Self-enforcing Trade Agreements, Dispute Settlement and Separation of Powers

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Abstract

If external enforcement of international trade agreements is either not available or infeasible and cooperation must therefore be enforced via promises of future cooperation, grim-trigger strategies supply the strongest punishments in many settings. This paper explores the implications for optimal punishments of the presence of an import-competing lobby who attempts to influence a politically-susceptible legislature to break the agreement once negotiated by the executive branch. Here, the optimal punishment must balance two, conflicting objectives: longer punishments help to enforce cooperation by increasing the costs to the government of defecting from the agreement, but because the lobby prefers the punishment outcome, this also incentivizes lobbying effort and with it political pressure to break the agreement. Thus the model generates new predictions for the optimal design of mechanisms for resolving the disputes that arise in the course of trade-agreement relationships.

1 Introduction

In the absence of strong external enforcement mechanisms for international trade agreements, we generally assume that cooperation is enforced by promises of future cooperation, or, alternatively, promises of future punishments for exploitative behavior. Typically, when repeated-game incentives

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are used to enforce cooperation and prevent players from defecting in a prisoner’s dilemma-style stage game, the strongest punishment available is the grim trigger strategy of defecting forever upon encountering a defection by one’s partner.

However, Klimenko, Ramey and Watson (2008) show that the typical grim trigger punishments are not useful for supporting cooperation when renegotiation is possible. They propose a notion of recurrent agreement that takes into account the possibility of renegotiation via one appealing solution: a dispute settlement institution (DSI) loosely patterned on the Dispute Settlement Body of the World Trade Organization that helps trading partners to credibly condition their negotiations on the state of their relationship and avoid the problems created by renegotiation.

In addition to the interference with grim-trigger punishments caused by renegotiation, a variety of concerns about welfare, realism and the like may lead us to explore alternative punishment sequences. I argue that in some important cases, we can derive optimal (non-grim-trigger) punishments directly from the players’ incentive constraints.

Relative to the existing literature, this paper incorporates a separation-of-powers policy-making process as in Buzard (2013) with endogenous lobbying along the lines of Grossman and Helpman (1994 and 1995) into such a repeated-game setting with a DSI to take account of the threat to cooperation posed by renegotiation. The structure is therefore similar to that of Bagwell and Staiger (2005) with endogenously-determined political economy weights and power over the policy-making process modeled as shared between executive and legislative branches of the government as in Milner and Rosendorff (1997) and Song (2008).

Here, welfare-maximizing executives use the trade-agreement as a kind of political commitment device: by setting tariffs to optimally reduce lobbying incentives, they help the legislatures resist political pressure they would otherwise face to break the agreement[1]. Given that all actors have perfect information about the effect of lobbying effort on the outcome of political process, the executives maximize social welfare by choosing the lowest tariffs that make it unattractive for the lobbies to

[1] The commonly-made assumption that the executive is less protectionist than the legislature is a special case of the finding that susceptibility to special interests generally declines with the size of one’s constituency. One simple illustration from the realm of trade policy is the following: a legislator whose district has a large concentration of a particular industry does not take into account the impact of tariffs on the welfare of consumers in other districts, while the executive, whose constituency encompasses the whole country, will internalize these diffuse consumption effects. For a detailed argument, see Lohmann and O’Halloran (1994).
Thus the problem with the lobby has an extra constraint relative to the standard problem. The standard constraint on the key repeated-game player, which here is the legislature, is loosened by increasing the punishment length because defections become relatively more unattractive. However, this new constraint due to the presence of lobbying becomes tighter as the punishment becomes more severe because the lobby prefers punishment periods. Because the tariffs during punishment, and thus its profits, are higher than during a cooperative period, the lobby has increased incentive to exert effort as the punishment lengthens.

The optimal punishment length must balance these two competing forces. Where the balance falls depends in large part on how influential the lobby is in the legislative process. If the lobby has very little power, the optimal punishment converges to that of the model without a lobby: longer punishments are better because the key constraint is the legislature’s. As the lobby becomes stronger, the optimal punishment becomes shorter because the lobby’s incentive becomes more important.

The recurrent agreement approach to renegotiation in a WTO-like environment following Kliemenko, Ramey and Watson 2008 (hereafter KRW) complements contributions by Cotter and Mitchell (1997), Ludema (2001) and Beshkar (2010a) that study “renegotiation-proof” trade agreements.

Related is also a growing literature on renegotiation in one-shot models of trade agreements, ranging from the limited forms of renegotiation in Bagwell and Staiger (1999) and Beshkar (2010b) to the richer models of renegotiation in Beshkar (2013) and Maggi and Staiger (2012a) and (2012b). These consider a substantively different question: instead of being concerned with the undermining of cooperation due to the renegotiation of punishments, they assume perfect external enforcement and ask questions about the optimality of various agreement and institutional designs given that the parties have some ability to renegotiate their agreement within the one-shot interaction.


2With no uncertainty of any kind, there will thus be no trade disputes in equilibrium. Political uncertainty can be easily added to the model as in Buzard 2013, in which case lobbying effort is typically non-zero and there is a positive probability of dispute in equilibrium.
I begin by describing the stage game in detail. Section 3 sets out the dispute settlement institution and the set-up of the repeated game. I then describe the structure and properties of optimal trade agreements in Section 4. Section 5 explores the forces shaping optimal dispute resolution procedures and Section 6 concludes.

2 Stage Game

I employ a three good model with two countries: home (no asterisk) and foreign (asterisk). The countries trade two goods, X and Y, where $P_i$ denotes the home price of good $i \in \{X, Y\}$ and $P_i^*$ denotes the foreign price of good $i$. Good $N$ is non-traded and assumed to be the numeraire. In each country, the demand functions are taken to be identical for both traded goods, respectively $D(P_i)$ in home and $D(P_i^*)$ in foreign and are assumed strictly decreasing and twice continuously differentiable, while demand for the numeraire, $D_N$ is assumed perfectly elastic at $P_N = 1$.

The supply functions for good X are $Q_X(P_X)$ and $Q_X^*(P_X^*)$ and are assumed strictly increasing and twice continuously differentiable for all prices that elicit positive supply. I also assume $Q_X^*(P_X) > Q_X(P_X)$ for any such $P_X$ so that the home country is a net importer of good X. The production structure for good Y is taken to be symmetric, with both demand and supply such that the economy is separable in goods X and Y. It is assumed that the production of each traded good requires the possession of a sector-specific factor that is available in inelastic supply and is non-tradable so that the income of owners of the specific factors is tied to the price of the good in whose production their factor is used. The non-traded good is produced with labor alone with constant returns to scale technology so that $Q_N = l_N$.

For simplicity, I assume each government’s only trade policy instrument is a specific tariff on its import-competing good: the home country levies a tariff $\tau$ on good X while the foreign country applies a tariff $\tau^*$ to good Y. Local prices are then $P_X = P_X^W + \tau$, $P_X^* = P_X^W$, $P_Y = P_Y^W$ and $P_Y^* = P_Y^W + \tau^*$ where a W superscript indicates world prices and equilibrium prices are determined by the market clearing conditions

\[
M_X(P_X) = D(P_X) - Q_X(P_X) = Q_X^*(P_X^*) - D(P_X^*) = E_X^*(P_X^*)
\]

\[
E_Y(P_Y) = Q_Y(P_Y) - D(P_Y) = D(P_Y^*) - Q_Y^*(P_Y) = M_Y^*(P_Y^*)
\]
where $M_X$ are home-county imports and $E_X^*$ are foreign exports of good $X$ and $E_Y$ are home-county exports and $M_Y^*$ are foreign imports of good $Y$.

It follows that $P_X^{W}$ and $P_Y^W$ are decreasing in $\tau$ and $\tau^*$ respectively, while $P_X$ and $P_Y^*$ are increasing in the respective domestic tariff. This gives rise to a standard terms-of-trade externality. As profits and producer surplus (identical in this model) in a sector are increasing in the price of its good, profits in the import-competing sector are also increasing in the domestic tariff. This economic fact, combined with the assumptions on specific factor ownership, is what motivates political activity.

I next describe the politically-relevant actors. In order to focus attention on protectionist political forces, I assume that only the import-competing industry in each country is politically-organized and able to lobby and that it is represented by a single lobbying organization. Each country’s government is composed of two branches: an executive who can conclude trade agreements and a legislature that has final say on trade policy. In summary, the political process is modeled as involving three players in each country: the lobby, the executive, and the legislature.

The stage-game timing is as follows. First, the executives set trade policy cooperatively in an international agreement. In the context of the repeated game, this can be construed as concluding an agreement in the first period and then potentially renegotiating at the beginning of each subsequent period, or as forming a new agreement each period. After the trade agreement is concluded in each period, the lobbies attempt to persuade the legislators in their respective countries to break the trade agreement. Next, the legislatures decide whether to abide by the agreement or to provoke a trade war. In the event that the trade agreement does not remain in force, there is a final stage of lobbying and voting to set the trade-war tariffs. Once all political decisions are taken, producers and consumers make their decisions.

I assume complete information, so the appropriate solution concept is subgame perfect Nash equilibrium. As this game is solved by backward induction, it is intuitive to start by describing the incentives of the legislatures, whose decisions I model as being taken by a median legislator. As the economy is fully separable and the economic and political structures are symmetric, I focus here on the home country and the $X$-sector. The details are analogous for $Y$ and foreign.

\footnote{As in Buzard 2013, the model can be extended to account for political uncertainty; there information about that uncertainty is taken to be symmetric and so subgame perfect Nash equilibrium remains the appropriate solution concept.}
The per-period welfare function of the home legislature is

\[ W_{ML} = CS_X(\tau) + CS_Y(\tau^*) + \gamma(e) \cdot PS_X(\tau) + PS_Y(\tau^*) + TR(\tau) \]  

(1)

where \( CS \) is consumer surplus, \( PS \) is producer surplus, \( \gamma(e) \) is the weight placed on producer surplus (profits) in the import-competing industry, \( e \) is lobbying effort, and \( TR \) is tariff revenue. Here, the weight the median legislator places on the profits of the import-competing industry, \( \gamma(e) \) is affected by the level of lobbying effort.\(^4\)

**Assumption 1.** \( \gamma(e) \) is continuously differentiable, strictly increasing and concave in \( e \).

Assumption \(^1\) formalizes the intuition that the legislature favors the import-competing industry more the higher is its lobbying effort, but that there are diminishing returns to lobbying activity.

Lobbying affects only the weight the legislature places on the profits of the import-competing industry. These profits are higher in a trade war than under a trade agreement, so given Assumption \(^1\), \( \gamma \) is increasing in lobbying effort, implying that the legislature becomes more favorably inclined toward the high trade-war tariff and associated profits as lobbying increases and therefore more likely to break the trade agreement.

Given the legislature’s preferences, the home lobby chooses its lobbying effort (\( e_b \) to influence the break decision and \( e_{tw} \) to influence the trade war tariff) to maximize the welfare function:

\[ U_L = [\pi(\tau_{tw}) - e_{tw}][\text{Trade War}] + \pi(\tau^a)[\text{Trade Agreement}] - e_b \]  

(2)

where \( \pi(\cdot) \) is the current-period profit and \( \tau^a (\tau_{tw}) \) is the home country’s tariff on the import good under a trade agreement (war). I use the convention throughout of representing a vector of tariffs for both countries \((\tau, \tau^*)\) as a single bold \( \tau \).

I assume the lobby’s contribution is not observable to the foreign legislature. The implication is that the lobby can directly influence only the home legislature, and so the influence of one country’s lobby on the other country’s legislature occurs only through the tariffs selected.\(^5\)

\(^4\)The standard PFS modeling would specify \( W_{ML} = C + aW \), but as will be seen when we come to the preferences of the executive, this is not sufficiently general for the purposes of this model. Although complex, an isomorphism can be made between the two forms in a special case as discussed in Buzard 2013.

In the first stage, the executives choose the trade agreement tariffs $\tau^a = (\tau^a, \tau^a)$ via a negotiating process that I assume to be efficient. This process therefore maximizes the joint payoffs of the trade agreement:

$$W_E(\tau^a) = W_E(\tau^a) + W_E^*(\tau^a)$$  \hspace{1cm} (3)

I model the executives’ choice via the Nash bargaining solution where the disagreement point is the executives’ welfare resulting from the Nash equilibrium in the non-cooperative game (i.e. in the absence of a trade agreement) between the legislatures.

The executives are assumed, for simplicity, to be social-welfare maximizers. Therefore the home executive’s welfare is specified as follows:

$$W_E = CS_X(\tau) + CS_Y(\tau^*) + PS_X(\tau) + PS_Y(\tau^*) + TR(\tau)$$

Note that this is identical to the welfare function for the legislature aside from the weight on the profits of the import industry, which is not a function of lobbying effort. This assumption does not require that the executives are not lobbied; only that their preferences are not directly altered in a significant way by lobbying over trade—that they do not sell protection in order to finance their re-election campaigns. In the case of the post-war United States, where the Congress has consistently been significantly more protectionist than the President, this seems to reasonably reflect the political reality. For trade policy, where there are concentrated benefits but harm is diffuse, there are good reasons for this to be the case. Because the President has the largest constituency possible, delegating authority to the executive branch may simply be a mechanism for “concentrating” the benefits since consumers seem unable to overcome the free-riding problem. In fact, a strong argument can be made that power over trade policy has been delegated to the executive branch precisely because it is less susceptible to the influence of special interests (Destler 2005).

Therefore, in line with both the theoretical and empirical literature, I will assume that $\gamma(e) \geq 1$ for all $e$. That is, even for the least favorable outcome of the lobbying process, the legislature will be at least slightly more protectionist than the executive.

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6 If political uncertainty is present, the joint payoffs must take into account the possibility that the trade agreement will be broken. In the case of certainty, agreement will always be maintained on the equilibrium path and so this specification is sufficient.

7 It is trivial to relax this assumption.
**Assumption 2.** $\gamma(e) \geq 1 \ \forall e.$

Assumption 2 ensures that $\tau^a < \tau^lw$, and more generally, that the legislature’s incentives are more closely aligned with the lobby’s than are those of the executive. This is not essential but simplifies the analysis and matches well the empirical findings that politicians with larger constituencies are less sensitive to special interests (See Destler 2005 and footnote I above).

Although the political process here matches most closely that of the United States in the post-war era, I believe the model or one of its extensions is applicable for a broad range of countries for which authority over the formation and maintenance of trade policy is diffuse and subject to political pressure either at home or in a trading partner.

## 3 Repeated Game

### 3.1 Dispute Settlement Institution

Following KRW, I will assume that the countries submit themselves to an external Dispute Settlement Institution (DSI) for the purposes of overcoming the renegotiation problem: that is, the incentive to renegotiate out of punishment phases that destroys the ability of the punishments to enforce cooperation. One way to (informally) make adherence to the DSI incentive compatible is to imagine that many trading partners use the DSI and that all will punish a country who deviates in any bilateral agreement.

The DSI is assumed to keep records of the negotiated agreements, complaints, and violations, and to settle disputes when agreements are violated. The simple DSI employed here conditions the interaction of the countries in the following manner: The DSI keeps records in terms of two possible states of the trade relationship, “cooperative” and “dispute.” At the start of any period, it is assumed that either there is no dispute pending, or else the DSI is in the process of resolving a dispute triggered by a violation in some prior period. I refer to the former situation as the “cooperative state,” or state $C$. If a dispute is pending, then the period begins in the “dispute state,” or state $D$. When a tariff

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8 In particular, the binary decision by the legislature about whether to abide by or break the trade agreement is modeled on the “Fast Track Authority” that the U.S. Congress granted to the Executive branch almost continuously from 1974-1994 and then again as “Trade Promotion Authority” from 2002-2007.

9 See KRW Section 5.1 for more details.
agreement is violated, the DSI switches the state from \( C \) to \( D \), and a dispute settlement process (DSP) begins, as described below. When settlement is achieved, the DSI switches the state from \( D \) back to \( C \).

Importantly, the DSI cannot be directly manipulated by the countries involved in a dispute, so countries continue to negotiate agreements and choose tariffs as before, except their negotiation can be conditioned on the DSI’s state. Therefore, the negotiation problem that countries face following a dispute history may be different than the negotiation problem they face following a cooperative history.

Rather than developing a detailed model of the DSP, KRW treat the DSP as a “black box,” where the key feature is that settlement occurs with delay. For a period that begins in the \( D \) state, the dispute is resolved, and the state is switched to \( C \), with probability \( p \) where \( p \) is exogenous and is meant to capture the idea that dispute resolution may entail costs including delay. I will follow an alternative and equivalent convention by assuming that the state is switched back to the “cooperative state” \( T \) periods after a dispute is initiated.

Thus the timing of actions is the following. If the countries are in state \( C \) at the start of period \( t \), they choose any agreement that is supportable in state \( C \) and communicate the agreement to the DSI. As long as their tariff choices adhere to the agreement, they remain in state \( C \) at the start of period \( t + 1 \). If one or both countries defect from the agreement, however, a dispute arises, and the state is switched to \( D \) at the start of period \( t + 1 \).

If the countries are in state \( D \) at the start of period \( t \), the state will only be switched back to \( C \) if \( t - 1 \) was the \( T \)th period since the beginning of the dispute. In this case, the countries immediately negotiate an agreement supportable in the cooperative state and communicate it to the DSI. If the dispute is unresolved, the state remains \( D \) through the start of period \( t + 1 \), irrespective of what tariffs the countries select in the current period. In this event, the countries choose an agreement from among those that are supportable in the dispute state.

KRW define a recurrent agreement to be a subgame perfect equilibrium in which, in each period, the continuation value is consistent with this theory of negotiation. This requires, first, that countries agree to do as well as possible in each state; and second, that agreement is recurrent, in that continuation payoffs are always drawn from those that are supportable in the current state, but the countries are unable to alter the state as part of their agreement. The solution concept employed here is that of
the maximal recurrent agreement; that is, the recurrent agreement that maximizes the welfare of the executives, who I assume for simplicity are social welfare maximizers.

3.2 Trade Agreements with External Enforcement

Standard repeated-game models have one player in each country; here there are three, each with distinctive roles that mirror those laid out in the stage game. To review, in each period, the executives can re-negotiate the trade agreement, the legislature can break the trade agreement and set trade-war tariffs, and the lobby can choose whether or not to exert lobbying effort and how much effort to exert.

Given that each trading partner submits to the DSI, we can determine the repeated-game incentives in each state. The executives will jointly maximize social welfare given the state, but have no opportunity to affect the state other than to choose tariffs that are supportable. Thus the executives maximize joint welfare subject to the incentive constraints of the other players. Again, because of symmetry and separability, it suffices to restrict attention to the home country.

In state $D$, no action of either the lobby or the legislature will change the state; that is, the continuation payoffs will be the same regardless of their actions. When $x$ periods of punishment are remaining, the supportability condition for the legislature to adhere to any tariff $\tau^D$, given the level of lobbying effort $e$, is

$$W_{ML}(\gamma(e), \tau^D) + \delta V_{ML}^D \geq W_{ML}(\gamma(e), \tau^R(\tau^{*D}), \tau^{*D}) + \delta V_{ML}^D$$

where $V_{ML}^D$ is the continuation value of the median legislator in the dispute state and $\tau^R(\tau^{*D})$ is the home legislature’s best response to $\tau^{*D}$ given $\gamma(e)$.\(^{10}\) Since future payoffs will not be impacted by current actions, the legislature has no incentive to choose anything other than its static best response. Thus the only tariffs that can be supported in state $D$ are the static best responses to the lobby’s choice $e$.

The lobby faces an analogous problem. Because nothing the lobby does can impact the disposition of the DSP, it will choose the effort level that maximizes static profits. Thus the unique tariffs that are supported in state $D$ will be identical to those in the trade-war phase of the one-shot game. I label these $\tau^{tw} = (\tau^{tw}, \tau^{*tw})$.\(^{11}\)

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\(^{10}\)Note that separability of the economy implies that the best response tariff is independent of the trading-partner’s choice. In what follows I will therefore drop this dependence to simplify notation.

\(^{11}\)For details, see Buzard 2013 Section 3.1.
The supportability conditions in state \(C\) are quite different. I will assume that one legislature is randomly assigned the opportunity to break the agreement in any given period. This implies the following constraint on the trade agreement tariffs \(\tau^a\):

\[
W_{ML}(\gamma(e), \tau^a) + \delta V_{ML}^C \geq W_{ML}(\gamma(e), \tau^R, \tau^*^a) + \delta V_{ML}^D
\]

where \(V_{ML}^C\) is the continuation value of the median legislator in the cooperative state. If the punishment is \(T\) periods in the dispute state (where only trade-war tariffs can be chosen), then the only part of the continuation values that need be considered are the next \(T\) periods because after \(T\) periods, the relationship will revert back to cooperation in either state and so the continuation value will be the same from period \(T + 1\) on. When in state \(C\) in the future, the executives will choose the same trade-agreement tariffs because they will maximize welfare subject to the same supportability conditions; in state \(D\), the argument above shows that \(\tau^{tw}\) must be chosen. Therefore we have

\[
W_{ML}(\gamma(e), \tau^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} W_{ML}(\gamma(e), \tau^a) \geq W_{ML}(\gamma(e), \tau^R, \tau^*^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} W_{ML}(\gamma(e), \tau^{tw}) \tag{4}
\]

and

\[
\pi(\tau^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} \pi(\tau^a) \geq \pi(\tau^R) + \frac{\delta - \delta^{T+1}}{1 - \delta} [\pi(\tau^{tw}) - e_{tw}] - e_b \tag{5}
\]

Because the countries will not be able to do anything to change the disposition of the DSI after a dispute has been triggered, it is only these constraints for \(T\)-length punishments that must be checked; once a punishment has been triggered, the dispute-state incentive conditions are the relevant ones.

### 4 Trade Agreement Structure

We can write the executives’ joint problem as

\[
\max_{\tau^a} \frac{W_E(\tau^a)}{1 - \delta} \quad \text{subject to (7) and (8)} \tag{6}
\]

\[
\frac{\delta_{ML} - \delta_{ML}^{T+1}}{1 - \delta_{ML}} \left[ W_{ML}(\gamma(e), \tau^a) - W_{ML}(\gamma(e), \tau^{tw}) \right] \geq W_{ML}(\gamma(e), \tau^R, \tau^*^a) - W_{ML}(\gamma(e), \tau^a) \tag{7}
\]

\[\text{Note that } \delta + \delta^2 + \ldots + \delta^t = \sum_{t=1}^\infty - \sum_{t=1}^\infty \gamma = \frac{\delta}{1 - \delta} - \frac{\delta^{t+1}}{1 - \delta} = \frac{\delta - \delta^{t+1}}{1 - \delta}.\]
\[ e_b \geq \pi(\tau^R(e_b)) - \pi(\tau^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} \left[ \pi(\tau^w) - e_{tw} - \pi(\tau^a) \right] \]  

(8)

where Inequalities 7 and 8 are simple rearrangements of 4 and 5.

To understand how the executives optimally structure trade agreements, we must first examine the incentives of the lobbies and how the legislatures make decisions regarding breach of the trade agreement. The symmetric structure of the model permits restriction of attention to the home country.

I will consider the economically interesting case in which, for a given \( \delta \) and \( T \), there exists a non-trivial trade agreement in the absence of lobbying, that is, one in which the lowest supportable cooperative tariffs are strictly lower than the trade-war (i.e. non-cooperative) level. Call the trade-agreement tariffs in the absence of lobbies \( \tau^a_{NL} \). If \( \tau^a_{NL} = \tau^w \), the lobby has no incentive to be active and the extra constraint implied by the presence of the lobby does not bind.

In state \( C \), the lobby has a two-stage problem. First, for the given \( \tau^a \), \( \delta \) and \( T \), it calculates the minimum \( e_b \) required to induce the legislature to break the trade agreement. Call this minimum effort level \( \overline{e}(\tau^a) \). This calculation of precise indifference is possible because I have assumed here that the political process is certain—that is, all actors know precisely how lobbying effort affects the identity of the median legislator through \( \gamma(e) \).

The \( e_b \) required to break the agreement will produce a “cheater’s payoff” of \( \pi(\tau^R(\overline{e})) \). The lobby will then compare its current and future payoffs from inducing a dispute net of lobbying effort (that is, \( \pi(\tau^R(\overline{e})) + \delta V_L^D - \overline{e} \)) to the profit stream from the trade agreement with no lobbying effort \( (\pi(\tau^a) + \delta V_L^C) \). With the appropriate substitutions and rearrangements, this is just Condition (8) evaluated at \( \overline{e} \). If the former is larger, it induces the cheapest possible break; if the latter is larger, the lobby chooses to be inactive and the agreement remains in force.

The executives maximize social welfare by choosing the lowest tariffs such that the trade agreement they negotiate remains in force. Thus they must raise tariffs to the point that makes the lobby indifferent between exerting effort \( \overline{e}(\tau^a) \) and disengaging completely\(^{13} \) provided that this also satisfies the legislative constraint. By construction, the legislative constraint will always be satisfied. Because \( \overline{e}(\tau^a) \) is calculated to make the median legislator indifferent between cooperating and initiating a dispute, when the lobby is disengaged \( (e_b = 0) \) the median legislator cannot prefer to break the

\(^{13}\)Here I assume that the lobby does not exert effort when indifferent; if one were to assume the opposite, tariffs would have to be raised an extra \( \varepsilon \).
agreement since her preferred tariff is lower than at $\bar{e}(\tau^a)$.

**Lemma 3** in the Appendix demonstrates that the solution to the executives’ problem is well-defined. This combined with the immediately preceding discussion demonstrates the following result.

**Result 1.** In the case of political certainty, the equilibrium trade agreement will induce zero lobbying effort and will never be subject to dispute. The executives will choose the minimum tariff level that induces the lobby to choose $e_b = 0$.

At the equilibrium tariffs, the lobby’s constraint will bind, while the legislature’s will not. The cost of provoking a dispute, however, is derived from the legislature’s constraint, which is then made slack when the lobby is disengaged.

### 4.1 Trade Agreement Properties

Following Result 1, we know that the lobby first uses Expression 7 at equality to determine $e(\tau^a)$: that is, how much it has to pay for any $\tau^a$ in order to induce the legislature to choose noncooperation. We can re-write this condition as

$$\frac{\delta_{ML} - \delta_{ML}^{T+1}}{1 - \delta_{ML}} [W_{ML}(\gamma(\bar{e}), \tau^a) - W_{ML}(\gamma(\bar{e}), \tau^{tw})] - [W_{ML}(\gamma(\bar{e}), \tau^R(\tau^a), \tau^{*a}) - W_{ML}(\gamma(\bar{e}), \tau^a)] = 0$$

(9)

With $\bar{e}(\tau^a)$ determined, the executives use Expression 8 at equality to determine $\tau^a$ (there are analogous expression for $\tau^{*a}$ throughout that can be ignored by symmetry): that is, the trade agreement tariff that is just high enough to induce the lobby to disengage, causing the equilibrium outcome to be $e_b = 0$ and the trade agreement tariff to remain in place. We can rewrite this condition as

$$\bar{e}(\tau^a) - [\pi(\tau^R(\bar{e}(\tau^a))) - \pi(\tau^a)] = \frac{\delta_{L} - \delta_{L}^{T+1}}{1 - \delta_{L}} [\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a)] = 0$$

(10)

Labeling the left sides of Equations 9 and 10 as $\Omega(\cdot)$ and $\Pi(\cdot)$, for notational convenience, these equations can be represented as

$$\Omega(\bar{e}(\delta_{ML}, \gamma, \tau^a), \delta_{ML}, \gamma, \tau^a) = 0$$

(11)

\footnote{Note that all expressions also depend on the fundamentals of the welfare function—$D, Q_X, Q_Y$—but these are suppressed for simplicity.}

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\[ \Pi(\tau^a(\delta_L, \delta_{ML}, \gamma), \overline{e}(\delta_{ML}, \gamma, \tau^a), \delta_L, \delta_{ML}, \gamma) = 0 \]  

Although explicit expressions for the solution functions \( \overline{e}(\cdot) \) and \( \tau^a(\cdot) \) cannot be arrived at without imposing further assumptions, significant intuition can be derived implicitly. An overview of the results will be provided here, while the mathematical details are in the Appendix.

The central question is the link between the two equations, that is, the relationship between \( \tau^a \) and \( \overline{e} \), and it is straightforward.

**Corollary 1.** An increase in the trade agreement tariff, ceteris paribus, leads to an increase in the minimum lobbying effort (\( \overline{e} \)) required to break the trade agreement.

Proof: See the Appendix.

Because the legislature’s preferred tariff at \( \tau \) must be higher than the trade agreement tariff in order for the legislature to be indifferent between the trade agreement and a trade war, a higher \( \tau^a \) is closer to the legislature’s ideal point, and thus the lobby must pay more to make the legislature willing to break it.

We can also make predictions about the effects of changes in patience levels of both the lobby and median legislator. For this purpose, in Equations 9 and 10 I have differentiated the discount factor, labeling the lobby’s \( \delta_L \) and the median legislator’s \( \delta_{ML} \).

**Corollary 2.** As the lobby becomes more patient (\( \delta_L \) increases), the trade agreement tariff also increases, ceteris paribus.

Proof: See the Appendix.

When the lobby becomes more patient (\( \delta_L \) increases), the equilibrium trade agreement tariff must be raised because the lobby pays \( e_b \) in the current period in order to attain the benefit of higher tariffs under a trade war in future periods; the cheater’s tariff \( \tau^R \) (which is lower than \( \tau^{tw} \)) in the current period is not its primary goal but is used to incentivize the legislature. Thus the lobby’s incentives to exert effort must be reduced by increasing the trade agreement tariff, thus reducing the profit gap between the trade war and the trade agreement.

\[ ^{15} \]

In the Appendix, I explain that it must be assumed that trade agreement tariffs are set such that the lobby would have incentive to exert effort to sustain a trade war. Restricting attention to this case is not trivial; see the conclusion for discussion of a planned extension that more fully explores the trade-war phase.
Turning to the patience of the median legislator, we start with the effect on the minimum lobbying effort level.

**Corollary 3.** As the median legislator becomes more patient ($\delta_{ML}$ increases), the minimum lobbying effort ($\overline{e}$) required to break the trade agreement decreases ceteris paribus.

Proof: See the Appendix.

A more patient median legislator weighs the punishment in the future for deviating more heavily relative to the gain from the cheater’s payoff in the current period for any given level of effort. The lobby must compensate by putting forth more effort in the current period to bend the median legislator’s preferences toward high tariffs.

What does an increase in $\delta_{ML}$, leading to an increase in $\overline{e}$, imply for the optimal trade agreement tariff? All of the math is in the Appendix, but the intuition is straightforward.

**Corollary 4.** As the median legislator becomes more patient ($\delta_{ML}$ increases), the trade agreement tariff decreases ceteris paribus.

Proof: See the Appendix.

This result contrasts with Corollary 2. When the median legislator becomes more patient, the executives are able to decrease the trade agreement tariff because the cutoff lobbying expenditure increases. This is because the lobby must now pay more to convince the legislature to choose short-run gains in the face of future punishment, so a wider profit gap between the trade war and trade agreement tariffs is consistent with disengaging the lobby.

Here the result comes through the legislature’s indifference condition instead of directly from the lobby’s indifference condition, but the intuition is the same: the trade agreement tariff is determined as whatever it takes to quell the lobby’s willingness to exert effort to break the agreement.

Let’s turn to another variable that impacts the equilibrium trade agreement in important ways: the weight the median legislator places on the profits of the import-competing sector. This political weighting function, $\gamma(e)$, is endogenous to many of the decisions underpinning the equilibrium, for simplicity, let’s examine the effect of an exogenous change in $\gamma$. First, on the cutoff effort level:

**Corollary 5.** Exogenous positive shifts in the political weighting function $\gamma(e)$ reduce the minimum lobbying effort ($\overline{e}$) required to break the trade agreement, ceteris paribus.
In accordance with intuition, if there is a shift in the political weighting function so that the legislature weights the profits of the import-competing sector more heavily for a given amount of lobbying effort, the lobby will have to exert less effort in order to induce a trade disruption.

This translates in a straightforward way to an impact on the trade agreement tariff.

**Corollary 6.** *Exogenous positive shifts in the political weighting function \( \gamma(e) \) lead to higher trade agreement tariffs, ceteris paribus.*

Proof: See the Appendix.

This makes a lot of sense given that an upward shift in the political weighting function in effect means that the lobby becomes more powerful, that is, it has a larger impact on the median legislator for a given level of effort. This is why the minimum effort level required to break the trade agreement is reduced, and therefore why the trade agreement tariff must be increased: when the lobby has to pay less to break the agreement, the agreement must be made more “agreeable.”

### 5 Optimal Dispute Resolution

In an environment without lobbying, KRW show that social welfare increases (that is, trade-agreement tariffs can be reduced) as punishments are made stronger. This can be seen here if we restrict attention to the legislature’s constraint:

\[
\frac{\delta - \delta^{T+1}}{1 - \delta} \left[ W_{ML}(\gamma(e), \tau^a) - W_{ML}(\gamma(e), \tau^{tw}) \right] \geq W_{ML}(\gamma(e), \tau^R, \tau^{*a}) - W_{ML}(\gamma(e), \tau^a)
\]

This constraint is made less binding as \( T \) increases—that is, as we raise the number of periods of punishment. The intuition is straightforward: the per-period punishment is felt for more periods as the one period of gain from defecting remains the same. Thus larger deviation payoffs can be supported as \( T \) increases.

**Lemma 1.** *The slackness of the legislative constraint is increasing in \( T \).*

Thus the environment with no lobby gave no model-based prediction about the optimal length of punishment. Longer is better, but there are renegotiation constraints that must be taken into account that are outside of the model as well as other concerns.
The lobby’s constraint works in the opposite direction in relation to $T$:

$$e_b \geq \pi(\tau^R) - \pi(\tau^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) \right]$$

Here, the lobby benefits in each dispute period, and so the total profit from a dispute is increasing in $T$. Thus we have

**Lemma 2.** The slackness of the lobbying constraint is decreasing in $T$.

The interaction of the impact of the length of the punishment on these two constraints is quite nuanced; in many cases, adding the lobbying constraint provides a prediction for the optimal $T$.

As the executives choose the smallest $\tau^a$ that makes the lobby indifferent at $\bar{e}(\tau^a)$, we must analyze the lobby’s constraint (Expression 8) evaluated at $\bar{e}(\tau^a)$ to determine the optimal $T$. The derivative of this constraint with respect to $T$ is

$$\left[ \frac{\partial \gamma}{\partial e} \left[ \frac{\pi(\tau^R) - \pi(\tau^a)}{1 - \delta} \right] + \frac{\delta^{T+1} \ln \delta}{1 - \delta} \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) \right] \right]$$

If this expression is negative for all $T$, the constraint is most slack at $T = 0$, which might seem to be at odds with incentivizing cooperation by the legislature; however, unless the legislature is so biased toward the lobbying industry that its preferred tariff at $e = 0$ is above the trade-agreement level (i.e. the trade agreement level that is required to disengage the lobby), the legislature will have no incentive to defect from the agreement.

On the other hand, if this expression is positive for all $T$, the constraint is most slack as $T$ approaches infinity and so we are in a case similar to that of the model without lobbying where a ad-hoc renegotiation constraint determines the upper bound on the punishment length. Here, the legislative constraint outweighs concerns about provoking lobbying effort. Perhaps most interesting are intermediate cases where the optimal $T$ is interior—that is, the punishment length optimally balances the need to punish legislators for deviating with that of not rewarding lobbies too much for provoking a dispute.

The intuition is clearest if we examine the case of perfectly patient actors, that is, let $\delta \to 1$. This essentially removes the influence of the period of cheater’s payoffs in which the interests of the legislature and the lobby are aligned (both do better in the defection stage) and exposes the differences between them in the dispute phase. In the limit, the derivative of the constraint with respect to $T$
becomes

\[
\begin{align*}
\frac{\partial \gamma}{\partial e} \left\{ \left[ \pi(\tau^R) - \pi(\tau^a) \right] + T \left[ \pi(\tau^{tw}) - \pi(\tau^a) \right] \right\} - \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) \right]
\end{align*}
\]  

(14)

\( \bar{e} \) is determined so that \( W_{ML}(\gamma(\bar{e}), \tau^a) - W_{ML}(\gamma(\bar{e}), \tau^{tw}) \) is always positive, so the numerator of the fraction is positive. The trade-agreement tariff is always lower than both the trade war tariff and the best-response (cheater’s) tariff and \( \frac{\partial \gamma}{\partial e} \) is positive by Assumption 1 so the denominator is always positive. Note that the only influence of \( T \) on the entire expression is through this denominator, so the value of the expression is decreasing in \( T \).

The second term, the change in the lobby’s gain from a break in the trade agreement, can for extremely large values of \( \tau^a \) be negative. If this is so, the entire expression is positive and the optimal \( T \) is the largest value possible. Intuitively, the lobby has nothing to gain from causing a break, so the legislature’s incentives are the only ones of concern.

In the case of interest where the lobby potentially has an interest in breaking the agreement, the right-hand term is positive. Here where we’ve taken \( \delta \to 1 \), the rate of change of the lobby’s gain is constant.

Depending on the relative magnitudes, the overall expression may be positive for small \( T \) and then become negative, or it may be negative throughout. In the former case, the optimal \( T \) will be interior, while in the latter it will be zero. The expression cannot be positive for all values of \( T \), so it cannot be optimal to have arbitrarily long punishments when the players approach perfect patience.

**Result 2.** In the case of political certainty with perfectly patient players, the optimal punishment scheme precisely balances the future incentives of the lobby and legislature. It always lasts a finite number of periods and may be of zero length if the influence of lobbying on legislative preferences is extraordinarily strong (\( \frac{\partial \gamma}{\partial e} \) is sufficiently high).

The key intuition for distinguishing between the situations described in Result 2 comes from examining the properties of the political process. If \( \frac{\partial \gamma}{\partial e} \) is moderate, the positive term in Expression 14 is more likely to dominate in the beginning and lead to an interior value for the optimal \( T \), whereas extremely large values for \( \frac{\partial \gamma}{\partial e} \) make it more likely that the boundary case occurs. For a given effort level, this derivative will be smaller when the lobby is less influential; that is, when a marginal increase in \( e \) creates a smaller increase in the legislature’s preferences. Thus when the lobby is less powerful (\( \frac{\partial \gamma}{\partial e} \) is smaller), longer punishments are desirable. If the lobby is very influential, the same length of
punishment will have a larger impact on the legislature’s decisions (the impact on the gain accruing
to the lobby does not change). This tips the balance in favor of shorter punishments.

This intuition generalizes for all $\delta$ as in Expression (13). Here the second-order condition is more
complicated and can be positive if $\frac{\partial \gamma}{\partial e}$ is very small. That is, if the lobby has very little influence in
the legislature, it is conceivable that welfare will be maximized by making $T$ arbitrarily large.

**Result 3.** *In the case of political certainty, if non-trivial cooperation is possible in the presence
of a lobby, the optimal punishment scheme is finite when the influence of lobbying on legislative
preferences is sufficiently strong ($\frac{\partial \gamma}{\partial e}$ is sufficiently high).*

This helps to complete the comparison to the standard repeated-game model without lobbying.
There, grim-trigger (i.e. infinite-period) punishments are most helpful for enforcing cooperation (cfr.
KRW’s Proposition 4). I have shown here that shorter punishments are most often optimal in the
presence of lobbying. This is because long punishments incentivize the lobby to exert more effort to
break trade agreements.

However, the model with no lobbies and one with very strong lobbies can be seen as two ends
of a spectrum parameterized by the strength of the lobby. The optimal punishment will lengthen as
the political influence of the lobby wanes and the desire to discipline the legislature becomes more
important relative to the need to de-motivate the lobby.

6 Conclusion

I have integrated a separation-of-powers policy-making structure with lobbying into a theory of re-
current trade agreements. This theory takes seriously the idea that the threat of renegotiation can
undermine punishment when cooperation is meant to be enforced through repeated interaction alone.
Assuming that countries can bind themselves to condition their negotiations on the state designation
of a dispute settlement institution allows punishments to become incentive compatible.

I have shown here that, given complete information about the outcome of the lobbying and po-
itical process, the executives maximize social welfare by choosing the lowest tariffs that make it
unattractive for the lobbies to exert effort toward provoking a trade dispute. Thus the problem with
the lobby adds this extra constraint to the standard problem. While the constraint on the key repeated-
game player, which here is the legislature, is loosened by increasing the punishment length, this new
constraint due to the presence of lobbying becomes tighter as the punishment becomes more severe. This happens because the lobby prefers punishment periods in which tariffs, and thus its profits, are higher. It thus has increased incentive to exert effort as the punishment lengthens.

In a model with only the legislature, welfare increases with the punishment length. Here, this result only occurs if the lobby is sufficiently weak. As the lobby’s political influence grows, the optimal punishment length becomes shorter—in the race between incentivizing the legislature and the lobby, the need to de-motivate the lobby begins to win. This suggests that a key consideration when designing the length of dispute settlement procedures is how to optimally balance the incentives of those capable of breaking trade agreements with the political forces who influence them, given the strength of that influence.

Future work is planned in at least two, related directions. In order for disputes to occur in equilibrium, political uncertainty must be added to the model as in Buzard 2013 (alternatively, asymmetric information could be introduced, or both). The model will then be able to address questions about the impact of political uncertainty on trade agreements and optimal dispute resolution mechanisms.

It will also be possible to explore whether accounting for the endogeneity of political pressure can explain the observed variation in the outcomes of dispute settlement cases (Busch and Reinhardt 2006) because, in this context, it becomes meaningful to ask when lobbies have the incentive to exert effort to perpetuate a dispute (hence removing the ad-hoc assumption imposed in the proof of Corollary 2). Once political uncertainty has been added to the model, this is a completely natural extension that helps display the range and flexibility of the base model presented here.
Lemma 3. A solution to the executives’ problem \((6)\) exists for all \(\delta\) and all \(T\).

Proof: The executives’ problem is to minimize \(\tau^a\) such that both the legislature’s and the lobby’s constraints are satisfied. If the solution to the problem in the absence of lobbies (i.e. with only the legislature’s constraint, and that evaluated at \(e_b = 0\)) cannot be satisfied for any \(\tau^a < \tau^{tw}\) (that is, \(\tau^{a\text{NL}} = \tau^{tw}\)), then the solution to \((6)\) will also be \(\tau^{tw}\).

Consider the case where \(\tau^{a\text{NL}} < \tau^{tw}\). I rewrite the constraints with the payoffs normalized and \(\delta = e^{-r\Delta}\) where \(r\) is the interest rate and \(\Delta\) is the period length:

\[
(1 - e^{-r\Delta}) \left[ W_{ML}(\gamma(e), \tau^R, \tau^{a\text{NL}}) - W_{ML}(\gamma(e), \tau^a) \right] - e^{-r\Delta} (1 - e^{-r\Delta T}) \left[ W_{ML}(\gamma(e), \tau^a) - W_{ML}(\gamma(e), \tau^{tw}) \right] \geq 0 \tag{15}
\]

\[
(1 - e^{-r\Delta}) e_b - (1 - e^{-r\Delta}) \left[ \pi(\tau^R) - \pi(\tau^a) \right] - e^{-r\Delta} (1 - e^{-r\Delta T}) \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) \right] \geq 0 \tag{16}
\]

The proof is via the Intermediate Value Theorem. I take as the leftmost boundary \(\tau^{a\text{NL}}\) (the trade agreement chosen by the executives when there is no lobby) because this is the lowest possible tariff the executives can achieve before the additional constraint implied by the presence of lobbies is added. By construction, \(\bar{e}(\tau^{a\text{NL}}) = 0\). The gain to the lobby of a break in the trade agreement here is

\[
(1 - e^{-r\Delta}) \left[ \pi(\tau^R(0)) - \pi(\tau^{a\text{NL}}) \right] + e^{-r\Delta} (1 - e^{-r\Delta T}) \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^{a\text{NL}}) \right]
\]

If this gain is non-positive, the lobby has no incentive to exert effort and so \(\tau^{a\text{NL}}\) is the solution to the executives’ problem\[^{16}\] If the lobby’s gain is strictly positive, the left-hand size of Expression \(^{16}\) is negative. The executives will have to raise the trade-agreement tariff to prevent the agreement from being broken in this case because the lobby’s constraint is not satisfied.

Next, look at the rightmost boundary, that is \(\tau^{a} = \tau^{tw}\). The executives have no incentive to set a trade-agreement tariff above the trade-war (i.e. non-cooperative) level, as this is the highest tariff-level they will have to face even if the trade-agreement is broken. When \(\tau^{a} = \tau^{tw}\), the legislature’s constraint becomes

\[
\left[ W_{ML}(\gamma(e_b), \tau^R, \tau^{tw}) - W_{ML}(\gamma(e_b), \tau^{tw}) \right] \geq 0
\]

[^{16}]: Note, in particular, that this is the case if both \(\Delta\) and \(\Delta T \to 0\).
To make this condition hold with equality, we need $\epsilon_b$ so that $\tau^R(\epsilon_b) = \tau^{tw}$; that is, $\bar{\epsilon} = \epsilon_{tw}$.

The lobby’s gain will then be

$$
(1 - e^{-r\Delta}) \left[ \pi(\tau^{tw}) - \pi(\tau^{-}) \right] + e^{-r\Delta} \left[ (1 - e^{-r\Delta T}) \left[ \pi(\tau^{tw}) - \epsilon_{tw} - \pi(\tau^{tw}) \right] \right] = -e^{-r\Delta} \left( 1 - e^{-r\Delta T} \right) \epsilon_{tw}
$$

Therefore, at $\tau^a = \tau^{tw}$, the left-hand side of Expression (16) is

$$
(1 - e^{-r\Delta}) \epsilon_{tw} + e^{-r\Delta} \left( 1 - e^{-r\Delta T} \right) \epsilon_{tw}.
$$

This is always positive, which only requires $\Delta T > 0$ (recall that the special case of both $\Delta$ and $\Delta T \to 0$ has already been treated above).

In order to apply the Intermediate Value Theorem, it is left to show that the left-hand side of Expression (16) is continuous in $\tau^a$. The lobby’s gain is continuous in the tariffs by the assumptions on profits in Section 2. Given Assumption 1 and the assumptions in Section 2, $\bar{\epsilon}$ is continuous by the Implicit Function Theorem. Because the sum of continuous functions is continuous, we have the desired result and the Intermediate Value Theorem can be applied to ensure that, under the conditions stated above, the left-hand side of Expression (16) attains zero on the interval $[\tau_{NL}^a, \tau^{tw}]$ at least once; the solution to Problem 6 is at the minimum such $\tau$.

Proof of Corollary 1

By the Implicit Function Theorem:

$$
\frac{\partial \epsilon}{\partial \tau^a} = -\frac{\partial \Omega}{\partial \tau^a} \frac{\partial \Omega}{\partial \epsilon} = -\frac{\partial \Omega}{\partial \epsilon} \frac{\partial \epsilon}{\partial \tau^a} = -\left[ 1 + \frac{\partial CS}{\partial \epsilon} \right] \frac{\partial W_{ML}(\gamma(\tau^a), \tau^a)}{\partial \tau^a}
$$

(17)

In order for Equation 9 to hold, $\gamma(\bar{\epsilon})$ must be such that the cheater payoff at $\tau^R(\gamma(\bar{\epsilon}))$ is higher than the payoff at $\tau^{tw}$ and that the same is true of $\tau^a$ and $\tau^{tw}$. Therefore $\bar{\epsilon}$ will be set so that the median legislator’s ideal point is to the right of $\tau^a$ (and to the left of $\tau^{tw}$), implying that the numerator is positive.

Turning to the denominator, $\gamma$ is assumed increasing in $\epsilon$ so $\frac{\partial \gamma}{\partial \epsilon}$ is positive. Both profit differences are negative since $\tau^a < \tau^{tw}$ and $\tau^R(\bar{\epsilon})$ is also smaller than the trade war tariff. The last term in the denominator is a marginal change in the maximand of the unilateral optimization problem, evaluated at the optimal unilateral tariff. Thus, by the Envelope Theorem, this term is zero. Therefore the
denominator is negative. Combined with the positive numerator and the leading negative sign, the expression is positive.

\[ \frac{\partial \tau}{\partial a} = - \frac{\partial \Pi}{\partial a} = - \frac{(T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \left[ \pi'(\tau^w) - c_{tw} - \pi(\tau^a) \right] + \frac{\partial \pi(\tau^a)}{\partial \tau} \left[ 1 + \delta_L - \delta_{T+1}^L \right] \]

(18)

First I will show that \( \frac{1 - (T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \) is positive. Focusing on the numerator and rearranging, we have

\[
1 - (T + 1)\delta_T^L + T\delta_{T+1}^L = (1 - \delta_T^L - T\delta_T^L) (1 - \delta_L) = (1 - \delta_L) \sum_{i=0}^{i=T-1} \delta^i - T\delta_T^L (1 - \delta_L)
\]

Therefore \( \frac{1 - (T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \) is positive.

The bracketed term must be positive in order for the lobby to have the incentive to lobby in the trade-war phase. I will assume that this is the case, but note that restricting attention to this case is not trivial; see the conclusion for discussion of a planned extension that more fully explores the trade-war phase.

\( \frac{\partial \pi(\tau^a)}{\partial \tau} \) has been show to be positive in the proof of Corollary 1 and profits are increasing in \( \tau^a \), so everything in the denominator is positive except for \( \frac{\partial \pi(\tau^R(\tau^a))}{\partial \tau} \). As \( \tau^R(\tau^a) \) is the solution to legislature’s problem of unilateral maximization of \( \tau \) given \( e \), when \( e \) changes in response to a differential change in \( \tau^a \), the change in \( \tau^R \) is zero by the Envelope Theorem. Thus the denominator is the sum of three positive terms, and \( \frac{\partial \pi(\tau^a)}{\partial \tau} \) is positive.

\[ \frac{\partial \Pi}{\partial \tau} = - \frac{\partial \Pi}{\partial ML} \frac{\partial ML}{\partial e} = - \frac{\partial \Pi}{\partial ML} \frac{1 - (T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \left[ W_{ML}(\gamma(\pi),\tau^a) - W_{ML}(\gamma(\pi),\tau^w) \right] \]

(19)

\[ \frac{\partial \Pi}{\partial ML} = - \frac{\partial \Pi}{\partial \Pi} \frac{\partial \Pi}{\partial \tau} = \frac{1 - (T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \left[ W_{ML}(\gamma(\pi),\tau^a) - W_{ML}(\gamma(\pi),\tau^w) \right] \]

Proof of Corollary 3

By the Implicit Function Theorem:

\[ \frac{\partial \Pi}{\partial \tau} = - \frac{\partial \Pi}{\partial ML} \frac{\partial ML}{\partial e} = - \frac{\partial \Pi}{\partial ML} \frac{1 - (T + 1)\delta_T^T + T\delta_{T+1}^T}{(1 - \delta_L)^2} \left[ W_{ML}(\gamma(\pi),\tau^a) - W_{ML}(\gamma(\pi),\tau^w) \right] \]

(19)
I have shown in the proof of Corollary 2 that the first term in the numerator is positive. The bracketed term is positive because \( \bar{\epsilon} \) is always determined via Equation 9 so that \( W_{ML}(\gamma(\bar{\epsilon}), \tau^a) - W_{ML}(\gamma(\bar{\epsilon}), \tau^{tw}) \) is positive: the trade-war tariff is the punishment relative to the trade agreement tariff. Therefore the numerator of the fraction is positive. The denominator is shown to be negative in the proof of Corollary 1. Therefore \( \frac{\partial \pi}{\partial \delta_{ML}} \) is positive. 

**Proof of Corollary 4**

Differentiating Equation 12 with respect to \( \delta_{ML} \), we have

\[
\frac{\partial \Pi}{\partial \tau^a} \frac{\partial \tau^a}{\partial \delta_{ML}} + \frac{\partial \Pi}{\partial \bar{\epsilon}} \frac{\partial \bar{\epsilon}}{\partial \delta_{ML}} + \frac{\partial \Pi}{\partial \gamma} \frac{\partial \gamma}{\partial \delta_{ML}} = 0
\]

There is no direct effect of \( \delta_{ML} \) on this equation, so \( \frac{\partial \Pi}{\partial \delta_{ML}} = 0 \). Thus

\[
\frac{\partial \tau^a}{\partial \delta_{ML}} = -\frac{\frac{\partial \Pi}{\partial \tau^a}}{\frac{\partial \Pi}{\partial \bar{\epsilon}} \frac{\partial \bar{\epsilon}}{\partial \delta_{ML}} + \frac{\partial \Pi}{\partial \gamma} \frac{\partial \gamma}{\partial \delta_{ML}}} = -\left(1 - \frac{\partial \pi(\tau^R(\bar{\epsilon}))}{\partial \bar{\epsilon}(\delta_{ML})}\right) \frac{\partial \bar{\epsilon}}{\partial \delta_{ML}} \frac{\partial \pi(\tau^R(\bar{\epsilon}))}{\partial \tau^a} + \frac{\partial \pi(\tau^a)}{\partial \tau^a} + \frac{\delta_{ML} - \delta^{tw}}{1 - \delta_{ML}} \frac{\partial \pi(\tau^a) \partial \gamma}{\partial \tau^a} = \frac{20}{24}
\]

By the same argument as in the proof of Corollary 2, the denominator is positive, and likewise by that argument, \( \frac{\partial \pi(\tau^R(\bar{\epsilon}))}{\partial \bar{\epsilon}(\delta_{ML})} \) is zero. So, because \( \frac{\partial \bar{\epsilon}}{\partial \delta_{ML}} \) is positive (Corollary 3), \( \frac{\partial \tau^a}{\partial \gamma} \) is negative.

**Proof of Corollary 5**

By the Implicit Function Theorem:

\[
\frac{\partial \bar{\epsilon}}{\partial \gamma} = -\frac{\frac{\partial \Omega}{\partial \bar{\epsilon}}}{\frac{\partial \Omega}{\partial \gamma}} = -\frac{\delta_{ML} \frac{\partial \pi^R(\tau^a)}{\partial \gamma} \frac{\partial \pi^{tw}(\tau^a)}{\partial \gamma} - [\pi(\tau^a) - \pi(\tau^{tw})] - \frac{\partial \pi^R(\tau^a)}{\partial \gamma} \frac{\partial \pi^{tw}(\tau^a)}{\partial \gamma} + \frac{\partial \pi(\tau^a)}{\partial \gamma} + \frac{\partial \pi^R(\tau^a)}{\partial \gamma} + \frac{\partial \pi^{tw}(\tau^a)}{\partial \gamma}]}{\frac{\partial \pi^R(\tau^a)}{\partial \gamma} \frac{\partial \pi^{tw}(\tau^a)}{\partial \gamma} - \frac{\partial \pi(\tau^a)}{\partial \gamma} - \frac{\partial \pi^R(\tau^a)}{\partial \gamma} + \frac{\partial \pi^{tw}(\tau^a)}{\partial \gamma}}\]

(21)

By the Envelope Theorem the final set of terms in both the numerator and denominator are zero: \( \tau^R \) is the optimal response to both \( \bar{\epsilon} \) and \( \gamma(\bar{\epsilon}) \). Thus we can factor \( \frac{\partial \gamma}{\partial \bar{\epsilon}} \) out of the denominator and cancel the rest, leaving \( -\frac{1}{\partial \bar{\epsilon}} < 0 \).

**Proof of Corollary 6**

Differentiating the executives’ condition, Equation 12 with respect to \( \gamma \), we have

\[
\frac{\partial \Pi}{\partial \tau^a} \frac{\partial \tau^a}{\partial \gamma} + \frac{\partial \Pi}{\partial \bar{\epsilon}} \frac{\partial \bar{\epsilon}}{\partial \gamma} + \frac{\partial \Pi}{\partial \gamma} = 0
\]
Because $\frac{\partial \Pi}{\partial \gamma} = 0$, we are looking for

$$\frac{\partial \tau^\alpha}{\partial \gamma} = -\frac{\partial \Pi}{\partial \gamma} \frac{\partial \sigma}{\partial e} = -\frac{\partial \sigma}{\partial e} \left(1 - \frac{\partial \pi(R(\tau))}{\partial e(\delta_{ML})}\right) \frac{\partial \pi}{\partial \gamma}
\frac{\partial e}{\partial \gamma} \left(\delta_{ML} - \frac{1}{1 - \delta_{L}} \frac{\partial \pi}{\partial \tau^\alpha}\right)
(22)$$

As shown in Corollary 5, $\frac{\partial \sigma}{\partial \gamma}$ is negative. The arguments given in the proof of Corollary 4 show that the rest of the numerator reduces to 1 and that the denominator is positive. Therefore $\frac{\partial \tau^\alpha}{\partial \gamma}$ is positive.
8 References


