Complementary Activities, Heterogeneous Firms, and Industry Structure*

Peter Arendorf Bache† Anders Laugesen

Working Paper
August 19, 2012

Abstract

We present conditions that are sufficient to couple complementary organisational activities at the firm-level with clear implications for the industry structure in a heterogeneous-firm model. We show how introduction of new activities and increased attractiveness of existing activities affect the industry structure in all dimensions by increasing the prevalence of all activities. The results accentuate the need to incorporate complementary organisational activities into an integrated model where firms face a multi-dimensional choice set. Through examples our predictions are shown to apply to several well-known models at the intersection of organisational economics and international trade.

Keywords: Firm Heterogeneity; Organisational Decisions; Complementarities; Supermodularity; Industry Structure

*The authors would like to thank Ron Davies, Thomas A. Gresik, Allan Sørensen, Siri Pettersen Strandenes, Mathieu Parenti along with seminar participants at the Kiel Institute for the World Economy, 8th Danish International Economics Workshop, 14th NOITS workshop, 8th NORIO workshop, and the 4th GIST Summer School for helpful comments.

†Corresponding author. Email: pbache@econ.au.dk. Both authors are affiliated with the Department of Economics and Business, Aarhus University, Fuglesangs Allé 4, 8210 Aarhus C, Denmark.

1
1 Introduction

A vast literature on heterogeneous firms in international trade has emerged in the wake of the seminal work by Melitz (2003). A part of this literature lets firms face decisions about more than one organisational activity. These activities include: exporting, the number of products marketed, investments in advertising, technology and quality upgrading, outsourcing, and offshoring. Many of these activities are modelled to be complementary, in the sense that undertaking one of them increases the payoff from undertaking another. An immediate consequence of such complementarity is that activities cannot be considered in isolation.

The present paper takes its offset in a Melitz (2003)-type model of heterogeneous firms and then provides conditions under which complementarity between different activities at the firm level translates into strong predictions for the industry structure. When activities are complementary and certain conditions hold, we show that the introduction of a new organisational activity, or increased attractiveness of an existing activity, increases the share of firms undertaking at least any given level of all other activities. The complementarity at the firm level thus implies that introducing or improving an activity advances the industry structure in all dimensions. These results suggest that in order to fully understand observed trends in the prevalence of a certain organisational activity, one should not only look at factors affecting that specific activity but also at factors affecting complementary activities.

A number of well-known models such as LIST MOST IMPORTANT conform to the conditions we set up and they are therefore subject to our general predictions for the industry structure. These examples are taken primarily from the trade literature, where the heterogeneous-firms setup we use have been advanced. However, there is nothing in our general formulation of the setup that limits the relevance of our results to trade issues. Indeed, the abstract activities we consider should be interpreted as being any complementary activities and decisions in the choice set of firms. This way, although deeply rooted in the trade literature, our study provides a framework for analysing industry-level implications of any firm-level complementary activities. To this end, we believe that the heterogeneous-firm approach has a lot to offer.

---

1See e.g. Antràs and Helpman (2004); Verhoogen (2008); Arkolakis (2010); Lileeva and Trefler (2008); Arkolakis and Muelend (2011); Bernard et al. (2011); Bustos (2011) for models where firms face a multi-dimensional choice set.
The present paper is related to Milgrom and Roberts (1990) and Milgrom and Roberts (1995) who conduct firm-level analysis of complementary organisational activities. In contrast to those studies, we analyse an industry equilibrium with heterogeneous firms and derive comparative statics on industry structure. This is our main contribution to the literature. In obtaining our results, we draw heavily upon results on supermodularity discussed in Topkis (1978), Milgrom and Roberts (1990), Milgrom and Shannon (1994), and Milgrom and Roberts (1995). By considering a supermodular\(^2\) profit function in a Melitz (2003)-type model, the present paper is also related to Mrazova and Neary (2011). Mrazova and Neary (2011) emphasise the role of supermodularity in the selection of firms into a single activity (market access) based on productivity and investigate the conditions under which supermodularity may arise. We take a different approach. We simply assume that the multiple activities faced by firm are complementary, i.e., that profits are supermodular in these activities. Then, given certain conditions, we derive comparative statics on the industry structure allowing for firm heterogeneity in multiple dimensions.

The remainder of the paper is organised as follows. Section 2 describes the basic model setup, the central assumptions, and derives our general results. Along the way we provide a series of examples based on the Bustos (2011) model of exporting and technology upgrading to build intuition. Section 3 considers existing models to which our results can be applied, and discusses some of these in detail. Section 4 offers some concluding remarks.

2 The Model

After paying an entry cost, firms enter a market characterised by a vector of fundamental parameters, \(\beta\), and the endogenous demand level, \(A\).\(^3\) Upon entry into the market, a firm draw a productivity level, \(\theta \in [\theta_0, \infty)\), and a vector of other firm characteristics, \(\gamma \in \Gamma\), from exogenous known distributions. After the realisations of \(\theta\) and \(\gamma\), the firm has to choose whether to start producing or to exit the market. If it chooses to produce, the firm has to make an organisational decision \(x = [x_1, \ldots, x_n]\), where \(x_i\) denotes the

\(^2\)Supermodularity is the formal equivalent of complementarity.

\(^3\)We call \(A\) the demand level throughout. You can think of \(A\) as a sufficient statistic for the market conditions faced by firms. Under an assumption of CES preferences, \(A\) would depend on the endogenous price index and total expenditure.
chosen level of activity \(i\). The level of an activity can be either a discrete or a continuous variable.\(^4\)

We let \(x \in X\) where \(X\) is the set of all conceivable, but not necessarily admissible, organisational decisions. The choice set \(X\) is a lattice which, loosely speaking, means that undertaking a given level of any activity may not prevent undertaking a given level of another activity but it may require undertaking a given level of another activity.\(^5\) The choice set of firms is restricted to a set of admissible choices \(S \subseteq X\) with \(S\) being a sublattice of \(X\).

In the theoretical part of her paper, Bustos (2011) presents a simple extension of the Melitz (2003) model that we will use as an example to strengthen the intuition for our approach and results.

**Example 1.** Take the standard Melitz (2003) model and allow firms to upgrade technology along the lines of Bustos (2011). The organizational decision of firms now face comprises two activities: whether to export or not and whether to upgrade technology or not. Exporting gives access to a foreign market in exchange for an additional fixed cost of exporting. A technology upgrade in this context means that productivity is scaled up, also in exchange for an additional fixed cost. Let \(1_{\text{ex}}\) be an indicator function for exporting and \(1_{\text{up}}\) be an indicator function for upgrading technology. A firm’s organisational decision can be characterised by \(x = (1_{\text{ex}}, 1_{\text{up}})\) and the set of all conceivable organisational decisions, \(X = \{(0,0), (0,1), (1,0), (1,1)\}\), is a lattice.

One possibility is to allow firms to choose among all elements of \(X\), i.e., to let \(S = X\). A possible restriction is to deny firms the opportunity to export. In this case of autarky, the set of admissible choices would be \(S' = \{(0,0), (0,1)\}\) which is a sublattice of \(X\). Another possible restriction is that firms need to upgrade technology in order to export. This gives the admissible choices \(S'' = \{(0,0), (0,1), (1,1)\}\) which is also a sublattice of \(X\). One case that yields a set of admissible choices that is not a sublattice of \(X\) is to deny firms access to simultaneous exporting and technology upgrading,

---

\(\footnote{A discrete activity could be the decision of whether to export or not. A continuous activity could be the share of the market reached as in Arkolakis (2010) or the range of products sold in a given market as in Bernard et al. (2011). The term activity should not be taken too literally, as it could also refer to e.g. the number of organisational layers in the firm considered by ?.}

\(\footnote{For a good introduction to lattice theory for the purpose of the present paper, see Milgrom and Roberts (1995).} \)
i.e., to restrict firms to \( S'' = \{(0,0),(1,0),(0,1)\} \). As previously mentioned, we do not allow admissible choice sets such as \( S'' \).

Firms naturally make their organisational decision, \( x \), to maximise profits, \( \pi \), under the constraint that \( x \in S \) while taking \( \theta, \gamma, \beta \), and \( A \) as given. Assumption 1 describes the basic structure we impose on our setup.

**Assumption 1.** Let an equilibrium exist and let the following be true:

i) Profits, \( \pi \), only depend on \( \theta \) and \( A \) through \( \Theta = A\theta \), i.e.,

\[
\pi = \pi(x, \Theta, \gamma, \beta).
\]  

(1)

ii) For all \( \gamma \in \Gamma \), the least productive firms choose not to produce.

iii) We restrict attention to equilibria where the organisational decision of the least productive active firms is constant for all \( \gamma \in \Gamma \). Further, we restrict attention to changes in \( \beta \) that does not directly affect the profits obtained by the least productive active firms.\(^6\)

iv) The distribution of productivities is Pareto and independent of \( \gamma \), i.e., letting \( F(\theta) \) be the c.d.f.,

\[
F(\theta) = 1 - \left(\frac{\theta_0}{\theta}\right)^k,
\]  

(2)

where \( k \) is the shape parameter of the Pareto distribution.

At first sight Assumption 1 may seem restrictive, or even odd, but it is satisfied in many studies belonging to the heterogeneous-firm trade literature initiated by Melitz (2003).\(^7\) Before we take a closer look at Assumption 1, let us continue with our example above.

**Example 1** (continued). By a slight change of the original notation, the profit function in Bustos (2011) can be rewritten as

\[
\pi(x, \Theta, \beta) = (1 + \tau^{1-\sigma})\text{I}_{ex}\mu \text{I}_{up}\Theta^{\sigma-1} - f - \text{I}_{ex}f_{ex} - \text{I}_{up}f_{up},
\]  

(3)

where \( x = (\text{I}_{ex}, \text{I}_{up}) \), \( \sigma \) is the constant elasticity of substitution of demand, \( f \) are the minimum fixed costs of production while \( f_{ex} \) and \( f_{up} \) are the fixed

\(^6\)Their profits are allowed to be indirectly affected by equilibrium effects through \( A \).

\(^7\)See Section 3 for a few of the vast number of possible references.
costs of exporting and upgrading, respectively. As the only source of firm heterogeneity is productivity, we disregard $\gamma$ in this case. The market parameters faced by all firms, $\beta$, are made up of the inverse iceberg trade cost, $\tau^{-1} < 1$ and the percentage gains to variable profits from upgrading technology, $\mu > 1$, i.e., $\beta = (\tau^{-1}, \mu)$.\(^8\)

Profits clearly only depend on productivity, $\theta$, and the demand level, $A$, through $\Theta = A\theta$. Bustos (2011) assumes that not all firms choose to produce after observing their productivity draw in order to get endogenous exit. Further, she assumes that the least productive active firms do not upgrade and do not export. This is done in order to obtain a non-degenerate sorting pattern in either dimension of the organisational decision. It is obvious that profits of the least productive active firms with $x = (0,0)$ are not directly affected by changes in $\beta$. Finally, Bustos (2011) assumes that productivities are Pareto distributed. In short, the model of Bustos (2011) conforms to Assumption 1.

Assumption 1.i means that profits only depend on productivity and the demand level through their product. This is restrictive but it is satisfied due to assumptions of CES preferences in most heterogeneous-firm trade models. $\Theta$ can be thought of as demand-adjusted productivity that takes into account that a given productivity is worth more the higher is demand.

Assumption 1.ii and 1.iii basically ensure selection as touched upon in the example above. Assumption 1.ii extends the standard assumption of non-degenerate selection into production such that it holds for all possible firm characteristics, $\gamma$. No draw of firm characteristics is therefore sufficiently favourable to ensure that a firm will choose to produce regardless of its productivity. The first part of Assumption 1.iii ensures that all equilibria considered in our comparative static analysis exhibit the same extent of non-degenerate selection, i.e., if the least productive active firms undertake a given level of a given activity in one equilibrium, given $\gamma$, then this is also the case in all other equilibria considered.\(^9\) In short, we do not consider comparative statics where non-degenerate self selection, with respect to a given level of a given activity, degenerates. As earlier mentioned, this is standard in the

\(^8\) $\mu$ is a positive monotone transformation of the factor that scales up productivity when technology is upgraded in Bustos (2011). It will become clear below why we use $\tau^{-1}$ and not $\tau$ as a market parameter in $\beta$.

\(^9\) The least productive active firms will generally differ in identity across the ex ante and ex post equilibria as the cutoff productivity level for positive production (the exit threshold) varies across the two equilibria.
international trade literature as heterogeneous-firm models with self selection into specific activities often do not consider degenerate cases.\textsuperscript{10} The first part of Assumption 1.iii extends this tradition to our case of multi-dimensional firm heterogeneity. The second part of Assumption 1.iii states that the least productive active firms are unaffected, at least directly, by changes in the fundamental market parameters, $\beta$. This seems restrictive but it is quite standard to consider changes in parameters that have a differential impact upon firms. Often this is done in a way where the least productive active firms are not affected directly but only through the equilibrium demand level effects that we allow for.

Assumption 1.iv states that the distribution of productivities is Pareto. This assumption is widely used as it is analytically tractable and enjoys some empirical support.\textsuperscript{11} For our purpose, a convenient property of the Pareto distribution is that the share of firms undertaking at least a given level of an activity depends only on the relevant cutoff relative to the exit cutoff.

The role of Assumption 1 will soon become clear. Let us now introduce Assumption 2 which together with Assumption 1 gives us our first proposition.

**Assumption 2.** The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in $x$, i.e., for all $x, y \in X$,

$$
\pi(x \vee y, \Theta, \gamma, \beta) + \pi(x \wedge y, \Theta, \gamma, \beta) \geq \pi(x, \Theta, \gamma, \beta) + \pi(y, \Theta, \gamma, \beta),
$$

where $x \vee y$ is the componentwise maximum of $x$ and $y$ and $x \wedge y$ is the componentwise minimum. Moreover, the profit function, $\pi(x, \Theta, \gamma, \beta)$, has increasing differences in $\Theta$, i.e., if $x \geq x'$, then $\pi(x, \Theta, \gamma, \beta) - \pi(x', \Theta, \gamma, \beta)$ is monotonely non-decreasing in $\Theta$.

Supermodularity of the profit function in the organisational decision, $x$, is the formal equivalent of saying that activities are complementary. That is, increasing the level of one activity increases the gains of increasing the level of other activities. Increasing differences in $\Theta$ mean that, given the demand level $A$, it becomes more attractive to undertake at least a given level of any activity as productivity increases.

\textsuperscript{10}Anyway, note that nothing in our setup prevents the scenario where sorting into one activity is degenerate ex ante as well as ex post. It is thus entirely possible that all firms for instance exports.

\textsuperscript{11}See e.g. Axtell (2001) for empirical support for Pareto distributed productivities.
Under the fairly standard assumptions summarised in Assumption 1, complementary organisational decisions, which are reinforced by productivity as in Assumption 2, lead us directly to Proposition 1.\footnote{We should consider a list of references where supermodularity is used as in Ass. 2. The reader must know that Ass. 2 is also fairly standard.}

**Proposition 1.** Let $\lambda_i(x_i, \beta, S)$ denote the share of active firms undertaking at least the level $x_i$ of activity $i$. Under Assumption 1 & 2, if $S$ and $S'$ are both sublattices of $X$ and $S' \subseteq S$, then we have that $\lambda_i(x_i, \beta, S) \geq \lambda_i(x_i, \beta, S')$ for all activities $i \in \{1, 2, ..., N\}$.

**Proof.** The proof is provided in section XXX.

Proposition 1 states that expanding the set of admissible organisational decisions (weakly) increases the share of firms undertaking at least a given level of any activity. Note that here we consider the overall share of firms in the industry. Consistent use of market or industry. Thus, we do not condition on firm characteristics, $\gamma$, but aggregate across these, perhaps unobservable, characteristics. Proposition 1 reveals that the firm-level complementarity imposed by Assumption 2 asserts itself at the industry level.

If we consider expanding the admissible choice set, $S$, in one dimension, say by opening the economy to trade and allowing exports, then under Assumption 1 and 2, this will advance the industry structure in all dimensions, in the sense that it increases the share of firms that undertake at least a given level of all activities.

We further illustrate the result in Proposition 1 by continuing the example above. It turns out that Assumption 2 is also satisfied in Bustos (2011) and therefore, Proposition 1 applies in this context.

**Example 1** (continued). Consider the profit function (3). It is easy to verify that this is supermodular in $x = (1_{ex}, 1_{up})$. Thus, exporting and technology upgrading are complementary in this setup. The intuition is straightforward. Upgrading technology allows you to produce more efficiently. Obviously, a more efficient technology is worth more when applied to a larger production, and as exporting increases the production volume of the firm, it makes technology upgrading more attractive. Conversely, exporting allows you to serve another market. Obviously this is worth more when that market can
be served more efficiently and exporting is therefore more attractive when technology is already upgraded.

The profit function (3) also exhibits increasing differences in $\Theta$. Technology upgrading (consistency in Bu11: t. u.) is worth more when the firm is productive to start with since productivity and variable profits are scaled up by fixed percentages, cf. (3). Moreover, exporting is worth more when productivity is higher.

Consequently, the Bustos (2011) framework conforms to both Assumption 1 and 2 and therefore, Proposition 1 applies. If the economies move from autarky to costly trade, this will not only imply that some firms begin to export but also that the share of active firms that choose to upgrade technology (weakly) increases. Conversely, making technology upgrading available for firms implies that a larger share of active firms will choose to export. As emphasised above, introducing exporting or technology upgrading to the admissible choice set of firms advances the industry structure in all two dimensions, in the sense that a larger share of firms undertake each activity.

An additional assumption; this time on the dependence of profits on the market parameters, $\beta$, will lead to a second proposition on comparative statics.

**Assumption 3.** The profit function, $\pi(x, \Theta, \gamma, \beta)$, has increasing differences in $\beta$, i.e., if $x \geq x'$, then $\pi(x, \Theta, \gamma, \beta) - \pi(x', \Theta, \gamma, \beta)$ is monotonely non-decreasing in $\beta$.

**Proposition 2.** Under Assumption 1–3, if $\beta \geq \beta'$, then we have $\lambda_i(x_i, \beta, S) \geq \lambda_i(x_i, \beta', S)$ for all activities $i \in \{1, 2, \ldots, N\}$.

**Proof.** The proof is provided in section XXX. $\square$

Proposition 2 implies that increasing the market parameters $\beta$ increases the share of firms undertaking at least a given level of any activity. Basically, Assumption 3 implies that increasing $\beta$ makes increasing the level of the $N$ activities more attractive as profits increase by more from doing so. As the activities are complementary, increasing the attractiveness of some of them leads to an increase in the share of firms that undertake at least a given level of any of them. While Proposition 1 is concerned with an expansion of the firms’ admissible choice set, Proposition 2 is concerned with an improvement in the attractiveness of already admissible decisions. We illustrate this distinction by returning to our example based on Bustos (2011).
Example 1 (continued). It can easily be verified that the profit function (3) exhibits increasing differences in $\beta = (\tau^{-1}, \mu)$. The model of Bustos (2011) therefore also conforms to Assumption 3 and Proposition 2 applies.

Increases in $\tau^{-1}$, through trade liberalisation, where the iceberg trade cost, $\tau$, are lowered, thus imply that not only will a larger share of active firms now choose to export, a larger share of active firms also chooses to upgrade technology. Analogously, letting the technology upgrade have a larger impact on productivity, not only implies that a larger share of active firms upgrades technology, in addition, a larger share of active firms will export. To conclude our example based on Bustos (2011) we note that introducing exporting or technology upgrading, or improving the attractiveness of either, advances the industry structure in all two dimensions. This industry-level implication of the assumed firm-level complementarity between exporting and technology upgrading is somewhat different from the firm-level predictions emphasised by Bustos (2011).

Proposition 1 and 2 highlight the importance of integrating complementary activity decisions into an integrated framework as we now have done. It also suggests that, in explaining industry-level trends observed in the data, e.g. the rise of offshoring during the last three decades, you should not only look at factors directly related to the offshoring decision of firms but also at factors related to complementary activity decisions. If offshoring is complementary to e.g. technology upgrading at the firm level, and the technology upgrading opportunities have been improved, then this could explain part of the rise in offshoring at (the firm and) industry levels of analysis.\footnote{What happens to x-star at the firm level? How our result on x-star, where equilibrium effects play a role, could differ from the x-star firm-levels in Milgrom and Roberts (1995).}

3 Concluding Remarks

To come. The same goes for the proof and a section on applications. For a newer version of this paper, see the website of the Department of Economics and Business at Aarhus University.\ldots
References


