Optimal consumption Taxes, Cross-Border Pollution and Economic Integration

by

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February 2013

Abstract

We consider a two-good, two small open economies model with cross-border pollution that is generated from consumption. Within this framework we examine i) the Nash-equilibrium consumptions taxes and compare them to the case where pollution is only local, ii) the cooperative equilibrium consumption taxes and we compare them to the Nash tax rates, and iii) cases where cooperative taxes are different between countries. Many results of the paper depend on whether pollution and the polluting good are complements or substitutes in consumption.

Keywords: Consumption taxes, cross-border pollution, Nash and cooperative equilibrium

JEL Classification: H21: Efficiency and optimal taxation

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1 Introduction

Consumption taxation is a longstanding area of academic interest in the economics literature and a central subject in policy design. The public finance literature has extensively examined the welfare and tax revenue implications of consumption taxation, e.g., Diamond and Mirrlees (1971), Browning (1985), as well as other related issues, e.g., the implications of moving consumption taxes on different commodities towards uniformity, e.g., Atkinson and Stiglitz (1976), Hatta (1977) and (1986).

At the same time, other strands of the economics literature have “tapped” into the issue of consumption taxation. For example, the issue of reforming an open economy’s structure of indirect taxes away from the more distortive trade taxes, i.e., import tariffs and export taxes, and towards less distortive domestic taxes such as consumption taxes. This has been motivated by the quest of open economies to improve national welfare and maintain or even raise government tax revenues in the process of trade liberalization during the recent decades, which resulted to large reductions in trade taxes, e.g., Michael et al. (1993), Abe (1995), Keen and Ligthart (2002), Lahiri and Nasim (2005), Emran and Stiglitz (2005) and Boadway and Sato (2009), Haibara (2012).1

Recently, in many, particularly developed countries, viable concerns have been raised about the negative environmental implications of the expanding global economic activity.2 In light of this concerns, various environmental policy instruments such as emissions taxes or (non-) tradable emissions permits have been implemented in order to restrict the environmentally "bad"

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1 Another direction of a theoretical as well as policy debate is whether taxation of final commodities should follow the so-called “destination” or “origin” principle e.g. see among others Georgakopoulos and Hitiris (1992), Bovenberg (1994), Keen and Lahiri 1998, Karakosta et al. (forthcoming). This debate over the two taxation principles has become even more prevalent in the recent decades with the expanion of regional economic integration.

2 For example, a frequently raised theoretical and policy related question has been "Is free trade good for the environment?", e.g., see Antweiler et al. (2001).
activities and to promote the environmentally "good" activities. In this vein, the use of consumption taxes has been frequently proposed as a viable alternative policy measure to environmental taxes in order to contain environmental damages. For one thing, this argument has been promoted following the realization that environmental tax revenues as share of the GDP are relatively low, even among rich-developed countries with long tradition in the use of environmental taxes, e.g., the EU. For example, Stamatova and Steurer (2011) report that in the EU-27 environmental tax revenues as share of the GDP rebounded to a 4.2 percent following five years of continuous decline and a historical low of 3.8 percent in 2008.\(^3\) On the other hand, with consumption expenditures by households and non-profit organizations servicing households representing approximately 58 percent of the GDP in the EU-27 in 2011, the use of consumption taxes, for environmental purposes, pose an advantage over direct pollution taxes of a much larger taxation base. On such grounds, Albrecht (2006) proposes: "... A green tax reform could therefore also be based on existing consumption taxes instead of direct pollution taxes ...", p.94. Equally important it is argued that consumption taxes may require less complex bureaucratic structures and institutional innovation, and thus, may pose easier implementation compared to emissions taxes.

On the grounds of such real world considerations, a small but growing economics literature considers the use of consumption taxes as an alternative mean to pure environmental policy instruments such as emission taxes and (non-) tradable emissions permits, of taxing consumption activity, when the latter entails the creation of pollution emissions. This literature, by and large, considers the welfare, tax revenue and environmental implications of reforms in trade and domestic taxes, e.g., consumption and or production taxes, in pollution ridden open economies, e.g., Copeland (1994), Beghin et al. (1997), Turunen-Red and Woodland (2004), Kayalica and Kayalica (2005), Haibara and Ohta (2011), Michael and Hatzianayotou (forthcoming).

The present study contributes to this literature, by a constructing a model of two small open economies with cross-border pollution generated from con-

\(^3\)In only four countries environmental tax revenues as share of the GDP exceeded 5 percent, Bulgaria, Malta, Slovenia and the Netherlands.
sumption. Consumption taxes are levied by the government in each country. Within this framework we first derive the Nash-equilibrium consumption taxes and we compare them to the case where consumption pollution is local. A comparative statics analysis is conducted in order to examine how these Nash consumption taxes change when the rates of pollution per unit of consumption, or of cross-border pollution change. Second, we derive the cooperative equilibrium consumption taxes and we compare them to the Nash tax rates. Finally we identify conditions under which the cooperative consumption taxes differ in the two countries.

2 The Model

Consider a model with two small open economies, Home and Foreign, with a representative household, producing and consuming two traded goods. Good 1 is assumed to be the numeraire and good 2 the non-numeraire whose world price \( q \) is fixed and equal to unity. In both countries, consumption of good 2 generates pollution which affects negatively household utility in both countries. A unit of consumption of good 2, in Home generates \( \alpha \) units of pollution, and in Foreign it generates \( \alpha^* \) units of pollution. Hereon, asterisks denote Foreign’s variables. The governments in the two countries levy a consumption tax, \( \tau \) and \( \tau^* \) respectively, on good 2. Thus, \( p = 1 + \tau \) and \( p^* = 1 + \tau^* \) denote, respectively, the consumer prices in the two countries. The production side of each country is represented by the GDP function, \( R(q) \) and \( R^*(q) \), respectively. Production conditions bear no impact on the analysis to follow, since the countries are assumed to be small open economies and there are no distortions on the production side of either country. Let \( E(p, r, u) \) be the minimum expenditure in Home required to achieve a given level of utility \( u \) at consumer price \( p \) and level of overall pollution \( r \). Its derivative with respect to \( p \) (i.e., \( E_p \)), is the Hicksian demand function for good 2, and \( E_{pp} < 0 \); its derivative with respect to \( r \) (i.e., \( E_r \)), is positive and captures the households’ marginal willingness to pay for reducing pollution,

\[ E_p = \partial E / \partial p \quad \text{and} \quad E_{pp} = \partial E_p / \partial p. \]

\[ ^4\text{All subscripts denote partial derivatives, e.g.,} \quad E_p = \partial E / \partial p \quad \text{and} \quad E_{pp} = \partial E_p / \partial p. \]
e.g., see Copeland (1994), and it is assumed that $E_{rr} > 0$. $E_{pr}$ denotes the relationship between good 2 and pollution in consumption. If positive (negative), the two are complements (substitutes) in consumption.\(^5\) Finally, $E_u$ denotes the inverse of the marginal utility of income and for simplicity is set equal to one. Similarly, $E(p^*, r^*, u^*)$ denotes Foreign’s minimum expenditure function.

Pollution generated in one country crosses the borders and affects consumers in the other country. Thus total pollution affected consumers in Home ($r$) and Foreign ($r^*$) are respectively denoted as:

\[
    r = \alpha E_p + \theta \alpha^* E_{p^*}^* \quad \text{and} \quad r^* = \alpha^* E_{p^*}^* + \theta^* \alpha E_p
\]  

where $\theta$ is the percentage of the pollution generated in Foreign that affect consumers in Home, and similarly, $\theta^*$ is the percentage of pollution generated in Home that affects consumers in Foreign.

The income-expenditure identity for Home requires that expenditure equals income from production plus the consumption tax revenue which is assumed to be lump sum distributed. Thus we have

\[
    E(u, p, r) = R(q) + \tau E_p(u, p, r)
\]  

Similarly, Foreign’s income-expenditure identity is defined as

\[
    E^*(u^*, p^*, r^*) = R^*(q) + \tau^* E_{p^*}^*(u^*, p^*, r^*)
\]

For analytical convenience, the following assumption is used throughout the analysis.

Assumption 1: All income effects fall on the numeraire commodity, i.e.,

\[
    E_{pu} = E_{p^*, u^*} = E_{ru} = E_{r^*, u^*} = 0
\]

\(^5\)Alternatively, it is to say that if $E_{pr} > 0 (< 0)$, then the polluting good 2 and clean environment are substitutes (complements) in consumption. For example, pollution and hiking activity can be considered as substitutes in consumption, while pollution and air-conditioning devices can be considered as complements. On the other hand, assuming $E_{pr} = 0$ implies that the polluting good and pollution (clean environment) are independent in consumption.
Differentiating equations 2 and 3 we get that

\[ E_u du = \tau E_{pp} d\tau + [-E_r + \tau E_{pr})dr \] (4)

\[ E_u^* du^* = \tau^* E_{ppr}^* d\tau^* + [-E_r^* + \tau^* E_{pr}^*)dr^* \] (5)

Differentiating equations (1), we obtain the effects in the levels of pollution \( r \) and \( r^* \) due to changes in the tax rates, in the rates of cross-border pollution, and in the rates of pollution per unit of consumption in the two countries as follows:

\[
\begin{bmatrix}
\delta & -\theta \alpha^* E_{pr}^* \\
-\theta \alpha^* E_{pr} & \delta^*
\end{bmatrix}
\begin{bmatrix}
dr \\
dr^*
\end{bmatrix}
= \begin{bmatrix}
\alpha E_{pp} \\
\theta^* \alpha E_{pp}
\end{bmatrix}
\begin{bmatrix}
d\tau \\
d\tau^*
\end{bmatrix}
+ \begin{bmatrix}
\theta \alpha^* E_{pr}^* \\
\alpha^* E_{pr}^*
\end{bmatrix}
\begin{bmatrix}
d\tau^* \\
d\tau^*
\end{bmatrix}
\]

\[ + \begin{bmatrix}
\alpha^* E_{pr}^* \\
0
\end{bmatrix}
d\theta
+ \begin{bmatrix}
0 \\
\alpha E_{pr}^*
\end{bmatrix}
\begin{bmatrix}
d\theta^* \\
d\alpha^*
\end{bmatrix}
+ \begin{bmatrix}
E_{pr}^* \\
\theta E_{pr}^*
\end{bmatrix}
\begin{bmatrix}
d\alpha \\
d\alpha
\end{bmatrix} \] (6)

where \( \delta = 1 - \alpha E_{pr} \) and \( \delta^* = 1 - \alpha^* E_{pr}^* \) are both positive under the plausible assumption that an increase in the consumption tax that reduces consumption, given everything else, reduces the consumption generated pollution. Using the system of equations (6) we get that

\[
\Delta dr = \alpha E_{pp} H d\tau + E_p H d\alpha + \delta^* \alpha^* E_{pr}^* d\theta
\]

\[ + \theta^* E_{pr}^* d\tau^* + \theta E_{pr}^* d\alpha^* + \alpha E_p \theta^* \alpha^* E_{pr}^* d\theta^* \] (7)

\[
\Delta dr^* = \theta^* \alpha E_{pp} H d\tau + \theta^* E_p H d\alpha + \alpha^* E_{pr}^* \theta^* \alpha E_{pr} d\theta^* +
\]

\[ + \alpha^* E_{pr}^* H^* d\tau^* + E_{pr}^* H^* d\alpha^* + \delta \alpha E_p d\theta^* \] (8)

where \( H = [1 - (1 - \theta^*)\alpha^* E_{pr}^*] \) and \( H^* = [1 - (1 - \theta^*)\alpha E_{pr}] \) are both positive.\(^6\) The determinants of the left hand side of equations (6), denoted as \( \Delta = 1 - \alpha^* E_{pr}^* - \alpha E_{pr} + (1 - \theta^*)\alpha E_{pr}\alpha^* E_{pr}^* \), is also positive under the

\(^6\) Note that \( H > \delta \) when \( \theta = \theta^* = 0 \) and \( H = 1 \) when \( \theta = \theta^* = 1 \). If \( E_{pr} > 0 \), then \( H > \delta \). If \( E_{pr} < 0 \), then \( H > 0 \). Similarly for \( H^* \).
assumption that an increase in the consumption tax at Home that reduces its consumption, reduces its total pollution. Substituting equation 7 into equation 4, and 8 into 5, after some manipulations we get

$$
\Delta u = A_r d\tau + A_r d\tau^* = E_{pp}[\tau^*\delta^* - \alpha E_r H]d\tau - [E_r - \tau E_{pr}]\theta \alpha E_{pp}^* d\tau^* \quad (9)
$$

$$
\Delta u^* = B_r d\tau^* + B_r d\tau = E_{pp}^*[\tau^*\delta - \alpha E_r^* H^*]d\tau^* - [E_r^* - \tau^* E_{pr^*}]\theta^* \alpha E_{pp}^* d\tau \quad (10)
$$

Equation (9) shows that an increase in Home’s consumption tax rate affects utility through a negative tax revenue effect, i.e., $\Delta^{-1}E_{pp}^*\tau^*$, due to lower consumption and consumption tax revenue. The term $-\Delta^{-1}E_{pp}^*\alpha E_r H$, captures the effect of the higher $\tau$ on Home’s utility through the induced changes in the level of pollution. That is, the higher $\tau$ which lowers $r$, exerts through this term a positive impact on welfare. The second right-hand-side term of this equation captures the impact of a higher $\tau^*$ on Home’s utility. A higher $\tau^*$ which reduces Home pollution due to lower cross-border pollution, on the one hand, has a positive direct impact on $u$, i.e., $-\Delta^{-1}E_r \theta \alpha E_{pp}^*$, and on the other hand, an ambiguous indirect effect, i.e., $\Delta^{-1}E_{pp}^* \theta \alpha E_{pp}^*$, through the change in consumption tax revenue, depending on the sign of $E_{pp}^*$. That is, a higher $\tau^*$, ceteris paribus, lowers cross-border pollution to Home. If the polluting good and pollution are complements in consumption, i.e., $E_{pp}^* > 0$, then consumption of good 2 falls and so do consumption tax revenues, entailing a negative impact on Home’s utility. If the polluting good and pollution are substitutes in consumption, i.e., $E_{pp}^* < 0$, the opposite result holds. Similar interpretations can be given for the terms in equation (10).

### 3 Nash Equilibrium

First, we consider the case where each country chooses its consumption tax to maximize its own welfare taking as given the policy of the other country. The reaction functions of the two countries are given by
\[\Delta \frac{\partial u}{\partial \tau} = A_r = E_{pp}\{\tau[1 - \alpha^*E_{p,r,*}] - \alpha E_r [1 - (1 - \theta^*)\alpha^*E_{p,r,*}]\} = 0 \quad (11)\]

\[\Delta \frac{\partial u^*}{\partial \tau^*} = B_{r^*} = E_{p,p^*}\{\tau^*[1 - \alpha E_{p,r}] - \alpha^*E_{r,*} [1 - (1 - \theta^*\alpha)E_{p,r}]\} = 0 \quad (12)\]

Using in equation (11), we obtain Home’s Nash consumption tax rate as

\[\tau^N = \frac{\alpha E_r [1 - (1 - \theta^*)\alpha^*E_{p,r,*}]}{[1 - \alpha^*E_{p,r,*}]} \quad (13)\]

Equation (13) shows that the Nash equilibrium consumption tax is positive. In the case where any of \(\theta, \theta^*\) and or \(E_{p,r,*}\) are zero, then the Nash consumption tax reduces to \(\tau^N = \alpha E_r\), which corresponds to the Nash tax rate when cross-border pollution is absent. \(^7\) When there is perfect or imperfect cross-border pollution between the two countries (i.e., \(0 < \theta^* \leq 1; 0 < \theta \leq 1\)) then, ceteris paribus, the Nash consumption tax with cross border pollution is higher (lower) to the Nash rate in the absence of cross border pollution if \(E_{p,r,*}\) is positive (negative). Intuitively, note that a consumption tax, by reducing the consumption of this good, affects negatively welfare by reducing tax revenue and positively by reducing pollution. An increase in the Home consumption tax, reduces local consumption and pollution, thus less pollution is transmitted abroad. If \(E_{p,r,*} > 0\), then in Foreign lower pollution reduces the consumption of the polluting good since this good and pollution are complements in consumption. This in turn results in less cross-border pollution from Foreign to Home, thus affecting positively Home’s welfare. This positive effects calls for higher Nash consumption tax. Similarly, setting \(B_{r^*} = 0\) in equation (12), we obtain Foreign’s Nash consumption tax.

The second order conditions for welfare maximization are satisfied if

\(^7\)When pollution is generated from production, the existence of cross-border pollution does not affect the formula of the Nash equilibrium emission tax, see e.g., Hatzipanayotou et al (2005).
\[
\Delta \frac{\partial^2 u}{\partial \tau^2} = A_{rr} = E_{pp}[\delta^* - \alpha^2 H^2 E_{pp} E_{rr} \Delta^{-1} - \alpha HE_{rp}]
\] (14)

\[
\Delta \frac{\partial^2 u^*}{\partial \tau^* r^*} = B_{rr^*} = E_{pp^*}[\delta^* - \alpha^* 2 H^* 2 E_{pp^*} E_{rr^*} \Delta^{-1} - \alpha^* H^* E_{rp^*}]
\] (15)

are negative. Sufficient but not necessary conditions for these to be satisfied is that pollution and the polluting good 2 are substitutes in consumption in each country, i.e., \(E_{rp} \leq 0\) in Home and \(E_{rp^*} \leq 0\) in Foreign. In the case \(E_{rp}\) and \(E_{rp^*}\) are positive, it is assumed that are sufficiently small so that the second order conditions for welfare maximization are satisfied. Differentiating the reactions functions of Home (i.e., equation (11)) and of Foreign (i.e., equation (12)), with respect to \(\tau^*\) and \(\tau\) respectively, we get

\[
\Delta \frac{\partial A_\tau}{\partial \tau^*} = A_{rr^*} = -\alpha \alpha^* \theta HE_{rr} E_{pp} E_{pp^*} \Delta^{-1} < 0
\] (16)

\[
\Delta \frac{\partial B_\tau^*}{\partial \tau} = B_{rr^*} = -\alpha \alpha^* \theta^* H^* E_{rr^*} E_{pp} E_{pp^*} \Delta^{-1} < 0
\] (17)

The slope of Home’s reaction function (i.e., \((d\tau^*/d\tau)|_{A_{\tau^*}=0} = -A_{rr^*}/A_{rr}\)) and that of Foreign’s (i.e., \((d\tau^*/d\tau)|_{B_{\tau^*}=0} = -B_{rr^*}/B_{rr}\)) are both negative, indicating that the Nash consumption taxes are strategic substitutes. Stability of the Nash equilibrium requires that the Home reaction function is steeper.

**Proposition 1** Consider two countries and assume the existence of consumption generated cross-border pollution between them.

- If in Foreign pollution and the polluting good are substitutes (complements) in consumption, then the Home Nash equilibrium consumption tax rate is lower (higher) compared to the case where cross-border pollution is absent.

- If in Foreign the pollution and the polluting good are independent in consumption, then the existence of cross-border pollution does not affect the Nash equilibrium consumption tax in Home.
3.1 Comparative Statics

In this subsection we consider the effects of changes in the rates of pollution per unit of consumption in the two countries, and in the rates of cross-border pollution, on the Nash equilibrium consumption tax rates. Totally differentiating equations (11) and (12), we get

$$
\begin{bmatrix}
A_{\tau\tau} & A_{\tau\alpha} \\
B_{\tau\tau} & B_{\tau\alpha}
\end{bmatrix}
\begin{bmatrix}
d\tau^N \\
d\tau^N^*
\end{bmatrix}
= -
\begin{bmatrix}
A_{\tau\alpha} \\
B_{\tau\alpha^*}
\end{bmatrix}
d\alpha -
\begin{bmatrix}
A_{\tau\alpha^*} \\
B_{\tau\alpha}
\end{bmatrix}
d\alpha^*
- \begin{bmatrix}
A_{\tau\theta} \\
B_{\tau\theta^*}
\end{bmatrix}
d\theta - \begin{bmatrix}
A_{\tau\theta^*} \\
B_{\tau\theta}
\end{bmatrix}
d\theta^* \tag{18}
$$

3.1.1 The effect of changes in rates of pollution per unit of consumption

Using the system of equations (18) we get the effect of changes in Home’s rate of pollution per unit of consumption (i.e., $\alpha$) on the Nash consumption taxes in both countries as follows

$$
\Omega \frac{d\tau^N}{d\alpha} = -B_{\tau\tau^*}A_{\tau\alpha} + A_{\tau\tau^*}B_{\tau\alpha} \tag{19}
$$

$$
\Omega \frac{d\tau^*N}{d\alpha} = -B_{\tau\alpha^*}A_{\tau\tau} + B_{\tau\tau^*}A_{\tau\alpha} \tag{20}
$$

where $\Omega = A_{\tau\tau^*}B_{\tau\tau^*} - B_{\tau^* \alpha}A_{\tau^*}$ is the determinant of the left hand side of equation (18) and is positive for the stability of the Nash equilibrium. Also $B_{\tau^* \alpha} = -\alpha^* \theta^* E_{p^*} E_r^* [\theta \delta^{-1} E_{p^*} E_r^* + H^* E_r^* E_{p^*} \Delta^{-1}]$ and $A_{\tau\alpha} = -HE_{pp}[E_r + \alpha HE_{rr} E_p \Delta^{-1}] > 0$. In general, the effect of an increase in $\alpha$ on the Nash consumption taxes is ambiguous. For improving the understanding of these results, we consider some special cases. First, let pollution and the polluting goods be very strong substitutes, i.e., $E_{p^*}$ is a large negative number in absolute terms, and $\theta$ be close to one, so that $B_{\tau^* \alpha} < 0$, then an increase in $\alpha$ increases the Nash consumption tax in Home and decreases it in Foreign.

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8\textbf{8} \theta \simeq 1 \text{ implies near (or) perfect cross-border pollution. This is a commonly used assumption in the relevant literature, e.g., see Vlassis (2012).}
This case can be explained using Figures 1 and 2. As noted in equations (16) and (17), the reaction functions of both countries have negative slopes. Point A on the diagram depicts the initial Nash equilibrium. Since $A_{\tau \alpha}$ is positive, an increase in \( \alpha \) moves the Home reaction function $RR$ upwards (i.e., \( (dR^*/d\alpha) |_{\alpha=0} = -A_{\tau \alpha}/A_{\tau^*} > 0 \)). Since the sign of $B_{\tau^* \alpha}$ is ambiguous, the Foreign reaction function $R^*R^*$ moves upwards or downwards depending on the sign of $E_{pr}$. If $E_{pr}$ is negative and $|E,\delta^{-1}E_{pr}| > E_{rr}E_{rr}\Delta^{-1}$, then $B_{\tau^* \alpha}$ is negative and the Foreign reaction function moves downwards resulting in a lower Nash consumption tax in Foreign and a higher one in Home. This is shown in Figure 1 where the initial Nash equilibrium is at A and the new Nash equilibrium is at point $B$. If, however, $E_{pr}$ is positive, then $B_{\tau^* \alpha}$ is positive and the Foreign reaction function moves upwards and the new Nash equilibrium could be at a point such as C, D or E shown in Figure 2. If the new Nash equilibrium is at C, then, the result is a lower Foreign and a higher Home Nash consumption tax. If instead the new Nash equilibrium is at D, then the result is a higher Nash consumption tax in both countries. It is even possible that the new Nash equilibrium is at a point like E with a lower Home and a higher Foreign Nash consumption tax. Intuitively, when $\alpha$ increases, Home reaction is to increase its Nash consumption tax and the best response by Foreign, on the one hand is to reduce its Nash tax rate. On the other hand, when $\alpha$ increases, more pollution gets into Foreign. In Foreign when pollution increases i) $E_r^*$ increases, since $E_r^*\tau^*$ is positive, calling for a higher tax and ii) If $E_{pr}$ is positive, then Home consumption of the polluting good increases, which in turn for given rate $\theta$ raises pollution in Foreign calling for a higher Nash consumption tax in that country. If, however, $E_{pr}$ is negative, Home consumption of the polluting good falls, and pollution in Foreign declines, calling for a lower Nash consumption tax in the country. The final result depends on the relative size of each of these effects.

Consider another special case where either $\theta$ or $\theta^*$ is zero, which implies that either $A_{\tau \tau^*}$ or $B_{\tau^* \alpha}$ is zero. Then, an increase in $\alpha$, increases Home’s Nash consumption tax rate. The same holds true when initially the pollution per unit of consumption is very small, i.e., $\alpha$ and thus $B_{\tau \tau^*}$ is close to zero. Similarly, when $\alpha$ increases, then the optimal response by Foreign
is to increase its Nash consumption tax rate when initially \( \alpha \) is very small (close to zero), and \( E_{pr} \) is either non-negative or a sufficiently small negative number so that \( B_{r^*\alpha} \) is positive.

Finally, consider the special case where \( \alpha = \alpha^* = 1, \theta = \theta^* = 1 \) and countries exhibit identical reactions in the sense that \( E_{pr}^* = E_{rr}, E_{pp} = E_{pp}^* = E_{rr}, E_{r}^* = E_{r} \) etc. In this case we have that \( A_{rr} = B_{r^*r^*} = E_{pp} \Delta - E_{pp}E_{rr} \Delta^{-1} < 0, A_{r^*r} = B_{r^*r} = -E_{rr}E_{pp} \Delta^{-1} < 0, A_{r^*\alpha} = -E_{pp}[E_{r} + E_{rr}E_{r} \Delta^{-1}] > 0, B_{r^*\alpha} = -E_{pp}[E_{r} \delta^{-1}E_{pr} + E_{rr}E_{r} \Delta^{-1}] \). Substituting into equation (19) and (20), we get

\[
\Omega \frac{d\tau^N}{d\alpha} = -E_{pp}^2[E_{r} \Delta + E_{rr}E_{r} - E_{pp}E_{rr}E_{r} \delta^{-1}] > 0
\] (21)

\[
\Omega \frac{d\tau^{*N}}{d\alpha} = E_{pp}^2[\Delta \delta^{-1}E_{r}E_{pr} + E_{rr}E_{r} + E_{pp}E_{rr}E_{r} \delta^{-1}] \] (22)

Equation (21) shows that an increase in the rate of pollution per unit of consumption at Home increases its Nash consumption tax, but the effect on Foreign’s Nash rate is ambiguous and it depends among other things on the sign and size of \( E_{pr} \).

If \( E_{pr} \) is negative and \( |E_{r} \delta^{-1}E_{pr}| > E_{rr}E_{r} \Delta^{-1} \), then \( B_{r^*\alpha} \) is negative and Foreign’s reaction function, see Figure 2, moves downwards resulting in a lower Nash consumption tax in Foreign. If \( E_{pr} \) is positive, then \( B_{r^*\alpha} \) is positive, and Foreign’s reaction function moves upwards, intersecting Home reaction function at C or D resulting either in a lower or a higher Nash consumption tax in Foreign.

### 3.1.2 The effect of changes in the rate of cross border pollution

Using the system of equation (18) we get the effect of changes in the rate of cross border pollution (\( \theta \)) on the Nash equilibrium consumption taxes in both countries as follows.

\[
\Omega \frac{d\tau^N}{d\theta} = -A_{r^*\theta}B_{r^*r^*} + A_{r^*r}B_{r^*\theta}
\] (23)
\[ \Omega \frac{d\tau^N}{d\theta} = -A_{\tau\tau} B_{\tau\theta} + A_{\tau\theta} B_{\tau\tau} \]  \hspace{1cm} (24)

where \( B_{\tau\theta} = -\alpha\alpha^* H E_{pr} E_{pr}^* \cdot E_{pr}^* + \alpha^* H E_{pr}^* E_{pr}^* \cdot \Delta^{-1} \), \( A_{\tau\theta} = -\alpha\alpha^* E_{pp}[\theta^* E_{r_r} E_{pr}^* + H\delta E_{r_r} E_{pr}^* \Delta^{-1}] \). In general, the effects of changes in \( \theta \) on the Nash tax rates is ambiguous. Like in the previous discussion, we consider some special cases for better understanding these effects. First, consider the case where \( E_{pr}^* < 0 \) and thus \( B_{\tau\theta} < 0 \), and \( E_{pr}^* \) is either positive or has a sufficiently small negative value so that \( A_{\tau\theta} > 0 \). In this case, an increase in \( \theta \) increases Home’s Nash consumption tax rate and it reduces that of Foreign. This case can be explain using Figure 1. Since \( A_{\tau\theta} > 0 \), the increase in \( \theta \) shifts upwards Home’s reaction function, and since \( B_{\tau\theta} < 0 \), Foreign’s reaction function moves downwards. The new Nash equilibrium is at point B with a higher Home consumption tax and a lower tax in Foreign. If, however, \( E_{pr}^* \) and \( E_{pr}^* \) are both positive, then both reaction functions shift upwards resulting in a new Nash equilibrium such as C with a lower Foreign and a higher Home consumption tax, or such as D with higher consumption taxes in both countries, or even to a point like E with a higher consumption tax in Foreign and a lower one in Home, see fig 2.

Next, consider the special case where there is no cross-border pollution from Home to Foreign (i.e., \( \theta^* = 0 \)). In this case \( B_{\tau\theta} = 0 \), and an increase in \( \theta \) increases Home’s Nash consumption tax rate. If initially the rate of cross border pollution from the Foreign into Home is very small (i.e., \( \theta \) is close to zero), then \( A_{\tau\theta} \approx 0 \) and an increase in this rate increases Home’s Nash tax rate if \( E_{pr}^* \geq 0 \).

Finally, consider again the special case where \( \alpha = \alpha^* = 1, \theta = \theta^* = 1 \) and where countries are identical in the sense that \( E_{pr}^* = E_{pr}, E_{pp} = E_{pp}^*, E_{r_r}^* = E_{r_r}, E_{r_r}^* = E_{r_r} \) etc. In this case we have that \( A_{\tau\theta} = -E_{pp}[E_r E_{pr} + \delta E_{r_r} E_{p} \Delta^{-1}] \) and \( B_{\tau\theta} = -E_{pr} E_{pp}[E_r + E_p E_{r_r} \Delta^{-1}] \). Substituting these expression into equations (23) and (24), we get the effect of changes in \( \theta \) on the Nash consumption taxes in both countries as follows

\[ \Omega \frac{d\tau^N}{d\theta} = E_{pp}^2 [\delta E_{r_r} E_{p} + \Delta E_{r} E_{pr} - E_{p} E_{r_r} E_{pp} \Delta^{-1}] \]  \hspace{1cm} (25)
\[ \frac{\partial \tau^N}{\partial \theta} = E_{pr}^2 [E_{pr} (E_{rr} E_p + \Delta E_r) + E_p E_{rr}^2 E_{pp} \Delta^{-1}] \] (26)

A decrease in the \( \theta \) has an ambiguous effect on both Nash consumption tax rates. Sufficient condition for the decrease in \( \theta \) to decrease Home’s Nash consumption tax is that \( E_{pr} > 0 \). The effect on Foreign’s Nash tax still remains ambiguous.\(^9\) If \( E_{pr} \) is positive, then both \( A_{\tau \theta} \) and \( B_{r \tau, \theta} \) are positive, and a decrease in \( \theta \) moves downwards the Home and Foreign reaction functions. The new Nash equilibrium could result in lower consumption taxes for both countries or in a lower for Home and a higher for Foreign. If \( E_{pr} \) is negative but sufficiently small in absolute terms so that \( A_{\tau \theta} \) is still positive, then the decrease in \( \theta \) moves downwards the Home reaction function and upwards the Foreign resulting in a new Nash equilibrium with a lower tax rate in Home and a higher tax in Foreign.

**Proposition 2** Consider two countries and assume the existence of consumption generated cross-border pollution between them.

- An increase in pollution per unit of consumption in Home, increases that country’s Nash consumption tax and decreases it in Foreign if pollution and the polluting good are strong substitutes and \( \theta \) is close to one so that \( B_{r \tau, \alpha} < 0 \).

- An increase in the rate of cross-border pollution into Home, increases the Nash consumption tax in Home and decreases it in Foreign if i) \( E_{pr} \) is negative and ii) \( E_{pr}^{*r} \) is either positive or has sufficiently small negative value so that \( A_{\tau \theta} > 0 \).

### 4 Cooperative Equilibrium

In this section we consider the case where each country chooses its consumption tax rate in order to maximize the joint welfare of both countries. That is

\(^9\)Note that in this special case where \( \theta = 1 \), i.e., the rate of cross-border pollution has its maximum value, we consider a reduction of its rate.
\[
\frac{\partial u}{\partial \tau} + \frac{\partial u^*}{\partial \tau} = A_t + B_t = 0 \quad \text{and} \quad \frac{\partial u^*}{\partial \tau^*} + \frac{\partial u}{\partial \tau^*} = B_{t^*} + A_{t^*} = 0 \quad (27)
\]

Using equation (9) and (10) we get that

\[
\begin{bmatrix}
\delta^* \\
\alpha^* \theta E_{pr}^a
\end{bmatrix}
\begin{bmatrix}
\tau^c \\
\tau^{c*}
\end{bmatrix}
= \begin{bmatrix}
\alpha (HE_r + \theta^* E^*_{r^*}) \\
\alpha^* (H^* E^*_{r^*} + \theta E_r)
\end{bmatrix} \quad (28)
\]

From equations (28) we get the cooperative consumption tax for each country as follows:

\[
(\Delta / \delta) \tau^c = \alpha [(HE_r + \theta^* E^*_{r^*}) - \alpha^* \theta^* E^*_{r^*} (H^* E^*_{r^*} + \theta E_r) \delta^{-1}] \quad (29)
\]

\[
(\Delta / \delta^*) \tau^{c*} = \alpha^* [(H^* E^*_{r^*} + \theta E_r) - \alpha \theta E_{pr} (HE_r + \theta^* E^*_{r^*}) \delta^{*-1}] \quad (30)
\]

### 4.1 Comparison of the Nash and Cooperative Equilibrium

It is well established that, in the presence of a negative international externality - - cross-border pollution, cooperative taxes are higher than Nash taxes, since they internalize its damaging impact to all parties involved. Now, in the context of our model, we are set to identify the cases where the cooperative consumption taxes are indeed higher than the Nash taxes, but we also look for the cases where it is possible that cooperative taxes are lower than Nash taxes.

From equation (13) Home’s Nash consumption tax rate can be re-written as

\[
\delta^* \tau^N = \alpha HE_r \quad (31)
\]

Note that \((\Delta / \delta) = \delta^* - (\alpha \alpha^* \theta \theta^* E_{pr} E^*_{r^*}) \delta^{-1}\). Thus if \(E_{pr}\) and \(E^*_{r^*}\) have the same sign, then \((\Delta / \delta) < \delta^*\). Comparing equations (29) and (31), we observe that if \(E^*_{r^*} \leq 0\), then always the cooperative consumption tax
is greater than the Nash consumption tax rate. Intuitively, when Home increases the consumption tax, its consumption and pollution is reduced and thus the level of pollution transmitted into Foreign falls. Since in Foreign pollution and the consumption of the polluting good are substitutes, the country’s consumption of this good increases, thus raising its pollution. Since Home cares also about the welfare of Foreign, it raises its consumption tax more in order for even less pollution to be transmitted into Foreign. In the case where no pollution is transmitted into Foreign (i.e., \( \theta^* = 0 \)), then always and irrespectively of the size of \( \theta \), \( \tau^c = \tau^N \). When \( \theta = 0 \) and \( \theta^* \neq 0 \), then \( (\Delta/\delta) = \delta^*, \delta^* = H, \) and \( \delta = H^* \). In this case \( \tau^c = \alpha E_r + \alpha \theta^* E_r^* \). That is, the cooperative consumption tax is always greater than the Nash consumption tax.

In the case where pollution and the consumption of the polluting good are complements in consumption in Foreign (i.e., \( E_{pr,r}^* > 0 \)), then it is possible that the cooperative consumption tax rate is smaller than the corresponding Nash tax rate. This is more likely to occur when \( \theta, \alpha^* \) and \( E_{pr,r}^* \) are large. Intuitively, in this case when Home increases its consumption tax, local consumption and pollution fall, less pollution is transmitted into Foreign and since pollution and the polluting good are complements in consumption, consumption and pollution in Foreign is reduced a lot since \( \alpha^* \) and \( E_{pr,r}^* \) are large. Thus, a lot less pollution is transmitted to Home. In this case it is possible that the cooperative consumption tax is lower than the Nash tax.

Consider, for example, the special case where there is perfect transboundary pollution (i.e., \( \theta = \theta^* = 1 \)). In this case \( H = H^* = 1 \) and \( \Delta = 1 - \alpha^* E_{pr,r}^* - \alpha E_{pr} \) and it can be shown that \( \tau^c = \alpha (E^c_{r*} + E_r) \) and the difference between the cooperative and Nash equilibrium consumption tax is

\[
\tau^c - \tau^N = \alpha (E^c_{r*} - \delta^{*-1} E_r \alpha^* E^*_{pr,r*})
\]  

(32)

It is clear from the above expression that when pollution and the polluting good are complements in consumption in Foreign (i.e., \( E_{pr,r}^* > 0 \)), that the cooperative tax in Home can be lower than its Nash tax. This outcome is more likely when \( \alpha^* \) and \( E_{pr,r}^* \) are large and \( E_r \) is much larger than \( E^*_r \).
Proposition 3 Consider two countries and assume the existence of consumption generated cross-border pollution between them

- If in Foreign, pollution and the polluting good are substitutes or independent in consumption, then the Home cooperative consumption tax is always greater than its corresponding Nash rate.

- If in Foreign, pollution and the polluting good are complements in consumption, then the Home cooperative consumption tax can be lower than its corresponding Nash rate.

4.2 Comparison of the Cooperative Consumption Taxes of the Two Countries

Now we examine cases where although each country cares also about the other country’s welfare, and thus choose their consumption tax so as to maximize their joint welfare, the optimal policy, however, calls for different consumption tax rates. We consider two cases, i) when the two countries differ in the rate of cross border pollution, and ii) when they differ in the rate of pollution per unit of consumption.

4.2.1 Differences in the rate of cross border pollution

First, consider the case where the two countries are identical in some respects. That is \( E_r^* = E_r \), and \( \alpha^* = \alpha \), but \( \theta = 0 \) and \( \theta^* \neq 0 \). In this case \( \delta^* = H, \delta = H^*, (\Delta/\delta) = \delta^* \) and \( (\Delta/\delta^*) = \delta \). Under these assumptions, equations (29) and (30) give that \( \tau^{*c} = \alpha^* E_r^* \) and \( \tau^c = \alpha E_r + \alpha \theta^* E_r^* = \alpha E_r (1 + \theta^*) \). Thus, even though the two countries are the same in some respects, and choose their policy to maximize their joint welfare, the country that transmits pollution into the other country will choose higher cooperative consumption tax. For example, if \( \theta^* = 0.5 \), then its cooperative consumption tax will be 50% higher that the corresponding one of the other country.\(^{10}\) The intuition is

\(^{10}\)This can be the case where we have only downstream transmission of pollution. That is, the pollution is generated in one country and is transmitted into the other either with water (i.e., rivers) or with air if the wind is usually coming from one direction.
simple. Since consumption at Home pollutes Foreign, when Home decides on its optimal consumption tax, its has to take into account the damage that it causes to Foreign.

### 4.2.2 Differences in the rate of pollution per unit of consumption

Next, consider the case where the two countries differ in the pollution per unit of consumption. Assume for example that $\theta = \theta^* = 1$. In this case $H^* = H = 1$. The optimal cooperative policy for Home calls for a consumption tax $\tau_c^H = \alpha(E^*_r + E_r)$, while for Foreign it calls for a tax $\tau_c^F = \alpha^*(E^*_r + E_r)$. Thus, given everything else, we can say that the country with a low rate of pollution per unit of consumption imposes a small cooperative consumption tax while the country with a high rate of pollution per unit of consumption imposes a high cooperative consumption tax.\(^{11}\)

**Proposition 4** Consider two countries which cooperate in their choice of consumption taxes, and assume the existence of consumption generated cross-border pollution between them.

1. If the two countries differ only in the rate of cross border pollution, then the country that transmits pollution into the other chooses a higher cooperative consumption tax rate.

2. If the two countries differ only in the rate of pollution per unit of consumption, then the country with the higher rate of pollution per unit of consumption chooses a higher cooperative consumption tax.

A common result of the tax competition literature is that the first best allocation is achieved by harmonizing the tax policies in the two countries. In the present model, however, with cross border pollution, this may not be achieved by coordinating only consumption taxes. Coordination in other

\(^{11}\)Let petrol be the polluting good. One country can have relatively low pollution per unit of consumption if its residents, responding to strict legislation or incentives, use cars with efficient engines (relatively new and well maintained cars) and efficient burners (new and well maintained) to heat their homes.
policies may also be needed in order to achieve the first best. For example, if the pollutions per unit of consumption in the two countries are different, then measures have to be taken in both countries in order to equalize these and then bring consumption taxes towards uniformity.\footnote{Cremer and Gahvari (2005) using a two country model with transboundary pollution generated from production, concluded that harmonizing the commodity taxes above their Nash rates while leaving the Nash emission taxes unaffected, may increase pollution and reduce welfare.}

5 Concluding Remarks

This paper builds a two-good, two-small open economies model. The consumption of good 2 in each country generates pollution which is transmitted across borders and affects negatively consumers in both countries. Within this model, we find among other things, that Home Nash equilibrium consumption tax rate is affected by the rates of cross-border pollution into both, by the rate of pollution per unit of consumption in both countries and depends also on the relationship in consumption between pollution and the polluting good in Foreign (i.e., complements or substitutes). The effect of an increase in Home’s rate of pollution per unit of consumption, or its rate of cross-border pollution, has in general an ambiguous effect on both countries Nash consumption tax rates. These effects depend, among other things, on the initial rates of pollution per unit of consumption, on the rates of cross border pollution and on the relationship in consumption between pollution and the polluting good in both countries. The paper identify cases where unambiguous results can be obtained.

The paper derives the cooperative equilibrium consumption taxes and compares them with those of the Nash equilibrium. For example, if pollution and the polluting good are substitutes or independent in consumption in one country, then the cooperative consumption tax of the other country is greater than its Nash tax rate. If, however, pollution and the polluting good in one country are complements in consumption, then the cooperative consumption tax of the other country can be lower compared to its Nash consumption tax.
Finally, even if two countries may differ only in the rate of pollution per unit of consumption, or in the rates of cross-border pollution, then their cooperative consumption taxes can be different. This last result may carry important policy implications, particularly within the context of tax harmonization and economic integration. For example, within the common market in EU, consumption taxation is effectively “origin” based (pay taxes in the location of purchase). As a result, when producer prices are the same and there is cross-border shopping, consumers would purchase the products from the country where consumption taxes are lower. This in effect makes a necessity the consumption tax harmonization and equalization of consumer prices across countries when they move to deeper stages of economic integration. Yet, the present analysis concludes that in the presence of an international externality, e.g., cross-border pollution, when countries differ with respect to some of their characteristics, e.g., the rates of pollution per unit of consumption, then tax harmonization may not be the optimal policy. In this case more actions may be needed for the tax harmonization to be optimum. For example, before the countries decide on harmonizing their consumption taxes they should try to harmonize their legislation and take other harmonizing measures in order for the pollution per unit of consumption in all the countries to be the same.

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