On Blending Competitive Trade Models

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1. Introduction

The three most frequently used models found in the theory of international trade when pure competition is assumed are those labeled the Ricardian model, the Heckscher-Ohlin model, and the Specific-Factors Model. Whereas the Ricardian model typically deals with a single factor of production (usually assumed to be labor) that can differ in its productivity from country to country, the latter two models are focused on industrial and country comparisons in the ratio(s) in which various factors of production are required for different commodities and the ratio(s) in which factors of production are found in the endowment bundles of different countries.[1] The Ricardian model, the small-scale version of the Heckscher-Ohlin model (the (2x2) version with two factors and two commodities), as well as the small-scale version of the Specific-Factors model, (the (3x2) version in which in each sector labor is combined with a factor used only in that sector while labor is mobile between sectors), have been subject to detailed algebraic analysis of a comparative statics variety. This type of analysis often has used the differential calculus to examine how small shocks (e.g. commodity price changes, factor endowment changes**,** or changes in taste patterns or technology) affect commodity outputs and the internal distribution of income among factors of production as well as income distribution between countries.

One of the basic characteristics of the existence of international trade is that countries are not required to produce the entire array of commodities that are locally consumed. Instead, international exchange allows a great degree of specialization among countries in the pattern of production in each, with the basic result that production patterns tend to focus on commodities in which a country has a comparative advantage. As a consequence, although active production in any country may be limited to a small number (such as a pair of commodities or, in the basic Ricardian model just a single best commodity), larger changes may induce a *change* in the *pattern* of production, and such a change is not covered by the techniques of calculus. Just as a diagrammatic representation is so useful (compared with the calculus) in depicting the gains from trade compared with autarky, so also such techniques can easily be deployed to illustrate required changes when patterns of production in each country must be altered

[1] Later in the paper I also discuss the Ricardian model with a changed assumption of a variety of labor types found in each country, as developed in Ruffin (1988).

following a large shock to equilibrium. Such shocks require alterations both in small scale as well as in larger-scale versions of Heckscher-Ohlin and Specific-Factors models, alterations that lead to *blends* of the two models as well as a use of the Ricardian analysis.[2]

2. The (3x2) Specific-Factors Model

The (3x2) version of the Specific-Factors Model is the most simple example in which a country produces at least two commodities but makes use of at least three inputs. The key to its simplicity is that each commodity makes use only of two of the inputs, which dictates that both sectors make use of a single input in common as well as an input not used in the other sector. Put differently, there is a single factor (usually taken to be labor) that is completely *mobile* between sectors as well as a pair of factors, each of which has absolutely *no* mobility between sectors. In general situations any particular factor of production may have some mobility among sectors, and the specific-factors setting is one that takes the degree of mobility to extremes – factors are either completely specific to a sector or completely mobile between (or among) sectors. As will be seen, the pair of distinctions among commodities as to relative factor requirements and relative degree of flexibility in technology are indeed relevant in the specific-factors model, but the important focus is on the degree of mobility possessed by inputs in moving from sector to sector.

A condition of equilibrium in competitive models is that if production of a commodity is positive, unit costs must be equal to the commodity price. Let the commodities be denoted by numbers, so that, letting the a_{ij}'s denote input/output coefficients:

(1) $a_{L1}w + a_{K1}r_1 = p_1$

(2) $a_{L2}w + a_{K2}r_2 = p_2$

Letting a "hat" (^) represent a relative change, e.g. $\hat{x} = \frac{dx}{x}$, and recognizing that cost minimization requires (for small changes) that the distributive share average of the relative changes in input/output coefficients is a second-order small, equations (3) and (4) represent the competitive profit conditions of change, where θ_{ij} is factor i's share in industry j:

(3) $\theta_{L1}\hat{w} + \theta_{K1}\hat{r}_1 = \hat{p}_1$

(4) $\theta_{L2}\hat{w} + \theta_{K2}\hat{r}_2 = \hat{p}_2$

[2] Earlier attempts at discussing these issues are found in Jones and Marjit (1992) and Jones (2008).

In words, the relative change in unit costs equals the relative change in commodity price. As a consequence each commodity price change is flanked by the relative changes in appropriate factor returns. This leads directly to the following inequality rankings if the price of the first commodity increases relatively to that of the second:

(5) $$\hat{r}_1 > \hat{p}_1 > \hat{w} > \hat{p}_2 > \hat{r}_2$$

Thus if commodity 1's relative price rises, the nominal wage change is trapped between the two rates of change of commodity prices, whereas the return to the fixed factor used in the first sector must increase in *real* terms, while that of the other fixed factor must decline in *real* terms. These returns to the fixed factors are, of course, better thought of as *rents*.

With labor the only mobile factor, a crucial concern asks about the re-allocation of labor between the two sectors as a result of the commodity price alterations. Attention thus focuses on the condition that the overall labor supply is fully employed:

(6) $$a_{L1}x_1 + a_{L2}x_2 = L$$

Note that each output is restricted by the quantity of the specific factor used in that sector:

(7) $$a_{Kj}x_j = K_j$$

Expressed in terms of rates of change,

(8) $$\hat{x}_j = -\hat{a}_{Kj} + \hat{K}_j$$

Now differentiate the full employment condition for labor, (6), and then substitute for the output changes shown in equation (8) to obtain:

(9) $$\lambda_{L1}(\hat{a}_{L1} - \hat{a}_{K1}) + \lambda_{L2}(\hat{a}_{L2} - \hat{a}_{K2}) = \hat{L} - \lfloor \lambda_{L1}\hat{K}_1 + \lambda_{L2}\hat{K}_2 \rfloor$$

(Each λ_{Lj} denotes the fraction of the labor force allocated to the j^{th} sector). The relative changes in the labor/capital ratios used in each sector depend upon changes in the wage rate relative to the change in the commodity price in each sector. In words, such a relationship reveals the elasticity of demand for labor in each sector, or, in more basic terms, the elasticity of the marginal product curve for labor in each sector. Denote such an elasticity by γ_{Lj}:

(10) $$\gamma_{Lj} \equiv -\frac{(\hat{a}_{Lj} - \hat{a}_{Kj})}{\hat{w} - \hat{p}_j}$$

Substitute these expressions into equation (9) in order to solve for the equilibrium change in the wage rate:

$$(11) \qquad \hat{w} = [\beta_1 \hat{p}_1 + \beta_2 \hat{p}_2] - \left(\frac{1}{\gamma_L}\right)\left[\hat{L} - \left(\lambda_{L1}\hat{K}_1 + \lambda_{L2}\hat{K}_2\right)\right]$$

where β_j equals $\lambda_{Lj}\, \gamma_{Lj} / \gamma_L$, the sum of the β's is unity, and γ_L is the economy's overall elasticity of demand for labor, the weighted average of sector elasticities, $(\lambda_{L1}\, \gamma_{L1} + \lambda_{L2}\, \gamma_{L2})$.

Equation (11) displays not only the effect of commodity price changes on the wage rate, but also the effect at constant commodity prices of any change in factor endowments. In particular, any increase in the endowment of labor lowers the wage rate, while any increase in either capital supply serves to increase the wage rate. Assuming now that factor endowments stay constant, the effect of any change in commodity prices on the wage rate reveals that, as in (5), the wage rate increases by a smaller relative amount if a single commodity price increases. Furthermore, the extent of such a change depends upon two features of the comparison between the technologies used to produce the two commodities as well as the importance of the commodity whose price increases in the aggregate value of the national income.[3] In particular:

$$(12) \qquad \beta_1 = \theta_1 \{i_1\, s_1\},$$

where the term i_1 refers to the expression λ_{L1}/θ_1, a comparison of the fraction of the labor force used in the first sector to the overall importance of the first sector in the national income, and thus is greater than unity only if the first sector is relatively labor-intensive. The term, s_1, is greater than unity only if the technology in the first sector is *relatively flexible*, and is defined as the comparison between the elasticity of demand for labor in the first sector and the economy's overall elasticity of demand for labor, (γ_{L1} / γ_L). In addition to these two characteristics of technology it is also the case that an increase in p_1 has a more powerful effect in raising the wage rate the more important in the economy is the value of output in the first sector.

This simple form of the specific-factors model was developed formally in Jones (1971) (see also Samuelson (1971)), and, in a less formal setting, by Haberler in the 1930's. As Samuelson often described the model, it is the one most closely associated with partial equilibrium analysis. Later I describe the ease with which it generalizes to higher dimensions.

[3] For details see pp. S – 20, 21 in Caves, Frankel and Jones, (10th edition).

3. The (2x2) Version of the Heckscher-Ohlin Model

Although both Heckscher and Ohlin discussed the relationship between trade and factor endowments in general terms, it was primarily Paul Samuelson who developed the more simple (2x2) Heckscher-Ohlin model. This model was considered the standard general equilibrium model to be used in the trade area for decades. Before pursuing the fate of this model in higher dimensions, it may be useful to consider its basic features in the (2x2) version, especially as they differ in many respects from the Specific-Factors (3x2) model. Indeed, I would argue that focusing on the differences between these two basic models in low dimensional settings is a useful exercise to appreciate the similarity and shared properties in higher dimensions, leading to particular blends of the two basic models.

The key difference between these basic models is that whereas the Specific-Factors model has more factors of production (3) than commodities produced (2), the Heckscher-Ohlin small version has the *same* number, (2), of factors as commodities. A major consequence of this is that there are as many factor prices to be determined as there are competitive profit conditions of equilibrium. Whereas the pair of price equal cost equations for the specific factors model, equations (1) and (2), and the corresponding profit equations of change, (3) and (4), could not be solved uniquely for factor rewards if commodity prices (or changes in prices) are known, at least not without depending as well on factor endowments or their changes, the (2x2) Heckscher-Ohlin model exhibits zero profit equations (13) and (14) that *do* link wage changes and the return to (the single type of) capital *uniquely* to the pair of commodity price changes.[4]

(13) $$\theta_{L1}\hat{w} + \theta_{K1}\hat{r} = \hat{p}_1$$

(14) $$\theta_{L2}\hat{w} + \theta_{K2}\hat{r} = \hat{p}_2$$

Although it is possible to solve separately for the wage change and the change in the return to capital, it may be more convenient merely to subtract (14) from (13) (recognizing that the distributive shares in each industry sum to unity) to obtain (15):

(15) $$|\theta|(\hat{w} - \hat{r}) = (\hat{p}_1 - \hat{p}_2)$$

Because each commodity price change in (13) and (14) is trapped between changes in the wage rate and return to capital, and because the first sector is assumed to be labor-intensive, any

[4] I am assuming that the first commodity is relatively labor-intensive compared with the second commodity, regardless of factor prices. This is tantamount to assuming the determinant of distributive factor shares in this equation set, $|\theta|$, or $(\theta_{L1} - \theta_{L2})$, is positive.

increase in the price of the first commodity compared to that of the second commodity leads to the following set of inequalities in factor returns:

(16) $\hat{w} > \hat{p}_1 > \hat{p}_2 > \hat{r}$

This ranking should be compared with that for the (3x2) specific-factors setting when the relative price of the first commodity increases: In the specific-factors model, once again the big winner and big loser in the rankings are both specific factors, but the wage change itself is trapped between the two commodity price changes, the ranking in (5). In the (3x2) setting there is no *magnification effect* for mobile labor, while there is such an effect for the wage rate in the Heckscher-Ohlin ranking shown by (16). There is an additional, more subtle, difference to be found between these two basic, low-dimensional models. Although in both the increase in the relative price of the labor-intensive commodity has the effect of raising the nominal wage rate, in the Specific-Factors model the greater is the labor-intensity of the commodity whose price has increased, the greater will be the increase in the wage rate. Just the reverse is the result in the Heckscher-Ohlin case. Admittedly, such a price change raises the wage rate by a magnified amount in the (2x2) Heckscher-Ohlin setting, but by a dampened amount in the Specific-Factors Model. However, consider the comparison of equations (11) and (12) for the Specific-Factors model with that of equation (15) for the Heckscher-Ohlin model.[5] In the Specific-Factors model the wage rate increase is larger the greater is the labor-intensity of the labor-intensive sector, while in Heckscher-Ohlin a greater gap in the comparison of labor-intensities results in a smaller wage rate increase.

4. Simple Diagrams for the Basic Models

As a prelude to the use of diagrams in higher dimensional cases, I review here the diagrams that best portray the scenario of pre- and post-trade for both the (3x2) and (2x2) models.

I start with the Specific-Factors Model with two commodities. Not reproduced here is the fairly standard 4-quadrant diagram found in many textbooks.[6] Such a diagram is used to illustrate how the production-possibilities curve in the first quadrant is derived using the total product curves for each commodity (quadrants II and IV) and the constraint that the labor force

[5] Equation (15) refers to the change in the wage rate compared with the change in the rental rate. For a formal solution for the wage rate change by itself solve equations (13) and (14) to obtain the solution for $\hat{w}$, holding p_2 constant: $\hat{w} = \{\theta_{K2}/|\theta|\}\, p_1\}\ \hat{p}_1$. If for given intensities in the second commodity the labor-intensity in the first sector would be larger, the denominator becomes larger, and the increase in the wage rate smaller.

[6] E.g. the diagram on p. 94 of the 10th edition of Caves, Frankel, Jones, *World Trade and Payments.*

is fully employed (quadrant III) and, not shown explicitly, that the quantity of the specific factor in each sector is given and constant. Shown here in Figure 1 is the oft-used diagram that illustrates how the equilibrium wage rate is determined by the intersection of two "front-to-front" curves showing the *value* of the marginal physical product of labor schedule for each sector. The equilibrium nominal wage rate is at point *A*, and the allocation division of labor at point *B*. Special notice should be focused on what is measured on the vertical axis. It is the *nominal* wage rate, with the prices of both commodities assumed given.[7] If the price of the first commodity should increase, say by 10%, the $VMPP_L^1$ schedule would be shifted *upwards* by 10%. As a consequence, the *equilibrium* wage rate can be seen to increase by a smaller percentage, as reflected in the positive fractional values for the β's in equations (11) and (12). If, instead, the endowment of the specific factor in the first industry were to be increased by 10**%,** the $VMPP_L^1$ schedule would be shifted to the *right* by 10%, and the equilibrium wage rate increased (perhaps by more than 10% if the two curves were sufficiently inelastic).

Whereas the Specific-Factor model displayed technology for each sector by using the marginal product schedule (physical or value), motivated by the immobility of the specific factors between sectors, the Heckscher-Ohlin model makes use of bowed-in isoquants for each sector. Here also there is both a *physical* version (the unit isoquant) and a *unit value* version. The latter depends upon knowledge of commodity prices. To save space, and in anticipation of the higher-dimensional cases, Figure 2 illustrates the concept of the *Hicksian Composite Unit-Value Isoquant* in a 4-commodity case. This is a concept used frequently in international trade theory, and it assumes that the technologies that are displayed are those in use in a particular country (and not necessarily found in common in other countries), whereas the commodity prices used are typically those ruling in *world* markets.[8] The composite unit-value isoquant in Figure 2 is the inner locus, *DCBGA* (extended at both ends), composed of individual unit-value isoquants and connecting tangency cords. What these cords reveal is that for given endowment proportions (say with endowment rays passing through *DC* or *BA),* full employment of both factors would necessitate production of *two* commodities. For example, point *E* shows a bundle of capital and labor that would produce a dollar's worth of commodity 1. There is no way of producing a dollar's worth of any other *single* commodity using less labor and capital than shown by point *E*. However, a point such as *G* illustrates how a blend of around 50 cents worth of the second commodity using techniques shown by point *B* and around 50 cents worth

[7] A curve showing the *marginal product of labor* schedule would have, on the vertical axis, the wage rate divided by the given commodity price, which would not be appropriate in Figure 1 since commodity price units differ between commodities.

[8] If unit-value isoquants for these four different commodities assumed domestic prices as given in an autarky situation in which all four commodities are produced, they would all be tangent to a common budget line with slope reflective of the autarky local wage/rent ratio.

of the first commodity using techniques shown by point *A* would require less capital and labor than would point *E* that produces only the first commodity. Note also that this diagram reveals that the technology that the country possesses for producing the third commodity is inferior to that possessed by some other country (or countries). For this country to produce a dollar's worth of commodity 3 more labor and capital would be used than is required by producing an alternative point on the Hicksian composite unit-value isoquant.

Unit-value isoquants combine knowledge of two important variables (technology and world commodity prices), whereas the two-quadrant diagram in Figure 3 illustrates these separately, focusing here on the two-commodity case. The two positively sloped schedules in the top diagram show how the capital/labor ratio that would be utilized to produce each commodity is dependent upon prevailing wage/rent ratios. The arbitrary assumption that the first commodity is produced with relatively labor intensive techniques for any factor price ratio is embodied by the locus for the first commodity lying everywhere below that for the second commodity for any commonly faced factor price ratio.[9] Points *H* and *F* represent two possible endowment proportions, say in two different countries. If, say, Home has endowment ratio, *H*, its wage/rental rate must lie in the range *AB*, while another country, Foreign, assumed here to share the same technological knowledge, must have the overlapping factor price ratio shown by the range *NQ*. The dependence of each country's wage/rental ratio on the commodity price ratio, e.g. as reflected in equation (15), is illustrated in the lower part of Figure 3 by the downward-sloping locus, drawn in order to illustrate two important points: Any increase in the relative price of the first commodity must, if the country is actively producing both commodities, serve to increase the relative wage rate because the first commodity is assumed to be labor-intensive. Secondly, there is the *magnification effect* formally shown in equation (15) by the (positive) *fractional* value for $|\theta|$ or by the ranking shown in equation (16). Finally, note that in the (2x2) Heckscher-Ohlin model the transformation curve in either country is not that much different from a downward-sloping line (as in the Ricardian model). For Home the transformation schedule, whose slope reflects the commodity price ratio, allows incomplete specialization *only* for the price range *CD*, while for Foreign both commodities can be produced only if the commodity price ratio is in range *JM*.[10] The famous Factor-Price Equalization

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[9] This rules out the possibility of factor-intensity reversal, which might occur if the two loci in the top part of Fig. 3 crossed each other. A word of warning: In later diagrams that illustrate growth possibilities in an economy capable of producing more than two commodities the axes are reversed to emphasize how factor endowment proportions help to *determine* factor price ratios.

[10] Wolfgang Mayer (1974) provides a well-cited treatment of the specific-factor setting as a short-run model and the (2x2) model as a long-run model, and illustrates the connection between transformation schedules in the two models, with the (2x2) schedule being an envelope of the various (3x2) curves as one kind of capital is over time converted to the other kind as output patterns differ. The surprise that I find with his illustration in Fig. 1 (p. 959)

Theorem (see Samuelson, 1948, 1949) for these two countries would hold only if commodity prices allow both countries to be incompletely specialized by lying in the *JD* range.

Note that the lower dimensional version of the Specific Factors model has dimension (3x2), while the lower dimensional version of the Heckscher-Ohlin model has dimension (2x2). With reference once again to Figure 3 suppose, for the Home country, that the relative price of the first commodity should increase beyond 0D (in the lower quadrant of Figure 3). Then this country becomes completely specialized in producing the first commodity, and the dimension passes from (2x2) to a (2x1) model. This latter model can be referred to as a *degenerative* case of the Specific-Factors model, and such a change provides a basic example of a *blending* of two types of model. (The dependence of the w/r ratio on the commodity price ratio in cases in which Home is completely specialized could be captured in Figure 3 by adding, in the lower quadrant, a vertical cord connecting point A to the curve in the lower quadrant corresponding to price ratio C, as well as a cord from this curve for the price ratio D to higher values of p_1/p_2).

5. The Multi-Commodity International Trade Scenario

In a Heckscher-Ohlin setting the move to higher dimensions is difficult in one direction but easy in another. The difficulty arises in situations in which more than two inputs are involved in any production process.[11] Until later I will assume that activities in the Heckscher-Ohlin setting use only labor and capital, both of which are mobile among sectors. Concentrate here on the case in which many commodities can be produced and many countries engaged in trade. This is useful in two principle ways. First, it allows a much more realistic setting than does the two-commodity version by revealing how much the move from autarky to international trade allows a great degree of *concentration* of productive activities whereas discussions for the basic 2x2 model mostly emphasize that trade can allow different *proportions* of the two commodities produced than the ratio in which they are consumed. (This point is made more obviously in Ricardian models of trade in which the concept of comparative advantage in production for trade makes its first appearance). Second, it is a setting in which the peculiar characteristics of growth and development in an economy open to trade are exposed: If a country can greatly

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is in the *range* of slopes of the long-run (2x2) model – almost the entire range of relative price from zero to infinity - compared with the limited CD range in Figure 3 above. (Such a large range appears also in the Krugman, Obstfeld, Melitz text, e.g. p. 84.) Neary (1978) also emphasizes the short-run and long-run interpretation.

[11] Later I emphasize that the difficulty is not in having a large number of inputs *in the economy*. This will become obvious in general higher dimensional models of the Specific-Factors type, where every activity uses labor plus an input used *only* in that activity. By contrast, the plethora of different degrees of substitution among factors if many are used in a *single* activity, and of the possibility of complementarity among some factors if more than two are involved in production, are not discussed here, but mentioned briefly later.

limit what it *produces* without sacrificing its *consumption* desires, in the process of growth it may make different decisions as to *which* commodities to produce. For open developing economies the vision of fairly balanced growth among productive sectors of the economy makes little sense if consumers can obtain their needs more economically by importing, and pay for these by concentrating on producing those commodities in which the country has the greatest comparative advantage. And the answer to the question of *which* commodities to produce will systematically change as capital/labor endowment proportions are altered with development.

Figure 2 illustrated such systematic changes. With the capital/labor endowment ratio shown by the slope of a ray from the origin, development at the given set of commodity prices would involve producing only the first commodity if such a ray (not drawn) is flatter than 0A, and with further increases in the endowment proportions at these prices the country would produce commodities 1 and 2 (e.g. with a dollar's worth produced with input point G), and then produce only the second commodity with further growth until some of commodity 4 can be produced as well. How many commodities need it produce? If all commodities are traded, the answer is no more than the number of productive inputs. The earlier discussion of the Hicksian composite unit-value isoquant, the convex hull of the unit-value isoquants (at world prices) in Figure 2, shows the tangent cords where two commodities are produced, as well as the strictly bowed-in sections along which full employment of both productive factors can be obtained by producing only a single commodity. As long as world prices do not mirror the details of technology in any single country, it would be accidental for more than two unit-value isoquants to be tangent to any single cord.[12] Thus a country need not produce more commodities than the number of its productive factors. Could it produce fewer? Of course it can, as Ricardian models loudly proclaim, as well as in the (2x2) Heckscher-Ohlin setting whenever prices are such that the economy is driven to complete specialization. For example the Home country in Figure 3 could not produce any of the first commodity if its relative price should be lower than 0C.

Figure 4 is like a five-commodity version of the top diagram in Figure 3, but the axes have been reversed, to highlight that with growth the equilibrium relative wage rate becomes endogenous.[13] And more has been changed: Other commodities have been added, and the technological relationship between wage/rental ratios and the capital/labor ratios that would

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[12] An exception would have to be made for a country large enough in world trade that even in a two-factor setting world prices would have to allow such a country to produce at least some of the other commodities. Of course here I do not include an important and realistic possibility, one in which obstacles to international trade (natural or man-made) result in a set of "non-tradeables" being locally produced since otherwise local demands could not be met. The issue of producing many commodities when non-tradeables are also produced are discussed later, and also in Jones and Marjit (1992) as well as Jones (2012).

[13] An early use of this kind of diagram is found in Jones (1974).

be adopted if each commodity were produced are illustrated by the upward-sloping dashed curves. The solid sections along these curves as well as the horizontal solid sections between curves illustrate the pattern of production that would take place for any given endowment proportions if world prices are given and unchanged. The horizontal lines are relevant if the economy produces a pair of commodities, such as commodities 1 and 2 along the cord at point G in Figure 2 if, in Figure 4, the endowment proportions are at point I. Along these flats in Figure 4 an economy produces a pair of commodities and, as a consequence, the given commodity prices determine the factor prices (and their ratio). In a sense, the solid locus shows how even though the wage/rental ratio is often improved (at given commodity prices) by growth in the capital/labor endowment ratio, there are also regions in which such growth leaves factor prices unchanged. If two countries share the same technology and world prices, but have different endowments which, nonetheless, allow them to produce the same pair of commodities, their wages and rents on capital will be the same, reflecting the Samuelson Factor-Price Equalization Theorem.

In this setting with many commodities, the sets of endowments for which a pair of commodities is produced are interposed with regions in which only a single commodity is produced.[14] This represents the *degenerate* case of the specific-factors setting referred to earlier, with the (3x2) dimensions replaced by the (2x1) dimensions. (Of course with only a single commodity produced, there is no distinction between mobile and specific factors.) Not only is the diagram useful in comparing the situation between two countries that share a common technology but differ in endowment capital/labor ratios, it also reveals how, if a country develops and its K/L ratio rises, both the Heckscher-Ohlin model (2x2) and the degenerate case of the Specific Factors model can be blended to reveal how the bundle of a pair of commodities produced can change over time, with always an intermediate step of complete specialization. Furthermore, Figure 4 is useful in revealing possible reactions to an increase in the world price of the second commodity. What happens to factor returns? It depends upon whether the country has endowment proportions such that it initially produces commodities 1 and 2 (such as for an economy with endowments given by point I), in which case the second commodity is relatively the more capital-intensive of the pair of commodities produced and the wage/rental ratio would fall. Alternatively, an economy producing both commodities 2 and 3 would, since commodity 2 is relatively labor-intensive, experience a magnified increase in the wage rate and the return to capital would fall. In this setting it would be incorrect to rule out "Heckscher-Ohlin behavior" if a pair of countries sharing the same technology and both producing the commodity whose price has increased respond in one

[14] The exception to this remark would be found in the extreme case in which all isoquants are right-angled, i.e. do not allow smooth substitution between labor and capital. In such an event, the positively sloped dashed lines depicting technology in Figure 4 would be vertical.

country with a wage fall and in the other with a wage increase. Indeed, this kind of "factor-intensity reversal" is seen to be quite possible even if the technologies by themselves have rising curves (in a Figure 4 scenario) that do not intersect.

In Figure 4 an increase in p_2 enlarges the range of endowments for which a country is specialized to the second commodity (to region AB along the technology curve for the second commodity after the price increase). If an economy's factor endowment has it specialized to producing only commodity 2 before and (of course) after the price increase both the wage rate and the return to capital would increase by the same relative amount as the commodity price, so that the wage/rental rate does not change (unless the K/L ratio changes). The standard (2x2) Heckscher-Ohlin model describes the consequences on the internal distribution of income for a developing country as it grows by increasing its capital/labor endowment: Of course factor prices do not change if commodity prices are constant (and the country remains incompletely specialized to the same pair of commodities). As Figure 4 emphasizes, increased endowments of capital relative to labor only serve to increase the relative wage rate when a country changes its *pattern* of production by becoming completely specialized in what was its capital-intensive commodity before growth. Of course changes in the relative price of a commodity that is produced always changes factor returns, but in a manner highlighted in the (2x2) version of the model only if the economy is *incompletely* specialized.

One of the main points made in the brief exposition of the Heckscher-Ohlin (2x2) model's determinants of factor prices, e.g. in Equation (15), is that commodity prices completely determine factor prices if the number of inputs (2) exactly matches the number of commodities produced (2). As opposed to this, in the simple (3x2) version of the Specific-Factors model other features besides the factor-intensity ranking of the pair of technologies are also involved in determining factor prices when commodity prices change.[15] Nonetheless, a diagram such as Figure 4 can be used to suggest how *both* features of technology as well as the relative importance of the commodity that has risen by a *finite* amount in price help to explain what happens to wages and rents in the (2x2) case when initially a country is incompletely specialized. Consider the latter feature first, and suppose that, in Figure 4, the economy produces both the first and second commodities, but primarily the second, at point C. If the price of the second commodity rises by a finite amount, as shown by the downward shift in the flat for 1 and 2 to point A (as well as the upward shift in the flat if producing 2 and 3 to point B), the economy's wage rate does indeed fall, but *not* to the flat anchored at A. Instead, the economy becomes completely specialized to the second commodity. Once this happen, the

[15] For example, as illustrated by Equations (11) and (12).

wage/rent ratio no longer falls – the nominal wage and the return to capital *both* increase proportionally to the increase in p_2 once specialization is complete.[16]

Now focus on the role of technology. To avoid cluttering Figure 4, consider in Figure 5 the situation in which the price of commodity 2 is once again increased, but, initially, Foreign produces 2 and 3 at point D. (Ignore for a moment the curve 2'). For a sufficiently large relative increase in p_2 Foreign's relative wage increases by a magnified amount to the flat anchored at B (as in Equation (15)), since the second commodity is labor-intensive compared with the third. However, suppose instead that the technology for producing the second commodity is more flexible – shown by curve 2' instead of curve 2. If Foreign were initially producing both 2 and 3 at point D, it would, after a sufficiently large increase in p_2, not experience such a strong response in the wage/rental ratio – to point E instead of a point on the flat through B. That is, for very small price changes from starting point D the change in flexibility would not matter, but it would for larger changes. Although the analysis of the (2x2) model isolates only the factor-intensity ranking as important in linking commodity prices to factor prices, the other feature of technology (relative flexibility) eventually comes into its own with finite price changes.

Turn, now, to the role of the factor intensity ranking between two industries. As explained in the previous section, in the (3x2) specific factor model a given increase in the price of a commodity raises the wage rate by more the more labor-intensive are techniques used to produce that commodity compared with the other. By contrast, in the (2x2) Heckscher-Ohlin model the *extent* of the wage increase is *smaller* the greater is the gap in labor intensities between sectors. Yes, the labor-intensity ranking is important in both model settings, but it is better for labor if the intensity difference is small and the price of the labor-intensive commodity increases in the Heckscher-Ohlin model. In Figure 5 suppose the upward sloping curve for the third commodity were closer to that for the second commodity (with good 2 still being relatively labor-intensive). With the initial point (such as D) reflecting both commodities initially produced, a given price increase for commodity 2 raises the "flat" even higher than the one supported by point B.

6. Technological Progress and Factor-Saving Bias

The preceding section illustrated, in Figure 4, how the "blended" diagram relating the wage/rental ratio to changes in the capital/labor endowment ratio is altered when there is an increase in the world price of one of the commodities, say commodity 2. The range of

[16] Details for finite price changes are discussed in Jones (2008).

endowment ratios for which the country is completely specialized in the second commodity is increased to the range shown by the AB section along the schedule for the second commodity. Suppose, now, that instead of a price change, there is technological progress experienced by the second commodity. At the initial level of wages and rents suppose that the "Hicksian extent of cost reduction at initial prices" is a given amount, comparable to the increase in p_2 that leads to the increased AB range of specialization in producing the second commodity shown in Figure 4. If so, the diagram to illustrate the effect of technical progress would look much like Figure 4's price change *if* technical progress is "Hicks-neutral". What is meant by this kind of "neutrality" is that the upward-sloping dashed curve 2 in Figure 4 would itself not be altered – i.e. for any given wage/rental ratio the ratio of capital to labor used to produce the second commodity would not be changed, although the wage rate and return to capital would each be increased by the same relative amount, *viz*. the Hicksian measure of technical progress. However, technical progress could, instead, be *biased* in the sense that at the initial factor price ratio the techniques for producing the second commodity might become more or less labor-intensive than shown by the upward-sloping curve for the second commodity in Figure 4. Such a biased technological progress for commodity 2 is illustrated by the leftward shift in the technology curve (2) in Figure 6. The upward sloping dashed section shows the capital/labor techniques that would be used in producing the second commodity if progress has a capital-saving bias. The two horizontal dashed sections show, respectively, the new Heckscher-Ohlin section in which both commodities 1 and 2 are produced as well as the section in which production of the second commodity is undertaken along with some production of the third.[17]

The heavy broken line in Figure 6 (like that in Figure 4) shows the blended locus initially of the relationship between factor endowment ratios and factor-price ratios. The dashed section illustrates the consequence on the production schedule for commodity 2 of technological progress that is capital-saving in character – along the upward sloping section commodity 2 is the only commodity produced, but with a new technology that requires a smaller amount of capital per unit of labor. What is the effect of this biased technological change for commodity 2 on wages and rentals and the ratio of the two? That depends in part on the initial production pattern (assuming it is in the Heckscher-Ohlin horizontal section in which both commodities 2 and 1 are being produced). If technical progress were only infinitesimally large (as in typical discussions in which calculus is used (or "hat-algebra")), it could still be biased towards being capital-saving. A standard result in the theory of international trade (e.g. in Jones, 1965) is that in the (2x2) Heckscher-Ohlin model the effect on factor prices depends *only* upon the relative extent of the progress and not at all on the bias. This is a result that has bothered a number of labor economists, and indeed they should be bothered since bias *does* matter if the change is of

[17] This diagram is Figure 3 in Jones (2008).

finite dimension. As Figure 6 reveals, the manner in which factor prices are affected if bias is present and the change is finite depends as well on the initial pattern of production. Points A, A', and A" depict three initial positions in which the country is incompletely specialized in production of commodities 1 and 2. If production is at point A, consisting mostly of labor-intensive commodity 1, the technical progress that is biased towards the saving of capital nonetheless conforms to the result expected if there were not bias: Commodity 2, in which progress takes place, reduces the wage/rental rate because the second commodity is the capital-intensive commodity of the pair being produced. (The wage rate change by itself is negative, just as if p_2 by itself is lowered). However, if the relative output of the second commodity were greater, such as for initial point A', the wage/rental ratio could be increased (from point A' to point B'). Indeed, a sufficiently large initial relative production of commodity 2, such as at A", could result in a change in the production pattern shown by B", with relatively heavy production of the second commodity joined by some production of commodity 3. The before and after comparison of production patterns reveals that the Heckscher-Ohlin setting of incomplete specialization is maintained, but with a switch in the identity of the commodity that is produced along with the second commodity. The *reduction* of capital (relative to labor) required in producing the second commodity (i.e. the nature of the *bias* in technical change) means that the existing capital stock can partly be used in producing the even greater capital/labor proportions required to produce the third commodity.

7. Specific Factors: An Extended Version Blending with Heckscher-Ohlin

So far the Specific-Factors scenario envisaged each specific factor engaged in a single activity. [18] This vision can be expanded by imagining a *sector* that consists of a number of activities or industries, *all using the same kind of capital*, a kind, however, that is not used in other sectors. In other words, each type of capital is *sector*-specific, not specific to just one industry. This is the kind of idea used in a special interpretation of a (3x3) model introduced some years ago by Fred Gruen and Max Corden (1970) to capture some features of the Australian economy. Theirs was a 2-*sector* model consisting of manufacturing and agriculture. Manufacturing was represented by a single industry, textiles, produced by labor and capital.

[18] An interesting different approach from the one discussed in this section is provided by Roy Ruffin (2001). The Specific- Factors model and the Heckscher- Ohlin model are "blended" in a different way: Each "specific" factor is used in a particular industry only if its "rents" for being specific there are positive. Otherwise it can be used in the other sector. In the 2-commodity case Ruffin illustrates by drawing a bowed-out transformation schedule. In the "middle" range the transformation schedule is that of a Specific-Factors model, with two types of (specific) labor and mobile capital, whereas nearer the axes the "rents" to the type of labor "specific" to the commodity shown along the other axis would be negative, and that type of labor joins with the other type in common usage, as in the Heckscher-Ohlin (2x2) model.

Agriculture, by contrast, was a sector consisting of a *pair* of industrial activities: wheat and wool. Whereas labor was used in (and was mobile among) all three industries, agriculture used a specific kind of sector specific "capital" (land), mobile only between industries (wheat and wool) in the agricultural sector. This kind of structure was generalized to higher dimensions in Jones and Marjit (1992) and is used here as an example of blending the basic ideas of the Specific-Factors model and the Heckscher-Ohlin model.

To introduce these ideas start not with a free-trade situation but, instead, with an economy initially in autarky, cut off from a world market in which different technologies and taste patterns have supported an array of commodity prices different from those in the autarkic equilibrium. Production in the autarkic economy is undertaken in many *sectors*, each of which has a number of industries all using labor (used throughout the economy) as well as a kind of capital specific to that sector. Diagrammatically, in the autarky equilibrium each sector can be visualized by a separate kind of capital on the vertical axis, labor on the horizontal and a set of unit-value isoquants for all industries in that sector. Unlike the Hicksian composite unit-value isoquant depicted in Figure 2, market prices in autarky have adjusted such that *all* the unit-value isoquants in each sector are tangent to a common "budget line", whose slope is given by – w/r_j for this, the j^{th} sector. Commodity prices are brought into equilibrium by matching up local demand and supply for each industry, with additional balance between aggregate labor demand and supply determining the wage rate. This value for *w*, along with equilibrium prices, p_j^i for the price of each industry, *i*, in the jth sector, determine the various specific-factor rentals, r_j. (For a commodity to be produced in autarky, its price must allow for the same positive return to sector-specific capital in that industry as for other industries in that sector).

This setting provides a more robust example than found in the previous settings of how opening an economy to international trade can greatly reduce the quantity of goods in production compared with autarky. Each sector now faces world competitive markets for each of the industries, and the kind of comparison typically made in Ricardian models (the search for the item with the greatest comparative advantage) yields, let us assume, a "winner" in each sector – the commodity that yields the highest return to sector-specific capital. If such were the case, the economy would end up with an [(n+1) x n] specific-factors setting – *n* commodities produced with *n* specific capitals and mobile labor. However, this need not be the outcome! Think back to the Gruen/Corden setting: The agricultural sector supported a *pair* of industries, wheat and wool, and indeed the dimensions of the model were 3 (inputs) and 3 (commodities). So also in this case there *may* exist a single sector that has two industries surviving, making this economy have what we call a "Heckscher-Ohlin *nugget*", i.e. with trade the *sector* collapses to a (2x2) Heckscher-Ohlin setting, and the economy, with trade, has the same number of industries as it does factors of production, *n+1.*

Consider, now, a diagrammatic illustration for an economy engaged in free trade and having a number of "sector-specific" forms of capital. The procedure leading to Figure 4 is no longer available, because there is no over-all unit for "K" – each sector has a different kind of capital. Nonetheless, all sectors face the same wage rate in equilibrium and with commodity prices and the endowments of all specific factors given all the separate "rents" earned by the various types of sector-specific capital would be determined. Therefore concentrate, first, on *each sector's* demand curve for labor. This is not a smooth negatively-sloped locus. Instead, it is a step-function of a type illustrated in Figure 7[19]. Note that at a sufficiently high wage rate, any rate higher than point *A*, no industry in sector *j* would survive. The reason? With commodity prices determined in world markets local demand for all industries in this sector could be supplied by imports, since at such a high wage rate the specific factor used in sector *j* would, if employed, yield a negative rate of return.[20] For a wage rate slightly lower than shown by point *A*, there emerges a single "best" industry in Sector *j,* and as the wage rate falls (with a concomitant increase in r_j), substitution possibilities for producing this best industry encourage more labor-intensive means of production, much as in Figure 4's depiction for an economy with only one type of specific capital.[21] However, at a lower wage rate a more labor-intensive industry might join the previous "best" industry, and a *range* of labor demanded in this sector can support the same wage rate, corresponding, in Figure 4, with a leftward movement along a flat, (which would be a rightward movement in Figure 7). When one "best" industry finally yields completely to a more labor-intensive one, a further lowering of the wage rate is associated with a greater demand for labor in this new single industry until an even newer (more labor-intensive) industry joins the previous industry, with a new flat appearing in the labor demand schedule. Thus open trade does indeed lead to a great degree of specialization in each sector, but not necessarily to the extreme of a single surviving industry. Which single industry or which pair of industries survives after trade depends upon the value of the wage rate in equilibrium.

The kind of demand curve for labor in the j^{th} sector illustrated in Figure 7 can be visualized for all other sectors. The *equilibrium* value of the wage rate depends upon the summation of labor demands from all sectors, and this *aggregate* demand locus for labor is shown in Figure 8. I have assumed that although there may be a number of flat segments for each sector (with its given amount of sector-specific capital), these occur at different wage rates from those for any

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[19] This kind of diagram has previously been utilized by Deardorff (1984).

[20] In the event that some commodities are non-tradeable, there could be one or more such industries that could command a commodity price sufficiently high that sector-specific capital in sector *j* would earn a positive return. Later I discuss the issues involved if non-tradeables are allowed in this model.

[21] A downward move in the wage rate (with a given quantity of capital but an increase in the labor supply) is, in Figure 4, associated with a *leftward* movement along the horizontal axis (i.e. a reduction in K/L).

other sectors. In Figure 8 two possible vertical aggregate supply curves for labor are illustrated. If the labor supply is L', the equilibrium wage rate is shown at point A, implying that for this economy there is some *single* sector in which there is a *nugget* of two industries that survive competition (as in the Gruen/Corden agricultural sector with wheat and wool). The economy as a whole has (n+1) factors of production, and is producing, with trade, an equal (n+1) number of commodities, a version of a Heckscher-Ohlin model.[22] However, if the labor supply curve were to expand to L", the equilibrium wage rate would fall (as shown by point B), but there would only be a single industry in every sector that survives competition. The economy would be characterized by an [(n+1) x n] Specific-Factors model. That is, this economy exhibits either a Heckscher-Ohlin structure or a Specific-Factors structure. It all depends – upon the particular technology for all commodities in all sectors as well as on the endowment of labor and of every type of specific capital. With growth of endowments and/or changes in world prices the model type is *endogenous*. Furthermore, even if the economy exhibits a producing nugget before and after change (i.e. it is Heckscher-Ohlin before and after) the particular industries that are producing as well as the sector in which the nugget operates may differ. The visualization of a *flat* along which factor prices stay the same is captured both in the kind of diagram like Figure 4 for a Heckscher-Ohlin (2xn) setting as well as in a multi-commodity model in which a number of sectors have their own type of capital but all use a homogeneous kind of mobile labor so that, as in Figure 8, a diagram relating aggregate demand for labor with total supply can depict the economy's equilibrium.

There is a phenomenon that is at the core of general equilibrium models, especially for economies engaged in trade: If one industry in an economy experiences a price increase (or perhaps a technological improvement), how are other industries affected? In the international trade literature the phrase introduced to reveal that, given the importance of relative ranking embodied in the theory of comparative advantage, a price rise for one industry could easily spell trouble for other industries in the economy is the *Dutch Disease*.[23] The two variants discussed above, the [(n+1) x n] version of the Specific-Factors model and the [(n+1) x (n+1)] version of the Heckscher-Ohlin model, allow radically different responses to a world increase in the price of a traded commodity. In the Specific-Factors model a price rise for any commodity produced would result in a wage increase (less than proportionate), and this spreads to *all* other sectors of the economy, resulting in a loss of labor and reduction in the return to capital In these sectors. The favored industry, as a consequence, expands at the expense of *all* other producing industries. This is the Dutch Disease writ large. A radically different outcome is a possibility in the Heckscher-Ohlin version in which one sector of the economy contains a (2x2)

[22] I assume that rates of return to all specific capitals are now positive.
[23] See, for example, the discussion in Corden and Neary (1982), and Neary and van Wijnbergen (1986)..

nugget. Suppose the commodity whose price increases is produced in some other sector, not containing the nugget. Production of that commodity expands with the inflow of labor released by the nugget. No *other* industry outside the nugget is affected because the wage rate and all returns to capital (other than in the sector in which there has been a price rise) do not change since the wage rate is determined *only* by the prices in the pair of industries produced in the nugget. The labor-intensive commodity that is produced in the nugget is the loser. The other (capital-intensive) commodity produced in the nugget actually expands – an expression of the famed Rybczynski effect (1955) whereby an economy (here the nugget) that experiences a loss of labor at constant commodity prices (in the nugget) will suffer a loss in output of the labor-intensive industry and an actual expansion of output in the capital-intensive industry in the nugget. The labor-intensive industry in the nugget is the *only* industry adversely affected in Dutch-Disease fashion.[24] This example illustrates, via the capital-intensive industry in the nugget, that a price rise in one industry (outside the nugget) can result in a *complementary* expansion in another industry.

To round out this discussion of the differences between these two versions of higher dimensional cases, suppose that the externally-provided price increase (in the Heckscher-Ohlin version) is for one of the industries *in* the nugget. If the favored industry is the labor-intensive industry in the nugget, the wage rate increases and *all* other industries contract – once again a scenario in which Dutch Disease is extreme. However, if the favored industry is the capital-intensive one in the nugget, the wage rate falls, thus allowing all industries outside the nugget to expand, with consequent enormously spread complementary effects outside the nugget, with the single labor-intensive industry in the nugget supplying labor to all other industries in the economy.

To summarize all these possibilities: (i) Strong Dutch Disease results, with many industries that have not benefitted from a price increase suffering an output fall, are most likely to exist if the price rise is for the labor-intensive industry in the nugget in the Heckscher-Ohlin [(n+1) x (n+1)] scenario. (ii) A less extreme version (in terms of outputs) of the Dutch Disease is that found in the Specific-Factor model, the [(n+l) x n] case, in which any producing industry's price rise has a dampened effect in raising the wage rate, which nonetheless hurts *all* other sectors. (iii) The weakest Dutch Disease result is found in the Heckscher-Ohlin version if the price rise is for the capital-intensive industry in the nugget, where only the nugget's labor-

[24] In the Gruen/Corden (3x3) model, if the price of textiles in the manufacturing sector increases, the agricultural sector loses labor, and this actually results in an expansion in the less labor-intensive activity in agriculture, namely wool production. In their case the domestic price of textiles increases because of a tariff on imports of textiles, and, since Australia is a large producer of wool on world markets, such a tariff serves to *worsen* Australia's terms of trade. Such a result seemed paradoxical in the literature on tariff theory, where a country, large enough to have an influence on world prices, would benefit by an improvement in its terms of trade when it levies a tariff.

intensive industry suffers and *all* other producing industries expand – a strong example of complementarity. (iv) If the price rise does not take place in the nugget of the Heckscher-Ohlin version, factor prices are unaffected, as are outputs in all other industries except for those in the nugget. The labor-intensive industry in the nugget contracts, while the capital-intensive industry actually expands in complementary fashion.

8. The Produced Mobile Factor Structure

One of the advantages of the Specific-Factors structure is the ease with which it can be generalized to any number of sectors. As in the earlier treatment of the (3x2) case, the focus can rest upon the market-clearing equation of change for the single mobile factor, say labor. The multi-commodity version of the equilibrium change in the wage rate, caused either by commodity-price changes or endowment changes, is the *n*-commodity version of equation (11), as well as to the form for each coefficient for price changes exhibited in equation (12). The *n*-dimensional structure is as easy to manipulate and understand as is the two-commodity case. Such a remark cannot be made of an *n*-commodity version of the Heckscher-Ohlin model without severe conditions being imposed on the production structure. One set of conditions that has been explored leads to an (nxn) structure that shows how *similar* a version of the Heckscher-Ohlin model can be made.[25] Indeed, one single modification in the multi-commodity specific factors setting is sufficient to convert the model to a Heckscher-Ohlin setting in which the *strong form* of the Stolper/Samuelson (1941) theorem is obtained for any commodity price change: Any commodity price rise will increase by a magnified amount the reward to the single factor used most intensively in the industry whose price has increased and, as well*, lower* the return to all other factors. The modification: Suppose that instead of raw labor serving as the single mobile factor in a multi-commodity specific factors model, the mobile factor is, instead, *produced* by all of the "specific" factors, in *any* of the normal ways assumed for constant-returns-to-scale production functions. This, in effect, reduces the number of independent inputs to *n*, the same as the number of produced commodities.

The proof of this kind of result for the Produced Mobile Factor structure involves only a simple but intensive use of all the competitive profit equations of change (such as those for the (2x2) Heckscher-Ohlin case illustrated in eqs. (13) and (14).) Let the first commodity experience a price increase, with all other commodity prices constant, and start with the (n-1) equations of change for all commodities, $j \neq 1$:

[25] Details of the model are provided in Jones and Marjit (1985). A discussion of ways of generalizing the Stolper/Samuelson result is found in Jones (2006).

(17) $$\theta_{jj}\hat{w}_j + \theta_{Mj}\hat{w}_M = 0, \qquad j = 2.....n$$

where *M* refers to the mobile factor and *j* to the specific factor used in the j^{th} sector. From these one can conclude that $\hat{w}_M$ has sign that is opposite to $\hat{w}_j$ for all *j* ≠ 1. Next, consider the competitive profit equation of change for the activity that uses all "specific" factors in producing the mobile factor:

(18) $$\theta_{1M}\hat{w}_1 + \sum_i \theta_{iM}\hat{w}_i = \hat{w}_M \qquad \text{for i = 2 through n.}$$

From this equation one can conclude that $\hat{w}_1$ and $\hat{w}_M$ have the same sign and, furthermore, that in absolute value $\hat{w}_1$ exceeds $\hat{w}_M$. Finally, consider the remaining competitive profit equation of change, which refers to changes in the activity producing the first commodity:

(19) $$\theta_{11}\hat{w}_1 + \theta_{M1}\hat{w}_M = \hat{p}_1$$

Because $\hat{p}_1$ is positive by assumption, and since $\hat{w}_1$ and $\hat{w}_M$ have the same sign, they must both be positive, with $\hat{w}_1$ the greater of the two. Summing up,

(20) $$\hat{w}_1 > \hat{p}_1 > \hat{w}_M > 0 > \hat{w}_j \qquad \text{for all j} \neq 1.$$

Concentrating just on the "specific" factors, the so-called strong Stolper/Samuelson results hold: An increase in any single commodity price raises by a magnified amount the return to the factor used most intensively in producing the commodity and lowers the return to all other factors (except the mobile factor, which is now a produced factor).

The *produced mobile factor structure* is indeed a special case of the Heckscher-Ohlin model, and it does satisfy the strong Stolper/Samuelson condition, which many (*n* x *n*) models do not.[26]

9. More on the Multi-Commodity Model: The Ruffin-Ricardo Model

By making use of the Ricardian-based model introduced by Roy Ruffin (1988) there is another scenario that leads to an easy-to-handle multi-commodity *and* multi-factor model. Ricardo's model envisaged a number of different labor types, but each type resided in a different country. While maintaining the notion that the world's labor supply was indeed

[26] The argument presented here leads to the following mathematical result. Suppose **A** represents a *positive diagonal matrix* and **B** is a non-negative matrix of *rank 1* (of the same dimension as **A**). Add these two matrices and take the inverse. The result is that $(\mathbf{A} + \mathbf{B})^{-1}$ is a matrix, **C**, with strong Stolper/Samuelson properties. That is, **C** is a matrix of positive diagonal elements all greater than unity and negative off-diagonal elements.

heterogeneous, Ruffin assumed that residing in each country was a *mixture* of various labor types.[27] Each type of labor has a given productivity in producing each commodity, as in Ricardo. However, and this is crucial, as in Ricardo each laborer on its own can produce *any* commodity. That is, it is assumed that there is *no joint production on the input side.* Any commodity can be produced by a single type of labor; there is no necessity of having a proper balance among factors producing a commodity since each works on its own. Furthermore, not only is labor of different types possessive of different levels of productivity, countries differ from each other in their relative endowments of different types. What emerges is a clean blend of Ricardian models with Heckscher-Ohlin models. Even in the (2x2) Heckscher-Ohlin model no commodity can be produced with a single input – a combination of the two types of input is required, although the particular intensities depend upon factor prices.

In the Ruffin-Ricardo model, free trade ensures factor-price equalization. It does not matter where each labor type lives, its productivity is the same in any country, and if commodity prices are equalized by trade so also will be the array of factor returns in different countries. As for production, the transformation schedule in any country, say in a two-commodity setting, will look much like that of a *world* transformation surface in the original Ricardian model, exemplified by a broken line along each segment of which a single country's labor is shifting from the production of one commodity towards that of the other.[28] The resulting shape of the transformation schedule (or surface) depends both upon the productivity of each type of labor in each commodity as well as a country's relative endowment of each kind of labor. If each country hosts a complete variety of labor types, with free trade their transformation surfaces would exhibit the same allocation of commodity production to each type of labor, so that any labor type will be producing the same commodity (or commodities) in all countries for any given set of world commodity prices. The difficulties in analyzing production scenarios in the general case of many factors and commodities are removed in the Ruffin setting in which any factor can produce any commodity without the co-operation of any other factor or factors. Once a move to required joint production on the input side with three factors is made, a pair of factors might be complements and/or different pairs of factors may differ greatly in their substitutability with each other. The Ruffin-Ricardo model frees up the multi-factor models from these possible difficulties.

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[27] In this model there is *not* assumed to be any movement between countries of any labor type.
[28] This is illustrated in the Ruffin (1988) article. The higher dimensional three-commodity case is discussed in McKenzie (1954) and Jones (1961).

10. How Concentrated does Trade make Production?: A Hybrid Model

Throughout our discussion of various competitive trade models a major generalization emerges: International trade allows consumption of a wide selection of commodities while permitting a great concentration of production for export. However, the extent of concentration in Ricardian models (especially) and in the two-factor Heckscher-Ohlin model seem realistically too drastic: The country can concentrate its production of tradeables to one or two commodities. Although a general model of production with many commodities and many productive factors can be developed, (and it would indeed expand the number of commodities that might be producible in a competitive equilibrium), it would not be of much use if the comparative static solutions of output and factor price changes when the economy is subject to shocks are difficult to ascertain without detailed information on the structure of the economy. However, it is possible to create a simple hybrid model, a blend of the many-commodity, two-factor (labor and capital) Heckscher-Ohlin model with the easier-to-analyze specific-factor setting.[29]

In the two-factor, many-commodity Heckscher-Ohlin model described in previous sections, it was assumed that capital as well as labor could (costlessly) become mobile from sector to sector should conditions in the market (such as the array of commodity prices) change to suggest an alteration in what becomes the "best" single or pair of commodities to produce. Although retaining the assumption that labor is homogeneous and mobile, suppose now that once capital is created, it is specific in its use, so that the commodity that was the previous year's best choice may no longer be the same in the current period. If so, current production of the previous period's winner can nonetheless take place, even if it were no longer to be the recipient of *new* capital formation. New capital takes specific forms once the choice of best locale for returns for this period's investment is made, and such capital then becomes useful in current *and* future periods as an input factor to the productive process for a particular commodity.

The setting of this model is useful in describing possible patterns of production over time. Thus suppose that echoes of the past are present in an initial setting in which there exists positive production of a set of commodities in which currently there would not be any new capital formation because the return to capital there is exceeded by that in some other sector or a pair of sectors. Given a fresh set of commodity prices in the current period, if new capital formation is to take place, *which* sectors of the economy should be the recipients? The optimal answer to this query depends upon (i) how much new capital formation takes place and (ii) how

[29] This model is developed in R. W. Jones, "Specific Factors and Heckscher-Ohlin: an Intertemporal Blend," *The Singapore Economic Review*, v. 52, #1 (2007), pp. 1-6.

much labor is allocated to join with this new investment. (The analogy is with Figure 2, where the endowment ray determines the pattern of production. Here it is the ratio of investment to the amount of labor working with it that determines the pattern of new capital formation). As to the first question I shall be completely simplistic and assume that there is a given amount of labor that is used every period to produce new capital.[30] This sidesteps the very important macro-economic question concerning the determinants of the aggregate value of new net investment. To answer the second question it is necessary to balance two further sources of demand for labor, above and beyond the quantity used to *produce* new investment: (i) All sectors that have in the past been recipients of specific capital will continue to produce (as long as the rental on that type of specific capital is positive), and, of course, will require current labor as well. How much labor each such sector demands depends upon the wage rate (with all commodity prices set on world markets). For each wage rate it is possible to add up the quantity of labor required in each such sector (call the total L_S, for *specific* labor demand). The L_S curve is downward sloping with respect to the wage rate. (ii) As well, labor is demanded in order to join new investment in producing the best type of good(s) at this year's prices. This labor demand (call it L_I, for labor used with new investment) is locally downward sloping with respect to the wage rate only if new capital is all used in a single sector. However, if new capital is to be used in a pair of industries, the demand for labor to accompany new capital formation lies anywhere on a flat where both these goods are produced. That is, at that wage/rental ratio, the demand curve for labor is horizontal along a flat.

These two sources of demand are brought together in Figure 9. This should remind you of Figure 1 for wage rate determination in the specific-factors model). Here the total labor supply is net of the pre-determined amount used to produce new capital. In Figure 9 the equilibrium wage rate (w_E) shows the allocation of labor (at point *A*) between the sectors that have specific capital from the past (L_S)[31] and the new sector receiving all current investment. If the L_S curve had intersected the L_I curve along a flat, new investment would be poured into a pair of industries, in the form of capital appropriate to the specific needs of each such sector.

One important characteristic revealed in this analysis is that new investment could take place in a sector that in a previous period had been the recipient of capital formation. And if there have been improvements over time in the efficiency of capital in this sector, the newer type of capital could receive a higher rental than capital of older vintage. Eventually some types

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[30] Further to simplify, assume new capital requires only labor in its production.

[31] This includes the labor used with pre-existing capital in this period's best sector.

of specific capital could not earn any positive rents, and would be scrapped.[32] Furthermore, this period's winner might be an import-competing sector as opposed to an export sector, depending upon the past accumulation of capital in that sector as well as new capital formation. This hybrid model provides a useful base for describing the passage over time of investments, the growth or decline in separate sectors, and the impact of commodity price changes on the wage rate and, thus, on the rents that can be obtained by the type of capital specific in each sector. Such rentals on specific capital represent, in competitive models, the amounts left over from total revenues after labor has been paid. This is a model in which history matters.

11. Middle Products

A further example of ways in which Heckscher-Ohlin models have been blended with those of Specific Factors is provided by the work of Sanyal and Jones (1982), in which all productive activity in an economy is undertaken either in what is termed the *Input Tier* or the *Output Tier*. The Input Tier is one in which factors of production (labor and certain specific factors), produce items that can be traded in world markets. Such trade allows Items that are available in world markets to be used as *inputs* along with labor to produce commodities in the *Output Tier* of the economy, and these represent final local consumption. In this scenario International Trade takes place in the *middle* of a country's production spectrum, as illustrated in Figure 10, and all consumption items appear as outputs in the Output Tier.[33] There are two specific factors in the Input Tier, denoted K_A and K_B (and many more could easily be added), and each works with labor assigned to the Input Tier (L_I) to produce a pair of outputs called *middle products*, some of which are used as further inputs at home, and the rest are traded on world markets for a different array of inputs (goods in process or intermediate goods). Items taken from the world market (items 1 through 8) are specific inputs that are combined with labor in the Output Tier (L_O) to produce final consumer goods (labeled x_1 through x_8). These consumer goods have local prices determined by the wage rate and world prices for all middle products used in the Output Tier. Commodities x_1 through x_8 comprise the set of non-tradeables. Note that the production structure in the Output Tier is of the Specific-Factors type, with each consumer good produced with one of eight different middle products and labor. However, with given prices for traded middle products, international trade in effect transforms the eight specific middle products into a single aggregate, thus turning the Output Tier into a Heckscher-Ohlin model with two factors

[32] There is an older literature in economic theory dealing with capital of different "vintages" and distinguishing between "putty" type capital and "clay" type capital. The "clay" type of capital in our treatment is sector-specific, after determination of the sector to receive general "putty" type capital. See Johansen (1959) and Solow (1962).
[33] This diagram appears as Figure 3 in Jones and Marjit (2009).

(labor and this aggregate of middle products) and, in the case shown in Figure 10, eight commodities. Whereas Peter Neary (1978) relied on time and depreciation to convert specific capital into a single mobile input, a similar conversion is made more rapidly in the Middle Products scenario with the use of international trade.

This description, how international trade helps to convert the original Specific-Factors structure of the Output Tier into a Heckscher-Ohlin structure with only two basic inputs, *viz.* labor and traded middle products, differs somewhat from our earlier description (as illustrated in Figure 4). In Figure 4's depiction, with only two basic inputs (labor and capital), the number of commodities produced may be limited to a single commodity (along the rising portions of the dark locus) or, at most, a pair of commodities. By contrast, in Figure 10 I show in the Output Tier that eight commodities are produced. The difference is that in the Middle Products scenario, all commodities produced in the Output Tier serve as final *non-traded* consumption commodities. All international trade takes the form of converting the Input Tier's output of middle products into a variety of specific inputs required to produce non-tradeables that are consumed. Indeed, a further transformation could be developed. The labor used as inputs in the Output Tier are assigned to eight different tasks, and the different technologies that may be used in each could lead to a differentiated set of skills learned by labor in the Output Tier. That is, whereas originally the Output Tier could be described as a specific-factors model with many specific factors and labor, which by trade is converted into a Heckscher-Ohlin model, the further differentiated experience of the labor force might be described as converting a Heckscher-Ohlin model back into a specific-factors setting, a setting that now has a single aggregate mobile factor (because of trade leading to imports of middle products for items 1 through 8 at given world prices) and a collection of eight *specific* types of labor.[34]

12. Non-Traded Goods

This discussion of Middle Products assumes that all commodities that are consumed represent value added in the Output Tier as local labor converts traded goods (middle products) into non-traded final consumption items. Return now to the previous discussions of blends of Specific Factors models and Heckscher-Ohlin models to investigate new issues that arise if some of the produced products do not enter international trade but instead are protected from the international markets by the existence of trade barriers that are natural (e.g. high transport costs) or man-made (e.g. prohibitive barriers to trade such as high tariffs). Focus attention on the situation faced by a small open economy that faces given prices for all traded commodities.

[34] More details are found in Jones and Marjit (2009).

Local demand and supply considerations nonetheless affect the prices of commodities that are not traded on world markets. Figure 11 illustrates a possibility for the transformation curve of an economy with given technology and endowment base facing given world prices for traded commodities 1 – 4, as well as producing a single non-traded good, N. Assume that N is more labor-intensive than any of the producible traded goods. The transformation curve reflects the supply side of the market. (Note, for example, that if there were no resources used to produce non-tradeables, the country would produce both commodities 3 and 4. Because I have assumed the non-tradeable commodity N is more labor-intensive than any of tradeables 1-4, as they descend from good 4 to good 1 the move is to higher ratios of labor intensity). Local demand could be illustrated by an indifference curve (not drawn) tangent, say, at point A, in which case the economy would produce traded goods 2 and 3 as well as the amount of the non-traded good, N. The transformation curve has *flats* for regions such as point A at which a pair of traded goods is locally produced, which determines the pair of input prices (wages and rentals), thus establishing, as well, the price of the non-traded commodity, N. For regions in which only a single traded good is produced, demand factors help to determine the relative price of the non-traded commodity given world prices for all the traded commodities (and thus to determine factor prices as well).

Now consider a case in which there is a change in the price of one of the traded commodities. To simplify, suppose this represents an improvement in the country's terms of trade reflected in a price *decrease* in a commodity imported, but *not* of a commodity the country produces or would produce even if the country's endowment *were* different – i.e. a price drop for a commodity in which the country's technology is sufficiently inferior to that in the producing country abroad that Home competition is impossible. Such a setting is helpful for exposing the basic property of demand relationships – such a price change in world markets has both an *income effect* and a *substitution effect* on all commodities consumed at Home. Although all other prices for traded commodities are assumed to remain unchanged, the price of non-tradeable N might rise, or fall, or remain unchanged. Figure 12 captures the possibilities. The supply curve for N consists of rising sections as well as flats, and its shape is consistent with that of the transformation schedule in Figure 11.[35] With the fall in the price of a tradeable commodity not produced at Home (but imported), the substitution effect works to shift demand away from all commodities that are substitutes in consumption with the tradeable whose price falls, while the income effect works in the opposite direction (assuming no commodities are inferior in demand behavior). For a finite fall in the price of the tradeable item, Figure 12 illustrates how the demand curve for non-tradables shifts to the right (say to D_N') if income effects are more powerful, and to the left (say to D_N") if substitution effects

[35] This situation is discussed in more detail in Jones (2012).

outweigh income effects. The boundary value of demand elasticity leading to no change (i.e. keeping the demand curve at D_N in Figure 12) is, of course, that showing unit elasticity, as in the Cobb-Douglas case. In Figure 12 the possible shifts in the D_N curve are shown large enough to bring about a change in the price of non-tradeable N. In general, the price of the non-tradeable does not change if the economy continues to produce the *same* pair of commodities.[36] A sufficiently large discrepancy between income and substitution effects may lead either to a fall or to an increase in the relative price of non-tradeables, leading to changes as well in some production patterns as well as to changes in the amounts of various exports and imports.

This discussion of the role of non-tradeables suggests that the demand relationships involved in the case in which the world price of one of the (non-locally produced) tradeables goes *down* leads the non-tradeable sector to act like a *reservoir* from which other sectors can extract labor if substitution effects in demand outweigh income effects (i.e. demand is elastic) or as a *sponge* soaking up labor from other sectors if income effects dominate. Alternatively, if the price of a locally-produced and exported tradeable commodity increases (in a specific-factors setting), as pointed out by Corden and Neary (1982) the rate of return to capital in the non-tradeable sector may rise (if income effects are dominant), even possibly by a greater amount than in the favored export sector.

13. Concluding Remarks

In pursuit of competitive models appropriate for comparative statics in open-economy development scenarios, I have illustrated examples that blend certain aspects of the three basic competitive international trade models: The Specific-Factors model, the Heckscher-Ohlin model and the basic (and earlier) Ricardian model. Blends of these models become appropriate when *finite* changes are considered, because one consequence of international trade is that the production array in a trading equilibrium encourages a great deal of concentration, with the result being that production *patterns* may change with alternating changes in the rankings by comparative advantage. Such finite changes can often be identified and analyzed using diagrammatic techniques when the calculus (so useful in trade theory when small shocks are analyzed) no longer proves sufficient when capturing changes in equilibrium patterns of production. This paper has discussed a number of different ways in which *blends* of the standard neo-classical competitive models are useful for the purpose, but these presumably do not exhaust the possibilities.

[36] Note that the price change in any commodity that is tradeable and consumed has the effect of shifting the set of indifference curves between tradeables and the non-tradeable, because It shifts the weights used in aggregating the various commodities composing the aggregate over all tradeables.

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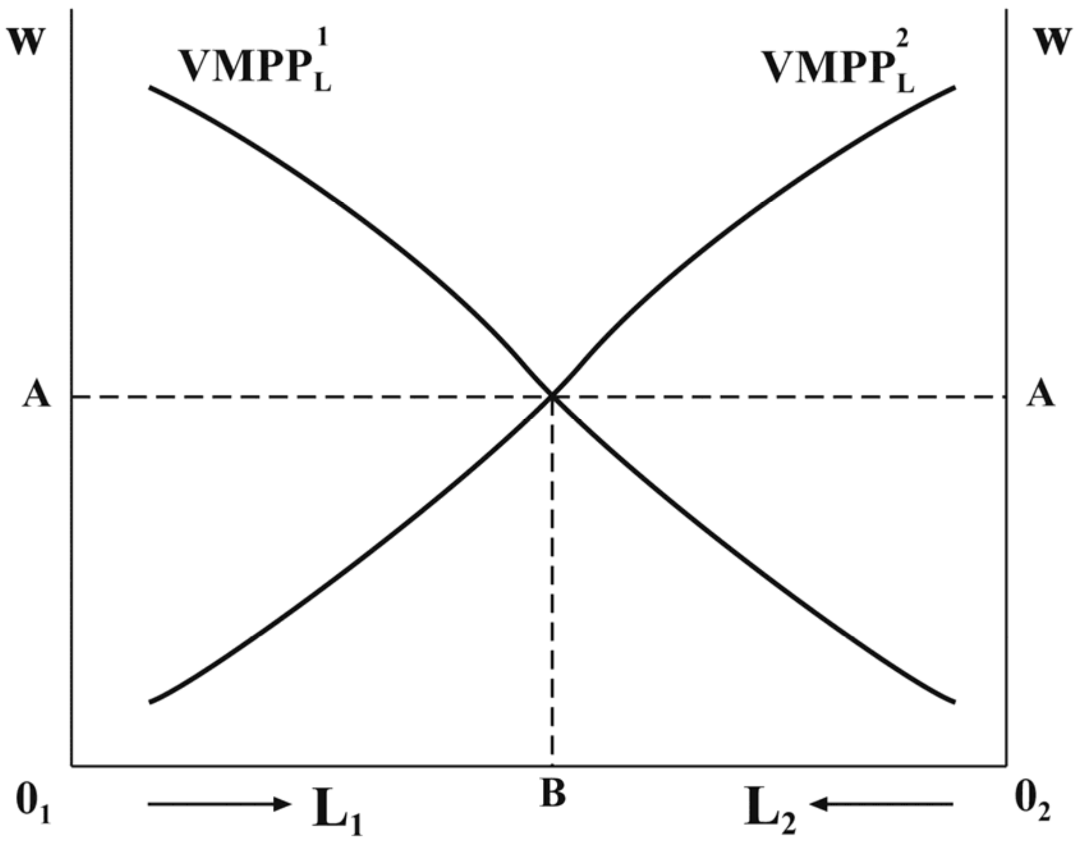

Figure 1: Labor Market: Specific Factors Model

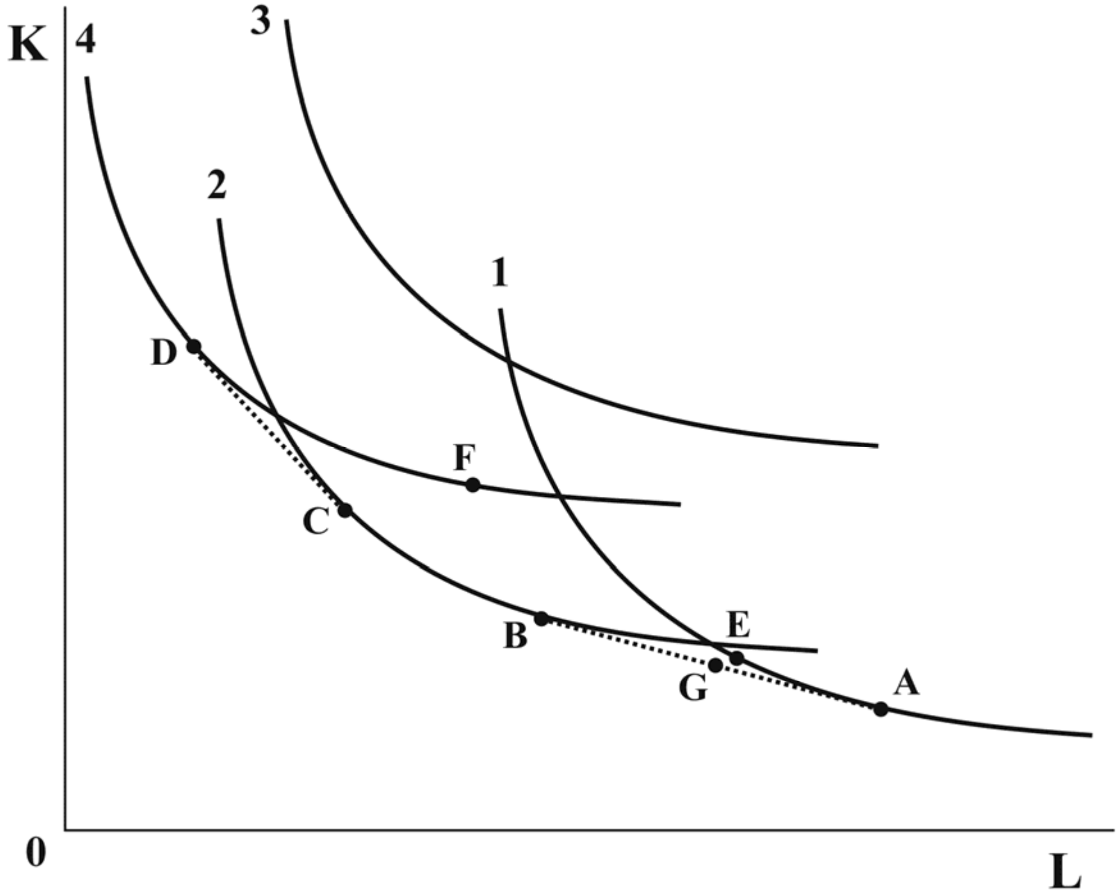

Figure 2: The Hicksian Composite Unit-Value Isoquant

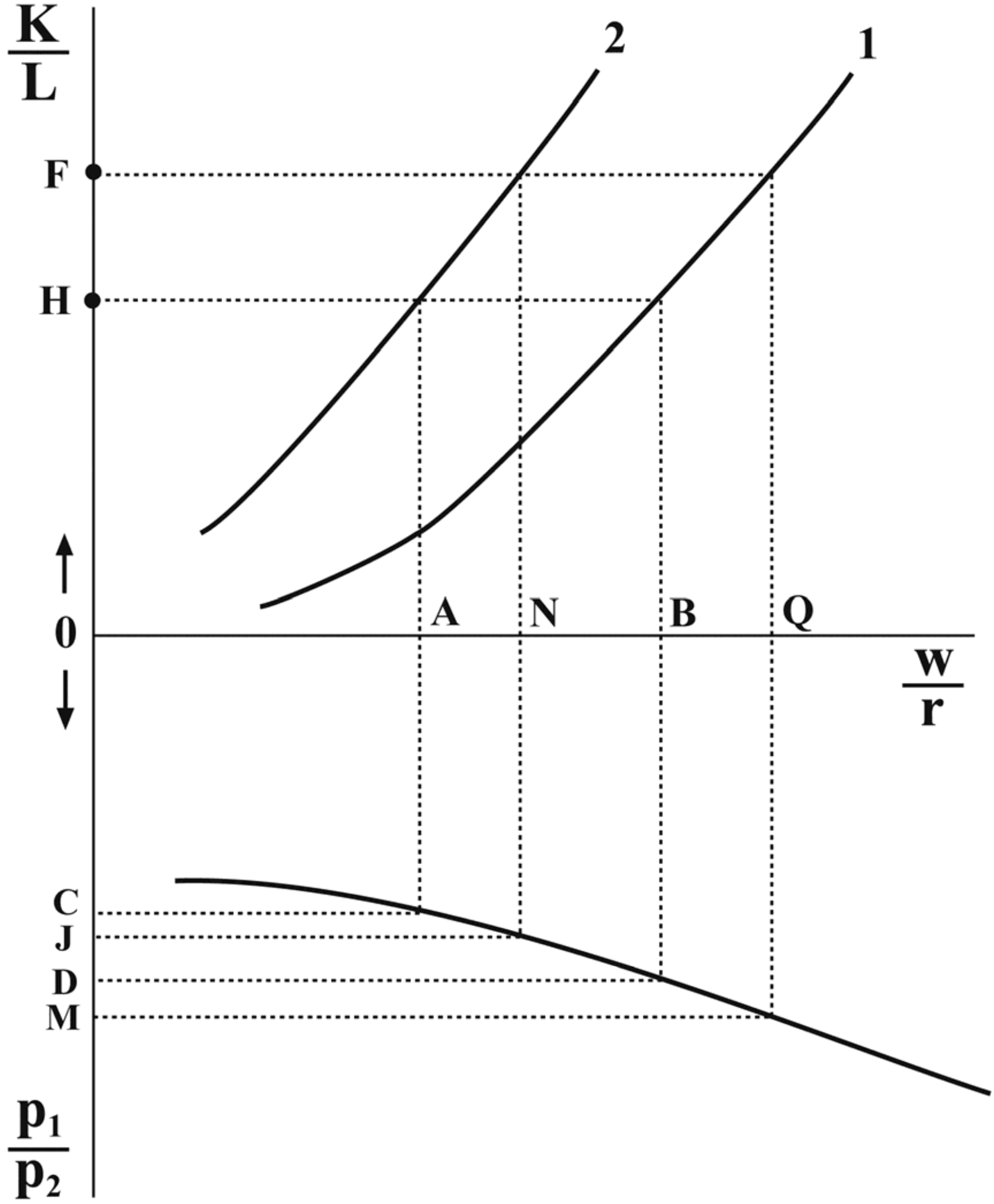

Figure 3: Heckscher-Ohlin 2x2 Model

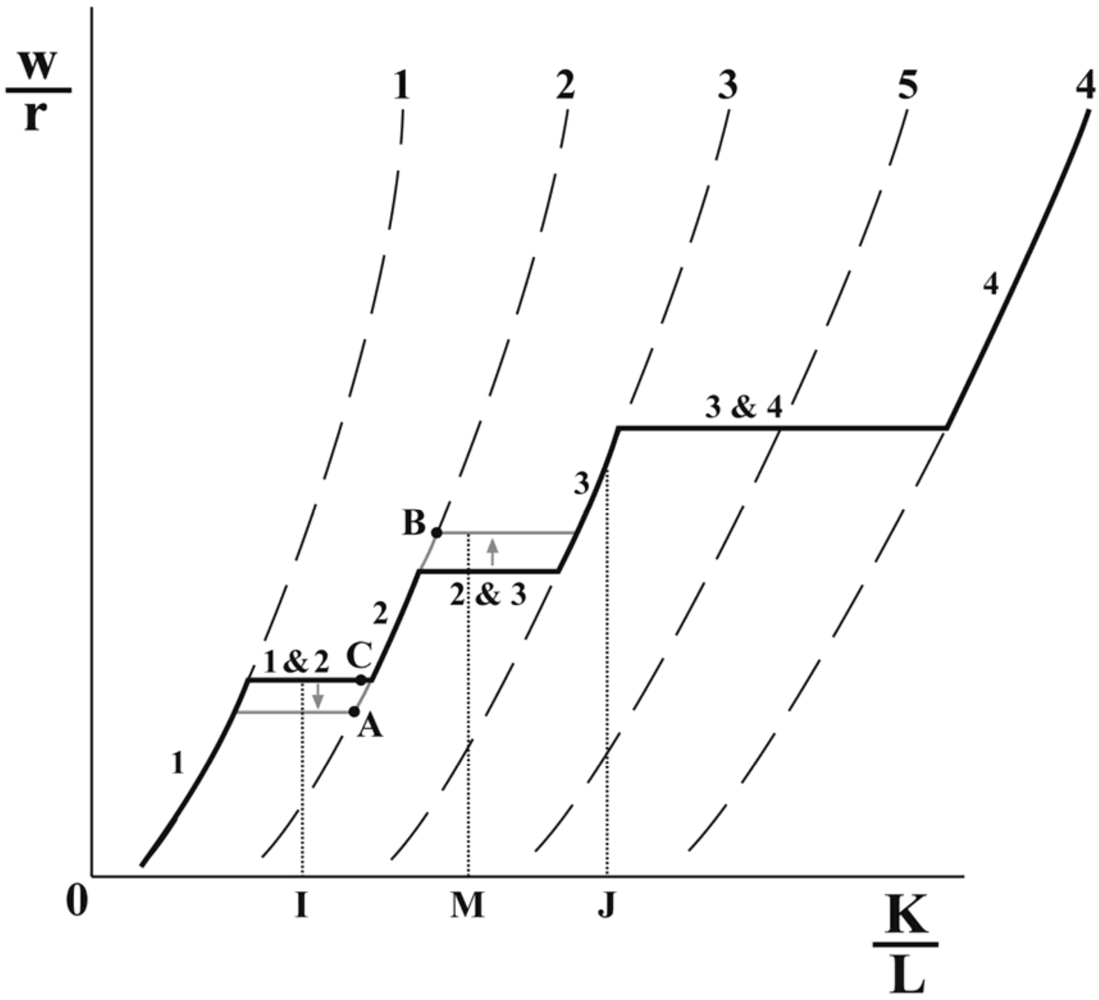

Figure 4: Factor Endowments and Factor Prices

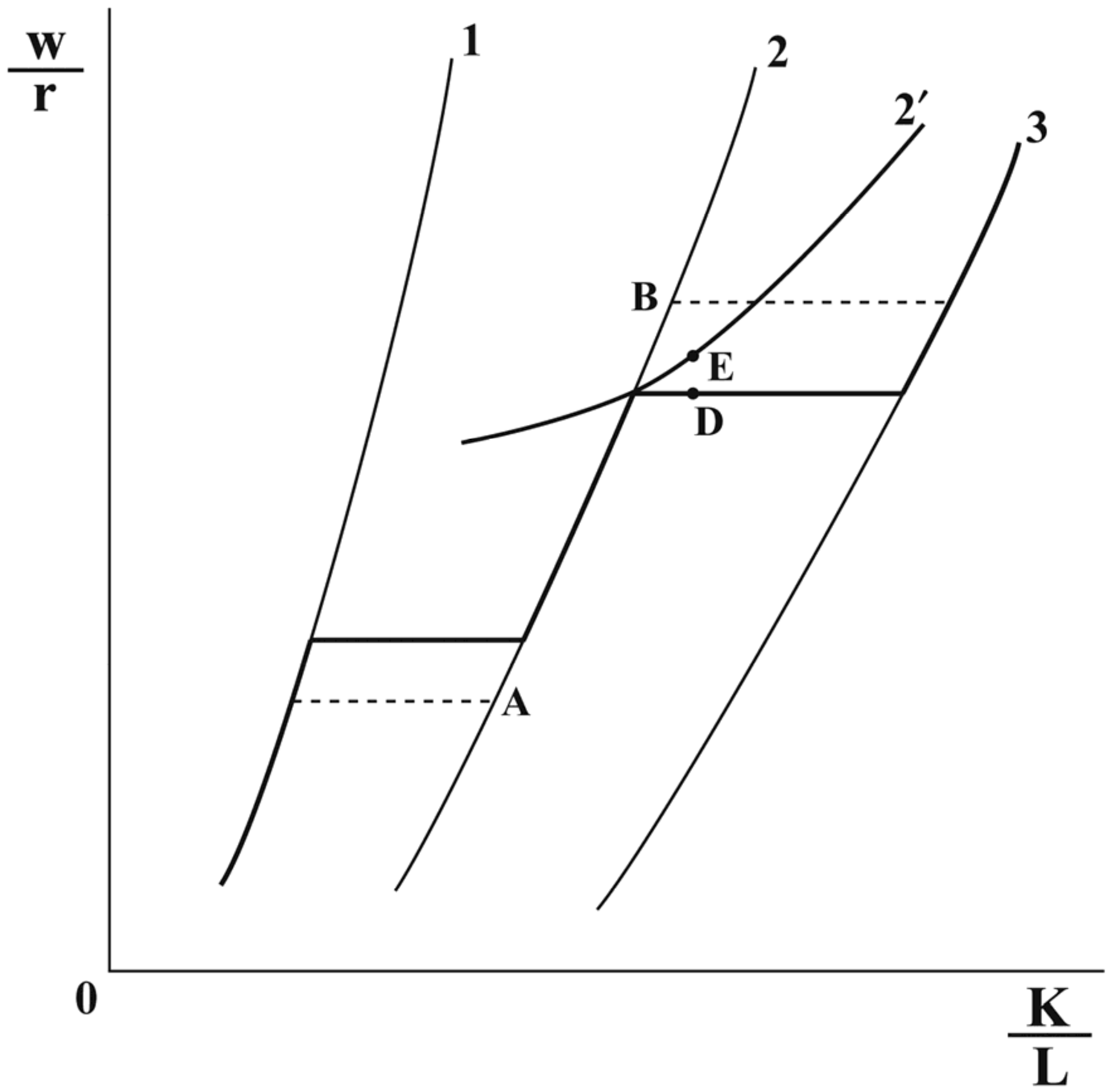

Figure 5: Flexibility in Technology

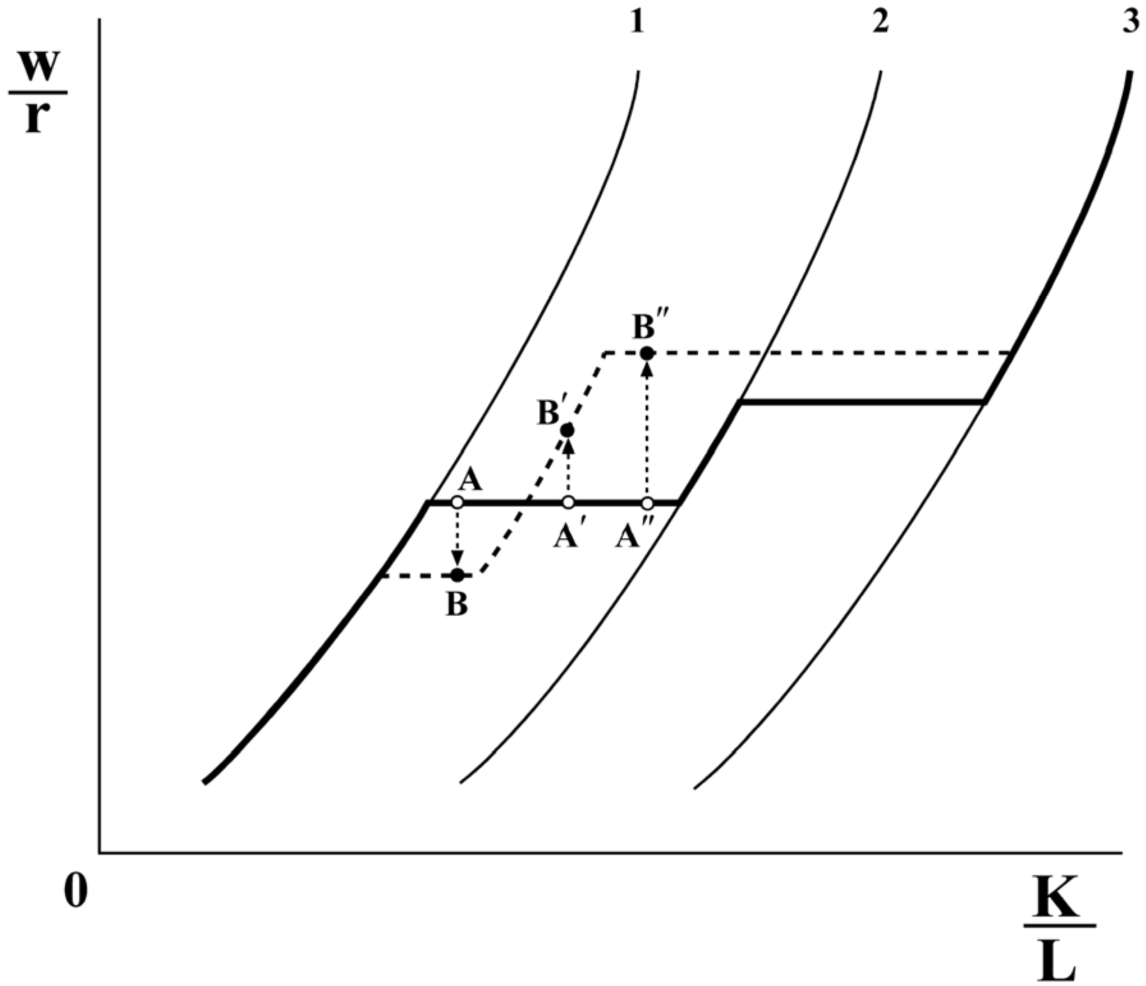

Figure 6: Biased Technological Progress

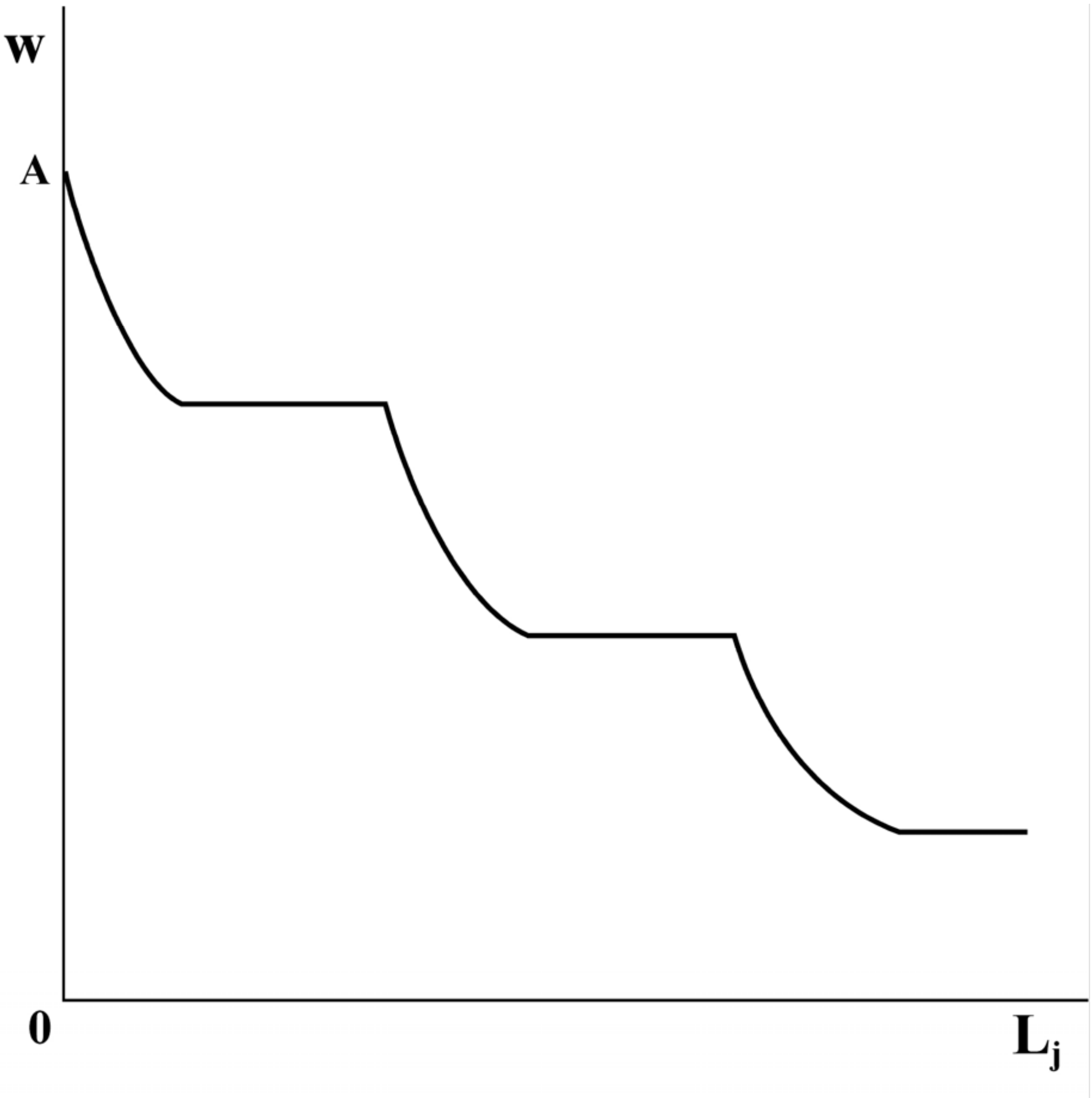

Figure 7: Labor Demand in Sector j

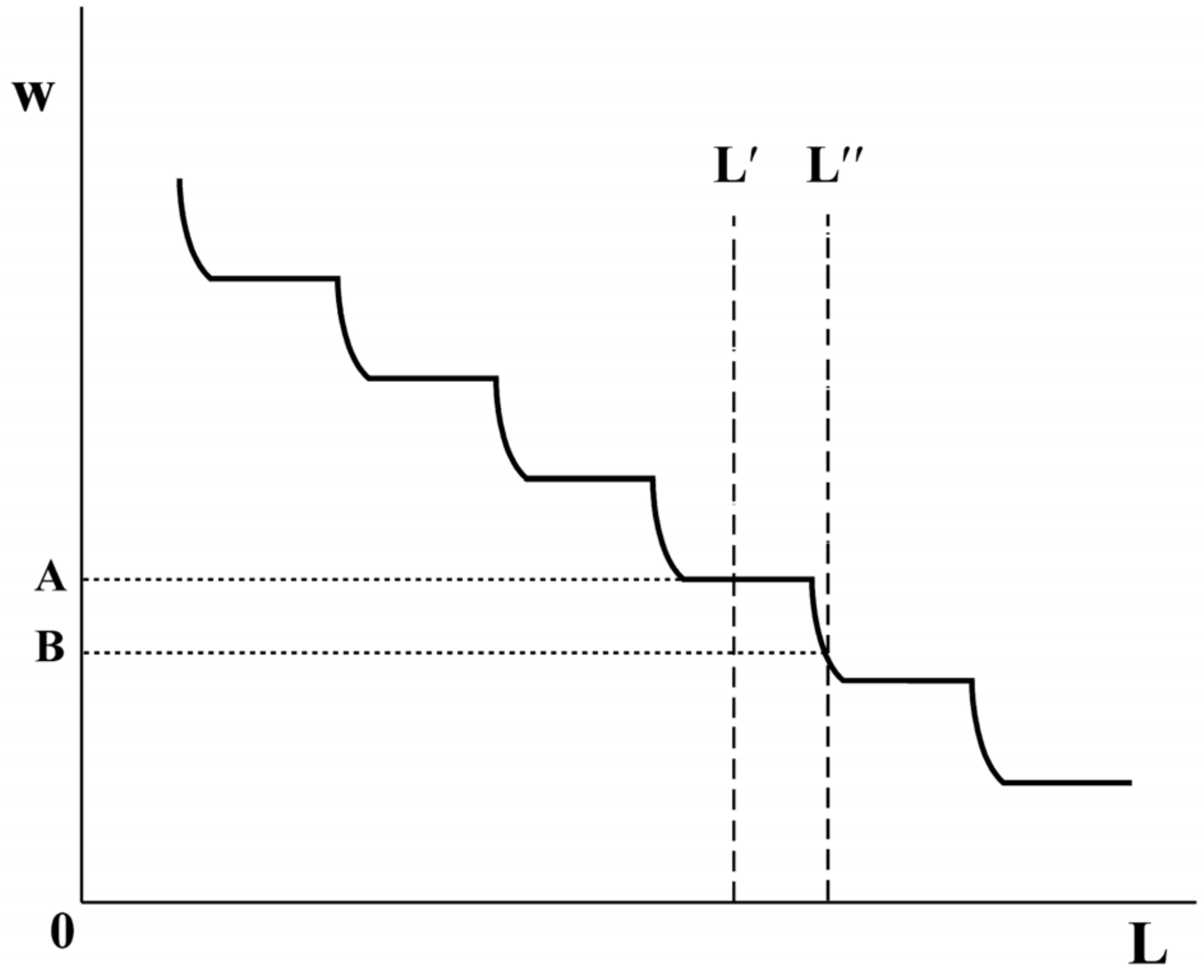

Figure 8: Aggregate Demand and Supply of Labor

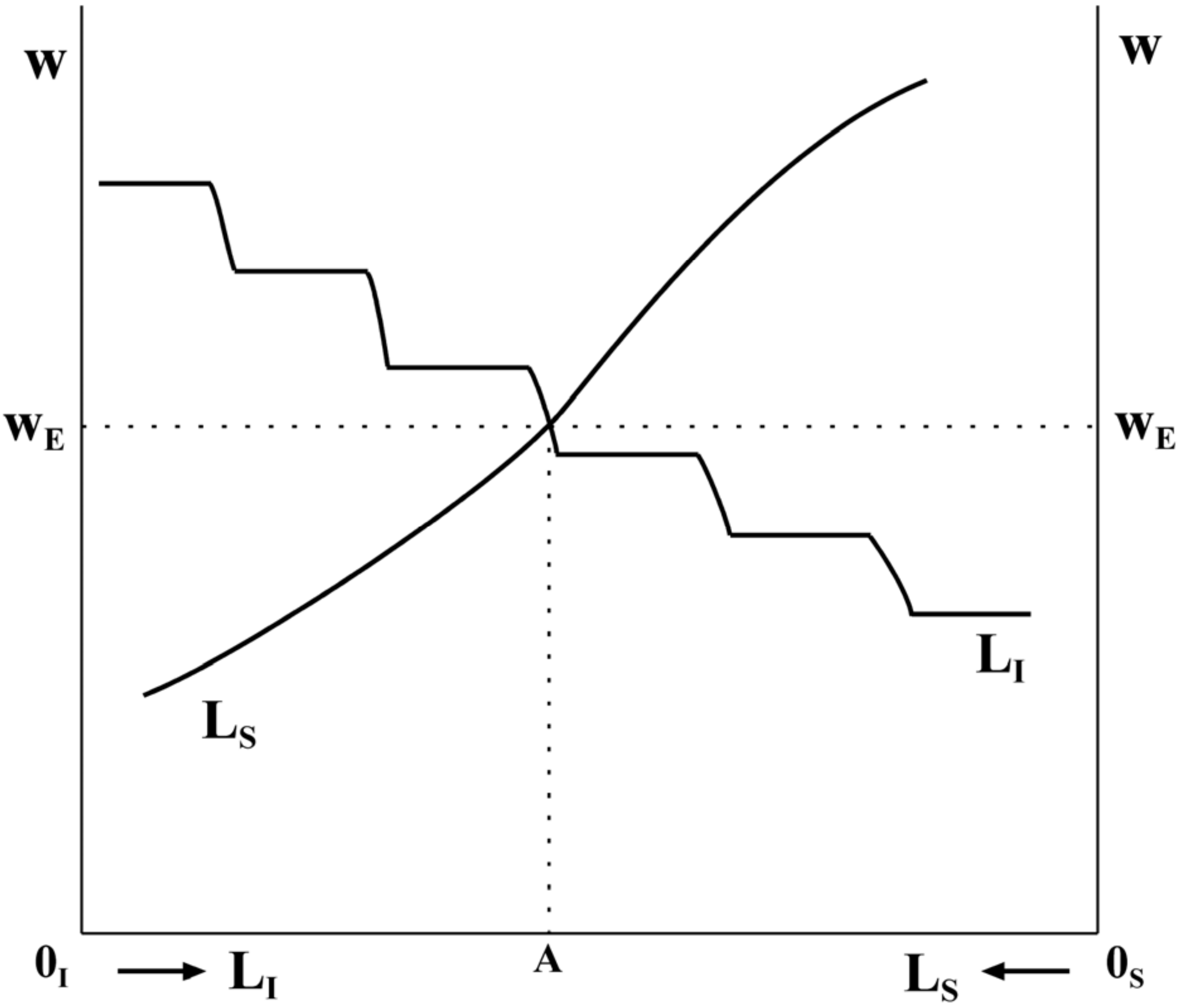

Figure 9: Labor Allocation Between New Best Sectors (I) and Older Producing Sectors (S)

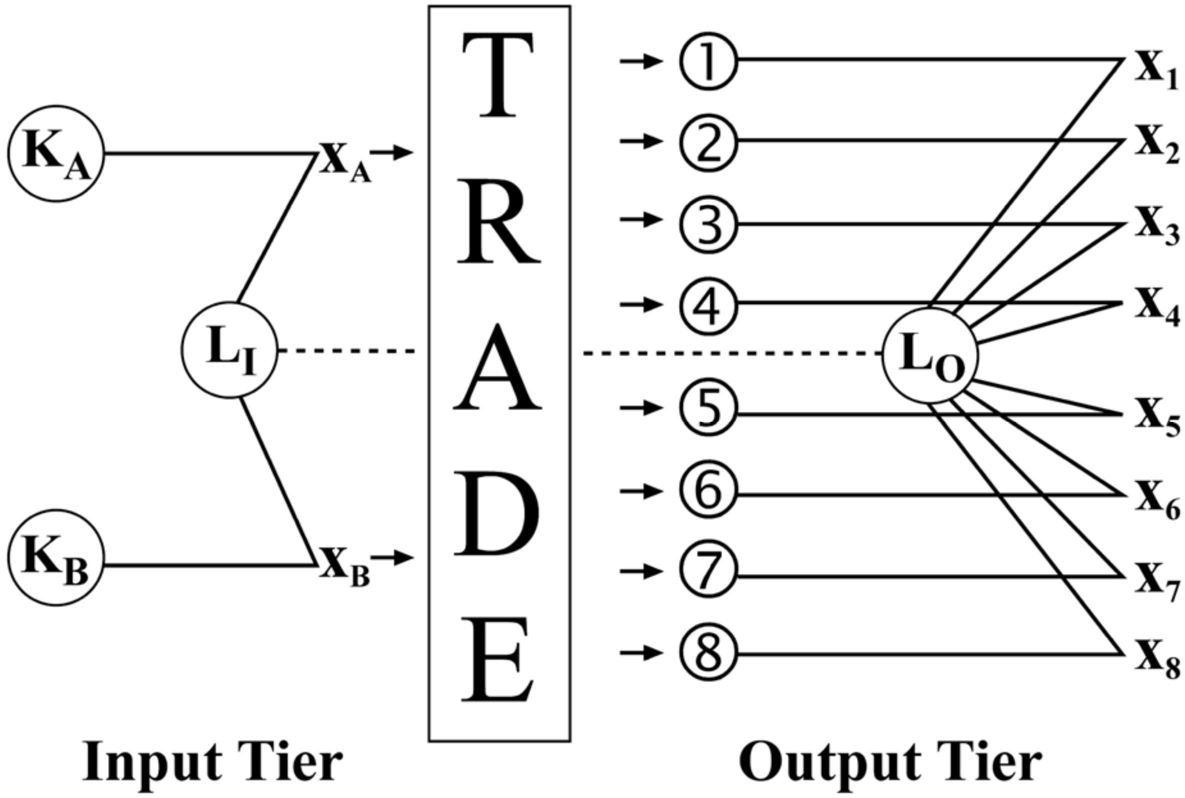

Figure 10. Trade in Middle Products

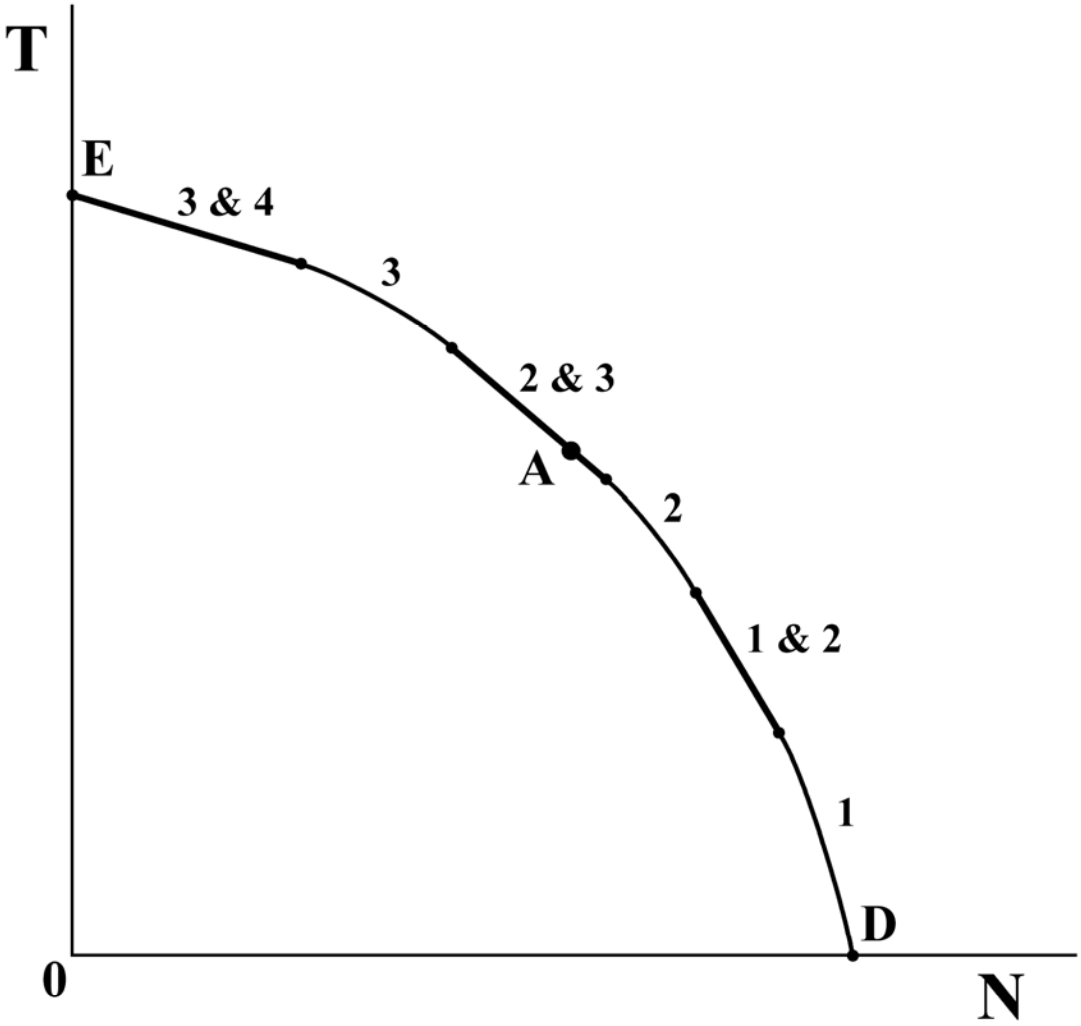

Figure 11: Transformation Curve: Traded and Non-Traded Goods

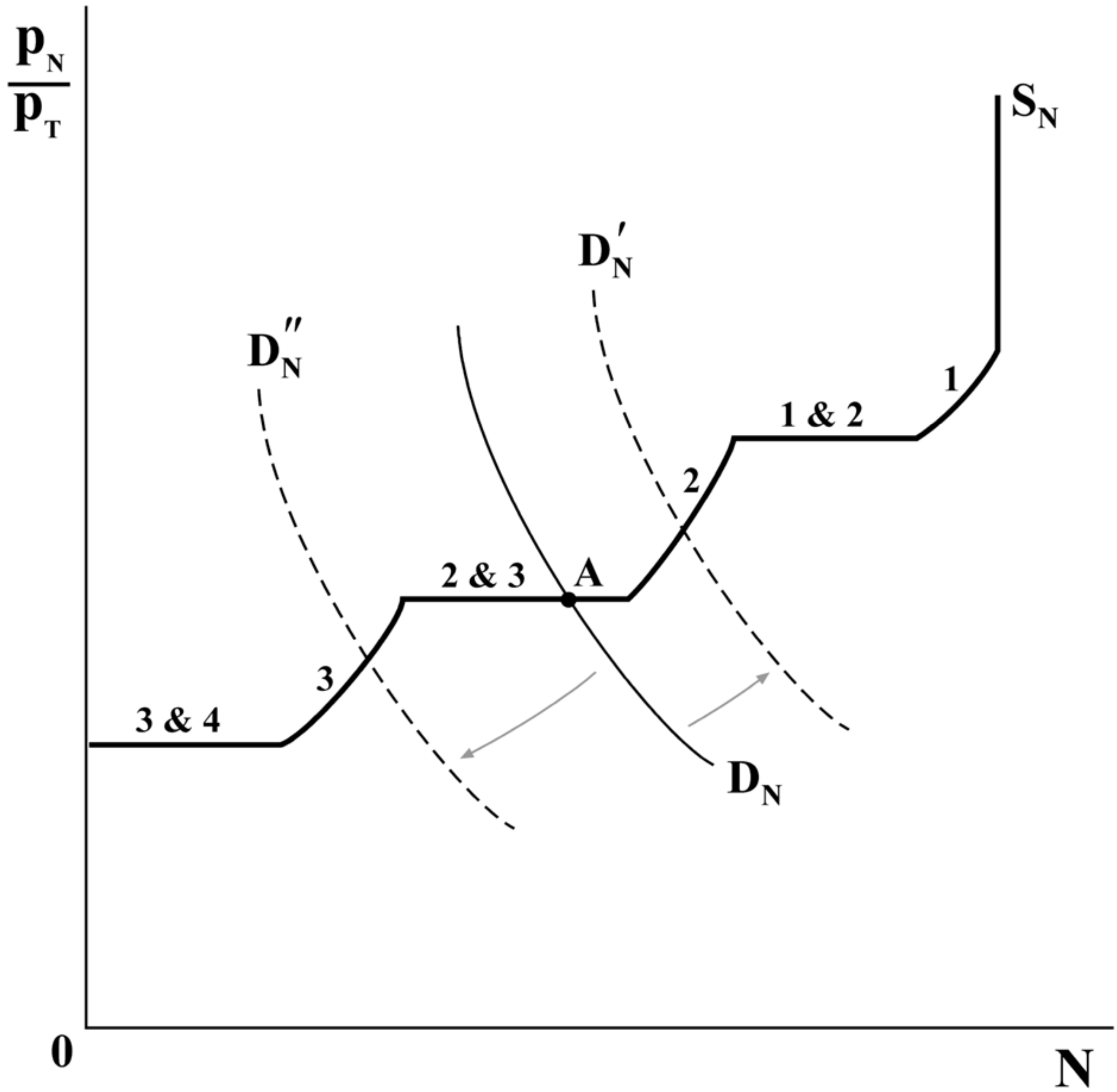

Figure 12: The Market for Non-Tradeables