Firm Organization and Trade in Tasks*

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Abstract

We incorporate trade in tasks a là Grossman and Rossi-Hansberg (2008) into the theory of firm organization of Marin and Verdier (2012) to examine how offshoring affects the way firms organize. We test the predictions of the model based on firm level data of Austrian and German firms with subsidiaries in Eastern Europe and we show that the data are consistent with the theory. We find that offshoring of production labour leads firms to reorganize to a more decentralized hierarchy. Offshoring of skilled managers relaxes the 'war for talent' but toughens competition and thus has an ambiguous impact on firm organization.

Keywords:

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1 Introduction

In this paper, we explore how offshoring of different types of labor affects the internal organization of a firm. Our model is an extension of the model of firm organization in Marin and Verdier (2012) with two additional ingredients. First, we consider a small open economy with imperfect competition a là Demidova and Rodríguez-Clare (2009). Second, we incorporate in the model trade in tasks a là Grossman and Rossi-Hansberg (2008). These two ingredients allow us to develop a tractable framework to analyze the effects of offshoring (trade in tasks) on firm organization.

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We consider the following framework. There are two industries and two factors (workers and managers). One industry produces a homogenous good and is perfectly competitive. The other industry is a monopolistically competitive industry a là Krugman (1980). Each firm producing a variety of the differentiated product has a certain type of organization: centralized or decentralized (the structure of the firm organization is borrowed from Marin and Verdier (2012)). There is free entry into the industry. Workers (low-skilled labor) are used in production of both products, while managers (high-skilled labor) are only used for entry into the industry. In other words, to operate in the market, a firm needs a manager.

As a benchmark environment, we consider a small open economy. The price of the homogenous good is equal to the world price and, thereby, exogenous. To model a differentiated sector in a small open economy, we follow Demidova and Rodriguez-Clare (2009). In particular, we assume that the number of imported varieties and their prices are exogenously given. In addition, domestic firms producing varieties of the differentiated product face an exogenous foreign demand for their products.

We distinguish between offshoring of production labor and managerial labor. We find that offshoring of production labor in a differentiated sector unambiguously increases firm's real profits. As a result, firms become less centralized. The intuition behind this finding is as follows. Since labor is offshored only in the differentiated sector, the wage level of low-skilled labor does not change (it is pinned down by the world price of the homogenous good). Therefore, we can divide the impact of offshoring on real profits into two effects. First, there is a positive effect (the productivity effect) associated with a decrease in the marginal cost of production. Second, there is a negative effect (the competition effect) arising due to the fact that all other domestic firms also became more productive. We show that the positive effect is always stronger than the negative effect. This is due to the fact that we consider an open economy. The productivity effect is enhanced by the presence of the foreign market, while the competition effect is weakened by the presence of foreign firms (whose productivity does not change).

We also explore how offshoring of managerial labor affects the firm organization. We find that manager offshoring has an ambiguous impact on firm's real profits and firm organization. The intuition behind is as follows. On the one hand, manager offshoring can increase the total expenditure in the economy and, therefore, raise firm's profits. On the other hand, offshoring of managerial labor decreases the cost of entry into the market, which in turn increases the number of domestic firms operating in the market, reducing firm's profits. Which effect dominates depends on the parameters in the model. In economies with a sufficiently large exposure to foreign competition, manager offshoring is likely to lead to a rise in real profits and changes in the firm organization (from more to less centralized firms).

We then test the predictions of the model based on firm level data of Austrian and German firms with subsidiaries in Eastern Europe and we show that the data are consistent with the theory. We find that offshoring of production labour leads firms to reorganize to a more decentralized hierarchy. Offshoring of skilled managers has a positive impact on firm decentralization when the level of foreign competition is high and a negative impact otherwise.

2 The Model

We consider a small open economy with two goods and two factors of production: skilled and unskilled labor. The utility function of a representative consumer is given by

$$U(X,Y) = X^{a}Y^{1-a},\ a \in (0,1), \tag{1}$$

where Y is a homogenous good and X is a differentiated good:

$$X = \left[\int_{i\in\Omega} x(i)^{\rho} di + \int_{i'\in\Omega_m} x_m(i')^{\rho} di'\right]^{1/\rho} \text{ and } 0 < \rho < 1.$$

Here Ω and Ω_m represent the set of domestic and foreign varieties, respectively.

The homogenous good is produced in a perfectly competitive environment with a linear technology that requires only unskilled labor. Domestic varieties of the differentiated good are produced under monopolistic competition with free entry.

2.1 Firm Organization

In modeling the internal organization of a firm producing a variety of the differentiated product, we follow Aghion and Tirole (1997) and Marin and Verdier (2012). We assume that the firm consists of an owner (the principal P) and a manager (the agent A). In particular, in each firm the principal hires a skilled manager to start a firm and employs unskilled workers to produce.

We assume that there are a number of alternative ways to run the firm that differ in terms of production costs and, therefore, payoffs. However, only two of them are worth doing from the perspective of the principal and the manager. One project has the lowest cost of production and, thereby, yields the highest possible profit B. The other project is the "best project" for the manager yielding the highest possible non pecuniary benefit b for the manager. Thus, there is a potential conflict of interest between the principal and the manager. The best project for the principal is not the best project for the manager. Here B and b are supposed to be known ex ante, but the parties do not know ex ante which project yields such payoff.

To gather information on the payoffs of the projects, the principal uses a low skilled labor monitoring technology. Specifically, by investing some amount of unskilled labor L, the principal learns all the payoffs with probability $E = \min(1, \sqrt{L})$ and remains uninformed with probability $1 - E$.[1] Similarly, by exerting some effort ke ($k < b$), the agent learns the payoff of all projects with probability $e \in [0, \bar{e}]$ and remains uninformed with probability $1 - e$. We assume that the principal is risk neutral and that the agent is infinitely risk averse with respect to income. As a result, the agent is not responsive to monetary incentives and receives a fixed wage q.

We also assume that, among available projects, there are some with very high negative payoffs to both the principal and the agent. This assumption implies that choosing a random project without being informed is not profitable. In particular, if the principal and the agent do not know the payoffs, there is no production. Thus, private information about the payoffs gives decision control to the informed party that, in this case, has "real power" rather than "formal power" in the firm.

We distinguish between three types of the internal organization of a firm: a P-organization, an A-organization, and an O-organization. In the P-organization, the principal has formal power. In the A-organization, the principal delegates formal power to the manager. Finally, in the O-organization, the principal also has formal power, but the manager puts zero effort into learning the payoffs of the available projects (one can think of the O-organization as the P-organization with zero effort put in by the manager). Thus, the principal chooses between the three modes of firm organization to maximize her utility.

We introduce the following notation in the paper. We denote c_B as the marginal cost of production when the best project for the principal is implemented. Similarly, c_b is the marginal cost when the best project for the manager is chosen. An assumption that $c_B < c_b$ creates a conflict of interest in the model. We also denote αB ($\alpha \in [0, 1]$) as the principal's benefit when the best for the manager project is implemented and βb ($\beta \in [0, 1]$) as the manager's benefit

[1] The idea behind using unskilled labor to monitor the payoffs is explained by that the principal has managerial overload and there is a conflict of interest between the principal and the manager.

when the best for the principal project is implemented. Here α and β capture the degree of conflict between the principal and the manager.

2.1.1 The P-organization

Under the P-organization, the principal has formal power. In this case, if the principal is fully informed about the payoffs, then the best for the principal project is implemented and the principal's monetary payoff is B, while the manager receives βb. If the principal is uninformed and the manager is informed, then the manager has real power and suggests her best project (which is accepted by the principal). The principal receives a monetary payoff αB and the manager receives private benefit b. If both the parties remain uninformed, there is no production.

Hence, the expected payoffs of the principal and the agent are given by

$$\begin{aligned} u_P &= EB + (1-E)e\alpha B - wE^2, \\ u_A &= E\beta b + (1-E)eb - ke. \end{aligned}$$

Here w is the wage rate of unskilled labor (wE^2 is the principal's cost of learning the project payoffs). The first order conditions of the parties with respect to efforts E and e are

$$\begin{aligned} \text{Principal} &: \quad B(1-\alpha e) = 2wE, \\ \text{Agent} &: \quad \begin{cases} e = \bar{e} & \text{if } k \leq b(1-E), \\ e = 0 & \text{otherwise.} \end{cases} \end{aligned}$$

As can be seen, the principal invests in monitoring more the higher the monetary payoff of the best for her project, the larger the conflict of interest between the principal and the manager (the lower α), and the lower the manager's effort e. The agent puts in more effort the higher the benefit from the best for her project and the lower the principal's interference (the lower E). Thus, the principal's control over the firm comes at cost of lower agent's initiative.

Marin and Verdier (2012) show that the equilibrium levels of effort under the P-organization are

$$\begin{aligned} E_P^* &= \frac{B(1-\alpha\bar{e})}{2w}, \; e_P^* = \bar{e} \quad \text{if } B/w \leq \tilde{B}_P \\ E_P^* &= \frac{B}{2w}, \; e_P^* = 0 \qquad\quad \text{if } B/w > \tilde{B}_P, \end{aligned} \tag{2}$$

where

$$\tilde{B}_P = \frac{2(1-k/b)}{1-\alpha\bar{e}}.$$

Note that the case with zero effort put in by the manager corresponds to the O-organization.[2] Thus, it is straightforward to show that the expected utility of the principal under the P-organization is

$$u_P^* = w\left(E_P^*\right)^2 + e_P^* \alpha B. \tag{3}$$

2.1.2 The A-organization

Under the A-organization, the principal delegates formal power to the manager. In this case, if both the parties are informed, then the best for the manager project is implemented. When the principal is informed and the agent is uninformed, the principal suggests her preferred project and, thereby, has real power. The expected payoffs of the principal and the agent are

$$\begin{aligned} v_P &= e\alpha B + (1-e)EB - wE^2, \\ v_A &= eb + (1-e)E\beta b - ke. \end{aligned}$$

The first order conditions of the parties with respect to efforts E and e are

$$\begin{aligned} &\text{Principal:} && B(1-e) = 2wE, \\ &\text{Agent:} && \begin{cases} e = \bar{e} & \text{if } k \leq b(1-E\beta), \\ e = 0 & \text{otherwise.} \end{cases} \end{aligned}$$

As can be seen, the advantage of delegating formal power to the manager is that the manager has more incentives to become informed. In contrast, the principal has fewer incentives to invest in monitoring the projects and, as a result, the principal looses not only formal power, but also real power. The equilibrium values of E and e are

$$\begin{aligned} E_A^* &= \frac{B(1-\bar{e})}{2w},\ e_A^* = \bar{e} \quad \text{if } B/w \leq \tilde{B}_A \\ E_A^* &= \frac{B}{2w},\ e_A^* = 0 \quad \text{if } B/w > \tilde{B}_A, \end{aligned}$$

where

$$\tilde{B}_A = \frac{2(1-k/b)}{\beta\left(1-\bar{e}\right)}.$$

Hence, the expected utility of the principal under the A-organization is

$$v_P^* = w\left(E_A^*\right)^2 + e_A^* \alpha B. \tag{4}$$

[2] The O-organization can be thought of as a single managed P-firm (run by the principal) without an internal hierarchy. The skilled agent is employed but is not doing anything useful, since the agent's effort is assumed not to be contractible.

2.1.3 The Choice of Firm Organization

We now explore how the decision whether to delegate formal power to the manager or not depends on the firm's real payoff B/w. In particular, the following proposition holds (see Marin and Verdier (2012) for details).

Proposition 1 *Assume that*

$$\tilde{B}_P = \frac{2(1-k/b)}{1-\alpha\bar{e}} < \bar{B} = \frac{4\alpha}{2-\bar{e}} < \tilde{B}_A = \frac{2(1-k/b)}{\beta\,(1-\bar{e})}.$$

It follows that, for $B/w < \tilde{B}_P$, the principal chooses the P-organization. For $\tilde{B}_P \leq B/w < \bar{B}$, the principal prefers the A-organization. Finally, for $B/w \geq \bar{B}$, the O-organization (the P-organization with zero effort put in by the manager) yields the highest utility.

As can be seen, a trade-off between control and initiative arises at intermediate levels of profits. In this case, the principal delegates formal power to the manager to keep her initiative. At high levels of profits, the principal's stakes are so high that she puts a lot effort in monitoring the projects, which in turn leads to zero effort put in by the manager under any type of firm organization. As a result, the O-organization is optimal. At low levels of profits, the principal's stakes are small and, therefore, she monitors and intervenes little. The manager puts in the maximum effort and the P-organization is optimal (as there is no need to keep the manager's initiative by delegating her formal power).

2.2 Product Markets and Trade Environment

We now introduce product market competition and trade in the model. In particular, we consider a small open economy where the number and the prices of foreign varieties are taken as given. In addition, we assume that there is some exogenous foreign demand for domestic varieties, which is given by $A_m/p(i)^\sigma$ (where A_m is some parameter).[3]

Domestic demand for home and foreign varieties of the differentiated good X is

$$\begin{aligned} x(i) &= \frac{aRP^{\sigma-1}}{(p(i))^\sigma}, \\ x_m(i') &= \frac{aRP^{\sigma-1}}{(p_m(i'))^\sigma}, \end{aligned}$$

[3] In fact, here we adopt the framework in Demidova and Rodriguez-Clare (2009), who introduce trade in a small economy with heterogenous firms.

where R is the total expenditure in the economy, $p_m(i')$ is the price of an imported variety i', and P is the CES price index given by

$$P^{1-\sigma} = \int_{i\in\Omega} p(i)^{1-\sigma} di + \int_{i'\in\Omega_m} p_m(i')^{1-\sigma} di'.$$

Here σ is the elasticity of substitution. Without loss of generality, we assume that $p_m(i') = p_m$ for any i'. Then,

$$P^{1-\sigma} = \int_{i\in\Omega} p(i)^{1-\sigma} di + n^* (p_m)^{1-\sigma}, \tag{5}$$

where n^* is the number of foreign varieties in the market (which is exogenous).

Demand for the homogenous product is

$$Y = \frac{(1-a)R}{p_Y},$$

where p_Y is the world price of the good. It is assumed that the homogeneous good is produced with a linear one-to-one technology (requiring only unskilled labor). Hence, the wage rate of unskilled labor is pinned down by the world price:

$$w = p_Y.$$

We assume that the marginal cost of production of a firm producing variety i is $wc(i)/Z_X$, where $c(i)$ stands for the part of the cost that depends on which project is implemented. If the best for the principal project is implemented, then $c(i) = c_B$, otherwise, $c(i) = c_b$ (recall that $c_b > c_B$). The variable Z_X, in turn, describes the "productivity" gains from offshoring some production tasks abroad. Specifically, Z_X is strictly more than one, if some part of the production is offshored, and equal to one, if the firm does not offshore (we specify Z_X in the next section). Thus, given the demand for domestic varieties, the price of variety i is

$$p(i) = \frac{\sigma}{\sigma-1}\frac{w}{Z_X}c(i),$$

This implies that the firm's total profits (taking into account sales abroad) are

$$\pi(i) = C\left(aRP^{\sigma-1} + A_m\right)\left(\frac{w}{Z_X}c(i)\right)^{1-\sigma},$$

where $C = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$.

2.2.1 Trade in Tasks

To model offshoring, we adopt the framework in Grossman and Rossi-Hansberg (2008). In particular, we assume that production in the differentiated sector involves a continuum of tasks (of measure one) and performing each task requires $c(i)$ units of labor. Production of each task can be offshored abroad. The cost of offshoring task $j \in [0, 1]$ is $\gamma t(j)$, where $t(j)$ is increasing and continuously differentiable, implying that it is more costly to offshore high-indexed tasks.

It is profitable to offshore task j if and only if the cost of producing it domestically is higher than the cost of offshoring. That is,

$$wc(i) > \gamma t(j) w^* c(i),$$

where w^* is the cost of unskilled labor abroad. The latter implies that tasks with index $j \in [0, I_X]$ are offshored, while the other tasks are performed domestically. Here I_X solves[4]

$$w = \gamma t(I_X) w^*. \tag{6}$$

Given the possibility of offshoring, the marginal cost of a firm producing variety i is

$$MC_i = wc(i)\,(1 - I_X) + w^* c(i) \int_0^{I_X} \gamma t(j) dj.$$

Taking into account (6), we have

$$MC_i = wc(i) \left(1 - I_X + \left(\int_0^{I_X} t(j) dj \right) / t(I_X) \right).$$

From the definition of Z_X,

$$MC_i = \frac{w}{Z_X} c(i).$$

This means that the productivity gains from offshoring represented by Z_X are

$$Z_X = \frac{1}{1 - I_X + \left(\int_0^{I_X} t(j) dj \right) / t(I_X)} < 1.$$

As can be seen, Z_X is increasing in I_X. The more tasks are offshored, the more productive the firms. If there is no offshoring ($I_X = 0$), then Z_X is equal to one and the marginal cost is $wc(i)$.

[4] Note that to guarantee the interior solution of (6), we need to assume that

$$\frac{1}{t(1)} < \gamma \frac{w^*}{w} < \frac{1}{t(0)}.$$

The condition states that the cost of offshoring of tasks with lower indexes should be sufficiently low, while the cost of offshoring of tasks with higher indexes should be sufficiently high. In this case, only a certain positive fraction of tasks is offshored.

2.3 The Equilibrium

Recall that the profits of a firm producing variety i are

$$\pi(i) = C\left(aRP^{\sigma-1} + A_m\right)\left(\frac{w}{Z_X}c(i)\right)^{1-\sigma}.$$

If the implemented project is the best project for the principal, the marginal cost of production is c_B. This implies that the highest possible principal's benefit is

$$B = C\left(aRP^{\sigma-1} + A_m\right)\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}. \tag{7}$$

Moreover, it is straightforward to see that

$$\alpha = \left(\frac{c_b}{c_B}\right)^{1-\sigma} < 1.$$

Depending on the parameters in the model, there are three types of equilibria (with the P-organizations, the A-organizations, and the O-organizations). Each equilibrium is characterized by the free entry condition and the factor markets clearing conditions. In the next subsection, we consider the equilibrium with the P-organizations.

2.3.1 Equilibrium with P-organizations

The free entry condition means that the expected principal's benefits are equal to the cost of starting a firm. To start a firm, the principal has to hire a skilled worker as a manager. Therefore, in the case of the P-organization, the free entry condition implies that

$$w\left(E_P^*\right)^2 + e_P^*\alpha B = q,$$

where q is the wage rate of the manager and the left-hand side is the principal's benefits (see (3)). Taking into account the expressions for E_P^* and e_P^* (see (2)), the free entry condition can be rewritten in the following way:

$$\frac{\left(1-\bar{e}\alpha\right)^2}{4}\left(\frac{B}{w}\right)^2 + \bar{e}\alpha\frac{B}{w} = \frac{q}{w}. \tag{8}$$

Let us denote n as the number of firms entering the market. Then, under the P-organization, $E_P^* n$ firms implement projects that are best for principals, $\left(1 - E_P^*\right)e_P^* n$ firms implement projects that are best for managers, and the rest leave the market (as both the principal and the manager remain uninformed). Hence, taking into account that some tasks are offshored abroad (specifically, only $1 - I_X$ tasks are performed domestically), demand for unskilled labor in the differentiated sector is

$$L_X = n(1 - I_X)\left[E_P^* c_B x_B + (1 - E_P^*) e_P^* c_b x_b\right],$$

where x_B and x_b are outputs of firms with marginal cost c_B and c_b, respectively. Then, the unskilled labor market clearing condition is

$$L_X + Y^S + n(E_P^*)^2 = L, \tag{9}$$

where Y^S is the production of good Y, $n(E_p^*)^2$ is labor used by principals to monitor the payoffs of projects, and L is the total endowment of unskilled labor.

The demand for skilled labor is equal to the number of firms entering the market. Thus, the market clearing condition for skilled labor is

$$H = n, \tag{10}$$

where H is the endowment of skilled labor in the economy. As the wage rate of unskilled labor w is pinned down by the world price of the homogenous good and Z_X is exactly determined by the relative wage w/w^* and the cost of offshoring $t(j)$, the equilibrium values of q and B can be found from (8) and (7). Finally, the amount produced in the homogenous sector is determined by (9). Thus, we can find all the endogenous variables in the model.

Note that to avoid specialization, the parameters in the model must be such that Y^S is positive in the equilibrium. Moreover, to be consistent with the P-organization equilibrium, the equilibrium value of B/w must be less than $\tilde{B}_P$ (see *Proposition* 1).

The equilibria with A- and O-organizations can be described in an exactly similar way.

3 Firm Organization and Offshoring

We now explore the relationship between offshoring of production tasks and the type of firm organization chosen by the principal. In particular, we examine how a uniform decrease in the cost of offshoring (lower γ) affects the real profits B/w. The idea behind this exercise is the relationship between the type of firm organization and the real profits stated in *Proposition 1*. *Proposition 1* suggests that the level of firm decentralization (the level of formal power delegated to a manager) has a hump shape as a function of the real profits. Thus, understanding of the relationship between the cost of offshoring and real profits sheds a light on the connection between offshoring and firm organization.

Since the results we formulate below hold for any type of firm organization, without loss of generality, we consider the equilibrium with the P-organizations described in the previous section. Recall that from (7), the highest principal's benefits are

$$B = C\left(aRP^{\sigma-1} + A_m\right)\left(\frac{w}{Z_X}c_B\right)^{1-\sigma},$$

where R is the total expenditure the economy given by $wL + qH$. The latter implies that

$$\frac{B}{w} = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(aP^{\sigma-1}\left(L + \frac{q}{w}H\right) + \frac{A_m}{w}\right).$$

The price index is

$$P^{1-\sigma} = \int_{i\in\Omega} p(i)^{1-\sigma}di + n^*\left(p_m\right)^{1-\sigma}.$$

As in the P-equilibrium $E_P^* n$ domestic firms implement projects with cost c_B and $(1 - E_P^*)\, e_P^* n$ firms implement projects with cost c_b, the price index can be written as follows:

$$P^{1-\sigma} = n\left(\frac{1}{\rho}\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(E_P^* + (1 - E_P^*)\, e_P^*\alpha\right) + n^*\left(p_m\right)^{1-\sigma},$$

where $\rho = (\sigma - 1)/\sigma$. Moreover, using the expressions for E_P^* and e_P^* in (2), it is straightforward to show that

$$E_P^* + (1 - E_P^*)e_P^*\alpha = \bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}.$$

Thus, the price index is equal to

$$P^{1-\sigma} = n\left(\frac{1}{\rho}\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right) + n^*\left(p_m\right)^{1-\sigma}.$$

Hence, substituting for $\frac{q}{w}$ (see (8)) and taking into account that $n = H$, we derive

$$B/w = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{a\left(L + \left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2 + \bar{e}\alpha\frac{B}{w}\right)H\right)}{H\left(\frac{1}{\rho}\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right) + n^*\left(p_m\right)^{1-\sigma}} + \frac{A_m}{w}\right). \tag{11}$$

As w is pinned down by the world price p_Y and Z_X depends only on the relative wage and the cost of offshoring, equation (11) is sufficient to find the equilibrium value of the real profits B/w. Note that given B/w, the relative wage of skilled labor, q/w, can be found from the free entry condition. In the Appendix, we show that the solution of (11) exists and is unique.

Notice that in the closed economy (when $A_m = 0$ and $n^* = 0$), we have

$$B/w = \frac{Ca\rho^{1-\sigma}\left(\frac{L}{H} + \frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2 + \bar{e}\alpha\frac{B}{w}\right)}{\left[\bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right]}, \tag{12}$$

which can be treated as a special case of the model in Marin and Verdier (2012). In this case, the real profits do not depend on the marginal cost of production and, thereby, on the cost of offshoring.

3.1 Offshoring of Production Tasks

We then explore how changes in γ affect the equilibrium value of B/w. Recall that

$$Z_X = \frac{1}{1 - I_X + \left(\int_0^{I_X} t(j)dj\right)/t(I_X)},$$

where I_X is determined from $w = \gamma t(I_X)w^*$. As w is exogenous, the only effect of γ on B/w is through changes in Z_X. In particular, lower offshoring cost γ results in higher productivity gains Z_X. Thus, we need to explore how a rise in Z_X affects the real profits. The following proposition holds.

Proposition 2 *In the P-equilibrium, a rise in Z_X leads to a higher value of the real profits in equilibrium.*

Proof. The proof is directly followed from equation (11). Specifically, it is straightforward to see that a rise in Z_X increases the right-hand side of the equation for any value B/w. As a result, the equilibrium value of B/w rises. ■

The intuition behind this finding is as follows. There are two effects of a rise in Z_X on the real profits. The direct productivity effect is a decrease in the marginal cost (lower $wc(i)/Z_X$) that increases firm's real profits. The indirect market size effect is a decrease in the market share each firm gets in the domestic market (lower $RP^{\sigma-1}$) caused by that all other firms become more productive. This in turn reduces firm's real profits. As can be seen from the proposition, the positive direct effect is stronger than the negative indirect effect. This is due to that we consider an open economy. In the case of an open economy, the effect of lower marginal cost on the profits is enhanced by the presence of the foreign market (characterized by A_m). Moreover, the market size effect is weakened by the presence of foreign firms whose productivity does not change: i.e., more productive domestic firms take some share of the market from foreign firms. As can be seen from (12), in the closed economy the two effects are exactly cancelled out.

As B/w rises and becomes closer the cutoff $\tilde{B}_P$ (see *Proposition* 1), the P-equilibrium becomes "closer" to the A-equilibrium and, to some extent, firms become less centralized. In particular, if B/w exceeds the cutoff $\tilde{B}_P$, firms switch from the P-organization to the A-organization where the manager has formal power. Similarly, in the equilibrium with the A-organizations, lower cost of offshoring implies that the A-equilibrium becomes closer to the O-equilibrium and, therefore, firms become more centralized. Hence, the theory suggests a hump-shaped relationship between offshoring and firm organization.

3.2 Offshoring of Managerial Tasks

In this section, we ask whether offshoring of different types of labor leads to the same implications for firm organization. In particular, we examine how offshoring of managerial labor affects the firm's real profits and, thereby, the level of firm's decentralization. We assume that, to start a firm, a continuum of tasks (of measure one) performed by a manager is involved and some of these tasks can be offshored abroad. Performing each task requires one unit of managerial labor. Tasks that are not offshored are performed by a domestic manager who is paid according to the number of performed tasks. As before, the domestic manager also monitors the payoffs from available projects, as she receives non pecuniary benefits from implemented projects. It is assumed that the "foreign" manager does not receive any benefits from implemented projects and, therefore, does not have incentives to monitor the payoffs. That is, the foreign manager only performs some offshored tasks that are necessary to start a firm.

As in the previous section, we analyze the P-equilibrium in the model. We assume that the fraction of tasks that can be offshored is exogenously given by $I_S < 1$. Endogenizing I_S leads to unnecessary complexity of the analysis and does not substantially change the qualitative results. The cost of managerial labor abroad is q^*. Note that offshoring is profitable only if the cost of foreign labor is cheaper than the cost of domestic labor: i.e., $q > q^*$. We assume that q^* is sufficiently low that the constraint on the number of tasks that can be offshored is binding: domestic firms find it profitable to offshore all the tasks they can offshore. Specifically, we assume that q^* is such that

$$C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\frac{A_m}{w} > 2\frac{\sqrt{(\bar{e}\alpha)^2+\frac{q^*}{w}(1-\bar{e}\alpha)^2}-\bar{e}\alpha}{(1-\bar{e}\alpha)^2}. \tag{13}$$

The latter inequality guarantees that q is strictly greater than q^* in equilibrium where all the tasks are offshored (see details in the Appendix).

When some managerial tasks are offshored abroad, the cost of entry is equal to $q(1-I_S)+q^*I_S$. Hence, the free entry condition can be written as follows

$$\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}=\frac{q(1-I_S)+q^*I_S}{w}.$$

which implies that

$$\frac{q}{w}=\frac{1}{1-I_S}\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*I_S}{w}\right). \tag{14}$$

The market clearing condition for skilled labor is now given by

$$n(1-I_S)=H \iff n=\frac{H}{1-I_S}. \tag{15}$$

To simplify the notation, we denote the level of foreign competition $n^*(p_m)^{1-\sigma}$ by IM. Hence, taking into account (14) and (15), the equilibrium value of B/w is determined by

$$B/w=C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w}+\frac{a\left(L(1-I_S)+\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*I_S}{w}\right)H\right)}{H\left(\frac{1}{\rho}\frac{w}{Z_X}c_B\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right]+(1-I_S)IM}\right). \tag{16}$$

We then examine how changes in the number of managerial tasks that can be offshored affect the real profits. Equation (16) results in a certain implicit relationship between B/w and I_S in equilibrium. In particular, the following proposition holds.

Proposition 3 *There exists such a value of IM denoted by IM_P that, in the P-equilibrium, B/w is increasing in I_S if and only if*

$$IM > IM_P. \tag{17}$$

Proof. In the Appendix. ■

Recall that the real profits can be written as follows:

$$\frac{B}{w}=C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(aP^{\sigma-1}\left(L+\frac{q}{w}H\right)+\frac{A_m}{w}\right).$$

Thus, offshoring of managerial tasks has two effects on the real profits. First, as the right-hand side in (14) is increasing in I_S, higher I_S can increase the relative cost of managerial labor q/w, which raises firm's real profits through a rise in the real total expenditure (given by $L+qH/w$).[5] This effect is reminiscent of the productivity effect in Grossman and Rossi-Hansberg (2008). Second, a rise in I_S reduces the cost of entry into the market and, thereby, increases the number of domestic firms n. This decreases the price index in the economy, which, in turn, reduces firm's real profits. Note that the effect on the price index is weaker, the higher is the competition from abroad (measured by IM). Thus, if IM is sufficiently high, the positive effect prevails over the negative and the real profits rise.

Proposition 3 suggests that the impact of offshoring of managerial labor on firm organization depends on the level of foreign competition. If the foreign competition is sufficiently tough, then offshoring of managerial labor results in firm decentralization (the P-equilibrium becomes "closer" to the A-equilibrium). Otherwise, offshoring of managerial labor leads to even more centralized firms. Note that the analogue of *Proposition* 3 can be formulated in the case of A- and O-equilibria as well.

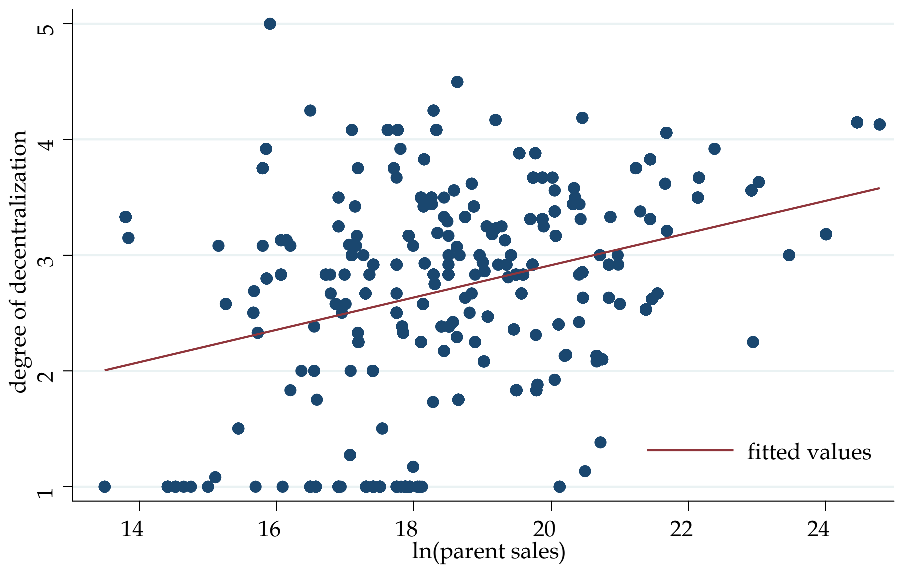
4 Empirical Analysis

In this section, we test the predictions of the model based on firm level data of Austrian and German firms with subsidiaries in Eastern Europe and we show that the data are consistent with the theory.

[5] The derivative of the right-hand side in (14) with respect to I_S is given by

$$\frac{\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*}{w}}{(1-I_S)^2}>0,$$

as $q>q^*$.



Sample of 2123 German and Austrian multinational firms. The variable on the Y-axis is an index that measures the degree of decentralization in decision making with values between 1 (decisions are completely made by the CEO) and 5 (decisions are completely made at the divisional level). The variable on the X-axis is the natural logarithm of the parent firm.

Table 1: OFFSHORING OF PRODUCTION LABOUR

VARIABLES	(1) OLS		(2) OLS		(3) OLS		(4) OLS		(5) OLS	
intrafirm trade	0.00366***	0.167	0.00372***	0.171	0.00380***	0.173	0.00332***	0.152	0.00332***	0.154
	[0.000448]		[0.000481]		[0.000480]		[0.000460]		[0.000503]	
ln parent sales	0.143***	0.343	0.129***	0.307	0.140***	0.332	0.139***	0.334	0.118***	0.281
	[0.0161]		[0.0165]		[0.0161]		[0.0168]		[0.0174]	
foreign competition			0.496***	0.234					0.468***	0.222
			[0.0807]						[0.0876]	
domestic competition					0.205***	0.122			0.0687	0.0409
					[0.0701]				[0.0766]	
distance to the technological frontier							-3.45e-08	-0.0303	-1.01e-07***	-0.0792
							[4.02e-08]		[3.03e-08]	
industry dummies	yes		yes		yes		yes		yes	
home country dummies	yes		yes		yes		yes		yes	
host country dummies	yes		yes		yes		yes		yes	
observations	724		699		713		708		680	
adjusted R-squared	0.118		0.171		0.131		0.119		0.177	

The dependent variable is an index that measures the degree of decentralization in decision making with values between 1 (decisions are completely made by the CEO) and 5 (decisions are completely made at the divisional level). Our measure for the degree of offshoring is intrafirm trade weighted by the parent firm's sales. Robust standard errors are reported in brackets; the righter columns contain standardized beta coefficients; *** p<0.01, ** p<0.05, * p<0.1. All regressions additionally include a constant, industry dummies and dummies for the home and host countries. The foreign and domestic competition regressors are dummies obtained from survey data and take the value 1 if the firm faces intense or very intense competition and 0, else. The distance to the technological frontier is measured by the difference between the productivity of the top 95% percentile firm within the firm's industry and the firm's own productivity level.

Table 2: OFFSHORING OF SKILLED MANAGERS

VARIABLES	(1) OLS	(2) OLS	(3) OLS
intrafirm trade	0.00800***	0.00778***	0.00807***
	[0.00104]	[0.00102]	[0.000981]
ln parent sales	0.221***	0.245***	0.284***
	[0.0326]	[0.0309]	[0.0320]
offshoring of managers	0.230*	-0.829***	-0.825***
	[0.137]	[0.233]	[0.249]
offsh. of man. * foreign comp.		1.374***	1.379***
		[0.263]	[0.276]
foreign competition	0.970***	0.0351	0.0221
	[0.146]	[0.226]	[0.241]
distance to the			6.68e-07**
technological frontier			[3.09e-07]
industry dummies	yes	yes	yes
home country dummies	yes	yes	yes
host country dummies	yes	yes	yes
observations	302	302	301
adjusted R-squared	0.249	0.315	0.323

The dependent variable is an index that measures the degree of decentralization in decision making with values between 1 (decisions are completely made by the CEO) and 5 (decisions are completely made at the divisional level). Our measure for the degree of offshoring is intrafirm trade weighted by the parent firm's sales. Robust standard errors are reported in brackets; *** $p<0.01$, ** $p<0.05$, * $p<0.1$. All regressions additionally include a constant, industry dummies and dummies for the home and host countries. The foreign and domestic competition regressors are dummies obtained from survey data and take the value 1 if the firm faces intense or very intense competition and 0, else. The distance to the technological frontier is measured by the difference between the productivity of the top 95% percentile firm within the firm's industry and the firm's own productivity level.

5 Conclusion

To be written.

Appendix

Existence and Uniqueness of the Equilibrium

In this subsection of Appendix, we show that there exists a unique solution of (11) with respect to B/w. Let us define $F(B/w)$ as the right-hand side of (11). Then, B/w solves

$$B/w = F(B/w).$$

It can be shown that $F(B/w)$ behaves as a linear function (of B/w) when B/w tends to infinity. The slope of this function is equal to $Ca\rho^{1-\sigma}/2$. Remember that $C = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$ and $\rho = \frac{\sigma-1}{\sigma}$. Then, the slope of $F(B/w)$ in a neighborhood of infinity is $a/2\sigma$, which is strictly less than one (as $a < 1$ and $\sigma > 1$). Thus, for low values of B/w, $F(B/w) > B/w$ (as $F(0) > 0$), while, for high values of B/w, $F(B/w) < B/w$. This immediately implies that the solution of (11) exists.

Note that equation (11) can be transformed in a quadratic equation of B/w and, therefore, cannot have more than two solutions. Taking into account the properties of function $F(B/w)$, one can see that equation (11) cannot have exactly two solutions as well. As a result, (11) has a unique solution.

When Offshoring is Profitable

Notice that $q > q^*$ if and only if

$$\frac{q(1-I_S)+q^*I_S}{w} > \frac{q^*}{w}.$$

The left-hand side of the inequality is the real cost of entry into the market if all the tasks are offshored. That is,

$$\frac{q(1-I_S)+q^*I_S}{w} = \frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2 + \bar{e}\alpha\frac{B}{w}.$$

Thus, $q > q^*$ if and only if

$$\begin{aligned}\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2 + \bar{e}\alpha\frac{B}{w} &> \frac{q^*}{w} \iff \\ \frac{B}{w} &> 2\frac{\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2} - \bar{e}\alpha}{(1-\bar{e}\alpha)^2}.\end{aligned}$$

As can be inferred from the equilibrium condition for B/w (see (11)), B/w is always strictly greater than $C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\frac{A_m}{w}$. Hence,

$$\begin{aligned}C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\frac{A_m}{w} &> 2\frac{\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2} - \bar{e}\alpha}{(1-\bar{e}\alpha)^2} \implies \\ \frac{B}{w} &> 2\frac{\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2} - \bar{e}\alpha}{(1-\bar{e}\alpha)^2} \implies \\ q &> q^*.\end{aligned}$$

The Proof of Proposition 3

Consider equation (16), which is given by

$$B/w = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w}+\frac{a\left(L\left(1-I_S\right)+\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*I_S}{w}\right)H\right)}{H\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right]+\left(1-I_S\right)IM}\right),$$

where IM is the measure of foreign competition given by $n^*\left(p_m\right)^{1-\sigma}$. Let us denote the right-hand side of the equation as $F(B/w, I_S)$. That is,

$$F(B/w, I_S) = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w}+\frac{a\left(L\left(1-I_S\right)+\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*I_S}{w}\right)H\right)}{H\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right]+\left(1-I_S\right)IM}\right).$$

Then, the equilibrium value of B/w solves

$$B/w = F(B/w, I_S).$$

It can be shown that

$$F'_{I_S}(B/w, I_S) = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}aH\frac{G(B/w)}{\left(H\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right]+(1-I_S)IM\right)^2},$$

where

$$\begin{aligned} G(B/w) &= -\left(L+\frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right] \\ &\quad +IM\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*}{w}\right). \end{aligned}$$

Let us denote $(B/w)^*$ as the positive solution of

$$IM\left(\frac{(1-\bar{e}\alpha)^2}{4}\left(\frac{B}{w}\right)^2+\bar{e}\alpha\frac{B}{w}-\frac{q^*}{w}\right) = \left(L+\frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha+\frac{(1-\bar{e}\alpha)^2}{2}\frac{B}{w}\right].$$

It is straightforward to see that $G(B/w) > 0$ if and only if $B/w > (B/w)^*$. Hence, we can conclude that a rise in I_S raises $F(B/w, I_S)$ if and only if $B/w > (B/w)^*$. In other words, if the equilibrium value of B/w is greater than $(B/w)^*$, then a further rise in I_S increases $F(B/w, I_S)$ and, thereby, B/w. Otherwise, $F(B/w, I_S)$ and B/w go down with a rise in I_S.

A direct implication of these findings is that B/w is increasing in I_S on $[0, 1)$ if and only if $(B/w)^0 > (B/w)^*$, where $(B/w)^0$ is the solution of

$$B/w = F(B/w, 0).$$

That is, $(B/w)^0$ is the equilibrium value of B/w when $I_S = 0$ (there is no offshoring of managerial labor). Next, we find the condition when $(B/w)^0 > (B/w)^*$.

Using the definition of $(B/w)^0$, it is straightforward to show that $(B/w)^0 > (B/w)^*$ if and only if $F((B/w)^*, 0) > (B/w)^*$. We have that

$$F((B/w)^*, 0) = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w} + \frac{a\left(L + \left(\frac{(1-\bar{e}\alpha)^2}{4}\left((B/w)^*\right)^2 + \bar{e}\alpha\,(B/w)^*\right)H\right)}{H\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}(B/w)^*\right] + IM}\right).$$

As $G((B/w)^*) = 0$,

$$\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\left[\bar{e}\alpha + \frac{(1-\bar{e}\alpha)^2}{2}(B/w)^*\right] = \frac{IM\left(\frac{(1-\bar{e}\alpha)^2}{4}\left((B/w)^*\right)^2 + \bar{e}\alpha\,(B/w)^* - \frac{q^*}{w}\right)}{\left(L + \frac{q^*}{w}H\right)}.$$

Hence, we derive that

$$F((B/w)^*, 0) = C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w} + \frac{a\left(L + \frac{q^*}{w}H\right)}{IM}\right).$$

Thus, B/w is increasing in I_S on $[0, 1)$ if and only if

$$C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w} + \frac{a\left(L + \frac{q^*}{w}H\right)}{IM}\right) > (B/w)^*. \tag{18}$$

The next step is to consider an explicit expression for $(B/w)^*$. We introduce the following notation:

$$\begin{aligned}
D_0 &= IM\frac{(1-\bar{e}\alpha)^2}{4}, \\
D_1 &= IM\bar{e}\alpha - \left(L + \frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\frac{(1-\bar{e}\alpha)^2}{2}, \\
D_2 &= \left(L + \frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\bar{e}\alpha + IM\frac{q^*}{w}.
\end{aligned}$$

Then, $(B/w)^*$ solves

$$D_0\left((B/w)^*\right)^2 + D_1\,(B/w)^* - D_2 = 0,$$

which implies that

$$(B/w)^* = \frac{\sqrt{D_1^2 + 4D_0D_2} - D_1}{2D_0} > 0.$$

Thus, inequality (18) is equivalent to

$$C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\left(\frac{A_m}{w} + \frac{a\left(L + \frac{q^*}{w}H\right)}{IM}\right) > \frac{\sqrt{D_1^2 + 4D_0D_2} - D_1}{2D_0} \iff$$

$$C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\frac{A_m}{w} > \frac{1}{IM}\left(2\frac{\sqrt{D_1^2 + 4D_0D_2} - D_1}{(1-\bar{e}\alpha)^2} - Ca\left(L + \frac{q^*}{w}H\right)\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\right). \tag{19}$$

Let us denote the right-hand side of inequality (19) as $K(z)$ where z is $\frac{1}{IM}$. Then,

$$K(z) = 2\frac{\sqrt{(\bar{e}\alpha - K_1 z)^2 + \left(K_2 z + \frac{q^*}{w}\right)(1-\bar{e}\alpha)^2} - (\bar{e}\alpha - K_1 z)}{(1-\bar{e}\alpha)^2} - K_3 z,$$

where

$$\begin{aligned}
K_1 &= \left(L + \frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\frac{(1-\bar{e}\alpha)^2}{2}, \\
K_2 &= \left(L + \frac{q^*}{w}H\right)\left(\frac{wc_B}{Z_X\rho}\right)^{1-\sigma}\bar{e}\alpha, \\
K_3 &= Ca\left(L + \frac{q^*}{w}H\right)\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}.
\end{aligned}$$

Next, we explore the properties of the function $K(z)$. It is straightforward to see that $K(0) > 0$. The derivative of $K(z)$ with respect to z is given by

$$K'(z) = \frac{-2K_1(\bar{e}\alpha - K_1 z) + K_2(1-\bar{e}\alpha)^2}{(1-\bar{e}\alpha)^2\sqrt{(\bar{e}\alpha - K_1 z)^2 + \left(K_2 z + \frac{q^*}{w}\right)(1-\bar{e}\alpha)^2}} + \frac{2K_1}{(1-\bar{e}\alpha)^2} - K_3.$$

Hence,

$$K'(0) = \frac{-2K_1\bar{e}\alpha + K_2(1-\bar{e}\alpha)^2}{(1-\bar{e}\alpha)^2\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2}} + \frac{2K_1}{(1-\bar{e}\alpha)^2} - K_3.$$

Since $-2K_1\bar{e}\alpha + K_2(1-\bar{e}\alpha)^2 = 0$,

$$K'(0) = \frac{2K_1}{(1-\bar{e}\alpha)^2} - K_3 > 0,$$

as $Ca\rho^{1-\sigma} < 1$. Thus, $K(z)$ is increasing in the neighborhood of zero. Moreover, $K'(\infty)$ is also positive, implying that $K(\infty) = \infty$. As, for any constant A, the equation $K(z) = A$ has at most two solutions and $K(\infty) = \infty$, we can conclude that $K(z)$ is an increasing function in z.

This in turn means that the right-hand side of inequality (19) is always positive and decreasing in IM with the value at infinity being equal to

$$K(0) = 2\frac{\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2} - \bar{e}\alpha}{(1-\bar{e}\alpha)^2}.$$

As we assume that $C\left(\frac{w}{Z_X}c_B\right)^{1-\sigma}\frac{A_m}{w} > 2\frac{\sqrt{(\bar{e}\alpha)^2 + \frac{q^*}{w}(1-\bar{e}\alpha)^2} - \bar{e}\alpha}{(1-\bar{e}\alpha)^2}$ (see (13)), there exists such a value of IM (we denote it as IM_P) that inequality (19) holds if and only if $IM > IM_P$.