Economics Working Papers
2013-14

Trade Liberalisation and Vertical Integration

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Abstract

We build a three-country model of international trade in final and intermediate goods and study the relation between different types of trade liberalisation and vertical integration. Firms are heterogeneous with respect to both productivity and factor intensity as observed in data. Final-good producers face decisions on exporting, vertical integration of intermediate-input production, and whether the intermediate-input production should be offshored to a low-wage country. We find that due to firm-level complementarities, the shares of final-good producers that pursue either vertical integration, offshoring, or exporting are all increasing when intermediate- or final-goods trade is liberalised and when the cost of vertical integration is reduced. At the same time, one will observe individual firms that shift away from either vertical integration, offshoring, or exporting. All these results hold for a class of productivity distributions to which the Pareto distribution belongs.

Keywords: International Trade; Firm Heterogeneity; Incomplete Contracts; Vertical Integration; Offshoring; Exporting; Trade Liberalisation

JEL Classifications: D23; F12; F61; L23

*The authors thank Ron Davies, Hartmut Egger, Allan Sørensen, and seminar participants in Aarhus, Bayreuth, Dublin, Kiel, Reykjavik, and Saint Petersburg for helpful comments. The authors are grateful to the Tuborg Foundation for financial support.

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1 Introduction

We develop a three-country model of trade in final and intermediate goods in order to investigate the relation between vertical integration and liberalisations of international trade in final and intermediate goods. The relationship between final-good producers and intermediate-good suppliers is characterised by incomplete contracts. Final-good producers face a joint decision on the three activities vertical integration, offshoring, and exporting.\(^1\) Importantly, the model is characterised by firm-level complementarities between these three activities, in the sense that undertaking one of the activities raises the gains from undertaking the others. Our main contribution is to derive a series of strong and testable results that illustrate how these complementarities have clear implications for the industry composition. In particular, we find that the shares of final-good producers that pursue either vertical integration, offshoring, or exporting, i.e., the prevalences of these activities, are all increasing when intermediate- or final-goods trade is liberalised and when the cost of vertical integration falls. Meanwhile, one observes individual firms shifting away from either vertical integration, offshoring, or exporting under the comparative statics mentioned above. This observation is compatible with rising prevalences of the three activities because some low-productivity firms, which do not undertake any of these activities, endogenously shut down due to fiercer competition.

These main findings relate to the ongoing discussion about the relationship between trade liberalisation, or more generally competition, and vertical integration. While elements of the popular press and the seminal studies by McLaren (2000) and Antràs and Helpman (2004) (henceforth AH) have accentuated a negative relation between trade liberalisation and vertical integration, other theoretical contributions like Grossman and Helpman (2004), Ornelas and Turner (2008), and Conconi et al. (2012) have shown that the relationship between trade liberalisation and vertical integration is often ambiguous.\(^2\) This paper relates to all these studies by unveiling a clear positive

\(^1\)We define vertical integration (outsourcing) as the acquisition of an intermediate input from an affiliated (unaffiliated) supplier. Offshoring refers to the phenomenon that production of this intermediate input takes place in the low-wage South.

\(^2\)In relation to this, Grossman and Helpman (2002) and Acemoglu et al. (2010) show that the relation between competition and vertical integration can be ambiguous. These theoretical results have been corroborated by the empirical evidence in Aghion et al. (2006) and Acemoglu et al. (2010). Recent work by Alfaro et al. (2012) finds a negative relation
relationship between different types of trade liberalisation and the prevalence of vertical integration. Further, we show that this finding is compatible with ambiguities at lower levels of aggregation. First, some firms shift away from vertical integration in the wake of trade liberalisation. Second, the share of firms undertaking vertical integration domestically may decrease or increase, partly depending on whether intermediate- or final-goods trade is being liberalised.

Our model builds on the two prominent models of international trade by Melitz (2003) and AH. In an important departure from AH, we accentuate a natural complementarity between the activities vertical integration and offshoring. In addition, our model serves as a natural extension of AH for several reasons. First, the tradeoffs governing the integration and offshoring decisions in the AH model, which does not include a possibility of exporting, can reasonably be expected to depend on the export status of the firm. One reason is a complementarity between the activities offshoring and exporting for which Amiti and Davis (2011), Bas (2012), and Kasahara and Lapham (2013) provide tentative evidence. Another reason is that the export decision partly determines the scale of the firm, which is likely to affect the decision to vertically integrate. Consistent with these speculations, Kohler and Smolka (2011) note that Spanish exporters are more likely than non-exporters to pursue vertical integration and offshoring.

Second, we extend the AH model to allow firms within the same industry to be heterogeneous with respect to headquarter intensity as well as productivity. Headquarter intensity refers to the elasticity of output with respect to headquarter services, which are one input into production. Heterogeneity in headquarter intensity serves two main purposes. Primarily, allowing for heterogeneity in headquarter intensity is a direct theoretical modelling response to the empirical findings of for instance Corcos et al. (forthcoming). These authors reveal that factor intensities like capital and labor between trade liberalisation and vertical integration in firm-level data.

The underlying forces behind this complementarity are present in the AH model but are obscured due to the desire to force the model to generate a rich equilibrium sorting pattern based on productivity alone.

Further, Bernard et al. (2012) note that across U.S. industries, there is a positive correlation (0.87 and significant) between the share of exporters and the share of importers of intermediate inputs.

Acemoglu et al. (2010) note that vertically integrated firms are larger than non-integrated firms, which indicates that size may play a role in the decision to integrate.
skill intensity—commonly used empirical proxies for headquarter intensity, cf. Antràs (forthcoming)—exhibit considerable variation across firms within narrowly defined industries. Corcos et al. (forthcoming) also argue that firm-level capital and skill intensities are important determinants of the decision to vertically integrate since the probability of intra-firm importing increases in these firm-level intensities conditional on firm productivity and importing. These new empirical observations indicate a need for extending the influential open-economy property-rights theory of the firm, pioneered by Antràs (2003) and AH and building on the work of Grossman and Hart (1986), to include firm-specific headquarter intensities, and our contribution is a first attempt in this regard. Furthermore, allowing for heterogeneity in headquarter intensity as well as in productivity is a natural way to obtain an industry equilibrium with the wide variety of organisational decisions observed by e.g. Tomiura (2007) and Kohler and Smolka (2011). An interesting and reassuring aspect of our model is that the sorting of firms into activities based on their productivity and headquarter intensity is broadly consistent with the empirical findings of Corcos et al. (forthcoming). However, we also illustrate that one has to be careful when applying a conventional intuition about the relation between headquarter intensity and vertical integration to a firm-level analysis of vertical integration and intra-firm importing in an open-economy context.

Third, our model departs from its AH backbone in dispensing with the common assumption of Pareto-distributed firm-specific productivity parameters in favour of a broader class of distributions that includes the Pareto.

Despite the apparent complexity of our model, it remains surprisingly tractable. The main reason is the firm-level complementarities inherent in our model. Drawing on the analysis of Bache and Laugesen (2013), these complementarities allow us to keep track of both the sorting of firms into

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6In the French firm-level data used by Corcos et al. (forthcoming), the capital and skill intensities of production exhibit much more variation within than across industries. Further, within-industry heterogeneity in factor intensity is even more pronounced than in its productivity counterpart. Bernard et al. (2003) also argue that industry is a poor indicator of firm factor intensity among U.S. manufacturers.

7As mentioned by Antràs (forthcoming) and Corcos et al. (forthcoming), this extension should make the open-economy property rights model more suitable for future analyses based on the firm-level sourcing data which are starting to appear. Empirical tests of the open-economy property-rights theory of the firm have broadly provided empirical support to the model but mostly been based on industry-level data on intra-firm trade; see e.g. Nunn and Trefler (forthcoming) and the survey by Antràs (forthcoming).
activities and comparative statics. Compared to Bache and Laugesen (2013) who analyse a framework that is much more generally formulated, the specific setup we analyse in this paper allows us to go a step further and show that the prevalences of the three activities we consider are strictly increasing as opposed to nondecreasing. That is, to arrive at strictly monotone comparative statics for the industry composition. Another innovation relative to Bache and Laugesen (2013) is to analyse the prevalence of firm strategies that combine various activities such as vertical FDI. Finally, in the present paper, we are able to derive results on the sorting of firms in more than just the productivity dimension.

2 Model

We build a three-country heterogeneous-firms trade model with two symmetric northern (N) countries that interact through intra-industry trade in differentiated final goods. In addition to the differentiated-goods industry, which is monopolistically competitive, each northern country contains a perfectly-competitive homogeneous-good industry. Our analysis shall focus on the former. The third country is South (S) which does neither demand nor produce differentiated goods. While South also has a perfectly-competitive industry producing the homogeneous good, South basically serves as a possible production site for intermediate inputs to production. Offshoring denotes the phenomenon that a northern final-good producer decides to let its production of intermediate inputs take place in South. The attraction of producing in South is its perfectly elastic supply of labour at the relatively low wage, \( w_S \). The northern wage is normalised to unity, \( w_N = 1 \), such that \( w_S < 1 \) is also the relative wage. In general equilibrium, this wage difference is justified by a labour productivity difference in the production of the freely-traded homogeneous good, \( q_0 \), which is produced and consumed in all three countries.

The preferences of the representative consumer in each N country are represented by the utility function,

\[
U = q_0 + \log \left[ \int_{i \in \omega} q(i)^\alpha di \right]^{1/\alpha}, \quad 0 < \alpha < 1,
\]

As mentioned in the introduction, our model builds upon Melitz (2003) and AH.
where \( q(i) \) denotes the quantity consumed of variety \( i \) of the differentiated goods. Each final-good producer produces a single unique variety and \( \omega \) denotes the endogenous measure of available varieties. Demand for variety \( i \) is given by the demand function,

\[
q(i) = Ap(i)^{-\sigma},
\]

where \( p(i) \) is the price, \( \sigma = 1/(1 - \alpha) \), and the demand shifter \( A \) is taken as given by firms while being endogenous in the aggregate.

### 2.1 Firm Entry

Prospective final-good firms in \( N \) pay \( f_E \) units of local labour in order to enter the monopolistically-competitive industry and to draw a productivity, \( \theta \), from a known distribution. This distribution, \( F(\theta) \), is \( C^1 \) on the interior of its domain which is unbounded from above. We let the distribution of \( \log \theta \) have nonincreasing hazard rate. Note that this is the case under the common assumption that \( \theta \) is Pareto distributed since this implies that \( \log \theta \) is distributed with constant hazard rate.\(^9\) Simultaneously with the realisation of \( \theta \), firms also realise their idiosyncratic characteristic \( \eta \) which also affects their technology of production. For reasons which will become clear below, we refer to \( \eta \) by the term headquarter intensity. \( \eta \) is independently distributed from \( \theta \) according to the distribution \( G(\eta) \) which is strictly increasing on its domain, \( \eta \in (0, 1) \). Upon the realisation of \((\theta, \eta)\), firms make their optimal decisions, \( klx \), where \( k \in \{O, V\}, l \in \{N, S\}, x \in \{D, X\} \), or exit the industry. The decision comprises three sub-decisions. First, a make-or-buy decision concerning procurement of an intermediate input, \( m \). This determines the ownership structure, \( k \), which can be either vertical integration (henceforth integration), \( V \), or outsourcing, \( O \). Second, an offshoring decision concerning the location of the production of the intermediate input. Through offshoring, this input may be produced in \( S \) and used in final-good production in \( N \). This determines \( l \in \{N, S\} \) where \( l = S \) under offshoring and \( l = N \) otherwise. The intermediate input will never be shipped between northern countries in equilibrium. Third, we have a decision about exporting. The export status, \( x \), can either be \( X \) for exporter or \( D \) for nonexporter or domestic firm. If no choice of \( klx \) entails positive profits, the firm exits the industry and forfeits its cost of entry.

\(^9\)Note that \( \log \theta \) being distributed with nonincreasing hazard rate implies that \( \log(\theta^\varepsilon) \) is distributed with nonincreasing hazard rate for \( \varepsilon > 0 \).
2.2 Production

Production of final-good variety \( i \) is given by
\[
q(i) = \theta(i)\zeta(i)h(i)^{\eta(i)}m(i)^{1-\eta(i)},
\]
where \( \zeta(i) \equiv \eta(i)\left(1 - \eta(i)\right)^{-(1-\eta(i))} \). \( h(i) \) and \( m(i) \) denote investments in headquarter services and a manufacturing intermediate input, respectively.\(^{10}\)

In the following, we focus attention to a given final-good variety and drop the index \( i \). Investments in headquarter services, \( h \), are undertaken by the particular final-good producer, \( H \), itself. Investments in intermediate inputs, \( m \), are undertaken by an intermediate-input supplier, \( M \). Due to a perfectly elastic supply of homogeneous \( M \) in all countries, matching is always unproblematic for \( H \). One unit of either input, \( h \) or \( m \), is produced from one unit of local labour in the country of its production. In contrast to \( m \), \( h \) can only be produced in the \( N \) country where \( H \) entered.

Trade in final and intermediate goods is costly. Iceberg costs of international final-goods trade are \( \tau > 1 \). We include the intermediate-goods iceberg trade costs, \( \tau_m > 1 \), in \( w_S \) such that \( w_S = \bar{w}_S\tau_m \) where \( \bar{w}_S \) is the relative southern wage net of trade costs that we pin down by the homogeneous good. Finally, production implies fixed costs, \( f_{klx} \), where
\[
f_{klx} = f_k + \mathbb{I}_S(l)f_S + \mathbb{I}_X(x)f_X. \tag{2}
\]

The assumption \( 0 < f_O < f_V \) has been used extensively in the previous literature ranging from Grossman and Helpman (2002) and AH to Antràs and Chor (forthcoming) and we simply follow suit. \( \mathbb{I}_S(l) \) and \( \mathbb{I}_X(x) \) are indicator functions for offshoring and exporting, respectively.\(^{11}\) As in Amiti and Davis (2011), offshoring and exporting both imply discrete increases in fixed costs since \( f_S, f_X > 0 \). All fixed costs are denominated in northern labour.

\(^{10}\)We use the term headquarter intensity for \( \eta(i) \) since \( \frac{\partial q(i)}{\partial h(i)} = \eta(i) \). Skill, R&D, and advertisement intensities have earlier been used as empirical proxies for \( \eta(i) \) since \( h(i) \) is often thought of as a white-collar input to production. Antràs (2003) uses capital intensity as an empirical proxy for \( \eta(i) \) since \( h(i) \) is intensive in capital in this model.

\(^{11}\)That is, \( \mathbb{I}_S(S) = \mathbb{I}_X(X) = 1 \) and \( \mathbb{I}_S(N) = \mathbb{I}_X(D) = 0 \).
3 The Decision

Under exporting, the final-good production is distributed across the two northern markets. The final-good producer, $H$, faces the following problem of maximising total revenue for a given level of production,

$$R(\theta, \eta, h, m, x; A) = \max_{q_D, q_X} A^{1/\sigma} \left[ q_D^\alpha + 1_X(x) (q_X/\tau)^\alpha \right]$$

s.t. \( q_D + q_X \leq \theta \zeta h^n m^{1-\eta}, \)

where \( q_D \) and \( q_X \) denote the quantities produced for, respectively, the domestic and the export markets when only \( q_X/\tau \) units of final goods arrive for sale on the export market. The optimal allocation entails a revenue of

$$R(\theta, \eta, h, m, x; A) = A^{1/\sigma} \theta^\alpha h^{\alpha \eta} m^{\alpha (1-\eta)} \zeta^\alpha (1 + \tau^{1-\sigma}) 1_X(x)(1-\alpha). \quad (3)$$

The next step is to analyse the equilibrium investments in $h$ and $m$. To this end, we impose an assumption of complete asset specificity meaning that the inputs, $h$ and $m$, are completely tailored to the production of the particular variety under scrutiny and useless elsewhere. Importantly, we assume that only the decision, $klx$, and not the subsequent production of $h$ and $m$, is contractible.\(^{12,13}\) After $klx$ is chosen, $H$ and $M$ simultaneously and noncooperatively determine their respective relationship specific investments, $h$ and $m$, while foreseeing future Nash bargaining over final-good revenue. Through a process of Nash bargaining, $H$ reaps the fraction $\beta_k$ of equilibrium revenue while $M$ reaps the complementary fraction. Following Grossman and Hart (1986) and Antràs (2005), we impose the following assumption about

\(^{12}\)Since Coase (1937), it has been known that firm boundaries are indeterminate in a world of complete contracts. Because we want to determine firm boundaries, we resort to an assumption of incomplete contracting where input investments are ex-post observable to the transacting parties but not verifiable by third parties. For an interesting extension into partial contractibility, see Antràs and Helpman (2008).

\(^{13}\)AH assume that $k$ and $l$ are contractible sub-decisions but their model does not include the possibility of exporting. We assume that parties can contract on export status as exports usually leave a paper trail. This trail is e.g. created from dealings with customs and shipping agencies.
relative bargaining power.\textsuperscript{14}

\[ \beta_V > \beta_O = 1/2. \]  

The inequality in (4) and the implied input-investment incentives are key for the tradeoff between integration and outsourcing. More on this below. The well-known intuition behind (4) is that integration entails more extensive property rights for \( H \) and thus improved bargaining power. When choosing their investments in \( h \) and \( m \), \( H \) and \( M \) foresee the share of the revenue, (3), they will receive as a consequence of the Nash bargaining. Hence, by backwards induction, the optimal decision, \( k_{lx} \), solves the programme,

\[
\begin{align*}
\max_{k \in \{O,V\}, \ell \in \{N,S\}, x \in \{D,X\}} & \quad R(\theta, \eta, h_{klx}, m_{klx}, x; A) - h_{klx} - w_l m_{klx} - f_{klx} \\
\text{s.t.} & \quad h_{klx} = \arg \max_h \beta_k R(\theta, \eta, h, m_{klx}, x; A) - h, \\
& \quad m_{klx} = \arg \max_m (1 - \beta_k) R(\theta, \eta, h_{klx}, m, x; A) - w_l m.
\end{align*}
\]  

In (5), we have implicitly used that \( H \) extracts all rents from \( M \) through a participation fee which assures that \( M \) is left at its outside option of zero. Consequently, the decision simply maximises joint bilateral profits. By combining (3) with the Nash equilibrium input investments from the constraints in (5) and the fixed costs of production in (2), we arrive at the profits of \( H \) as a function of the decision \( k_{lx} \),

\[ \pi_{klx}(\Theta, \eta; A) = A \Theta \psi_k(\eta) \gamma_l(\eta)(1 + \tau^{1-\sigma})^{1 \times(x)} - f_{klx}, \]

where \( \Theta = \theta^\sigma \), \( \gamma_l(\eta) = w_i^{(1-\eta)(1-\sigma)} \), and

\[ \psi_k(\eta) = \frac{1 - \alpha [\beta_k \eta + (1 - \beta_k)(1 - \eta)]}{[\alpha \beta_k^{\sigma}(1 - \beta_k)^{(1-\eta)}]^{\sigma-1}}. \]

Note that \( \gamma_N = 1 \) and \( \psi_O = (1 - \alpha/2)(\alpha/2)^{\sigma-1} \) are independent of \( \eta \). Immediately upon entry and the realisation of \((\theta, \eta)\), \( H \) chooses the decision,

\textsuperscript{14}AH provide microfoundations for the inequality in (4) which is based on differences in outside options. In their notation, we assume that \( \delta^N = \delta^S \), i.e., an equally large fraction of final-good production is lost under bargaining break-down under integration whether or not offshoring occurs. This simplifying assumption is shared with Antràs (2003). AH assume that \( \delta^N \geq \delta^S \), i.e., break-down is no less costly under offshoring.
klx, that maximises (6) given that these profits are positive. In case optimal profits,
\[ \pi^*(\Theta, \eta; A) = \max_{klx} \pi_{klx}(\Theta, \eta; A), \] (7)
are negative, \( H \) exits the industry and forfeits the fixed cost of entry. We assume that, for all \( \eta \), at least some low-productivity firms choose not to produce. In equilibrium, the following free-entry condition holds since the pool of potential entrants is unbounded.
\[ f_E = \int \int \max\{0, \pi^*(\Theta, \eta; A)\} dF(\theta) dG(\eta). \] (8)
The industry’s demand level, \( A \), is implicitly determined by (8) as a function of all parameters.\(^{15}\)

### 3.1 Input Sourcing and Headquarter Intensity

Before discussing the sorting pattern and the results of our model, we want to make clear how headquarter intensity affects the attractiveness of offshoring and integration. First, we notice that variable profits, \( A\Theta \psi_k(\eta)\gamma_l(\eta)(1 + \tau^{1-\sigma})^{Lx(x)} \), are always increasing in offshoring because \( \gamma_S(\eta) > \gamma_N = 1 \) for \( \eta \in (0, 1) \). The reason is that offshoring lowers the marginal cost of the intermediate-input production. All else equal, the gains from offshoring are higher the smaller is \( \eta \) since a lower headquarter intensity implies that the intermediate good is more important in the production of the final good. Formally, \( \gamma_S(\eta) \) is continuous and strictly decreasing in \( \eta \) with \( \gamma_S(\eta) \to 1 \) as \( \eta \to 1 \).

Next, we take a careful look at the decision to integrate. The equilibrium input investments in \( h \) and \( m \) are always suboptimally low compared to the first best, perfect-contracting, input investments. This holds because \( H \) and \( M \) each cover the full marginal costs of their investments while they reap only a fraction of the marginal gains from these investments; see (5). The result is that either ownership structure obtains only a fraction of the variable profits that would arise under perfect contracting. The factor \( \psi_k(\eta) \) in variable profits represents the efficiency of the relationship between \( H \) and

\(^{15}\)Given the implicit restriction on the distribution of productivities that expected profits are finite, the existence and uniqueness of an equilibrium and \( A \) follow from the continuity and strict monotonicity of (7) in \( A \), \( \pi^* < 0 \) when \( A = 0 \), \( \pi^* \to \infty \) when \( A \to \infty \), and the intermediate value theorem.
This depends on the choice of $k$ since integration assigns $H$ a larger share of revenue and $M$ a smaller share compared to outsourcing. Thus, integration improves $H$’s incentive to invest in $h$ but worsens $M$’s incentives to invest in $m$. Consequently, integration is a more efficient relationship than outsourcing when headquarter intensity is high but a less efficient relationship when headquarter intensity is low. This is the intuition behind the following lemma which is based on Proposition 3 and its proof in Antràs and Helpman (2008).

**Lemma 1.** $\psi_V(\eta)$ is continuous and strictly increasing in $\eta$ and there exists a unique headquarter intensity, $\eta^* \in (0, 1)$, where $\psi_O = \psi_V(\eta^*)$. Thus, $\eta > \eta^* \Leftrightarrow \psi_V(\eta) > \psi_O$ and $\eta < \eta^* \Leftrightarrow \psi_V(\eta) < \psi_O$.

Note that offshoring involves a tradeoff between higher fixed costs and lower marginal costs for all values of $\eta$. Integration on the other hand, only involves a tradeoff between higher relationship efficiency and higher fixed costs for $\eta \in (\eta^*, 1)$. For lower headquarter intensities, outsourcing achieves higher relationship efficiency and lower fixed costs than integration. Thus, no firms with $\eta \in (0, \eta^*)$ choose to integrate.

### 3.2 Complementarities and Sorting

Complementarities between the activities faced by firms are central to our analysis of firms’ decisions. First off, offshoring and exporting are complementary for all firms. This is because exporting involves additional sales and offshoring effectively reduces the marginal cost. Higher sales are worth more when the goods are produced more cheaply and vice versa. Next, consider the interaction between integration and the two other activities. Remember that whenever integration is considered by firms ($\eta > \eta^*$), it is because integration lets the firms obtain a larger share of the variable profits that would be possible under perfect contracting. Reducing the share of variable profits lost due to incomplete contracting is obviously worth more when these variable profits are higher. This is the case both when serving an additional market through exporting and when the marginal cost is lowered through offshoring. In total, whatever the $\eta \in (0, 1)$, the activities that firms actually consider are complementary (fixed costs are additively separable).\(^\text{16}\) Undertaking one

\(^{16}\)Formally, imposing the partial ordering, $V > O$, $S > N$, and $X > D$, the complementarities discussed follow from the profit function being supermodular in $lx$ for $\eta \in (0, \eta^*)$ and in $k lx$ for $\eta \in (\eta^*, 1)$. 

activity increases the gains from undertaking others. It should be noted that
due to sorting considerations, integration and offshoring are not modelled
to be complementary in the AH model. However, such a complementarity
arises quite naturally when focusing on the essence of these activities, cf. the
discussion above.\textsuperscript{17}

The activities integration, offshoring, and exporting are not only comple-
mentary to each other, they are also complementary to productivity. Inte-
gration (when considered) and exporting are complementary to productivity
for the same reason that these activities are complementary to offshoring;
lower marginal costs increase the gains from these activities. Offshoring is
complementary to productivity since the isoelastic demand function means
that scaling down marginal cost by a given factor, as implied by offshoring,
is worth more when productivity is high.\textsuperscript{18,19}

These properties of the profit function imply that given the headquarter
intensity, $\eta$, the sorting of firms into activities obeys a cutoff rule. For all $\eta \in (0, 1)$, there exist productivity cutoffs for offshoring and exporting such
that all firms with higher (lower) productivities do (do not) undertake the
activity in question. In addition, there exists a similar productivity cutoff
for integration for $\eta \in (\eta^*, 1)$. Conditioning on $\eta$, the three activities will
therefore each be associated with a productivity premium consistent with
empirical studies.\textsuperscript{20} We assume that, for all $\eta$, not all firms undertake either
integration, offshoring, or exporting. Hence, as in AH, the least productive
active firms choose the decision $klx = OND$. Further, in order to observe a

\textsuperscript{17}Compared to AH, we obtain a supermodular profit function through the combination
of additively separable fixed costs and the parameter restriction $\delta_N = \delta_S$ mentioned in
footnote 14. When $\delta_N = \delta_S$, the sorting pattern presented by AH can only be achieved
if the fixed costs of the firm are strictly supermodular in the offshoring and integration
decision.

\textsuperscript{18}See Mrazova and Neary (2013) for a detailed discussion of the complementarity be-
tween offshoring and productivity in the AH model.

\textsuperscript{19}Formally, the complementarity between productivity and either offshoring or export-
ing is seen from the profit function having increasing differences in $(l, x; \theta)$ when we use
the partial ordering of activities mentioned in footnote 16. The complementarity between
productivity and integration on $(\eta^*, 1)$ follows from the profit function having increasing
differences in $(k; \theta)$ on this interval.

\textsuperscript{20}There is a productivity premium associated with integration in data; see Federico
(2010) and Kohler and Smolka (2011). For evidence about a size and age premium for
integration, see Acemoglu et al. (2010). Importers of intermediate goods are more pro-
ductive than nonimporters; see Bernard et al. (2012) who also discuss the well-established
exporter productivity premium.
wide variety of organisational forms, we assume that for $\eta$ close to 0 and 1, the productivity cutoff for offshoring and integration, respectively, is lower than that for exporting.\footnote{Loosely speaking, this means that when integration and offshoring provide the greatest gains for firms, these activities are more attractive than exporting. Thus, exporting is sufficiently expensive not to be the most attractive activity for all headquarter intensities. This guarantees that we will observe nonexporters which offshore and nonexporters which integrate in equilibrium. This assumption is not central for our results; see Section 4.}

### 3.3 Intra-Firm Importing

Interestingly, our model delivers a pattern of firm sorting into activities that is broadly consistent with two empirical findings of Corcos et al. (forthcoming) who use firm-level import data from France to investigate the determinants of the choice between intra-firm and arms'-length importing. First, they show that, conditional on headquarter intensity (proxied by capital and skill intensity) and offshoring, higher firm-level productivity makes intra-firm importing more likely relative to arms'-length importing. Second, they find that conditional on firm-level productivity and offshoring, higher firm-level headquarter intensity makes intra-firm importing more likely relative to arms'-length importing. Corcos et al. (forthcoming) interpret these empirical results as firm-level support for the key predictions of the open-economy property rights theory of the firm concerning intra-firm importing.

The first empirical finding of Corcos et al. (forthcoming) squares perfectly with the cutoff rule described above and the implied sorting pattern in the productivity dimension. The compliance of our model with the second empirical finding mentioned above is a bit more subtle.

**Proposition 1.** Consider the sorting of firms into decisions in a given equilibrium. Among the offshoring firms with a given headquarter intensity, the share that also integrates is higher the higher is the headquarter intensity. However, considering firms with a given productivity, we may observe that one firm undertakes arms'-length importing while having higher headquarter intensity than another firm that undertakes intra-firm importing.

**Proof.** See Appendix B.

The first part of Proposition 1 is reminiscent of an industry-level result obtained by AH. In contrast to this, our result covers the full spectrum of
headquarter intensities within a single industry. To understand the second part of Proposition 1, consider the choice of integration conditional on offshoring. A higher $\eta$ increases the relationship-efficiency of integration relative to outsourcing, which works in the favour of intra-firm importing. However, a higher $\eta$ also decreases the attractiveness of offshoring, which reduces the incentive to integrate due to the complementarity. If this latter effect is sufficiently strong, it is indeed possible that a higher $\eta$ means that firms shift from intra-firm importing to arms'-length importing.\footnote{The adverse complementarity effect is smaller when the North-South wage gap decreases. Hence, the sorting pattern of firms in our model is more likely to resonate perfectly with the second finding of Corcos et al. (forthcoming) when the wage gap becomes smaller.} Proposition 1 illustrates that one has to be careful when applying the conventional intuition about the relation between $\eta$ and integration, which was discussed in Section 3.1, to a firm-level analysis of intra-firm importing in an open-economy context.\footnote{We note that Proposition 1 is not dependent on the inclusion of exporting into the model. This follows from Appendix B.}

4 Comparative Statics

Apart from generating a two-dimensional pattern of firm sorting that is broadly consistent with empirical evidence, our model gives rise to strong predictions based on comparative static analysis of the industry composition.

**Definition 1.** The prevalence of a given activity is the share of active final-good firms that pursue this activity. The prevalence of vertical FDI is the share of active final-good firms that both integrate and offshore.

**Proposition 2.** Reductions in $(f_V, w_S, f_S, \tau, f_X)$ imply that the prevalences of integration, offshoring, and exporting strictly increase. Reductions in $(f_V, w_S, f_S, \tau)$ imply that the prevalence of vertical FDI strictly increases.

*Proof.* See Appendix A.

Proposition 2 illustrates a strong industry-level interdependence among the activities arising from the firm-level complementarities. In general, increasing the attractiveness of any of the three activities makes the industry

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\footnote{The adverse complementarity effect is smaller when the North-South wage gap decreases. Hence, the sorting pattern of firms in our model is more likely to resonate perfectly with the second finding of Corcos et al. (forthcoming) when the wage gap becomes smaller.}
composition of firms shift towards all three activities becoming more prevalent. These results may seem to be an obvious consequence of the complementarities of the model. That the results are by no means trivial is illustrated by the following proposition.

**Proposition 3.** Reducing the costs associated with any one of the activities integration, offshoring, or exporting induces some individual firms to shift away from the other two activities.

*Proof.* See Appendix C.

Our main findings in Propositions 2 and 3 relate to the ongoing discussion about the relation between trade liberalisation and integration. In this strand of literature, the results vary almost as much as the modelling strategies. Our contribution in this regard is twofold. First, we unveil a clear positive relation between trade liberalisations and integration at the industry level. In particular, reductions in fixed or variable costs of trade in final or intermediate goods induce a shift in the industry composition towards integration (Proposition 2). Second, this is not incompatible with shifts towards outsourcing at the firm level (Proposition 3).

To see how Propositions 2 and 3 are compatible, consider trade liberalisation. Apart from their direct effects on firms’ decisions (for a given \(A\)), trade liberalisations result in fiercer competition (a lower equilibrium demand level, \(A\)). The reason is that, given the demand level, liberalisation of either kind increases the profits for all firms weakly and for some firms strictly. Thus, for the free-entry condition, (8), to hold, competition must toughen and the demand level, \(A\), falls. This tends to reduce the size of firms and thereby discourage integration. However, it also forces the least productive firms, which do not integrate, to shut down. With a nonincreasing hazard rate of log-productivities, the net result is an increase in the prevalence of integration.

Even though Proposition 2 implies a clear positive relation between all four types of trade liberalisation and integration, it should be noted that the particular type of trade liberalisation can matter for the effect on integration once we address the share of firms which undertake northern integration, i.e., the share of active final-good firms that choose the strategy \(VNx, x \in \tau_m \bar{w}_S\).
\{D,X\}. The formal result, which is stated below, again illustrates the point that firm-level complementarities may manifest themselves more clearly at higher levels of aggregation.

**Proposition 4.** Let productivities be Pareto distributed. Then liberalisations of intermediate-goods trade reduce the prevalence of northern integration while liberalisations of final-goods trade may increase the prevalence of northern integration.

**Proof.** See Appendix D.

While the first part of Proposition 4 is in accordance with a finding of AH, it is important to note that our main result in Proposition 2 is not. AH find that the prevalence of outsourcing rises when intermediate-goods trade is liberalised (w_s decreases). This incongruity does not depend on our inclusion of exporting, so we abstract from this activity for the moment. In order to accord with what is observed in data, a model has to generate an equilibrium where all possible combinations of the outsourcing and offshoring decisions, \( kl \in \{ON,OS,VN,VS\} \), are observed. Since AH want to achieve this (in headquarter-intensive industries) with productivity as the only source of heterogeneity, they cannot allow integration and offshoring to be complementary at the firm level. Letting firms be heterogeneous with respect to both productivity and headquarter intensity means that we are able to analyse the implications of a clear-cut complementarity between the sourcing decisions faced by firms while generating the desired diversity of observed organisational forms. We contribute by showing that this plausible complementarity implies an unambiguously positive relationship between trade liberalisations and the prevalence of integration.

Finally, we analyse the effects of a change in the barriers to entry through a change in \( f_E \). Like the previously analysed trade liberalisations, a decrease in \( f_E \) implies a fall in the demand level, \( A \), due to free entry. As a consequence, the productivity cutoffs for integration, offshoring, exporting, and vertical FDI increase meaning that firms shift away from these activities. However, as before, low-productivity firms are being pushed out and the nonincreasing hazard rate of log-productivities imply that the net effects on the prevalences of the activities are positive. Further, if the distribution of log-productivity has strictly decreasing hazard rate, the prevalences mentioned above strictly increase; see Bache and Laugesen (2013). These results
relate to an existing literature on the relation between competition and integration mentioned in footnote 2 and illustrate again the discrepancy between firm- and industry-level comparative statics.

Although our assumptions are not necessarily controversial, let us briefly discuss how certain assumptions can be relaxed without affecting Proposition 2 crucially. First off, assume that outsourcing does not result in symmetric but rather generalised Nash bargaining, i.e., $\beta_O$ is not necessarily one half. Proposition 2 still holds in this case if one assumes that productivities are Pareto distributed.\textsuperscript{26} Next, relaxing the assumption that $G(\eta)$ is strictly increasing simply means that we cannot be sure that the effects in Proposition 2 are strictly positive. Nevertheless, we still know that they are nonnegative, cf. Appendix A. Dispensing with the assumption that some firms with high and low $\eta$ integrate and offshore, respectively, without exporting simply means that we cannot be sure that the effects of reducing $f_X$ on the prevalences of integration and offshoring are strictly positive, nor that the effect on the prevalence of exporting of reducing $(f_V, w_S, f_S)$ is strictly positive. These effects will however still be nonnegative while all other strict results still hold. Finally, it should also be noted that, if one removes one of the activities integration, offshoring, or exporting from the choice set of firms, Proposition 2 is still valid for the remaining activities and the relevant cost reductions.\textsuperscript{27} Thus, the strong industry-level interdependence of e.g. exporting and integration implied by Proposition 2 is not lost by not allowing firms to offshore.

5 Concluding Remarks

Our main contribution is to obtain strong and testable results about the interdependencies among integration, offshoring, exporting, and vertical FDI at the industry level of analysis. Of particular interest in light of the existing literature is the clear positive relationship between trade liberalisations and the prevalence of vertical integration. Notably, these results are compatible

\textsuperscript{26}The reason is that, in this case, $\psi_O$ depends on $\eta$ which means that the exit productivity cutoff does so as well. Then we need a constant hazard rate of the log-productivity distribution to use the results of Bache and Laugesen (2013) and argue that the prevalence of any given activity is nondecreasing in $\nu \equiv (-f_V, -w_S, -f_S, -\tau, -f_X)$ which is the first step of the proof in Appendix A. From here, the second step proceeds as before.

\textsuperscript{27}This can be shown by repeating the steps of the proof in Appendix A ignoring one of the activities.
with ambiguities at lower levels of aggregation. Central to our analysis are the firm-level complementarities we identify. Apart from the introduction of exporting, our model is a natural extension of Antràs and Helpman (2004) for the following reasons. First, we allow firms within the same industry to be heterogeneous with respect to both productivity and headquarter intensity. This is a response to recent empirical evidence which reveals that industry is a poor indicator for headquarter intensity and that firm-level headquarter intensity is an important determinant of the decision to vertically integrate. Allowing for heterogeneity in headquarter intensity is shown to be relatively uncomplicated even for a general distribution of headquarter intensities. We believe that the inclusion of firm-specific headquarter intensities makes the open-economy property rights theory of the firm more suitable for future empirical tests based on the firm-level data sets with input-sourcing information which are starting to appear. Reassuringly, we obtain a sorting pattern broadly in line with Corcos et al. (forthcoming). Further, we emphasise that one has to be careful when applying the conventional intuition about the relationship between headquarter intensity and vertical integration at the firm level in an open-economy context. Second, we dispense with the common assumption of Pareto-distributed firm productivities and utilise instead a broader class of distribution functions to which the Pareto distribution belongs.

### A Proof of Proposition 2

As an intermediate step, we establish that increases in $\nu \equiv (-f_V, -w_S, -f_S, -\tau, -f_X)$ lead to nondecreasing prevalences of integration, offshoring, and exporting. This is done by drawing upon the results, in particular Proposition 3, of Bache and Laugesen (2013) (henceforth BL). To do so, we adopt the following ranking of the values of the three choice variables in $ktx: V > O, S > N, and X > D$. Remember that the profit function reads

$$\pi_{ktx}(\Theta, \eta; A, \nu) = A\Theta \psi_k(\eta)\gamma(\eta)(1 + \tau^{1-\sigma})x(x) - f_{ktx}.$$  

Note that any increase in $\nu$ must imply that $A$ falls since the profits of all firms weakly increase given the level of $A$, the profits of some firms strictly increase given $A$, and we have free entry. In the terminology of BL, an increase in $\nu$ leads to comparative statics using the carrot. Our assumption
that the least productive active firms with any $\eta$ choose $klx = OND$ means that the productivity cutoff for being active,

$$\Theta_{\text{exit}}(\eta; A, \nu) \equiv \inf\{\Theta : \pi_{OND}(\Theta, \eta; A, \nu) > 0\} = \frac{f_O}{A^\psi_O},$$

is invariant across $\eta$ and importantly, is not affected directly by changes in $\nu$ but only indirectly through changes in $A$.

Let $s_j^{\eta \leq \eta^*}$ and $s_j^{\eta > \eta^*}$ be the shares of active final-good firms with $\eta$ less than or greater than $\eta^*$, respectively, that end up undertaking activity $j \in \{V, S, X\}$. The overall share of active firms undertaking activity $j$ is then given by

$$s_j = G(\eta^*)s_j^{\eta \leq \eta^*} + [1 - G(\eta^*)]s_j^{\eta > \eta^*}.$$

Since $\eta^*$ is not affected by changes in $\nu$, we have shown that $s_j$ is nondecreasing in $\nu$ if we can establish that $s_j^{\eta \leq \eta^*}$ and $s_j^{\eta > \eta^*}$ are nondecreasing in $\nu$. Consider first the latter. For $\eta > \eta^*$, the profit function, (9), is supermodular in $(k, l, x)$ and has increasing differences in $(k, l, x; \Theta)$, $(k, l, x; A)$, and $(k, l, x; \nu)$. Since our setup conforms to all the other conditions for invoking Proposition 3 of BL for $\eta > \eta^*$, we conclude that $s_j^{\eta > \eta^*}$ is nondecreasing in $\nu$ with $j \in \{V, S, X\}$. Next, consider $s_j^{\eta \leq \eta^*}$. For these values of $\eta$, we know that no firms choose integration and we can treat $k$ as exogenously fixed at $k = O$. Thus $s_j^{\eta \leq \eta^*} = 0$. Further, for $\eta \leq \eta^*$, the profit function with $k = O$ is supermodular in $(l, x)$ and has increasing differences in $(l, x; \Theta)$, $(l, x; A)$, and $(l, x; \nu)$. We can therefore use Proposition 3 of BL to conclude that $s_j^{\eta \leq \eta^*}$ with $j \in \{V, S, X\}$ is indeed nondecreasing in $\nu$ as well. It follows that $s_j$ is nondecreasing in $\nu$ for $j \in \{V, S, X\}$.\textsuperscript{28}

The second step in this proof is to show that $s_j$, $j \in \{V, S, X\}$, is strictly increasing in $\nu$. For this purpose we note that following BL, the effect of $\nu$ on $s_j$ can be split into two parts: the total direct effect of changes in $\nu$ (for a given $A$) and the total indirect effect due to the change in $A$. Both of these are nonnegative. This follows from BL and was implicitly used in the first step of this proof.\textsuperscript{29} We will show that the effect of $\nu$ on $s_j$ is

\textsuperscript{28}The assumption that $G(\eta)$ is strictly increasing is redundant for this intermediate result.

\textsuperscript{29}Following BL, it is easy to show that $s_j$, $j \in \{V, S, X\}$, is strictly increasing in $\nu$ when the distribution of log $\Theta$ (log $\theta$) has a strictly decreasing hazard rate. We would however like to allow for Pareto-distributed productivities, $\theta$, i.e., a distribution of log $\Theta$ with constant hazard rate.
strictly positive by arguing that the total direct effect is strictly positive. Since the direct effect of \( \nu \) on the productivity cutoff for being active is zero as argued above,\(^30\) the total direct effect on \( s_j \) is determined by the direct effect on the share of all firms (not just active) that undertake activity \( j \), i.e., the direct level effect in the language of BL. This share is determined by a cutoff, \( \Theta_j(\eta; A, \nu) \), giving the threshold productivity for undertaking activity \( j \) depending on the headquarter intensity, \( \eta \).\(^31\) The direct effects of increases in \( \nu \) on the cutoffs \( \Theta_V(\eta; A, \nu) \), \( \Theta_S(\eta; A, \nu) \), and \( \Theta_X(\eta; A, \nu) \) are nonpositive for all \( \eta \); see BL. Hence, if we can establish that these effects are strictly negative, each for an interval of \( \eta \), we are done.\(^32\)

First, we establish that \( \Theta_V \), \( \Theta_S \), and \( \Theta_X \) are continuous functions of \( \eta \) on the intervals \( (\eta^*, 1) \), \( (0, 1) \), and \( (0, 1) \), respectively. We will only provide details on how to show this for \( \Theta_V \) as showing it for \( \Theta_S \) and \( \Theta_X \) will be completely analogous. Let us define \( \Theta_V \) properly. To do so, let

\[
\pi_k(\Theta, \eta; A, \nu) = \max_{lx} \pi_{kIx}(\Theta, \eta; A, \nu).
\]

Note that \( \pi_k \) is continuous in \((\Theta, \eta)\) since \( \pi_{kIx} \) is. Now, on \((\eta^*, 1)\), \( \Theta_V(\eta; A, \nu) \) is given by

\[
\pi_V(\Theta_V, \eta; A, \nu) - \pi_O(\Theta_V, \eta; A, \nu) = 0. \tag{10}
\]

Since the LHS of (10) is continuous in \((\Theta, \eta)\) and strictly increasing in \( \Theta \),\(^33\) it follows from the implicit function theorem that \( \Theta_V \), as determined by (10), is continuous in \( \eta \).

Next, we establish that \( \Theta_V \to \infty \) as \( \eta \to \eta^* \) from above and \( \Theta_V \) is bounded from above as \( \eta \to 1 \). Let \( \Theta_{V\mid Ix}(\eta; A, \nu) \) be implicitly defined by

\[A\Theta[\psi_V(\eta)\gamma_{I1}(\eta)(1+\tau^{1-\sigma})^1x(x_1) - \psi_O\gamma_{I2}(\eta)(1+\tau^{1-\sigma})^1x(x_2)] - (f_{Vlx_1} - f_{Olx_2}) = 0, \tag{11}\]

where \( l_1x_1 \) and \( l_2x_2 \) are the optimal choices of \( lx \) under \( k = V \) and \( k = O \), respectively. Since \( V > O \), we have \( l_1 \geq l_2 \) and \( x_1 \geq x_2 \). Further, since \( \psi_V(\eta) > \psi_O \) for \( \eta \in (\eta^*, 1) \), the square bracket on the LHS of (11) is strictly positive on \((\eta^*, 1)\) and thus the LHS of (10) is strictly increasing in \( \Theta \) on \((\eta^*, 1)\).

---

\(^{30}\)In the language of BL, the direct selection effect is zero.

\(^{31}\)Note that \( \Theta_V \) is infinite for \( \eta \in (0, \eta^*) \). The following discussion of \( \Theta_V \) will therefore only concern its behaviour on \((\eta^*, 1)\).

\(^{32}\)Remember that \( G(\eta) \) is strictly increasing on \( \eta \in (0, 1) \).

\(^{33}\)That the LHS of (10) is weakly increasing in \( \eta \) follows from \( \pi_{kIx} \) having increasing differences in \((k, l, x; \Theta)\) since this implies that \( \pi_k \) has increasing differences in \((k; \Theta)\). That it is in fact strictly increasing can be seen by writing out (10) to get
the equation

$$\pi_{V|lx}(\Theta_{V|lx}, \eta; A, \nu) - \pi_{OLx}(\Theta_{V|lx}, \eta; A, \nu) = 0.$$  \hspace{1cm} (12)

Let $\Theta_V(\eta; A, \nu) = \min_{lx} \Theta_{V|lx}$. Then for firms with productivities below $\Theta_V$, outsourcing is the optimal ownership structure since regardless of the firms’ choices of $lx$, profits are higher under $O$ than under $V$. This is due to the LHS of (12) being strictly increasing in $\Theta$. Thus, $\Theta_V$ represents a lower bound on $\Theta_V$ for all $\eta$. In a similar fashion, $\hat{\Theta}_V = \max_{lx} \Theta_{V|lx}(\eta; A, \nu)$ is an upper bound on $\Theta_V$. Now, since the LHS of (12) is increasing in $lx$ and $\Theta$ by the supermodularity and increasing differences properties of $\pi_{klx}$, respectively, we have $\underline{\Theta}_V = \Theta_{V|SX}$ and $\hat{\Theta}_V = \Theta_{V|ND}$.\footnote{The intuition for $\min_{lx} \Theta_{V|lx} = \Theta_{V|SX}$ (max$_{lx} \Theta_{V|lx} = \Theta_{V|ND}$) is that integration is promoted as much (little) as possible by other complementary activities in this case.} Since the lower bound on $\Theta_V$, $\Theta_{V|SX} \to \infty$ as $\eta \to \eta^*$ from above, the same holds for $\Theta_V$. Further, as the upper bound on $\Theta_V$, $\Theta_{V|ND}$ is bounded from above as $\eta \to 1$, so is $\Theta_V$.\footnote{Since $\Theta_{V|SX} = \frac{\nu \gamma V}{\nu \gamma V(\eta) - \psi_O}$ and $|\psi_V(\eta) - \psi_O| \to 0$ as $\eta \to \eta^*$, it follows that $\Theta_{V|SX} \to \infty$ as $\eta \to \eta^*$ from above. Further, since $\Theta_{V|ND} = \frac{\nu \gamma V}{\nu \gamma V(\eta) - \psi_O}$, $\psi_V(\eta) > \psi_O$ for $\eta > \eta^*$, and $\psi_V(\eta)$ is increasing in $\eta$, $\Theta_{V|ND}$ is bounded from above as $\eta \to 1$.} By similar lines of arguments, one can establish that $\Theta_S \to \infty$ as $\eta \to 1$, $\Theta_S$ is bounded from above as $\eta \to 0$, and that $\Theta_X$ is bounded from above both as $\eta \to 0$ and as $\eta \to 1$.

Our assumption that some firms with $\eta$ close to 0 choose to offshore without exporting implies that $\Theta_S < \Theta_X$ for $\eta$ sufficiently close to 0. Further, the assumption that some firms with $\eta$ close to 1 choose to integrate without exporting implies that $\Theta_V < \Theta_X$ for $\eta$ sufficiently close to 1. Combined with the properties derived above an application of the intermediate value theorem implies that $\Theta_V$ and $\Theta_X$, $\Theta_V$ and $\Theta_S$, and $\Theta_S$ and $\Theta_X$ intersect in the interior of $(\eta^*, 1)$, $(\eta^*, 1)$, and $(0, 1)$, respectively. This means that $V$ and $S$ share the same cutoff productivity for at least one value of $\eta$, as does $V$ and $X$ and $S$ and $X$. The final steps of the proof show that this implies that these joint cutoffs must each be relevant for an interval of $\eta$, and that for each of the activities, $j \in \{V, S, X\}$, at least one of its joint cutoffs is strictly decreasing conditional on $A$ when $\nu$ increases.

Consider the joint cutoffs for activity $V$. First off, we have established that for $\eta$ sufficiently close to (but above) $\eta^*$, $\Theta_V > \Theta_S, \Theta_X$ and that for $\eta$ sufficiently close to 1, $\Theta_V < \Theta_S, \Theta_X$. That is, $\Theta_V$ must be equal to respectively $\Theta_S$ and $\Theta_X$ for at least one value of $\eta$. Suppose that $\Theta_V$ and $\Theta_S$
not equal on an interval of \( \eta \). Then there must exist an \( \eta = \eta' \) for which \( \Theta_V(\eta'; A, \nu) = \Theta_S(\eta'; A, \nu) = \Theta' \) with \( \Theta_V > \Theta_S \) for \( \eta \) just below \( \eta' \) and \( \Theta_V < \Theta_S \) for \( \eta \) just above \( \eta' \). First, suppose that \( \Theta_X(\eta'; A, \nu) \neq \Theta' \). That is, for \( (\Theta, \eta) \) sufficiently close to \( (\Theta', \eta') \) all firms choose the same export status, \( x \). Then we must have that \( \pi_{V_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{O_{N_x}}(\Theta', \eta'; A, \nu) = \pi_{O_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{V_{N_x}}(\Theta', \eta'; A, \nu) \).\(^{36}\) However, this cannot be the case since \( \pi_{O_{Sx}} - \pi_{O_{N_x}} = 0 \) implies that \( \pi_{V_{Sx}} - \pi_{V_{N_x}} > 0 \). Second, suppose \( \Theta_X(\eta'; A, \nu) = \Theta' \). Then we must have that

\[
\pi_{V_{Sx}}(\Theta', \eta'; A, \nu) = \pi_{O_{N D}}(\Theta', \eta'; A, \nu) = \pi_{V_{N_{x1}}}(\Theta', \eta'; A, \nu) = \pi_{O_{S_{x2}}}(\Theta', \eta'; A, \nu)
\]  

(13)

for some \( x_1, x_2 \).\(^{37}\) But as before, this cannot be true. To see why, suppose (13) holds for \( x_1 = D \) and \( x_2 = X \). Then \( \pi_{O_{Sx}} - \pi_{O_{N D}} = 0 \) implies that \( \pi_{V_{Sx}} - \pi_{V_{N D}} > 0 \) and we have a contradiction. If (13) holds for \( x_1 = X \) and \( x_2 = D \), then \( \pi_{V_{N_{x}} - \pi_{O_{N D}} = 0} \) implies \( \pi_{V_{Sx}} - \pi_{O_{S_{D}}} > 0 \) and again we have a contradiction. If \( x_1 = x_2 = D \), \( \pi_{V_{N_{D}} - \pi_{O_{N D}} = 0} \) implies \( \pi_{V_{S_{D}} - \pi_{O_{S_{D}}} > 0} \). But this means that \( \pi_{V_{S_{D}}(\Theta', \eta'; A, \nu)} \) is strictly higher than \( \pi_{O_{N D}(\Theta', \eta'; A, \nu)} \), and due to continuity, firms with \( \eta = \eta' \) and \( \Theta \) just below \( \Theta' \) find \( klx = V_{S_{D}} \) more profitable than \( klx = O_{N D} \). But these firms should choose \( O_{N D} \) if \( \Theta_V(\Theta', \eta'; A, \nu) = \Theta_S(\Theta', \eta'; A, \nu) = \Theta_X(\Theta', \eta'; A, \nu) = \Theta' \) and we have a contradiction. Finally, if \( x_1 = x_2 = X \), then \( \pi_{V_{S_{x}} - \pi_{V_{N_{x}}} = 0} \) implies \( \pi_{O_{S_{x}} - \pi_{O_{N_{x}}} < 0} \) but by \( \pi_{O_{S_{x}}} = \pi_{O_{N_{D}}} \), we thus have \( \pi_{O_{N_{x}}} > \pi_{O_{N D}} \), which, by a similar argument as before, is incompatible with \( \Theta_V(\Theta', \eta'; A, \nu) = \Theta_S(\Theta', \eta'; A, \nu) = \Theta_X(\Theta', \eta'; A, \nu) = \Theta' \). Thus, we must have that \( \Theta_V = \Theta_S \) holds for an interval of \( \eta \) since otherwise we arrive at a contradiction. Arguments similar to those used above imply that the same must hold for \( \Theta_V \) and \( \Theta_X \) and for \( \Theta_S \) and \( \Theta_X \).

To conclude the proof of the first part of the proposition, note that when \( \Theta_S = \Theta_V \), this joint cutoff is given by

\[
\pi_{V_{Sx_1}}(\Theta_V, \eta; A, \nu) - \pi_{O_{N_{x_2}}}(\Theta_V, \eta; A, \nu) = 0
\]  

(14)

for some \( x_1 \geq x_2 \). But the LHS of (14) is strictly increasing in both \( \Theta \) and \( (-f_V, -w_S, -f_S) \) which means that, given \( A \), \( \Theta_V \) is strictly decreasing

---

\(^{36}\)Right at \((\Theta', \eta')\) firms are indifferent between \(V_{Sx} \) and \(O_{N_{x}} \) which gives the first of the equalities. For \( \eta \) just below \( \eta' \), \( \Theta_S \) gives indifference between \( O_{N_{x}} \) and \( O_{S_{x}} \). As \( \Theta_S \) is continuous, this indifference at \( \Theta_S \) extends to \( \eta = \eta' \) which gives the second equality. The last equality follows from a similar argument using \( \Theta_V \) for \( \eta \) just above \( \eta' \).

\(^{37}\)The argument is completely analogous to before.
in $(-f_V, -w_S, -f_S)$. This holds for an interval of $\eta$ as argued above. As $\Theta_V = \Theta_S$ on this interval of $\eta$, the same can be said about $\Theta_S$. Further, when $\Theta_V = \Theta_X$, it is given by

$$
\pi_{Vl_1X}(\Theta_V; \eta; A, \nu) - \pi_{Ol_2D}(\Theta_V; \eta; A, \nu) = 0
$$

for some $l_1 \geq l_2$. As the LHS of (15) is strictly increasing in $\Theta$ and $(-f_V, -\tau, -f_X)$, $\Theta_V$ and $\Theta_X$ are strictly decreasing in $(-f_V, -\tau, -f_X)$, given $A$, on an interval of $\eta$. Finally, when $\Theta_S = \Theta_X$, it is given by

$$
\pi_{k_1SX}(\Theta_S; \eta; A, \nu) - \pi_{k_2ND}(\Theta_S; \eta; A, \nu) = 0
$$

for some $k_1 \geq k_2$. As the LHS of (16) is strictly increasing in $\Theta$ and $(-w_S, -f_S, -\tau, -f_X)$, $\Theta_S$ and $\Theta_X$ are strictly decreasing in $(-w_S, -f_S, -\tau, -f_X)$, given $A$, on an interval of $\eta$. Combining these results leads you to conclude that whenever $\nu \equiv (-f_V, -w_S, -f_S, -\tau)$ increases, $\Theta_V$, $\Theta_S$, and $\Theta_X$ each strictly decreases on some interval of $\eta$ given $A$. As argued above, this gives us the first part of the proposition.

That the prevalence of vertical FDI (the share of all active final-good firms that undertake both $V$ and $S$) is strictly increasing in $(-f_V, -w_S, -f_S, -\tau)$ is quite simple to show at this point. Note that vertical FDI only occurs for $\eta > \eta^*$. First, let us define the productivity cutoff for vertical FDI,

$$
\Theta_{VS}(\eta; A, \nu) = \max\{\Theta_V(\eta; A, \nu), \Theta_S(\eta; A, \nu)\}.
$$

This productivity cutoff is clearly nonincreasing in $(-f_V, -w_S, -f_S, -\tau)$ given $A$. By the same arguments as above, we need to show that, given $A$, $\Theta_{VS}$ is strictly decreasing in $(-f_V, -w_S, -f_S, -\tau)$ on an interval of $\eta$ in order to prove the last part of the proposition. We have already established that $\Theta_V = \Theta_S$ on an interval of $\eta$ for which they are strictly decreasing in $(-f_V, -w_S, -f_S)$ for a given $A$. This means that the same holds for $\Theta_{VS}$ on this interval of $\eta$. Finally, since $\Theta_V > \Theta_S, \Theta_X$ for all $\eta$ sufficiently close to, but above, $\eta^*$, $\Theta_{VS} = \Theta_{V|S\eta}$ for these $\eta$. It is easy to verify that $\Theta_{V|S\eta}$ is strictly decreasing in $-\tau$ given $A$, and we are therefore done.

### B Proof of Proposition 1

This proof draws on results and definitions from Appendix A and should be read in extension of this. Denote by $s_{V|S}(\eta)$ the share of final-good firms
with headquarter intensity $\eta$ which integrate given that they are offshoring. To prove that $s_{V|S}$ is increasing in $\eta$, we first show that if $s_{V|S}$ is one for some $\eta$, then it must be one for all higher values of $\eta$. If this is not the case, there must exist an $\eta' \in (\eta^*, 1)$ such that $\Theta_S(\eta') = \Theta_V(\eta') = \Theta'$ and for $\eta$ just above $\eta'$, $\Theta_S < \Theta_V$. For $\eta$ just above $\eta'$, $\Theta_S$ must thus be given by
\[
\pi_{OSx_1}(\Theta_S, \eta; A, \nu) - \pi_{ONx_2}(\Theta_S, \eta; A, \nu) = 0
\]
for some $x_1 \geq x_2$. This gives us
\[
\Theta_S(\eta; A, \nu) = \frac{f_{OSx_1} - f_{ONx_2}}{A\psi_O[\gamma_S(\eta)(1 + \tau^{1-\sigma})^{X(\eta)} - (1 + \tau^{1-\sigma})^{X(\eta)}]}
\]  
(17)

for $\eta$ at and just above $\eta'$. Further, we have that $\pi_{V|Sx_3}(\Theta', \eta'; A, \nu) = \pi_{OSx_1}(\Theta', \eta'; A, \nu) = \pi_{ONx_2}(\Theta', \eta'; A, \nu)$ where $Vx_3$ is undertaken by firms with $\eta = \eta'$ and productivity just above $\Theta'$. It must hold that $x_3 \geq x_1$. Now, define $\Theta_V$ as
\[
\pi_{V|Sx_3}(\Theta_V, \eta; A, \nu) - \pi_{ONx_2}(\Theta_V, \eta; A, \nu) = 0.
\]  
(18)

This gives us
\[
\Theta_V(\eta; A, \nu) = \frac{f_{V|Sx_3} - f_{ONx_2}}{A[\gamma_S(\eta)]\psi_V(\eta)(1 + \tau^{1-\sigma})^{X(\eta)} - \psi_O(1 + \tau^{1-\sigma})^{X(\eta)}]}
\]  
(19)

Differentiating (17) and (19) with respect to $\eta$ gives
\[
\frac{\partial \Theta_S}{\partial \eta} = -\Theta_S \frac{1}{1 - \gamma_S^{-1}(1 + \tau^{1-\sigma})^{X(\eta)} - \psi_O(1 + \tau^{1-\sigma})^{X(\eta)}} \frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S}
\]  
(20)

and
\[
\frac{\partial \Theta_V}{\partial \eta} = -\Theta_V \frac{1}{1 - \gamma_S^{-1}(1 + \tau^{1-\sigma})^{X(\eta)} - \psi_O(1 + \tau^{1-\sigma})^{X(\eta)}} \left(\frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V}\right),
\]  
(21)

where $\xi \equiv \psi_V(\eta)/\psi_O > 1$ since we consider $\eta > \eta^*$. From (20) and (21) expressions one can see that $\frac{\partial \Theta_S}{\partial \eta} (\eta'; A, \nu) < \frac{\partial \Theta_S}{\partial \eta} (\eta'; A, \nu)$ when using that $\gamma_S(\eta)$ and $\psi_V(\eta)$ are strictly decreasing and strictly increasing in $\eta$, respectively.\footnote{First, this follows directly if $\frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} \geq 0$. If $\frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} < 0$, it follows from noting that $0 > \frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S} + \frac{\partial \psi_V}{\partial \eta} \frac{1}{\psi_V} > \frac{\partial \gamma_S}{\partial \eta} \frac{1}{\gamma_S}$, $0 < \gamma_S^{-1}(1 + \tau^{1-\sigma})^{X(\eta)} - \gamma_S^{-1}(1 + \tau^{1-\sigma})^{X(\eta)} < \gamma_S^{-1}(1 + \tau^{1-\sigma})^{X(\eta)} - 1X(\eta) < 1$, and that for $\eta = \eta'$, $\Theta_S = \Theta_V$. However,} But this means that for $\eta$ just above $\eta'$, we have $\Theta_S > \Theta_V$. However,
from (18), this implies that for firms with \( \eta \) just above \( \eta' \) and \( \Theta \) just below \( \Theta_S \),
\[
\pi_{V_{Sx_1}}(\Theta_S, \eta; A, \nu) - \pi_{ON_{x_2}}(\Theta_S, \eta; A, \nu) > 0,
\]
which is contradictory to these firms choosing \( klx = ONx_2 \). Thus, there
must exist an \( \eta^{**} > \eta^* \) such that for a given \( \eta \) greater than \( \eta^{**} \) all firms
that offshore also integrate and for a given \( \eta \) less than \( \eta^{**} \) not all firms
that offshore also integrate. We now know that \( s_{V|S}(\eta) = 0 \) for \( \eta \in (0, \eta^* \) and
\( s_{V|S}(\eta) = 1 \) for \( \eta \in [\eta^{**}, 1) \). What remains to be shown is that \( s_{V|S}(\eta) \) is
monotone increasing in \( \eta \) on \( (\eta^*, \eta^{**}) \). To do so, note that on this interval,
\( \Theta_V > \Theta_S \). That is, \( \Theta_V \) is given by
\[
\pi_{V_{Sx_4}}(\Theta_V, \eta; A, \nu) - \pi_{OS_{x_5}}(\Theta_V, \eta; A, \nu) = 0
\]
for some \( x_4 \geq x_5 \). This gives us
\[
\Theta_V(\eta; A, \nu) = \frac{f_{V_{Sx_4}} - f_{OS_{x_5}}}{A\gamma_S(\eta)[\psi_V(\eta)(1 + \tau^{1-\sigma})^1x_{(x_4)} - \psi_O(1 + \tau^{1-\sigma})^1x_{(x_5)}]}.
\]
(22)
Further, \( \Theta_S \) is given by,
\[
\pi_{OS_{x_6}}(\Theta_S, \eta; A, \nu) - \pi_{ON_{x_7}}(\Theta_S, \eta; A, \nu)
\]
for some \( x_6 \geq x_7 \). This gives us
\[
\Theta_S(\eta; A, \nu) = \frac{f_{OS_{x_6}} - f_{ON_{x_7}}}{A\psi_O[\gamma_S(\eta)(1 + \tau^{1-\sigma})^1x_{(x_6)} - (1 + \tau^{1-\sigma})^1x_{(x_7)}]}.
\]
(23)
Note that \( x_5 \geq x_6 \). Introducing \( B(\eta) \equiv [\gamma_S(\eta)(1 + \tau^{1-\sigma})^1x_{(x_6)} - (1 + \tau^{1-\sigma})^1x_{(x_7)}] \), the cutoffs (22) and (23) can be written as
\[
\Theta_V(\eta; A, \nu) = \frac{(f_{V_{Sx_4}} - f_{OS_{x_5}})[(1 + \tau^{1-\sigma})^1x_{(x_6)} - (1 + \tau^{1-\sigma})^1x_{(x_7)}\gamma_S(\eta)^{-1}]}{AB(\eta)[\psi_V(\eta)(1 + \tau^{1-\sigma})^1x_{(x_4)} - \psi_O(1 + \tau^{1-\sigma})^1x_{(x_5)}]}
\]
(24)
and
\[
\Theta_S(\eta; A, \nu) = \frac{f_{OS_{x_6}} - f_{ON_{x_7}}}{A\psi_O B(\eta)},
\]
(25)
respectively. First note that if one keeps \( B(\eta) \) constant, then (24) is decreasing
in \( \eta \) for constant \( (x_4, x_5, x_6, x_7) \). Next, for \( \eta \in (\eta^*, \eta^{**}) \), we can express
\( s_{V|S}(\eta) \) as
\[
s_{V|S}(\eta) = \frac{1 - \tilde{F}(\log \Theta_V)}{1 - \tilde{F}(\log \Theta_S)},
\]
(26)
where \( \tilde{F}(\log \Theta) = F(\Theta^{1/(\sigma - 1)}) \) is the distribution of \( \log \Theta = (\sigma - 1) \log \theta \). Note first that for given \((x_4, x_5, x_6, x_7)\), any effect of an increase in \( \eta \) that does not go through \( B(\eta) \) will increase \( s_{V|S}(\eta) \) since it reduces \( \Theta_V \). Next, we show that the effect of \( \eta \) through \( B \) on \( s_{V|S} \) is positive as well. To see that this is true, rewrite (26) to get

\[
s_{V|S}(\eta) = \exp \left\{ - \int_{\log \Theta_S}^{\log \Theta_V} \lambda(\Theta) \, d\Theta \right\},
\]

where \( \lambda(\Theta) \) is the hazard rate of \( \tilde{F} \). Now, differentiating (27) with respect to \( B(\eta) \) gives us

\[
\frac{\partial s_{V|S}}{\partial B} = s_{V|S} \left( \lambda(\log \Theta_S) \frac{\partial \log \Theta_S}{\partial B} - \lambda(\log \Theta_V) \frac{\partial \log \Theta_V}{\partial B} \right).
\]

It is clear from (24) and (25) that \( \frac{\partial \log \Theta_V}{\partial B} = \frac{\partial \log \Theta_S}{\partial B} < 0 \). Further, since the distribution of \( \log \Theta \) has nonincreasing hazard rate, we know that \( \lambda(\log \Theta_V) \leq \lambda(\log \Theta_S) \) as we consider an interval where \( \Theta_V > \Theta_S \).\(^\text{39}\) Thus, \( s_{V|S} \) is decreasing in \( B \) which means that the effect of \( \eta \) through \( B \) on \( s_{V|S} \) is positive for given \((x_4, x_5, x_6, x_7)\). Thus, in total, we have shown that on the interval \((\eta^*, \eta^{**})\), \( s_{V|S} \) is increasing in \( \eta \) whenever \((x_4, x_5, x_6, x_7)\) does not change. However, we know that if \((x_4, x_5, x_6, x_7)\) changes at some point as \( \eta \) increases, then at this point, the value of \( s_{V|S} \) is unaffected since \( \Theta_V \) and \( \Theta_S \) are continuous in \( \eta \). Further, just above and below such points, \( s_{V|S} \) is increasing in \( \eta \). Thus, we can conclude that \( s_{V|S} \) is increasing in \( \eta \) on \((\eta^*, \eta^{**})\), and we are done.

The reason we cannot say that individual firms will unambiguously find intra-firm importing more attractive as \( \eta \) increases conditional on productivity is that \( \eta \) has two opposing effects on the RHS of (22). One is that \( \psi_V \) increases and the other is that \( \gamma_S \) decreases. If the latter effect dominates for some \( \eta \in (\eta^*, \eta^{**}) \), then we see that \( \Theta_V \) is above \( \Theta_S \) and increasing in \( \eta \). But then, for some level of productivity, a higher \( \eta \) will cause firms to shift from operating with \( kl = VS \) to \( kl = OS \), i.e., higher headquarter intensity causes a shift from intra-firm importing to arms'-length importing at the firm level.

\(^{39}\) As \( \log \theta \) has nonincreasing hazard rate and \( \sigma > 1 \), \( \log \Theta = \log \theta^{\sigma - 1} = (\sigma - 1) \log \theta \) has nonincreasing hazard rate as well.
C Proof of Proposition 3

For $\eta$ sufficiently close to 0, the cutoffs for offshoring and exporting are given by

$$\pi_{OSD}(\Theta_S, \eta; A, \nu) - \pi_{OND}(\Theta_S, \eta; A, \nu) = 0$$

and

$$\pi_{OSX}(\Theta_X, \eta; A, \nu) - \pi_{OSD}(\Theta_X, \eta; A, \nu) = 0,$$

respectively. Next, for $\eta$ sufficiently close to 1, the cutoffs for integration and exporting are given by

$$\pi_{VND}(\Theta_V, \eta; A, \nu) - \pi_{OND}(\Theta_V, \eta; A, \nu) = 0$$

and

$$\pi_{VNX}(\Theta_X, \eta; A, \nu) - \pi_{VND}(\Theta_X, \eta; A, \nu) = 0,$$

respectively. Now, any increase in $(-w_S, -f_S, -\tau, -f_X)$ results in a reduction in $A$. Consider a reduction in the fixed cost associated with integration, $f_V$. This does not affect the cutoff conditions (28) and (29) directly but does so indirectly through $A$. Since LHS of both equations are strictly increasing in $(\Theta, A)$, the result of the decline in $A$ is a strict increase in the cutoffs given by (28) and (29) for the relevant $\eta$’s. That is, some firms shift away from offshoring and exporting. Next, consider a reduction in the costs associated with offshoring, ($w_S, f_S$). Since the LHS of (30) and (31) are not directly affected by these changes and strictly increase in $(\Theta, A)$, the result is a strict increase in the cutoffs given by these two equations for the relevant $\eta$’s. That is, some firms shift away from integration and exporting. Finally, consider a reduction in $(\tau, f_X)$. Using the same line of arguments again implies that the cutoffs given by (28) and (30) are strictly increasing for the relevant $\eta$’s. That is, some firms shift away from integration and offshoring.

D Proof of Proposition 4

First we prove that increasing $(-w_S, -f_S)$ always reduces the prevalence of northern integration. To do so, consider for the moment the initial equilibrium and note that it follows from Appendix B that if $\Theta_V \leq \Theta_S$ for some $\eta = \eta'$, then $\Theta_V \leq \Theta_S$ for all $\eta \geq \eta'$. Next, suppose that $\Theta_V < \Theta_S$, then

$$\Theta_S(\eta; A, \nu) = \frac{f_{VSx_1} - f_{VNX_2}}{A(\gamma_S(\eta))(\tau^1-\sigma)I_X(x_1) - (\tau^1-\sigma)I_X(x_2))};$$

27
for some $x_1 \geq x_2$, and

$$\Theta_V(\eta; A, \nu) = \frac{f_{V,Nx_3} - f_{ONx_4}}{A[\psi_V(\eta)(1 + \tau^{-1}\sigma)1_{x}(x_3) - \psi_O(1 + \tau^{-1}\sigma)1_{x}(x_4)]},$$

for some $x_3 \geq x_4$ where $x_2 \geq x_3$. Thus, when $\Theta_V < \Theta_S$, the ratio is given by

$$\frac{\Theta_S}{\Theta_V} = \frac{(f_{VSx_1} - f_{V,Nx_2})[1 + \tau^{-1}\sigma]1_{x}(x_3) - \psi_V(\eta)^{-1}(1 + \tau^{-1}\sigma)1_{x}(x_4)]}{(f_{V,Nx_3} - f_{ONx_4})[\psi_V(\eta)(1 + \tau^{-1}\sigma)1_{x}(x_3) - (1 + \tau^{-1}\sigma)1_{x}(x_2)]}.$$

(32)

Since $\gamma_S(\eta)$ and $\psi_V(\eta)$ are decreasing and increasing in $\eta$, respectively, it is obvious that the ratio in (32) is increasing in $\eta$ for given $(x_1, x_2, x_3, x_4)$. Further, as (32) is continuous at points where $\Theta_V < \Theta_S$, we have that $\Theta_V < \Theta_S$ for all $\eta > \eta''$. Let $\eta''$ be given by

$$\eta'' \equiv \inf \{ \eta : \Theta_V < \Theta_S \} = \inf \{ \eta : 1 < \frac{\Theta_S}{\Theta_V} \}.$$

Then we can express the prevalence of northern integration, $s_{V,N}$, as

$$s_{V,N} = \frac{\int_{\eta''}^{1} [F(\Theta_S) - F(\Theta_V)] dG(\eta)}{1 - F(\Theta_{exit})}.$$

Let us show that $\eta''$ is increasing in $(-w_s, -f_s)$. By (32), which is valid for $\eta \geq \eta''$, it is obvious that changes in $A$ do not affect $\eta''$. Further, given $A$, increases in $(-w_s, -f_s)$ reduce $\Theta_S$ and does not affect $\Theta_V$ whenever $\Theta_V < \Theta_S$. It follows that $\eta''$ must rise in $(-w_s, -f_s)$. Next, using the Pareto distribution with shape parameter $z(\sigma - 1)$ for $F$, $s_{V,N}$ can be expressed as

$$s_{V,N} = \frac{\int_{\eta''}^{1} [(A\Theta_V)^{-z} - (A\Theta_S)^{-z}] dG(\eta)}{(A\Theta_{exit})^{-z}}.$$

(33)

Now, note that $\eta''$ increases in $(-w_s, -f_s)$, that $A\Theta_V$ and $A\Theta_{exit}$ are unaffected by changes in $(-w_s, -f_s)$ for $\eta > \eta''$, and that $A\Theta_S$ is decreasing in $(-w_s, -f_s)$. Thus, $s_{V,N}$ decreases in $(-w_s, -f_s)$.

Second, to show that $s_{V,N}$ can be increasing when final-goods trade is liberalised, assume that $\Theta_X < \Theta_S$ for $\eta \geq \eta''$. Note that this implies that $A\Theta_X = A\Theta_V$ for an interval of $\eta$ in $(\eta'', 1)$.\(^{40}\) Under these assumptions, an

\(^{40}\)For $\eta = \eta''$, $\Theta_S = \Theta_V$. 28
increase in $-f_X$ has no effect on $\eta^{**}$, $A\Theta_S$, or $A\Theta_{exit}$ in (33). However, for the interval of $\eta$ in $(\eta^{**}, 1)$ where $A\Theta_X = A\Theta_V$, $A\Theta_V$ is decreasing and $A\Theta_V$ is nonincreasing otherwise. This means that $s_{VN}$ increases in $-f_X$.

We can also illustrate that $s_{VN}$ can be unambiguously increasing in $-\tau$ if we relax the assumption that $\Theta_X > \Theta_V$ for $\eta$ sufficiently close to 1. Assume that $\Theta_X < \Theta_V$ for all $\eta$. Then $x_1 = x_2 = x_3 = x_4 = X$ in (32) which means that $\eta^{**}$ and $\frac{\Theta_S}{\Theta_V}$ for $\eta \geq \eta^{**}$ are unaffected by $\tau$. Expressing $s_{VN}$ as

$$s_{VN} = \int_{\eta^{**}}^{1} (A\Theta_V)^{-z}[1 - (\frac{\Theta_S}{\Theta_V})^{-z}] \, dG(\eta),$$

it now follows that $s_{VN}$ is increasing in $-\tau$ since $A\Theta_{exit}$ is unaffected and $A\Theta_V$ is decreasing.

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