A Simple Theory of Trade, Finance, and Firm Dynamics*

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Abstract We propose a stylized monopolistic competition model of international trade where firms differ with respect to the expected economic lifetime of their innovations. Upon entry, they receive a commonly observed signal which is updated over time. Jointly with partial irreversibility of investment, this generates heterogeneity in effective discount rates and, thus, in the cost of finance. In line with evidence, the model predicts a negative correlation between firms' financing costs and their age. Over a firm's life-cycle, per period net profits and the export participation probability grow. With multiple asymmetric export markets, firms gradually expand their market coverage and total sales. Exporters are less likely to exit than purely domestic firms. Asymptotically, trade liberalization reduces overall general equilibrium exit rates. Belief updating entails excessive financing of incumbents relative to entrants and too much exporting. A confidence crisis modeled by belief reversion causes an over proportional decrease in exports, thereby offering a novel interpretation of the recent trade slump.

Keywords: International Trade, Monopolistic Competition, Heterogeneous Firms, Heterogeneous Fixed Costs, Bayesian Updating

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1 Introduction

Recent theoretical work pioneered by Melitz (2003) has shed light on the role of productivity heterogeneity for the effect of international trade on firm behavior and aggregate outcomes. Given the presence of fixed costs, only more productive firms sort into exporting, and a reduction of trade costs increases aggregate productivity. Similar selection effects can be derived from firm-level differences in perceived product quality (Baldwin & Harrigan, 2010) or the degree of tradability of output (Bergin & Glick, 2009). The core prediction of these models, namely that more competitive firms are more likely to be exporters, enjoys massive empirical support. A smaller strand of theoretical work introduces heterogeneity regarding fixed market access costs into the Krugman (1980) framework while keeping marginal revenues constant across firms (Schmitt & Yu, 2001).

Unrecognized in the recent trade literature, firms also differ with respect to their exit probabilities, at least as perceived by financial markets. Ashcraft & Santos (2009) study data on credit default swaps and document a remarkable degree of heterogeneity amongst firms with respect to their perceived risk of business discontinuation. The Melitz (2003) model does not capture this stylized fact, since at each period, all firms are equally likely to be hit by a death shock. Plant death is important for aggregate statistics: Bernard and Jensen (2007) show that plant deaths account for more than half of gross job destruction in U.S. manufacturing. In our model, we continue to view business discontinuation as a discrete exogenous shock.[1] But we allow for firms to differ with respect to the probability of such a death shock arising. Upon innovating a new product, firms trigger uncertain, public observable signals about the viability of their new product (i.e., their type), yielding beliefs that are correct in expectation and are updated according to the Bayesian law in case of firm survival. In the presence of partial irreversibility of investment,[2] this assumption implies

[1]We are silent about the exact source of the shock. It may due to the the sudden disappearance of demand due to the emergence of a cheaper perfect substitute of the firm's variety or due to a technology shock causing the immediate depreciation of the firm's assets.

[2]Impulliti, Irarrazabal & Oppromolla (2011) use a Melitz (2003) model with a stochastic evolution of productivity and irreversibility of investment. They provide a rich discussion of the empirical importance of

firm-level differences with respect to their cost of finance.

As in Melitz (2003), firms are identical ex ante. The financial markets are risk neutral and perfectly competitive. At the beginning of each period, producers must invest a fixed cost which cannot be recovered but which depreciates at the end of the period. So, assuming without loss of generality that funds are available at a zero baseline interest rate, a firm's effective financing costs are equal to its per-period exit probability. If a firm survives the period, market participants update their beliefs downwards. So, as time elapses, the funding of fixed costs activities (such as exporting) becomes gradually cheaper.

Firms' marginal revenues remain constant over time, so that the model enjoys the tractability of Schmitt & Yu (2001). However, despite its simplicity, the setup generates additional insights that are not available in the Melitz (2003) framework. As only firms with sufficiently low exit hazards enter foreign markets, exporter are on average longer-lived than domestic firms. Trade liberalization allows those formerly domestic firms with lowest effective interest rates to take up exporting while domestic firms facing high interest rates are forced to exit. So, trade liberalization lowers the expected average survival time of exporters but increases that of domestic firms. Due to a composition effect, in the overall economy, expected average survival increases. Hence, liberalization leads to higher ex post stability of firms, but effects differ between exporters and domestic firms.

The model also yields insights about firm and firm-generation dynamics. Recent literature studies the dynamic behavior of firms in open economies. Impulliti, Irarrazabal & Oppromolla (2011) work with productivity shocks and irreversible investment in an otherwise standard Melitz (2003) model. Fajgelbaum (2011) stresses labor market frictions. Burstein & Melitz (2011) analyze the role of innovation. These papers have in common that they want to explain the obvious stylized fact that firms are not typically born as exporters but evolve into exporters over time. In our model, dynamics are driven by two very simple mechanisms; the cleansing mechanism: *inferior firms are more likely to default*, and the

sunk costs in trade related applications.

updating mechanism: *trust in firms increases in firm age.*

The cleansing mechanism yields firm generation dynamics. As firms with high exit probability default more likely, the type distribution of firm generations evolves over time. Average exit probabilities of firm generations decrease w.r.t. their age, yielding decreasing average discount rates, increasing average net profits and an increasing fraction of exporters. The updating mechanism is driven by type uncertainty and the resulting Bayesian updating, yielding similar firm specific dynamics as the cleansing mechanism implies for firm generations. The older a firm, the lower the discount rate it is being assigned, yielding lower costs of finance, increasing net profits and increasing probability of exporting. Besides, as firms anticipate these life cycle patterns, there are some firms that enter the domestic market realizing negative profits initially.[3] In contrast, on the export market such early entries do not occur as active firms can wait until belief updating pushes their discount rate below the threshold ensuring positive profits.

Even though belief updating is rational on individual firm level, joint analysis of cleansing and updating mechanism reveals that it leads to misvaluation of aggregate magnitudes. As start-up beliefs are correct in expectation, perceived average exit probabilities and true average exit probabilities of firm generations do not differ initially. But as the evolution of true average exit probabilities is solely driven by the cleansing mechanism and the evolution of perceived average exit probabilities is driven by the cleansing *and* the updating mechanism, the older the firm generation gets, the further perceived and true magnitudes drift apart. Thus, average discount rates of incumbents are inefficiently small, yielding excessive financing of incumbents relative to entrants (innovators). As incumbents and entrants compete for workforce, this yields insufficient entry of new firms. A corollary of this is that belief updating implies excessive exporting: If a firm enters the export market by a misjudgment of its type, it will in expectation default before accumulated profits balance exporting fixed costs, yielding a negative welfare effect.

[3] Belief updating requires that the firm is active, i.e., producing, and therefore observed by market participants.

The predictions of our model are consistent with a number of empirical stylized facts: (i) firm survival and export status are positively correlated (Greenaway et al., 2009), the link between the two running through access to finance (Goerg and Spaliara, 2009); (ii) over longer horizons of time, about 40% of total export growth occurs at the extensive margin (Bernard and Jensen, 2004); (iii) over time, firms gradually expand the number of export markets that they serve (Lawless, 2009); (iv) export activities are heavily persistent due to the existence of sunk costs (Das et al., 2007).

We use the model to study a crisis of confidence, in which market participants revise their beliefs, i.e., they delete a portion of the updating history. Since type beliefs of exporters are on average farther away from true types, this revision leads to a stronger decline in exports (and, by trade balance) of imports relative to domestic sales. Credit conditions of large old firms (exporters) deteriorate more strongly than of small young ones. These observations are in line with the effect of the Lehman Brothers crash on September 15, 2008. This shock led to a tightening of credit restrictions, in particular of large firms, and to a collapse of trade.[4]

The remainder of this paper is structured as follows. Section 2 describes the basic framework; Section 3 derives our core results under the simplifying assumption that firms' expected life times are known with certainty after entry; Section 4 extends the analysis to the more realistic case of uncertain default probabilities. Section 5 concludes.

2 Setup

Households: We consider $n + 1$ symmetric countries. Each country is populated by a representative household of size L, who supplies labor inelastically, and who cares about the quantity of a final good C according to a linear utility function. Hence, per capita utility is $u = C/L$.

[4] We are not attempting a full quantitative analysis of the crisis but rather wish to highlight a novel theoretical mechanism that may have played a role along more standard determinants such as the strong decline in demand.

Production: In each country, there is a mass M of monopolistically competitive producers of differentiated intermediate inputs, indexed by ω. These inputs are assembled by a perfectly competitive final goods sector into the final good Y according to the CES production function:

$$Y = \Big(\int q(\omega)^{\rho} d\omega\Big)^{1/\rho} = C + I, \rho \in (0,1). \tag{1}$$

The final good $Y = C + I$ can be either consumed by households or used as investment by firms. While the final good is freely tradable, differentiated inputs are subject to standard iceberg trade costs $\tau \geq 1$. Standard manipulation yields optimal input demands of final goods producers and associated expenditures:

$$q(\omega) = Q\Big(\frac{p(\omega)}{P}\Big)^{-\sigma} \text{ and } r(\omega) = R\Big(\frac{p(\omega)}{P}\Big)^{1-\sigma}, \tag{2}$$

with $\sigma = 1/(1-\rho) > 1$ and Dixit Stiglitz aggregates P, Q and $R = PQ$. Input goods are produced via a one-to-one technology, $q = l$, with labor l being the only factor of production. As firms do not differ in productivity they charge identical prices, p_d on the domestic and p_x on the export markets:

$$p_d = \frac{w}{\rho}, \text{ and } p_x = \tau p_d, \tag{3}$$

where w denotes the wage rate. Thus, domestic per period *operating* profits and revenues are identical for all firms and are given by:

$$\pi_d = (p_d - w)q_d = \Big(\frac{wq_d}{\sigma - 1}\Big), \text{ and } r_d = p_d q_d = \sigma\pi_d, \tag{4}$$

with analogous expressions for exporters.

Heterogeneity: Firm heterogeneity is introduced via firm specific per period exit probabilities $\delta \in [0,1]$ distributed with pdf $g(\delta)$ and cdf $G(\delta)$. In chapter 3 we assume that start-up investments reveal true types δ of firms, thereby deactivating the updating mechanism and isolating the dynamics generated by the cleansing mechanism. Then, from chapter 4 onwards, we drop this assumption, introducing perceived types $\hat{\delta}$ that evolve according to Bayes' law and analyze the full dynamics triggered by the cleansing and the updating mechanism.

Financial Market: We consider a risk neutral, perfectly competitive financial market and normalize the interest rate required by households to zero. Thus, in case of revealed types, a firm δ is charged a per period rate of $\delta\alpha$ for a loan with nominal α, yielding zero expected profits for creditors.[5] Analogously, in case of type uncertainty, a firm of perceived type $\hat{\delta}$ is charged $\hat{\delta}\alpha$.

Timing: We analyze an infinitely repeated game of symmetric information. Each period $t \in \mathbb{N}$ consists of three stages: $s = 1$: Inactive firms may turn active by sinking f_I units of the final output good into research and development. This effort yields a new variety of the differentiated input for sure, but the viability of the innovation δ is drawn from $g(\delta)$ and differs across firms. The market receives signals that reveal true firm types δ (chapter 3), or that yield certain beliefs of firm types $\hat{\delta}$ (chapter 4). $s = 2$: Active firms consider to either turn inactive or to sell on the domestic market (which requires fixed domestic market access of f_d), or to additionally engage in exporting (which requires fixed export market access costs of f_x). Both f_d and f_x are measured in units of the final output good. $s = 3$: Active firms may be forced to exit the market by idiosyncratic shocks, that arrive according to their per period exit probability δ, and turn inactive. Survivors remain active, generate profits and conduct loan rate repayments. In case of type uncertainty (chapter 4), beliefs are updated contingent on firm survival.

[5] Here we restrict our analysis to sunk fixed costs, that can not be recovered subsequent to firm default. One could additionally introduce a component that is not sunk. As additional insights are small – if more units of final good are needed for investment $I \uparrow$, aggregate consumption decreases $C = Y - I \downarrow$, but idiosyncratic interest rates of firms are not affected – we simply assume sunkness of fixed costs for the purpose of technical simplicity.

Aggregation: A long run equilibrium is characterized by a mass M and a type distribution $h(\delta)$ of active firms and a mass M_x and a type distribution $h_x(\delta)$ of exporters in every country. As all active firms charge the same domestic price p_d and all exporters charge the same price p_x for their exports:

$$1 = P = \Big(\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_d^{1-\sigma} M h(\delta) d\delta + n \int_0^1 p_x^{1-\sigma} M_x h_x(\delta) d\delta\Big)^{1/(1-\sigma)}$$

$$= (M p_d^{1-\sigma} + n M_x p_x^{1-\sigma})^{1/(1-\sigma)}, \tag{5}$$

by choosing P as numeraire of our model. Analogously we get $Q = (M q_d^{\rho} + n M_x q_x^{\rho})^{1/\rho}$ and $R = M r_d + n M_x r_x$.

3 Baseline Model

In this section we focus on the cleansing mechanism and its impact on firm generation dynamics. The updating mechanism is switched off by assuming perfect observability of firm types. Additionally, we assume $g(0) = 0$, i.e. no firm shall be able to survive all possible shocks. We denote expected values with respect to a certain distribution χ by $E_{\chi}(\cdot)$ and impose the technical assumption $E_g(1/\delta) \in (1, \infty)$.[6]

Zero Cut-Off Profit Conditions: Market access costs f_d and f_x are modeled as flow fixed costs which occur at the beginning of each period and which are sunk. So, in case of firm default they are lost and in case of firm survival firms repay them at the end of the period, and apply for new loans at the beginning of the next period. As the financial market is risk neutral and perfectly competitive an active firm of type δ faces per period loan rates of

[6]The restriction $E_g(1/\delta) < \infty$ is equivalent to requiring that the density $g(\delta)$ converges faster than linearly towards zero as its argument δ converges against the boundary $\delta \to 0$. The restriction $E_g(1/\delta) > 1$ precludes convergence towards the degenerate density that assigns all probability to the outcome $\delta = 1$.

$\delta P f_d = \delta f_d$, plus $n\delta f_x$ in case of exporting. Thus, domestic entry occurs only if per period operating profits π_d dominate per period loan rates δf_d, yielding $\pi_d = \delta_d^* f_d$, with δ_d^* denoting the domestic cut-off type. Analogously we get $\pi_x = \delta_x^* f_x$, with the exporting cut-off type δ_x^*. As per period operating profits earned at each market do not depend on firm type we have:

$$\pi_d = \delta_d^* f_d \quad \text{and} \quad \pi_x = \delta_x^* f_x, \tag{6}$$

for all firms. Importantly, per period *net* profits do depend on firm types as loan rate repayments δf_d for domestic market entry and δf_x for foreign market entry are type-dependent. Thus, a firm of type $\delta \leq \delta_d^*$ realizes per period net profits of:

$$\pi^n(\delta) = \begin{cases} \pi_d^n(\delta) = \pi_d - \delta f_d = (\delta_d^* - \delta) f_d & \text{if } \delta \in (\delta_x^*, \delta_d^*], \\ \pi_d^n(\delta) + n\pi_x^n(\delta) = (\delta_d^* - \delta) f_d + n(\delta_x^* - \delta) f_x & \text{if } \delta \in (0, \delta_x^*]. \end{cases} \tag{7}$$

Dividing domestic and exporting per period profits and applying (2) and (3), we get a one-to-one correspondence between cut-off types δ_x^* and δ_d^*:

$$\frac{\delta_x^* f_x}{\delta_d^* f_d} = \frac{\pi_x}{\pi_d} = \tau^{1-\sigma} \Rightarrow \delta_x^* = \tau^{1-\sigma} \frac{f_d}{f_x} \delta_d^*. \tag{8}$$

To ensure that all active firms serve their domestic market and only a subset of domestically active firms engages in exporting, we assume $f_x \geq f_d$.

Free Entry Condition: As firm types are unobservable ex ante, firms are not able to offer banks the repayment of a fixed nominal in order to be granted the loan needed for carrying out the start-up investment f_I. If, for example, the firm turns out to be of the domestic cut-off type δ_d^*, it will realize zero per period net profits and hence will not be able to deduct any positive rate payments. Therefore, firms offer the repayment of a type dependent nominal $\alpha(\delta)$ that has to be less than their expected total net profits $\alpha(\delta) \leq \Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\delta) =$

$\pi^n(\delta)/\delta$. Banks accept only if they do not incur losses in expectation. As start-up investment costs f_I are sunk and as only a fraction $G(\delta_d^*)$ of new firms is able to enter the market, this yields $E_g(\alpha(\delta)|\delta \leq \delta_d^*) \geq f_I/G(\delta_d^*)$. As banks face perfect competition, this inequality is binding. Free entry of firms drives down profits until nominal and expected total net profits coincide $\alpha(\delta) = \pi^n(\delta)/\delta$, leaving firms with with zero profits and yielding:

$$E_g(\pi^n(\delta)/\delta|\delta \leq \delta_d^*) = f_I/G(\delta_d^*). \tag{9}$$

In the Appendix we prove that cut-off values δ_d^* and δ_x^* exist and are uniquely determined by (7), (8) and (9). Moreover, we prove following correspondence of cut-off values with τ:

$$\tau \downarrow \Rightarrow \ \delta_d^* \downarrow, \ \delta_x^* \uparrow . \tag{10}$$

As direct implication of this correspondence, we get:

Proposition 1 (Trade Liberalization and Firm Churning) *Trade liberalization yields lower average firm churning, while churning of exporters increases.*

Long Run Distribution of Active Firms: In expectation, low-δ-firms drop out from the market later than high-δ-firms. Thus, the long run distribution of active firms $h(\delta)$ differs from the distribution of entering firms $g(\delta)$. Every period a certain measure M_e of g-distributed firms tries to enter the market (henceforth denoted as firm generation), yielding a certain measure $M_e g(\delta)$ of entrants per type δ. Let $i(\delta)$ denote the aggregate mass of incumbents of type δ, then firms of type δ accumulate until the measure of entrants $M_e g(\delta)$ coincides with the measure of defaulting firms $\delta i(\delta)$, yielding $i(\delta) = M_e g(\delta)/\delta$. Thus, the type distribution of one active firm equals:

$$h(\delta) = \begin{cases} \frac{g(\delta)/\delta}{\int_0^{\delta_d^*} g(\delta)/\delta \, d\delta} & \text{if } \delta \in (0, \delta_d^*], \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Correspondingly the type distribution of one exporter equals $h(\delta|\delta \leq \delta_x^*)$. As $h(\delta)$ shifts mass towards low values of δ, average turnover of firms entering the market $E_g(\delta|\delta \leq \delta_d^*)$ is higher than average market turnover $E_h(\delta)$. Cutting it short, we face following mechanism:

Proposition 2 (Cleansing Mechanism) *The older a firm generation, the lower its average exit probability.*

As loan rates, size of net profits and entry into exporting are determined by firms exit probabilities, we can directly infer

Proposition 3 (Firm Generation Effects) *The older a firm generation, the lower its average loan rate, the higher its average net profit and the higher the fraction of exporters among its members.*

We close the model by determining firm masses and per period consumption.

Firm Masses: In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\delta_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the relative mass of exporting firms $M_x = H(\delta_x^*)M$, with $H(\delta)$ denoting the cdf of the long run distribution of firm types $h(\delta)$, we get:

$$M = wL/[(\sigma - 1)(f_d\delta_d^* + f_x nH(\delta_x^*)\delta_x^*)]. \tag{12}$$

For a step-by-step derivation of (12) refer to the Appendix.

Welfare: We can determine the equilibrium wage rate w from $P = 1$ and receive aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + nH(\delta_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}. \tag{13}$$

As utility is linear in consumption, (13) constitutes a measure of welfare. A detailed derivation of (13) is provided in the Appendix. From the measure of entering firms and the fixed costs they have to bear, we can directly determine the quantity of the final product spent for start-up investments and market entries every period:

$$I = (f_I + f_d G(\delta_d^*) + n f_x G(\delta_x^*)) M_e. \tag{14}$$

From (2) and (3) we get $\tau p_x = \tau^{1-\sigma} q_d < q_d$. Thus, trade liberalization increases the number of available varieties in every country. Moreover, trade liberalization increases average productivity: Relations (10) yield that trade liberalization forces firms with low net profits out of the market ($\hat{\delta}_d^* \downarrow$) shifting production towards more efficient firms. As per period net profits constitute the difference of per period profits (that are independent of firm type) and per period fixed costs (that decrease in length of firm life), trade liberalization raises $Y - I = C$ and we get:

Proposition 4 (Trade Liberalization and Welfare) *Trade liberalization increases welfare.*

4 Uncertain Firm Types

In this section, we discuss variations and applications of our simple baseline model from above. First, we introduce type uncertainty, leaving everything else unchanged (subsection 4.1), then we discuss consequences of a confidence crisis (subsection 4.2) and conclude with the analysis of the asymmetric country case (subsection 4.3). Henceforth start-up investments trigger uncertain signals, yielding perceived types $\hat{\delta}_0 \in [0, 1]$. When referring to the cross section of firms we drop the age indicating subscript and denote perceived types with $\hat{\delta}$. Perceived types $\hat{\delta}$ constitute expected values of their corresponding belief $\delta \sim b_{\hat{\delta}}(\delta)$, i.e. $E_{b_{\hat{\delta}}}(\delta) = \hat{\delta}$. Again, we impose the technical assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1, \infty)$ for all $\hat{\delta}$. Moreover, we assume starting perceived types $\hat{\delta}_0$ to be distributed with the true type pdf g introduced in chapter 3. Thus, average perceived types and average true types of firm generations do not differ initially. Turning to the perceived type evolution of individual firms, we can frame a very simple mechanism. Every period a firm survives, its perceived type is being updated according to Bayes' law until it is hit by a shock and forced to exit the market. As updating is only triggered by good news (firm survival), we get $\hat{\delta}_0 > \hat{\delta}_1 > \cdots > \hat{\delta}_t > \ldots$ for all periods a firm survives, with $\hat{\delta}_t$ denoting its perceived type in its t^{th} period subsequent foundation.

Proposition 5 (Updating Mechanism) *The older a firm, the lower its perceived exit probability.*

4.1 Symmetric Countries

Except from the type uncertainty introduced above, the setup from chapter 3 remains unchanged.

Zero Cut-Off Profit Conditions: As loans for market access costs are negotiated on a per period basis, firms face rate payments $\hat{\delta}_t f_d$ (plus $n\hat{\delta}_t f_x$ in case of exporting) that always reflect current firm status $\hat{\delta}_t$. Thus, the older a firm the lower its rate payments. As firms anticipate this life cycle pattern, the entry decision arises from comparing present value of expected future profits with present value of expected future costs. Consider a firm with initial perceived type $\hat{\delta}_0$, then present value of expected future profits from domestic activity equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi(\hat{\delta}_t)) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x) = E_{b_{\hat{\delta}_0}}(1/\delta)\pi_d + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)n\pi_x$, with $t(\hat{\delta}_0)$ denoting the period of entry into exporting in case of survival. Introducing a weighted probability of survival until entry into exporting $\psi(\hat{\delta}_0)$, defined via $E_{b_{\hat{\delta}_0}}(\psi(\hat{\delta}_0)/\delta) := E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)$ the present value of expected future profits can be rewritten as $E_{b_{\hat{\delta}_0}}(1/\delta)(\pi_d+\psi(\hat{\delta}_0)n\pi_x)$. The present value of expected future costs from domestic entry equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d) = E_{b_{\hat{\delta}_0}}(1/\delta)\bar{\delta}(\hat{\delta}_0)f_d$, with $\bar{\delta}(\hat{\delta}_0)$ denoting the expected life time average perceived type of a firm with start-up perceived type $\hat{\delta}_0$.[7] For the cut-off value $\hat{\delta}_d^*$ both expressions coincide, yielding $\pi_d +$

[7]As $\hat{\delta}_t$ decreases monotonically in t, the expected amount of cleared entry costs $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) <$

$\psi(\hat{\delta}_d^*)n\pi_x = \bar{\delta}(\hat{\delta}_d^*)f_d$ with $\bar{\delta}(\hat{\delta}_d^*) < \hat{\delta}_d^*$. As domestic entry yields initial loan rates $\hat{\delta}_0 f_d$, firms with start-up perceived types $\hat{\delta}_0 \in [\bar{\delta}(\hat{\delta}_d^*), \hat{\delta}_d^*)$ realize negative profits initially, speculating on positive profits in future periods. Differently, in case of exporting, firms can wait until their perceived type is low enough and realize positive per period net profits from the first period onwards. As domestic and exporting per period operating profits do not depend on firm type we get:

$$\pi_d = \bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x \quad \text{and} \quad \pi_x = \hat{\delta}_x^* f_x. \tag{15}$$

Thus, a firm of age t and perceived type $\hat{\delta}_t$ realizes a per period net profit of:

$$\pi^n(\hat{\delta}_t) =$$

$$\begin{cases} \pi_d^n(\hat{\delta}_t) = \pi_d - \hat{\delta}_t f_d = (\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x & \text{if } \hat{\delta}_t \in (\hat{\delta}_x^*, \hat{\delta}_d^*], \\ \pi_d^n(\hat{\delta}_t) + n\pi_x^n(\hat{\delta}_t) = (\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t)f_d + (1 - \psi(\hat{\delta}_d^*))(\hat{\delta}_x^* - \hat{\delta}_t)n f_x & \text{if } \hat{\delta}_t \in (0, \hat{\delta}_x^*]. \end{cases} \tag{16}$$

Dividing domestic and exporting per period profits and applying (2) and (3), we get a one-to-one correspondence between $\hat{\delta}_x^*$ and $\hat{\delta}_d^*$:

$$\begin{aligned} \frac{\hat{\delta}_x^* f_x}{\bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x} \quad &= \quad \frac{\pi_x}{\pi_d} = \frac{q_x}{q_d}\left(\frac{p_x - \tau w}{p_d - w}\right) = \tau^{1-\sigma} \\ &\Rightarrow \quad \hat{\delta}_x^* = \tau^{1-\sigma}\frac{\bar{\delta}(\hat{\delta}_d^*)f_d}{(1 + n\psi(\hat{\delta}_d^*))f_x}. \end{aligned} \tag{17}$$

Summarizing, the updating mechanism from proposition 5 yields:

Proposition 6 (Firm Specific Effects) *Net profits of firms and ex ante probability of exporting increase in firm age. Some firms face negative per period net profits from domestic activity initially, while entry into export occurs only in case of positive per period net profits.*

Free Entry Condition: In line with the known firm type case, firms offer the repayment of their signal dependent expected total net profits $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi^n(\hat{\delta}_t))$ and risk neutral,

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$E_{b_{\hat{\delta}_0}}(1/\delta)\hat{\delta}_0 f_d < \infty$ is finite by assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1,\infty)$. Thus, there exists a unique $\bar{\delta}(\hat{\delta}_0) \in (0, \hat{\delta}_0)$ fulfilling $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d)$.

perfectly competitive banks grant loans until expected profits coincide with expected costs:

$$E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi^n(\hat{\delta}_t))|\hat{\delta}_0 \leq \hat{\delta}_d^*) = f_I/G(\hat{\delta}_d^*). \tag{18}$$

In the Appendix we prove that cut-off values $\hat{\delta}_d^*$ and $\hat{\delta}_x^*$ exist and are uniquely determined by (16), (17) and (18). Moreover, we prove following correspondence of cut-off values and τ and n:[8]

$$\tau \downarrow \Rightarrow \; \hat{\delta}_d^* \downarrow, \; \hat{\delta}_x^* \uparrow . \tag{19}$$

yielding identical trade liberalization effects on firm churning as in the known firm type case (proposition 1).

Long Run Distributions of Active Firms: First we determine the steady state distribution of true types and then, second, the steady state distribution of perceived types. Every period a certain measure M_e of firms with g-distributed true types tries to enter the market. As true types are not observable, even high-δ-firms may enter, if their start-up signal is sufficiently good, i.e. if $\hat{\delta}_0 \leq \hat{\delta}_d^*$, yielding the modified distribution of entrants $j(\delta) = \int_0^{\hat{\delta}_d^*} b_{\hat{\delta}_0}(\delta)g(\hat{\delta}_0)d\hat{\delta}_0$. Let $i(\delta)$ denote the aggregate mass of incumbents of type δ, then firms of true type δ accumulate until the measure of entrants $M_e j(\delta)$ coincides with the measure of defaulting firms $\delta i(\delta)$ yielding $i(\delta) = M_e j(\delta)/\delta$. Thus, we get long run true type distributions of one active firm:

$$h(\delta) = \frac{j(\delta)/\delta}{\int_0^1 j(\delta)/\delta \; d\delta}. \tag{20}$$

Perceived types of entrants are distributed with $g(\hat{\delta}_0|\hat{\delta}_0 \leq \hat{\delta}_d^*)$ and evolve according to the Bayesian updating process subsequently. Thus, if we fix a perceived type $\hat{\delta}$ and want to determine the density of incumbents for this perceived type, we have to consider two components: new entrants with perceived type $\hat{\delta}_0 = \hat{\delta}$ and older firms that started with a start-up

[8] It is not possible to determine a similar relationship of f_x with cut-off values, as this relationship is not independent from distributional assumptions and concrete choices of parameter values.

perceived type $\hat{\delta}'_0 > \hat{\delta}$ and happen to be assigned a current perceived type $\hat{\delta}$ by Bayesian updating. Let $\hat{\delta}_{-t} > \hat{\delta}$ denote the start-up perceived type that coincides with $\hat{\delta}$ after t periods of Bayesian updating. Then, the entry density of perceived type $\hat{\delta}_{-t}$ equals $M_e g(\hat{\delta}_{-t})$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)$ yielding the perceived type density of incumbents $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t) M_e g(\hat{\delta}_{-t})$.[9] Thus, we get long run perceived type distributions of one domestic firm:

$$\hat{h}(\hat{\delta}) = \begin{cases} \frac{\hat{j}(\hat{\delta})}{\int_0^{\hat{\delta}_d^*} \hat{j}(\hat{\delta})\, d\hat{\delta}} & \text{if } \hat{\delta} \in (0, \hat{\delta}_d^*], \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Misvaluation of Active Firms: Even though it is rational to update beliefs on firm level it leads to misvaluation in aggregate figures. As start-up signals $\hat{\delta}_0$ for individual firms are correct in expectation, perceived average exit probabilities and true average exit probabilities of new born firm generations do not differ. Initially, both coincide with the expected value of the true type distribution g. Subsequently, both decline with respect to generation age by excess exit of high-δ-types according to the cleansing mechanism (proposition 2). But as the descent of aggregate perceived types is amplified by the updating mechanism (proposition 5), perceived and true figures differ from the second period onwards and keep drifting apart the older the firm generation gets. As this misjudgment yields loan rates that are too small in expectation, too many firms turn exporters.[10] Active and inactive firms compete at the labor market for work force in order to produce and to embark new markets or to conduct the start-up investment respectively. Thus, this comparative favorisation of incumbents yields too little entry of new firms. Summarizing, the joint impact of the cleansing mechanism from proposition 2 and the updating mechanism from proposition 5 implies:

Proposition 7 (Firm Generation Effects) *The older a firm generation, the further per-*

[9] As $g(0) = 0$, only perceived types $\hat{\delta} > 0$ are possible. And as $\lim_{t\to\infty}(\hat{\delta}_d^*)_t = 0$, there always exists a finite t s.t. $(\hat{\delta}_d^*)_t < \hat{\delta}$. Thus, $T(\hat{\delta})$ is finite for all $\hat{\delta} > 0$.

[10] If a firm with true type $\delta > \hat{\delta}_x^*$ enters the export market by a misjudgment of its type $\hat{\delta}_t \leq \hat{\delta}_x^*$, it will (in expectation) default before sunk entry costs nf_x are balanced by accumulated per period net profits.

ceived and true average exit probabilities deviate, yielding too little loan rates for incumbents. Thus, the steady state exhibits excessive exporting and insufficient start-up investment.

Firm type generation effects from the known type case (proposition 3) carry over to the uncertain firm type case.

Firm Masses: In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\hat{\delta}_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the mass of exporting firms $M_x = \hat{H}(\hat{\delta}_x^*)M$, with $\hat{H}(\hat{\delta})$ denoting the cdf of the long run distribution of perceived firm types $\hat{h}(\hat{\delta})$, we get:

$$M = wL/[(\sigma-1)(f_d\bar{\delta}(\hat{\delta}_d^*) + nf_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*))]. \tag{22}$$

For a step-by-step derivation refer to the Appendix.

Welfare: We determine the equilibrium wage rate w from $P = 1$ and receive aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + n\hat{H}(\hat{\delta}_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}. \tag{23}$$

As utility is linear in consumption, (23) constitutes a measure of welfare. A detailed derivation of (23) is provided in the Appendix. The final product is spent for start-up investments, market entry of new firms and for foreign market entry of incumbents that turn exporters by Bayesian updating. Let $(\hat{\delta}_x^*)_{-t}$ denote the start-up perceived type that coincides with $\hat{\delta}_x^*$ after t periods of updating, then (conditional on survival) all firms with start-up perceived types $\hat{\delta}_0 \in ((\hat{\delta}_x^*)_{-(t-1)}, (\hat{\delta}_x^*)_{-t}]$ will turn exporters in their t^{th} period. As the entry density of a perceived type $\hat{\delta}_0$ equals $M_e g(\hat{\delta}_0)$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_0}}((1-\delta)^t)$, the measure of firms of age t that turn exporters by Bayesian updating every period equals $\int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0$. Adding all possible ages[11] $t = 1, 2, \ldots, T(\hat{\delta}_x^*) < \infty$ we receive:

$$
\begin{aligned}
I &= (f_I + f_d G(\hat{\delta}_d^*) + n f_x G(\hat{\delta}_x^*)) M_e \\
&\quad + n f_x \Sigma_{t=1}^{T(\hat{\delta}_x^*)} \int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t) M_e g(\hat{\delta}_0) d\hat{\delta}_0. \qquad (24)
\end{aligned}
$$

Similar to the known type case trade liberalization forces firms with low net profits out of the market ($\hat{\delta}_d^* \downarrow$) shifting production towards firms with higher net profits. But as loan rates (that depend on perceived types) and real per period fixed costs (that depend on real types) differ systematically, this shift does not always improve average efficiency of the economy. As we prove in the Appendix the welfare result from proposition 4 does not hold in general if real firm types are unobservable.

Proposition 8 (Trade Liberalization and Welfare) *In case of uncertain firm types, trade liberalization can have a negative welfare effect.*

4.2 Crisis of Confidence

On September 15, 2008, the collapse of Lehman Brothers sparked the most severe world-wide recession after World War II. While it is not clear, whether the failure of the investment bank was a consequence or rather the cause of the recession, many belief that the severance of the problems was hugely aggravated (Bacchetta et al., 2010). While we are not trying to give a quantitative assessment of the crisis, our model allows to shed light on the different effects of a belief revision on small as compared to large firms. It captures, admittedly in a a very stylized way, the facts that (i) exports dropped much more than GDP in most countries and in the world (see Behrens et al., 2010, for a discussion) and that (ii) large firms saw their financing conditions deteriorate more strongly than small ones (ifo, 2012).

[11] $T(\hat{\delta}_x^*)$ denotes the number of periods of Bayesian updating a firm with highest possible start-up perceived exit probability $\hat{\delta}_d^*$ needs to turn exporter. As $\lim_{t\to\infty} \hat{\delta}_t = 0$ for all $\hat{\delta}$, $T(\hat{\delta}_x^*)$ has to be finite.

Belief Revision: We consider a shock that prompts all agents to return to former beliefs, i.e. some firm survival informations is deleted.[12] There are several natural ways to model a belief revision. A belief revision could prompt all agents to return to their beliefs a certain number of periods ago, it could prompt all agents to delete a certain fraction of firm survival histories, or in the extreme case prompt all agents to return to start-up perceived types of firms. As start-up beliefs constitute lower bounds for belief revisions, the shock does not force any firms to exit domestic markets. On the contrary some firms will stop exporting.

Proposition 9 (Crisis of Confidence) *Implementing a crisis of confidence by a belief revision, it forces some firms to exit foreign markets while leaving the number of domestic firms unaltered.*

4.3 Asymmetric Countries

By incorporating country heterogeneity w.r.t. fixed market entry costs, we generate multi-level growth into exporting. The older a firm the more export destinations it will serve. To avoid unnecessary technical complications, we consider infinite countries $\iota \in [0,1]$, each being of zero measure. A foreign firm faces fixed costs f_ι upon market entry in country ι. Countries are ordered according to the size of their entry costs, i.e. $\iota < \kappa \Rightarrow f_\iota < f_\kappa$. To circumvent the special case of all firms only serving the market of country $\iota = 0$, which arises due to our simplifying assumption of free tradability of final goods, we introduce an additional layer of final product. The final good produced by countries shall henceforth be referred to as country good. Those country goods are then used to produce the "new" final good at zero transformation costs according to the standard CES-production function (25). Both, country goods and final goods are traded freely. This setup extension nests all previous results, as all countries produce identical amounts of country goods in the symmetric country

[12] Entry and exit informations are excluded from the revision, as neither defaulted firms can be reanimated, nor new born firms can be extinguished by a change in belief.

case. Under this additional stage of production the (normalized) price index of the final good is given by:

$$1 = P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota\Big)^{1/(1-\sigma)}, \tag{25}$$

with

$$P_\iota = \Big(\int_0^1 \Big(\int_{\omega_{\kappa,\iota}\in\Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)}, \tag{26}$$

denoting the price index of country ι, where $\Omega_{\kappa,\iota}$ denotes the set of intermediate goods imported from country κ. All payments, such as wage payments, loan rates or fixed costs, are still measured in units of final good. Whenever results are independent of country type, we suppress the country indicating subscript.

Uniform Wage Rate: In case of infinite, zero measure countries, costs or profits a firm faces within one country are infinitesimal and hence negligible. Only costs or profits a firm faces within a positive measure of countries will influence its actions. Thus, all firms that conduct the start-up investment will enter domestic markets, as this entrance at infinitesimal entry costs entails a positive probability of entry into a positive measure of foreign countries, yielding positive expected profits. Hence, true and perceived types of entrants are distributed with pdf g in all countries. Besides domestic entry fees, also domestic profits are infinitesimal and hence negligible. Thus, firm actions (the choice of export destinations and export prices) solely depend on perceived firm type and are independent of firm location. As neither the distribution, nor the action of firms depend on their location, the aggregate production of intermediate inputs by firms located in one country, is identical for all countries. Thus, by trade balance, all countries are compensated with identical amounts of final good yielding identical wages in all countries.

Zero Cut-Off Profit Conditions: Firms will enter a foreign market ι as soon as per period profit π_ι dominates per period costs $\hat{\delta}_t f_\iota$, yielding the first zero cut-off profit condition

$\pi_\iota = \hat{\delta}_\iota^* f_\iota$, with $\hat{\delta}_\iota^*$ denoting the cut-off type for entry into market ι. Dividing per period profits, we get the second zero cut-off profit condition $\hat{\delta}_\iota^* = (f_\kappa/f_\iota)\hat{\delta}_\kappa^*$ for all $\iota, \kappa \in [0,1]$. Thus, $f_\iota < f_\kappa$ yields $\hat{\delta}_\iota^* > \hat{\delta}_\kappa^*$, i.e. the higher the market entry costs the smaller the set of perceived firm types that will enter. Let $\kappa(\hat{\delta}_t)$ denote the "last" country a firm of perceived type $\hat{\delta}_t$ will export to, i.e. the country with cut-off value $\hat{\delta}_\kappa^* = \hat{\delta}_t$. Then, a firm of perceived type $\hat{\delta}_t$ will export to all countries $\iota \in [0, \kappa(\hat{\delta}_t)]$. The lower the firms perceived exit probability $\hat{\delta}_t$ the greater its measure of export destinations, until, for $\hat{\delta}_t \leq \hat{\delta}_1^*$ it exports to all countries.

Free Entry Condition: Free entry of firms ensures that expected future profits $E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)}(\pi_\kappa - \hat{\delta}_t f_\kappa)d\kappa))$ coincide with costs for the start-up investment f_I. As we prove in the Appendix, zero cutoff and free entry conditions determine cut-off values $\hat{\delta}_\iota^*$ uniquely. From the ordering of cut-off values ($\iota < \kappa \Rightarrow \hat{\delta}_\iota^* > \hat{\delta}_\kappa^*$) and the updating mechanism (proposition 5) we receive following proposition:

Proposition 10 (Firm Specific Effects) *The measure of export destinations increases in firm age.*

Long Run Distributions of Active Firms: As firms of all types enter, the true type distribution of incumbents equals $h(\delta) = (g(\delta)/\delta)/(\int_0^1(g(\delta)/\delta)d\delta)$ and the perceived type distribution equals $\hat{h}(\hat{\delta}) = \hat{j}(\hat{\delta})/\int_0^1 \hat{j}(\hat{\delta})d\hat{\delta}$, with $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)M_e g(\hat{\delta}_{-t})$.

Firm Masses: All firms that conduct the start-up investment enter and firm exit occurs with respect to the true type distribution. Thus, the steady state correspondence of firm masses of entrants and incumbents equals $M_e = E_h(\delta)M$. Firm export status depends on perceived firm type. Thus, the mass of firms within a certain country that export to country κ equals $M_\kappa = \hat{H}(\hat{\delta}_\kappa^*)M$. Additionally, taking into account the labor market clearing

$L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we receive:

$$M = wL/[(\sigma - 1)\int_0^1 \hat{H}(\hat{\delta}_\kappa^*)\hat{\delta}_\kappa^* f_\kappa d\kappa]. \tag{27}$$

For a step-by-step derivation refer to the Appendix.

Welfare: Determining the equilibrium wage rate w from $P = 1$, we receive aggregate per period consumption:

$$C = Lw/P = Lw = L(\rho/\tau)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(\sigma-1)}. \tag{28}$$

Again, a detailed derivation is provided in the Appendix. In line with section 4.1 aggregate per period investment consists of the units of final product needed for start-up investment, $f_I M_e$, the units needed for direct market entry, $\int_0^1 G(\hat{\delta}_\iota^*) f_\iota d\iota M_e$, and the units needed for entry into market ι by Bayesian updating, $f_\iota \Sigma_{t=1}^{T(\hat{\delta}_\iota^*)} \int_{(\hat{\delta}_\iota^*)_{-(t-1)}}^{(\hat{\delta}_\iota^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t) M_e g(\hat{\delta}_0) d\hat{\delta}_0$. As the last term arises for all markets, we get:

$$\begin{aligned} I &= f_I M_e + \int_0^1 G(\hat{\delta}_\iota^*) f_\iota d\iota M_e \\ &\quad + \int_0^1 \Big(f_\iota \Sigma_{t=1}^{T(\hat{\delta}_\iota^*)} \int_{(\hat{\delta}_\iota^*)_{-(t-1)}}^{(\hat{\delta}_\iota^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t) M_e g(\hat{\delta}_0) d\hat{\delta}_0\Big) d\iota, \end{aligned} \tag{29}$$

which completes the characterization of the general equilibrium under type uncertainty in an asymmetric country setting.

5 Conclusion

Upon innovation, firms are uncertain as to the viability of their new product. Market expectations about the lifetime of an innovation determine the effective costs of finance for firms. So, if some fraction of firms' investment needs are irreversible, firms differing with respect to the perceived probability of death shocks face different financing possibilities. International trade interacts with this heterogeneity: firms with lower perceived default probabilities are more likely to be exporters, lower trade costs make the expected survival rates of domestic firms smaller but those of exporters larger; firm survival is longer in open compared to closed economies. All this facts are well supported by empirical evidence.

In contrast to firm-level heterogeneity in productivity or product quality, a firm's life expectancy cannot be easily inferred from its production process or its sales statistics. Rather, it is more likely that market participants only receive a noisy signal about the true type of a firm. Conditional on survival of the firm, market participants update their beliefs. This process has important further implications for firm behavior and aggregate outcomes. First, it implies that the financial conditions faced by firms improve over time. Second, due to this, firms will be gradually growing as they enter more and more markets. Third, the updating process leads to an excessive expansion of large incumbents to the expense of startups, so that the number of existing firms tends to be too small. Fourth, a sudden reversal of beliefs leads to reduction in economic activity, but the collapse of trade flows is larger than that of total income. Again, these facts square well with empirical facts.

The main advantage of the framework is its simplicity. As long as firms are homogeneous with respect to variable components of revenue, aggregation is very simple. This allows an analytical characterization of firm dynamics without making assumptions on the form of distribution functions. It also makes further extensions of the model possible. One interesting avenue for further research would be to add a more complete description of financial frictions to the model or to allow for a second source of heterogeneity, possibly of the form used in Melitz (2003).

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Appendices

Appendix A: Baseline Model

Existence and Uniqueness of Cut-Off Values: Starting with (9) and applying (7) in the third step of the calculation, we get:

$$\begin{aligned} f_I/G(\delta_d^*) &= E_g(\pi^n(\delta)/\delta|\delta \leq \delta_d^*) \\ &= E_g(\pi_d^n(\delta)/\delta|\delta \leq \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n\pi_x^n(\delta)/\delta|\delta \leq \delta_x^*) \\ &= E_g((\delta_d^* - \delta)f_d/\delta|\delta \leq \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n(\delta_x^* - \delta)f_x/\delta|\delta \leq \delta_x^*) \\ &= f_d(\delta_d^* E_g(1/\delta|\delta \leq \delta_d^*) - 1) + nf_x(G(\delta_x^*)/G(\delta_d^*))(\delta_x^* E_g(1/\delta|\delta \leq \delta_x^*) - 1), \end{aligned}$$

yielding:

$$f_d G(\delta_d^*)(\delta_d^* E_g(1/\delta|\delta \leq \delta_d^*) - 1) + nf_x G(\delta_x^*)(\delta_x^* E_g(1/\delta|\delta \leq \delta_x^*) - 1) = f_I. \tag{30}$$

Replacing δ_x^* by δ_d^* via (8), the left hand side of (30) is a continuous function of δ_d^* that equals 0 for $\delta_d^* = 0$ and is strictly positive for $\delta_d^* = 1$ as $E(1/\delta) > 1$. Thus, it is always possible to choose $f_I > 0$ sufficiently small in order to ensure the existence of a solution of (30). Uniqueness follows from proof by contradiction: Assume there are at least two different domestic cut-off values $\delta_d^\natural < \delta_d^\flat$ solving (30). Then net per period profits of firms $\delta \in (\delta_d^\natural, \delta_d^\flat)$ have to be less or equal to net per period profits of firm $\delta^\flat$, which yields a contradiction as net per period profits strictly decrease in δ.

Derivation of (10): Equation (8) exhibits a direct effect $\tau \downarrow \Rightarrow \delta_x^* \uparrow$. As $\delta_x^* \uparrow$ yields $\delta_x^* \downarrow$ via (30) and as there is no direct effect of τ on (30), we get correspondence (10).

Derivation of (12): Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) +$

$nM_x(r_x - \pi_x) = M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x))$. Transforming r_d and r_x according to (4) and (6) and replacing π_d and π_x via (6) we get:

$$\begin{aligned} wL &= M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x)) = M(\sigma\pi_d - \pi_d + nH(\delta_x^*)(\sigma\pi_x - \pi_x)) \\ &= M((\sigma - 1)\delta_d^* f_d + nH(\delta_x^*)(\sigma - 1)\delta_x^* f_x) = M(\sigma - 1)(\delta_d^* f_d + nH(\delta_x^*)\delta_x^* f_x) \end{aligned}$$

yielding (12).

Derivation of (13): From $1 = P = (Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$\begin{aligned} w &= w/P = w(Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{\sigma-1}} = w(Mp_d^{1-\sigma} + nH(\delta_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\ &= (w/p_d)(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} = \rho(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}. \end{aligned}$$

Together with $C = Lw/P$, this implies (13).

Appendix B: Uncertain Firm Types (Symmetric Countries)

Existence and Uniqueness of Cut-Off Values: Starting with (18) and applying (16) in the fourth step of the calculation, we get:

$$
\begin{aligned}
f_I/G(\hat{\delta}_d^*) &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi^n(\hat{\delta}_t))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d^n(\hat{\delta}_t) + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_t))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d^n(\hat{\delta}_t)) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_{t(\hat{\delta}_0)+t}))|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*)-\hat{\delta}_t)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x)) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t(\hat{\delta}_x^* - \hat{\delta}_{t(\hat{\delta}_0)+t})n f_x)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*)-\bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x)) \\
&\quad +E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*)-\bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x)E_{b_{\hat{\delta}_0}}(1/\delta) \\
&\quad +n f_x(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*)-\bar{\delta}(\hat{\delta}_0))f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x \\
&\quad +\psi(\hat{\delta}_0)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x)E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*),
\end{aligned}
$$

yielding:

$$
G(\hat{\delta}_d^*)E_g([(\bar{\delta}(\hat{\delta}_d^*)-\bar{\delta}(\hat{\delta}_0))f_d+(\psi(\hat{\delta}_0)(\hat{\delta}_x^*-\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))-\psi(\hat{\delta}_d^*)\hat{\delta}_x^*)n f_x]E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \le \hat{\delta}_d^*) = f_I, \quad (31)
$$

with $t(\hat{\delta}_0)$ denoting the period in which a firm of start-up perceived type $\hat{\delta}_0$ would enter the export market in case of survival. Existence and uniqueness of the solution $\hat{\delta}_d^*$ follows from analog arguments as in the known firm type case.

Derivation of (19): From (17) we get $\partial\hat{\delta}_x^*/\partial\tau < 0$, yielding $\tau \downarrow \Rightarrow \hat{\delta}_x^* \uparrow$. As $\psi(\hat{\delta}_d^*) \le \psi(\hat{\delta}_0)$

yields the correspondence $\hat{\delta}_x^* \uparrow \Rightarrow \hat{\delta}_x^* \downarrow$ via (31), and as there is no direct effect of τ on (31), we get correspondence (19).

Derivation of (22): Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) + nM_x(r_x - \pi_x) = M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x))$. Transforming r_d and r_x via (4) and (15) and replacing π_d and π_x according to (15) and π_d and π_x according to (15), yields:

$$
\begin{aligned}
wL &= M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x) \\
&= M(\sigma\pi_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(\sigma\pi_x - \pi_x) \\
&= M((\sigma - 1)(\bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x) + n\hat{H}(\hat{\delta}_x^*)((\sigma - 1)\hat{\delta}_x^* f_x) \\
&= M(\sigma - 1)(f_d\bar{\delta}(\hat{\delta}_d^*) + nf_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*),
\end{aligned}
$$

yielding (22).

Derivation of (23): From $1 = P = (Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$
\begin{aligned}
w &= w/P \\
&= w(Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{\sigma-1}} \\
&= w(Mp_d^{1-\sigma} + n\hat{H}(\hat{\delta}_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\
&= (w/p_d)(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} \\
&= \rho(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}.
\end{aligned}
$$

Together with $C = Lw/P$, this implies (23).

Proof of Proposition 8: If a firm starts exporting by a misjudgment of its true type, expected profits from exporting $\Sigma_{t=0}^{\infty}(1 - \delta)^t\pi_x$ are dominated by costs f_x. In this case the firm uses up more units of final good of a country than it produces, yielding a negative welfare effect. By constructing a very special ex ante distribution g' of true and perceived firm types, we can increase the fraction of firms that enter by misjudgment of their type almost to 1. Let $(\hat{\delta}_x^*)_{-1}$ denote the value of the start-up perceived type that coincides with the exporting cut-off value after one period of updating and let $R = \int_{\hat{\delta}_x^*}^{(\hat{\delta}_x^*)_{-1}} g(\hat{\delta}_0)d\hat{\delta}_0$ denote the fraction of start-up firms $\hat{\delta}_0$ within $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$. Then, those start-up firms will enter foreign markets in their second period of operation, yielding a negative aggregate welfare effect, as perceived firm types are correct in expectation. By shifting probability density towards a value $\hat{\delta}_0'$ within the open interval $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$, we can push R arbitrarily close towards 1.[13] For some value of R' (close enough to 1) the negative welfare effect from export entry of firms belonging to this fraction will outweigh the possibly positive welfare effect from export entry of the residual $1 - R'$. Under such an ex ante distribution g' of true and perceived firm types a change to prohibitive variable trade costs $\tau \to \infty$ or to $n \to 0$ accessible foreign markets increases welfare. Hence, by the mean value theorem of differential calculus, there exists a τ' and a n' at which liberalizing trade yields negative welfare effects.

Appendix C: Uncertain Firm Types (Asymmetric Countries)

Existence and Uniqueness of Cut-Off Values: Using the zero cut-off profit conditions $\pi_\iota = \hat{\delta}_\iota^* f_\iota$ and $\hat{\delta}_\iota^* = (f_\kappa/f_\iota)\hat{\delta}_\kappa^*$ we can transform the free entry condition into an equation with only one unknown $\hat{\delta}_0^*$:

$$\begin{aligned} f_I &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} (\pi_\kappa - \hat{\delta}_t f_\kappa)d\kappa)) \\ &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((\hat{\delta}_\kappa^* - \hat{\delta}_t)f_\kappa)d\kappa)) \\ &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((f_0/f_\kappa)\hat{\delta}_0^* - \hat{\delta}_t)f_\kappa d\kappa)) \end{aligned} \tag{32}$$

[13] As this shifting of probability density draws $\hat{\delta}_x^*$ and $(\hat{\delta}_x^*)_{-1}$ closer together, $\hat{\delta}_0'$ has to belong to the interval subsequent the shifting of probability density. As $\hat{\delta}_x^* < (\hat{\delta}_x^*)_{-1}$ for all non-degenerate distributions g, such a $\hat{\delta}_0'$ always exists.

The right hand side of (32) is a continuous monotonically increasing function of $\hat{\delta}_0^*$. It equals zero for $\hat{\delta}_0^* = 0$, as in this case all cut-off values vanish $\hat{\delta}_\kappa^* = (f_0/f_\kappa)\hat{\delta}_0^* = 0$ and thus no firm will enter into exporting. If $\hat{\delta}_0^* = 1$ all firms will export to country $\iota = 0$ and to countries with similarly low market entry costs $\iota = 0 + \epsilon$.[14] Hence $\int_0^{\kappa(\hat{\delta}_0)}((f_0/f_\kappa)\hat{\delta}_0^* - \hat{\delta}_0)f_\kappa d\kappa > 0$ for all $\hat{\delta}_0$, yielding a strictly positive right hand side of (32). Thus, for all sufficiently small $f_I > 0$, there exists a unique solution $\hat{\delta}_0^*$ of (32).

Derivation of (27): From the labor market clearing condition $L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we get:

$$
\begin{aligned}
wL &= \int_0^1 M_\kappa w\tau q_\kappa d\kappa = \int_0^1 M_\kappa(p_\kappa q_\kappa - (p_\kappa - w\tau)q_\kappa)d\kappa \\
&= \int_0^1 M_\kappa(r_\kappa - \pi_\kappa)d\kappa = \int_0^1 M_\kappa(\sigma - 1)\pi_\kappa d\kappa \\
&= \int_0^1 M_\kappa(\sigma - 1)\hat{\delta}_\kappa^* f_\kappa d\kappa = \int_0^1 \hat{H}(\hat{\delta}_\kappa^*)M(\sigma - 1)\hat{\delta}_\kappa^* f_\kappa d\kappa \\
&= M(\sigma - 1)\int_0^1 \hat{H}(\hat{\delta}_\kappa^*)\hat{\delta}_\kappa^* f_\kappa d\kappa,
\end{aligned}
$$

yielding (27).

Derivation of (28): Determining the country index

$$
\begin{aligned}
P_\iota &= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota}\Big)d\kappa\Big)^{1/(1-\sigma)} \\
&= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p_\iota^{1-\sigma} d\omega_{\kappa,\iota}\Big)d\kappa\Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_\iota^{1-\sigma} M_\iota d\kappa\Big)^{1/(1-\sigma)} \\
&= p_\iota M_\iota^{1/(1-\sigma)} = w(\tau/\rho)(\hat{H}(\hat{\delta}_\iota^*)M)^{1/(1-\sigma)}
\end{aligned}
$$

and plugging it into $P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota\Big)^{1/(1-\sigma)} = w(\tau/\rho)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(1-\sigma)}$ we receive $C = Lw/P = L(\rho/\tau)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(\sigma-1)}$.

[14] We assume that market entry costs f_ι increase continuously in ι.