Trade, Economic Geography and the Choice of Product Quality

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Abstract

The present paper studies the effect of the choice of product quality on trade and location of firms. We build a quality-augmented model where consumers have preferences for the quality of a set of differentiated varieties. Firms do not only develop and sell manufacturing varieties in a monopolistic competitive market but also determine the quality level of their varieties by investing in research and development. We explore the price and quality equilibrium properties when firms are immobile. We then consider a footloose capital model where capital is allocated to the manufacturing firms in the region offering the highest return. We show that the larger region produces varieties of higher quality and that the quality gap increases with larger asymmetries in region sizes and with larger trade costs. Finally, the home market effect is mitigated when firms choose their product quality.

Keywords: Monopolistic Competition, Endogenous Quality, Economic Geography.

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1 Introduction

The present paper studies the effect of the choice of product quality on trade and location of firms. In particular, this paper discusses the role of the size of regions on firms' choice of location and product quality. It is well-know that firms' mobility fosters spatial polarization of economic activity (Krugman 1991). It is however less clear how differences in region sizes affect the quality produced in each region. Recently, Okubo and Picard (2008) highlight that firms endowed with higher qualities choose to locate in the larger region (selection effect). Yet, product quality is not an exogenous factor. Firms invest in research and development to improve their product quality and this investment is likely affect their decisions about plant locations. Such a relationship between quality and location is a topic that has lacked attention. The present paper therefore focuses on the incentives to invest in product quality and the impact of such investments on firms' location.

In this paper we build a quality-augmented version of Ottaviano *et al.* 's (2002) model where consumers have preferences for the quality of manufacturing varieties. As in Foster *et al.* (2008), the linear properties of the demand system of this model are particularly well suited for such an analysis. As is usual in the literature, each firm produces a distinct variety, competes under monopolistic competition and chooses its location in one of two regions. We consider a footloose capital model where capital is allocated to the firms and region offering the highest returns. The main difference with the literature is that firms are also able to determine their quality levels by investing in research and development.

We obtain the following results. We firstly consider the firms' choice of quality at a given spatial distribution of firms. We show that the larger region attracts the firms that produce varieties of higher quality and that the quality gap between regions increases with larger regional asymmetries and larger trade costs. Hence, the size of the local market is an important determinant of the average product quality and the added value of the goods that are produced in a particular region. In contrast to the international trade literature on product quality (see below for a reference to the literature), our result does not hinge on income effects but rather on a market size and competition effect. On the one hand, firms get higher returns from their investment when they locate in the region where demand is larger. On the other hand, investments in product quality foster competition and make the larger region more competitive. Hence, incentives to invest in quality are mitigated by a harsher competition in larger regions. Quite interestingly, we show that the co-agglomeration of firms and consumers in the same locale is good for average quality and bad for aggregate price index (good for cost of living). Although firms agglomerating in the larger region face a harsher competition, they benefit from a larger market which increases their incentives to invest in quality. Therefore, global quality rises.

We secondly consider the location choice of firms that simultaneously choose their quality. We assume an investment technology with decreasing returns. We show that the location equilibrium exists and is unique. In this model, the endogenous choice of product quality does not lead to convexities that generate multiple equilibria. In this location and quality equilibrium, the firms that choose to produce high quality varieties locate in the larger region. As standard in the economic geography literature, a fall in trade cost entices a larger number of firms to locate in the larger region. More interestingly, we show that firms invest more in quality on average and the quality gap decreases as trade costs fall. Removing trade barriers is always good for quality because firms have access to larger markets and more easily recoup their investment costs. This market access effect always dominates the negative effect that quality investments have on competition and profitability. We also show that market integration reduces regional disparities in terms of product quality. Better access to consumers increases the economic returns on quality investment. Finally we provide interesting results about the effect of investments in product quality on the spatial distribution of firms and the home market effect. Indeed, smaller costs of quality investments entice firms to agglomerate further in the larger region and increase the home market effect if firms agglomerate in the larger country. When firms are more dispersed, falls in the cost of quality investments has non monotone effects on the location of firms and on the home market effect.

Related literature This paper is closely related to several literature strands. First, quality and location is the focus of a well-known business literature about "sophistication" and "clustering". Porter (1990, p. 188) reports some qualitative evidence that investment in product quality turns out to be more important and more successful in regions with larger demand sizes. A typical example lies in the story of the two German designers of the rotary press, Koenig and Bauer, who returned from London (U.K.) to Bavaria (Germany) in 1818 to set up their first plant because this region was one amongst the world's largest market for printing press. German competitors in the press industry responded with differentiation strategies based on quality and reliability, which made Germany the country with the highest quality and highest price premium in this market. Similarly, the emergence of a US cluster in patient monitoring equipment after World War II is mainly explained by the fact that the US wealthy private hospitals had higher demands for sophisticated monitoring than any European country with socialized medicine. Finally, the emergence of the Japanese cluster in the robotic industry is also explained by the higher demand for robotics by the Japanese management who had significantly stronger engineering background.

Second, product quality and trade is also the topic of a recent strand of empirical trade literature. It is shown that the quality or the value of goods plays a crucial and important role in international trade pattern. For instance, using US commodity trade data, Schott (2004) finds that the unit value of trade within one product line is higher for high-wage countries. Hummels and Klenow (2005) find that richer countries export higher value goods. Hallak (2006) finds that rich countries import relatively more from the countries producing high quality goods.[1] Hence, there also exists quantitative evidence of heterogeneity of product quality in the trade patterns, which suggests that the study of the relationship between quality, trade and firms' location deserves a dedicated attention.

Third, academic research has produced a theoretical literature about product quality and trade based on vertical differentiation. This literature allows various authors to explain why higher quality products are more likely to be consumed and produced in high wage countries (Linden 1961, Falvey 1981, Falvey and Kierzkowski 1987 and Flam and Helpman 1987, Stockey 1991).[2] Murphy and Shleifer (1997) develop a model where

[1] Hummels and Klenow (2005) use import data from 76 countries at the six digit level of the Harmonized System and then find that the quality margin is a function of the exporter size. Hallak (2006) analyzes bilateral trade flows among 60 countries.

[2] Falvey (1981) models different endowments. As a consequence, he finds that capital abundant country exports capital-intensive high quality products whereas it imports labor-intensive low quality products. Then, Falvey and Kierzkowski (1987) added technology difference to dissimilar factor endowments. Flam

high quality products end up being produced in high human capital countries. Feenstra and Romalis (2006) extend the Heckscher Ohlin model to product qualities. Baldwin and Harrigan (2008) and Khandelwal (2007) study heterogeneous quality in international trade models. However, none of those models study the location of firms. The relationship between product quality and location choice is recently studied in Okubo and Picard (2008) who show that larger regions attracts better quality firms. This paper extends this idea in a model where product quality is a variable chosen by firms.

Finally, as this paper discusses firms' heterogeneity, it can be cast in the recent trade literature on cost heterogeneity (Melitz, 2003; Helpman, Melitz and Yeaple, 2004; Falvey *et al.,* 2004; Melitz and Ottaviano, 2008; Baldwin and Robert-Nicoud, 2008). This literature responds to extensive empirical research on trade behavior at firm level[3] and focuses on the impact of trade liberalization on the average productivity and on the trade behaviors of firms with heterogenous productivity. Yet, cost heterogeneity is not the only critical characteristic that explains trade patterns. For some authors, trade is better explained by demand or quality heterogeneity than by cost heterogeneity (see Baldwin 2005, Greenaway 1995 and Greenaway *et al.* 1995; Fukao *et al.* 2003; Foster *et al.* 2008). So, a deeper investigation of the impact of quality heterogeneity and on the regional determinants of product quality is welcome.

The paper is structured as it follows. Section 2 presents the model. Section 3 and 4 discuss the choice of quality and location.

2 The model

Our model extends the footloose capital model of Ottaviano and Thisse (2004) by allowing for consumer's preference for product quality. In this section, we present the basic

and Helpman (1987) use a Ricardian technology difference to model quality trade. Stockey (1991) suggests that consumers in the rich country consume more of the same quality products than consumers in the poor country.

[3] Many empirical studies give evidence on the relationship between trade and firm productivity through trade liberalization. See Bernard, Eaton, Jensen and Kortum (2003) Bernard, Jensen, and Schott (2004), Pavcnik (2002) and Tybout and Westbrook (1995).

model and characterize the market outcome for any given organizational structure and spatial distribution of firms. In this paper the timing is as follows: first firms simultaneously choose their quality and location, second they set their prices and finally consumers consume the goods produced by the firms. We solve this sequential game by backward induction.

2.1 Preferences

Consider a world with two regions, labeled H and F. Variables associated with each region will be subscripted accordingly. We assume that there is a mass L of consumers, with a share $1/2 \leq \theta_H < 1$ located in region H. In what follows, we refer to H as the large and to F as the small region.

All consumers in region $i = H, F$ have identical quasi-linear preferences over a homogenous good and a continuum of horizontally differentiated varieties, indexed by $v \in \mathcal{V}$. As in Ottaviano *et al.* (2002), the utility of a representative agent in region i is given by the following quadratic function:

$$U_i = \int_{\mathcal{V}} \widehat{\alpha}(v) q_i(v) \mathrm{d}v - \frac{\beta - \gamma}{2} \int_{\mathcal{V}} [q_i(v)]^2 \mathrm{d}v - \frac{\gamma}{2} \left[\int_{\mathcal{V}} q_i(v) \mathrm{d}v \right]^2 + q_i^o, \tag{1}$$

where $q_i(v)$ denotes the consumption of variety v in region i and q_i^o stands for the consumption of the homogenous good in that same region. As in Ottaviano *et al.* (2002), γ is a measure of the degree of substitution between varieties whereas $\beta - \gamma$ (> 0) measures the consumer bias toward a more dispersed consumption of varieties.

The new element in this model is the function $\widehat{\alpha}(v) : \mathcal{V} \equiv [0, 1] \rightarrow [\underline{\alpha}, \infty)$, $\underline{\alpha} \geq \overline{\alpha} > 0$, that reflects the *quality* of variety v. It measures the consumer's *willingness to pay* for this variety v; that is, the intensity of consumer's preferences for the differentiated product v with respect to the homogenous good. When $\widehat{\alpha}(v)$ is identical for all varieties, varieties are horizontally and symmetrically differentiated. When $\widehat{\alpha}(v)$ varies, the quality and therefore the willingness to pay varies across varieties so that goods are also vertically differentiated in the sense that consumers have a higher willingness to pay for the variety v than for v' if $\widehat{\alpha}(v) > \widehat{\alpha}(v')$. Note finally that the consumers have identical preferences: there is no a priori 'regional preferences'. We denote the *average quality* by $\alpha \equiv \int_{\mathcal{V}} \widehat{\alpha}(v) \mathrm{d}v$.

Each consumer in region $i = H, F$ maximizes his/her utility (1) subject to his/her budget constraint:

$$\int_{\mathcal{V}} p_i(v) q_i(v) \mathrm{d}v + p_i^o q_i^o \leq w_i + p_i^o \overline{q}^o, \tag{2}$$

where $p_i(v)$ denotes the consumer price of variety v and w_i stands for each individual's earning. Following Ottaviano *et al.* (2002), we assume that consumers own a sufficiently large endowment $\overline{q}^o > 0$ of the numéraire. Consequently, income effects are present in the demand for homogenous goods but are absent in the demand for manufacturing varieties. As will become clear in the sequel, free trade in the homogenous good market leads to price equalization across regions and makes this good a natural choice for the numéraire ($p_i^o = 1,\ i = H, F$).

We assume that all varieties are consumed. Maximizing the utility (1) subject to the budget constraint (2) yields the following first order condition:

$$\widehat{\alpha}(v) - (\beta - \gamma)\, q_i(v) - \gamma \int_{\mathcal{V}} q_i(\xi) d\xi - p_i(v) = 0 \tag{3}$$

Integrating the left hand side of this equality yields the average quality

$$\alpha = \beta \int_{\mathcal{V}} q_i(v) \mathrm{d}v + \int_{\mathcal{V}} p_i(v) \mathrm{d}v \tag{4}$$

The last two expressions allows us to derive the individual demand for variety v as the following linear formula:

$$q_i(v) = (b + c) \left[\widehat{\alpha}(v) - p_i(v)\right] + c \left(\mathbb{P}_i - \alpha\right) \tag{5}$$

where

$$\mathbb{P}_i = \int_{\mathcal{V}} p_i(v) \mathrm{d}v$$

is the manufacturing price index in region i and where b and c are the following positive coefficients

$$b = \frac{1}{\beta} \quad \text{and} \quad c = \frac{\gamma}{\beta\left(\beta - \gamma\right)} \tag{6}$$

The parameter b measures the price sensitivity of demand and the parameter c the degree of product substitutability. In particular, when $c \to 0$ varieties are perfectly differentiated, whereas they become perfect substitutes when $c \to \infty$.

2.2 Price equilibrium and trade costs

Production takes place in two sectors. In the first sector, the homogenous good is produced under perfect competition using one unit of labor per unit of output. We assume that this good can be costlessly traded between regions, which implies that its price is internationally equalized and equal to wages. Normalizing wages to one we get $p_i^o = w_i = 1$ for $i = H, F$, which justifies our previous choice of this good as the numéraire.

In the second sector, called the manufacturing sector, each firm produces under increasing returns to scale and sells a single differentiated manufacturing variety. Let $\mathcal{V}_i$ and n_i be the set and the mass of manufacturing firms located in region i. That is, $n_i = \mu(\mathcal{V}_i) \equiv \int_0^1 \mathrm{d}\mu_i(v)$ where $\mu(\mathcal{V}_i)$ is the measure of $\mathcal{V}_i$ and $\mu_i(v)$ is the measure of variety v if it is produced in region i ($\mu_H(v) + \mu_F(v) = 1$ and $\mu_H(v) * \mu_F(v) = 0$). In this section we derive the price equilibrium for a *given* location structure $(\mathcal{V}_H, \mathcal{V}_F)$ and for a given distribution of product quality $\widehat{\alpha}(\cdot)$ across firms. The average quality are therefore given by

$$\alpha = \int_0^1 \widehat{\alpha}(v) d\mu_H(v) + \int_0^1 \widehat{\alpha}(v) d\mu_F(v)$$

The demand for each variety in each market depends on the set of varieties produced domestically and on the set produced abroad. In accord with empirical evidences (e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that product markets are *segmented*. Firms are hence free to set prices specific to each national market they sell their product in. The profit of a manufacturing firm established in region i is given by

$$\Pi_i(v) = L\theta_i p_{ii}(v) q_{ii}(v) + L\theta_j (p_{ij}(v) - \tau) q_{ij}(v) - I\left(\widehat{\alpha}\left(v\right)\right) - r_i(v), \quad v \in \mathcal{V}_i \qquad (7)$$

where L is the total population, θ_i is the share of population in region i, $I\left(\widehat{\alpha}\left(v\right)\right)$ is the firm's investment in quality $\widehat{\alpha}(v)$, $r_i(v)$ is the remuneration of firm v's capital and $q_{ij}(v)$ and $p_{ij}(v)$ is the price and demand of variety v when it is produced in region i and consumed in region j. In the latter expression we have normalized w.l.o.g. the marginal cost of production to zero and assumed a unit transport cost τ paid in numéraire. By (5), the individual demand writes as

$$q_{ij}(v) = (b + c)\left[\widehat{\alpha}(v) - p_{ij}(v)\right] + c\left(\mathbb{P}_j - \alpha\right)$$

for all $i, j \in \{H, F\}$. Under monopolistic competition, firms are too small to affect the aggregate variables. So, they set their prices $p_{ii}(v)$ and $p_{ij}(v)$ taking the price indices $(\mathbb{P}_i, \mathbb{P}_j)$and the distribution of quality $(\widehat{\alpha}(\cdot))$ as givens. The optimal prices are computed as it follows:

$$p_{ii}(v) = \frac{(b+c)\,\widehat{\alpha}(v) + c\,(\mathbb{P}_i - \alpha)}{2(b+c)} \text{ and } p_{ij}(v) = p_{jj}(v) + \frac{\tau}{2} \tag{8}$$

which depend on the quality of the variety offered. As in Ottaviano and Thisse (2004), the price of exported goods $p_{ij}(v)$ are inflated by half of the transport cost.

At the equilibrium in the product market, the firm's prices $(p_{ii}(v), p_{ij}(v))$ are consistent with aggregate prices or price indices $(\mathbb{P}_i, \mathbb{P}_j)$. The latter are successively equal to

$$\begin{aligned}
\mathbb{P}_i &= \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 p_{ji}(v) d\mu_j(v) \\
&= \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 \left(p_{ii}(v) + \frac{\tau}{2}\right) d\mu_j(v) \\
&= \int_0^1 p_{ii}(v) dv + \int_0^1 \frac{\tau}{2} d\mu_j(v) \\
&= \frac{\alpha b + c\mathbb{P}_i}{2(b+c)} + \frac{\tau}{2} n_j
\end{aligned}$$

Solving for the fixed point yields

$$\mathbb{P}_i = \frac{\alpha b + (b+c)\tau n_j}{2b+c} \tag{9}$$

Hence, the price index depends only on the average quality and the number of firms in each region. It does not depend on the quality chosen by each firm and on the particular location of firms. This property stems from the fact that preferences are linear in the quality parameter. Equilibrium prices are then equal to

$$p_{ii}^*(v) = \frac{1}{2}\frac{2\alpha b + \tau n_j c}{2b+c} + \frac{\widehat{\alpha}(v) - \alpha}{2} \text{ and } p_{ij}^*(v) = p_{jj}^*(v) + \frac{\tau}{2} \tag{10}$$

Note that those prices are the ones presented in Ottaviano *et al.* (2002) if varieties have the same quality $(\widehat{\alpha}(v) = \alpha)$. Otherwise, firms selling higher quality products set higher prices. Indeed, each firm's prices increase with its idiosyncratic product quality. They however fall with larger average product quality $(\partial p_{ii}^*(v)/\partial \alpha < 0)$. The firm reacts to more attractive competing goods by lowering its prices. As in Okubo and Picard (2008), we note that prices are independent to the precise composition of local production. That is, each firm sets a price $p^*_{ii}(v)$ that depends only on the quality of its own variety $\widehat{\alpha}(v)$, on the average quality α and on the mass of firms (n_i, n_j) in each region. In other words, for any given profile of quality $\widehat{\alpha}(\cdot)$ that yields the average quality α, the prices $p^*_{ii}(v)$ and $p^*_{ij}(v)$ depend only on $\widehat{\alpha}(v)$ and (n_i, n_j) but not on the sets $(\mathcal{V}_i, \mathcal{V}_j)$. This independence of prices to the precise composition of local production turns out to be a useful property in the subsequent analysis of spatial selection (see Section 4).

Given the above prices, it is easy to show that firms' optimal output is equal to $q^*_{ii}(v) = (b+c)p^*_{ii}(v)$ and $q^*_{ij}(v) = (b+c)\left(p^*_{ij}(v) - \tau\right)$ so that the profit of firm v located in region i can be written as

$$\Pi_i(v) = L(b+c)\left[\theta_i(p^*_{ii}(v))^2 + \theta_j\left(p^*_{ij}(v) - \tau\right)^2\right] - I\left(\widehat{\alpha}\left(v\right)\right) - r_i(v) \tag{11}$$

In this analysis we have assumed that all firms export. To garantee this, we must impose that $q^*_{ij}(v) > 0 \iff p^*_{ij}(v) > \tau$ for all i, j and for all $n_i, n_j \in [0,1]$. That is,

$$\underline{\alpha} > \tau\left(1 + \frac{c}{2b}\right) \tag{12}$$

which puts either a lower bound on quality or a highest bound on trade costs.

We now analyze the firms' simultaneous choice of quality and location. The firms' location choice for a given set of qualities is analyzed in Okubo and Picard (2008). Those authors show that firms with best quality select the larger market. This result applies to this analysis. For the sake of exposition we make a separate discussion about the choice of quality in Section 3. The analysis of the simultaneous choice follows in Section 4.

3 Quality and trade

In this section we discuss a trade model where the spatial distribution of firms is given and where firms are able to choose their quality investments. We assume decreasing returns to investment in quality where each firm can improve its quality to a quality z by investing an amount

$$I(z) = I\ \ (z - \underline{\alpha})^2 \quad \text{if} \quad z \geq \underline{\alpha}$$

of numéraire. The investment cost is nil otherwise: $I(z) = 0$ if $z < \underline{\alpha}$. In this expression, the parameter $\underline{\alpha}$ is the costless product quality. Because of decreasing returns to scale in quality, higher product qualities require more than proportional investment levels. For the sake of convenience we define

$$I \equiv L(b+c)I_o/4$$

where L is the total consumer population and where I measures this return to investment on a per-consumer basis. This formulation is convenient because firms' investment incentives are proportional to the total size of market (i.e. L consumers) and because profit level are proportional to $(b+c)$. Finally we assume that the quality investment can be made only in the location where the firm is located. For instance, quality investments are related to specific management efforts to improve product quality, specific training of local labor force or specific manufacturing immobile equipments.

in physical capital

Under monopolistic competition, the firm is small in the product market and has no influence on other firms' prices and quality levels indices. Hence, the firm maximizes its profit $\Pi_i(v)$ with respect to its own quality level $\widehat{\alpha}(v)$ taking $\mathbb{P}_i$ and α as givens. We get

$$\frac{\partial \Pi_i(v)}{\partial \widehat{\alpha}(v)} = L(b+c)\left\{\left[\theta_i p_{ii}^*(v) + \theta_j \left(p_{ij}^*(v) - \tau\right)\right] - \left(\widehat{\alpha}\left(v\right) - \underline{\alpha}\right) I_o/2\right\} = 0 \tag{13}$$

and

$$d^2\Pi_i(v)/d\left(\widehat{\alpha}(v)\right)^2 < 0 \iff I_o > 1 \tag{14}$$

The optimal quality is finite only if decreasing returns to scale in quality investment are strong enough, which we assume from now. Indeed, if this condition were not satisfied, investment cost would rise at a smaller pace than the revenue increase associated to quality improvement and the optimal quality would be unbounded.

From expression (13), the optimal increase in quality

$$\widehat{\alpha}^*\left(v\right) - \underline{\alpha} = \frac{2}{I_o}\left[\theta_i p_{ii}^*(v) + \theta_j \left(p_{ij}^*(v) - \tau\right)\right] \tag{15}$$

is proportional to the average markup on the world population and inversely proportional to the cost of quality investment I_o. The firm balances the marginal cost and the marginal revenues of its quality investment. Under linear demand functions, outputs are proportional to markups and marginal revenues are also proportional to markups. The impact of access is clearly apparent: since trade costs reduce markups, marginal revenues and therefore incentives to invest in quality get smaller as the production site is located further away from consumers.

3.1 Relationship between quality and prices

It is instructive to study the optimal quality as a function of the prices of other firms. The following relationships between quality and prices can be used as a basis for empirical work. Plugging the prices (8) into (13), we compute the following optimal increase in quality:

$$\widehat{\alpha}^*(v) - \underline{\alpha} \equiv \alpha_i^* - \underline{\alpha} = \frac{\underline{\alpha}(b+c) - c\alpha + c(\theta_i \mathbb{P}_i + \theta_j \mathbb{P}_j) - \tau\theta_j(b+c)}{(I_o - 1)(b+c)} \quad v \in \mathcal{V}_i \tag{16}$$

This expression allows us to make the following observations. First, *ceteris paribus*, firms invest more in quality for smaller average quality and for less substitutable products (smaller α and c). Because of decreasing returns to quality investment, firms get higher returns from improving their own qualities in a world where the industry produces bad qualities and operate under weak competition. Second, the optimal quality increases with the *global average price,* which we define as $\theta_i \mathbb{P}_i + \theta_j \mathbb{P}_j$. High aggregate prices allow each firm to set higher prices and give it more incentives to invest in product quality. Third, investment in product quality diminishes with higher trade costs. This is because higher trade costs reduce market access and therefore the sales in the foreign region; incentives to invest in quality are then smaller. Fourth, from (16), changes in region sizes may have ambiguous impact on quality. Indeed,

$$\frac{d\widehat{\alpha}^*(v)}{d\theta_i} = \frac{c(\mathbb{P}_i - \mathbb{P}_j) + \tau(b+c)}{(I_o - 1)(b+c)} \quad v \in \mathcal{V}_i$$

is positive if and only if $c\mathbb{P}_i + \tau(b+c) > c\mathbb{P}_j$. Therefore, a shift of the consumer base from region j to region i entices firms located in region i to increase their quality only if they are already setting high enough local prices and if they are able to save on sufficiently important trade costs on their foreign consumers. This can be the case if the local market has high quality and weak competition.

Finally, the *quality gap* between regions is given by

$$\alpha_H^* - \alpha_F^* = \frac{\tau}{I_o - 1}\left(2\theta_H - 1\right) \tag{17}$$

which is simply proportional to the asymmetry in region sizes. As a result, *high quality products are necessarily produced in the larger region.* Furthermore, *higher trade costs increase the quality gap.* This is because the markups on firms' exports decrease with higher trade costs and reduce the returns to investment in quality. This effect has a stronger impact on the firms that have a larger share of export in their production, which are the firms located in the small market. Note that the quality gap is independent from the location of firms. We have seen above that the quality of a firm depends just on the global average prices $(\theta_i \mathbb{P}_i + \theta_j \mathbb{P}_j)$ but is not directly related to the distribution of firms (n_i, n_j). So, the relocation of firms does not alter on the quality gap.

3.2 Relationship between quality and location

We now study the optimal quality and prices as a function of firms' spatial distribution. We first compute the average quality $\alpha^* = n_H \alpha_H^* + n_F \alpha_F^*$. Using (16) and (10) we get the following relationship:

$$\alpha^* - \underline{\alpha} = 2b \frac{\underline{\alpha} - \tau\left(\theta_H n_F + \theta_F n_H\right)}{I_o\left(2b + c\right) - 2b} \tag{18}$$

where the denominator is positive because of the concavity assumption (14). We note the following points. First, the average quality increases with higher costless quality $\underline{\alpha}$. This is because a larger $\underline{\alpha}$ makes it cheaper to achieve any quality level. Second, the average quality decreases with larger trade cost τ. As explained above, higher trade costs reduce markups on exports and diminish the incentives to invest in quality. Finally, let us define the *co-agglomeration of firms with consumers* in region i as the increase in n_i when $\theta_i > 1/2$. Remembering that $n_F = 1 - n_H$ and $\theta_F = 1 - \theta_F$, one can readily check that the term $(\theta_H n_F + \theta_F n_H)$ decreases in n_H for any $\theta_H > 1/2$. Hence, average quality falls when more firms locate in region H (higher n_H). As a result, *co-agglomeration of firms with consumers in the same locale is good for average quality.* This result depends on two opposite effects. First, markups are larger when firms locate in the larger market and therefore spend less on trade costs, which entices them to invest more in quality.

Second, product market competition increases when firms collocate in the larger market, which decreases markups and incentives to invest in quality. In this model, the first effect dominates.

We can perform a similar analysis on the following measure of aggregate cost of living

$$\theta_i \mathbb{P}_i + \theta_j \mathbb{P}_j = \underline{\alpha} - 4\frac{[(b+c)I_o - 2b]\left[\underline{\alpha} - \tau\left(\theta_H n_F + \theta_F n_H\right)\right]}{I_o\left(2b+c\right) - b}$$

One can show that it increases with higher costless quality $\underline{\alpha}$ and that it increases with higher trade cost τ iff $I_o > 2b/(b+c)$. In the aggregate, prices rise if quality improvement starts from a larger costless quality level and if market access gets better. A better market access fosters quality and increase prices. Also, because $(\theta_H n_F + \theta_F n_H)$ decreases in n_H for any $\theta_H > 1/2$, the aggregate cost of living decreases when more firms locate in the larger region. Although co-agglomeration of firms with consumers improves average quality, it increases competition so that prices tend to fall. The competition effect is therefore stronger than the quality improvement effect!

The optimal quality is given by the following formula:

$$\alpha_i^* - \underline{\alpha} = \frac{2\underline{\alpha}b}{I_o\left(2b+c\right) - 2b} + \frac{c\tau I_o\left(\theta_H n_F + \theta_F n_H\right)}{\left(I_o - 1\right)\left(I_o\left(2b+c\right) - 2b\right)} - \frac{\tau\theta_j}{I_o - 1}$$

It can be shown that the optimal quality increases with larger costless quality $(d\alpha_i^*/d\underline{\alpha})$, *weaker* co-agglomeration of firms with consumers in the large region (α_i^* increases with $\theta_H n_F + \theta_F n_H$ which decreases in n_H) and larger local market (smaller θ_j). It reflects the trade-off discussed earlier. It can be shown that α_H^* increases with trade cost τ whereas α_F^* decreases with it.

We summarize our results in the following proposition.

Proposition 1 *(i) The larger region produces varieties of higher quality. (ii) The quality gap increases with larger asymmetries in region sizes and with larger trade costs. (iii) The aggregate cost of living falls as the larger region hosts more firms. (iv) Co-agglomeration of firms with consumers is bad for individual quality, good for average quality and bad for aggregate prices (thus good for cost of living).*

This proposition brings a contrasting perspective on global quality and firms' spatial distribution. Indeed, on the one hand, the asymmetry in consumers' spatial distribution is a cause spatial disparities in product quality. On the other hand, disparities in firms' spatial distribution lead to a rise in the average quality. This result is novel in the literature.

We are now equipped to discuss the relationship between prices and location.

3.3 Relationship between prices and location

Because $q_{ii}^* = (b+c)p_{ii}^*$ and $q_{ij}^* = (b+c)\left(p_{ij}^* - \tau\right)$, local sales and export sales move in the same direction as local and export prices. Given (10), a firm located in region i and choosing its optimal quality sets the following domestic and export prices:

$$p_{ii}^* = \frac{1}{2}\frac{2b\alpha^* + \tau n_j c}{2b+c} + \frac{\alpha_i^* - \alpha^*}{2} \quad \text{and} \quad p_{ij}^* = \frac{1}{2}\frac{2\alpha^* + \tau n_i c}{2b+c} + \frac{\alpha_i^* - \alpha^*}{2} + \frac{\tau}{2} \tag{19}$$

The two (first) terms of each expression reflect two location forces underlying the choice of quality. The first term represents the optimal price for a hypothetical 'average quality good' ($\alpha_i^* = \alpha^*$). It increases with average quality α^*. In the Ottaviano and Thisse (2005) model, the quality is constant and haspositive impacts on firms' agglomeration because an increase in its level diminishes the trade cost - price ratio and therefore the dispersion effect of trade costs. In the present model with endogenous choice of quality, the 'average quality good' depends on the location of firms and consumers. Hence, by (18), the co-agglomeration of firms with consumers increases average quality so that it inflates prices and outputs upwards. Indeed, firms locating in the larger region produce higher quality goods, set higher prices and push prices indices $\mathbb{P}_i$ up, which raises demand. Meanwhile trade costs have a smaller impact on margins and profits because a higher average quality diminishes the trade cost - price ratio and therefore the dispersion effect of trade costs. Therefore, *trade costs therefore exert a smaller dispersion force and agglomeration is more likely to take place.* This effect strengthens when decreasing returns to scale of quality investments I_o weaken.

The second term represents a markup for quality (resp. a discount if $\alpha_i^* < \alpha^*$) for the firm's quality advantage (resp. disadvantage). In this model without income effects, quality affects the consumer's individual demand in the same way in each region. As a result, the firm sets the same markup (resp. discount) for quality in both markets. The markup for quality of firms located in the larger region H can be expressed as

$$\frac{\alpha_H^* - \alpha^*}{2} = \frac{\tau}{2\left(I_o - 1\right)}\left(\theta_H - \theta_F\right)\left(1 - n_H\right) > 0$$

which increases with region size asymmetries (larger θ_H) but decreases with the co-agglomeration of firms in the domestic market (larger n_H). Markups for quality create a repulsion force for firms locating in the larger market because those firms see the individual benefit of their quality advantage reduced as more firms locate there. Firms located in the smaller region get a quality discount

$$\frac{\alpha_F^* - \alpha^*}{2} = -\frac{\tau}{2\left(I_o - 1\right)}\left(\theta_H - \theta_F\right) n_H < 0$$

whose absolute value increases with stronger region size asymmetries (larger θ_H) and with further co-agglomeration of firms in the domestic market (larger n_H). As more firms co-agglomerate in the larger region H, the firms in the smaller region F get an even worse quality discount and are enticed to quit this region. However, the overall effect is that *the quality markups and discounts exert a dispersion force on firms*, which counters too much agglomeration of firms in the larger locale. This effect also strengthens when decreasing returns to scale of quality investments I_o weaken, in particular when I_o is close to one.

To sum up, the choice of quality creates two conflicting forces: an agglomeration force because average price-trade cost ratio rises and a dispersion forces because the net benefit of quality advantage diminishes as more firms agglomerate in the larger region. Those forces are exacerbated by smaller decreasing returns to scale of quality investments. We now turn to the discussion of the possibility of bilateral trade.

3.4 Bilateral trade

Bilateral trade takes place when firms export from all regions. It may not be feasible for all values of economic parameters. In particular, firms located in the smaller region have lower quality and sell at a competitive disadvantage. Those firms are the first that stop exporting. The exports of such firms q_{FH}^* are proportional to $p_{FH}^* - t$; that is, to

$$q_{FH}^* \propto \frac{1}{2}\left(\frac{2b\underline{\alpha} + c\tau n_F}{2b + c} - \tau\right) + \frac{4b^2}{2\left(2b + c\right)}\frac{\underline{\alpha} - \tau\left(\theta_H n_F + \theta_F n_H\right)}{I_o\left(2b + c\right) - 2b} - \frac{\tau}{2\left(I_o - 1\right)}\left(\theta_H - \theta_F\right) n_H$$

which is a decreasing function of n_H because its derivative with respect to n_H is equal to

$$-\frac{1}{2}\frac{c\tau}{2b+c}\left[1+\frac{I_o\left(4b+c\right)-2b}{\left(I_o-1\right)\left(I_o\left(2b+c\right)-2b\right)}\left(\theta_H-\theta_F\right)\right]<0$$

Exports into the larger region fall if more firms have located and intensified competition there. One can show that $q_{FH}^{*}=0$ at

$$n_H=\overline{n}_H\left(I_o\right)\equiv\frac{2b}{\tau}\frac{\underline{\alpha}-\tau+\frac{2b}{I_o(2b+c)-2b}\left(\underline{\alpha}-\tau\theta_H\right)}{c+\left(\theta_H-\theta_F\right)\left(\frac{2b+c}{I_o-1}-\frac{4b^2}{I_o(2b+c)-2b}\right)} \tag{20}$$

where the second term in the denominator is positive because $I_o>1$. So, there exists bilateral trade if firms export from the smaller to the larger region, which occurs iff $n_H<\overline{n}_H\left(I_o\right)$.

Consider firstly the case where decreasing returns to scale for quality are very large (e.g. $I_o\to\infty$). In this case a^* tends to $\underline{\alpha}b$ and $a_i^*-a^*$ tends to zero so that we return to Ottaviano and Thisse's (2004) model. Using the previous formula, we get that there bilateral trade is feasible iff $n_H<\overline{n}_H\left(\infty\right)\equiv\frac{2b(\underline{\alpha}-\tau)}{\tau c}$, where $\overline{n}_H\left(\infty\right)$ is smaller than zero if $\tau>\underline{\alpha}$ and larger than one for $\tau<\tau^{\infty}\equiv\frac{2b}{2b+c}\underline{\alpha}$. So, bilateral trade is never feasible under the first condition whereas it is always feasible under the latter condition. For trade costs between τ^{∞} and $\underline{\alpha}$, bilateral trade is possible only if the larger region does not host too many manufacturing competitors.

Consider secondly the case where decreasing returns to scale for quality are small enough (e.g. $I_o\to 1$). Then we get $\overline{n}_H\left(1\right)=0$ and we can conclude that bilateral trade is never feasible. In this case, firms have more incentives to invest in quality in the larger region because they benefit from a better access to market and because the cost of quality investment does not increase that much as quality rises. Therefore, regional quality and markups diverge dramatically. As a result, large price discrepancies and large export differences take place. As a consequence, *exports from the smaller region fall to zero for any trade cost τ and any regional size asymmetries.*

Consider finally intermediate returns to scale I_o. Then, it is apparent that the threshold $\overline{n}_H\left(I_o\right)$ depends on region size asymmetries. Indeed because $\overline{n}_H$ decreases with larger θ_H, bilateral trade is less likely for larger region size asymmetries. Firms invest more in quality in the larger region and become more competitive relatively to the firms in the small region. Does stronger decreasing returns to scale in quality investment I_o reduce the set of economic configurations that support bilateral trade? It will reduce it if I_o reduces the set of feasible spatial distributions of firms, $[0, \overline{n}_H(I_o)]$. Since $\overline{n}_H(1) = 0$ is smaller than $\overline{n}_H(\infty)$ for any $\underline{\alpha} > \tau$, one may conclude that the set of spatial configurations that support bilateral trade shrinks for very large falls in returns to scale in quality investment. Yet, the function $\overline{n}_H(I_o)$ is not always monotone. Indeed, let $\underline{\alpha} > \tau$ and

$$\overline{\theta}_H \equiv \frac{1}{2} + \frac{b\left(2\underline{\alpha} - \tau\right)}{2\underline{\alpha}\left(4b + c\right) - 2\tau\left(3b + c\right)} < 1$$

Then, one can check that the function $\overline{n}_H(I_o)$ monotonically increases for all $I_o > 0$ if $\theta_H > \overline{\theta}_H$ and that $\overline{n}_H(I_o)$ firstly increases in I_o for small I_o and then decreases for large I_o if $\theta_H < \overline{\theta}_H$. This non-monotonicity property stems from the two above-mentioned agglomeration and dispersion forces of quality choice on firms location. When I_o is high enough, the agglomeration force dominates because average price-trade cost ratio rises more than quality markups falls. When I_o is close enough to one, the dispersion force dominates because the impact of quality markups on prices and profits becomes more important than the impact of 'average quality'. We summarize this discussion in the following proposition:

Proposition 2 *Suppose $\underline{\alpha} > \tau$. (i) Larger region size asymmetries θ_H reduce the set of spatial configurations supporting bilateral trade. (ii) A larger share of the manufacturing industry n_H in the larger region H also reduces the set of spatial configurations supporting bilateral trade. (iii) A fall in increasing returns to scale in quality investment I_o monotonically reduces the set of spatial configurations supporting bilateral trade for large enough region size asymmetries $\theta_H > \overline{\theta}_H$. If $\theta_H < \overline{\theta}_H$, such a fall has non-monotonous effects: it increases and then reduces this set as I_o falls.*

Finally, given the first order condition (13), the profit of a firm located in region i can be written as

$$\Pi_i^* = L(b+c)\left\{\left[\theta_i p_{ii}^{*2} + \theta_j\left(p_{ij}^* - \tau\right)^2\right] - \frac{1}{I_o}\left[\theta_i p_{ii}^* + \theta_j\left(p_{ij}^* - \tau\right)\right]^2\right\} - r_i \tag{21}$$

where the second squared bracket term represents the investment cost in quality. If the investment cost (I_o) is infinite, the second term vanishes and the expression of profit is equal to the one obtained by Ottaviano and Thisse (2004) where α is set to $\underline{\alpha}$. Note that in contrast to operational profits (first squared bracket), investment cost (second squared bracket) cannot be broken down between the two markets. The investment cost is spread across local and foreign sales and is therefore a quadratic function of the population-weighted markup (e.g. $\theta_i p_{ii}^* + \theta_j \left(p_{ij}^* - \tau\right)$). By contrast operational profits are given by population-weighted average of the profits (outputs times markups), which are expressed by the squares of prices.

3.5 Optimality of quality investments

We here finally study whether the equilibrium implies too large or too low quality levels. Note first that firms individually choose a higher quality than the quality that maximizes industry profits $\Pi^{tot} \equiv \sum_i \int_{\mathcal{V}_i} \Pi_i(v)dv$. Over-investment in quality takes place because firms do not internalize the negative effect of their quality increases on their competitors' sales. Indeed, an increase in average quality reduces the profit of any firm that does not simultaneously raise its quality level. This easiest way to show this is to study the simultaneous rise of each quality $\widehat{\alpha}^*(v)$ to $\widehat{\alpha}^*(v) + \varepsilon$ where ε is a sufficiently small positive real number. Then, we get

$$\frac{d\Pi^{tot}}{d\varepsilon} = -L\frac{c\,(b+c)}{2b+c}\frac{I_o}{2}\left(\alpha^* - \underline{\alpha}\right) < 0$$

(see Appendix 1). Therefore, lower quality levels should be called for by investors and industry lobbies. Because average quality increases with co-agglomeration of firms with consumers, co-agglomeration of firms with consumers exacerbates the effect of over-investment in quality.

By contrast, consumers would prefer the opposite outcome: higher quality levels. Firms are indeed not able to collect the whole consumer surplus from an increase of their quality so that they do not internalize the benefit of a better quality to infra-marginal consumers. More formally, using condition (3) one can write the consumer surplus of an individual located in region i as

$$U_i^* = \frac{1}{2}\int_{\mathcal{V}} \left[\widehat{\alpha}^*(v) - p^*(v)\right] q(v)dv + \overline{q}_i^o$$

If we raise again each quality $\widehat{\alpha}^*(v)$ to $\widehat{\alpha}^*(v)+\varepsilon$, we get (see Appendix 1)

$$\frac{dU_i}{d\varepsilon}=\frac{b(b+c)^2}{(2b+c)^2}\left(\alpha^*-\tau n_j\right)$$

which is positive because $\alpha^* \geq \underline{\alpha} > \tau$ under condition (12). Because the consumers residing in the region that hosts the larger number of firms benefit from a better market access, they gain more from this average quality increase ($dU_i/d\varepsilon > dU_j/d\varepsilon \iff n_i > n_j$).

Since a global quality increase affects industries and consumers in opposite ways, it is natural to ask whether such an increase would raise aggregate welfare. In Appendix 1 we compute get that at the equilibrium average quality α^* given by (18)

$$\frac{d}{d\varepsilon}\left(L\theta_i U_i + L\theta_j U_j + \Pi^{tot}\right) = 2b^2 I_o\left(2b+c\right)\frac{\underline{\alpha}-\tau\left(\theta_H n_F+\theta_F n_H\right)}{I_o\left(2b+c\right)-2b}$$

which is positive because $\underline{\alpha} \geq \tau \geq \tau\left(\theta_H n_F + \theta_F n_H\right)$. So, global welfare would be increased by a global increase of quality. We summarize this discussion in the following proposition:

Proposition 3 *The equilibrium average quality is set too high for shareholders and industry lobbies whereas it is set too low for consumer and global welfare.*

4 Quality and economic geography

In this section we study how firms' choices of location and quality shape the economic geography. It is well-know that firm or capital mobility fosters spatial polarization of economic activity. It is however less clear how differences in region sizes affect the quality produced in each region and the number of firms locating in each region. We here show that, compared to the case with exogenous quality levels, the home market effect can stronger under endogenous quality for small region size asymmetries but weaker for sufficiently large region size asymmetries.

We model a footloose capital model in which unit mass of capital is inelasticly supplied by a set of immobile capital owners. Because of the immobility of the capital owners and because of the absence of income effect in the demand for manufacturing goods, the residence place of those agents has no importance on product demands, profits and location of firms. For the sake of exposition, we assume that the capital market is perfectly competitive and that each firm requires one unit of capital in order to operate. As a result, the mass of varieties is equal to unity and the previous analysis holds.

The equilibrium in the capital market is obtained as follows. Capital owners allocate their capital to the firms that offer the highest return across regions. To obtain a unit of capital, each firm chooses the location that maximizes its profit and bids for capital up to the value that cancels its profit (21). As a result, we get two possible configurations. On the one hand, the whole capital flows in region H (resp. F) so that $n_H = 1$ (resp. $n_F = 1$) because firms producing in this region always offers a better return: $r_H > r_F$ (resp $r_F > r_H$). On the other hand, the capital spreads across regions so that $n_H \in [0, 1]$ because firms offer the same return in both regions: $r_H = r_F$. Therefore, the location of firms is given by the rent differential $\Delta r^* (n_H) = r_H - r_F$.

For the sake of simplicity, we focus on the case of bilateral trade. We also study the equilibrium where firms simultaneously choose their location and quality investment. This means that firms see the two decisions with the same degree of irreversibility.[4] For the previous section we know that quality investments depend on firms' locations. The rent differential writes as

$$\begin{aligned}\Delta r^* (n_H) = {} & L \left\{ \theta_H \left[(p^*_{HH})^2 - (p^*_{FH} - \tau)^2 \right] - \theta_F \left[(p^*_{FF})^2 - (p^*_{HF} - \tau)^2 \right] \right\} \\ & - \frac{1}{I_o} \left\{ \left[\theta_H p^*_{HH} + \theta_F (p^*_{HF} - \tau) \right]^2 - \left[\theta_F p^*_{FF} + \theta_H (p^*_{FH} - \tau) \right]^2 \right\}\end{aligned}$$

where optimal prices are functions of the individual and average quality (α^*_i,α^*) that depend on the location of firms. Bilateral trade imposes that the spatial equilibrium n_H satisfies condition (20).

After cumbersome simplifications, we get the rent differential

$$\Delta r^* (n_H) \propto [\Delta(\theta_H, I_o) - (n_H - 1/2)]$$

[4]Note first that the present model also applies to the sequential model where firms choose their locations before their quality investments. Second, Picard and Okubo (2008) study a same model quality is exogenous. Extending the latter analysis to a sequential model with a quality decision before the location choice adds up a coordination problem between quality and location decision time periods. Finally, the present model can be loosely interpreted as a dynamic model where, in each period, some firms die because their varieties become obsolete and are replaced by new firms that choose their location and quality.

where

$$\Delta(\theta_H, I_o) = \frac{2b\,(2\underline{\alpha} - \tau)}{c\tau}\,(\theta_H - 1/2)\,G\,(\theta_H - 1/2, I_o) \tag{22}$$

and

$$G\,(x, I_o) = \frac{(2b + c)\,I_o\,(I_o - 1)}{(I_o - 1)\,((2b + c)\,I_o - 2b) + ((4b + c)\,I_o - 2b)\ \ 4x^2}$$

The numerator and denominator of the function G are positive because $I_o > 1$. Because Δr^* decreases in n_H, the location equilibrium exists and is unique. Therefore the location equilibrium is given by

$$n_H^* = \min[\frac{1}{2} + \Delta(\theta_H, I_o), 1]$$

It is trivial to check that $\Delta(\theta_H)$ decreases with larger τ. Furthermore, using (18) and (17), it can readily be shown that

$$\frac{d\,(\alpha_H^* - \alpha_H^*)}{d\tau} = \frac{2\theta_H - 1}{I_o - 1} > 0 \quad \text{and} \quad \frac{d\alpha^*}{d\tau} \propto -\,(\theta_H n_F + \theta_F n_H) + \frac{\tau}{2}\,(2\theta_H - 1)\,\frac{dn_H^*}{d\tau} < 0$$

Therefore, whereas the quality gap falls when trade cost falls, average quality rises. The following proposition summarizes those results.

Proposition 4 *Under bilateral trade, the location equilibrium exists and is unique. In this equilibrium, high quality varieties are produced in the larger region. More firms locate in the larger region as trade cost falls. However, as the trade cost falls, the average quality rises and the quality gap between regions decreases.*

We now discuss how the spatial distribution of firms changes as region size asymmetries rise. In particular we study the home market effect according to which the market equilibrium may involve a more than proportionate share of industry in the region with larger population. That is, we measure the home market effect as $HME > 1$ where

$$HME = \frac{n_H^* - 1/2}{\theta_H - 1/2} = \frac{2b\,(2\underline{\alpha} - \tau)}{c\tau} G\,(\theta_H - 1/2, I_o)$$

For this discussion we take the home market effect under homogenous quality $\underline{\alpha}$ as the point of comparison (that is, $I_o \to \infty$ so that $G(x, \infty) = 1$). One can show that the function $G(x, I_o)$ decreases in x for all $x \in [0, 1/2]$, that it is larger than one for $x = 0$ and equal to one if $x = \sqrt{b(I_o - 1)/[2(4b + c)I_o - 4b]}$. This yields the following proposition.

Proposition 5 *Under bilateral trade, the home market effect decreases with stronger region size asymmetries θ_H. It is stronger than the home market effect under homogenous quality $\underline{\alpha}$ for small region size asymmetries whereas it is weaker for sufficiently large region size asymmetries.*

We now study the impact of a fall in the decreasing returns to scale of quality investment I_o on the location of firms. One can show that $G(x, 1) = 0$ and $G(x, \infty) = 1$ and that $G(x, I_o)$ increases in I_o for any

$$I_o < \overline{I}_o(x) \equiv \begin{cases} 1 + \frac{2x^2(2b+c)+\sqrt{2b(2b+c)(1-4x^2)x^2}}{b-(8b+2c)x^2} & \text{if} \quad x^2 < b/[2(4b+c)] \\ \infty & \text{if} \quad x^2 \geq b/[2(4b+c)] \end{cases}$$

This allows us to establish the following proposition.

Proposition 6 *Under bilateral trade, a fall in the decreasing returns to scale of quality investment I_o has a non monotone effect on firms location asymmetries n_H and home market effect (HME) if region size asymmetries are small enough ($(\theta_H - 1/2)^2 < b/[2(4b+c)]$). As I_o falls to one, location asymmetries n_H and home market effect (HME) first increase and then decrease. Otherwise, if region size asymmetries are large enough, location asymmetries and home market effect always diminish as I_o falls.*

Hence, a fall in decreasing returns to scale of quality investment I_o can have non monotone effect. This result must be related to Proposition 2 and stems from the agglomeration and dispersion forces of the choice of quality on firms' location. For large I_o, the agglomeration effect dominates as a fall in I_o increases 'average quality' more than it decreases the quality markups. For small enough I_o, the dispersion effect dominates as a fall in I_o affects more negatively the quality markups than it affects 'average quality'. Hence, firms tend to agglomerate more for large I_o and disperse more for smaller I_o. It is therefore not possible to state the technological improvement in quality investment in quality unambiguously raise or diminish countries' inequalities.

5 Extension: No trade

We here discuss the choice of location and quality of firms when trade costs are too high so that trade is infeasible. We assume that it is still too costly to build an additional plant in a second region. Consumers residing in region $i \in \{1,2\}$ have the same preference over varieties but the set of varieties available in their region is restricted to $\mathcal{V}_i$. The varieties produced in their countries have quality parameter $\widehat{\alpha}(v) : \mathcal{V}_i \subseteq [0,1] \rightarrow [\underline{\alpha}, \infty)$, $\underline{\alpha} \geq \overline{\alpha} > 0$ where $\underline{\alpha}$ is the costless quality level. Consumers maximize their utility under their budget constraint and have individual demands equal to

$$q_i(v) = (b+c)\left[\widehat{\alpha}(v) - p_i(v)\right] + c\left(\mathbb{P}_i - \mathbb{A}_i\right)$$

where $\mathbb{P}_i = \int_{\mathcal{V}_i} p_i(v)\mathrm{d}v$ and where $\mathbb{A}_i$ denotes the *local average quality* $\int_{\mathcal{V}_i} \widehat{\alpha}(v)\mathrm{d}v$. The profit of a firm producing variety v is given by

$$\Pi_i(v) = L\theta_i p_i(v) q_i(v) - I\left(\widehat{\alpha}\left(v\right)\right) - r_i(v), \quad v \in \mathcal{V}_i$$

Each firm v maximizes its price and quality level so that

$$p_i(v) \equiv p_i = \frac{(b+c)\,\alpha_i + c\left(\mathbb{P}_i - \mathbb{A}_i\right)}{2\,(b+c)} \quad \text{and} \quad \widehat{\alpha}(v) \equiv \alpha_i = \underline{\alpha} + 2p_i/I_o$$

Prices and quality is homogenous in the region. The market price and quality is given by the the fixed point of the system of two equations $\mathbb{P}_i = n_i p_i$ and $\mathbb{A}_i \equiv n_i \alpha_i$. Solving this system yields the following market price and quality levels:

$$p_i^* = \frac{(b+c-cn_i)\,\underline{\alpha}}{2\,(b+c) - cn_i - \frac{2}{I_o}\,(b+c-cn_i)} > 0$$

$$\alpha_i^* = \underline{\alpha} + \frac{2\,(b+c-cn_i)\,\underline{\alpha}}{(2\,(b+c) - cn_i)\,I_o - 2\,(b+c-cn_i)} > \underline{\alpha}$$

Firms' optimal output is equal to $q_i^*(v) = (b+c)p_i^*$ so that the profit of firm v located in region i can be written as

$$\Pi_i^* = L(b+c)\theta_i\,(p_i^*)^2\left(1 - \frac{1}{I_o}\right) - r_i$$

The rent differential is then given by

$$\Delta r^*\,(n_H) = L(b+c)\left(1 - \frac{1}{I_o}\right)\left[\theta_H\,(p_H^*)^2 - \theta_F\,(p_H^*)^2\right]$$

Firms locate in region H iff $\Delta r^*\,(n_H) \geq 0$; that is, iff $(1+\varkappa)\,p_H^* \geq p_F^*$ where $\varkappa \equiv \sqrt{\theta_H/\theta_F} - 1 \geq 0$ increases with larger θ_H. The location and quality equilibrium n_H^* solves the equation

$$\varkappa = \frac{c\,(b+c)\,(2n_H - 1)}{(b+c-cn_H)\left[(2b+c+cn_H) - \frac{2}{I_o}\,(b+cn_H)\right]}$$

where the denominator of the ratio is always positive. Because $\varkappa \geq 0$, it must be that $n_H^* \geq 1/2$. Moreover, the RHS can be shown to be a monotonically increasing function of n_H on the interval $[1/2, 1]$. Therefore the location and quality equilibrium is unique and implies that more firms locate in the larger country when the share of consumers in the latter increases. Note the location and quality equilibrium is independent of the level of costless quality $\underline{\alpha}$. Finally one can show that n_H^* increases with smaller I_o. Hence, firms agglomerate more when the decreasing return to scale in quality investment I_o falls.

Proposition 7 *Under no trade, an increase in region asymmetries raises the number of firms locating in the larger region. A fall in the decreasing returns to scale of quality investment I_o exarcerbates location asymmetries n_H.*

6 Conclusion

The present paper studies the effect of the choice of product quality on trade and location of firms. In this model consumers have preferences for the quality of a set of manufacturing varieties. Firms do not only develop and sell manufacturing varieties in a monopolistic competitive market but also determine the quality level of their varieties by investing in research and development. We show that the larger region produces varieties of higher quality and that the quality gap increases with larger asymmetries in region sizes and with larger trade costs. In a footloose capital model we find that the home market effect can be reversed when firms choose their product quality as the number of firms in the larger region does not necessarily increase with the size of that region. This is *work in progress* and further analysis is expected. In particular we envisage to study the welfare of investment in quality and to study the case of core-periphery models where workers (rather than capital) is mobile across regions.

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7 Appendix 1

Let us raise each quality $\widehat{\alpha}^*(v)$ to $\widehat{\alpha}^*(v) + \varepsilon$ where ε is a sufficiently small positive real number. We then get

$$\begin{aligned}\frac{d\Pi^{tot}}{d\varepsilon} &= \sum_i \int_{\mathcal{V}_i} \left(\frac{\partial \Pi_i^*(v)}{\partial \widehat{\alpha}(v)} + \frac{\partial \Pi_i^*(v)}{\partial \alpha} \right) dv \\ &= -L \frac{c\,(b+c)}{2b+c} \sum_i \int_{\mathcal{V}_i} \left[\theta_i p_{ii}^*(v) + \theta_j \left(p_{ij}^*(v) - \tau \right) \right] dv \\ &= -L \frac{c\,(b+c)}{2b+c} \frac{I_o}{2} \left(\alpha^* - \underline{\alpha} \right) < 0\end{aligned}$$

where we apply the envelop theorem in the first line ($\partial \Pi_i^*(v)/\partial \widehat{\alpha}(v) = 0$ by (13)), we use $\partial p_{ij}^*(v)/\partial \alpha = -c/\left(2b+c\right) < 0$ in the second line and we use (15) and integrate over varieties in the third line.

The consumer surplus is given by

$$U_i^* = \frac{1}{2} \int_{\mathcal{V}} \left[\widehat{\alpha}^*(v) - p^*(v) \right] q(v) dv + \overline{q}_i^o$$

which can be broken down by country as

$$U_i^* = \frac{(b+c)}{2} \int_{\mathcal{V}_i} \left[\widehat{\alpha}^*(v) - p_{ii}^*(v) \right] p_{ii}^*(v) dv + \frac{(b+c)}{2} \int_{\mathcal{V}_j} \left[\widehat{\alpha}^*(v) - p_{ji}^*(v) \right] \left[p_{ji}^*(v) - \tau/2 \right] dv + \overline{q}_i^o$$

If we raise again each quality $\widehat{\alpha}^*(v)$ to $\widehat{\alpha}^*(v) + \varepsilon$, we successively get

$$\begin{aligned}\frac{dU_i}{d\varepsilon} &= \frac{1}{2} \int_{\mathcal{V}_i} \frac{d}{d\varepsilon} \left[\left(\widehat{\alpha}^*(v) - p_{ii}^*(v) \right) q_{ii}(v) \right] dv + \frac{1}{2} \int_{\mathcal{V}_j} \frac{d}{d\varepsilon} \left[\left(\widehat{\alpha}^*(v) - p_{ji}^*(v) \right) q_{ji}(v) \right] dv \\ &= \frac{b+c}{2} \int_{\mathcal{V}_i} \frac{d}{d\varepsilon} \left[\left(\widehat{\alpha}^*(v) - p_{ii}^*(v) \right) p_{ii}(v) \right] dv + \frac{b+c}{2} \int_{\mathcal{V}_j} \frac{d}{d\varepsilon} \left[\left(\widehat{\alpha}^*(v) - p_{ji}^*(v) \right) \left(p_{ji}^*(v) - \tau \right) \right] dv \\ &= \frac{b+c}{2\left(2b+c\right)} \left[\int_{\mathcal{V}_i} \left[\widehat{\alpha}^*(v) b + c p_{ii}(v) \right] dv + \int_{\mathcal{V}_j} \left[\widehat{\alpha}^*(v) b + c p_{ji}^*(v) - \tau\left(b+c\right) \right] dv \right]\end{aligned}$$

where we use $dp_{ji}^*(v)/d\varepsilon = b/\left(2b+c\right) < \frac{1}{2}$. Furthermore, we can firstly integrate $\widehat{\alpha}^*(v)$ over the sets $\mathcal{V}_i$ and $\mathcal{V}_j$, secondly substitutes for the values of prices and then simplify to get:

$$
\begin{aligned}
\frac{dU_i}{d\varepsilon} &= \frac{(b+c)}{2\,(2b+c)}\left[\alpha b+\int_{\mathcal{V}_i} cp^*_{ii}(v)dv+\int_{\mathcal{V}_j}\left[cp^*_{ji}(v)-(b+c)\,\tau\right]dv\right] \\
&= \frac{(b+c)}{2\,(2b+c)}\left[\begin{array}{c} \alpha b+\int_{\mathcal{V}_i} c\frac{1}{2}\frac{2\alpha b+\tau n_j c}{2b+c}+c\frac{\widehat{\alpha}(v)-\alpha}{2}dv \\ +\int_{\mathcal{V}_j}\left[c\frac{1}{2}\frac{2\alpha b+\tau n_j c}{2b+c}+c\frac{\widehat{\alpha}(v)-\alpha}{2}-(2b+c)\,\frac{\tau}{2}\right]dv \end{array}\right] \\
&= \frac{(b+c)}{2\,(2b+c)}\left[\alpha b+c\frac{1}{2}\frac{2\alpha b+\tau n_j c}{2b+c}-(2b+c)\,\frac{\tau}{2}n_j\right] \\
&= \frac{(b+c)}{2\,(2b+c)}\left[2b\alpha\frac{b+c}{2b+c}-2b\tau n_j\frac{b+c}{2b+c}\right] \\
&= \frac{2b(b+c)^2}{2\,(2b+c)^2}\,(\alpha-\tau n_j)>0
\end{aligned}
$$

which is positive because $\alpha^* \geq \underline{\alpha} > \tau$ under condition (12).

Finally,

$$
\begin{aligned}
\frac{d}{d\varepsilon}\left(L\theta_i U_i+L\theta_j U_j+\Pi^{tot}\right) &= L\frac{2b(b+c)^2}{2\,(2b+c)^2}\left(\alpha^*-\tau\left(\theta_i n_j+\theta_j n_i\right)\right)-L\frac{c\,(b+c)}{2b+c}\frac{I_o}{2}\left(\alpha^*-\underline{\alpha}\right) \\
&= L\frac{(b+c)}{2\,(2b+c)^2}\left[\begin{array}{c} 2b(b+c)\underline{\alpha}-2\tau b(b+c)\left(\theta_i n_j+\theta_j n_i\right) \\ +\left(2b(b+c)-c\,(2b+c)\,I_o\right)\left(\alpha^*-\underline{\alpha}\right) \end{array}\right]
\end{aligned}
$$

Using (18) we have

$$
\frac{d}{d\varepsilon}\left(L\theta_i U_i+L\theta_j U_j+\Pi^{tot}\right)=2b^2 I_o\,(2b+c)\,\frac{\underline{\alpha}-\tau\left(\theta_H n_F+\theta_F n_H\right)}{I_o\,(2b+c)-2b}>0
$$

because $\underline{\alpha} > \tau \geq \tau\left(\theta_H n_F+\theta_F n_H\right)$.