International Technology Competition:
The Comeback of the United States during 1980s*

Ufuk Akcigit
University of Pennsylvania & NBER

Sina T. Ates
University of Pennsylvania

Giammario Impullitti
Cambridge University

Preliminary and Incomplete
April 5, 2012

Abstract

Regarding the innovative activity in the US and the rest of the world, there are two strong observations: First, there is a clear convergence in the number of patents (and also citations) issued by US and non-US based firms over the course of the 1970s until the mid-1980s. Second, in the first half of the 1980s, R&D subsidies and tax credits were introduced both at the federal and state level in the US and a halt of the technological convergence followed. In this paper, we ask the following questions: How sizable is this catch-up pattern that we observe? How detrimental is it for the US producer and consumers? Did R&D Tax Credit Policies of the 80s affect international technology competition? If yes, what have the welfare consequences of those policies been? We build a Schumpeterian endogenous growth model in which two countries compete over the leadership in different product lines where optimal R&D decision of firms depend on the technology gap between countries, a novel feature of our model. Another distinct feature of our work is that it introduces R&D tax credits into a structural model to study the effect of this policy on the global patterns of innovation. In the absence of policy intervention, the model is able to replicate the convergence observed in the data. Innovation policies help a country to improve the quality in its product lines faster, but more importantly, it boosts the business stealing effect: firms in that country seize the leadership, and thus profits, of the product lines they operate in more frequently. The model allows us to decompose the growth into these effects, as well as to analyse the welfare implications of the innovation policy.

Keywords: Innovation, technology competition, research and development policy, convergence, endogenous growth, spillover.

JEL classification: ...

*We thank IAES conference participants and Christopher A. Lainez for useful comments. We are indebted to Daniel J. Wilson for sharing and helping with his data.
# Contents

1 Introduction ................................................. 3

2 Literature Review ........................................... 4

3 Empirical Facts .............................................. 7

4 Model ........................................................ 12
   4.1 Preferences ............................................. 13
   4.2 Technology ............................................. 14
   4.3 R&D and Innovation .................................... 14
   4.4 Equilibrium ............................................. 15
   4.5 Value Functions ........................................ 17
   4.6 Transitional Dynamics ................................. 18
   4.7 Consumption and Welfare ............................... 19
   4.8 R&D Subsidy Policies ................................. 20

5 Calibration ................................................... 21
   5.1 Data and Moments .................................... 21
   5.2 Moment Matching .................................... 22

6 Results ....................................................... 23
   6.1 Validation Test ....................................... 24
   6.2 R&D Tax Credits of the 80s ............................ 24
   6.3 Welfare ............................................... 25
       6.3.1 Closed Economy Case ............................ 27
   6.4 Growth Decomposition ................................ 27

7 Conclusion .................................................. 29

A Additional Empirical Material ............................. 30

B Derivations .................................................. 30
   B.1 Lemma ................................................ 30
   B.2 Law of Motions ...................................... 30

C Algorithm .................................................... 32
1 Introduction

Technological progress and innovation has been the major source for the US economic leadership in the world. However, there is an increasing concern both among the economists, politicians and the media that the US is losing its innovativeness and leadership in the global technology competition. US currently ranks 17th among the OECD countries in terms of its public support for private R&D and both sides from the political spectrum argue for increased support for R&D and innovation. The heated debate on international technology competition and its implication for economic policy stimulates many important research questions: Is there a technology catch-up by foreign countries and if so, how sizable is it? How detrimental is the catch-up for the US producer and consumer? Does R&D Tax Credit Policy (RTC) affect the international technology competition and if so, in what way? What are the trade-offs generated by RTCs and what are the welfare consequences of these policies? Finally, how should the RTCs be designed to boost innovation and restore US global leadership? This paper sheds light on the aforementioned questions by studying the effects of the US R&D Tax Credit of the 1980s.

Many developed countries are engaged in competition against other countries for the global technology leadership. Governments are trying to support their national companies in this race through R&D tax incentives. In the US, the first Federal R&D Tax Credit (FRDC) was introduced in 1981 through the Economic Recovery Tax Act. This was followed by State R&D Tax Credits (SRTC), Minnesota being the first to introduce SRTC in 1982, one year after the FRTC. The goal of these policies were to increase the international competitiveness of the US companies and economy (NSF, 2005). The credit is simply designed to reduce the after-tax cost of R&D, thereby encouraging firms to invest and innovate more.

The objectives of this paper are (1) to produce new empirical facts on international technology competition using USPTO patent data and OECD R&D data, (2) to build a theoretical model motivated by those empirical facts, (3) to calibrate the model and study the quantitative effects of the RTCs of the 80s on firms’ R&D incentives and consumer welfare, and (4) to study the optimal design of RTCs.

Our empirical analysis indicates a very strong catch-up in terms of the patent counts. Between 1975 and 1985, the ratio of foreign patent counts to the sum of US and foreign patent counts increased steadily by 33%. While US was the global technology leader in 80% of the technology classes in 1975, in 10 years this fraction went down to 60%. In the meantime, the fraction of technology classes in which the US held a neck-and-neck position with foreigners increased from 10% to 20% and the fraction of technology classes in which the US was a lagging increased from 10% to 20%. During the same period, the US was also lagging in terms of its R&D investment. While the ratio of R&D spending to GDP was 1.86% in the US, it was around 2.07% in the competitor countries.\(^1\) The early 1980s has been a turning point for the US. In 1981, US adopted its first FRDC and starting in 1982, different states started to adopt SRTCs which clearly accelerated after 1985. Thanks to these aggressive R&D incentives, the

\(^1\)As competitors, we refer to Canada, France, Germany, Italy, Japan, Spain and the UK. The numbers are calculated using a weighted combination of the data for these countries which is explained in detail in Section 5.
convergence in the ratio of patent counts stopped and the US inventors regained the world leadership in a variety of technology fields. By the end of 1995, US restored its leadership in 65% of the technology classes.

The aforementioned facts motivate our theoretical framework with the following features: in our model, (1) two countries compete step-by-step in multiple technology classes for the global leadership, (2) the strategic interaction that we introduce allows countries to invest differently depending on whether the country is neck-and-neck, a leader or a follower, (3) countries trade just in intermediate goods yet not in the asset markets, hence leadership in more technology classes implies higher income and per-capita consumption yet foreign innovation also benefits the domestic consumer, (4) countries impact the level of domestic innovation by R&D subsidies.

We solve our model with its transitional dynamics and calibrate its parameters using various data sources. The calibrated model allows us to do two interesting exercises. First, we simulate the closed economy version of our model with the same initial conditions and assess the gain from openness. The trade-off in our model is that, openness brings faster innovations to US customers, however, business stealing makes the US firms lose their market shares in some industries and therefore reduce the household income.

In the second exercise we simulate the model without the R&D policies of the 80s to assess the welfare gains or losses created by the policy change. The introduction of the R&D subsidy in our model allocates resources from current consumption to R&D. Hence, consumers lose from the current consumption in exchange of higher future consumption. This is the standard trade-off that also exist in closed economy endogeneous growth models. The additional channel in an open economy setup is the business stealing effect, which also increases the domestic income. In our calibration, we quantify these distinct channels.

The rest of the paper is organized as follows. Section 2 reviews the related literature, section 3 describes the empirical facts on international technology competition, section 4 introduces the theoretical framework, section 5 outlines the calibration procedure, section 6 provides the quantitative results and section 7 concludes.

2 Literature Review

This paper is related to two different, yet related, literatures. The literature on international technology diffusion and convergence, and the line of research analyzing the welfare effects of international technology diffusion and its implications for innovation policy.

Empirical research has shown that there are substantial productivity differences in technology, even among similar countries (e.g. Trefler, 1993, and Hall and Jones, 1999, Klenow and Rodriguez-Claire, 2005) and that technology diffusion is the key factor behind the dynamics of cross-country incomes differences. Although technology diffuses across countries’ boarders, the world seem to be quite far from the global technology environment of the Neoclassical growth model. Eaton and Kortum (1999) for instance, estimate that the world is two-thirds away from technological autarky towards a global technological scenario. A substantial body of theoretical and empirical work has studied the channels and mechanisms of technological change and its diffusion across countries. The main channels of international technology diffusion are trade (import, export, FDI) and international knowledge spillovers. Our paper focuses
on both trade and knowledge spillovers as channels of technology flows across countries. The possibility that today’s innovation “stands on the shoulder’s” of past successful researchers, documented in the empirical results of Caballero and Jaffe (1993) for a closed economy, can be expected to take place in an open economy as well. An increase in the stock of global R&D can increase the productivity of researchers all over the world. Coe and Helpman (1995) and Coe, Helpman, and Offmeister (2009), among others, have found evidence of sizable international knowledge spillovers, reporting positive effects of foreign R&D on domestic TFP in a sample of OECD countries. Trade in final and intermediate goods can be an important vehicle of knowledge spillovers, as discussed in the extensive survey by Keller (2004).

Eaton and Kortum (1999) build a multi-country version of the search model of innovation and quality-ladder growth by Kortum (1997) with diffusion and adoption lags. They focus on the steady-state where all countries are growing at the same rate, with countries that are quicker to adopt innovations from all sources taking the lead in productivity levels. They fit the model to five leading OECD countries (the US, Japan, Germany, the UK and France) in the late 1980s in order to provide a quantitative assessment of the extent of international technology diffusion and of the importance of each country as an innovation source and destination. In order to measure diffusion patterns, they use international patents data. In the model the innovating firm has to decide where to patent the idea, and the decision is related to the likelihood that this idea will be used in that country. Hence, they use data on international patenting as evidence of the source of ideas and where those ideas are more likely to be used (diffused). They find that countries generally adopt from one half to three forth of the ideas generated abroad. Since they focus on the steady state and do not analyze the convergence process, they fit the model to the late 1980s when these countries have roughly converged to similar growth rates. Hence, their steady-state analysis cannot account for the faster growth of Germany, Japan and France in the previous decades that brought to that convergence.2

Helpman (1993) presents a Schumpeterian North-South model of trade and growth in which the advanced country’s firms innovate and Southern firms copy those innovation. Technology diffusion from the North to the South is determined by the strength of the property right protection in the South and by the intensity of imitation. Transitional dynamics is performed to analyze the welfare effect of tightening IPR policy in each country, and to study the effects of faster diffusion triggered by more intense imitation.3 Howitt (2000) builds a multi-country Schumpeterian growth model without scale effects in which convergence is obtained through international knowledge spillovers. In each country innovation builds on a world-wide technological frontier which is in turn fuelled by innovation from all countries. Global convergence to the same long-run growth rate is obtained, and savings and policy differences affect relative productivity and income levels.4

---

2Eaton and Kortum (1997) provides a first step towards analyzing the transitional dynamics. The out of steady-state dynamics of technology diffusion is not fully derived since the diffusion dynamics is obtained conditioning on the observed path of R&D observed in the data, while they should be obtained endogenously from simulating the model.

3One limit of this model is that the intensity of imitation is exogenous, while in reality this is an economic activity that responds to incentives. See Dinopoulou and Segerstrom (2007) and Gustafsson and Segerstrom (2011) for recent extensions of the North-South trade models where imitation is endogenized but the analysis is confined to the steady state.

4Eaton and Kortum (2006) develop a two-country version of the Ricardian model in Eaton and Kortum (2002) introducing innovation and technology diffusion via imitation. Their goal is to analyze the effects on innovation of two modes of globalization: an increase in the speed of technology diffusion and a reduction in
Our paper contributes to this literature along the following dimensions: first, as Howitt (2000), we present a model of technology diffusion, innovation, and convergence dynamics, but we introduce heterogeneous industries in order to account for the sectoral variation in technology diffusion observed in the data. As Eaton and Kortum (1999) and (2006) we build a quantitative model of the global economy but we introduce heterogeneous industries and we fully analyze the transitional dynamics of diffusion.

Besides modelling the mechanisms of convergence through technology diffusion, our paper also analyzes the effects of diffusion on the leading country’s welfare. There is an emerging literature studying the effects of technological progress in a country on its trading partners’ technology and welfare. Eaton and Kortum (2002) multi-country quantitative Ricardian model shows that increasing productivity in the US and Germany produces sizable welfare gains for other OECD countries. A set of recent papers has analyzed the effects of China’s technological catch-up on its trading partners’ welfare (see e.g. Hsie and Ossa, 2011, Di Giovanni, Levchenko, and Zhang, 2012, and Levchenko and Zhang, 2011). These papers do not model technological change, and they focus on the effects of exogenous productivity growth in one country on its trading partners’ welfare. Impullitti (2010) provides a first step towards a more comprehensive analysis of the effects of international technology diffusion on innovation, growth and welfare. He sets up a multi-country version of the quality ladder growth model in which international technology diffusion is interpreted as an increase in the set of industries in which domestic and foreign firms compete in innovation to obtain the leadership in the global market. The model is fitted to 12 OECD countries, and uses the geographical distribution of R&D investment across manufacturing industries to back out an empirical measure of international technology diffusion. The data show a rapid technology diffusion from the early 1970s to the early 1990s: the US is the undisputed leader in the early 1970s and sees its leadership substantially eroded by massive entry of Japanese and European firms in global R&D races in many sectors of the economy in the following years. When the observed dynamics of technology diffusion is fed into a quantitative version of the model it produces negligible welfare gains for the leading country, the US, suggesting that

The second line of research the paper relates to is that on optimal innovation policy in open economy. Our analysis of the optimal innovation policy response to technological catch-up performed in these papers introduces a state-dependent dimension in the research on strategic trade and industrial policy. The strategic policy literature pioneered by Spencer and Brander (1983) and Grossman and Eaton (1986), analyzed the strategic motive to use tariffs and subsidies (to production and innovation) to protect the rents and the market shares of domestic firms in a imperfectly competitive global economy. Impullitti (2010) discussed above, introduces technology dynamics in this class of models and study the impact of increasing foreign competition triggered by faster diffusion on the incentives to subsidize R&D in the leading country. Acemoglu and Akcigit (2011) analyze the role of technological distance in an
industry for the optimal protection of property rights in a closed economy. They extend the model of growth through step-by-step innovation of Aghion, Harris, and Vickers (1997) and Aghion et al. (2001) to a full general equilibrium economy, consider a continuum of industry-level technology gaps (step sizes), and analyze the link between these gaps and the optimal patent protection. Their main result suggests that the optimal policy involves state-dependent intellectual property rights (IPR) protection, where industries with technological leaders in high-gaps industries receiving stronger protection.

Our paper adds both theoretical and quantitative contributions to this literature. On the theory side, we endogenize the dimension of technology diffusion analyzed in Impullitti (2010) and characterize the transitional dynamics of convergence, therefore providing a careful assessment for the welfare effects on international technology diffusion. Moreover, we introduce sectoral heterogeneity which allows us both to model the sectoral variation in diffusion patterns observed in the data, and to explore the welfare properties of state-dependent R&D subsidies, involving a policy structure that links the subsidy rate to the technology gap observed in each sector. Our model builds on Acemoglu and Akcigit (2011) introducing international trade and competition between domestic and foreign producers in each product line. Moreover, while they limit the analysis to the steady state, we consider the entire transition path generated by technological convergence and changes in state-dependent innovation policies. Since we are focusing on optimal innovation policy, a complete characterization of the dynamic path of welfare is key feature for both our theoretical and quantitative analysis. Aghion and Howitt (2009) consider two-country version of the step-by-step model similar to ours to analyze the effects on profits, wages and growth of opening up to trade. We depart from their analysis first by focusing on international technology diffusion and convergence, rather than focusing on the comparative statics of trade openness. Secondly, we introduce state-dependent R&D subsidy and perform a quantitative analysis of the dynamic link between subsidies and technological convergence.

Finally our paper contributes to the theoretical and empirical literature on R&D subsidies. (TBW: Laincz (2009), Akcigit, Hanley and Serrano-Velarde (2012), Acemoglu et al (2012), Bloom, Griffith and Van Reenen (2002), Guenther (2005), Hall (2001), etc.)

3 Empirical Facts

This section presents some empirical regularities regarding international technology competition and R&D activity. The facts related to the former, which motivate our paper, show the trends in patenting activity of foreign countries relative to the US. Before going into the policy analysis, we want our model to explain these trends so that they provide an external validity check for our model. Then, we will examine the R&D activity in the US to provide a causal relationship between this and the overall performance of a country in the technological race. This will explain why changes in R&D policies might be a valid channel to analyse the shifts in international patterns of technological competition.

beneficial for the global economy, there exists a threshold level of competition below which the leading country experiences welfare losses under cooperation. Along the same line of research a recent paper by Cozzi, Chu, and Galli (2012) extends the Schumpeterian model of distance to frontier and growth of Acemoglu et al. (2006) to study the link between the stage of development of a country and its optimal IPR policy. They find that at an early stage of development, it is optimal to adopt a weak IPR protection to facilitate imitation. At a later stage of development, it becomes optimal to strengthen IPR protection to encourage domestic innovation.
Fact 1: Technological Convergence until mid-80s

There is a striking change in the relative position of foreign countries relative to the US in the worldwide technological competition over the course of 1970s until mid-80s. Both in the aggregate and sectoral level, we observe a clear pattern of catching-up which we measure using patent and citation counts.

Figure (1) shows the yearly change in the proportion of patents registered in the US by foreigners using NSF data on patent counts. It also depicts a similar ratio for the citations those patents received. Both lines show an obvious, increasing trend which means that the growth in the number of foreign-based patents is higher than the one in the US counterpart.

![Figure 1: Ratio of patents and associated citations registered in the US by foreigners over the total, 1965-1985](image)

The right panel in Figure (2) brings the analysis down to the level of patent classes (IPC4) using the same data set. What it delineates is the percentage of sectors (broadly defined by patent classes) “owned” by the US- and foreign-based firms over years as well as the percentage of sectors where they are in a “neck&neck” position. The ownership of a sector is defined by having more patents than a certain share of patents registered for the particular sector. The situation we call neck&neck arises when the difference of the shares of patents held by two countries is less than a threshold. When generating this picture, we term sectors where the difference is less than 15% as neck&neck sectors. This implies that a sector is dominated (owned) by the firms of a country if their share is above 57.5%. The second panel replicates the similar figure using the number of citations attached to those patents in the left panel. Both panels show the declining trend in the percentage of all sectors where US firms are dominating. This observation demonstrates the relative strengthening of foreign competitors in the technological competition.

Fact 2: R&D Tax Credit during 80s and the Halt of Convergence

Before analysing how the trends presented above changed after the introduction of R&D tax credits (on both state and federal level), let us give some details about this policy which started
Figure 2: Proportion of sectors owned by the US and foreigners, resp., 1965-1985

with the federal credit established in 1981. Figure (3) shows the evolution of average rate of tax credits over the states together with the number of the states that offer a tax credit.

Figure 3: State level evolution of R&D tax credits

Figure (3) highlights that the average effective tax credit for R&D on the state level reached a little more than 6% as of 2006, nearly half of the federal one. The number of states following such a policy rose to 32. Now, the question is: how did the the variables shown in the previous subsection behave after these policy changes? Figures (4) and (5) address this question. The former shows that the ratio of sectors dominated by the US-owned patents stopped decreasing
and started to show an increasing trend. The same change is to be observed in citation counts. Figure (5) depicts a related pattern: ratio of patents (and citations) by foreign registrars in total patents registered comes to a halt and starts to decrease. These turns in innovation patterns which overlap with the introduction of tax credits in the US leads us to the hypothesis that the latter caused the former. Now, we will test this argument quantitatively.

Figure 4: Proportion of sectors owned by the US and foreigners, resp., 1965-1995

Figure 5: Ratio of patents and associated citations registered in the US by foreigners over the total, 1965-1995

To highlight the relationship between tax credits and innovative behavior of firms we will
Table 1: The Effect of R&D Tax Credit on Innovation (excl. Federal Credits)

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>ln(R&amp;D$_t$)</th>
<th>ln(R&amp;D$_{t-1}$)</th>
<th>ln(Patents$_t$)</th>
<th>ln(Patents$_{t-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(R&amp;D$_{t-1}$)</td>
<td>-</td>
<td>0.631</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(Patents$_t$)</td>
<td>-</td>
<td>-</td>
<td>0.499</td>
<td>(72.83)***</td>
</tr>
<tr>
<td>ln(State credit$_t$)</td>
<td>3.153</td>
<td>0.524</td>
<td>2.948</td>
<td>1.203</td>
</tr>
<tr>
<td>Year Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

exploit the state level variation in the dates when credit policies came into action. The basic firm level regression equation for which we obtain firm level data using COMPUSTAT is as follows:

\[
\ln Y_{jst} = \text{const.} + \ln Y_{jst-1} + \ln SC_{st} + \psi_j + \psi_t + u_t
\]

where \(\psi_j\) and \(\psi_t\) represent firm and year dummies, respectively, and \(u_t\) is the error term. \(SC_{st}\) is the tax credit level in the state \(s\) where firm \(j\) operates. For the dependent variable \(Y\) we use both R&D and patent counts. We utilize two different specifications for this regression which differ in the inclusion of the lagged value of the dependent variable. The results are summarized in Table (1). All versions (represented by columns of the table) reveal the positive effect of state level R&D tax credits on the firms’ innovative activities. This effect is also robust to the existence of lagged values of the dependent variable in the regression.\(^7\)

Just before passing to the next subsection, we want to make one point clearer. Up to now, the figures that motivated our analysis were using relative measures. As an example, Figure (5) might raise the question whether the total count of patents registered decreased or the result was mainly driven by a pattern created by foreign registrations. Against such a doubt, we end this subsection with Figure (6). As it clearly delineates, in the pre-tax-credit period there is a considerable increase in the number of patents of foreign applicants while the US-counterpart did not change sizeably. In the aftermath of policy changes, the activity in the US responds with a strong increasing trend so that even though the foreign side continued its rising pattern we obtained the above figures.

**Fact 3: R&D Elasticity of Output**

This section examines the effect of R&D on output. Again, we will use state level differences in tax credit policies as exogenous variation in R&D (patenting) in order to capture its impact on output via instrumental variable (IV) estimation. In the first stage, we regress R&D on state level tax credits as we did previously. In particular, we use the following relationship:

\[
\ln(R&D_{jt}) = \text{const.} + \ln(State credit$_{st}$) + \psi_s + \psi_t + u_t
\]

\(^7\)A version of the regression analysis which includes also the federal credits can be found in the appendix.
where \( j, s, t \) denote the firm, the state and the year, respectively. Then, we use the estimates of this regression as IV for R&D in the following equation:

\[
\ln(Sales_{jt}) = const. + \ln(R&D_{jt}) + u_t. \tag{3}
\]

The results are shown in Table (2). They imply a significant positive effect of R&D on firms’ output.

| Table 2: IV Regression of Sales on R&D Spending |
|-----------------------------|-----------------------------|
| **First Stage** | **Second Stage** |
| Dep. Var. | \( \ln(R&D_t) \) | Dep. Var. | \( \ln(Sales_t) \) |
| \( \ln(State\ credit_t) \) | 2.920 | \( \ln(R&D_t) \) | 0.407 |
| | (5.59)*** | | (16.96)*** |
| Year Dummy | Yes | Year Dummy | Yes |
| Sector Dummy | Yes | Sector Dummy | Yes |

4 Model

The model consists of two countries indexed by \( c \in \{A, B\} \). Each country has access to the same final good production technology. There is a continuum of intermediate goods indexed by \( j \in [0, 1] \) used in final good production. Final good is used for consumption, production of intermediate goods and innovation. There is free trade in intermediate goods and final goods sectors and no trade in assests. Trade in intermediate and final goods allow us to focus on technology competition and business stealing between two countries. Lack of trade in assets will rule out international borrowing and lending and will make the two countries grow in
different rates during the transition. Households will have linear utility to simplify the interest rate determination which is required since our model will focus on transitional dynamics.

In each production line for intermediate goods there are two firms, one domestic and one foreign, competing in quality for global technology leadership. Firms innovate by investing resources to improve the quality of their product and in equilibrium only the firm producing the top quality of a product will supply it to the world economy. Since innovation success is a random process the global economy will feature a distribution of firms supplying heterogeneous quality products. There are two channels of interdependency linking the countries: trade in intermediate goods and trade in ideas. The latter consists in technology diffusion through international knowledge spillovers.

The model builds on the step-by-step innovation frameworks of Aghion, Harris, and Vickers (1997) and Aghion et al. (2001). More precisely it follows Acemoglu and Akcigit (2012) in considering a full general equilibrium economy and in analyzing state-dependent innovation policies, but it restricts the focus to an economy with 1-step gap. The two main novel features of this model are that first, it introduces international trade and competition between domestic and foreign producers in each product line. Secondly, it does not limit the analysis to the steady state but considers the entire transition path generated by changes in state-dependent innovation policies.

4.1 Preferences

Consider the following continuous time economy. Both countries admit a representative household with linear utility:

\[ U(t) = \int_{t}^{\infty} \exp(-\rho (\tau - t))C_c(\tau) \, d\tau \]  

(4)

where \( C_c(t) \) represents consumption at time \( t \), \( \rho > 0 \) is the discount rate. The budget constraint of a representative household in country \( c \) at time \( t \) is

\[ r_c(t)A_c(t) + L_c w_c(t) = p_c(t)C_c(t) + \dot{A}_c(t) \]  

(5)

where \( r_c(t) \) is the return to asset holdings of the household, \( L_c \) is the amount of fixed factor (could be labor or land) in country \( c \), \( w_c(t) \) is the fixed factor income, \( p_c(t) \) is the price of the consumption good in country \( c \). Household in country \( c \) owns all the firms in \( c \), therefore the asset market clearing condition requires that the sum of firm values has to be equal to the asset holdings

\[ A_c(t) = \int_{0}^{1} V_{cj}(t) \, dj \]

where \( j \) indexes the firms (and the product lines; see below). We assume full home bias in asset holding, an assumption that is robustly supported by the empirical evidence in the 1980s and 1990s. For instance, in 1989, 92% of the US stock market was held by US residents, and Japan, UK, France and Germany show similar shares, 96%, 92% 89% and 79% respectively. A similar picture can be observed till the early 2000s when the home bias started to decline (see e.g. Coeurdacier and Rey, 2012).

\[ \text{Note that this model will not be a environment to analyse the gains from trade. Here we take existence of trade as given. Our question is, given that two comparable countries do already trade, what are the implications of R&D policies in this environment on international technology competition.} \]
4.2 Technology

The final good, which is to be used for consumption, R&D expenditure and the input cost of the intermediate good production, is produced in both countries according to the following technology in a perfectly competitive markets:

$$Y_c(t) = \frac{L_c^\beta}{1-\beta} \int_0^1 \left[ \sum_{cj \in \{A,B\}} q_{cj}(t)^\beta k_{cj}(t)^{1-\beta} \right] dj$$

(6)

Here, $L_c$ is the amount of fixed factor in $c$, $k_j$ refers to the intermediate good $j \in [0,1]$ and $q_j$ is the the quality level of $k_j$, and $\beta$ is the share of fixed factor in total output. We assume the fixed factor $L_c$ immobile across countries whereas the intermediate goods can be obtained from any country without any additional cost. We normalize $L_c = 1$ in both countries to reduce notation. Note that both countries produce each variety $j$ and they are perfect substitutes after adjusting for their qualities. As a result, final-good producers will choose to buy its input from the country that has a higher quality of the same variety. Final good producers in both countries have access to the same technology and this will allow us to focus on the heterogeneity on the intermediate goods sector. The fact that final good sectors are produced in perfectly competitive markets and that marginal costs are the same in both countries imply that the price of the final output in both countries will be the same. We normalize that price to 1 without any loss of generality.

4.3 R&D and Innovation

In each product line $j$, one firm from each country $c \in \{A,B\}$ (indexed by $cj$) compete for the market leadership à la Bertrand. Each one of these infinitely-lived firms has the same marginal cost of production, $\eta$, yet they differ in terms of their quality of output, $q_{cj}$. We will say that country $A$ is the leader in $j$ if

$$q_{Aj}(t) > q_{Bj}(t)$$

and the follower if

$$q_{Aj}(t) < q_{Bj}(t).$$

Firms are in a neck-and-neck position when $q_{Aj}(t) = q_{Bj}(t)$. The quality $q_{Aj}(t)$ improves through successive innovations in $A$ or spillovers from $B$. Each time there is an improvement, the existing quality increases by $(\lambda - 1) q_{Aj}$, i.e.

$$q_{Aj}(t + \Delta t) = \lambda q_{Aj}(t).$$

We assume the following initial condition for qualities, $q_{Aj}(0) = q_{Bj}(0) = 1$. As a result, the state of a firm can be summarized by a single integer $m_A$ such that

$$\text{Technology gap between } A \text{ and } B \text{ in } j = \frac{q_{Aj}(t)}{q_{Bj}(t)} = \frac{\lambda^{n_A(t)}}{\lambda^{n_B(t)}} = \lambda^{m_A(t)},$$

where $n_A(t)$ is the total number of quality improvements that has taken place in country $A$ and $m_A(t) = n_A(t) - n_B(t)$ is the difference between the two countries in terms of those occurrences. Note that $m_A(t) = 0$ when the two firms are neck and neck. To preserve tractability, we assume
that the maximum technology gap between the countries cannot be more than 1 step due to technology spillovers. Thus

\[ m_c(t) \in \{-1, 0, 1\} \text{ for } c \{A, B\}. \]

Firms invest in R&D in order to obtain market leadership through improving the quality of their products. Let \( z_{cj} \) and \( x_{cj} \) denote the amount of R&D investment and the resulting Poisson arrival rate of innovation by country \( c \) in \( j \). The production function of innovations takes the following form,

\[ x_{cj}(t) = \left( \gamma_c \frac{z_{cj}(t)}{\alpha c q_{cj}(t)} \right)^\frac{1}{\gamma_c}, \quad c \in \{A, B\}. \]

Note that \( q_{cj} \) in the denominator captures the fact that a quality is more costly to improve if it is more advanced. This production function implies the following cost function for generating an arrival rate of \( x_{cj} \)

\[ C(x_{cj}, q_j(t)) = q_j(t) \frac{\alpha c}{\gamma_c} x_{cj}. \]

Let us denote the innovation arrival rates of the leader, follower and neck-and-neck form by \( x_{1j}(t), x_{-1j}(t) \) and \( x_{0j}(t) \), respectively. We adopt similar notation for the quality levels \( q_{1j}(t), q_{-1j}(t) \) and \( q_{0j}(t) \). Then the law of motion for the firm level qualities can be summarized as

\[
\begin{align*}
q_{1j}(t+\Delta t) &= \begin{cases} 
\lambda q_{1j}(t) \text{ with probability } x_{1j}(t) \Delta t \\
q_{1j}(t) \text{ with probability } 1 - x_{1j}(t) \Delta t
\end{cases} \\
q_{-1j}(t+\Delta t) &= \begin{cases} 
\lambda q_{-1j}(t) \text{ with probability } x_{-1j}(t) \Delta t + x_{1j}(t) \Delta t \\
q_{-1j}(t) \text{ with probability } 1 - x_{-1j}(t) \Delta t - x_{1j}(t) \Delta t
\end{cases} \\
q_{0j}(t+\Delta t) &= \begin{cases} 
\lambda q_{0j}(t) \text{ with probability } x_{0j}(t) \Delta t \\
q_{0j}(t) \text{ with probability } 1 - x_{0j}(t) \Delta t
\end{cases}
\end{align*}
\]

Note that when a firm is the leader or a neck and neck, then its quality improves only through its own R&D whereas a follower also benefits from the leader’s innovation due to spillovers. Here the assumption is that when an innovation arrives the second to previous leading technology becomes obsolete and freely available to the follower in the product line. Since in this economy the leader and the follower belong to different countries by construction, the knowledge spillover implies a technology flow across the country’s boarders.

### 4.4 Equilibrium

Next, we will define the Markov Perfect Equilibrium of the model. Then we will build up the value functions for the intermediate producers and derive their closed form solutions along with the R&D decisions. These will help us characterize the evolution of the world economy over time.

**Definition 1 (Allocation)** An allocation for this world economy consists of interest rate \( r(t) \), country-specific fixed factor price \( w_c(t) \), country-specific aggregate output, consumption, R&D expenditure and intermediate input expenditure \( \{Y_c(t), C_c(t), Z_c(t), K_c(t)\} \) and last, intermediate good prices, quantities, and innovation arrival rate \( \{p_j(t), k_j(t), x_{cj}(t)\} \) in country \( c \), product line \( j \).
We start with the maximization problem of the household. The Euler equation of the
household problem determines the interest rate in the economy as
\[ r(t) = \rho. \]
Next, we turn to the maximization problem of the final good producer. Using the production
function \( (6) \), the final good producers generate the following demand for the fixed factor \( L_c \) and intermediate good \( j \)
\[
w_c(t) = \frac{\beta}{1-\beta} L_c^{\beta-1} \int_0^1 q_j^\beta(t) k_j(t)^{1-\beta} dj\]
\[ p_j(t) = L_c^\beta q_j^\beta(t) k_j(t)^{-\beta}, \forall j \in [0, 1]. \]
where \( q_j(t) \equiv \max \{ q_{cj}(t), q_{cj'}(t) \} \) and \( k_j(t) \) is its quantity. At this point, let us make the
following assumption to guarantee that the latest innovator in a product line can charge the
unconstrained monopoly price.

**Assumption 1** Throughout the paper, we assume the following inequality to hold
\[
\lambda \geq \left( \frac{1}{1-\beta} \right)^{\frac{1-\beta}{\eta}}
\]
We denote the constant marginal cost of producing an intermediate variety by \( \eta \). Then, the profit maximization problem of the monopolist in product line \( j \) becomes
\[
\pi(q_j(t)) = \max_{k_j \geq 0} \left\{ L^\beta q_j^\beta(t) k_j(t)^{1-\beta} - \eta k_j(t) \right\}, \forall j \in [0, 1].
\]
The optimal quantity and price for intermediate variety \( j \) follows from the first order conditions
\[
k_j^*(t) = \left[ \frac{1-\beta}{\eta} \right]^{\frac{1}{\beta}} q_j(t) \text{ and } p_j^*(t) = \frac{\eta}{1-\beta}, \]
(7)
where we used the fact that \( L_c = 1 \). The realized price is a constant mark-up over the marginal
cost and is independent of the individual product quality. Thus, the profit for each intermediate
good is
\[
\pi^*(q_j(t)) = \pi q_j(t),
\]
where \( \pi \equiv \eta^{\frac{\beta-1}{\beta}} (1-\beta)^{\frac{1-\beta}{\beta}}. \) Under these parameter choices the output becomes
\[
Y_c(t) = Y(t) = \left[ \frac{1-\beta}{\eta} \right]^{\frac{1-\beta}{\beta}} \frac{Q(t)}{1-\beta}, \]
(8)
where \( Q \) denotes the quality index of the world defined as
\[
Q(t) \equiv \int_0^1 q_j(t) dj.
\]
Finally the fixed factor price is
\[
w_c(t) = w(t) = \left[ \frac{1-\beta}{\eta} \right]^{\frac{1-\beta}{\beta}} \frac{\beta}{1-\beta} Q(t).
\]
(9)
Below, the asterisk indicates an equilibrium value.
4.5 Value Functions

Now, we can write the value functions for the leader, neck-and-neck and laggard firms, which are denoted by the subscripts 1, 0, −1, respectively. For expositonal concerns, we will abstract from the inclusion of R&D tax credits for now. To begin with, the value function of the leader in \( j \) that has a quality \( q_j = q \) is expressed as follows,

\[
rV_{1A} (q) - \dot{V}_{1A} = \max_{x_{1A}} \left\{ 2\pi q - \theta_{Rc} q - q\alpha_A x_{1A}^{\gamma_A} + x_{1A} [V_{1A} (\lambda q) - V_{1A} (q)] + x_{-1B} [V_{0A} (q) - V_{1A} (q)] \right\}.
\]

This value function is very intuitive. There is a discounting at the rate \( r = \rho \). The first term on the right is the profit collected at every instant by the leader (note that it sells to producers in both countries), minus the fixed cost in R&D \( \theta_{Rc} q \). The second term is the R&D cost by the leader. As a result, the leader innovates at the rate \( x_{1A} \) which generates a change in firm value of \( V_{1A} (\lambda q) - V_{1A} (q) \). Finally, the follower firm in country \( B \) innovates at the rate \( x_{-1B} \) in which case both firms end up being neck-and-neck.

Similarly the value function for the follower firm in \( A \) is equal to

\[
rV_{-1A} (q) = \max_{x_{-1A}} \left\{ -\theta_{Rc} q - q\alpha_A x_{-1A}^{\gamma_A} + x_{-1A} [V_{0A} (\lambda q) - V_{-1A} (q)] + x_{1B} [V_{-1A} (\lambda q) - V_{-1A} (q)] \right\}.
\]

This value function has similar interpretation as before. However two additional points are worth emphasizing. First, the follower firm makes zero profit due to Bertrand competition. Second, when the leader firm innovates in country \( B \), in that case the state of the follower also increases from \( q \) to \( \lambda q \) due to spillovers.

Finally the value function for the neck and neck firms can be expressed as

\[
rV_{0A} (q) = \max_{x_{0A}} \left\{ -\theta_{Rc} q - q\alpha_A x_{0A}^{\gamma_A} + x_{0A} [V_{1A} (\lambda q) - V_{0A} (q)] + x_{0B} [V_{-1A} (\lambda q) - V_{0A} (q)] \right\}.
\]

Note that in the neck-and-neck situation, Bertrand competition leads to no profits. All these value functions are written for the firms which produce in country \( A \). The counterparts for the ones in country \( B \) is found just by switching the country subscripts.

**Definition 2 (Equilibrium)** A Markov Perfect Equilibrium of this world economy is an allocation

\[
\{ r^*(t), w^*_c(t), p^*_c(t), k^*_c(t), x^*_c(t), Y^*_c(t), C^*_c(t), Z^*_c(t), K^*_c(t) \}_{c \in \{A,B\}, j \in [0,1]}
\]

such that (i) the sequence of prices and quantities \( p^*_c(t), k^*_c(t) \) satisfy (7) and maximize the operating profits of the incumbent firm in the intermediate good product line \( j \); (ii) the R&D decision \( x^*_c(t) \) maximizes the expected profits of firms taking wages \( w^*_c(t) \), aggregate output \( Y^*_c(t) \), the R&D decisions of other firms \( x^*_c(t) \) and government policy \( \tau^*_c(t) \) as given; (iii) labor allocation \( L^*_c(t) \) is the profit maximizing labor choice of the final good producers; (iv) \( V^*_c(t) \) is as given in equation (8), and (v) wages \( w^*_c(t) \) and interest rates \( r^*(t) \) clear the labor and asset markets at every \( t \).
Lemma 1 The value functions are linear in quality such that \( V_{m, A}(q) = q v_{m, A} \) and \( \dot{V}_{mc} = \partial V_{mc} / \partial t = 0 \) for \( m \in \{-1, 0, 1\} \) where

\[
rv_{1A} = \max_{x_{1A}} \left\{ 2\pi - \theta_{Re} - \alpha_A x_{1A}^2 + x_{1A} v_{1A} (\lambda - 1) + x_{-1B} [v_{0A} - v_{1A}] \right\}, \tag{10}
\]

\[
rv_{-1A} = \max_{x_{-1A}} \left\{ -\theta_{Re} - \alpha_A x_{-1A}^2 + x_{-1A} [\lambda v_{0A} - v_{-1A}] + x_{1B} v_{-1A} (\lambda - 1) \right\}, \tag{11}
\]

\[
rv_{0A} = \max_{x_{0A}} \left\{ -\theta_{Re} - \alpha_A x_{0A}^2 + x_{0A} [\lambda v_{1A} - v_{0A}] + x_{0B} [v_{-1A} - v_{0A}] \right\}. \tag{12}
\]

Proof. See the appendix.

Lemma (1) has the implication that the quality level \( q \) drops out of the value functions. This, in turn, indicates that the values of firms do not change over time; as a result, optimal R&D levels are independent of \( q \) and constant. Thus, the optimal R&D levels are as follows:

\[
x_{-1A} = \left( \frac{\lambda v_{0A} - v_{-1A}}{\gamma_A \alpha_A} \right)^{\frac{1}{\gamma_A - 1}}, \quad x_{0A} = \left( \frac{\lambda v_{1A} - v_{0A}}{\gamma_A \alpha_A} \right)^{\frac{1}{\gamma_A - 1}} \quad \text{and} \quad x_{1A} = \left( \frac{(\lambda - 1) v_{1A}}{\gamma_A \alpha_A} \right)^{\frac{1}{\gamma_A - 1}}. \tag{13}
\]

4.6 Transitional Dynamics

The decisions of firms will affect the leadership of a country in product lines. Let \( \mu_{mc} \) denote the share of product lines in which the state of country \( c \) is \( m \in \{-1, 0, 1\} \) such that

\[
\mu_{-1c} + \mu_{0c} + \mu_{1c} = 1. \tag{14}
\]

For instance, \( \mu_{1A} \) measures the portion of the intermediate product lines where the firms of country \( A \) are leaders. The low of motions for these values are:

\[
\dot{\mu}_{1A} = \dot{\mu}_{-1B} = \mu_{0A} x_{0A} - \mu_{1A} x_{-1B} \]

\[
\dot{\mu}_{0A} = \dot{\mu}_{0B} = \mu_{-1A} x_{-1A} + \mu_{1A} x_{-1B} - \mu_{0A} [x_{0A} + x_{0B}] \tag{15}
\]

Note that it is sufficient to express the low of motion for \( \mu_{1A} \) and \( \mu_{0A} \) since \( \dot{\mu}_{-1A} = -\dot{\mu}_{1A} - \dot{\mu}_{0A} \). These flow equations have very simple intuitions. Country \( A \) will gain more leadership when its neck and neck firms in \( \mu_{0A} \) of the product lines innovate at the rate \( x_{0A} \). Similarly, country \( A \) will lose leadership when the follower firms in \( \mu_{1A} \) product lines innovate at the rate \( x_{-1B} \). In the second line, country \( A \) will become neck and neck with country \( B \) in \( \mu_{-1A} \) product lines when the follower in country \( A \) are successful or in \( \mu_{1A} \) product lines, the follower in country \( B \) is successful. On the other hand, country \( A \) will loose its neck and neck product lines if the firm in \( A \) or \( B \) is successful.

Note that these equilibrium innovation arrival rates are time-invariant. Equations (14) and (15) allow us to express the above system of differential equations in the following form

\[
\dot{y} = Ay + b
\]

where

\[
y \equiv \begin{bmatrix} \mu_{1A} \\ \mu_{0A} \end{bmatrix}, \quad A \equiv \begin{bmatrix} -x_{-1B} & x_{0A} \\ x_{-1B} - x_{-1A} & -(x_{-1A} + x_{0A} + x_{0B}) \end{bmatrix}, \quad \text{and} \quad b \equiv \begin{bmatrix} 0 \\ x_{-1A} \end{bmatrix}.
\]
Let $\mathbf{T}$ be the matrix whose columns are the eigenvectors of $\mathbf{A}$. Define a new variable $z$ such that $y = \mathbf{T}z$. Then,

$$\dot{z} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}z + \mathbf{T}^{-1}\mathbf{b} = \mathbf{D}z + \mathbf{h}$$

where $\mathbf{D}$ is the diagonal matrix with eigenvalues on the main diagonal. This means that the system of equations became one that is comprised of independent first order ODEs which we can solve separately. We provide the remaining details of the solution of this system of differential equations in the Appendix.

### 4.7 Consumption and Welfare

In this section, we characterize the evolution of consumption of households in country $c$. We use the budget constraint of the representative household in country $c$ to pin down its consumption level

$$C_c(t) = \rho A_c(t) + w_c(t) - \hat{A}_c(t). \tag{16}$$

We already found the value of the fixed factor price in (9) as $w_c(t) = \frac{\beta}{1-\beta}Q(t)$. Let us denote

$$Q_{1c}(t) = \int_0^1 1_{[j=1c]}q_j(t)\,dj, \quad Q_{0c}(t) = \int_0^1 1_{[j=0c]}q_j(t)\,dj \quad \text{and} \quad Q_{-1c}(t) = \int_0^1 1_{[j=-1c]}q_j(t)\,dj.$$

Determining consumption requires the determination of $A$ and $\hat{A}$. We turn to the asset market clearing condition to determine these values

$$A_c(t) = \int_0^1 V_{cj}(t)\,dj = v_{1c}Q_{1c}(t) + v_{0c}Q_{0c}(t) + v_{-1c}Q_{-1c}(t) \tag{17}$$

and

$$\hat{A}_c(t) = v_{1c}\hat{Q}_{1c}(t) + v_{0c}\hat{Q}_{0c}(t) + v_{-1c}\hat{Q}_{-1c}(t). \tag{18}$$

Thus, using (16), (17) and (18) we find the consumption in country $c$ as

$$C_c(t) = \begin{cases} \rho [v_{1c}Q_{1c}(t) + v_{0c}Q_{0c}(t) + v_{-1c}Q_{-1c}(t)] \\ +\beta/(1-\beta)Q(t) \\ - [v_{1c}\hat{Q}_{1c}(t) + v_{0c}\hat{Q}_{0c}(t) + v_{-1c}\hat{Q}_{-1c}(t)] \end{cases}, \tag{19}$$

the total income as

$$I_c(t) = \begin{cases} [(2\pi - \alpha_c x_{1c}^\gamma)Q_{1c}(t) - \alpha_c x_{0c}^\gamma Q_{0c}(t) - \alpha_c x_{-1c}^\gamma Q_{-1c}(t)] \\ +\beta/(1-\beta)Q(t) - \theta Re [Q_A(t) + cQ_{0c}(t) + Q_{-1c}(t)] \end{cases}, \tag{20}$$

and the total R&D expenditure as

$$RnD_c(t) = \alpha_c x_{1c}^\gamma Q_{1c}(t) + \alpha_c x_{0c}^\gamma Q_{0c}(t) + \alpha_c x_{-1c}^\gamma Q_{-1c}(t) \tag{21}$$

where

$$Q(t) = Q_{1A}(t) + Q_{0A}(t) + Q_{1B}(t) = Q_{1A}(t) + Q_{0A}(t) + \lambda Q_{-1A}.$$
Finally note that
\[
\begin{align*}
\dot{Q}_{1A}(t) &= x_{1A}(\lambda - 1)Q_{1A}(t) - x_{-1B}Q_{1A}(t) + x_{0A}\lambda Q_{0A}(t) \\
\dot{Q}_{0A}(t) &= (x_{-1A} + \delta)\lambda Q_{-1A}(t) + (x_{-1B} + \delta)Q_{1A}(t) - (x_{0B} + x_{0A})Q_{0A}(t) \\
\dot{Q}_{-1A}(t) &= x_{1B}(\lambda - 1)Q_{-1A}(t) + x_{0B}Q_{0A} - x_{-1A}Q_{-1A}(t)
\end{align*}
\]

In matrix notation, we can summarize this expression as follows:
\[
\begin{bmatrix}
\dot{Q}_{-1A}(t) \\
\dot{Q}_{0A}(t) \\
\dot{Q}_{1A}(t)
\end{bmatrix} =
\begin{bmatrix}
x_{1B}(\lambda - 1) - x_{-1A} & x_{0B} & 0 \\
\lambda(x_{-1A} + \delta) & -(x_{0B} + x_{0A}) & (x_{-1B} + \delta) \\
0 & \lambda x_{0A} & x_{1A}(\lambda - 1) - x_{-1B}
\end{bmatrix}
\begin{bmatrix}
Q_{-1A}(t) \\
Q_{0A}(t) \\
Q_{1A}(t)
\end{bmatrix}
\]

Proposition 1 The common growth rate of production for the two economies is 
\[
g^* = (\lambda - 1) \left[ \mu^*_A x^*_1 + \mu^*_{-1A} x^*_1 + x^*_0 \right]
\]
or more precisely\[
\frac{\dot{Q}}{Q} = (\lambda - 1) \frac{Q_{1A} x^*_1 + \lambda Q_{-1A} x^*_1 + Q_{0A}(x^*_0 + x^*_0)}{Q_{1A} + Q_{0A} + \lambda Q_{-1A}}\]

Proof. See appendix (straightforward from Acemoglu-Akcigit, 2012). \(\blacksquare\)

### 4.8 R&D Subsidy Policies

In this section, we describe the effect of R&D subsidies on both countries.\(^9\) We assume that country \(c\) provides a subsidy of \(\tau_c\) on firm’s R&D spending. It finances this amount through the lump sum tax imposed on household. Then the value functions in (10) – (12) are modified as
\[
\begin{align*}
rv_{1A} &= \max_{x_{1A}} \left\{ 2\pi - (1 - \tau_c) \left[ \alpha_A x^*_1 + Q_{1A} (x_{-1A} + \theta_{Rc}) \right] + x_{1A}v_{1A} (\lambda - 1) + x_{-1B} [v_{0A} - v_{1A}] \right\}, \\
rv_{-1A} &= \max_{x_{-1A}} \left\{ - (1 - \tau_c) \left[ \alpha_A x^*_1 + Q_{-1A} (x_{-1A} + \theta_{Rc}) \right] + x_{-1A} [v_{0A} - v_{-1A}] + x_{1B}v_{-1A} (\lambda - 1) \right\} \\
rv_{0A} &= \max_{x_{0A}} \left\{ - (1 - \tau_c) \left[ \alpha_A x^*_1 + \theta_{Rc} \right] + x_{0A} [v_{1A} - v_{0A}] + x_{0B} [v_{-1A} - v_{0A}] \right\}.
\end{align*}
\]

The resulting budget balance of the government implies
\[
T_A(t) = \tau_A \left\{ \alpha_A \left[ x^*_1 Q_{1A}(t) + x^*_0 Q_{0A}(t) + x^*_1 Q_{-1A}(t) \right] + \theta_{Rc} \right\}
\]
and the household budget in that case is expressed as
\[
\rho A_c(t) + w_c(t) - T_A(t) = C_c(t) + \dot{A}_c(t).
\]

\(^9\)When solving and simulating our model we use R&D subsidy rates, a measure which captures the changes in R&D tax credits but enters the value functions conveniently. The details about their calculation is provided in section 5.
Therefore the new consumption level is expressed as

\[
C_c(t) = \left\{ \begin{array}{l}
- T_A(t) \\
\rho [v_{1c} Q_{1c}(t) + v_{0c} Q_{0c}(t) + v_{-1c} Q_{-1c}(t)] \\
+ \beta / (1 - \beta) Q(t) \\
- \left[ v_{1c} Q_{1c}(t) + v_{0c} Q_{0c}(t) + v_{-1c} Q_{-1c}(t) \right] \end{array} \right\}.
\] (29)

The government subsidies will now affect consumption and welfare in several directions. There is a negative effect of R&D subsidies on consumption which comes from the fact that R&D subsidies have to be financed through taxation which in turn lowers the current consumption as it is indicated in the first line of (29). However, R&D subsidies generate two positive gains through innovation dynamics: First, an increase in the subsidy rates encourages more innovation and hence increases the speed of quality improvements. Second, and somewhat more interestingly, it also generates a compositional gain since county A starts to have more leadership in the world after the subsidy. In the following section, we will analyse these effects quantitatively.

5 Calibration

We now turn to the calibration procedure. First of all, the data counterparts of the two countries in our model will be US and and a foreign country which will be a weighted combination of the following 7 countries (unless the data are missing): Canada, France, Germany, Italy, Japan, Spain and UK. To weight the data for these countries we use the patents registries by these countries in the US in 1975. The weight of a particular country corresponds to the share of patents, whose owner belongs to this country, in the total foreign-based patent registries in the US. In the following analysis, country A will represent the US and country B the foreign country.10

We calibrate the model to a set of moments which we obtain from the data that spans over 1975-1985. The reason why we picked this range for calibration is the behavior of the sector ownership graph in Figure (4). Later on, using these estimates, we will test the performance of our model simulating it for the next 10 years and we will show that it is able to capture the changing pattern in Figure (4).

5.1 Data and Moments

We start with parameters calibrated externally. We set the interest rate, which in our model, is equal to \( \rho \), the time preference, due to the linear utility specification, to match the real monthly rate averaged over 1975-1985. We obtain this monthly rate by taking the difference of the federal funds rate and the year-over-year change in median CPI for which we used the data of Cleveland FED.

A crucial set of parameters is the R&D subsidy rates. The numbers we use are those calculated in Impullitti(2010) which only lack Canada (to take this into account, the weights used to organize this data are calculated again after dropping Canada). This data go back

10In the figures that compare the two countries in section 6, the former is presented by a blue line whereas the latter by a red one. The legend uses \( FN \) for the competitors, if applicable.
Table 3: Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Internally Calibrated</th>
<th>Externally Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_A$</td>
<td>54.03</td>
<td>$r$</td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>109.23</td>
<td>$r_A^{75}$</td>
</tr>
<tr>
<td>$\gamma_A$</td>
<td>1.95</td>
<td>$r_B^{75}$</td>
</tr>
<tr>
<td>$\gamma_B$</td>
<td>2.286</td>
<td>$\tau_A^{86}$</td>
</tr>
<tr>
<td>$\theta_A$</td>
<td>0.477·10^{-3}</td>
<td>$\tau_B^{86}$</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>2.136·10^{-3}</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.162</td>
<td>2.25%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.932</td>
<td>17.6%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.099</td>
<td>14.0%</td>
</tr>
<tr>
<td>$\bar{q}_{1A}$</td>
<td>0.001</td>
<td>26.3%</td>
</tr>
<tr>
<td>$\bar{q}_{-1A}$</td>
<td>0.182</td>
<td>14.7%</td>
</tr>
</tbody>
</table>

until 1979. Given that the rates do not fluctuate much for the countries in the sample before mid 80s (except the US for which there is a rise after 1981), we take the numbers before 1979 be the same with the one in 1979. For the calibration part, the subsidy rates for both countries are 1975-1985 averages which is again weighted for the foreign countries. When we simulate the model for the next ten years, we will recalculate the subsidy rates to match the averages across 1986-1995. Doing these, we also recalculate the weights of foreign countries the same way but using 1986 patent counts. The right panel of Table (3) summarizes these estimates.

The rest of the parameters which are shown in Table (3) we will calibrate internally. At the current stage, we use 9 moments for this purpose. The first four are coming from the patent data and reflect the pattern observed in Figure (4). We use 1975 values of the share of sectors dominated by the US and the share of the sectors where the two countries are in a neck&neck position in terms of patent counts as the initial conditions to pin down the constant terms of the differential equations which represent the evolution of these variables. The corresponding values in 1985 provide the targets which these differential equations reach in 10 periods.

The next two moments are the average growth rates of the real GDP per capita in both countries. In our model, these correspond to the income growth rates of the countries. We obtain the data for real GDP per capita from Worldbank database. The get the numbers for the other two moments, R&D intensities, we utilized Main Science and Technology Indicators (MSTI) database of OECD. There are two minor issues with this data set. First, we use the non-defense R&D intensity numbers and these miss for Japan. However, Science and Engineering Indicators reports of NSF, based on MSTI data, provide estimates of this variable for Japan which we use to amend our calculations with the OECD data. The second problem is that MSTI starts in 1981. That is why for this variable we use averages across 1981-1985.\footnote{Another very minor point is that the data is missing for UK in 1982 and 1984. These are generated by linear interpolation using the numbers of the previous and next years.}

The last target we are trying to match is the initial GDP per capita ratios of these countries in 1975.

5.2 Moment Matching

- Based on the moments we have, the estimation procedure proceeds as follows:

  - Solve for the value functions using a system of 6 nonlinear equations and 6 unknowns. Here, the unknowns are $v_{mc}, c \in \{A,B\}, m \in \{-1,0,1\}$. Using the resulting values, calculate $x_{mc}$, the optimal R&D decisions, using the first order conditions of the firm decision problem.
Table 4: Calibration Results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Estimate</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{1A}$ in 1985</td>
<td>0.5491</td>
<td>0.5491</td>
<td>0</td>
</tr>
<tr>
<td>$\mu_{0A}$ in 1985</td>
<td>0.2118</td>
<td>0.2103</td>
<td>-0.0015</td>
</tr>
<tr>
<td>GDP. Growth US</td>
<td>2.162%</td>
<td>1.432%</td>
<td>-0.73%</td>
</tr>
<tr>
<td>GDP. Growth Foreign</td>
<td>2.348%</td>
<td>1.698%</td>
<td>-0.65%</td>
</tr>
<tr>
<td>R&amp;D Intensity US</td>
<td>1.857%</td>
<td>1.857%</td>
<td>0</td>
</tr>
<tr>
<td>R&amp;D Intensity Foreign</td>
<td>2.065%</td>
<td>2.655%</td>
<td>0.59%</td>
</tr>
<tr>
<td>GDP Ratio 1975</td>
<td>21%</td>
<td>4.88%</td>
<td></td>
</tr>
</tbody>
</table>

- Create the transition matrices and calculate the differential equations describing the evolution of $\mu_{1A}$ and $\mu_{0A}$. Get the constants using the initial conditions.

- Now we calculate the initial average quality levels, $Q$, of both countries. To this end, we use the following trick. Let us define $\bar{q}_{mc}$ such that $Q_{mc} = \mu_{mc}\bar{q}_{mc}$. Initially, we normalize $\bar{q}_{mB} = 1 \forall m$. Due to the definition of being in neck&neck position, we have $\bar{q}_{0A} = 1$. Then, given $\bar{q}_{1A}$ and $\bar{q}_{-1A}$, we calculate the initial levels of $Q_{mA}$. Note again that this rewriting of $Q$, we only use for the initial levels. Given the initial levels, we can create the transition matrix $\mathbf{Tr}_A$ for $Q_A$ as in (25). Once we have $Q_A$ for every $t$, it is straightforward to calculate $Q_B$ and $Q_w$, worldwide average quality.

- Given the average quality levels, we calculate consumption, income and R&D expenditures using equations (19), (20), and (21). Then, we obtain income growth rates, average R&D intensities and the initial consumption ratio.

- Check the differences from the moments and iterate.

The preliminary results are summarized in Table (4). What we can say at this stage is that the model captures the evolution of share of sectors in which US has the leadership or is in a neck&neck situation in terms of the relative patent counts pretty well. It also does a good job with respect to relative ranking of countries in output growth rate and R&D intensity measures. However, its estimates fall short of the growth rate when it comes to absolute values. Moreover, R&D intensity in the foreign countries overshoot. At last the performance in matching the initial output ratio is fairly inadequate.

6 Results

In this section, we present the resulting patterns of this economy. We start with the evolution of “leadership” and “neck&neck” shares of the US. Figure (7) shows the actual patterns in bars and the estimated one with a line. As noted above, the model matches the shares in the end.
period 10, which translates to 1985 in the data, very well. The success is not just about the end points but also the differential equation generating this figure also captures the catching-up process. In short, we will show that this performance remains solid when we simulate the next 10 years.

The next two figures show the resulting behavior of consumption and profits accruing to each country. For the 1975-1985 period, the relevant parts of the graphs are to the left of the dotted black line. Consistent with the evolution of the shares, profits generated by the firms of each country that are leaders in their product lines converge among the countries. Figure (6) describes this shift. The repercussion on the consumption pattern is that the consumption per capita in the foreign country also catches up with the one in the US while they both continue to increase, as can be seen in Figure (6). Both graphs extend 10 more years to the right of the black line as if there was no changes in R&D tax credit policies in any of the relevant countries just to make the patterns clearer.

6.1 Validation Test

Before checking the predictions of the model in terms of consumption for the period following the changes in the R&D tax credit policies we want to assess how well this model captures the changing patterns in the international technology competition. Figure (9) gives a clear picture of its success. It is able to reflect the downturn in the catching-up process and the rise of the US in the international arena of innovative activity thanks to the help of increasing subsidies. Thus, this figure assures us about the strength of the model for further analysis.

6.2 R&D Tax Credits of the 80s

That the US regains its power in the international technology competition reflects itself in the shares of sectors in which the firms operating in the US seizes the “leadership”. This results in an increase in the share of profits flowing to the US as Figure (6.2) exhibits. A comparison of the path after period 10 to the counterpart in Figure (6) how a possible further loss of profits
In consumption terms, the shift in profits leads to the halt of convergence. Of course, at the time of an increase in the subsidy rates, consumption drops due to the rise in taxes collected from households to subvent R&D. However, as the effect of the policy takes place, US consumption policy recovers fast as depicted in Figure (6.2).

6.3 Welfare

Now, we want to run two experiments regarding the consumption patterns of the US economy based on our estimates. First, we will compare the actual pattern to the hypothetical case of no change in R&D subsidies in any country. Then, we compare the hypothetical economy in the previous case to another one in which the US economy is taken to be a closed one.

12 The parts of the figures to the left of the black line show the initial 10 years and is thus the same as in the previous counterpart of these figures.
Let us start with the calculation of welfare in consumption equivalent terms. First rewrite the consumption of country $c$ in terms of $Q_c$ which was defined earlier:

$$C_c(t) = \begin{cases} -T_A(t) \\ \rho \left[ v_{1c}Q_{1c}(t) + v_{0c}Q_{0c}(t) + v_{-1c}Q_{-1c}(t) \right] \\ + \beta/ (1 - \beta) Q(t) \\ - \left[ v_{1c}Q_{1c}(t) + v_{0c}Q_{0c}(t) + v_{-1c}Q_{-1c}(t) \right] \end{cases}$$

$$= \begin{cases} -\tau_A \alpha_A \left[ x_{-1A}^{2A}Q_{-1A}(t) + x_{0A}^{2A}Q_{0A}(t) + x_{1A}^{2A}Q_{1A}(t) \right] \\ \rho \left[ v_{-1c}Q_{-1c}(t) + v_{0c}Q_{0c}(t) + v_{1c}Q_{1c}(t) \right] \\ + \beta/ (1 - \beta) \left[ \lambda Q_{-1A}(t) + Q_{0A}(t) + Q_{1A}(t) \right] \\ - \left[ v_{-1c}Q_{-1c}(t) + v_{0c}Q_{0c}(t) + v_{1c}Q_{1c}(t) \right] \end{cases}$$

$$= \begin{cases} -\tau_A \alpha_A \left[ x_{-1A}^{2A} + x_{0A}^{2A} + x_{1A}^{2A} \right] + \rho \left[ v_{-1c}Q_{-1c}(t) + v_{0c}Q_{0c}(t) + v_{1c}Q_{1c}(t) \right] \\ + \beta/ (1 - \beta) \left[ \lambda 1 1 \right] - \left[ v_{-1c}Q_{-1c}(t) + v_{0c}Q_{0c}(t) + v_{1c}Q_{1c}(t) \right] \end{cases} Q_c(t)$$

$$= \Lambda Q_c(t)$$

where $\Lambda$ is a constant raw vector.

The expression above allows us to compare the welfare levels in two different cases: the counterfactual case that there is no change in subsidy rates as in Figure (6) and the actual case as in Figure (6.2). The consumption equivalent welfare change in 25 years, which we denote by $\delta_{25}$, is obtained as follows:

$$(1 + \delta_{25}) \int_0^{25} e^{-rt} C^{ns}(t) dt = \int_0^{25} e^{-rt} C^{ys}(t) dt$$

where $ns$ denotes the no subsidy case and $ys$ the actual case of changing subsidies.

---

13The number 25 is selected to reflect the period 1985-2010, i.e. the period between the policy shift in our model and present time.
6.3.1 Closed Economy Case

In this hypothetical situation, we assume all the estimated parameters to be true for the US but it is alone in producing all the intermediate goods which it needs for the final good production. When we start to simulate it we suppose that the country has exactly the same average quality \( Q_{-1A} + Q_{0A} + Q_{1A} \) as in 1985, i.e. right before we move subsidy rates in the actual situation. We keep the tax rate in the base level as in 1975. Moreover, there is no change in any calculation related to the final good side.

Intermediate goods side changes because now there is no foreign competition in the production lines, i.e. per line, there exists one single domestic form. The incentive for this form to innovate is just to raise the profits. There is no threat of losing the product line thus no incentive to protect it. Thus, the value function of the form becomes

\[
rv_A = \max_{x_A} \left\{ \pi - \theta_R x_A - \alpha_A x_A^{\gamma_A} + x_A v_A (\lambda - 1) \right\}
\]

where we used small latters for the value following the intuition of Lemma (1). Note that now we have \( \pi \) instead of \( 2\pi \) as the world economy consists of just one country. The resulting optimal R&D level is as follows:

\[
x_A = \left( \frac{(\lambda - 1) v_{1A}}{\gamma_A \alpha_A} \right)^{\frac{1}{\lambda - 1}}.
\]

Putting this value back into the value function allows us to solve for optimal R&D:

\[
rv_A = \frac{r}{(\lambda - 1) \alpha_A \gamma_A x_A^{\gamma_A - 1}} = \pi - \theta_R + \alpha_A (\gamma_A - 1) x_A^{\gamma_A}.
\]

The evolution of the average product quality depends just on domestic innovation which happens with Poisson arrival rate \( x_A \). Thus we have the usual expression for the growth rate:

\[
g_Q = g_Y = \frac{Q}{Q} = (\lambda - 1) x_A.
\]

With these values in hand, we can calculate consumption in terms of \( Q_A \) following the same steps as before.

For our two hypothetical experiments, the consumption equivalent comparisons yield the results presented in Table (5) where “Benchmark” refers to the simulation of the US economy as an open one, but without any subsidy changes neither home or abroad. The first row reveals the importance of the response of the US using the R&D tax credits and its resulting restrengthening in international technological race. The second line reflects the importance of trade in intermediate goods, thus ideas and inventions. Even though we assumed that the US does not respond as its worsening situation prevail this situation is still preferred over the closed case. This implies that the positive effects of the foreign invention on the domestic quality dominate the adverse business stealing effect which is nonexistent in the closed economy case.

6.4 Growth Decomposition

In this part of the analysis, we want to break the effect of R&D tax credit on the income growth of the US into its two components: quality improvement and business stealing effects.
Table 5: Consumption Equivalent Welfare Analysis. Changes in US Consumption

<table>
<thead>
<tr>
<th>Experiment</th>
<th>New Case</th>
<th>Old Case</th>
<th>Gain 1985-2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Both Countries move</td>
<td>Benchmark</td>
<td>3.68%</td>
</tr>
<tr>
<td>2</td>
<td>Benchmark</td>
<td>Closed</td>
<td>15.29%</td>
</tr>
</tbody>
</table>

The former refers to the resulting effect of the policy change that makes US firms innovate more and increase their qualities more frequently in isolation from the changes in product line ownership. One example of such an innovation would be one that was done by a leading US firm. The second effect refers to the incremental seizure rate of leadership by the US firms in product lines where the operation is in the hands of the other country. In this way, US firms captures profits of more and more product lines.

Before presenting the decomposition results, we want to examine the processes underlying these two effects. Business stealing effect is generated by the movements of $\mu_{mA}$, the share of sectors where US firms are in position $m \in \{-1, 0, 1\}$. To observe the quality improvements in isolation of the changes in the competition environment, we will divide $Q_{mA}$ into its parts as we did in the algorithm presented above to obtain the underlying average quality variable $\bar{q}_{mA}$.

The growth rates of these processes are presented in Table 6. Two situations are compared: actual and benchmark over the period 1985-1995. The first one represents the world in which we have policy change as in the real world. The latter is the hypothetical case in which, given the values are in 1985 values, the economy evolves under the assumption of no policy shift in the next 10 years, just as in the same setting of the initial 10 years. We look at the upper panel which shows the values for the US. In the measure of quality, we observe that the growth in neck&neck and leader quality is higher in the actual case. A similar comparison tells that leadership share of the US obtains a positive growth rate instead of the negative one of the benchmark case. With patterns similar to our expectations, these values indicate an acceleration in both components of growth. The natural question, then, is: What are their relative contributions?

Table 6: Growth Rates, 1985-1995

<table>
<thead>
<tr>
<th></th>
<th>$\bar{q}_c$ Actual</th>
<th>Benchmark</th>
<th>$\mu_c$ Actual</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\bar{q}_{-1A}$</td>
<td>0.36%</td>
<td>0.57%</td>
<td>$\mu_{-1A}$</td>
</tr>
<tr>
<td></td>
<td>$\bar{q}_{0A}$</td>
<td>1.34%</td>
<td>0.66%</td>
<td>$\mu_{0A}$</td>
</tr>
<tr>
<td></td>
<td>$\bar{q}_{1A}$</td>
<td>2.55%</td>
<td>1.42%</td>
<td>$\mu_{1A}$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\bar{q}_{-1B}$</td>
<td>2.55%</td>
<td>1.42%</td>
<td>$\mu_{-1B}$</td>
</tr>
<tr>
<td></td>
<td>$\bar{q}_{0B}$</td>
<td>1.34%</td>
<td>0.66%</td>
<td>$\mu_{0B}$</td>
</tr>
<tr>
<td></td>
<td>$\bar{q}_{1B}$</td>
<td>0.36%</td>
<td>0.57%</td>
<td>$\mu_{1B}$</td>
</tr>
</tbody>
</table>

Table 7 addresses this question. For the same period as above we compare the previous two cases and an additional one, which we call counterfactual. We will use this case to isolate the quality source of growth from the additional business stealing effect due to movements in

---

14Note that the numbers for the competitors in the lower panel are symmetric to the ones above. This completely makes sense as the qualities and shares are the same when countries are in neck&neck position whereas $m = -1$ for one country indicates the case $m = 1$ for the other in a given product line.
Thus, in this case we calculate $Q$ levels using $\bar{q}$ levels of the actual case but $\mu$ values of the benchmark case. Comparing the incremental rate in growth in this case to the benchmark, we calculate what ratio of the difference in growth rates between actual and benchmark setting this value constitutes. The result is the share of quality improvement whereas the source of the rest is business stealing. Table 7 reveals that a disproportionately large share of growth, 81%, comes from the latter effect for the US, whereas the same source impacts the income growth of the competing country very negatively.

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Benchmark</th>
<th>Counterfactual</th>
<th>Business Stealing</th>
<th>Quality Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>1.43%</td>
<td>1.17%</td>
<td>1.22%</td>
<td>81%</td>
<td>19%</td>
</tr>
<tr>
<td>$g_B$</td>
<td>1.76%</td>
<td>1.31%</td>
<td>2.00%</td>
<td>-53%</td>
<td>153%</td>
</tr>
</tbody>
</table>

7 Conclusion

In this paper we analyzed the international technology competition between the United Stated and the frontier countries in the world. Our analysis focused particularly on an interesting historical episode in which the United States was initially being caught up and losing its leadership position in many technology fields. Between 1965 and 1985, its leadership rate decreased from roughly 79% down to 54%. However, mid-80s featured very aggressive R&D policies by the US which then helped the country rebuild its leadership in most of the fields. Our quantitative analysis measured the welfare gains from the R&D policies of the 80s. These policies not only increased the speed of quality improvements for consumers, but also generated business stealing which then additionally contributed to the income growth in the US. Our analysis suggested that this gain could be up to 3.68% in 25 years following the policy shift in terms of the consumption equivalence relative to the case without the policy change. Finally, we also studied the gains from openness. As a novel feature, in our model openness generated two features: (i) market size effect and (ii) escape competition effect. These two effects make the innovators in an open economy invest much more heavily than in a closed economy. As a result, we found that, for the same period as in the previous comparison, the US economy grew 1.2% while being open, whereas this number would have been 0.6% had the economy been a closed economy.

Our future work includes the study of optimal R&D policy in which the government makes the subsidy a function of the level of the industry. In particular, the next question we wish to ask is: Should the R&D subsidy be geared towards more advanced or less advanced industries? We hope that this analysis will shed additional light on the discussion on the optimal innovation policy in open economies.
APPENDIX

A  Additional Empirical Material

This section presents the counterpart of regression analysis in subsection 3 which also includes federal analysis. The results are shown in Table 8. In all specifications except the last one, federal credits have positive and significant coefficients as expected. The results are qualitatively the same with the exception of last regression.

Table 8: The Effect of R&D Tax Credit on Innovation (incl. Federal Credits)

<table>
<thead>
<tr>
<th>Dep. Var.:</th>
<th>ln(R&amp;D&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>ln(R&amp;D&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>ln(Patents&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>ln(Patents&lt;sub&gt;t&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(R&amp;D&lt;sub&gt;t-1&lt;/sub&gt;)</td>
<td>- 0.641</td>
<td>- (113.16)***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ln(Patent&lt;sub&gt;t-1&lt;/sub&gt;)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.559 (77.22)***</td>
</tr>
<tr>
<td>ln(State credit&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>7.555 (28.72)***</td>
<td>0.731 (3.26)**</td>
<td>4.255 (16.74)***</td>
<td>0.148 (0.55)</td>
</tr>
<tr>
<td>ln(Federal credit&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>3.940 (28.26)***</td>
<td>1.930 (16.61)***</td>
<td>0.563 (4.18)***</td>
<td>-0.341 (-2.41)***</td>
</tr>
<tr>
<td>Year Dummy</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Firm Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

B  Derivations

B.1  Lemma

Proof of Lemma 1. Optimal R&D decisions based on the values Lemma 1 suggests are given in equation (13) from which we can back out \( v_{cm} \). Since neither of these values depend on any variable which move with time, \( \dot{v}_{cm} = 0 \), thus \( \dot{V}_{cm} = 0 \). Moreover, multiplying all the equations given in the lemma give the original value functions with \( \dot{V} \) set to zero. Therefore, we conclude \( V_{cm} = qv_{cm} \). ■

B.2  Law of Motions

Derivations for Equations (22) – (24). Let us also denote

\[ \bar{q}_{1c}(t) \equiv \frac{Q_{1c}(t)}{\mu_{1c}}, \quad \bar{q}_{0c}(t) \equiv \frac{Q_{0c}(t)}{\mu_{0c}} \quad \text{and} \quad \bar{q}_{-1c}(t) \equiv \frac{Q_{-1c}(t)}{\mu_{-1c}}. \]
Note that
\[ Q_{1c}(t + \Delta t) = \int_0^1 \left[ 1_{[j=1c]} \left( \frac{\lambda q_j(t)}{\lambda x_{1c}\Delta t} \right) x_{1c} \Delta t + 1_{[j=0c]} x_{0c} \Delta t \lambda \tilde{q}_{0c}(t) \right] dj \]
\[ = \left[ \lambda x_{1c} \Delta t \int_0^1 1_{[j=1c]} q_j(t) dj + (1 - x_{1c} \Delta t - x_{1c} \Delta t - \delta \Delta t) \int_0^1 1_{[j=1c]} q_j(t) dj \right] \]
\[ = \left[ \lambda x_{1c} \Delta t Q_{1c}(t) + (1 - x_{1c} \Delta t - x_{1c} \Delta t - \delta \Delta t) Q_{1c}(t) \right] \]
\[ \frac{Q_{1c}(t + \Delta t) - Q_{1c}(t)}{\Delta t} = \lambda x_{1c} Q_{1c}(t) - (x_{1c} + x_{1c} + \delta) Q_{1c}(t) + x_{0c} \lambda \tilde{Q}_{0c}(t) \]
\[ \dot{Q}_{1c}(t) = (\lambda - 1) x_{1c} Q_{1c}(t) - (x_{1c} + \delta) Q_{1c}(t) + x_{0c} \lambda \tilde{Q}_{0c}(t) \]

And
\[ Q_{0c}(t + \Delta t) = \int_0^1 \left[ 1_{[j=0c]} \left( 1 - x_{0c} \Delta t - x_{0c} \Delta t \right) q_j(t) \right] dj \]
\[ = \left[ (1 - x_{0c} \Delta t - x_{0c} \Delta t) \int_0^1 1_{[j=0c]} q_j(t) dj + (x_{-1c} + \delta) t_{1c} \left( t \right) \int_0^1 1_{[j=1c]} dj \right] \]
\[ = \left[ (1 - x_{0c} \Delta t - x_{0c} \Delta t) Q_{0c}(t) + \left( x_{-1c} + \delta \right) t_{1c} \left( t \right) \right] \]
\[ \frac{Q_{0c}(t + \Delta t) - Q_{0c}(t)}{\Delta t} = \left( x_{-1c} + \delta \right) \lambda Q_{-1c}(t) + \left( x_{-1c} + \delta \right) Q_{1c}(t) - \left( x_{0c} + x_{0c} \right) Q_{0c}(t) \]
\[ \dot{Q}_{0c}(t) = \left( x_{-1c} + \delta \right) \lambda Q_{-1c}(t) + \left( x_{-1c} + \delta \right) Q_{1c}(t) - \left( x_{0c} + x_{0c} \right) Q_{0c}(t) \]

and
\[ Q_{-1c}(t + \Delta t) = \int_0^1 \left[ 1_{[j=-1c]} \left( 1 - x_{-1c} + \delta \right) t_{1c} \left( t \right) \right] dj \]
\[ = \left[ \int_0^1 1_{[j=-1c]} \left( 1 - x_{-1c} + \delta \right) t_{1c} \left( t \right) dj + \int_0^1 1_{[j=-1c]} x_{1c} \Delta t \lambda \tilde{q}_{0c}(t) \right] \]
\[ = \left[ (1 - x_{-1c} + \delta) t_{1c} \left( t \right) \right] \]
\[ \frac{Q_{-1c}(t + \Delta t) - Q_{-1c}(t)}{\Delta t} = x_{1c} \left( \lambda - 1 \right) Q_{-1c}(t) + x_{0c} \left( x_{0c} \right) Q_{0c} - \left( x_{-1c} + \delta \right) Q_{-1c}(t) \]
\[ \dot{Q}_{-1c}(t) = x_{1c} \left( \lambda - 1 \right) Q_{-1c}(t) + x_{0c} \left( x_{0c} \right) Q_{0c} - \left( x_{-1c} + \delta \right) Q_{-1c}(t) \]
Differential Equation.

\[ \dot{z} = Dz + h \]
\[ = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} z + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \]

Then

\[ \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} c_1 \exp(d_1 t) - h_1/d_1 \\ c_2 \exp(d_2 t) - h_2/d_2 \end{bmatrix} \]
\[ = \begin{bmatrix} c_1 \exp(d_1 t) \\ c_2 \exp(d_2 t) \end{bmatrix} - \begin{bmatrix} h_1/d_1 \\ h_2/d_2 \end{bmatrix} \]
\[ = \begin{bmatrix} c_1 \exp(d_1 t) \\ c_2 \exp(d_2 t) \end{bmatrix} - D^{-1}h. \]

To find the constants, we use \( z(t = 0) = T^{-1}y(t = 0) \), the initial distribution in 1975:

\[ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} z_1(t = 0) \\ z_2(t = 0) \end{bmatrix} + D^{-1}h. \]

Now, with all these values, we can calculate \( y(t = 10) = Tz(t = 10) \), the model estimate for the distribution in 1985 and compare it to the one obtained in the data.

C Algorithm

Country A represents US case.

Given

- R&D intensities
  Income growth rates
  \( \mu_{1A,1985} \) and \( \mu_{0A,1985} \) (also 1975 values)
  interest rates, profits (and maybe other stuff like taxes)

guess \( \{\gamma_A, \gamma_B, \alpha_A, \alpha_B, \lambda, q_{1A}, q_{-1A}\} \). This starts the outer loop. Inside this loop:

1. Calculate optimal R&D values using an inner loop.

   - Guess 6 unknowns: \( \{v_{1A}, v_{1B}, v_{0A}, v_{0B}, v_{-1A}, v_{-1B}\} \).
   - Calculate \( \{x_{1A}, x_{1B}, x_{0A}, x_{0B}, x_{-1A}, x_{-1B}\} \) implied by FOC (13).
   - Using these, go back to the 6 value functions. Try to minimize the difference between left and right hand sides.
   - Once this inner loop converges, calculate the final set of R&D decisions, \( \{x_{1A}, x_{1B}, x_{0A}, x_{0B}, x_{-1A}, x_{-1B}\} \).
2. Solve for \( \{\mu_1, \mu_0\} \) in 10 periods (1975-1985) as explained in Transitional Dynamics section.

3. In this step, I find \( Q_A \) which is defined in Consumption and Welfare section for 10 periods to calculate R&D intensities and productivity growth rates.

   - Take the guesses \( \{\tilde{q}_1, \tilde{q}_0\} \) with \( \tilde{q}_0 = 1 \).

\[
Q_A(t = 0) = \begin{bmatrix}
Q_{-1A}(t) \\
Q_0A(t) \\
Q_1A(t)
\end{bmatrix} = \begin{bmatrix}
\tilde{q}_{-1A} \mu_{-1A} \\
q_{0A} \mu_{0A} \\
q_{1A} \mu_{1A}
\end{bmatrix}
\]

   - Starting with this, calculate \( Q_A \) using the transition equation (25) and

\[
Q_A(t + 1) = Q_A(t) + \dot{Q}_A(t).
\]

   - Once \( Q_A \) is obtained for each period, we have

\[
Q_B = \begin{bmatrix}
Q_{-1B} \\
Q_0B \\
Q_1B
\end{bmatrix} = \begin{bmatrix}
Q_{1A} / \lambda \\
Q_0A(t) \\
Q_{-1A} \lambda
\end{bmatrix}
\]

and for the world quality:

\[
Q(t) = Q_{1A}(t) + Q_0A(t) + Q_{1B}(t) = Q_{1A}(t) + Q_0A(t) + \lambda Q_{-1A}.
\]

   - Income vectors of countries (horizontal vectors):

\[
I_c = \{ [0 \ 0 \ 1] \pi - \alpha_c [ x_{-1c}^\gamma \ x_0c^\gamma \ x_1c^\gamma ] \} Q_c + \beta / (1 - \beta) Q
\]

   - Growth rates \( g_{-c} \) are year-over-year percentage changes.

4. Now, I calculate intensities (the division of vectors are element-by-element):

\[
R&D_c = \frac{1}{10} \sum_{t=1}^{10} \frac{\alpha_c [ x_{-1c}^\gamma \ x_0c^\gamma \ x_1c^\gamma ]}{I_c} Q_c
\]

5. Compare values found in steps 2, 3 and 4 to the original ones. Check for bounds (assumption 1, etc.). Update the guesses for the outer loop.