These two indirect effects on s_i and s_a through a change in A are reversed when A decreases but are obviously still opposing. Therefore, in order to ensure that the net effect of a change in A on H_i is nonpositive like the total direct effect and the total foreign indirect effect, we need to impose more

[14] This is the reason why Assumption 3 is satisfied in Section 2.

[15] It follows from Lemma 1 and (6) that θ_i is nonincreasing in A_f given A and (β, S). Therefore, s_i is nondecreasing in A_f given A and (β, S).

[16] It follows from Lemma 1 and (6) that θ_i is nonincreasing in A given A_f and (β, S). Therefore, s_i is nondecreasing in A given A_f and (β, S).

structure on our setup. Note that a nonpositive total domestic indirect effect in (8) is equivalent to

$$\frac{s_i(x_i; A', A'_f, \beta'', S'')}{s_a(A', \beta'', S'')} \leq \frac{s_i(x_i; A'', A'_f, \beta'', S'')}{s_a(A'', \beta'', S'')}. \tag{9}$$

When F is C^1, we have that $1 - F(\theta) = e^{-\int_{\theta_0}^{\theta} \lambda_\theta(u)\,\mathrm{d}u}$ where λ_θ denotes the hazard rate of the distribution of θ. Using this observation, (9) can be expressed as

$$e^{-\int_{\theta_a(A',\beta'',S'')}^{\theta_a(A'',\beta'',S'')} \lambda_\theta(u)\,\mathrm{d}u} \leq e^{-\int_{\theta_i(x_i;A',A'_f,\beta'',S'')}^{\theta_i(x_i;A'',A'_f,\beta'',S'')} \lambda_\theta(u)\,\mathrm{d}u}. \tag{10}$$

To tackle this condition, the following assumption comes in handy.

Assumption 4. *The absolute value of the percentage change in θ_a induced by a change in A is weakly larger than the absolute value of the percentage change in θ_i induced by the same change in A.*

Assumption 4 often follows readily from Assumption 1 given that: (i) the marginal active firms do not export; (ii) consumer preferences are CES; (iii) firms face fixed costs of undertaking the various activities. In particular, it is straightforward to illustrate that Assumption 4 holds within Melitz and Redding (2014) and also within the AFT model with nontrivial selection into the two complementary activities exporting and offshoring (see their Section 3.5 and our Section 2). The intuition for Assumption 4 is as follows. First, θ_i may not depend on A whereas θ_a always depends on A. Second, if θ_i depends on A, but not on A_f, then the percentage changes in θ_a and θ_i, induced by a change in A, are equal. Third, θ_i may also depend on both A and A_f in an additively separable fashion. But, in this case, the additive separability gives us the result in Assumption 4. This implies that the absolute value of the change in $\log\theta_a$ induced by the change in A weakly exceeds the absolute value of the change in $\log\theta_i$. A change of integrand in (10) gives us

$$e^{-\int_{\log\theta_a(A',\beta'',S'')}^{\log\theta_a(A'',\beta'',S'')} \lambda_{\log\theta}(u)\,\mathrm{d}u} \leq e^{-\int_{\log\theta_i(x_i;A',A'_f,\beta'',S'')}^{\log\theta_i(x_i;A'',A'_f,\beta'',S'')} \lambda_{\log\theta}(u)\,\mathrm{d}u}, \tag{11}$$

where $\lambda_{\log\theta}$ is the hazard rate of the distribution of log-productivity.

Let us first analyse a weak decrease in A, i.e., an enhancement of competition in the home country. Since $\theta_i \geq \theta_a$ and since the change in $\log\theta_a$

induced by the change in A weakly exceeds the change in $\log\theta_i$ and these changes are nonnegative if competition is enhanced, the condition (11) is fulfilled when A is nonincreasing if the hazard rate of log-productivity is nonincreasing. Recall that log-productivity is distributed with constant hazard rate when productivity is Pareto distributed.[17] A constant hazard rate of log-productivity implies by definition that the density at any level of log-productivity is constant relative to the probability mass above it. This means that the percentage changes (induced by a change in A) in the share of active firms, s_a, and the share of firms undertaking at least a given level of activity i, s_i, are equal if the changes in the log-thresholds, $\log\theta_a$ and $\log\theta_i$, are equal. Relative to this case with a total domestic indirect effect equal to zero, Assumption 4 plus a nonincreasing hazard rate of log-productivity both work in the direction of a nonpositive total domestic indirect effect when A weakly decreases. To see this, note that relative to the case with constant hazard rate of log-productivity (productivity is Pareto distributed), a nonincreasing hazard rate puts more (relative) probability density at $\log\theta_a$ relative to $\log\theta_i$ since $\theta_a \leq \theta_i$. By Assumption 4, the change in $\log\theta_a$ weakly exceeds the change in $\log\theta_i$ induced by the decrease in A. These effects both work in favour of an unambiguously nonpositive total domestic indirect effect and MCS for the industry composition when A falls. In this case, both s_a and s_i will decrease, all else equal, but since log-productivity is distributed with nonincreasing hazard rate, and by Assumption 4, the effect on s_a will be weakly larger than the effect on s_i. In other words: we are assured a sufficient mass of exiting firms and this does the trick because the exiting firms choose relatively low levels of the n activities. It should also be noted that an increasing hazard rate of log-productivity does not work with (11) in general because the change in $\log\theta_a$ induced by the change in A may equal the change in $\log\theta_i$. This rules out the possibilities of log-normally- of Frechet-distributed productivity parameters, cf. footnote 17, if one wants the total domestic indirect effect to be nonpositive.

Summing up, Lemma 1 and Assumption 3 jointly guarantee that the total direct effect in (8) is nonpositive. We have also found that the total

[17] $\log\theta$ being distributed with constant hazard rate, $\lambda_{\log\theta}$, implies that $F_{\log\theta}(\log\theta) = 1 - e^{-\lambda_{\log\theta}(\log\theta - \log\theta_0)}$, where $F_{\log\theta}$ denotes the c.d.f. of $\log\theta$. Rearranging gives $F(\theta) = 1-(\theta_0/\theta)^{\lambda_{\log\theta}}$. Thus $F(\theta)$ is given by the Pareto distribution. The normal distribution has a monotone increasing hazard rate. This also holds for the Gumbel distribution. Recall that log-productivity is distributed Gumbel when productivity is Frechet distributed as in Eaton and Kortum (2002) and Bernard et al. (2003).

foreign indirect effect in (8) is nonpositive if and only if ΔA_f induced by the increase in (β, S) and the potential foreign response is nonnegative. Finally, we have found that the total domestic indirect effect (8) is nonpositive when ΔA is nonpositive and the hazard rate of log-productivity is nonincreasing. Consequently, we have the following main result.

Proposition 1. *Increases in (β, S) induce MCS for the industry composition in the home country if the distribution of log-productivity has a nonincreasing hazard rate and the induced changes in A and A_f are nonpositive and nonnegative, respectively.*

In the more symmetry-oriented model by Bache and Laugesen (2014a), it is often straightforward to figure out whether an increase in (β, S) enhances or dampens competition in one country. In the current context, things are more complicated because of the asymmetries and multiple free-entry conditions. Anyway, the following is known with certainty in the case where (β_f, S_f) is held constant and some, but not all, firms export. If A is nonincreasing as a response to the increase in (β, S), then A_f is nondecreasing and vice versa. This follows readily from the foreign free-entry condition, (4), since the profits of the foreign exporters are nonincreasing when A is nonincreasing. This implies that A_f must be nondecreasing such that the total foreign indirect effect in (8) is nonpositive as needed when one key condition for a nonpositive total domestic indirect effect (A is nonincreasing) is fulfilled. Checking the sign of the endogenous responses of the two demand levels should not pose problems in a concrete and nested model context.[18] Further, if (β_f, S_f) is held constant, one only needs to check the sign of one demand level response. We emphasise, however, that Proposition 1 does not depend on (β_f, S_f) being held constant. Hence, the proposition may also hold under e.g. bilateral trade liberalisations. Finally, consider the case where the home country is a small open economy with positive production of differentiated goods. In this case, unilateral developments in the home country (increases in (β, S)) will not affect the demand levels in one or potentially more foreign countries implying that it becomes straightforward to sign the effect on the domestic demand level via the free-entry condition, (3).[19]

[18] See for instance the analysis of an increase in China's *sourcing potential* in Section 6.1 in AFT which is shown to enhance U.S. competition through decreases in the U.S. price index and the U.S. demand level.

[19] It follows from the analysis in Bache and Laugesen (2014a) that trade liberalisations,

4.2 The Foreign Industry Composition

Let us henceforth focus on the case where A is nonincreasing and A_f is nondecreasing as a result of the increase in (β, S) and where we see MCS for the industry composition in the home country if log-productivity is distributed with nonincreasing hazard rate (Proposition 1). The question is: what happens to the foreign industry composition in this case? As shown in Appendix C, the answer is given by the following proposition.

Proposition 2. *A weak decrease in (β_f, S_f) implies that $H_{if}(x_{if}; \beta_f, S_f)$ is nondecreasing for all levels, x_i, of all activities, $i = 1, \ldots, n$, if the distribution of log-productivity in foreign has a nonincreasing hazard rate and the changes in A and A_f are nonpositive and nonnegative, respectively.*

It is interesting to see that the developments mentioned by Propositions 1 and 2 are reverse and may easily co-exist. This is for instance the case when the conditions behind Proposition 1 hold and (β_f, S_f) is held constant. Relative to this scenario, a weak decrease in (β_f, S_f) only strengthens the developments in foreign and may also make the needed demand level adjustments more likely.

5 Example: the Melitz and Redding (2014) Model

The activities export status (given by an indicator function for active exporting) and total labour demand for variable production are complementary to each other in Melitz (2003).[20] This obviously also holds for the extension to asymmetric countries in the model by Melitz and Redding (2014) which is nested in the model below provided that wages in the (just two) countries are determined through a homogeneous good. For the Melitz and Redding (2014) model we have the following results. Conditional on an increase in competition in a country which unilaterally liberalises trade through a decrease in the fixed costs of exporting, the liberalising country experiences MCS for its industry composition if productivity is distributed Pareto. Importantly, the needed increase in the level of competition is confirmed by the results in Section 2.4 of Demidova and Rodriguez-Clare (2013). Hence, in

like those in Section 2, will decrease the domestic demand level under the small open economy assumption.

[20] See e.g. Section 4.5 in Bache and Laugesen (2014a).

the liberalising country, the firm-size distribution makes a shift to the right (first-order stochastic dominance) and the fraction of exporters increases. The industry-level implications are again reverse in the nonliberalising country implying that the firm-size distribution exhibits a shift to the left and the fraction of exporters decreases. Moreover, as firm revenue is monotonically increasing in labour demand for variable production (q/φ in the original notation), average firm revenue increases in the liberalising country (we see a shift to the right for the distribution of firm revenue) and decreases in the nonliberalising country.

6 Concluding Remarks

We perceive our results as striking. Developments in one country can impact the other country in a way not previously emphasised by the trade literature. However, our results are so far conditional on movements in the endogenous demand levels. It would of course be attractive to determine whether these movements appear or not within a fairly general model like ours. Throwing light on this issue will allow us to better understand the important question of how the responses of (and also characteristics of) a trade partner to trade liberalisations in one country will affect both countries. Moreover, such an analysis will be useful for pointing out the need and effects of global policy coordination. We intend to investigate these issues in the future. Until new results on these matters have been obtained, we propose to check for the movements in A and A_f by looking at the effects of comparative statics in specific models (like e.g. the AFT model) nested within our framework. Under such a strategy, it should be straightforward to check the induced movements in A and A_f. Lastly, we point out that the induced changes in A and A_f are unambiguous following an unilateral trade liberalisation under the assumption of a small open economy. In such a case, unilateral trade liberalisation is very likely to cause asymmetric MCS for the industry compositions.

A Mathematical Appendix

Let $X \subseteq \mathbb{R}^n$ and $T \subseteq \mathbb{R}^m$ be partially ordered sets with the component-wise order.[21] For two vectors, $x', x'' \in \mathbb{R}^n$, we let $x' \vee x''$ denote the component-wise maximum and $x' \wedge x''$ denote the component-wise minimum.[22] The set X is a lattice if for all $x', x'' \in X$, $x' \vee x'' \in X$ and $x' \wedge x'' \in X$. The set $S \subseteq X$ is a sublattice of X if S is a lattice itself. For two sets, $S', S'' \subseteq \mathbb{R}^n$, we say that S'' is higher than S' and write $S' \leq_s S''$ if for all $x' \in S'$ and all $x'' \in S''$, $x' \vee x'' \in S''$ and $x' \wedge x'' \in S'$. If a set becomes higher, then we say that the set is increasing.

Let X be a lattice. The function $h : X \times T \to \mathbb{R}$ is supermodular in x on X for each $t \in T$ if for all $x', x'' \in X$ and $t \in T$,

$$h(x', t) + h(x'', t) \leq h(x' \wedge x'', t) + h(x' \vee x'', t). \tag{12}$$

Supermodularity of h in x implies that the return from increasing several elements of x together is larger than the combined return from increasing the elements separately.[23] This follows from the fact that a higher value of one subset of the elements in x increases the value of increasing other subsets of elements. Supermodularity thus implies that the elements of the vector x are (Edgeworth) complements. If h is smooth, supermodularity is equivalent with $\partial^2 h/\partial x_i \partial x_j \geq 0$ for all i, j where $i \neq j$. By (12), it follows that any function h is trivially supermodular in x when x is a single real variable. The function $h(x, t)$ has increasing differences in (x, t) if for $x' \leq x''$, $h(x'', t) - h(x', t)$ is monotone nondecreasing in t. Increasing differences mean that increasing t raises the return from increasing x and vice versa. If h is smooth, increasing differences are equivalent with $\partial^2 h/\partial x_i \partial t_j \geq 0$ for all i, j. The following monotonicity theorem is due to Topkis (1978).

Theorem 1. *Let $X \subseteq \mathbb{R}^n$ be a lattice, $T \subseteq \mathbb{R}^m$ be a partially ordered set, S be a sublattice of X, and $h : X \times T \to \mathbb{R}$. If $h(x, t)$ is supermodular in x on X for each $t \in T$ and has increasing differences in (x, t) on $X \times T$, then $\arg\max_{x \in S} h(x, t)$ is monotone nondecreasing in (t, S).*

[21] For $x' = (x'_1, \ldots, x'_n) \in \mathbb{R}^n$ and $x'' = (x''_1, \ldots, x''_n) \in \mathbb{R}^n$, $x' \leq x''$ if $x'_i \leq x''_i$ for $i = 1, ..., n$ and $x' < x''$ if $x' \leq x''$ and $x' \neq x''$.

[22] That is, $x' \vee x'' = (\max\{x'_1, x''_1\}, \ldots, \max\{x'_n, x''_n\})$ and $x' \wedge x'' = (\min\{x'_1, x''_1\}, \ldots, \min\{x'_n, x''_n\})$.

[23] To see this, rewrite (12) into $[h(x', t) - h(x' \wedge x'', t)] + [h(x'', t) - h(x' \wedge x'', t)] \leq h(x' \vee x'', t) - h(x' \wedge x'', t)$.

If the set of maximisers only contains one element, this unique maximiser is nondecreasing in (t, S). In the remainder of this paper, we restrict attention to cases where the set of maximisers is a nonempty and complete sublattice.[24] This implies that the set of maximisers has greatest and least elements, and Theorem 1 implies that these greatest and least elements are nondecreasing functions of (t, S). We follow the convention of focusing on the greatest element in the set of maximisers, effectively treating this maximiser as unique.[25]

B The Firm Level of Analysis

Let us define the equilibrium decision of a firm conditional on being active as

$$\tilde{x}^*(\theta, \beta, S) \equiv x^*(A\theta, A_f\theta, \beta, S). \tag{13}$$

From the RHS of (13), it is clear that changes in (β, S) have a direct effect on firm decisions for given demand levels, A and A_f, but such changes also have two indirect effects through changes in the two demand levels. This dichotomy allows us to decompose the total effect on $\tilde{x}^*$ from changing (β', S') to (β'', S'') where either β or S could remain unchanged. Define $\Delta\tilde{x}^* \equiv \tilde{x}^*(\theta, \beta'', S'') - \tilde{x}^*(\theta, \beta', S')$ and note that

$$\begin{aligned} \Delta\tilde{x}^* = {} & \underbrace{x^*(A'\theta, A'_f\theta, \beta'', S'') - x^*(A'\theta, A'_f\theta, \beta', S')}_{\text{Direct effect}} \\ & + \underbrace{x^*(A''\theta, A'_f\theta, \beta'', S'') - x^*(A'\theta, A'_f\theta, \beta'', S'')}_{\text{Domestic indirect effect}} \\ & + \underbrace{x^*(A''\theta, A''_f\theta, \beta'', S'') - x^*(A''\theta, A'_f\theta, \beta'', S'')}_{\text{Foreign indirect effect}}, \end{aligned}$$

where A' and A'_f relate to (β', S') (and (β'_f, S'_f)) and A'' and A''_f relate to (β'', S'') (and (β''_f, S''_f)). It follows from Lemma 1 that an increase in (β, S) always has a nonnegative direct effect on the equilibrium decision, $\tilde{x}^*$. The increase in (β, S) provides firms an incentive to increase their levels of at least one activity. The inherent complementarities among activities ensure

[24] General sufficient conditions for this are found in Milgrom and Shannon (1994).

[25] We share this approach with Bagwell and Ramey (1994) and Holmstrom and Milgrom (1994). All results also hold when one focuses on e.g. the least element.

that this is manifested in an increase in $\tilde{x}^*$, all else equal. Whereas the direct effect of an increase in (β, S) on $\tilde{x}^*$ is unambiguously nonnegative irrespective of how (β, S) increases, the signs of the indirect effects critically depend on whether competition is enhanced or dampened in the two countries. By Lemma 1, the sign of a particular indirect effect is equivalent to the sign of the change in the relevant A. Thus, an indirect effect is aligned with the direct effect when competition is dampened (such that the relevant demand level increases) but opposed to the direct effect when competition is enhanced (the relevant demand level decreases). The observations make it straightforward to devise sufficient conditions for MCS at the firm level but also make it clear that these conditions are not generally fulfilled.

C The Foreign Industry Composition

Similar to (7) we have that

$$H_{if}(x_{if}; \beta_f, S_f) = 1 - \frac{s_{if}(x_{if}; A_f, A, \beta_f, S_f)}{s_{af}(A_f, \beta_f, S_f)}.$$

Similar to (8) we have that

$$\begin{aligned}\Delta H_{if} = & \underbrace{\frac{s_{if}(x_{if}; A'_f, A', \beta'_f, S'_f)}{s_{af}(A'_f, \beta'_f, S'_f)} - \frac{s_{if}(x_{if}; A'_f, A', \beta''_f, S''_f)}{s_{af}(A'_f, \beta''_f, S''_f)}}_{\text{Total direct effect in foreign}} \\ & + \underbrace{\frac{s_{if}(x_{if}; A'_f, A', \beta''_f, S''_f)}{s_{af}(A'_f, \beta''_f, S''_f)} - \frac{s_{if}(x_{if}; A''_f, A', \beta''_f, S''_f)}{s_{af}(A''_f, \beta''_f, S''_f)}}_{\text{Total domestic indirect effect in foreign}} \\ & + \underbrace{\frac{s_{if}(x_{if}; A''_f, A', \beta''_f, S''_f)}{s_{af}(A''_f, \beta''_f, S''_f)} - \frac{s_{if}(x_{if}; A''_f, A'', \beta''_f, S''_f)}{s_{af}(A''_f, \beta''_f, S''_f)}}_{\text{Total foreign indirect effect in foreign}}.\end{aligned}$$

First, take the total foreign indirect effect in foreign. This effect is clearly nonnegative when A is nonincreasing since $s_{if}(x_{if}; A''_f, A', \beta''_f, S''_f) \geq s_{if}(x_{if}; A''_f, A'', \beta''_f, S''_f)$ in this case.

Next, take the total domestic indirect effect in foreign. This effect is nonnegative if

$$\frac{s_{if}(x_{if}; A'_f, A', \beta''_f, S''_f)}{s_{af}(A'_f, \beta''_f, S''_f)} \geq \frac{s_{if}(x_{if}; A''_f, A', \beta''_f, S''_f)}{s_{af}(A''_f, \beta''_f, S''_f)},$$

or, equivalently, if

$$e^{\int_{\log \theta_{af}(A''_f, \beta''_f, S''_f)}^{\log \theta_{af}(A'_f, \beta''_f, S''_f)} \lambda_{f \log \theta}(u) \, \mathrm{d}u} \geq e^{\int_{\log \theta_{if}(x_{if}; A''_f, A', \beta''_f, S''_f)}^{\log \theta_{if}(x_{if}; A'_f, A', \beta''_f, S''_f)} \lambda_{f \log \theta}(u) \, \mathrm{d}u}, \tag{14}$$

where $\lambda_{f \log \theta}$ denotes the hazard rate of log-productivity in foreign. By Assumption 4, it is clear that the inequality (14) is satisfied if log-productivity is distributed with nonincreasing hazard rate in foreign while A_f is nondecreasing. Moreover, (14) will not hold in general under a strict increase in A_f if log-productivity is distributed with strictly increasing hazard rate. This because the log-changes in θ_{af} and θ_{if} may be equal.

Finally, take the total direct effect in foreign and note that $s_{af}(A'_f, \beta'_f, S'_f) \leq s_{af}(A'_f, \beta''_f, S''_f)$ if (β_f, S_f) is nonincreasing (by Assumption 3). Further, it is also clear that $s_{if}(x_{if}; A'_f, A', \beta'_f, S'_f) \geq s_{if}(x_{if}; A'_f, A', \beta''_f, S''_f)$ when (β_f, S_f) is nonincreasing. Hence, if an assumption like Assumption 3 holds in foreign and if (β_f, S_f) is nonincreasing, then the total direct effect in foreign is also nonnegative. This completes the proof.

References

Amiti, M., Davis, D.R., 2012. Trade, Firms, and Wages: Theory and Evidence. Review of Economic Studies 79, 1–36.

Antràs, P., Helpman, E., 2004. Global Sourcing. Journal of Political Economy 112(3), 552–580.

Antràs, P., Fort, T.C., Tintelnot, F., 2014. The Margins of Global Sourcing: Theory and Evidence from U.S. Firms. Working Paper.

Arkolakis, C., 2010. Market Penetration Costs and the New Consumers Margin in International Trade. Journal of Political Economy 118(6), 1151–1199.

Arkolakis, C., Costinot, A., Donaldson, D., Rodríguez-Clare, A., 2012. The Elusive Pro-Competitive Effects of Trade. Working Paper.

Arkolakis, C., Muendler, M.A., 2011. The Extensive Margin of Exporting Products: A Firm-level Analysis. Working Paper.

Athey, S., Schmutzler, A., 1995. Product and Process Flexibility in an Innovative Environment. RAND Journal of Economics 26(4), 557–574.

Bache, P.A., Laugesen, A., 2014a. Monotone Comparative Statics for the Industry Composition. Working Paper.

Bache, P.A., Laugesen, A., 2014b. Trade Liberalisation and Vertical Integration. Working Paper.

Bagwell, K., Ramey, G., 1994. Coordination Economies, Advertising, and Search Behavior in Retail Markets. American Economic Review 84(3), 408–517.

Bernard, A.B., Eaton, J., Jensen, J.B., Kortum, S., 2003. Plants and Productivity in International Trade. American Economic Review 93(4), 1268–1290.

Bernard, A.B., Jensen, J.B., Redding, S.J., Schott, P.K., 2012. The Empirics of Firm Heterogeneity and International Trade. Annual Review of Economics 4, 283–313.

Bernard, A.B., Redding, S.J., Schott, P.K., 2011. Multiproduct Firms and Trade Liberalization. The Quarterly Journal of Economics 126(3), 1271–1318.

Bustos, P., 2011. Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms. American Economic Review 101(1), 304–340.

Caliendo, L., Rossi-Hansberg, E., 2012. The Impact of Trade on Organization and Productivity. The Quarterly Journal of Economics 127(3), 1393–1467.

Davis, D.R., Harrigan, J., 2011. Good Jobs, Bad Jobs, and Trade Liberalization. Journal of International Economics 84, 26–36.

Demidova, S., Rodriguez-Clare, A., 2013. The Simple Analytics of the Melitz Model in a Small Economy. Journal of International Economics 90, 266–272.

Eaton, J., Kortum, S., 2002. Technology, Geography, and Trade. Econometrica 70(5), 1771–1780.

Helpman, E., Itskhoki, O., 2010. Labour Market Rigidities, Trade and Unemployment. Review of Economic Studies 77, 1100–1137.

Helpman, E., Itskhoki, O., Redding, S.J., 2010. Inequality and Unemployment in a Global Economy. Econometrica 78(4), 1239–1283.

Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Export Versus FDI with Heterogeneous Firms. American Economic Review 94(1), 300–316.

Holmstrom, B., Milgrom, P., 1994. The Firm as an Incentive System. American Economic Review 84(4), 972–991.

Kohler, W., Smolka, M., 2011. Sourcing Premia with Incomplete Contracts: Theory and Evidence. The B.E. Journal of Economic Analysis & Policy 11(1), 1–39.

Mayer, T., Melitz, M., Ottaviano, G.I., 2014. Market Size, Competition, and the Product Mix of Exporters. American Economic Review 104(2), 495–536.

Melitz, M.J., 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. Econometrica 71(6), 1695–1725.

Melitz, M.J., Ottaviano, G.I.P., 2008. Market Size, Trade, and Productivity. Review of Economic Studies 75, 295–316.

Melitz, M.J., Redding, S.J., 2014. Heterogeneous Firms and Trade, in: Handbook of International Economics. Elsevier.

Milgrom, P., Shannon, C., 1994. Monotone Comparative Statics. Econometrica 62(1), 157–180.

Mrazova, M., Neary, J.P., 2013. Selection Effects with Heterogeneous Firms. Working Paper.

Topkis, D.M., 1978. Minimizing a Supermodular Function on a Lattice. Operations Research 26, 305–321.