Welfare Evaluation in a Heterogeneous Agents Model: How Representative is the CES Representative Consumer?

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August 17, 2012

Preliminary Draft: Do not cite without permission

Abstract

The aim of the present paper is to investigate the impact of trade liberalizations on welfare in a model with heterogeneous consumers. I introduce consumer heterogeneity à la Anderson-DePalma-Thisse (1992). Each consumer purchases a single product among the set of available varieties; her choice rests on the observable product characteristics and her idiosyncratic taste preferences. I characterize individual utilities to generate the aggregate behavior of a CES representative consumer.

The standard welfare equivalence between the CES representative consumer and the discrete choice model breaks down in presence of asymmetric trade liberalizations. In fact, asymmetric liberalizations produce differential gains among heterogeneous consumers. I show that there exists no feasible income transfer such that the gains from trade are equally distributed across individual consumers and, thus, comparable to the potential gains experienced by the CES representative consumer. Intuitively, aggregation creates an insurance mechanism: the CES representative consumer softens the impact of trade liberalizations reallocating consumption among the available varieties. Individual consumers, instead, purchase a single product and do not internalize the effects of liberalization on prices of other available varieties. In addition, in presence of decreasing marginal utilities, consumers losing/gaining less from trade liberalizations require aggregate compensations larger than the total income collected from those experiencing the biggest welfare improvements. Finally, I offer a simple measure to quantify the welfare discrepancy between the CES representative consumer and the heterogeneous consumers models.

Key words: Discrete Choice Models, CES Representative Consumer, Trade Liberalizations, Compensating Variations.

JEL classification: D11, D60, F10

1 Introduction

The identification of alternative channels for the Welfare Gains from Trade exhibits a synchronous evolution to the research of more comprehensive explanations for the observed trade patterns. Gains from specializing in comparative-advantaged industries, qualified by Ricardo (1870) and Heckscher-Ohlin (1921), have been supplemented by variety, scale and pro-competitive effects in models of

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increasing returns and imperfect competition.[1] More recently, models with heterogeneous firms expose productivity gains due to market share reallocations from the least to the most productive plants.

The variety of possible mechanisms, however, is not able to inflate the empirical estimates of those gains. Arkolakis, Costinot and Rodriguez-Clare (forthcoming) show that the quantitative predictions are not responsive to the micro-economic features of production. In models of perfect[2] and monopolistic[3] competition, the welfare gains are fully captured by the share of expenditure on domestic goods and a gravity-based estimate of the trade elasticity. The macro-level characteristics, instead, have possibly large impacts on such quantitative predictions: extending a model to include multiple sectors, multiple factor of productions or tradable intermediate goods expands the gains up to a factor of 5.

This paper poses a related problem, characterizing the sensitivity of welfare quantification to demand micro-foundations. In particular, I explore the significance of micro-level foundations for the case of aggregate CES preferences. The choice to focus on the CES representative consumer is justified by two reasons. First, most trade models adopt the CES utility structure because CES preferences generate an inner motive to trade.[4] Second, the positive and normative equivalence between the CES representative consumer and a continuum of the individual agents is a well known result. Anderson, De Palma and Thisse (1992) prove that the demand pattern and the welfare of individual agents, each purchasing a single product among the set of available varieties, based on the product observable characteristics and their idiosyncratic taste preferences, aggregate to the demand and welfare of a CES representative consumer.

However, the normative equivalence between the CES representative consumer and the Discrete Choice Model breaks down in presence of asymmetric trade liberalizations. In fact, asymmetric liberalizations produce differential gains among heterogeneous consumers. I show that there exists no feasible income transfer such that the gains from trade are equally distributed across individual consumers and comparable to the potential gains experienced by the CES representative consumer. Intuitively, aggregation creates an *insurance mechanism*: the CES representative consumer mitigates the impact of trade liberalizations reallocating consumption among the available varieties. Individual consumers, instead, purchase a single product; they cannot internalize the trade liberalization effects on prices of available varieties not included in their optimal consumption basket. In addition, decreasing marginal utilities with respect to income complete the characterization of the infeasibility result: in presence of decreasing marginal utilities, consumers losing/gaining less from trade liberalizations require larger compensations than the income collected from those experiencing bigger welfare improvements.

This paper contributes to the literature on the evaluation of Welfare Gains from Trade. The theoretical literature generally abstracts from demand-side heterogeneity. Most models relies upon the characterization of a CES representative consumer, more or less implicitly referring to Anderson, De Palma and Thisse (1992) normative and positive foundation of the CES representative consumer.

Only recently, two empirical contributions (Sheu (2010) and Sun (2011)) stress the importance of consumer heterogeneity on welfare quantification in response to trade liberalizations. Both papers

[1] See Helpman and Krugman (1987) for a comprehensive reference.

[2] See Armington (1969) and Eaton and Kortum (2002).

[3] See Krugman (1980) and Melitz (2003).

[4] Domestic and imported products are considered imperfect substitutes due to their differing countries of origin; see Armington (1969).

show that the empirical estimates of Welfare Gains from Trade are lower when they are based upon the logit random coefficients demand system (if compared to those derived from a CES representative consumer model). Sheu (2010) attributes the gap in the welfare calculations to the restrictive substitution structure of the CES model.

My work provides a theoretical foundations to the empirical findings from Sheu (2010) and Sun (2011). The substitution structure of the CES model, hypothesized by Sheu (2010), is at the core of the *insurance mechanism*. However, this element alone is not sufficient to justify the welfare discrepancy between the two models. A second necessary element is the consumer heterogeneity in the marginal utility of income: if individual marginal utilities were constant and equal across all consumers,[5] transfers compensating for differential utility changes after asymmetric trade liberalization would be feasible.

A final contribution of this paper is to characterize a simple measure to quantify the welfare discrepancy between the CES representative consumer and the heterogeneous consumers framework. Herriges and Kling (1999) compare different approaches to compute welfare changes in random utility models in presence of non-linear income effects. They analyse the McFadden's GEV sampler, a linear model, the representative consumer framework and the computation of bounds on the welfare change of interest and they show that the researcher faces a trade-off between computational ease and potential bias in choosing among the possible approaches. The measure I propose is a transfer function, which represents an exact and complementary welfare measure, offering a specific relative (to the representative consumer) perspective. Its calculation requires only knowledge of prices changes and the elasticity of substitution among goods of the representative consumer.

The paper is organized as follows. Section 2 describes the Model and summarizes the positive and normative equivalence. Section 3 develops a proof of the welfare result. The robustness analysis is performed in Section 4. Section 5 analyses the consequence for Trade Policy Decisions and Section 6 concludes.

2 A Discrete Choice Model of Consumer Behaviour

Suppose that an economy is populated by a unit mass of statistical indipendent and identical consumers. Each consumer is endowed with an income y that she decides to allocate across two commodities. Commodity 1 and 2 are sold at prices p_1 and p_2. In addition to prices, the consumer's choice is informed by some taste preferences. Let ε_1 and ε_2 the idiosyncratic valuation of commodity 1 and 2; ε_i, $i = 1, 2$ summarize all the relevant unobservable (to the modeller) information that guide consumer's choice towards a particular good. I assume that $\varepsilon_i \sim \textit{Gumbel}\,(\gamma, \mu)$, $i = 1, 2$: each consumer extracts independent draws ε_i, $i = 1, 2$ from a Type-I Extreme Value Distribution. Thus, a particular consumer j is identified by $(\varepsilon_1, \varepsilon_2)$; she, then, solves the following problem

$$\max_{x_i, i=1,2} U^j = \max_{x_i, i=1,2} \sum_{i=1,2} [\ln x_i + \varepsilon_i] \qquad \text{s.to} \quad \sum_{i=1,2} p_i x_i = y \tag{1}$$

where U^j is the utility function of consumer j and $\sum_{i=1,2} p_i x_i = y$ represents its budget constraint. The linearity of the problem implies that the consumer will spend all of his income y on the commod-

[5] This obtains if the underlying utility functions of the individual consumers are log-linear, where the numéraire good is chosen to be the good entering the utility linearly.

ity which will allow her to reach the highest utility level[6]. A consumer choosing i has the following indirect utility

$$V^j = U^j\big|_{i=\arg\max U^j} = \ln y + \varepsilon_i - \ln p_i$$

Aggregating across all consumers choosing a particular commodity, the commodity space can be partitioned in the area of consumers choosing commodity 1 and the area of consumers choosing commodity 2. The measure of the area of consumers choosing commodity i is given by

$$Pr\,(i) = \frac{(p_i)^{-\frac{1}{\mu}}}{\sum_{i=1,2} (p_i)^{-\frac{1}{\mu}}}$$

The aggregate demand for variety i is, then, obtained multiplying the measure of consumers choosing variety i by the individual demand for commodity i, constant across all consumers

$$X_i = Pr\,(i) \cdot \frac{y}{p_i} \tag{2}$$

The aggregate indirect utility is given by

$$V = \ln y + \mu \ln \left[p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} \right] \tag{3}$$

(2) and (3) represent the demand and the level of welfare of a representative consumer with a CES utility over the available alternatives and income y.[7]

3 Welfare Effects of Asymmetric Price Changes

The equivalence between the CES representative consumer and the population of idiosyncratic consumers breaks down in presence of asymmetric price liberalization. In fact, asymmetric price changes produce differential gains across consumers choosing different varieties. The indirect utility of the representative consumer cannot be used as a meaningfull measure of the individual consumer welfare: in fact, I will prove in what follows that there does not exist a system of income transfers such that the change in utility of the representative consumer captures the change in utility experienced by each individual. In order to fix ideas, I will consider an asymmetric price change such that the price index remains constant.[8] In particular, suppose that $\tilde{p}_1 < p_1$ and $\tilde{p}_2 > p_2$ such that

$$p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} = \tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}$$

[6]The case in which a consumer is indifferent between the two commodities can be ruled out under the continuum assumption on the distribution of $(\varepsilon_1, \varepsilon_2)$.

[7]See Anderson, de Palma and Thisse (1992) for a proof of this result.

[8]The analysis extend to the case in which the price index varies. In this case, the total price variation can be decomposed into two sets of prices for the specific varieties such that the first set changes by a proportional amount, characterizing a new reference price index, while the second set leaves the new reference price index invariant. In fact, suppose that $\tilde{p}_1 < p_1$ and $\tilde{p}_2 < p_2$, with $P \equiv p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}$. Then, define $(\hat{p}_1, \hat{p}_2)$ such that

$$\theta^{-\frac{1}{\mu}} \left[p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} \right] = \hat{p}_1^{-\frac{1}{\mu}} + \hat{p}_2^{-\frac{1}{\mu}} = \tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}$$

Then, either $\tilde{p}_1 < \hat{p}_1$ and $\tilde{p}_2 > \hat{p}_2$ or $\tilde{p}_1 > \hat{p}_1$ and $\tilde{p}_2 < \hat{p}_2$; the price index associated to both sets of prices is the same.

The price change induces some switching patterns across consumers; in particular, those consumers with $\ln p_2 - \ln p_1 \leq \varepsilon_2 - \varepsilon_1 \leq \ln \tilde{p}_2 - \ln \tilde{p}_1$ switch to commodity 1. The set of all individual consumers can be partitioned across three areas:

- Consumers choosing commodity 1 before and after the price change. Let

$$A = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_2 \leq \varepsilon_1 + \ln p_2 - \ln p_1\}$$

  be the set of those consumers. Then, the measure of those consumers coincides with the fraction of consumers choosing commodity 1 before the price change

$$\mu(A) = \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}}$$

  For each $j \in A$, the utility change amounts to $\Delta V^j = \ln p_1 - \ln \tilde{p}_1$. The total change over A

$$\Delta V^A = (\ln p_1 - \ln \tilde{p}_1) \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}}$$

- Consumers choosing commodity 2 after the price change. Let

$$B = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_2 \geq \varepsilon_1 + \ln \tilde{p}_2 - \ln \tilde{p}_1\}$$

  be the set of those consumers. Then, the measure of those consumers coincides with the fraction of consumers choosing commodity 2 after the price change

$$\mu(B) = \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}}$$

  For each $j \in B$, the utility change amounts to $\Delta V^j = \ln p_2 - \ln \tilde{p}_2$. The total change over B

$$\Delta V^B = (\ln p_2 - \ln \tilde{p}_2) \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}}$$

- Consumers switching from commodity 2 to commodity 1 after the price change. Let

$$C = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \ln p_2 - \ln p_1 \leq \varepsilon_2 \leq \varepsilon_1 + \ln \tilde{p}_2 - \ln \tilde{p}_1\}$$

  be the set of those consumers. The utility change differs across $j \in C$. For a particular consumer j, $\Delta V^j = \varepsilon_1 - \varepsilon_2 + \ln p_2 - \ln \tilde{p}_1$. The distribution of changes $\varepsilon_1 - \varepsilon_2$ can be easily derived

$$\begin{aligned} F(x) &= \int_{-\infty}^{\infty} Prob\left[\varepsilon_1 \leq x + \varepsilon_2\right] f(\varepsilon_2)\, \mathrm{d}\varepsilon_2 \\ &= \frac{1}{1 + \exp\left(-\frac{x}{\mu}\right)} \end{aligned}$$

Aggregating across all consumers $j \in C$, the change in utility

$$\begin{aligned}\Delta V^C &= \int_{\ln \tilde{p}_1 - \ln \tilde{p}_2}^{\ln p_1 - \ln p_2} (x + \ln p_2 - \ln \tilde{p}_1) \frac{1}{\mu} \frac{\exp\left(-\frac{x}{\mu}\right)}{1 + \exp\left(-\frac{x}{\mu}\right)} \mathrm{d}x \\ &= \frac{\ln \frac{p_1}{\tilde{p}_1}}{1 + \left(\frac{p_1}{p_2}\right)^{-\frac{1}{\mu}}} - \frac{\ln \frac{p_2}{\tilde{p}_2}}{1 + \left(\frac{\tilde{p}_1}{\tilde{p}_2}\right)^{-\frac{1}{\mu}}} + \ln \frac{\tilde{p}_1}{p_1} + \mu \left[\ln\left(\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}\right) - \ln\left(p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}\right)\right]\end{aligned}$$

Summing across the three areas, the aggregate change in utility coincides with the change in utility of the representative consumer,

$$\begin{aligned}\Delta V^A + \Delta V^B + \Delta V^C &= (\ln p_1 - \ln \tilde{p}_1) \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} + (\ln p_2 - \ln \tilde{p}_2) \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}} \\ &\quad + \left[\frac{p_2^{-\frac{1}{\mu}}}{p_2^{-\frac{1}{\mu}} + p_1^{-\frac{1}{\mu}}} - 1\right] \ln \frac{p_1}{\tilde{p}_1} - \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_2^{-\frac{1}{\mu}} + \tilde{p}_1^{-\frac{1}{\mu}}} \ln \frac{p_2}{\tilde{p}_2} \\ &= \ln \frac{p_1}{\tilde{p}_1} \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} - \ln \frac{p_1}{\tilde{p}_1} \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} = 0\end{aligned}$$

Different groups of consumers, however, experience opposite change in individual welfare. In particular, all consumers $j \in A$ experience an increase in utility ($\ln p_1 - \ln \tilde{p}_1 > 0$) while for $j \in B$, the utility will be lower ($\ln p_2 - \ln \tilde{p}_2 < 0$); the change in utility might be positive or negative for $j \in C$, depending on the realizations of the idiosyncratic taste shocks.

The representative consumer is able to capture the microeconomic effects of a price change if and only if a transfer such that resources are reallocated from those who gain to those who lose is feasible, leaving all consumers at least as well off as before the change occurred.

The above utility decomposition suggests that a linear transfer would be able to produce such a reallocation. However, income, which is the only available resource to be reallocated, enters the utility function logarithmically. This implies that a linear change in utility is possible if and only if income changes exponentially. Such changes are not feasible: exponential change in income exhaust the available resources faster. Thus, there are not sufficient resources to be reallocated from the consumers enjoying a higher utility to those experiencing lower utility level. The following theorem summarizes this result

Theorem 1. *(*Asymmetric Price Variation*). Consider an economy populated by a unit mass of statistical independent and identical consumers solving (1). Suppose that a price change occurs such that*

$$p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} = \tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}} \tag{4}$$

Then, there does not exist a system of income transfers such that all consumers are at least as well off as before the change in prices occurred.

The proof builds the required reallocations across the partition, characterized above, induced by the individual choice of the optimal variety before and after the price change, showing that the aggregate reallocation is not feasible.

Proof. Across the areas of individual choice of the optimal variety,

- Consumers $j \in A$ experience a higher utility after the price change; thus, a negative income transfer would leave their utility unchanged after the price change. For $j \in A$, the new income level is determined by

$$\begin{aligned}\varepsilon_1 - \ln p_1 + \ln y &= \varepsilon_1 - \ln \tilde{p}_1 + \ln y^A \\ \ln y^A - \ln y &= \ln \tilde{p}_1 - \ln p_1 \\ y^A - y &= \left[\frac{\tilde{p}_1}{p_1} - 1\right] y\end{aligned}$$

  Clearly $y^A - y < 0$. As the change in utility is common across all consumers $j \in A$, the total transfer required to make the consumer as well off as before the price change is determined by

$$T^A = \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} \left[y^A - y\right] = \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_1}{p_1} - 1\right] y$$

- Consumers $j \in B$ experience a lower utility after the price change; thus, they require a positive income transfer to be as well off as before the price change. For each $j \in B$, the income level after the price change is determined by

$$\begin{aligned}\varepsilon_2 - \ln p_2 + \ln y &= \varepsilon_2 - \ln \tilde{p}_2 + \ln y^B \\ \ln y^B - \ln y &= \ln \tilde{p}_2 - \ln p_2 \\ y^B - y &= \left[\frac{\tilde{p}_2}{p_2} - 1\right] y\end{aligned}$$

  The aggregate transfer across $j \in B$ is given by

$$T^B = \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_2}{p_2} - 1\right] y$$

- The changes in utility among consumers $j \in C$ differ in sign. In particular, the decision to switch from commodity 1 to commodity 2 can be attributed to two effects, which have different signs. On the one hand, the reduction in price of commodity 1 would have positive impact on utility. On the other hand, a higher price for commodity 2 acts to reduce utility. The total utility change depends on which effect dominates. The first effect is relatively more important for those consumers which are marginally indifferent between commodity 1 and commodity 2 before the price changes: they would experience a higher utility and thus, the income transfer necessary to make them as well off as before the change is negative. The second effect has a dominating impact on those consumers which are marginally indifferent between commodity 1 and commodity 2 after the change: they, thus, require an increase in income for an unchanged level of utility.[9] For $j \in C$, the individual income transfer is given by

$$y^C - y = \left[\frac{\tilde{p}_1}{p_2} e^{\varepsilon_2 - \varepsilon_1} - 1\right] y$$

The aggregate transfer is obtained averaging across individual transfers

$$\begin{aligned} T^C &= \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p}_2}{\tilde{p}_1}} y \left[\frac{\tilde{p}_1}{p_2} e^{\varepsilon_2 - \varepsilon_1} - 1\right] f\left(e^{\varepsilon_2 - \varepsilon_1}\right) \mathrm{d}\left(e^{\varepsilon_2 - \varepsilon_1}\right) \\ &= y\frac{\tilde{p}_1}{p_2}\left[\mathrm{B}\left(\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right) - \mathrm{B}\left(\left(\frac{\tilde{p}_2}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right)\right] \\ &- y\left[\frac{\tilde{p}_1^{-\frac{1}{\mu}}}{\tilde{p}_2^{-\frac{1}{\mu}} + \tilde{p}_1^{-\frac{1}{\mu}}} - \frac{p_1^{-\frac{1}{\mu}}}{p_2^{-\frac{1}{\mu}} + p_1^{-\frac{1}{\mu}}}\right] \end{aligned}$$

An income transfer is feasible iff

$$T = T^A + T^B + T^C \leq 0$$

For $y = 1$, the transfers can be directly compared to the utility changes across sets of consumers

- For consumers $j \in A$,

$$T^A \geq -\Delta V^A \quad \text{as} \quad \left[\frac{\tilde{p}_1}{p_1} - 1\right] \geq \ln \frac{\tilde{p}_1}{p_1}$$

- Analogously, for consumers $j \in B$,

$$T^B \geq -\Delta V^B \quad \text{as} \quad \left[\frac{\tilde{p}_2}{p_2} - 1\right] \geq \ln \frac{\tilde{p}_2}{p_2}$$

- For $j \in C$,[10]

$$T^C \geq -\Delta V^C \quad \text{as} \quad \left[\frac{\tilde{p}_1}{p_2} e^x - 1\right] \geq \left(x + \ln \frac{\tilde{p}_1}{p_2}\right) \quad \forall x$$

The inequality implies that a transfer is never feasible if $y = 1$

$$T^A + T^B + T^C > -\left(\Delta V^A + \Delta V^B + \Delta V^C\right) = 0 \tag{5}$$

The inequality (5) is strict as the set such that $\left[\frac{\tilde{p}_1}{p_2} e^x - 1\right] > \left(x + \ln \frac{\tilde{p}_1}{p_2}\right)$ has a positive measure.

In the case $y \neq 1$, the changes in income are magnified by the initial income level. In particular if $y > 1$, the positive changes in income are larger than the changes in utility, as will the negative transfers; the opposite applies for $y < 1$. However, a transfer will never be feasible

$$y\left(T^A + T^B + T^C\right) > 0, \quad \forall y > 0$$

□

[9] Faced with a new income level, the consumers $j \in C$ might decide to switch back to commodity 2. I assume $\frac{\tilde{p}_2}{\tilde{p}_1} > 1$, which is a sufficient condition such that the choice of all consumers $j \in C$ remains commodity 1.

[10] See the Mathematical Appendix for a full derivation of the inequality for $j \in C$.

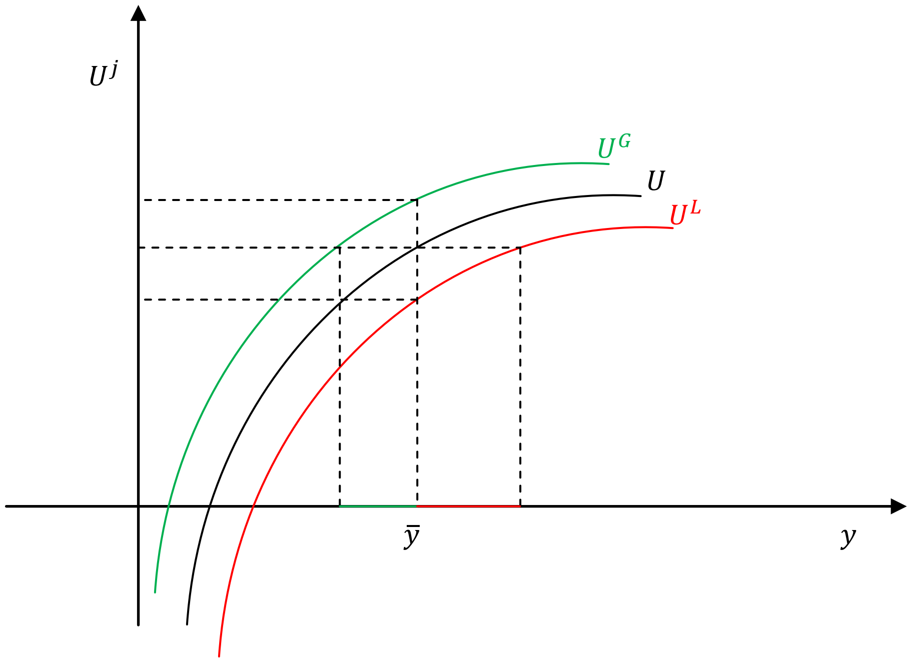

Figure 1: Income transfers across consumers with compensating utility changes and decreasing marginal utilities. The income transfer associated to the consumer who gains (difference between the black and the green curve) is always lower than the income transfer associated to the consumer who loses (difference between the black and the red curve), since all consumers share the same initial income level $\bar{y}$.

Figure (1) show the intuition behind the technical result. The aggregate change in utility across all consumers perfectly compensate. Thus, it is sufficient to be represented as two *aggregate consumers*[11] with opposite changes in utility. In presence of decreasing marginal utility, the income collected from the aggregate consumer who gains is lower than the income to be distributed to the consumer that suffer a loss in utility if they both share the same income level.

3.1 A Slutsky Compensation Mechanism

The compensation mechanism analysed in the previous section is characterized by income transfers satisying the condition of keeping constant utility level after the price change, i.e. *hicksian transfers*. In this section, I will consider alternative transfers such that the old consumption choice is still affordable to each individual consumers, i.e. *slutsky transfers*. This section proves the infeasibility result for *slutsky transfers*. In fact, *hicksian transfers* represent the compensation scheme minimizing the income to be reallocated.

Theorem 2. *(*Asymmetric Price Variation - Slutsky Compensation Mechanism*). Consider an economy populated by a unit mass of statistical independent and identical consumers solving (1).*

[11]The construction of the two representative consumers aggregates across individual consumers depending on the direction of their utility change. This interpretation holds independently from the sizes of the two groups.

Suppose that a price change occurs such that

$$p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} = \tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}$$

Then, there does not exist a system of income transfers such that the old consumption choice is available for all consumers, i.e. a Slutsky compensation mechanism is available.

Proof. It is convenient to conduct the analysis across the three areas defined above.

- For $j \in A$, the initial choice is common across all consumers. In fact, each of them decides to purchase $c_1^j = \frac{y}{p_1}$. The income level y_A that makes the old consumption basket affordable is defined by

$$c_1^j = \frac{y}{p_1} = \frac{y_A}{\tilde{p}_1} = \tilde{c}_1^j$$

  Thus, the individual income transfer coincides with the hicksian characterization

$$y_A - y = \left[\frac{\tilde{p}_1}{p_1} - 1\right] y$$

- The initial choice of consumers $j \in B$ is given by

$$c_2^j = \frac{y}{p_2}$$

  After the price change, the initial consumption choice is still affordable if each consumer's available income is

$$y_B : \quad \frac{y}{p_2} = \frac{y_B}{\tilde{p}_2}$$

  Also in this case, the individual transfer coincides with the hicksian definition

$$y_B - y = \left[\frac{\tilde{p}_2}{p_2} - 1\right] y$$

- The Slutsky compensation scheme differs for consumers $j \in C$. The new income level required such that the initial consumption choice is affordable is defined by

$$y_C : \quad \frac{y}{p_2} = \frac{y_C}{\tilde{p}_2}$$

  However, if $\frac{\tilde{p}_2}{\tilde{p}_1} > 1$,[12] each consumer will choose commodity 1.[13]

The total transfer required by the Slutsky compensation scheme is given by

$$T = y\left\{\left[\frac{\tilde{p}_1}{p_1} - 1\right]\frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} + \left[1 - \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}}\right]\left[\frac{\tilde{p}_2}{p_2} - 1\right]\right\}$$

[12]The condition $\frac{\tilde{p}_2}{\tilde{p}_1} > 1$ is not particularly restrictive since the price change I analyze is such that $\tilde{p}_1 < p_1$ and $\tilde{p}_2 > p_2$.

[13]In fact, each consumer $j \in C$ will enjoy an higher utility when choosing commodity 1. In fact, the consumer chooses between

$$\tilde{c}_1^j = \frac{y_C}{\tilde{p}_1} = \frac{\tilde{p}_2}{\tilde{p}_1}\frac{y}{p_2} \quad \text{vs} \quad \tilde{c}_2^j = \frac{y_C}{\tilde{p}_2} = \frac{y}{p_2}$$

where $\tilde{c}_i^j$ denotes the consumption of commodity i after the price change by consumer j. Clearly $\tilde{c}_1^j > \tilde{c}_2^j$ if $\frac{\tilde{p}_2}{\tilde{p}_1} > 1$.

The total transfer is not feasible. In fact, let $P = p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}$. Then,

$$T \leq 0 \quad \Leftrightarrow \quad \frac{p_1^{-\frac{1}{\mu}}}{P} \geq \frac{\left[\frac{\tilde{p}_2}{p_2} - 1\right]}{\frac{\tilde{p}_2}{p_2} - \frac{\tilde{p}_1}{p_1}}$$

From the assumption,

$$\frac{\tilde{p}_2}{p_2} > 1 \quad \text{and} \quad \frac{\tilde{p}_1}{p_1} < 1$$

Thus,

$$\frac{\left[\frac{\tilde{p}_2}{p_2} - 1\right]}{\frac{\tilde{p}_2}{p_2} - \frac{\tilde{p}_1}{p_1}} > 1$$

but

$$0 \leq \frac{p_1^{-\frac{1}{\mu}}}{P} \leq 1$$

This implies that $T > 0$. □

An alternative proof can be constructed comparing the income transfers implied by the Slutsky mechanism to the hicksian transfer scheme. Those mechanisms coincides over $A \cup B$, but they differ for $j \in C$. In fact, the Slutsky mechanism requires positive transfers for all consumers $j \in C$, while the trasfers defined according to the hicksian mechanism might be positive or negative. Let

$$\varepsilon_2 - \ln p_2 + \ln y = \varepsilon_1 - \ln \tilde{p}_1 + \ln y_C^h$$
$$y_C^s : \quad \frac{y}{p_2} = \frac{y_C^s}{\tilde{p}_2}$$

be the income levels associated to the hicksian and the Slutsky compensation mechanism. Then,

$$y_C^h - y < 0 \quad \text{for} \quad \ln \frac{p_2}{p_1} \leq \varepsilon_2 - \varepsilon_1 < \ln \frac{p_2}{\tilde{p}_1} y_C^h - y > 0 \qquad \text{for} \quad \ln \frac{p_2}{\tilde{p}_1} \leq \varepsilon_2 - \varepsilon_1 \leq \ln \frac{\tilde{p}_2}{\tilde{p}_1}$$

For $\varepsilon_2 - \varepsilon_1 = \ln \frac{\tilde{p}_2}{\tilde{p}_1}$, $y_C^h - y = y_C^s - y$; for all other consumers, $y_C^h - y < y_C^s - y$. Thus, the income transfer associated to the Slutsky compensation mechanism must be even larger than the hicksian income transfers.

3.2 An extension to n goods

In presence of n commodities, the main complication lies in establishing the switching patterns among goods, when a price change occurs. A similar analysis can be performed if sub-aggregating the varieties in groups with two available alternatives. For example, for each additional commodity, we might consider the choice between commodity n and all other $n - 1$ commodities. Then, the relevant income transfer is given by

$$
\begin{aligned}
T^{n-1} = & \frac{p_n^{-\frac{1}{\mu}}}{p_n^{-\frac{1}{\mu}} + P_{-n}^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_n}{p_n} - 1 \right] y + \frac{\tilde{P}_{-n}^{-\frac{1}{\mu}}}{\tilde{p}_n^{-\frac{1}{\mu}} + \tilde{P}_{-n}^{-\frac{1}{\mu}}} \left[\frac{\tilde{P}_{-n}}{P_{-n}} - 1 \right] y + \\
& + y \frac{\tilde{p}_n}{P_{-n}} \left[\mathrm{B} \left(\left(\frac{P_{-n}}{p_n} \right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu \right) - \mathrm{B} \left(\left(\frac{\tilde{P}_{-n}}{\tilde{p}_n} \right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu \right) \right] + \\
& - y \left[\frac{\tilde{p}_n^{-\frac{1}{\mu}}}{\tilde{P}_{-n}^{-\frac{1}{\mu}} + \tilde{p}_n^{-\frac{1}{\mu}}} - \frac{p_n^{-\frac{1}{\mu}}}{P_{-n}^{-\frac{1}{\mu}} + p_n^{-\frac{1}{\mu}}} \right]
\end{aligned}
$$

where P_{-n} is the price index for the $n-1$ commodities. Then, the aggregate welfare comparison requires tracking all transfer function across all successive groups with two available alternatives.

4 Functional Form Robustness

The welfare result depends on two assumptions: non-constant marginal utility in income and dependence of the indirect utility on a subset of the available varieties. This section explores the robustness of the former assumption, considering different utility specifications.

4.1 A log-linear case

Assume that the indirect utility function of an agent choosing variety i is given by[14]

$$V^j\big|_{i=\arg\max U^j} = y + \varepsilon_i - \ln p_i$$

In presence of an price change satisfying (4), the income transfers across the different areas of choice exactly compensate. In fact,

- For $j \in A$, the total income transfer is given by

$$T^A = (\ln p_1 - \ln \tilde{p}_1) \frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}}$$

- For $j \in B$, the total income transfer is given by

$$T^B = (\ln p_2 - \ln \tilde{p}_2) \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}}$$

[14]This indirect utility specification follows from the assumption of a log-linear utility; a similar form applies also to any other quasi linear utility specification.

- Aggregating across all consumers $j \in C$, the total income transfer is given by

$$
\begin{aligned}
T^C &= \int_{\ln \tilde{p}_1 - \ln \tilde{p}_2}^{\ln p_1 - \ln p_2} (x + \ln p_2 - \ln \tilde{p}_1) \frac{1}{\mu} \frac{\exp\left(-\frac{x}{\mu}\right)}{1+\exp\left(-\frac{x}{\mu}\right)} \mathrm{d}x \\
&= \frac{\ln \frac{p_1}{\tilde{p}_1}}{1+\left(\frac{p_1}{p_2}\right)^{-\frac{1}{\mu}}} - \frac{\ln \frac{p_2}{\tilde{p}_2}}{1+\left(\frac{\tilde{p}_1}{\tilde{p}_2}\right)^{-\frac{1}{\mu}}} + \ln \frac{\tilde{p}_1}{p_1} + \mu \left[\ln \left(\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}} \right) - \ln \left(p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}} \right) \right]
\end{aligned}
$$

As for the total utility change, the aggregate transfer is always feasible, i.e. $T^A + T^B + T^C = 0$.

4.2 Multiplicative Function

Suppose that the consumers can choose among two varieties and enjoy a linear utility,

$$
U = \sum_{i=1,2} \alpha_i e^{\varepsilon_i} x_i
$$

The linearity of the utility function implies that the two varieties are perfect substitutes up to observable characteristics, summarized by $\alpha_j \in \mathbb{R}$, and an idiosyncratic valuation of the good, captured by e^{ε_i}, $i = 1, 2$. Let p_i be the unit price of commodity i, $i = 1, 2$. The solution to the maximization problem implies that each consumer chooses the commodity with the lowest characteristics-adjusted price,

$$
V = \max \left\{ \frac{\alpha_1 e^{\varepsilon_1}}{p_1}, \frac{\alpha_2 e^{\varepsilon_2}}{p_2} \right\}
$$

Suppose that $\tilde{p}_1 < p_1$ and $\tilde{p}_2 > p_2$ satisfying (4). The Transfer Function arising in the present case is analogous to the transfer under log-logit utilty

$$
\begin{aligned}
T = &\frac{p_1^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + p_2^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_1}{p_1} - 1 \right] y + \frac{\tilde{p}_2^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \tilde{p}_2^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_2}{p_2} - 1 \right] y + \\
&+ y \frac{\tilde{p}_1}{p_2} \left[\mathrm{B} \left(\left(\frac{p_2}{p_1} \right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu \right) - \mathrm{B} \left(\left(\frac{\tilde{p}_2}{\tilde{p}_1} \right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu \right) \right] + \\
&- y \left[\frac{\tilde{p}_1^{-\frac{1}{\mu}}}{\tilde{p}_2^{-\frac{1}{\mu}} + \tilde{p}_1^{-\frac{1}{\mu}}} - \frac{p_1^{-\frac{1}{\mu}}}{p_2^{-\frac{1}{\mu}} + p_1^{-\frac{1}{\mu}}} \right]
\end{aligned}
$$

As proven before, the aggregate transfer function is always positive.

Intuitively, in presence of marginal utilities of income which are not equal across all consumers and if all agents have the same income, the redistribution policy implies that units of income are reallocated from those consumers who have higher marginal utilities to those who have lower marginal utilities. Thus, the income required for the compensation will never be sufficient, delivering the *infeasibility* result.

5 Implications for Trade Policy

The theoretical result I derive applies to all cases inducing changes in the price of domestic varieties, given a set of available varieties. Trade liberalizations represent an important application of the welfare result as most unilateral trade liberalizations provide an exogenous source of price variation of available commodities. Thus, data on price changes are immediately available to quantify the welfare discrepancy between the CES representative consumer model and the Discrete Choice Model, as well as an index (the price index[15]) of welfare for the CES representative consumer model.

Beyond the advantages to characterize welfare in presence of heterogeneous consumers, the result developed in this paper suggests that trade liberalizations might induce a welfare reduction among consumers, if tariff changes are unevenly distributed across industries. The optimal policy requires that tariff cuts are never to be coupled with tariff increases and all tariffs are to be reduced by the same proportion in all industries whose commodities are imperfect substitutes from the perspective of consumers. An estimation of cross-industry elasticities of substitution is, thus, necessary for the formulation of Trade Policy Analysis.

All derivations in the paper adopt a general form for the welfare function, which assigns equal weights to all groups of consumers. Changing those weights will generate different prescriptions for the optimal trade policy, depending on the specific assumption on the distribution of such weights. Unfortunately, the adoption of a specific policy does not necessarily requires that the *hicksian* transfer function characterized in the present contribution would be implemented. Thus it is difficult to formulate an empirical recipe to estimate the weights across categories of consumers. However, the best is to simulate the change in welfare across different groups of consumers when modifying the distribution of weights, before defining the lines of Trade Policy.

6 Conclusions

This paper analyses the significance of micro-level foundations of demand on the analysis of Welfare Gains from Trade. In particular, I focus on the CES representative consumer for two reasons. First, most trade models adopt the CES utility structure because CES preferences generate an inner motive to trade. Second, the positive and normative equivalence between the CES representative consumer and a continuum of the individual agents is a well known result developed by Anderson, de Palma and Thisse (1992).

I show that the welfare equivalence between the CES representative consumer and the discrete choice model breaks down in presence of asymmetric trade liberalizations. In fact, asymmetric liberalizations produce differential gains among heterogeneous consumers. I prove that there exists no feasible income transfer such that the gains from trade are equally distributed across individual consumers and, thus, comparable to the potential gains experienced by the CES representative consumer. Intuitively, aggregation creates an insurance mechanism: the CES representative consumer softens the impact of trade liberalizations reallocating consumption among the available varieties. Individual consumers, instead, purchase a single product and do not internalize the effects of liberalization on prices of other available varieties. Moreover, in presence of marginal utilities of income

[15]In fact, the CES price index and the transfer function require similar data availability.

not equal across all consumers and if all agents have the same income, the redistribution policy implies that units of income are reallocated from those consumers who have higher marginal utilities to those who have lower marginal utilities. Thus, the income required for the compensation will never be sufficient, delivering the *infeasibility* result. This result is robust to alternative utility specifications inducing different marginal utilities across heterogeneous consumers.

Finally, I introduce a simple measure quantifying the welfare discrepancy between the CES representative consumer and the heterogeneous consumers framework. This measure has the advantage to be exact and requires little data (prices changes and the elasticity of substitution among goods) to be computed.

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A Mathematical Appendix

A.1 Basic Derivations

This section develops the complete derivations for the basic model. The probability of choosing commodity i is given by

$$\begin{aligned} Pr\left(i\right) &= \int_{-\infty}^{\infty}\int_{\left\{\varepsilon_i:\varepsilon_j\leq\varepsilon_i+\ln\frac{p_j}{p_i}\right\}} f\left(\varepsilon_j\right)f\left(\varepsilon_i\right)\mathrm{d}\varepsilon_i\mathrm{d}\varepsilon_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbb{1}_{\left\{\varepsilon_i:\varepsilon_j\leq\varepsilon_i+\ln\frac{p_j}{p_i}\right\}} f\left(\varepsilon_j\right)f\left(\varepsilon_i\right)\mathrm{d}\varepsilon_i\mathrm{d}\varepsilon_i \\ &= \int_{-\infty}^{\infty} Pr\left[\varepsilon_j\leq\varepsilon_i+\ln\frac{p_j}{p_i}\right]f\left(\varepsilon_i\right)\mathrm{d}\varepsilon_i \\ &= \int_{-\infty}^{\infty}\frac{1}{\mu}\exp\left[-\exp\left(-\frac{\varepsilon_i+\ln\frac{p_j}{p_i}}{\mu}-\gamma\right)\right]\exp\left[-\left(\frac{\varepsilon_i}{\mu}+\gamma\right)\right]\exp\left[-\exp\left(-\frac{\varepsilon_i}{\mu}-\gamma\right)\right]\mathrm{d}\varepsilon_i \\ &= \frac{\left(p_i\right)^{-\frac{1}{\mu}}}{\sum_{i=1,2}\left(p_i\right)^{-\frac{1}{\mu}}} \end{aligned}$$

The aggregate indirect utility is obtained as

$$\begin{aligned} V &= \int_{-\infty}^{\infty}\max\left\{U^j\left(\varepsilon\right)\right\}f_{\max}\left(\varepsilon\right)\mathrm{d}\varepsilon \\ &= \int_{-\infty}^{\infty}\frac{x}{\mu}\sum_{i=1,2}\exp\left(-\frac{\ln p_i}{\mu}\right)\exp\left(-\frac{x-\ln y}{\mu}-\gamma\right)\exp\left[-\exp\left(-\frac{x-\ln y}{\mu}-\gamma\right)\sum_{i=1,2}\exp\left(-\frac{\ln p_i}{\mu}\right)\right]\mathrm{d}x \end{aligned}$$

Let $z=\exp\left(-\frac{x-\ln y}{\mu}-\gamma\right)$; then, $z\to 0$ as $x\to\infty$ and $z\to+\infty$ as $x\to-\infty$. Moreover,

$$\mathrm{d}z = -\frac{1}{\mu}\exp\left[-\left(\frac{x-\ln y}{\mu}+\gamma\right)\right]\mathrm{d}x$$

Thus, using the proposed change of variables

$$\begin{aligned} V &= \left[p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right]\int_0^{\infty}\left[\ln y-\mu\left(\ln z+\gamma\right)\right]\exp\left[-z\left[p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right]\right]\mathrm{d}z \\ &= \left[p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right]\left[\frac{\left(\ln y-\mu\gamma\right)}{p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}}+\mu\frac{\ln\left[p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right]+\gamma}{p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}}\right] \\ &= \ln y+\mu\ln\left[p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right] \end{aligned}$$

where the cdf distribution of the maximum of Type-I Extreme Value random variables

$$
\begin{aligned}
F_{\max}(\varepsilon) &= Prob\left[\max\left\{\ln\frac{y}{p_1}+\varepsilon_1, \ln\frac{y}{p_2}+\varepsilon_2\right\} \leq x\right] = Prob\left[\ln\frac{y}{p_1}+\varepsilon_1 \leq x, \ln\frac{y}{p_2}+\varepsilon_2 \leq x\right] \\
&= Prob\left[\varepsilon_1 \leq x - \ln\frac{y}{p_1}\right] \cdot Prob\left[\varepsilon_2 \leq x - \ln\frac{y}{p_2}\right] = \prod_{i=1,2} \exp\left[-\exp\left(-\frac{x-\ln\frac{y}{p_i}}{\mu} - \gamma\right)\right] \\
&= \exp\left[-\exp\left(-\frac{x-\ln y}{\mu} - \gamma\right)\sum_{i=1,2}\exp\left(-\frac{\ln p_i}{\mu}\right)\right]
\end{aligned}
$$

A.2 Welfare Effects of Asymmetric Price Changes

A.2.1 Utility Change across Switching Consumers

Change in Utility across consumers switching from commodity 2 to commodity 1

$$
\begin{aligned}
\Delta V^C &= \int_{\ln\tilde{p}_1-\ln\tilde{p}_2}^{\ln p_1-\ln p_2} \left(x+\ln p_2-\ln\tilde{p}_1\right)\frac{1}{\mu}\frac{\exp\left(-\frac{x}{\mu}\right)}{1+\exp\left(-\frac{x}{\mu}\right)}\mathrm{d}x \\
&= \left|\frac{x+\ln\frac{p_2}{\tilde{p}_1}}{1+\exp\left(-\frac{x}{\mu}\right)}\right|_{\ln\tilde{p}_1-\ln\tilde{p}_2}^{\ln p_1-\ln p_2} - \left|\mu\ln\frac{1+x}{x}\right|_{\left(\frac{\tilde{p}_1}{\tilde{p}_2}\right)^{-\frac{1}{\mu}}}^{\left(\frac{p_1}{p_2}\right)^{-\frac{1}{\mu}}} \\
&= \frac{\ln\frac{p_1}{p_2}+\ln\frac{p_2}{\tilde{p}_1}}{1+\left(\frac{p_1}{p_2}\right)^{-\frac{1}{\mu}}} - \frac{\ln\frac{\tilde{p}_1}{\tilde{p}_2}+\ln\frac{p_2}{\tilde{p}_1}}{1+\left(\frac{\tilde{p}_1}{\tilde{p}_2}\right)^{-\frac{1}{\mu}}} - \ln\frac{p_1}{p_2} + \ln\frac{\tilde{p}_1}{\tilde{p}_2} + \ln\frac{\tilde{p}_2}{p_2} + \mu\left[\ln\left(\tilde{p}_1^{-\frac{1}{\mu}}+\tilde{p}_2^{-\frac{1}{\mu}}\right) - \ln\left(p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right)\right] \\
&= \frac{\ln\frac{p_1}{\tilde{p}_1}}{1+\left(\frac{p_1}{p_2}\right)^{-\frac{1}{\mu}}} - \frac{\ln\frac{p_2}{\tilde{p}_2}}{1+\left(\frac{\tilde{p}_1}{\tilde{p}_2}\right)^{-\frac{1}{\mu}}} + \ln\frac{\tilde{p}_1}{p_1} + \mu\left[\ln\left(\tilde{p}_1^{-\frac{1}{\mu}}+\tilde{p}_2^{-\frac{1}{\mu}}\right) - \ln\left(p_1^{-\frac{1}{\mu}}+p_2^{-\frac{1}{\mu}}\right)\right]
\end{aligned}
$$

A.2.2 Income Transfer across Switching Consumers

The aggregate transfer is obtained averaging across individual transfers for $j \in C$

$$
\begin{aligned}
T^C &= \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} y \left[\frac{\tilde{p}_1}{p_2} e^{\varepsilon_2-\varepsilon_1} - 1\right] f\left(e^{\varepsilon_2-\varepsilon_1}\right) \mathrm{d}\left(e^{\varepsilon_2-\varepsilon_1}\right) \\
&= \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} y \left[\frac{\tilde{p}_1}{p_2} x - 1\right] \frac{\frac{1}{\mu} x^{-\frac{1}{\mu}-1}}{\left[1+x^{-\frac{1}{\mu}}\right]^2} \mathrm{d}x \\
&= y\frac{\tilde{p}_1}{p_2} \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} \frac{\frac{1}{\mu} x^{-\frac{1}{\mu}}}{\left[1+x^{-\frac{1}{\mu}}\right]^2} \mathrm{d}x - y \frac{1}{\left[1+x^{-\frac{1}{\mu}}\right]} \Bigg|_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} \\
&= y\frac{\tilde{p}_1}{p_2} \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} \frac{\frac{1}{\mu} x^{-\frac{1}{\mu}}}{\left[1+x^{-\frac{1}{\mu}}\right]^2} \mathrm{d}x - y \frac{1}{\left[1+\left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}\right]} + y \frac{1}{\left[1+\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}\right]}
\end{aligned}
$$

Let $x^{-\frac{1}{\mu}} \equiv t$. Then, $\mathrm{d}t = -\frac{1}{\mu} x^{-\frac{1}{\mu}-1} \mathrm{d}x$ and

$$
\begin{aligned}
t &\to \left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}} \quad \text{as} \quad x \to \frac{\tilde{p_2}}{\tilde{p}_1} \\
t &\to \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}} \quad \text{as} \quad x \to \frac{p_2}{p_1}
\end{aligned}
$$

Thus, using the proposed variable change, it is easier to recognise that the above integral has the form of the incomplete beta function[16]

$$
\begin{aligned}
-\int_{\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}}^{\left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}} \frac{t^{-\mu}}{[1+t]^2} \mathrm{d}t &= \int_0^{\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}} \frac{t^{-\mu}}{[1+t]^2} \mathrm{d}t - \int_0^{\left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}} \frac{t^{-\mu}}{[1+t]^2} \mathrm{d}t \\
&= \mathrm{B}\left(\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right) - \mathrm{B}\left(\left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right)
\end{aligned}
$$

Thus,

$$
T^C = y\frac{\tilde{p}_1}{p_2}\left[\mathrm{B}\left(\left(\frac{p_2}{p_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right) - \mathrm{B}\left(\left(\frac{\tilde{p_2}}{\tilde{p}_1}\right)^{-\frac{1}{\mu}}, 1-\mu, 1+\mu\right)\right] - y\left[\frac{\tilde{p}_1^{-\frac{1}{\mu}}}{\tilde{p}_2^{-\frac{1}{\mu}} + \tilde{p}_1^{-\frac{1}{\mu}}} - \frac{p_1^{-\frac{1}{\mu}}}{p_2^{-\frac{1}{\mu}} + p_1^{-\frac{1}{\mu}}}\right]
$$

Finally, I need to compare

$$
T^C = \int_{\frac{p_2}{p_1}}^{\frac{\tilde{p_2}}{\tilde{p}_1}} \left[\frac{\tilde{p}_1}{p_2} z - 1\right] \frac{\frac{1}{\mu} z^{-\frac{1}{\mu}-1}}{\left[1+z^{-\frac{1}{\mu}}\right]^2} \mathrm{d}z \tag{6}
$$

[16] The Incomplete Beta Function is defined as

$$
\mathrm{B}(x, a, b) = \int_0^x t^{a-1} (1-t)^{b-1} \, \mathrm{d}t
$$

Using the change of variables $u = \frac{t}{1-t}$, it is possible to obtain a Beta function of the form given above.

and[17]

$$-\Delta V^C = \int_{\ln p_2 - \ln p_1}^{\ln \tilde{p}_2 - \ln \tilde{p}_1} (x - \ln p_2 + \ln \tilde{p}_1) \frac{1}{\mu} \frac{\exp\left(-\frac{x}{\mu}\right)}{1 + \exp\left(-\frac{x}{\mu}\right)} \mathrm{d}x \tag{7}$$

As for $j \in A \cup B$, the comparison is immediate if we are able to condition on the subset of consumers within C for which the utility/income change is constant; in particular, this is accomplished using a change of variables for either (6) or (7). I will apply the variable transformation to (6), as the idea of exhaustability of linear resources, due to exponential allocation might be clearer. Let $z = \exp(x)$ for (6)[18]; thus, under the variable change,

$$T^C = \int_{\ln p_2 - \ln p_1}^{\ln \tilde{p}_2 - \ln \tilde{p}_1} \left[\frac{\tilde{p}_1}{p_2} e^x - 1 \right] \frac{\frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right)}{\left[1 + \exp\left(-\frac{x}{\mu}\right)\right]^2} \mathrm{d}x$$

Then, the comparison between the income transfer and the utility change becomes quite easy,

$$T^C - \left(-\Delta V^C\right) = \int_{\ln p_2 - \ln p_1}^{\ln \tilde{p}_2 - \ln \tilde{p}_1} \left\{ \left[\frac{\tilde{p}_1}{p_2} e^x - 1 \right] - \left(x + \ln \frac{\tilde{p}_1}{p_2} \right) \right\} \frac{\frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right)}{\left[1 + \exp\left(-\frac{x}{\mu}\right)\right]^2} \mathrm{d}x$$

[17] Deriving the Integral $-\Delta V^C$ requires a variable change, from $z = \varepsilon_1 - \varepsilon_2$ to $w = \varepsilon_2 - \varepsilon_1$.
[18] For (7), the required change of variables is to set $x = lnz$.