

# Productivity and Exporting Dynamics in the Face of Trade Liberalization

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## Abstract

This paper analyzes firms' exporting and productivity dynamics in the face of trade liberalization. I motivate an empirical model where firms productivity development endogenously depends on its exporting decisions and the degree of trade liberalization. At the same time, firm-level productivity is an important determinant for a firm's export status due to fixed and sunk costs of exporting. The model is estimated structurally, where the static demand and production function parameters are estimated using the algorithm of Akerberg, Caves and Frazer (2006), while the dynamic exporting cost parameters are estimated using Bajari, Benkard and Levin's (2007) method for dynamic discrete choice models. The model is applied to data of Danish textile producing firms which experienced a phase of trade liberalization during the period 1993-2004. The estimation results indicate that both selection effects and learning by exporting are important phenomena. Moreover, trade liberalization positively influences firms' productivity trajectories and thereby reinforces selection into exporting.

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# 1 Introduction

In this paper I analyze exporting and productivity dynamics in the face of trade liberalization for Danish textile producers. The textile sector was facing a phase of strong trade liberalizations in the 1990ies until 2004 when the EU abolished all quota restrictions.<sup>1</sup> Existing studies show that this phase of trade liberalization had positive productivity effects for EU textile firms most likely related to an increased competitive pressure. De Loecker (2011) shows this relationship for Belgium textile producers. I use a model similar to De Loecker (2011) to analyze the productivity effects for Danish firms and additionally introduce the export market to this model. In particular, I allow a firm's exporting decision to affect firm-level productivity in a way similar to the model of Aw et al. (2011); i.e. exporting may endogenously affect a firm's productivity while firm-level productivity is an important determinant for a firm's exporting decision due to sunk entry and fixed costs of exporting. Moreover, due to its effect on productivity, trade liberalization may also indirectly affect selection into exporting.

I estimate this model structurally. The model can be divided into a static and a dynamic decision problem. The static part relates to estimating firm level productivity and industry-specific demand elasticities. I draw from the insights of Klette and Griliches (1996) and De Loecker (2011) while using the estimation algorithm of Akerberg et al. (2006) to obtain unbiased estimates of productivity and demand parameters. The dynamic part of the model involves the estimation of firm exporting decisions. This part of the decision problem is dynamic as firms have to pay sunk export market entry costs and fixed exporting costs. This paper therefore relates to recent studies in international trade that apply techniques for structurally estimating dynamic discrete choice models to firms' exporting decisions (Das et al., 2007; Kasahara and Lapham, 2008; Aw et al., 2011). I depart from these models by using a different estimation algorithm. In particular, I use the estimation routine suggested by Bajari et al. (2007, henceforth BBL). The estimation approach consists of two steps and is based on a simple rationale. In the first step of this approach, non-parametric estimates of the data are obtained describing how firms behave

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<sup>1</sup>Note that the EU reintroduced / extended certain quota restrictions even after 2004.

at every state. In the second step, the equilibrium conditions from the underlying model are imposed to uncover why firms behave in the observed way.

The rest of the paper is structured as follows. In the next section I outline the most important parts of the empirical model. Section 3 then describes the data and presents some reduced-form evidence on the relationships between trade liberalization and productivity as well between exporting and productivity. In section 4 I then present the estimation approach while section 5 contains the results. Section 6 concludes.

## **2 An empirical model of firm export participation**

In the following I present a model of firm level export participation and endogenous productivity dynamics in the face of trade liberalization. Following Das et al. (2007), I assume that firms engage in monopolistic competition both in the domestic and foreign market. I assume that marginal production costs are constant and that shocks to the domestic demand do not influence the optimal level of exports (i.e marginal costs are not affected by output shocks). Firms' demand schedules vary by industry, while firms are heterogenous in terms of marginal production costs implying varying export profit trajectories across firms. Moreover, as standard in these models, I focus on the decision to export and firms' productivity dynamics and abstract from the decision to enter or exit production. A firm's decision problem can be distinguished into a static and a dynamic problem. The general model setup follows Aw et al. (2011) in several ways, while I depart by allowing for a more complex production function, by estimating industry-specific demand elasticities, and by a different estimation approach both with respect to the static and the dynamic decision problem. On the other hand, I do not consider R&D investments in this study as they do.

### **2.1 Static decision**

A firm  $i$  faces a standard Cobb-Douglas production function where a unit of output  $Q_{it}$  depends on the choice of the inputs capital ( $K_{it}$ ), labor ( $L_{it}$ ), and materials ( $M_{it}$ ) as well

as a productivity shock ( $\omega_{it}$ ) and an error term ( $u_{it}$ ) capturing measurement error and idiosyncratic shocks to production.

$$Q_{it} = K_{it}^{\alpha_k} L_{it}^{\alpha_l} M_{it}^{\alpha_m} \exp(\omega_{it} + u_{it}). \quad (1)$$

On the demand side, the firm faces a standard CES demand system with different substitution patterns by industry  $d$ .

$$Q_{it} = Q_{dt} \left( \frac{P_{it}}{P_{dt}} \right)^{\eta_d} \exp(\xi_{jt}). \quad (2)$$

Hence, the demand for firm  $i$  depends on its own price ( $P_{it}$ ), the average price in the industry ( $P_{dt}$ ), an aggregate demand shifter ( $Q_{dt}$ ) and unobserved, sub-segment  $j$  specific demand shocks ( $\xi_{jt}$ ). The industry level  $d$  is the 2-digit NACE industry classification, as this is the most detailed level for which I can obtain industry prices  $P_{dt}$ . Klette and Griliches (1996) and De Loecker (2011) show how this demand system together with the standard Cobb-Douglas production function can be exploited to deal with the omitted price variable bias related to a potential correlation of inputs with omitted prices. In particular, we can solve equation (3) for the price  $P_{it}$  and then plug this term together with equation (1) into the expression for firm revenue  $R_{it} = P_{it}Q_{it}$ . Taking the logs of the resulting equation and defining log deflated revenue by  $\tilde{r}_{it} = r_{it} - p_{dt}$  (where lower case letters indicate logs) results in the following expression:

$$\tilde{r}_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \sum_d \beta_d q_{dt} + \omega_{it}^* + \xi_{jt}^* + u_{it} \quad (3)$$

Note that the coefficients here are reduced form coefficients as indicated by the use of  $\beta$  instead of  $\alpha$ . In particular,  $\beta_h = \left( \frac{\eta_d + 1}{\eta_d} \right) \alpha_h$  for  $h = k, l, m$ . Similarly, the unobserved productivity shock is pre-multiplied by  $\left( \frac{\eta_d + 1}{\eta_d} \right)$  leading to  $\omega^*$  and the unobserved demand shock is pre-multiplied by  $|\eta_d|^{-1}$  leading to  $\xi^*$ , where  $\frac{1}{|\eta_d|} = \beta_d$  is the inverse of the segment specific demand parameter and  $q_{dt}$  refers to segment level output. To be precise,  $q_{dt}$  is the market share weighted sum of segment-level output computed as  $q_{dt} = \sum_{i=1}^{N_d} m_{s_{idt}} \ln R_{it}$  where  $N_d$  denotes number of firms in industry  $d$ ,  $m_{s_{idt}}$  is firm  $i$ 's market share in industry

$d$  and  $R_{it}$  is firm  $i$ 's revenue (Klette and Griliches, 1996). As indicated in equation (3),  $q_{dt}$  is expanded to  $\sum_d \beta_d q_{dt}$  in order to estimate the equation jointly for all segments where  $d_{id}$  is a dummy variable switching on if firm  $i$  belongs to industry  $d$ .

Equation (3) will be used to obtain estimates of firm productivity and sector-specific demand elasticities which are required to model firm's export participation decision. I should note that the demand elasticities are obtained from a firm's total output implying that these elasticities are averages across the domestic and foreign markets. I use these elasticities also to model the export market. Note that I do not observe exporting profits directly in the data which is why I rely on a link between exporting profits and exporting revenue as suggested by Das et al. (2007). These authors show that under the broad assumption of profit maximizing firms, the following relationship holds:

$$\pi_{it}^* = \frac{1}{\eta_d} R_{it}^X, \quad (4)$$

where superscript  $X$  indicates the export market. Hence, given the estimated demand elasticities, I can use a firm's exporting sales to model exporting profits. Further note that given the assumption of constant marginal costs,  $R_{it}^X$  depends on the same firm-specific arguments as domestic (or total) output. In the dynamic empirical model of firm export participation, I follow Aw et al. (2011) and focus on the use of capital, besides firm productivity, as firm characteristics and neglect labor and materials. I think that this approach is warranted when considering that a firm's use of labor, capital, and materials are highly positively correlated<sup>2</sup> and the large additional computational burden that arises from also conditioning on materials and labor in the dynamic optimization problem. Moreover, I assume that productivity  $\omega_{it}$  has an average effect of magnitude  $\gamma_\omega$  on exporting profits. Gross exporting profits are therefore given by

$$\pi_{it}^* = r_{it}^X - \ln \eta_d = \gamma_k k_{it} + \gamma_\omega \omega_{it} + u_{it}^X, \quad (5)$$

where  $\gamma_h$  ( $h = k, \omega$ ) are unknown parameters, and  $u_{it}^X$  is an error term.

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<sup>2</sup>The correlation coefficients lie between 0.76 and 0.83.

## 2.2 Transition of the State Variables

Before describing the firm's dynamic optimization problem, I need to specify how the state variables transition over time. A standard assumption for productivity is that it evolves over time as a Markov process. As noted before, firm-level productivity is allowed to be endogenously affected by the firm's export participation and by trade liberalization. I therefore specify the law of motion of productivity as follows

$$\omega_{it} = g(\omega_{it-1}, e_{it-1}, qr_{st-1}) + v_{it} \quad (6)$$

where  $e_{it-1}$  is an indicator variable of firm  $i$ 's export market participation in year  $(t-1)$ ,  $qr_{st-1}$  measures the one year lagged degree of protection of sub-segment  $j$  in which firm  $i$  is active and  $v_{it}$  is an iid shock which accounts for the stochastic nature of the productivity process. Note that including  $e_{it-1}$  in  $g(\cdot)$  allows for the possibility of learning by exporting. Furthermore, I assume that capital follows a first order Markov process conditional on lagged export status and quota protection follows an exogenous first order markov process. Finally, note that I have to discretize all variables when modeling the dynamic decision process. I do so by grouping the variables into five categories according to their percentile distributions.

## 2.3 Dynamic Decision

The exporting decision has to be modeled in a dynamic framework as firms have to pay irreversible sunk costs ( $SC$ ) when entering the export market for the first time. Moreover, firms have to pay per period fixed costs ( $FC$ ) when they export. Gross exporting profits  $\pi_{it}^*(\cdot)$  therefore need to be adjusted by these costs. As standard in the literature of dynamic discrete choice models, I assume that there is a choice specific iid shock  $\varepsilon_{it}(e)$ . Firm  $i$ 's per period flow payoff from exporting is then given by

$$\Pi_{it}(\cdot) = \pi_{it}^*(\cdot) - e_{it-1}\gamma_{FC} - (1 - e_{it-1})\gamma_{SC} + \varepsilon_{it}(e_{it}), \quad (7)$$

where  $\pi_{it}^*(\cdot)$  are gross exporting profits and  $\gamma_{FC}$  and  $\gamma_{SC}$  are the log transformed fixed ( $\ln(FC)$ ) and sunk costs ( $\ln(SC)$ ) of exporting respectively. I assume that a firm's decision process is Markovian, implying that firms only condition on the current state vector and their private shocks when deciding whether to export (see e.g. Ryan, 2011). In particular, each firm's strategy  $\sigma_i(\mathbf{s}, \varepsilon_i)$  is a mapping from states and shocks to actions:  $\sigma_i : (\mathbf{s}, \varepsilon_i) \rightarrow e_i$ , where the vector  $\mathbf{s}$  contains all observable state variables. Further, given an infinite horizon, bounded profits and a discount factor  $\delta$  below unity, firm  $i$ 's expected exporting profits in a given state can be written recursively as:

$$V_i(\mathbf{s}; \boldsymbol{\sigma}) = E_{\varepsilon}[\Pi_i(\boldsymbol{\sigma}(\mathbf{s}, \boldsymbol{\varepsilon}), \mathbf{s}, \varepsilon_i) + \delta \int V_i(\mathbf{s}'; \boldsymbol{\sigma}) dP(\mathbf{s}' | \boldsymbol{\sigma}(\mathbf{s}, \boldsymbol{\varepsilon}), \mathbf{s}) | \mathbf{s}], \quad (8)$$

where bold letters indicate vector notation; i.e.  $\mathbf{s}$ ,  $\mathbf{e}$ , and  $\boldsymbol{\varepsilon}$  are vectors of observed states, actions, and private shocks respectively. Moreover,  $V_i$  is firm  $i$ 's value function; to be precise,  $V_i$  is firm  $i$ 's ex ante value function by reflecting expected profits before private shocks are realized. Given the above assumptions, in equilibrium we have that  $V_i(\mathbf{s}; \boldsymbol{\sigma}) \geq V_i(\mathbf{s}; \boldsymbol{\sigma}')$  implying that the profile  $\boldsymbol{\sigma}$  is a Markov perfect equilibrium (Bajari et al., 2007).

## 3 Data

### 3.1 Data Sources and Summary Statistics

The analysis is based on firms in textile producing industries (2-digit NACE codes 17 and 18) that were located in Denmark during the period 1993-2004. Most of the data are sourced from Denmark Statistic providing firm-level accounting data and customs data on the firm's international trading activities.

Furthermore, I merge data from the EU commission on import quotas to my data set. The EU commission provides online information on the implementation of quotas for textile products by foreign market for all years since 1993.<sup>3</sup> I construct a protection

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<sup>3</sup>The quota data can be downloaded from <http://trade.ec.europa.eu/sigl/querytextiles.htm>

variable  $qr_{jt}$  which varies by sub-segment  $j$  and by time  $t$  using this data on import quotas. As in De Loecker (2011), the average protection that applies to a product  $p$  in year  $t$  is given by

$$qr_{pt} = \sum_c a_{ct} qr_{cpt}$$

where  $qr_{cpt}$  is a dummy variable which is equal to 1 if the EU imposes a quota on imports of product  $p$  from country  $c$  in year  $t$  and  $a_{ct}$  is a weight of the supplying country  $c$  in year  $t$ . I use a country's GDP as weight as this is an indication for a country's production potential. To obtain a sub-segment specific protection variables, I then take the simple average of  $qr_{pt}$  across all products belonging to sub-segment  $j$ . Figure 1 presents plots of this variable for each 4-digit industry over time. The reduction in magnitude of this variable over time indicates the trade liberalization that firms in the textile producing sectors where facing. Note that the EU commission lists the quotas by product category and foreign market for which the quota applies. The products are classified into special categories which are linked to 8-digit CN codes by a correspondence table. I use the correspondence between 8-digit CN codes and the prodcom classification to merge the protection data by 4-digit NACE industry to my firm-level data set.

Table 1 provides summary statistics on the variables used in the empirical analysis. Moreover, table 2 presents the number of firms and their trading status by sub-segment  $j$  (4-digit NACE category).

### 3.2 A First Look at the Data

To motivate the consecutive analysis, I present some initial evidence on the relationship between exporting and productivity and trade liberalization and productivity. First, in table 3, I present the trade premium of exporting firms relative to non-exporting firms in terms of labor productivity. In particular, I regress the log of value added per employee on a dummy variable indicating firms' export participation. In column (i) I control for sub-segment industry dummies and obtain a trade premium of 49%. In column (ii) I consider time series evidence of the trade premium by estimating a firm fixed effect specification. The coefficient on the exporting dummy is again positive and



significant while it - unsurprisingly - reduces in magnitude. In columns (iii) and (iv) I repeat these estimations using the estimated TFP. While the coefficients are smaller compared to those in columns (i) and (ii), the conclusions are similar. Table 3 therefore suggests a relationship between exporting and productivity while it does not allow to make statements about the direction of causality. The structural model presented in the next section allows to draw conclusions about this issue.

In table 4 I next regress log labor productivity on the protection variable while I again first consider sub-segment fixed effects (column i) and then firm fixed effects (column ii). In either case a negative and significant effect is obtained. In columns (iii) and (iv) I again repeat the estimations using TFP as proxy for productivity. As before, the coefficients reduce in magnitude. It is important to stress that the negative sign is expected here as more protection implies a higher value for the variable  $qr_{jt}$ ; i.e. firm productivity and trade liberalization are positively correlated.

## 4 Estimation

### 4.1 Estimation of Productivity and Demand Elasticities

As mentioned before, I use equation (3) to estimate firm productivity and sector-specific demand elasticities. I assume that the unobserved, sub-segment specific demand shock  $\xi_{jt}$  can be decomposed into the following components:

$$\xi_{jt} = \xi_j + \xi_t + \tau qr_{jt} + \tilde{\xi}_{jt} \quad (9)$$

$\xi_j$  are sub-segment-specific dummy variables,  $\xi_t$  are year fixed effects,  $\tau$  is a parameter to be estimated in front of the sector specific protection variable  $qr_{jt}$  and  $\tilde{\xi}_{jt}$  is an iid demand shock. The resulting estimation equation then looks as follows:

$$\tilde{r}_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \sum_s \beta_d q_{dt} + \sum_j \kappa_j D_j + \tau qr_{jt} + \omega_{it}^* + v_{it}, \quad (10)$$

where  $v_{it} = u_{it} + \tilde{\xi}_{jt}$  captures idiosyncratic shocks to production ( $u_{it}$ ) and demand ( $\tilde{\xi}_{jt}$ ). I collect the sub-segment dummy variable as well as the year fixed effects in  $\kappa_j D = \sum_j \kappa_j D_j$ . Equation (10) can then be estimated using the common approaches to the estimation of production functions. As standard nowadays, I use a control function approach to estimate equation (10). Recent advancements in this literature point towards problems regarding the identification of the labor coefficient in the first stage of estimation routines à la Olley and Pakes (1996) and Levinsohn and Petrin (2003). I therefore use the Akerberg et al. (2006) approach to estimate equation (10) which specifically deals with this issue. In the following, I shortly outline the estimation algorithm.

I follow the insight from Levinsohn and Petrin (2003) and use intermediate inputs  $m_{it}$  as proxy for the unobserved productivity shock  $\omega_{it}$ . As De Loecker (2011) points out in a setting similar to this paper, the important point is to note that a firm's choice of materials is directly related to its productivity, capital stock and all demand variables which influence a firm's residual demand and therefore its optimal input choice. The material demand equation is then given by

$$m_{it} = m_t(k_{it}, \omega_{it}, qr_{jt}, q_{dt}, D, e_{it}). \quad (11)$$

Note that I also include firm  $i$ 's export status into equation (11) to allow for differences in input demand between exporting and non-exporting firms (De Loecker, 2010). De Locker (2011) shows that the monotonicity of input demand in productivity is preserved under the current assumption of monopolistic competition implying that there exist a function  $h(\cdot)$  which can proxy for productivity

$$\omega_{it} = h_t(k_{it}, m_{it}, qr_{jt}, q_{dt}, D, e_{it}). \quad (12)$$

The first step of the algorithm consists of estimating

$$\tilde{r}_{it} = \phi_t(l_{it}, k_{it}, m_{it}, qr_{jt}, q_{dt}, D, e_{it}) + v_{it}, \quad (13)$$

where

$$\phi_t(\cdot) = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_s q_{dt} + \kappa_j D + \tau q r_{jt} + h_t(\cdot). \quad (14)$$

I approximate the function  $h_t(\cdot)$  using a polynomial expansion of order four of it's argument and obtain  $\hat{\phi}_{it}$  and  $v_{it}$ . Hence, the first step allows me to compute productivity for any parameter value as  $\omega(\beta_l, \beta_k, \beta_m, \beta_d, \kappa_j, \tau) = \hat{\phi} - \beta_l l_{it} - \beta_k k_{it} - \beta_m m_{it} - \beta_s q_{dt} - \kappa_j D - \tau q r_{jt}$ . The second stage then relies on the law of motion of productivity (equation 7) in order to obtain the innovation to productivity  $v_{it}(\beta_l, \beta_k, \beta_m, \beta_d, \tau, \kappa_j)$  as the residual from non-parametrically regressing  $\omega(\beta_l, \beta_k, \beta_m, \beta_d, \kappa_j, \tau)$  on its lag and  $q r_{jt-1}$  and  $e_{it-1}$ . I can then specify moments to identify the parameters of interest (see De Loecker (2011) for more details).

$$E \left( v_{it}(\cdot) \begin{pmatrix} l_{it-1} \\ m_{it-1} \\ k_{it} \\ q_{st-1} \\ q r_{jt} \\ D \end{pmatrix} \right) = 0 \quad (15)$$

Note that I do not consider interactions between the sub-segment dummies and other variables, but instead include  $D$  in the non-parametric regression of  $\omega_{it}$  on  $\omega_{it-1}$ ,  $q r_{jt-1}$ , and  $e_{it-1}$  as suggested by de Loecker (2011). Finally, the sample analog of of equation (15) is given by

$$\frac{1}{T} \frac{1}{N} \sum_t \sum_i v_{it}(\beta) \begin{pmatrix} l_{it-1} \\ m_{it-1} \\ k_{it} \\ q_{st-1} \\ q r_{jt} \\ D \end{pmatrix} \quad (16)$$

which is minimized by GMM.

## 4.2 Estimation of Exporting Costs

The dynamic decision problem is estimated using Bajari, Benkard and Levin's (BBL, 2007) algorithm for dynamic discrete choice models. The algorithm is based on the above assumption of a Markov perfect equilibrium, i.e. each firm's behavior only depends on the current state and its current private shock. The estimation approach belongs to the class of conditional choice probability estimators (CCP) (Hotz and Miller, 1993) and specifically relates to Hotz et al. (1994) by making use of forward simulation. The algorithm consists of two parts; first the policy function and the law of motion for the state variables are estimated. In the second step, the structural parameters are estimated using the optimality condition for equilibrium.

In the current setup, the structural parameters of the model are the discount rate  $\delta$ , the profit function  $u_i$ , the transition probabilities of the states, and the distribution of the private shocks. In the following,  $\delta$  is treated as known and the private shocks are assumed to be drawn from a type 1 extreme value distribution. The profit functions are assumed to be known up to a finite parameter vector  $\theta$  and the transition probabilities are estimated directly from the data. The dynamic structural parameters of the profit function are obtained from imposing the equilibrium condition: If firms follow Markov strategies, then in equilibrium  $V_i(\mathbf{s}; \sigma) \geq V_i(\mathbf{s}; \sigma')$ . In the following I outline the estimation details for the two steps of the algorithm.

### 4.2.1 First step

In the first step, I estimate the coefficients of the gross export profit equation, the transition probabilities for the states, and the policy function. The obtained parameters together with estimates of the policy function and transition probabilities are then used to calculate the value function.

## Estimation of gross export equation

The gross export profit equation reflects a static optimization problem implying that its parameters can be estimated in the first step separately. This possibility is ensured by assuming conditional independence between gross profits and the choice specific shocks ( $\varepsilon_{it}(e)$ ) as pointed out by Aguirregabiria and Mira (2010). The conditional independence assumption consists of two parts. First, the unobserved states  $\varepsilon_{it}(e)$  and  $u_{it}^X$  have exogenous transition probabilities implying that their transitions do not depend the firm's current export decision. Second, conditional on the observed state variables  $\mathbf{s}$ ,  $\varepsilon_{it}(e)$  and  $u_{it}^X$  have independent transitions and  $\varepsilon_{it}(e)$  is not serially correlated (Aguirregabiria, 2010).

Despite the conditional independence assumption, estimating the gross export profit equation (5) by OLS would lead to a selection bias because  $u_{it}^X$  is not mean independent of the firm's exporting decision in period  $t$ . I therefore draw from the insight of Aguirregabiria (2010) and make the following two assumptions to address this issue. First, I assume that the error term  $u_{it}^X$  follows a first order Markov process:

$$u_{it}^X = f(u_{it-1}^X) + \zeta_{it}. \quad (17)$$

This assumption is standard. Second, I assume that the innovation  $\zeta_{it}$  is unknown to the firm when it makes the decision to export in period  $t$ . Note that this assumption is significantly weaker than assuming that  $u_{it}^X$  is unknown to the firm when it makes the exporting decision. In particular, this assumption allows the selection of the firm to depend on  $u_{it-1}^X$ . Therefore, if a firm is exporting at periods  $t$  and  $t - 1$ , the estimation equation becomes

$$\pi_{it}^* = \gamma_k k_{it} + \gamma_\omega \omega_{it} + f(\pi_{it-1}^* - \gamma_k k_{it-1} - \gamma_\omega \omega_{it-1}) + \zeta_{it}. \quad (18)$$

Note that given the second of the just stated assumptions, we have that

$$E(\zeta_{it} | k_{it}, k_{it-1}, \omega_{it}, \omega_{it-1}, \pi_{it-1}^*, e_{it} = e_{it-1} = 1) = 0$$

so that equation 18 can be estimated by least squares. In particular, I estimate the model by non-linear least square and approximate the function  $f(\cdot)$  by a polynomial series approximation ( $u_{it}^X = \rho_1 u_{it-1}^X + \rho_2 (u_{it-1}^X)^2 + \zeta_{it}$ ). The state variables in the dynamic programming problem are therefore  $e_{t-1}$ ,  $u_{it-1}^X$ ,  $k_{it}$ ,  $qr_{st}$  and  $\omega_{it}$ . I construct  $u_{it-1}^X$  based on the residuals from equation (18) by first estimating the density of the innovation  $\zeta_{it}$  using a Gaussian kernel as suggested by Aguirregabiria (2010) and then applying the law of motion of  $u_{it}^X$  given by equation (17).

### Estimation of transition probabilities

As mentioned before, the transition probabilities for productivity are estimated from equation (7) and that for  $u_{it}^X$  is given by equation (17). Note that in either case I draw the innovations from a uniform distribution on the interval  $(-1,1)$ . Moreover, I estimate the transition probabilities of the state variables  $k_{it}$  and  $qr_{st}$  using frequency estimators. Finally, the transition of  $e_{t-1}$  over time is given by the firms action in the previous period.

### Estimation of policy function

Given discrete choice data and the assumptions of choice-specific shocks entering additively ( $\Pi_i(\mathbf{e}, \mathbf{s}, \varepsilon_i) = \tilde{\Pi}_i(\mathbf{e}, \mathbf{s}) + \varepsilon_i(e_i)$ ), the choice-specific value function is given by

$$v_i(e_i, \mathbf{s}) = E_\varepsilon[\tilde{\Pi}_i(e_i, \mathbf{s}) + \delta \int V_i(\mathbf{s}'; \boldsymbol{\sigma}) dP(\mathbf{s}'|e_i, \mathbf{s})], \quad (19)$$

so that a firm  $i$  optimally chooses an action  $e_i$  which satisfies

$$v(e_i, \mathbf{s}) + \varepsilon_i(e_i) \geq v(e'_i, \mathbf{s}) + \varepsilon_i(e'_i). \quad (20)$$

This is the policy rule which needs to be estimated in the first stage. The choice specific value function  $v_i(e_i, \mathbf{s})$  can be estimated directly from the data making use of the result from Hotz and Miller (1993) who show that for any two action  $e_i$  and  $e'_i$ ,

$$v(e'_i, \mathbf{s}) - v(e_i, \mathbf{s}) = \ln(\Pr(e'_i|\mathbf{s})) - \ln(\Pr(e_i|\mathbf{s})), \quad (21)$$

where  $Pr(e_i|\mathbf{s})$  is the probability of observing choice  $e_i$  in state  $\mathbf{s}$ . Given that the state space is relatively large, I follow BBL's suggestion and estimate  $Pr(e_i|\mathbf{s})$  as flexibly parameterized functions of the actions and states; in particular, I use a second order polynomial expansion. The estimates then provide us with "cutoffs" for each combination of state variables. These cutoffs are essential for estimating the value function. I denote the cutoffs by  $cut(\mathbf{s})$  to emphasize their dependence on the state variables.

### Estimating the value function

After estimating the transition probabilities and equilibrium policy functions, we want to obtain the equilibrium value functions in order to estimate the structural parameters of the model. BBL suggest to use forward simulation (Hotz et al., 1994) for this purpose following these steps:

1. Starting at state  $\mathbf{s}_1 = \mathbf{s}$ , draw private shocks  $v_{i1}$  from the type 1 extreme value distribution
2. Calculate the specified action  $e_{i1} = \sigma_i(\mathbf{s}_1, \varepsilon_{i1})$  for each firm  $i$ , where  $\sigma_i(\mathbf{s}_1, \varepsilon_{i1}) = \mathbf{1}[\text{cut}(\mathbf{s}_1) + \varepsilon_{i1}(1) - \varepsilon_{i1}(0) > 0]$  with  $\mathbf{1}[\ ]$  being the indicator function (note, I make use of the linearity of the parameters in firms' profits here and calculate profits later).
3. Draw a new state  $\mathbf{s}_2$  using the estimated transition probabilities
4. Repeat steps 1-3 for  $T$  periods

The firm's discounted sum of profits is then averaged over many simulation paths  $n_s$  which yields an estimate for firm  $i$ 's value function denoted  $\hat{V}_i(\mathbf{s}; \boldsymbol{\sigma}; \boldsymbol{\theta})$ . Such an estimate can be obtained for any policy  $(\boldsymbol{\sigma}, \boldsymbol{\theta})$  pair including  $(\hat{\boldsymbol{\sigma}}, \boldsymbol{\theta})$  where  $\hat{\boldsymbol{\sigma}}$  is the estimated policy profile from the first stage.

Note that the parameters enter firms' profit functions linearly implying that  $\Pi_i(\mathbf{e}, \mathbf{s}, \boldsymbol{\varepsilon}; \boldsymbol{\theta}) = \boldsymbol{\Psi}_i(\mathbf{e}, \mathbf{s}, \boldsymbol{\varepsilon})\boldsymbol{\theta}$ , where  $\Psi_i^1 \dots \Psi_i^M$  is an M-dimensional vector of basis functions. The value functions can then be rewritten as:

$$V_i(\mathbf{s}; \boldsymbol{\sigma}; \theta) = E\left[\sum_{t=1}^{\infty} \delta^t \Psi_i(\cdot \boldsymbol{\sigma}(\mathbf{s}_t, \boldsymbol{\varepsilon}_t), \mathbf{s}_t, \boldsymbol{\varepsilon}_{it}) | \mathbf{s}_0 = \mathbf{s}\right] \cdot \theta = \mathbf{W}_i(\mathbf{s}; \boldsymbol{\sigma}) \cdot \theta$$

This can lead to substantial savings in computational time as  $W_i$  does not depend on the unknown parameters  $\theta$ . Hence, for any  $\sigma$ , one can use forward simulation to estimate  $W_i$  and then obtain  $V_i$  for any value and  $\theta$ . In the current setup, this amounts to

$$\begin{aligned} V_i(\mathbf{s}; \boldsymbol{\sigma}; \theta) &= \mathbf{W}^1(\mathbf{s}; \boldsymbol{\sigma}) - \mathbf{W}^2(\mathbf{s}; \boldsymbol{\sigma}) \cdot \gamma_{FC} - \mathbf{W}^3(\mathbf{s}; \boldsymbol{\sigma}) \cdot \gamma_{EC} + \mathbf{W}^4(\mathbf{s}; \boldsymbol{\sigma}) \\ &= E\left[\sum_{t=0}^{\infty} \delta^t \tilde{\Pi}_i(\mathbf{s}) | \mathbf{s}_0 = \mathbf{s}\right] \\ &\quad - E\left[\sum_{t=0}^{\infty} \delta^t \boldsymbol{\sigma}(\mathbf{s}_t, \boldsymbol{\varepsilon}) \boldsymbol{\sigma}(\mathbf{s}_{t-1}, \boldsymbol{\varepsilon}_t) | \mathbf{s}_0 = \mathbf{s}\right] \cdot \gamma_{FC} \\ &\quad - E\left[\sum_{t=0}^{\infty} \delta^t \boldsymbol{\sigma}(\mathbf{s}_t, \boldsymbol{\varepsilon}) (1 - \boldsymbol{\sigma}(\mathbf{s}_{t-1}, \boldsymbol{\varepsilon}_t)) | \mathbf{s}_0 = \mathbf{s}\right] \cdot \gamma_{EC} \\ &\quad + E\left[\sum_{t=0}^{\infty} \delta^t \boldsymbol{\varepsilon}(\boldsymbol{\sigma}(\mathbf{s}_t, \boldsymbol{\varepsilon})) | \mathbf{s}_0 = \mathbf{s}\right] \end{aligned}$$

where the first term is the static profit of firm  $i$  given that the current state is  $\mathbf{s}$ . This term is calculated based on the parameters of the gross export profit equation (5). The second and third terms are the expected present fixed and sunk entry costs of exporting respectively, and the last term is the expected present value of the realized private shocks with  $\boldsymbol{\varepsilon}(\boldsymbol{\sigma}(\mathbf{s}_t, \boldsymbol{\varepsilon}))$  being the shock related to the action chosen at time  $t$ .

#### 4.2.2 Second step

The second step of the algorithm makes use of the equilibrium condition; namely that for all firms  $i$  and states  $s$  and alternative Markov policies  $\boldsymbol{\sigma}'_i$ , in equilibrium the following condition holds:

$$V_i(\mathbf{s}; \boldsymbol{\sigma}; \theta) \geq V_i(\mathbf{s}; \boldsymbol{\sigma}'_i; \theta) \quad (22)$$

This equilibrium condition defines a set of parameters  $\theta$  that rationalize the strategy profile  $\boldsymbol{\sigma}$ . BBL propose a minimum distance estimator to estimate  $\theta$ . Following BBL, I let  $\varphi \in \Xi$  index the equilibrium conditions so that each  $\varphi$  denotes a particular  $(i, \mathbf{s}, \boldsymbol{\sigma}'_i)$



combination. Then, I define

$$g(\varphi, \theta, \alpha) = V_i(\mathbf{s}; \boldsymbol{\sigma}; \theta, \varrho) - V_i(\mathbf{s}; \boldsymbol{\sigma}'; \theta, \varrho).$$

Note that the dependence on  $\varrho$  is due to the fact that  $\boldsymbol{\sigma}$  and  $P$  are parameterized by  $\varrho$ . Next define the function

$$Q(\theta, \varrho) = \int (\min\{g(\varphi; \theta, \varrho), 0\})^2 dH(\varphi).$$

$H$  is a distribution over the set  $\Omega$  of inequalities. BBL point out that the true parameter vector  $\theta_0$  satisfies  $Q(\theta_0, \varrho) = 0 = \min Q(\theta, \varrho)$  and that  $\theta$  can be estimated by minimizing the sample analog of  $Q(\theta, \varrho)$ .

For this purpose, denote  $\Omega_k$  ( $k = 1 \dots n_I$ ) the set of inequalities from  $\Xi$  chosen by the econometrician. BBL suggest to choose firms and states at random and then consider alternative policies  $\sigma'$  which are slight perturbations of the estimated policy  $\sigma$  (e.g.  $\sigma' = \sigma + \epsilon$ ). For each chosen inequality, we then use forward simulation to construct the analog  $V_i$  terms. The suggested estimator then minimizes the following objective function

$$Q(\theta, \varrho) = \frac{1}{n_I} \sum_{k=1}^{n_I} (\min\{\hat{g}(\Omega_k; \theta, \varrho), 0\})^2. \quad (23)$$

## 5 Results

I present the results from estimating the production function in table 5. I present both the scaled parameter estimates  $\beta$  and the true technology parameters  $\alpha$ . These parameters are of common magnitude and imply slightly increasing returns to scale. Moreover, table 5 contains the coefficients on industry output ( $\beta_s$ ) whose inverse represent demand elasticities. The elasticities for the NACE two digit sectors 17 and 18 amount to 8.23 and 7.75 and are therefore fairly similar. The implied markups that firms charge over marginal production costs are then 15% and 14% respectively.

In table 6 I next present the estimation results for the law of motion of productivity

(equation 7). I present results based on continuous and discrete data; the results for the discrete data are the estimates that I use to model the law of motion of productivity in the dynamic part of the model. A couple of observations are worth noting here. First, as expected, firm productivity developments are very persistent as indicated by the large coefficient on lagged productivity. Second, I present evidence for learning by exporting. Firms that exported in the previous year are on average 2.2% more productive than non-exporting firms. Note that this estimate can accumulate to sizeable productivity effects in the long run. Finally, I find that trade liberalization indeed positively impacts firm productivity. The coefficient estimate suggests that removing all quota protection would lead to an average increase in firm productivity of 6.3%.

I next turn to the results for the export participation equation. In table 7 estimation results for the gross export profit equation are presented. As expected, larger firms in terms of capital and more productive firms have significantly higher export sales. Note in particular, the size of the productivity coefficient which indicates the important role of productivity for exporting. Moreover, the results show that the assumption of a first order Markov process is warranted given its persistency which is indicated by  $\rho_1$  and  $\rho_2$ . Note that I use these estimates to model the transition process of  $u_{it}^X$  over time.

Finally, in table 8 I present the estimation results for the dynamic parameters. First, note that these parameters are estimated very precisely. Moreover, as I am estimating a structural model, I can interpret these parameters in monetary values. The parameters suggest that sunk entry costs of exporting are indeed substantial amounting on average to 638.000 Euros. On the other hand, the estimates suggest that fixed costs are relatively low amounting to only 10.000 Euros on average.

## 6 Conclusion

This paper analyzes firms' exporting and productivity dynamics in the face of trade liberalization. I motivate an empirical model where firms productivity development endogenously depends on its exporting decisions and the degree of trade liberalization. At the same time, firm-level productivity is an important determinant for a firm's export status

due to fixed and sunk costs of exporting. The model is estimated structurally, where the static demand and production function parameters are estimated using the algorithm of Akerberg, Caves and Frazer (2006), while the dynamic exporting cost parameters are estimated using Bajari, Benkard and Levin's (2007) method for dynamic discrete choice models. The model is applied to data of Danish textile producing firms which experienced a phase of trade liberalization during the period 1993-2004. The estimation results indicate that both selection effects and learning by exporting are important phenomena. Moreover, trade liberalization positively influences firms' productivity trajectories and thereby reinforces selection into exporting.

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## Tables

Table 1: Summary Statistics

	Mean	St.Dev.	Min	Max
Log Sales	16.24	1.48	9.90	20.53
Log Capital	14.80	1.63	8.52	20.23
Log Labor	2.80	1.19	0.00	6.01
Log Materials	15.65	1.57	9.39	20.02
Protection	0.29	0.18	0.00	0.70
Export Dummy	0.72	0.45	0.00	1.00
Log Export Sales	14.77	2.51	6.48	20.35

Table 2: Export Status by Sub-segment

Industry	Export Dummy		Total
	0	1	
1710	18	46	64
1720	38	67	105
1730	69	46	115
1740	235	500	735
1751	21	103	124
1752	75	140	215
1754	24	57	81
1760	20	94	114
1771	4	30	34
1772	23	186	209
1821	25	81	106
1822	140	295	435
1823	15	187	202
1824	64	164	228
Total	771	1,996	2,767

## Figures

Table 3: Trade Premium

Dependent variable	Labor Productivity		TFP	
Export Dummy	0.492*** (0.026)	0.148*** (0.035)	0.153*** (0.008)	0.054*** (0.013)
Sub-Segment Dummies	yes		yes	
Firm Fixed Effects		yes		yes

Table 4: Productivity and Trade Liberalization

Dependent variable	Labor Productivity		TFP	
Protection Variable	-1.543*** (0.243)	-1.236*** (0.165)	-0.581*** (0.074)	-0.382*** (0.06)
Sub-Segment Dummies	yes		yes	
Firm Fixed Effects		yes		yes

Table 5: Production Function Parameters

	Reduced Form Coefficients		Implied Coefficients
	$\beta_l$ 0.126** (0.06)	$\alpha_l$	0.142
	$\beta_m$ 0.684*** (0.039)	$\alpha_m$	0.770
	$\beta_k$ 0.116*** (0.013)	$\alpha_k$	0.130
	$\beta_s$ -0.121** (0.056)	$\eta_{17}$	8.231
(NACE 17)			
	$\beta_s$ -0.129* (0.07)	$\eta_{18}$	7.750
(NACE 18)			
RTS			1.042

Block-bootstrapped standard errors in parenthesis

Table 6: Law of Motion Productivity

	Continuous Data	Discrete Data
$\omega$ (t-1)	2.530** (1.009)	0.791*** (0.066)
$\omega^2$ (t-1)	-0.176* (0.1)	-0.015 (0.011)
$e$ (t-1)	0.022** (0.009)	0.258*** (0.043)
$qr_{st}$ (t-1)	-0.063*** (0.015)	-0.055*** (0.013)
Constant	-3.213 (2.535)	0.816*** (0.104)

Clustered standard errors in parenthesis

Table 7: Gross Export Profit Equation

	Discrete Data
$k$	0.299*** (0.036)
$\omega$	1.511*** (0.118)
$\rho_1$	0.870*** (0.018)
$\rho_2$	0.022*** (0.006)

Clustered standard errors in parenthesis

Table 8: Dynamic Parameter Estimates

	Parameters	Euros
$\gamma_{FC}$	11.286*** (1.36)	10,696
$\gamma_{SC}$	15.376*** (3.848)	638,823

Standard errors in paranthesis;  
time horizon,# of simulation draws  
and # of alternative policies are set  
to 200; results are based on 10  
Monte Carlo runs;  $\delta = 0.95$

Figure 1: Trade Liberalization

