Gender Wage Gaps across Skills and Trade Openness

Preliminary version

Sarra Ben Yahmed*
Aix-Marseille Université (AMSE)

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Abstract

Several empirical studies have shown that the effect of openness on the gender wage gap depends on the skill requirement of the workplace. This paper offers a theoretical explanation to understand that finding.

We integrate a statistical discrimination framework with the labour assignment approach to give general conditions under which the matching between firms and workers gives rise to a wider gender wage gap at the upper tail of the distribution. Workers' characteristics vary in two dimensions, skills and labour market attachment. The inability to observe individual's labour market attachment induces employers to base partly their decision on group average. Firms simultaneously choose their technology and the workers they hire. Following the literature on labour and international trade, we assume that skills act as complements to technological upgrading. Exporting firms that are more productive, are also more skill-intensive and pay higher wages ; assuming further that worker's job commitment is a complement to technological upgrading, we find that a reduction in trade costs increases the gender wage gap among high-skill workers only, in line with empirical evidence. This model also sheds light on the link between the overall wage structure and gender wage inequality.

*GREQAM, 2 rue de la Charité, 13002 Marseille, e-mail:*sarra.ben-yahmed@univ-amu.fr*

1 Introduction

A few empirical studies have shown that the effect of openness on both the gender wage gap and the employment gap depends on the skill requirement of the job. Openness may improve women's access to paid employment and thus may increase total female wage earnings in an economy, but those gains are concentrated in low-skill low-wage jobs within industries. What is more, the wage gap in skilled positions may increase with trade openness. Despite a strong empirical interest, the channels of impact underlying this pattern have not been formalized. This paper seeks to offer an explanation for this phenomenon by relating skill-biased investments fostered by trade openness with discriminating behaviors that depend on jobs' characteristics.

It is well known that trade openness affects workers differently along the skill distribution ; the skill premium is found to increase with trade exposure in both developed and developing countries[1]. At the same time, evidence on women facing a glass ceiling is widespread and persistent in numerous countries. The adjusted gender wage gap is higher in the upper part of the wage distribution. The existence of a glass ceiling implies that women are more discriminated against in high wage jobs, either because they are paid less in those jobs either because employers do not promote women to top jobs[2]. Besides, as heterogeneity in wage levels across firms explain a large portion of variation in individuals wages, higher adjusted wage gaps at the top of distribution can also be accounted for by sorting of women into low-wage firms, a glass door effect[3]. Given that the incidence of gender discrimination is stronger at the top of the wage distribution and that trade openness tends to exacerbate wage inequality, one should expect a widening of the gender wage gap at the top of the wage distribution following trade liberalization. This paper provides

[1]Empirical assessments of the impact of trade openness on wage inequality are for instance, Bernard & Jensen (1997) on the US, Pavcnik et al. (2004) on Brazil, Brambilla et al. (2011) on several Latin-American countries and also Goldberg & Pavcnik (2007) for a literature review. Theoretical models that explain how trade increases the skill-premium are Neary (2003), Yeaple (2005), Bustos (2011) and Sampson (2012) among others.

[2]Recent examples in this literature include Nopo et al. (2010) for Latin American countries, de la Rica et al. (2008) for Spain and Albrecht et al. (2009) for the Netherlands.

[3]Meyersson Milgrom et al. (2001) show that segregation into low-wage occupations and low-wage establishments explain part of the wage gap in Sweden while Amuedo-Dorantes & De la Rica (2006) come to similar conclusion for Spain. Javdani (2012) applies the methodology of Pendakur & Woodcock (2010) on Canadian data to decompose the gender wage gaps along the distribution into a within-firm glass ceiling effect and a glass door effect i.e. the under-representation of women in high-wage firms.

a description of the mechanism and the conditions under which trade openness reinforces the glass ceiling effect.

A few empirical studies have pointed up the heterogeneous effects of trade on the relative position of women depending on the skill requirement of the workplace. Joekes (1995) argues that the positive impact of export oriented industrialization on female employment might be reversed when new technologies are introduced. Ozler (2000) uses plant-level data from Turkey and shows that trade liberalization has led to employment gains for women relative to men in manufacturing sector. However, women continue to be employed in low-skill and low-pay jobs within plants. Furthermore, among plants with a high female share, as well as among large establishments, investments in machinery and equipment lead to a decline in the female share. This finding backs the argument that employment gains for women following trade liberalization might be reversed as a consequence of technological upgrading. Ederington et al. (2009) find that sectoral exports reduce the female share of skilled workers and a reduction in tariff increases the demand for unskilled female labour. They further distinguish between exporters' and non-exporters' reactions to trade liberalization: following tariff cuts, exporting firms do not increase the share of skilled female employees while non-exporting firms do. Finally, Oostendorp (2004) looks at the impact of trade on the gender wage gap within narrowly defined occupations for more than 80 countries. Exploiting the changes in trade intensity within a given occupation-sector-country cell over time, he finds that an increase in the sectoral trade share narrows the occupational wage gap for unskilled labour only and the occupational gender wage gap is lower in unskilled occupations compare to skilled occupations, the difference being bigger in developing countries. We postulate that those results might be due to employers' anticipation of a lower labour market attachment of women which is more penalizing in occupations requiring high skills.

This paper offers a theoretical explanation for the finding that trade affects the gender wage gap differently across the skill distribution. We integrate a statistical discrimination framework with the labour assignment approach to develop a model where the matching between firms and workers gives rise to a gender wage gap. This work is also related to recent trade models using labour assignment to provide insights about the impact of globalization on the labour markets[4]. We especially build on Yeaple (2005) where heterogeneous

[4] See for example Ohnsorge & Trefler (2007), Costinot & Vogel (2010)

workers sort into high-tech and low-tech firms according to their comparative advantage. High-skill workers are always more productive than low-skill workers (absolute advantage) but they are more so in high-tech firms (comparative advantage). The most skilled workers are employed in the high-tech firms where the skill reward is higher. With trade openness, more firms invest in the high technology which increases the skill-premium.

In this model, we allow for two groups of workers, men and women, whose characteristics vary in two dimensions, skills and commitment. We assume that men and women have the same skill distribution that is perfectly observable by everyone. However, labour market commitment is unobservable and cannot be anticipated with certainty by the employer. The model features statistical discrimination in the sens that the inability to observe individual's true labor market commitment induces employers to base partly their decision on group average. In particular, employers discriminate against women because they have *on average* a lower labour market commitment[5]. Employers set different selection criteria for workers from groups with different average commitment. As employers pay worker-specific wages, a woman is hired at a lower wage compare to a man with identical skill to compensate for the potential loss in case of lower commitment. This setting has been inspired by Lazear & Rosen (1990) dynamic model of statistical discrimination where women face a lower promotion probability along the job ladder of a given firm because of learning in top jobs and a lower propensity for women to remain on the job. In our model, workers are sorted across firms rather than types of jobs ; moreover, the matching is not determined by learning but by technology differences that result from firms' endogenous investment decisions. We suggest a static setting where employers discount the productivity of women because of a lower expected availability during the period. Because job commitment is exogenously distributed among women and is not correlated with their skills, the discount factor is the same for every woman.

Firms have the possibility to pay a higher fixed investment cost to upgrade their technology and reduce variable cots. They make a simultaneous decision on technology investment and hiring. Employers calculate the expected workers productivity in each technology based on their observation of skills and their expectation about the worker's degree of commitment.

- The skill of a worker matters as skilled workers have a comparative advantage in the high-technology firms.

[5]Gender difference in labour market commitment stems from work interruptions typically due to maternity leave and child rearing. It also includes the impossibility to work overtime as well as lower energy on the job due to greater time spent on housework and childcare

- The expected commitment to the firm matters because job commitment is a complement to technology upgrading, i.e. workers who are strongly attached to the labour market have a comparative advantage in using the high-technology.

The impact of expected commitment on the wage increases with the skill of a worker and is bigger in high technology firms. This leads to higher gender wage gaps at the upper part of the skill/wage distributions. We are able to characterize the wage gap distribution.

We then shed light on the implications of international trade for the gender wage gap in the presence of such heterogeneity. This work is also related to recent trade models using labour assignment to provide insights about the impact of globalization on the labour markets[6]. We especially build on Yeaple (2005) where the most skilled workers are employed in the high-tech firms where the skill reward is higher as the sophisticated technology upgrades labour productivity. With trade openness, more firms invest in the high technology which increases the skill-premium. We additionnaly find that:

- Trade liberalization increases the gender wage gap at the upper tail of the skill/wage distribution as it induces more firms to adopt skill-intensive production technologies where job commitment acts as a strategic complement.

- The evolution of the gender wage gap within a sector depends on the distribution of exporters and their incentives to adopt the high technology.

The paper is consistent with the labour and trade literature that shows how trade openness associated with firm heterogeneity influence earning inequalities in the labour market. In particular, the recent trend of the literature has emphasized that exporters invest in skill-upgrading technologies, are bigger, more productive and pay higher wages[7]. We suggest that this is the case both within gender groups (along the skill level) and between groups (the gender wage gap). In this way, this work echoes previous papers linking the overall wage structure with changes in relative wages of different subgroups such as men and women[8]. Trade liberalizations provide particularly interesting episodes to

[6]See for example Costinot & Vogel (2010)

[7]See the influential paper by Bernard & Jensen (1997) for evidence on the exporter wage premium and skill-biased shift in labour demand in U.S. manufacturing during the 1980s. Bustos (2011) gives evidence based on Argentinian firm data that corroborates her theoretical predictions: reduction in trade costs favours investment in high-skill technologies by increasing potential export revenues and making costly investment worthy.

[8]The wage structure is the values the labour market attaches to skills and other productive characteristics. For studies of the US labour market, see Blau and Kahn, 1992, 1994, 2003

study the effects of the wage structure on gender wage differentials as it generates variation in wage dispersion.

2 Set up of the model

This model features heterogeneous workers and firms that differ after choosing their technology and hiring workers. The set up of the model is very close to the one in Yeaple (2005) where the economy is composed of one sector that produce a homogeneous good and another sector that produce a differentiated good with two available technologies. The present paper additionally puts forward the effect of statistical discrimination when part of the worker's productivity is unobservable.

2.1 Demand

Preferences are identical across all consumers who choose a quantity of a homogeneous good and a quantity of varieties of a differentiated good. The utility function is Cobb Douglas between the differentiated good X and the homogeneous good Y and presents a CES sub-utility over the varieties i of X. This function expresses a love of variety of consumers. Then

$$U = Y^{1-\beta} X^{\beta}$$
$$X = \left(\sum_i^N x_i^{\alpha} \right)^{\frac{1}{\alpha}}$$

where the elasticity of substitution across varieties of X is given by $\sigma = \frac{1}{1-\alpha}$. The price index of the differentiated good X is : $P_X = \left(\sum_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. If all prices are equal, the price index is $P_X = pN^{\frac{1}{1-\sigma}}$. It decreases with the number of varieties produced N and the elasticity of substitution σ. Consumers chose the share of their income M they will devote to the differentiated good by maximizing their utility subject to their revenue constraint. The price of the homogeneous good is normalized to one.

$$X = (\beta M)/P_X$$
$$Y = (1-\beta)M \quad (1)$$

Let us note $E = \beta M$ is the portion of income spent on the differentiated good. Consumers decide also how much of each variety they consume. As they value variety, they consume a positive amount of each symmetric variety, they spend the same amount on each variety :

$$x_i = \frac{E}{P_X}\left(\frac{p_i}{P_X}\right)^{-\sigma} \tag{2}$$

The demand for variety i takes into account the average price of good X. The term $\frac{E}{P_X}$ corresponds to the aggregate demand for X while the price differential $\frac{p_i}{P_X}$ models the competition effect between variety i and the other varieties.

2.2 Workers, observable and unobservable characteristics

We consider an economy where the workforce is heterogeneous in both skills and job commitment. There is a continuum of skills s distributed among the population according to a distribution function L over the support $[0; \bar{s}]$. $L(s)$ is the inelastic supply of labour with skill no greater than s. We assume that men and women have the same exogenous skill distribution $L_f(s) = L_m(s) = L(s)$. The mass of workers per group is normalized to one. As for the differences in job commitment, let us assume that there are two types of individuals, the highly-committed that spend the maximum time and effort in the firm over the period $e = \bar{e}$ and the low-committed ones for which $e = \underline{e}$ [9]. We simplify the model by assuming that men always demonstrate a high level of commitment $Pr_m(e = \bar{e}) = 1$ while women have a probability to favour labour market activity over their domestic activities equal to $Pr_f(e = \bar{e}) = \eta$ with $0 < \eta < 1$ [10]. There is no correlation between s and e which means that the probability of being highly committed to one's job is independent of one's skill level.

The skill of a worker can be perfectly observed by the employers however the level of job commitment is unobservable : employers cannot anticipate the time and energy a worker is going to put in the job. As women have had *on average* a lower labour market attachment, employers expect female workers to be less committed.

[9]In sociology, the preference theory developed in Hakim (2000) argues that differences in women's preferences for combination of domestic activities and paid employment explain differences in labour market attachment among women. She sorts women in three categories: home-centered, adaptative and work-centered. Only women belonging to the last two categories participate to the labour market. We model the difference between these two groups by an exogenous difference in job commitment.

[10]This amounts to a normalization of male probability of commitment. We actually just need that men are more likely to prefer work-centered lifestyles and thus have a higher probability to be highly committed.

Labour productivity is increasing with both s and e and depends also on the technology j in use. The productivity of a worker endowed with skills of level s and a level of commitment e when working with technology j is noted $\varphi_j(s,e)$. Because employers cannot observe e, they cannot know the exact productivity of a worker in advance. Employers form expectations based on the observable, the skill and the sex of the worker. We note $\tilde{\varphi}_{jg}(s)$ the *expected* productivity of a worker with skill s from group g as viewed by the employer prior hiring when technology j is used. As men's productivity is perfectly observable, $\tilde{\varphi}_{jm}(s) = \varphi_j(s,\bar{e})$. As for women's productivity, employers form identical expectations given by: $\tilde{\varphi}_{jf}(s) = E(\varphi_j(s,e)|\eta) = \eta\varphi_j(s,\bar{e}) + (1-\eta)\varphi_j(s,\underline{e})$. Employers anticipate different productivities for a man and a woman endowed with the same skill level and working with the same technology : $\tilde{\varphi}_{jm}(s) > \tilde{\varphi}_{jf}(s) \quad \forall j \in \{l,h\}$ and $\forall s \in]0;\bar{s}]$.

2.3 Production

In sector Y, the homogeneous good, firms produce under constant returns to scale and perfect competition using labour only. We assume that labour productivity does not depend on either workers' skills or effort in this sector and we set $\varphi_Y = 1$. We note c_y the unit cost of production equal to the wage per efficient unit of labour : $c_Y = \frac{w_Y}{\varphi_Y}$. Under perfect competition in both product and labour markets, firms set prices equal to their unit cost of production $p_Y = \frac{w_Y}{\varphi_Y}$. We choose sector Y as the numéraire $p_Y = 1$, consequently we have $c_Y = w_Y = \varphi_Y = 1$.

In sector X, the differentiated good sector, firms operate under imperfect competition and increasing returns to scale. We assume that the sector is characterized by horizontal product differentiation and monopolistic competition where N firms produce each a variety of the differentiated product. Firms have to pay a fixed investment cost to produce one variety. This innovation cost F acts as an entry barrier which ensures that each variety is produced by only one firm. As varieties are not perfect substitutes, firms enjoy some market power that enable them to make positive operating profits and pay the fixed cost. After choosing its technology, the firm can produce a variety of good X hiring labour; we differentiate male labour m and female labour f. The following assumptions characterize the technology and the productivity function.

Assumption A1. ***Fixed and variable costs***
Firms can invest in two different technologies indexed by $j = \{l,h\}$, to acquire the the high-

technology firms bear a higher fixed cost $F_h > F_l$ but benefit from a higher productivity of labour for a given skill level and commitment, $\varphi_h(s, e) > \varphi_l(s, e)$. If a worker has no particular skill, $s = 0$, his/her productivity is the same in sector X and Y : $\varphi_h(0, e) = \varphi_l(0, e) = 1$.

Firms choose the type of investment they make considering both its cost and the resulting gain in productivity. This specification is consistent with R&D being positively correlated with firm productivity (Klette & Kortum (2004) for example).

Assumption A2. ***Log-supermodularity in skills and technology***
Skills acts as a strategic complement to technology upgrading:

$$\frac{\varphi_h(s', e)}{\varphi_h(s, e)} > \frac{\varphi_l(s', e)}{\varphi_l(s, e)} \quad \text{for any } s' > s \quad \forall e$$

The productivity gain derived from hiring a worker with a higher skill is greater under technology h. This means that workers with higher skill levels have a comparative advantage in the sophisticated technology.

Assumption A3. ***Log-supermodularity in commitment and technology***
Job commitment and technology upgrading are complementary:

$$\frac{\varphi_h(s, \bar{e})}{\varphi_h(s, e)} > \frac{\varphi_l(s, \bar{e})}{\varphi_l(s, e)} \qquad \text{for any } \bar{e} > e \quad \forall s$$

This means that strongly committed workers have a comparative advantage in the high technology.

Assumption A4. ***Log-supermodularity in skills and commitment***
Job commitment and skills are complementary:

$$\frac{\varphi_j(s', \bar{e})}{\varphi_j(s, \bar{e})} > \frac{\varphi_j(s', e)}{\varphi_j(s, e)} \qquad \text{for any } \bar{e} > e \quad \text{and } s' > s \quad \forall j = \{l, h\}$$

A high skill level is more valuable when the worker's job commitment is high.

Assumptions A2 to A4 requires the productivity function to be non separable in s, e and j.

We further assume that labour markets are perfectly competitive and that employers are risk-neutral. Workers bear no search cost and wages are flexible. Firms take the wage rate per efficiency unit of labour as given. Writing $\tilde{c}_j$ the *expected* unit cost under technology j, firms set worker-specific wages in accordance with the expected productivity of each worker $w_g(s) = \tilde{c}_j \tilde{\varphi}_{jg}(s)$.

3 Closed economy Equilibrium

Firms operating with the same technology are symmetric, in particular they have the same expected productivity for a given worker of skill s and sex g, that we note $\tilde{\varphi}_{jg}$. Consequently the expected unit cost $\tilde{c}_j$ is identical across firms of identical technologies. As a result, the technology $j = \{h, l\}$ specifies all relevant firm's variable. We can thus solve firms' problem in sector X using two representative firms h and l. Total production cost for a firm j can be written as :

$$TC_j = \frac{1}{N_j} \sum_g \left(\int_{s \in S_{jg}} w_{jg}(s) l(s) ds \right)$$

where S_{jg} is the set of skills of workers belonging to group g employed by a firm of type j and N is the endogenous number of firms. Effective labour is used to produce q_j quantity of good X to be sold and to pay the fixed cost :

$$q_j + F_j = \frac{1}{N_j} \sum_g \left(\int_{s \in S_j} \varphi(s) l(s) ds \right)$$

We denote $\tilde{c}_j$ the *expected* cost per efficient unit of labour under technology j, $\tilde{c}_j = \frac{w_g(s)}{\tilde{\varphi}_{jg}(s)}$. The profit of a firm using technology j can written :

$$\pi_j = p_j q_j - c_j(q_j + F_j)$$

3.1 Profit maximization, under monopolistic competition

Firms maximize their expected profits with respect to quantities[11].

$$\pi_j = max_{q_j}\{p_j q_j - \tilde{c}_j(q_j + F_j)\}$$

The first-order condition for equilibrium is :

$$p_j = \frac{\sigma}{\sigma - 1}\tilde{c}_j \tag{3}$$

Under competitive labour markets and monopolistic competition, firms with technology j hire workers up to the point where the wage per efficiency unit of labor, $\tilde{c_j} = \frac{w_j(s,\eta)}{\varphi_j(s,\eta)}$, equals the marginal revenue product $p_j\frac{\sigma-1}{\sigma}$. Hence, employees working with the same technology are paid the same fraction of their respective expected productivity:

$$w_{jg}(s) = p_j\frac{\sigma - 1}{\sigma}\tilde{\varphi}_{jg}(s)$$

with $0 < \frac{\sigma-1}{\sigma} < 1$.

3.2 Sorting of workers across firms

We follow Yeaple (2005) where workers with different skills sort across h and l type firms. In this paper, workers not only differ in their observable skill s but also in their unobservable degree of job commitment. Following the literature on job assignment, we assume that workers know the wage they can earn if they are matched to a given firm j and go to the firm that offers the highest wage.

Proposition 1. ***Sorting of workers***
If higher skill workers have a comparative advantage in h-type firms then h-type firms hire the most skilled workers of each group g

This is because i) firms hire workers until the revenue derived from hiring an additional worker equals the cost of doing so and ii) a firm h will always be able to offer a higher wage to a given worker as long as i) is satisfied. We prove this result in the appendix. We show that if the positive assortative matching is not followed, the output value can increase by

[11]Under monopolistic competition without any strategic interactions, competition on prices or on quantities lead to the same equilibrium result.

switching the assignment of workers to firms [12]. This self-selection process means that the marginal worker -of each group- is indifferent between working in one firm or another.

The wage distribution for men and women

The wage function is $w_{jg}(s) = \tilde{c}_j.\tilde{\varphi}_{jg}(s)$ the cost per unit of efficient labour times the expected productivity of the worker. We can give an expression for the wage that depends on firms' technologies and the skill thresholds s_{jg} below which a worker from group $g = \{f, m\}$ is not hired by a firm $j = \{l, h\}$.

$$w_g(s) = \begin{cases} c_Y \varphi_Y = 1 & \text{if } s \leq s_{lg} \\ \tilde{c}_l \tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \leq s \leq s_{hg} \\ \tilde{c}_h \tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \leq s \end{cases} \tag{4}$$

Among each group $g = \{m, f\}$, workers with a skill level equal to the threshold s_{lg} is indifferent between working in sector Y and working in a firm l in sector X. Similarly a worker with a skill level s_{hg} is indifferent between working in a firm using either technology h or l : $\tilde{c}_l \tilde{\varphi}_{lg}(s_{hg}) = \tilde{c}_h \tilde{\varphi}_{hg}(s_{hg})$. Consequently, we can rank the unit cost of production :

$$\frac{\tilde{c}_l}{c_Y} = \frac{\varphi_Y(s_{lg})}{\tilde{\varphi}_{lg}(s_{lg})} = \frac{1}{\tilde{\varphi}_{lg}(s_{lg})} < 1 \qquad \text{and} \qquad \frac{\tilde{c}_h}{\tilde{c}_l} = \frac{\tilde{\varphi}_{lg}(s_{hg})}{\tilde{\varphi}_{hg}(s_{hg})} < 1 \tag{5}$$

Firms in the diversified sector have lower unit cost of production than firms in sector Y. Within sector X, firms using the low technology have higher unit cost than firms using the high technology.

Using the indifference condition for both groups, we can rank the skill threshold required to men and women.

$$\frac{\varphi_Y(s_{lf})}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{\tilde{\varphi}_Y(s_{lm})}{\tilde{\varphi}_{lg}(s_{lm})} \Leftrightarrow \frac{1}{\tilde{\varphi}_{lf}(s_{lf})} = \frac{1}{\tilde{\varphi}_{lm}(s_{lm})}$$

and

$$\frac{\tilde{\varphi}_{lm}(s_{hm})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{hf}(s_{hf})} \Leftrightarrow \frac{\tilde{\varphi}_{hf}(s_{hf})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{lm}(s_{hm})}$$

Using these two equations, we can state the following proposition :

Proposition 2. ***Ranking of male and female skill requirements***

[12] This sorting mechanism has been first suggested by Roy (1951) where workers self-select into the occupation that gives them the highest expected earnings.

i) s_{jf} is a function of s_{jm} and η

ii) Given that $Pr_f(e=\bar{e}) < Pr_m(e=\bar{e})$ and φ_l is increasing in e, employers using the technology l require women a higher skill level $s_{lf} > s_{lm}$

iii) Under the assumptions A2 and A3, the skill threshold to work for a firm h is higher for women $s_{hf} > s_{hm}$

The proof is developed in the appendix.

Consequently, a woman working in sector Y can have a greater skill level than a man working in a firm l; this holds for workers with skills comprised between the male and female threshold for entering sector X, $s_{lm} \leq s \leq s_{lf}$. Similarly, a female worker employed in a firm l can have a greater skill level than a men working in a firm h; this holds for workers with skills comprised between the male and female threshold for entering a firm h, $s_{hm} \leq s \leq s_{hf}$.

We can now describe the wage distribution for both men and women. The slope of the wage profile becomes steeper at each group-specific skill threshold s_{lg} and s_{hg} because the technology l enhances worker productivity compare to the technology used in sector Y and the technology h features stronger skill complementarity than the technology l. Within group, the skill of any worker using the technology l is lower than the skill of a worker using the technology h.

The gender wage gap

We can further give the distribution of the wage gap along the skill distribution.

Let us define the gender wage gap between a man and a woman of skill s as $WG(s) = \frac{w_m(s)}{w_f(s)}$

$$WG(s) = \begin{cases} 1 & \text{if } s \leq s_{lm} \\ \tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y} & \text{if } s_{lm} \leq s \leq s_{lf} \\ \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\ \frac{\tilde{c}_h}{\tilde{c}_l} \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\ \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s \end{cases}$$

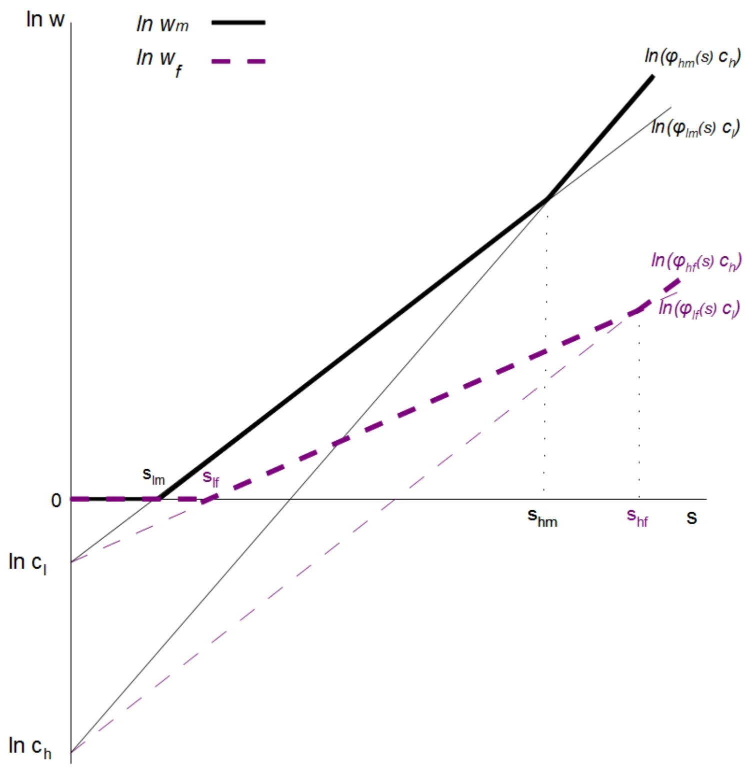

Figure 1: The wage distribution for men and women

Proposition 3. *Under assumptions A1, A2 and A4, the gender wage gap is increasing in the skill level.*

We provide a proof in the appendix.

3.2.1 Free entry

Investment in technology is unrestricted so that the number of firms adjust until profits using either technology are zero. For each type of technology, the unit cost under which total revenues equal total (labour) costs is :

$$\tilde{c_j} = \frac{\sigma}{\sigma - 1} P_X^{\frac{\sigma-1}{\sigma}} \left(\frac{E(\sigma - 1)}{F_j} \right)^{\frac{1}{\sigma}} \tag{6}$$

The different fixed costs generate two productivity cutoffs. Producing with the technology h requires a higher productivity $\tilde{c}_h < \tilde{c}_l$ to be able to make higher operating profits to pay for the higher fixed cost. Firms make their investment and human resources decisions jointly as the unit cost of producing with a given technology depends on the skill level of the workforce.

Relating the zero profit conditions for both types of firms, we obtain:

$$\frac{\tilde{\varphi}_{hg}(s_{hg})}{\tilde{\varphi}_{lg}(s_{hg})} = \left(\frac{F_h}{F_l} \right)^{\frac{1}{\sigma}} \tag{7}$$

This pins down the skill threshold to enter a firm h for both men and women as a function of the technologies' parameters. An increase in the fixed cost to invest in the high technology increase the skill threshold required to workers.

3.3 The number of firms

Female and male total labour supply is assumed to be fixed and is divided across the tree types of firms.

The numbers of high-technology and low-technology firms in sector X are given by :

$$N_h(q_h + F_h) = \int_{s \in S_{hf}} \tilde{\varphi}_{hf}(s) l(s) ds + \int_{s \in S_{hm}} \tilde{\varphi}_{hm}(s) l(s) ds$$

$$N_l(q_l + F_l) = \int_{s \in S_{lf}} \tilde{\varphi}_{lf}(s) l(s) ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm}(s) l(s) ds$$

Using the free entry condition :

$$N_h = \frac{1}{\sigma F_h} (\int_{s \in S_{hf}} \tilde{\varphi}_{hf}(s) l(s) ds + \int_{s \in S_{hm}} \tilde{\varphi}_{hm}(s) l(s) ds) \tag{8}$$

$$N_l = \frac{1}{\sigma F_l}(\int_{s \in S_{lf}} \tilde{\varphi}_{lf}(s) l(s) ds + \int_{s \in S_{lm}} \tilde{\varphi}_{lm}(s) l(s) ds) \tag{9}$$

The number of firm j depends on the four skill thresholds s_{jg} with $j = \{h, l\}$ and $g = \{m, f\}$.
The threshold s_{hm} is pined down by the free entry condition in sector X while the sorting of workers across the two types of firms relates s_{hm} to s_{hf}. The market clearing condition for good Y determines the skill threshold s_{lm}.

3.4 Market clearing in sector Y

Production of good Y is : $Y = \sum_g \int_0^{s_{lg}} l(s) ds$.
The demand for good Y, given by the Cobb-Douglas preferences, must equal the production of the good. Since Y is the numeraire $p_Y = 1$, the market clearing condition is :

$$Y = (1 - \beta) M$$

where M is total revenue which equals total wages (firms make no positive profits in equilibrium) : $M = \sum_g (\int_{s \in S_{Yg}} l(s) ds + \int_{s \in S_{lg}} w_{lg}(s) l(s) ds + \int_{s \in S_{hg}} w_{hg}(s) l(s) ds)$. Consumption of good Y is a function of the cost thresholds s_{lg} and s_{hg}.

Using equations (13) and (5), replacing M in the equation for the demand of good Y and equalizing demand and production for good Y we obtain :

$$\frac{\beta}{1-\beta} \tilde{\varphi}_l(s_{lm}) \sum_g \int_0^{s_{lg}} l(s) ds = \sum_g \left(\int_{s_{lg}}^{s_{hg}} \tilde{\varphi}_{lg}(s) l(s) ds + \frac{\tilde{\varphi}_l(s_{hm})}{\tilde{\varphi}_h(s_{hm})} \int_{s_{hg}}^{\bar{s}} \tilde{\varphi}_{hg}(s) l(s) ds \right) \tag{10}$$

Equation (10) defines the skill threshold below which individuals are working in sector Y.

4 The open economy

We assume that the domestic country trades with an identical foreign country so that we need to define the allocations and equilibrium in one country only[13]. Markets are

[13] Assuming that the trading partner is identical allows to consider only one set of skill thresholds, $s_{lf}, s_{lm}, s_{hf}, s_{hm}$, which are common to each country. Country differences, for example in the skill dis-

segmented because of a variable trade cost τ which includes fret and insurance costs along with tariffs. As a result, a firm may charge different prices on the domestic and foreign market. Besides, a firm incurs a fixed export cost F^t to start exporting as in Melitz (2003). F^t covers fixed market access costs such as setting up new distribution channels, shipping requirements as well as ensuring that the firm's goods conforms to foreign standards and regulatory environment. The fixed cost generates a selection of firms into exporting as it is established by the empirical literature. Regardless of the export decision, a firm always incurs the investment cost F_j . Because this overhead production cost is already incurred, a firm would not export and not produce for its domestic market. Indeed domestic sells yield always strictly higher operating profits compare to sells to foreign markets because of the additional fixed and variable costs.

The demand for a variety i of the differentiated good comes now from both domestic and foreign consumers who are assumed to have the same preferences:

$$x_i = p_i^{-\sigma} E P_X^{\sigma-1}$$
$$x_i^t = (p_i^t)^{-\sigma} E P_X^{\sigma-1} \tag{11}$$

where p_i is the price of variety i on the domestic market and p_i^t is the price of variety i when it is traded to a foreign market. E is the share of the income spent on goods X. The price index is now :

$$P_X = (\sum_i p_i^{1-\sigma} + \sum_k p_k^{t1-\sigma})^{\frac{1}{1-\sigma}}$$

where p_k^t is the price of variety k traded by a foreign firm and sold on the domestic market. Firms are subjected to per-unit iceberg trade cost τ. To address the foreign demand, a firm need to produce $q^t = \tau x_i^t$ as a share τ of the production is required for transportation.

Firms maximize their profits with respect to either price or quantity :

$$\pi_j = \max_{p_j} \{p_j q_j + I^t.(p_j^t \frac{q_j^t}{\tau}) - \tilde{c}_j(q_j + F_j + I^t.(q_j^t + F^t))\}$$

where I^t equals 1 if the firm exports and $q_j^t = \tau(p_i^t)^{-\sigma} E P_X^{\sigma-1}$. As marginal costs are constant, we can separate the profits they earn on each market. The pricing rule in the do-

tribution or in the technology, would give rise to different thresholds and equilibrium conditions that vary across countries.

mestic market implies, exactly as in the autarky case, that the marginal cost of production equates the marginal revenue.

$$p_j \frac{\sigma - 1}{\sigma} = \tilde{c}_j \tag{12}$$

Firms who export will set higher prices in the foreign markets that reflect the increased marginal cost due to the transportation cost τ that is completely supported by the consumer (the standard mill pricing strategy):

$$p_j^t \frac{\sigma - 1}{\sigma} = \tau \tilde{c}_j \Leftrightarrow p_j^t = \tau p_j$$

The marginal cost of production is still given by $\frac{w_{jg}(s)}{\tilde{\varphi}_{jg}(s)}$ so that the sorting of workers across firms stated in proposition 1 continues to hold, h-firms employ the workers with the highest skill level.

What are the firms that export ?
Profits of a j-firm that serves only the domestic market are :
$\pi_j = \tilde{c_j}^{1-\sigma} \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} E P_X^{\sigma-1} - \tilde{c_j} F_j$
Profits of a j-firm that serves both markets are :
$\pi_j^t = \tilde{c_j}^{1-\sigma} \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} E P_X^{\sigma-1} (1 + \tau^{1-\sigma}) - \tilde{c_j}(F_j + F^t)$
A firm of type j finds it profitable to export if $\pi_j \leq \pi_j^t$. Three cases arise :

i $F^t \tau^{\sigma-1} \geq F_h$, no firm export

ii $F_l \leq F^t \tau^{\sigma-1} \leq F_h$, h firms only export

iii $F^t \tau^{\sigma-1} \leq F_l$, both l and h firms export

We can see directly that if there are no fixed cost to export, $F^t = 0$, all firms are able to export and no level of variable cost $\tau > 1$ can generate the selection of the most productive firms into exporting. As the differences between exporters and non exporters -within sectors- are empirically pervasive, it is accepted that models with CES demand should assume a combination of fixed and variable trade costs to generate a sorting of firms according to their productivity.

From now on, we focus on the case 2 where only the high-technology firms are able to export. The free entry conditions determine which workers are employed by exporters. For

h firms, the zero-profit conditions implies :

$$\tilde{c}_h = (\sigma(F_h + F^t))^{\frac{-1}{\sigma}}(\frac{\sigma}{\sigma - 1})^{\frac{1-\sigma}{\sigma}} EP_X^{\sigma-1}(1 + \tau^{1-\sigma})^{\frac{1}{\sigma}}$$

We denote $\tilde{c}^a_j$ the marginal cost of j firms under autarky. The expected cost per efficient unit of labour that firms h can pay $\tilde{c}_h$ is larger under trade, $\tilde{c}_h > \tilde{c}^a_h$. This stems from the increase in market size that benefits to exporting firms.

The zero profit condition for l-firms is :

$$\tilde{c}_l = (\sigma F_l)^{\frac{-1}{\sigma}}(\frac{\sigma}{\sigma - 1})^{\frac{1-\sigma}{\sigma}}(EP_X^{\sigma-1})^{\frac{1}{\sigma}}$$

We can find the skill-threshold to enter a firm h for men and women by relating the two zero-profit conditions :

$$\frac{\tilde{c}_h}{\tilde{c}_l} = \left(\frac{(F_h + F^t)}{F_l(1 + \tau^{1-\sigma})}\right)^{\frac{1}{\sigma}}$$

From the above equation, we can see that the difference in marginal costs between the two types of firms is smaller under trade than under autarky, $\frac{\tilde{c}_h}{\tilde{c}_l < \frac{\tilde{c}^a_h}{\tilde{c}^a_l}}$.

Using the indifference conditions for the marginal workers of each group whose skill levels define the skill-threshold, $w_{lg}(s_{hg}) = w_{hg}(s_{hg}) \Leftrightarrow \frac{c^t_h}{c^t_l} = \frac{\tilde{\varphi}_{lg}(s_{hg})}{\tilde{\varphi}_{hg}(s_{hg})}$, we have :

Proposition 4. *When only h firms export,*

i) the skill threshold to enter a firm h is lower under trade compare to the autarky case for both groups, $s_{hg} < s^a_{hg} \quad$ for $g = \{l, h\}$. More workers are matched with a high technology firms under trade.

ii) the skill threshold to enter a firm h is still higher for women, $s_{hm} < s_{hf}$

iii) trade liberalization further reduces the skill requirement for both groups, $\frac{\partial(\tilde{c}_h/\tilde{c}_l)}{\partial\tau} > 0 \Rightarrow \frac{\partial s_{hg}}{\partial\tau} > 0 \quad \forall g$

Although the expression for $\tilde{c}_l$ does not change, its value changes with openness. The decrease in the skill threshold s_{hg} to enter h firms raises wages for the most skill workers; this in turn raises total income which corresponds to a higher demand for the non-trade good (this effect will be explicit when the general equilibrium effect is highlighted). Sector

Y thus demands more labour. Consequently, we have a higher skill threshold to enter the manufacturing industry under trade $s_l > s_l^a$ and the marginal production cost of firms l goes down $\tilde{c}_l < \tilde{c}_l^a$. Trade openness brings an increase in productivity in the manufacturing sector along with a higher demand for local services for instance, as a result some workers move from the manufacturing sector to the non-traded sectors; this is in line with general employment patterns.

Proposition 5. *When only h firms export,*

i) the skill threshold to enter a firm l is higher under trade compare to the autarky case for both groups, $s_{lg}^t > s_{lg}^a$

ii) the skill threshold to enter a firm l is still higher for women, $s_{lm}^t < s_{lf}^t$

iii) the effect of trade liberalization is to further increase the skill threshold above which workers are employed in the traded sector, $\frac{\partial(\tilde{c}_l/c_Y)}{\partial\tau} > 0 \Rightarrow \frac{\partial s_{lg}}{\partial\tau} > 0 \quad \forall g$

This is consistent with the stylized fact that the share of manufacturing employment among women is lower than among men and that trade openness does not reverse that trend.

The wage function has the same form than under autarcky but the values of the skill thresholds s_{lg} and s_{hg} as well as the cost thresholds $\tilde{c}_l$ and $\tilde{c}_h$ has changed :

$$w_g(s) = \begin{cases} c_Y\varphi_Y = 1 & \text{if } s \le s_{lg} \\ \tilde{c}_l\tilde{\varphi}_{lg}(s) & \text{if } s_{lg} \le s \le s_{hg} \\ \tilde{c}_h\tilde{\varphi}_{hg}(s) & \text{if } s_{hg} \le s \end{cases} \tag{13}$$

Changes in the wage distributions

The gender wage gap $\frac{w_m(s)}{w_f(s)}$ is now given by :

$$WG(s) = \begin{cases} 1 & \text{if } s \le s_{lm} \\ \tilde{c}_l\frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y} & \text{if } s_{lm} \le s \le s_{lf} \\ \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \le s \le s_{hm} \\ \frac{\tilde{c}_h}{\tilde{c}_l}\frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \le s \le s_{hf} \\ \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \le s \end{cases}$$

Proposition 6. *Following trade liberalization,*

i) the gender wage gap is reduced at the bottom of the skill distribution as $s_{lm} > s^a_{lm}$

ii) the gender wage gap widens at the top of the distribution given that $s_{hm} < s^a_{hm}$ *and the wage profile is steeper under technology* h

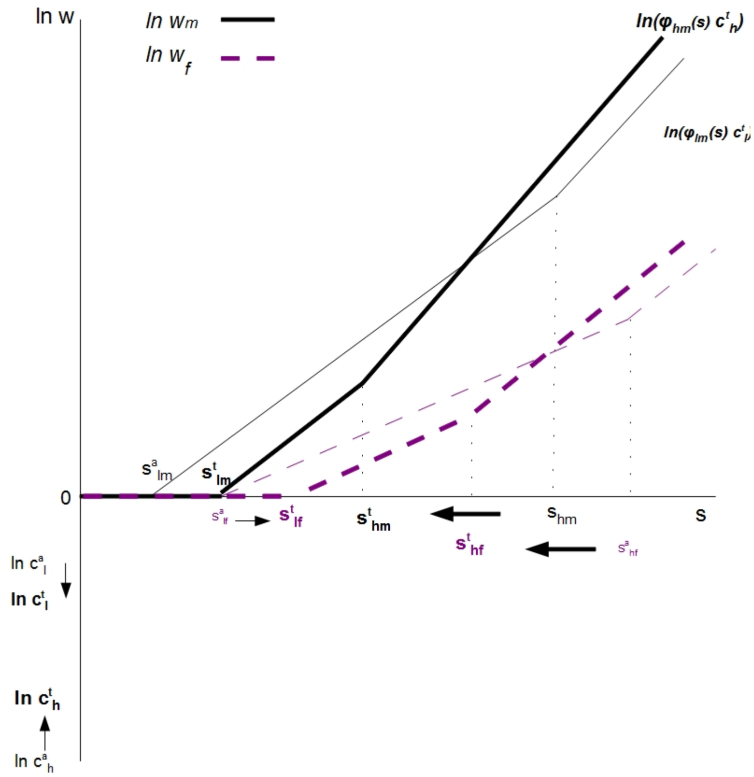

Figure 2: Changes in the gender wage gap with trade openness

4.1 The number of firms

In the case where only h firms export and using the free entry condition, the number of firms is given by :

$$N_h = \frac{1}{\sigma(F_h + F^t)}(\int_{s\in S_{hf}} \tilde{\varphi}_{hf}(s)l(s)ds + \int_{s\in S_{hm}} \tilde{\varphi}_{hm}(s)l(s)ds) \tag{14}$$

$$N_l = \frac{1}{\sigma F_l}(\int_{s\in S_{lf}} \tilde{\varphi}_{lf}(s)l(s)ds + \int_{s\in S_{lm}} \tilde{\varphi}_{lm}(s)l(s)ds) \tag{15}$$

More workers are hire by h firms under trade as the skill threshold is lower $s_{hg} > s^a_{hg} \quad \forall g$.

4.2 Market clearing in sector Y

Finally, the market clearing condition for good Y determines the new skill threshold s_{lm}. Good Y is not traded. The market clearing condition is still given by $Y = (1-\beta)M$,where M equals total wages in the open economy. Skilled workers' wages have increased following trade liberalization as more firms adopt the high-technology. $M = \sum_g(\int_{s\in S_{Yg}} l(s)ds + \int_{s\in S_{lg}} w_{lg}(s)l(s)ds + \int_{s\in S_{hg}} w_{hg}(s)l(s)ds)$.
Using (13) with the new skill thresholds,

$$M = \sum_g(\int_0^{s_{lg}} l(s)ds + \tilde{c}_l \int_{s_{lg}}^{s_{hg}} \tilde{\varphi}_{lg}(s)l(s)ds + \tilde{c}_h \int_{s_{hg}}^{\bar{s}} \tilde{\varphi}_{hg}(s)l(s)ds)$$

Using equation (5) and equalizing demand and production for good Y, we have :

$$\frac{\beta}{1-\beta}\tilde{\varphi}_l(s_{lm}) \sum_g \int_0^{s_{lg}} l(s)ds = \sum_g \left(\int_{s_{lg}}^{s_{hg}} \tilde{\varphi}_{lg}(s)l(s)ds + \frac{\tilde{\varphi}_l(s_{hm})}{\tilde{\varphi}_h(s_{hm})} \int_{s_{hg}}^{\bar{s}} \tilde{\varphi}_{hg}(s)l(s)ds \right) \tag{16}$$

This equation determines the skill threshold below which individuals are now working in sector Y and closes the model.

5 Conclusion

This paper offers a theoretical explanation for varying gender wage gap along the skill distribution and varying impacts of trade openness on the wage gap. We need three supermodularity assumptions on the labour productivity function to give general conditions under which we find the pattern observed in empirical studies. More precisely we show that if skills and job commitment are complements to technological upgrading and if skills and job commitment are complements, statistical discrimination based on job commitment expectations generates a higher gender wage gap at the upper part of the distribution. The closed economy part of the model thus puts forward one reason for the glass ceiling effect as well as the increase in residual wage disparity within gender groups as documented by numerous empirical assessments.

The analysis also provides insights into the impact of trade openness in a setting with intra-industry trade and monopolistic competition. First, we show that when trade openness induces skill-biased technological change, it increases the wage gap at the top of the skill distribution. Second, due to general equilibrium effects, the gender wage gap is reduced at the lower part of the distribution. Finally, the paper adds to the understanding of the interactions between the overall wage structure of an economy and the gender wage gap.

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A Sorting of heterogeneous workers across firms

We can prove by contradiction, that a high-technology firm hires workers with higher skill level compare to a low-technology firm

Consider two workers with skill $s_1 < s_2$. Let us assume that worker 1 is hired by a firm h and worker 2 is hired by a firm l.
Firm h pays worker 1 so that its profit is maximized :

$$\frac{\sigma - 1}{\sigma} p_h = \frac{w_h(s_1)}{\tilde{\varphi}_h(s_1)}$$

Firm l pays worker 2 so that its profit is maximized :

$$\frac{\sigma - 1}{\sigma} p_l = \frac{w_l(s_2)}{\tilde{\varphi}_l(s_2)}$$

Firm l would not increase its profit by hiring worker 1 at a wage just above the one paid by a firm l :

$$\frac{\sigma-1}{\sigma}p_l \leq \frac{w_h(s_1)}{\tilde{\varphi}_l(s_1)}$$

Firm h would not increase its profit by hiring worker 2 at a wage just above the one paid by a firm h :

$$\frac{\sigma-1}{\sigma}p_h \leq \frac{w_l(s_2)}{\tilde{\varphi}_h(s_2)}$$

Equations 1 and 3 implies that $\frac{p_l}{p_h} \leq \frac{\tilde{\varphi}_h(s_1)}{\tilde{\varphi}_l(s_1)}$

Equations 2 and 4 implies that $\frac{p_l}{p_h} \geq \frac{\tilde{\varphi}_h(s_2)}{\tilde{\varphi}_l(s_2)}$

Which implies that $\frac{\tilde{\varphi}_h(s_2)}{\tilde{\varphi}_l(s_2)} \leq \frac{\tilde{\varphi}_h(s_1)}{\tilde{\varphi}_l(s_1)}$. But this contradicts the assumption that more skilled workers have a comparative advantage in the high-technology.

B Ranking of of male and female skill requirements

The indifference condition states that :

$$\frac{\tilde{\varphi}_{hf}(s_{hf})}{\tilde{\varphi}_{hm}(s_{hm})} = \frac{\tilde{\varphi}_{lf}(s_{hf})}{\tilde{\varphi}_{lm}(s_{hm})}$$

$$\Leftrightarrow \frac{\eta\varphi_h(s_{hf},\bar{e}) + (1-\eta)\varphi_h(s_h,\underline{e})}{\varphi_h(s_{hm},\bar{e})} = \frac{\eta\varphi_l(s_{hf},\bar{e}) + (1-\eta)\varphi_h(s_h,\underline{e})}{\varphi_l(s_{hm},\bar{e})}$$

That we can rearrange

$$\Leftrightarrow \frac{\eta + (1-\eta)\frac{\varphi_h(s_{hf},\underline{e})}{\varphi_h(s_{hf},\bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_{hf},\underline{e})}{\varphi_l(s_{hf},\bar{e})}} = \frac{\varphi_l(s_{hf},\bar{e})}{\varphi_l(s_{hm},\bar{e})}\frac{\varphi_h(s_{hm},\bar{e})}{\varphi_h(s_{hf},\bar{e})}$$

Let us prove by contradiction that $s_{hf} > s_{hm}$.

Suppose that $s_{hf} = s_{hm} = s_h$, the condition is now

$$\frac{\eta + (1-\eta)\frac{\varphi_h(s_h,\underline{e})}{\varphi_h(s_h,\bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_h,\underline{e})}{\varphi_l(s_h,\bar{e})}} = \frac{\varphi_l(s_h,\bar{e})}{\varphi_l(s_h,\bar{e})}\frac{\varphi_h(s_h,\bar{e})}{\varphi_h(s_h,\bar{e})} \Leftrightarrow \frac{\varphi_h(s_h,\underline{e})}{\varphi_h(s_h,\bar{e})} = \frac{\varphi_l(s_h,\underline{e})}{\varphi_l(s_h,\bar{e})}$$

Which contradicts the supermodularity assumption between technology and commitment. So that the male and female skill requirements cannot be equal.

Suppose now that $s_{hf} < s_{hm}$. By supermodularity between technology upgrading and skills, we know that :

$$\frac{\eta + (1-\eta)\frac{\varphi_h(s_{hf},\underline{e})}{\varphi_h(s_{hf},\bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_{hf},\underline{e})}{\varphi_l(s_{hf},\bar{e})}} < 1$$

By supermodularity between technology upgrading and skills and the fact that labour productivity is increasing in skills, we know that:

$$\frac{\varphi_h(s_{hm},\bar{e})}{\varphi_h(s_{hf},\bar{e})} > \frac{\varphi_l(s_{hm},\bar{e})}{\varphi_l(s_{hf},\bar{e})} > 1$$

Combining the two we have :

$$\frac{\eta + (1-\eta)\frac{\varphi_h(s_{hf},\underline{e})}{\varphi_h(s_{hf},\bar{e})}}{\eta + (1-\eta)\frac{\varphi_l(s_{hf},\underline{e})}{\varphi_l(s_{hf},\bar{e})}} \frac{\varphi_l(s_{hm},\bar{e})}{\varphi_l(s_{hf},\bar{e})} < \frac{\varphi_h(s_{hm},\bar{e})}{\varphi_h(s_{hf},\bar{e})}$$

which contradicts the indifference condition. The female skill requirement to be hired by a high-tech firm cannot be lower than the male skill requirement.

C The gender wage gap is increasing in the level of skills

We explain here under which assumptions the gender wage gap increases with s. The expression for the wage gap is :

$$WG(s) = \begin{cases} 1 & \text{if } s \leq s_{lm} \\ \tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y} & \text{if } s_{lm} \leq s \leq s_{lf} \\ \frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{lf} \leq s \leq s_{hm} \\ \frac{\tilde{c}_h}{\tilde{c}_l} \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)} & \text{if } s_{hm} \leq s \leq s_{hf} \\ \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)} & \text{if } s_{hf} \leq s \end{cases}$$

There is no wage gap between men and women who work in sector Y as labour productivity in Y does not depend on either skills or job commitment. This is relevant for jobs involving routine tasks. When the workers' skill levels are comprised between s_{lm} and s_{lf}, the wage gap equals $\tilde{c}_l \frac{\tilde{\varphi}_{lm}(s)}{\varphi_Y}$ which is greater than 1 and increasing in s from (A1).

For that part to be increasing in s, we actually need that technology l features stronger complementarities with skills compare to the technology used in sector Y.
When φ is supermodular in skills and technology upgrading (A2), $\frac{\tilde{c}_h}{\tilde{c}_l}\frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lf}(s)}$ is increasing in s.
The supermodularity of φ in skills and commitment (A4) ensures that the ratios $\frac{\tilde{\varphi}_{lm}(s)}{\tilde{\varphi}_{lf}(s)}$ and $\frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{hf}(s)}$ are increasing in s.

D Changes in the Gender Wage Gap with trade openness

How has the gender wage gap changed compare to the autarky case? Changes in the gender wage gap $\frac{WG(s)}{WG^a(s)}$ is non-linear in s.
For $s \in [0; s^a_{lm}]$, there is no wage gap under either trade or autarky.
For $s \in [s^a_{lm}; s_{lm}]$, $\frac{WG(s)}{WG^a(s)} = \frac{1}{\tilde{c}^a_l \tilde{\varphi}_{lm}(s)}$, the wage gap is *lower* under trade as more men are employed in sector Y where there is no wage gap.
For $s \in [s_{lm}; s^a_{lf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_l}{\tilde{c}^a_l}$, the wage gap is *lower* under trade as the unit cost of l-firms has decreased.
For $s \in [s^a_{lf}; s_{lf}]$, $\frac{WG(s)}{WG^a(s)} = \tilde{c}_l \tilde{\varphi}_{lf}(s)$, the wage gap is *higher* under trade.
For $s \in [s_{lf}; s_{hm}]$, the wage gap is of the same magnitude under trade and autarky.
For $s \in [s_{hm}; s^a_{hm}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_h \tilde{\varphi}_{hm}(s)}{\tilde{c}_l \tilde{\varphi}_{lm}}$ which is greater than 1 as sorting implies that $\tilde{c}_h \tilde{\varphi}_{hm}(s) > \tilde{c}_l \tilde{\varphi}_{lm}$ for $s > s_{hm}$. The wage gap is *higher* under trade.
For $s \in [s^a_{hm}; s_{hf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{c}_h}{\tilde{c}^a_h}\frac{\tilde{c}_l}{\tilde{c}^a_l}$, the wage gap is *higher* under trade.
For $s \in [s_{hf}; s^a_{hf}]$, $\frac{WG(s)}{WG^a(s)} = \frac{\tilde{\varphi}_{hm}(s)}{\tilde{\varphi}_{lm}}$ which is greater than 1. The wage gap has increased with trade.

For $s \in [s^a_{hf}; \bar{s}]$, $\frac{WG(s)}{WG^a(s)} = 1$