Protectionism as insurance
Risk Aversion, terms of trade uncertainty and optimal trade policy

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Abstract

We develop a theoretical general equilibrium model of international trade, which incorporates uncertainty about both terms of trade and the production function. Uncertainty resolves after the resource allocation to the different sectors has been determined. We show that in such a setting, the resulting equilibrium allocation chosen by the producers is not welfare maximizing if producers and consumers have different risk-preferences. Specifically, if the terms of trade are the main source of uncertainty and consumers are risk averse, producers will overspecialize in the net-export good. We proceed by showing that this misallocation can justify trade policy as a means to assure a more diversified production structure. The diversified production pattern thus serves as an insurance against adverse trade shocks.
1. Introduction

While there is a large consensus among trade economists, that free trade is desirable, it is puzzling to see that a lot of trade barriers still prevail. The most widely accepted explanation of this was given in the seminal work by Grossman and Helpman (1994), who state that tariffs are the result of a bargaining process between lobbies and the government. In the present paper, we are giving an alternative explanation. Free trade exposes countries to a market that they cannot control and might increase the aggregate amount of volatility. A country might therefore wish to limit trade in order to be less exposed to trade induced forms of risk. On the other hand, trade might also be seen as a remedy to domestic volatility by giving access to foreign markets that are object to different shocks than the home country.

This paper develops two models of international trade and assesses how different sources of trade related uncertainty can justify trade policy. The first model considers a small country, that cannot influence the world market price. Uncertainty in this model is introduced through volatile terms of trade. In the second model, we consider a large country. In this model, the volatility in the world market price is a direct result of volatility in the production function at home and abroad.

When producers and consumers have different attitudes towards risk, the market equilibrium will not maximize the expected welfare if there is uncertainty. We show that risk averse consumers will opt for a more diverse production if terms of trade are volatile. Producers on the contrary will want to exploit their comparative advantage to a higher extent by specializing more in the production of the net-export good. This can change when the domestic production risk is taken into account, but the two groups of agents will always have different optimal production patterns, if their risk preferences differ. We find that trade policy can be used to attain a production point that takes the consumers preferences into account. We show that consumers expected utility increases if trade policy is implemented.

Our paper is related to several strands of literature. There are several papers assessing the influence of uncertainty on the optimal pattern of production and trade. Turnovsky (1974) investigates how uncertainty about the terms of trade or the production technology changes the pattern of specialization in a Ricardian model. He finds that, under certain conditions, a country might decide to deviate from complete specialization. By deviating from the Ricardian structure used by Turnovsky (1974), we are able to go beyond the extreme cases of specialization reversal that are considered in his work.

Anderson and Riley (1973) compare the optimal allocation of resources under certain and uncertain terms of trade. They find that, as risk-aversion increases, it becomes optimal to diversify production more, moving the production point closer to the autarky equilibrium. We are able to replicate this finding for our model. While Anderson and Riley (1973) do not discuss the political implications of their results any further, we use it as a rationale for protectionist trade policy. We infer that, if risk aversion makes it optimal to restrict trade, than interventionist trade policies will be welfare enhancing if risk neutral agents decide upon the production pattern. By allowing for domestic production shocks we than extend the scope of Anderson and Riley (1973), which allows
us to show that there is a volatility trade off between openness and reclusion.

Eaton and Grossman (1985) investigate whether trade policy can be desirable if the terms of trade are stochastic. They focus on a setting, where one the two production factors is mobile in the short run, but the other has to be allocated before uncertainty resolves. The second factor is indivisible, such that factor owners have to allocate their entire endowment to one sector. This way, once the terms of trade uncertainty resolves, some factor owners will earn above their expected returns, and others below. Trade policy can then be seen as a redistributive measure that effectively works like a tax on the factor owners that did better. This interpretation of trade policy is different to ours, since in our model, insurance takes place across states, whereas in Eaton and Grossman (1985) it occurs across agents.

In a recent paper develop and test a Dornbusch et al. (1977) style two country-general equilibrium model of international trade and introduce different sorts of uncertainty to it. The purpose of the paper is to show that small countries have a higher incentive to introduce tariffs on imported goods than large countries, in order to diversify their range of domestically produced goods. The reasoning behind this is that smaller countries will have a relatively small range of products that they produce. If there is a bad production shock abroad, the home country would have a comparative advantage in some goods that it does not produce under certainty. Since countries have to decide on their production pattern before comparative advantages and terms of trade are determined, the country would then need to import these goods at a high price. A tariff would expand the range it produces, so that the bad shock will have less pronounced ramifications. This diversification motive is more important for smaller countries, since they spend relatively more of their budget on imports. Thus, even if a small country’s tariff will have a smaller influence on the terms of trade, the optimal tariff is found to be higher for a smaller country. This finding is contrary to our results. For our model, we find that the terms of trade effect dominates the diversification effect. This is driven by the setup of our model, in which every country produces every available commodity. Diversification in our model means to produce less of the good one specializes in and more of the net-import good, whereas in Gaisford and Ivus (2012) it means to increase the range of production.

Mayer (1977) proposes a model in which a country faces the exogenous probability of an embargo. He argues that a protectionist trade policy can have a welfare enhancing effect in such a situation, because of the restructuring the production sector would need to perform, in order to satisfy autarky demand. This can be seen as an extreme scenario of our model, where the terms of trade shocks are so adverse, that a country will want to retrace to its autarky production point.

Cassing et al. (1986) argue that a change in the terms of trade has different influences on the different factor owners. They argue that risk-averse factor owners would advocate a terms of trade stabilizing tariff. We go beyond this, by showing that even if all consumers are endowed with the same factors, they would still be in favor of a tariff, if the production pattern is determined by the producers. Our paper is also related to a strand of the literature that investigates how uncertainty influences allocation decisions and optimal policy, notably the work of Krebs et.al. (2005), van Marrewijk and van

The remainder of the paper is structured as follows. In section 2 we present a theoretical model for a small country that faces uncertainty about its terms of trade with the world and determine the optimal trade policy. In section 3 we extend this model to a 2 country setting, in which the terms of trade form arise as a result of the allocation decisions and production shocks that occur in both countries. We present several numerical examples for different trade and production patterns and discuss which trade policy fits which case best. In section 4 we conclude.

2. Theoretical model for a small country

In this section we present the model that will serve as a benchmark to the more general model which we propose in section 3. We do this to get a better understanding of the different sources of uncertainty that determine whether a protectionist policy is desirable or not.

There are two sectors, manufacturing and agricultural, and one factor of production, capital. We assume that the small country that we consider has a comparative advantage in producing the manufacturing good, and will export some of its production to the world market. Producers maximize their expected profit by employing capital at its factor price. We assume that output is increasing in capital input, but that we have decreasing returns to scale. All producers have profit maximizing behavior. In the aggregate, this will lead to an efficient allocation of resources in expectation. This implies expected GDP maximization. The aggregate production functions of the economy are denoted by \( y_m(K_m) \) for the manufacturing sector and by \( y_a(K_a) \) for the agricultural sector. The GDP of the economy can be denoted as

\[
I(K_m, K_a, p) \equiv p y_m(K_m) + y_a(K_a)
\]

(1)

The economy has the following resource constraint.

\[
K = K_m + K_a
\]

(2)

Plugging these resource constraints into (1), the GDP can be expressed as

\[
I(K_m, p) = p y_m(K_m) + y_a(K - K_m)
\]

(3)

Capital is assumed to be mobile in the long run, but fixed in the short run. This means that the producers have to allocate capital, knowing that they cannot directly reoptimize, should the terms of trade be different from their expectations.

The relative world market price is assumed to be exogenously given. It is assumed to be distributed over the closed interval \([p_l, p_h] \) according to the density function \( f(p) \). The price for the agricultural good serves as the numeraire. We assume that even in the worst realization of the relative world price \( p_l \) there is no trade reversal. Thus we have
that $p_l > p_a$, where $p_a$ is the relative autarky price of the net-export good. The expected GDP can thus be written in the following way

$$E[I(K_m, p)] = \int_{p_l}^{p_h} I(K_m, p) f(p) dp$$

(4)

The function $I(K_m, p)$ has the following properties.

- $\frac{\partial I}{\partial p} > 0$: An increase in the relative world market price of the exported good increases the GDP
- $\frac{\partial I}{\partial K_m} \leq 0$, $\frac{\partial^2 I}{\partial K_m^2} < 0$: there exists a maximum w.r.t. $K_m$
- $\frac{\partial^2 I}{\partial K_m \partial p} > 0$: If the price of the manufacturing good rises, the marginal return to capital in the manufacturing sector rises.

The producers’ maximization behavior leads to a maximization of the expected GDP by an optimal allocation of capital. The FOC that determines the optimal capital allocation is

$$\int_{p_l}^{p_h} \frac{\partial I}{\partial K_m}(K^*_m) f(p) dp = 0$$

(5)

We are now going to show that $K^*_m$ is strictly larger than the allocation that risk-averse consumers would choose. From balanced trade it follows that the representative consumer’s income is equal to the GDP. Therefore her income ultimately results from the choices made by the producers. Thus, usually consumers will take income as given when maximizing their expected utility. We are now abstracting from this by considering which allocation were to arise if the consumer could decide upon it. Intuitively different risk preferences will result in different optimal allocations if there is uncertainty about the prices. We will now show that this is indeed the case in our model, if consumers are risk averse. The consumer’s indirect utility function $V(I, p)$ is increasing and concave in income and decreasing in prices, if we assume that consumption of all goods is strictly positive. If she can maximize her indirect utility over the allocation choice, the maximization problem would be

$$\max_{K_m} E[V(I(K_m), p)] = \int_{p_l}^{p_h} V(I, p) f(p) dp$$

(6)

The consumer’s FOC that corresponds to the producers’ FOC is

$$\int_{p_l}^{p_h} \frac{\partial V}{\partial I} \frac{\partial I}{\partial K_m} (K^*_m) f(p) dp = 0$$

(7)

Comparing the two FOCs, we can derive a first proposition, about the different optimal capital allocation the two groups are going to have.
Proposition 1. In our model \(K_n^m\), the capital that the producers are allocating to the export sector (i.e. the manufacturing sector in our example) will be strictly larger than \(K^*_m\), the capital that the representative consumer would allocate to the manufacturing sector in order to maximize her expected utility, if the consumer is risk averse and producers are risk neutral.

Proof. We can show that \(\frac{\partial^2 V}{\partial I \partial p} < 0\). Therefore, the first term in the integral is positive and decreasing in \(p\). Since \(\frac{\partial I}{\partial K_m}\) changes sign and is increasing in \(p^2\), it is negative for small \(p\). The first term thus gives a larger positive weight to the negative elements of the second term and a smaller positive weight to the positive elements. In order to make the argument clear, we are rewriting the integral as the limit of the sum of all the elements. For the producers, we know that

\[
I'_{K_m}(K_n^*, p_l) f(p_l) + \ldots + I'_{K_m}(K_n^*, p_h) f(p_h) = 0
\]  

(8)

Since \(I'(.)\) changes sign and is increasing in \(p\), we know that

\[
I'_{K_m}(K_n^*, p_l) < \ldots < I'_{K_m}(K_n^*, p_h) = 0 < \ldots < I'_{K_m}(K_n^*, p_h)
\]  

(9)

Since \(f(p) > 0 \ \forall p\), the distribution function does not change the sign of the terms in (8). If we now multiply every element in (8) with the corresponding marginal indirect utility, we get that

\[
V'(K_n^*, p_l)I_{K_m}(K_n^*, p_l) f(p_l) + \ldots + V'(K_n^*, p_h)I_{K_m}(K_n^*, p_h) f(p_h) < 0
\]  

(10)

since \(V'(.)\) gives more weight to the negative elements. Therefore the derivative of the expected utility of the consumer is negative at \(K_n^*\). It follows that the optimal \(K_m\) for the consumers has to be smaller than \(K_n^*\).

2.1. Introduction of a tariff

Since the producers’ investment decisions do not maximize the consumer’s utility, the question is whether there is scope for policy to improve the allocation for the consumers.

Proposition 2. With uncertainty about the terms of trade and a rigid production structure, a certain degree of protectionism will increase expected welfare. The optimal tariff will be strictly positive but non-prohibitive.

\[
\frac{dE[V]}{dt} \bigg|_{t=0} > 0
\]  

(11)

\[
\frac{dE[V]}{dt} \bigg|_{t = \frac{p_w-p_h}{p_n}} < 0
\]  

(12)

where \(t = \frac{p_w-p_h}{p_n}\) is the prohibitive tariff for which trade stops.

\[1\text{ see Appendix}\]

\[2\text{ See Appendix}\]
Proof. We want to show that the introduction of a tariff has a beneficial effect for the consumers, but that the optimal tariff will not lead to autarky. Therefore we take the derivative of the expected indirect utility w.r.t. to $t$.

$$
\frac{dE[V(.)]}{dt} = E[\frac{\partial V}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial V}{\partial I} \frac{\partial I}{\partial t} + \frac{\partial V}{\partial K} \frac{\partial K^*}{\partial p} \frac{\partial p}{\partial t}]
$$

$$
= E[\frac{\partial V}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial V}{\partial I} \frac{\partial I}{\partial t} + \frac{\partial V}{\partial I} \frac{\partial I}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial K^*}{\partial p} \frac{\partial p}{\partial t} E[\frac{\partial V}{\partial I} \frac{\partial I}{\partial K}]]
$$

$$
= E[\frac{\partial V}{\partial I} \left( -d_m \frac{\partial p}{\partial t} + \frac{\partial I}{\partial t} \frac{\partial I}{\partial p} \right)] + \frac{\partial K^*}{\partial p} \frac{\partial p}{\partial t} E[\frac{\partial V}{\partial I} \frac{\partial I}{\partial K}]
$$

where $d_m$ describes the domestic consumption of the industrial good. The last line follows from Roy’s Identity.

We observe a tariff that is such that it decreases the relative price of the industrial good, by increasing the absolute price of the import good. The home price of the agricultural good still serves as the numeraire price. Tariff revenue is redistributed to the consumer as a lump-sum transfer. The GDP after the introduction of the tariff can be written as

$$
I_t = p(t)y_m + ya + \frac{t}{1+t}
$$

where $t$ is the tariff and $M$ is the net import of the agricultural good. $\frac{t}{1+t}M$ describes the tariff revenue, evaluated at world market prices. The relative home price is changed such that

$$
\frac{\partial p}{\partial t} = -\frac{p_w}{(1+t)^2}
$$

Using $M = d_a - ya$, we can rewrite (14) as

$$
I_t = p(t)y_m + ya + \frac{t}{1+t}(d_a - ya)
$$

We need to take the partial derivative of this w.r.t. $t$ to plug it into (13).

$$
\frac{\partial I_t}{\partial t} = \frac{1}{(1+t)^2}M + \frac{t}{1+t} \frac{\partial d_a}{\partial t}
$$

Using the envelope theorem, we know that $\frac{\partial I_t}{\partial p} = y_m + \frac{t}{1+t} \frac{\partial d_a}{\partial p}$ Setting these two results back into (13) and using (15) we obtain

$$
\frac{dE[V(.)]}{dt} = E[\frac{\partial V}{\partial I} \left( \frac{1}{(1+t)^2}M - p_wX - \frac{tp_w}{(1+t)^2} \frac{\partial d_a}{\partial p} \right] + \frac{\partial K^*}{\partial p} \frac{\partial p}{\partial t} E[\frac{\partial V}{\partial I} \frac{\partial I}{\partial K}]
$$
where $X$ is the net export of the industrial good. The balanced trade condition for the economy is

$$M - p(t)X \equiv 0 \quad (19)$$

With $p(t) = p_w$ at $t = 0$ and $p(t) < p_w$ if $t > 0$, we see that $M - p_wX$ must be negative for positive tariffs. With $p$ being the relative price of the industrial good, it follows that $\frac{\partial d_a}{\partial p} > 0$ for homothetic preferences. Thus, the first term of (18) is negative for positive $t$.

We have already shown in the previous section that the equilibrium capital allocation is too extreme for the consumers, if there is uncertainty concerning the terms of trade. Thus $E\left[\frac{\partial V}{\partial I}|_t \frac{\partial I}{\partial K_m}\right] < 0$. We also know that $\frac{dK^*_m}{dp} > 0^3$. Therefore we have

$$dE[V(.)] dt = E\left[\frac{\partial V}{\partial I} \left( \frac{1}{(1+t)^2} (M - p_wX) - \frac{tp_w}{(1+t)^3} \frac{\partial d_a}{\partial p} \right) \right] \leq 0$$

$$- \frac{\partial K^*_m}{\partial p} E\left[ \frac{p_w}{(1+t)^2} \frac{\partial V}{\partial I} \frac{\partial I}{\partial K_m} \right] \leq 0$$

Now we are going to show, that the introduction of a tariff has a beneficial effect for the consumers. In order to do so, we evaluate equation (20) at $t = 0$.

$$dE[V(.)]|_{t=0} = E\left[\frac{\partial V}{\partial I} \frac{\partial I}{\partial K_m} \right] \frac{\partial K^*_m}{\partial t} > 0 \quad (21)$$

We can thus conclude that a certain amount of protectionism is superior to free trade.

Now, we are interested in showing that the optimal tariff will not be prohibitive, i.e. that some trade is better than autarky. We define the tariff which leads to autarky such that the resulting relative price $p = \frac{p_w}{(1+t)}$, is equal to the autarky price.

$$\frac{p_w}{(1+t)} = p_a$$

$$\Rightarrow t = \frac{p_w - p_a}{p_a} \quad (22)$$

Under autarky, we do not have uncertainty about the terms of trade. The producers’ FOC boils down to

$$\frac{\partial I}{\partial K_m} = 0 \quad (23)$$

^ See Appendix
and the consumer’s FOC to
\[ \frac{\partial V}{\partial I} \frac{\partial I}{\partial K_m} = 0 \] (24)
which leads to the same optimal \( K^*_m \). Thus, the second term in (20) becomes zero. The entire equation (20) changes to
\[ \frac{dE[V(.)]}{dt} |_{t=\frac{pw-pa}{pw}} = E\left( \frac{\partial V}{\partial I} \left( p_w^2 \left( M - p_w X - (p_w - p_a) \frac{\partial d_a}{\partial p} \right) \right) \right) < 0 \] (25)
A prohibitive tariff is therefore non-desirable. It manages to resolve the miss-allocation by taking away the uncertainty, but it creates inefficiencies elsewhere. We can thus conclude that the optimal tariff has to be positive, but non-prohibitive.

3. A two-country model with production shocks

In the previous section we have shown that a protectionist policy will be beneficial, if trade introduces uncertainty to an otherwise non-stochastic economy. We were not suggesting how the change in the terms of trade comes about, since it did not matter for our analysis. Now we are going to develop a model, which explicitly derives the terms of trade as a function of domestic and foreign production shocks. The market clearing price results once that all allocations are determined, and after the production shocks are revealed. The case for a protectionist trade policy becomes thus less clear. In the previous case, the intuition was somewhat straightforward. If consumers are risk averse and trade increases aggregate volatility, then it was to be expected that they would be in favor of a trade reducing policy.

With two sources of uncertainty, both domestic and foreign, the intuition is less clear. More trade could serve as a remedy against domestic production shocks, if the world market is less volatile than the home economy.

In this section we are going to show that there is some truth to it, and that the optimal trade policy depends on the magnitude and the origin of uncertainty.

In order to be able to calculate specific world market prices, we abstract from our more general form in the previous section. The consumers in both countries are assumed to have the same Cobb-Douglas utility function \( U = (x_1^{\alpha_m} x_2^{1-\alpha_m})^\beta \). The producers in the home country produce, using the following production functions
\[ y_{m1} = \varphi \sqrt{K_m} \]
\[ E[y_{a1}] = \frac{1}{2}(1 + \sigma_1)\sqrt{K_a} + \frac{1}{2}(1 - \sigma_1)\sqrt{K_a} \] (26)
\( \varphi \) is a productivity parameter which is used later on in order to model comparative advantage. \( \sigma \in [0,1] \) is the volatility in the agricultural sector for the home country. We
model it such that it takes the value $-\sigma$ or $\sigma$ with probability $\frac{1}{2}$. Therefore, $E[\sigma] = 0$. The subscript 1 identifies a variable to belong to home. Abroad, the production functions are assumed to be

\[
y_{m2} = \sqrt{K_m} \\
E[y_{a2}] = \frac{1}{2}(1 + \sigma_2)\sqrt{K_a} + \frac{1}{2}(1 - \sigma_2)\sqrt{K_a}
\]

(27)

The expected profit maximization problem in each country follows naturally.

### 3.1. A need for a different normalization

As Dierker and Grodal (1999) explain in their work, different price normalizations under uncertainty always imply a different weighting of the states of the world. There is no wrong or right way to normalize, but there are ways that make more sense than others.

In the previous chapter, this was not of much importance, since we were only interested in qualitative comparisons. Now however, we need to quantify how different shocks will influence the optimal allocations in order to be able to say anything about optimal trade policy.

The problem with using the numeraire as normalization is that the high relative price of the manufactured good in the good state of the world, makes the producers put a large emphasis on this state of the world. Since this problem does not arise for the consumers (since they effectively care not about prices, but about actual consumption), it biases our comparison of the allocations producers and consumers prefer. This means, that the difference in the optimal allocation for the two groups of agents does not only result from the different risk preferences, but also from different reactions to a particular normalization.

In order to assure that the differences between consumers and producers are only driven by differences in attitudes towards risk, we will thus choose a normalization that leads explicitly to the same weights for consumers and producers.

We define an appropriate normalization as one, where producers and consumers give the same weight to the different states. Under such a normalization, a risk neutral producer will choose the same allocation as a risk neutral producer would.

The representative consumer maximizes her expected indirect utility. With Cobb-Douglas utility $U = (x_1^\alpha x_2^{1-\alpha})^\beta$ and income $I$, the expected indirect utility can be expressed as

\[
E(V(I,p)) = E\left[\left(\alpha^\alpha (1 - \alpha)^{1-\alpha} \left(\frac{1}{p_1}\right)^\alpha \left(\frac{1}{p_2}\right)^{1-\alpha}\right)^\beta I^\beta\right]
\]

(28)

where $I$ is the GDP. If the consumer is risk neutral, the expected indirect utility becomes

\[
E(V(I,p)) = \alpha^\alpha (1 - \alpha)^{1-\alpha}E\left[\left(\frac{1}{p_1}\right)^\alpha \left(\frac{1}{p_2}\right)^{1-\alpha} I\right]
\]

(29)

Producer’s profit maximizing behavior leads to GDP maximization. Therefore, a normalization that will result in the same optimal allocations for both groups is one for
which

$$E(V(I,p)) = \alpha^\alpha (1-\alpha)^{1-\alpha} E[I]$$

(30)

Under this normalization, consumers effectively maximize expected GDP. So, our normalization has to be such that

$$\left( \frac{1}{p_1} \right)^\alpha \left( \frac{1}{p_2} \right)^{1-\alpha} = 1$$

$$\Rightarrow p_1^\alpha p_2^{1-\alpha} = 1$$

(31)

If we define \( \frac{p_1}{p_2} = p \) as the relative price, we have to weight each state of the world by \( \left( \frac{1}{p} \right)^\alpha \) to get the desired normalization. An example:

With our old normalization we had that

$$I = py_1 + y_2$$

(32)

Now this changes to

$$\tilde{I} = p^{1-\alpha} y_1 + \left( \frac{1}{p} \right)^\alpha y_2$$

(33)

The relative price remains unchanged, but this normalization fulfills our requirements.

### 3.2. Optimal allocations and equilibria under the new normalization

Producers maximize expected profit, given the normalized prices. This leads to expected GDP maximization. The expected GDP is given by

$$E(I) = E[p^{1-\alpha} y_m + \left( \frac{1}{p} \right)^\alpha y_a]$$

(34)

Setting in the production technology and resource constraints for the home country, this becomes

$$E(I) = E[p^{1-\alpha} \varphi \sqrt{K_{m1}} + \left( \frac{1}{p} \right)^\alpha \sqrt{K_1 - K_{m1}}]$$

(35)

where \( \varphi \) again measures the comparative advantage in producing the manufacturing good. We limit ourselves to a discrete version of uncertainty, where we have either a high or low productivity in the agricultural sector. Since these shocks also occur abroad\(^4\), this gives us four different scenarios. We can thus rewrite (35) as

$$E[I_1] = \frac{1}{4} (1+r) \left( p_{hh}^{1-\alpha} \varphi \sqrt{K_{m1}} + \left( \frac{1}{p_{hh}} \right)^\alpha (1+\sigma_1) \sqrt{K_1 - K_{m1}} \right)$$

$$+ \frac{1}{4} (1-r) \left( p_{hl}^{1-\alpha} \varphi \sqrt{K_{m1}} + \left( \frac{1}{p_{hl}} \right)^\alpha (1+\sigma_1) \sqrt{K_1 - K_{m1}} \right)$$

$$+ \frac{1}{4} (1-r) \left( p_{lh}^{1-\alpha} \varphi \sqrt{K_{m1}} + \left( \frac{1}{p_{lh}} \right)^\alpha (1-\sigma_1) \sqrt{K_1 - K_{m1}} \right)$$

$$+ \frac{1}{4} (1+r) \left( p_{ll}^{1-\alpha} \varphi \sqrt{K_{m1}} + \left( \frac{1}{p_{ll}} \right)^\alpha (1-\sigma_1) \sqrt{K_1 - K_{m1}} \right)$$

(36)

\(^4\)Foreign productivity influences the world market price
where $r$ measures the correlation between the two countries. Producers maximize their expected profit by choosing capital optimally. This gives us the FOC for the expected GDP-maximizing capital allocation for the home country

$$\begin{align*}
0 & = (1 + r) \left( (p_{1h}^{1-\alpha} + p_{1l}^{1-\alpha}) \sqrt{K_1 - K_{m1}^*} \varphi - ((\frac{1}{p_{1h}})^\alpha (1 + \sigma_1) + (\frac{1}{p_{1l}})^\alpha (1 - \sigma_1)) \sqrt{K_{m1}^*} \right) \\
& + (1 - r) \left( (p_{1h}^{1-\alpha} + p_{1l}^{1-\alpha}) \sqrt{K_1 - K_{m1}^*} \varphi - ((\frac{1}{p_{1l}})^\alpha (1 + \sigma_1) + (\frac{1}{p_{1h}})^\alpha (1 - \sigma_1)) \sqrt{K_{m1}^*} \right)
\end{align*}$$

(37)

In order to calculate the market clearing prices, we have to determine the allocations abroad as well. The FOC is very similar, the only things that change is that $\varphi_2 = 1$, where the subscript 2 denotes country 2. Their FOC is then

$$\begin{align*}
0 & = (1 + r) \left( (p_{2h}^{1-\alpha} + p_{2l}^{1-\alpha}) \sqrt{K_2 - K_{m2}^*} - ((\frac{1}{p_{2h}})^\alpha (1 + \sigma_2) + (\frac{1}{p_{2l}})^\alpha (1 - \sigma_2)) \sqrt{K_{m2}^*} \right) \\
& + (1 - r) \left( (p_{2h}^{1-\alpha} + p_{2l}^{1-\alpha}) \sqrt{K_2 - K_{m2}^*} - ((\frac{1}{p_{2l}})^\alpha (1 + \sigma_2) + (\frac{1}{p_{2h}})^\alpha (1 - \sigma_2)) \sqrt{K_{m2}^*} \right)
\end{align*}$$

(38)

What we still need in order to calculate the prices is an expression for relative world demand. With Cobb-Douglas preferences, $d_m(I, p) = \frac{\alpha I}{p^{1-\alpha}}$ and $d_a(I, p) = (1 - \alpha) I p^\alpha$, so relative demand does not depend on income. Therefore relative world demand is equal to relative domestic demand: $\frac{d_m(I, p)}{d_a(I, p)} = \frac{1 - \alpha}{\alpha} p$. In order to calculate the market clearing relative price in each state of the world, we set relative demand equal to relative supply

$$\frac{1 - \alpha}{\alpha} p_{ij} = \frac{(1 \pm \sigma_1) \sqrt{K_1 - K_{m1}^*} + (1 \pm \sigma_2 \sqrt{K_2 - K_{m2}^*}} \varphi \sqrt{K_{m1}^*} + \sqrt{K_{m2}^*}} {i, j \in (h, l)}$$

(39)

These four equations and the two optimal allocation problems have to be solved simultaneously. We cannot (for now) solve them analytically, but the numerical examples allow us already to derive a lot of information about the equilibrium variables.

Having determined the expressions that pin down the market equilibrium, we are now going to perform the same exercise as in the previous chapter. We will show that the consumer’s optimal allocation differs from the market outcome. Assuming a scenario where the risk-averse consumer can decide on her preferred allocation, she will choose one that maximizes her expected utility. The different resulting allocation will also change the world market price in each state, so that we have to solve for these again. Given the normalization the indirect utility depends only indirectly on the prices. The representative consumer maximizes

$$E[V(I(K_m, p))] = (\alpha^\alpha (1 - \alpha)^{1-\alpha})^\beta E[I(K_m, p)^\beta]$$

(40)

Taking the derivative w.r.t. to $K_m$ gives us the FOC

$$E[\frac{\partial I}{\partial K_m} I(K_m)^{\beta-1}] = 0$$

(41)
Setting in the four states of the world, this can be written as

\[
0 \overset{!}{=} (1 + r) \left( p_{1hc} \varphi \sqrt{K_1 - K_{c1m1}^*} - \left( \frac{1}{p_{hhc}} \right)^\alpha (1 + \sigma_1) \sqrt{K_{m1}^*} \right) I_{hhc}(K_{c1m1}^*)^{\beta - 1} + (1 - r) \left( p_{1hc} \varphi \sqrt{K_1 - K_{c1m1}^*} - \left( \frac{1}{p_{hlc}} \right)^\alpha (1 - \sigma_1) \sqrt{K_{m1}^*} \right) I_{lhc}(K_{c1m1}^*)^{\beta - 1} \right] + (1 - r) \left( p_{1hc} \varphi \sqrt{K_1 - K_{c1m1}^*} - \left( \frac{1}{p_{lhc}} \right)^\alpha (1 - \sigma_1) \sqrt{K_{m1}^*} \right) I_{lhc}(K_{c1m1}^*)^{\beta - 1} \right]
\]

The additional \( c \) subscript indicates that the equilibrium prices and GDP change due to the allocation chosen by the consumers.

We assume that abroad it is still the producers that decide upon the allocation of resources. Due to the change in price the FOC of the foreign producers changes to

\[
0 \overset{!}{=} (1 + r) \left( p_{1hc} \varphi \sqrt{K_1 - K_{c2m2}^*} - \left( \frac{1}{p_{hhc}} \right)^\alpha (1 + \sigma_2) + \left( \frac{1}{p_{lhc}} \right)^\alpha (1 - \sigma_2) \right) \sqrt{K_{m2}^*} \right) I_{hhc}(K_{c1m1}^*)^{\beta - 1} + (1 - r) \left( p_{1hc} \varphi \sqrt{K_1 - K_{c2m2}^*} - \left( \frac{1}{p_{hlc}} \right)^\alpha (1 - \sigma_2) + \left( \frac{1}{p_{lhc}} \right)^\alpha (1 + \sigma_2) \right) \sqrt{K_{m2}^*} \right) I_{lhc}(K_{c1m1}^*)^{\beta - 1} \right]
\]

The prime indicates that the optimal allocation of the foreign producers changes due to the change in world market prices. The market clearing prices are calculated as before and solve the following equation

\[
\frac{1 - \alpha}{\alpha} p_{ijc} = \frac{(1 \pm \sigma_1) \sqrt{K_1 - K_{c1m1}^*} + (1 \pm \sigma_2) \sqrt{K_2 - K_{m2}^*}}{\varphi \sqrt{K_{c1m1}^*} + \sqrt{K_{m2}^*}} \quad i, j \in (h, l)
\]

Here again we have a system of six equations in six unknowns that we have to solve simultaneously. We will do this numerically. From this we receive values for \( K_{c1m1}^* \) that we can compare to \( K_{m1}^* \).

### 3.3. Differences in optimal allocations

As in the previous version, we will now compare the different ideal allocations. We see that consumers tend to prefer less specialization than producers if the terms of trade volatility is considerably higher than the domestic production volatility. If the domestic production risk is higher than abroad, consumers will want to specialize and export more than the producers. The foreign production risk has a less direct effect on the consumers utility, since it’s effect on the world market price is not one to one. Therefore the home market risk dominates if the shocks are of similar size. We see that consumers tend to prefer less specialization than producers if the terms of trade volatility is considerably
higher than the domestic production volatility. If the domestic production risk is higher than abroad, consumers will want to specialize and export more than the producers. The foreign production risk has a less direct effect on the consumers utility, since its effect on the world market price is not one to one. Therefore the home market risk dominates if the shocks are of similar size. In the first column, an increase in correlation makes the worst case scenario more likely. In the worst case scenario (bad terms of trade, low productivity) it is better to produce more of the agricultural good, since the terms of trade decrease is the dominating effect for these parameters. In the third column we see that, as the second worst case scenario gets more likely (very low productivity but good terms of trade), it becomes better to specialize more on the export good. As the worst case scenario becomes more likely, specialization makes less sense.

### 3.4. Optimal trade policy

Given the results in the previous section, we would expect a country with a comparative advantage in the manufacturing sector to protect its agricultural sector if it has a less volatile production function than the world market.

To investigate whether this intuition holds true, we are going to introduce a tariff to our model. We assume that the value of the ad-valorem tariff is known before the allocation decisions have to be made. Therefore producers will take the new relative domestic price that the tariff implies into account, when making their decision.

The tariff changes the relative domestic prices such that the producers have to allocate

---

**Table 1: Allocations Difference $K_{m1}^* - K_{m1}^*$**

<table>
<thead>
<tr>
<th>$\sigma_2 = 0$</th>
<th>$\sigma_2 = 0.2$</th>
<th>$\sigma_2 = 0.4$</th>
<th>$\sigma_2 = 0.6$</th>
<th>$\sigma_2 = 0.8$</th>
<th>$\sigma_2 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 = 0$</td>
<td>0</td>
<td>-0.15</td>
<td>-0.39</td>
<td>-1.24</td>
<td>-1.84</td>
</tr>
<tr>
<td>$\sigma_1 = 0.2$</td>
<td>0.24</td>
<td>0.09</td>
<td>-0.35</td>
<td>-1.02</td>
<td>-1.68</td>
</tr>
<tr>
<td>$\sigma_1 = 0.4$</td>
<td>0.97</td>
<td>0.83</td>
<td>0.4</td>
<td>-0.29</td>
<td>-1.16</td>
</tr>
<tr>
<td>$\sigma_1 = 0.6$</td>
<td>2.18</td>
<td>2.07</td>
<td>1.71</td>
<td>1.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\sigma_1 = 0.8$</td>
<td>3.83</td>
<td>3.78</td>
<td>3.62</td>
<td>3.23</td>
<td>2.19</td>
</tr>
<tr>
<td>$\sigma_1 = 1$</td>
<td>5.69</td>
<td>5.74</td>
<td>6.05</td>
<td>6.47</td>
<td>7.28</td>
</tr>
</tbody>
</table>

$K_1 = 100$  $K_2 = 500$  $\phi = 2.5$  $R = 0$  $\beta = 0.2$  $\alpha = 0.5$

---

**Table 2: Allocations Difference $K_{m1}^* - K_{m1}^*$ for different correlations**

<table>
<thead>
<tr>
<th>$\sigma_1 = 0.2, \sigma_2 = 0.6$</th>
<th>$\sigma_1 = 0.4, \sigma_2 = 0.4$</th>
<th>$\sigma_1 = 0.6, \sigma_2 = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R = -1</td>
<td>-0.45</td>
<td>0.81</td>
</tr>
<tr>
<td>R = -0.5</td>
<td>-0.73</td>
<td>0.61</td>
</tr>
<tr>
<td>R = 0</td>
<td>-1.02</td>
<td>0.41</td>
</tr>
<tr>
<td>R = 0.5</td>
<td>-1.31</td>
<td>0.2</td>
</tr>
<tr>
<td>R = 1</td>
<td>-1.59</td>
<td>0</td>
</tr>
</tbody>
</table>

$\alpha = 0.5$  $K_1 = 100$  $K_2 = 500$  $\phi = 2.5$  $\beta = 0.2$
their resources in order to maximize

\[ E[I(t, p)] = E[(\frac{p^w}{1 + t})^{1-\alpha}y_m + (\frac{1 + t}{p^w})^\alpha y_a] \]  

(45)

where \( p^w \) indicates the world market relative price. The FOC for the producers thus becomes

\[
0 = (1 + r) (\frac{(p_{hlt}^w)}{1 + t})^{1-\alpha} + (\frac{(p_{llt}^w)}{1 + t})^{1-\alpha} \sqrt{K_1 - K_{t^*}^1} \varphi - ((\frac{1 + t}{p_{hlt}^w})^\alpha (1 + \sigma_1) + (\frac{1 + t}{p_{llt}^w})^\alpha (1 - \sigma_1)) \sqrt{K_{1^*}^m} \\
+ (1 - r) (\frac{(p_{hlt}^w)}{1 + t})^{1-\alpha} + (\frac{(p_{llt}^w)}{1 + t})^{1-\alpha} \sqrt{K_1 - K_{t^*}^1} \varphi - ((\frac{1 + t}{p_{hlt}^w})^\alpha (1 + \sigma_1) + (\frac{1 + t}{p_{llt}^w})^\alpha (1 - \sigma_1)) \sqrt{K_{1^*}^m} 
\]

(46)

We assume that the tariff proceedings are distributed directly to the consumers. The effective income for any state is thus

\[
\tilde{I}(t,p) = (\frac{p^w}{1 + t})^{1-\alpha}y_m + (\frac{1 + t}{p^w})^\alpha y_a + (1 + t) - \alpha \left( \frac{1}{p^w} \right)^\alpha M(I,p) \]  

(47)

The last term describes the tariff proceedings, evaluated at the domestic relative price of the imported good. We see that \( m \) depends on the income of the consumers, so we have to solve for it.

\[
M(I,p) = d_a(I,p) - y_a \\
= \frac{I(1 - \alpha)(p^w)^\alpha}{(1 + t)^\alpha} - y_a \]  

(48)

Setting this into equation (47) and solving for \( I \), we get

\[
I(p^w, t) = (\frac{1 + t}{1 + \alpha t}) \left( (p^w) - \alpha y_a \right) \]  

(49)

Assuming that the foreign government does not impose a tariff, we can calculate the equilibrium price for all the four states. Relative demand changes, since the consumers in the two countries face different domestic prices

\[
\frac{d_{a1} + d_{a2}}{d_{m1} + d_{m2}} = \frac{1 - \alpha}{\alpha} \frac{p^w}{(1 + t)^\alpha} I_1 + I_2(1 + t)^\alpha \]  

(50)

Relative supply only changes because allocations change due to the new domestic prices

The prices thus result from the following four equations

\[
\frac{1 - \alpha}{\alpha} \frac{p_{ij}}{(1 + t)^\alpha} I_{ij1} + I_{ij2}(1 + t)^\alpha \\
= \sqrt{K_1 - K_{m1}^*} (1 + \sigma_1) + \sqrt{K_2 - K_{m2}^*} (1 + \sigma_2) \sqrt{K_{1^*}^m + K_{2^*}^m} \quad j, k \in (h, t) \]  

(51)

5 prices and production technology have to be adjusted according to the state of the world
where the subscript \( t \) indicates the equilibrium prices and allocations that arise when a tariff is set.

For now, we cannot solve for the optimal tariff analytically. However, we can compare the expected indirect utility of the consumers in a setting with a tariff with the free trade expected indirect utility. We approximate the optimal tariff by finding the tariff that gives the maximal positive difference between the two utility measures. From this, we get the following *optimal* tariffs as a function of our parameters. We see that the size of the foreign country has a huge influence on the optimal tariff. This shows us that we absolutely need to control for the terms of trade effect when we want to evaluate to what extent uncertainty determines optimal trade policy. We see that increasing risk aversion increases the optimal tariff. This result becomes more pronounced when we choose a larger size for the foreign country.

The direction of the optimal tariff is as expected, the magnitude however is rather small. This is mainly due to the dominance of the terms of trade effect. The terms of trade effect in our setting is expected to be a lot higher than in reality, where the world market prices form as a result of the production decision of many countries. In reality, we would expect uncertainty to have a bigger influence on the optimal trade policy.

### 4. Conclusion

In this paper we have shown that trade policy can be beneficial if there is trade-related uncertainty and heterogeneous risk-preferences. In the small country case, where the only source of uncertainty are the terms of trade, we have shown unambiguously that
risk-averse consumers will prefer a more diversified production pattern. Furthermore, a non-prohibitive tariff will be optimal, since it changes the market equilibrium towards a less specialized production point, which reduces uncertainty.

In the second part, we have considered a model that allows us to draw a more realistic picture of the world. Here, volatility in the terms of trade is a result of production shocks both at home and abroad. For this model, results are more ambivalent. Optimal allocations and therefore trade policy depends on the relative magnitude of the domestic shock, as compared to the foreign shock. We can replicate our result from the first chapter if we choose domestic uncertainty to be low and the foreign one to be high. In a case where the domestic production functions is highly stochastic, the opposite can be observed, namely that consumers will want to specialize more than optimal in the production of their net-export good. However, our results are biased because we cannot disentangle the risk aversion effect from the terms of trade effect.

The predictions of our model fit the European case. It would be a reasonable assumption that the agricultural production function is less volatile in Europe than, say, in Africa. Europe will thus be expected to artificially sustain the agricultural sector in order to be protected against terms of trade fluctuations.

In order to be able to show this, we will continue our work by taking the model to the data, to see whether the existing tariff structure is in line with our predictions.
A. Appendix

A.1. Proof for $\frac{\partial^2 V}{\partial I \partial p} < 0$

With homothetic preferences, we can write $V(I, p)$ as $V = g(I)h(p)$. We know that $\frac{\partial V}{\partial I} = g'(I)h(p) > 0$. Taking the cross-derivative w.r.t. $p$ we get

$$\frac{\partial^2 V}{\partial I \partial p} = g'(I)h'(p) \quad (52)$$

From Roy’s Identity, we know that $-x_i = \frac{\partial V}{\partial p} \frac{\partial V}{\partial I}$. Therefore, if consumption is strictly positive, we have that $\frac{\partial V}{\partial p} < 0 \Rightarrow h'(p) < 0$. Thus

$$g'(I)h'(p) < 0 \quad (53)$$

Q.E.D.

A.2. Proof for $\frac{\partial^2 I}{\partial K_i \partial p}(K^*_i) > 0$

$$I(K^*_i) = py_i(K^*_i) + y_a(K^*_i) \quad (54)$$

Since this is an optimal value function, we can use the envelope theorem to get

$$\frac{\partial I(K_i)}{\partial p} = y_i(K^*_i) \quad (55)$$

taking the derivative of this w.r.t. $K_i$ gives

$$\frac{\partial^2 I(K_i)}{\partial p \partial K_i} = \frac{\partial y_a(K^*_i)}{\partial K_i} = \frac{\partial^2 I(K_i)}{\partial K_i \partial p} > 0 \quad (56)$$

Q.E.D.

A.3. Proof for $\frac{\partial K^*_i}{\partial p} > 0$

From the GDP maximization we have that

$$p \frac{\partial y_i}{\partial K_i} + \frac{\partial y_a}{\partial K_i} = 0 \quad (57)$$

taking the total derivative and using the envelope theorem, we get

$$\frac{\partial y_i}{\partial K_i} dp + \left(p \frac{\partial^2 y_i}{\partial K_i^2} + \frac{\partial^2 y_a}{\partial K_i^2} \right) dK_i = 0$$

$$\Rightarrow \frac{dK_i^*}{dp} = -\frac{\frac{\partial y_i}{\partial K_i}}{p \frac{\partial^2 y_i}{\partial K_i^2} + \frac{\partial^2 y_a}{\partial K_i^2}} > 0 \quad (58)$$

The denominator is negative as a result from the decreasing returns to scale assumption.

Q.E.D.
References


