Offshoring and Sequential Production Chains: A General-Equilibrium Analysis

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Abstract

In this paper, we develop a two-sector general equilibrium trade model which includes offshoring, sequential production and endogenous market structures. We analyze how relative factor endowments and various forms of globalization and technological change affect incomes, production patterns and market structures. We show that, against common belief, a reduction in trade costs may lower the range of tasks offshored even though the total volume of offshoring increases. Also, we show that more fluctuation in offshoring production costs reduces firms' average number of production stages, which might explain recent empirical observations.

Keywords: Offshoring, sequential production, global production chain, task trade.

JEL Classification: D24, F10, F23, L23.

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1 Introduction

Over the past years a lot of attention has been devoted to the determinants and consequences of the "second unbundling" in international trade (Baldwin 2006) – i.e., the international fragmentation of production.[1] Most analyses of this phenomenon are based on a specific idea of the production process, according to which individual tasks can be ordered with respect to the cost advantage of performing them abroad (including additional monitoring and communication costs resulting from foreign production), and the decision to offshore a certain task depends on the production and offshoring costs for that particular task. Some recent contributions, however, explicitly consider the fact that in many cases the production process is *sequential* – i.e., the individual steps follow a predetermined sequence that cannot be modified at will.[2] If such a sequence is combined with transport costs of shifting intermediate goods across borders, firms may find it disadvantageous to offshore certain steps even if they could be performed at much lower costs abroad. The reason is that the domestic country may have a cost advantage with respect to adjacent steps, and the costs of shifting back and forth intermediate goods may more than eat up potential cost savings from fragmenting the production process. As Harms et al. (2012) show, this constellation has important implications for observed offshoring patterns: first, it may explain why – despite large international discrepancies in factor prices – the actual extent of offshoring is still relatively low. Second, it suggests that minor changes in relative costs and/or transport costs may result in major changes in the offshoring activities: firms that used to perform most of their activities in the home country may suddenly decide to shift a large share of their production process abroad. Finally, it implies that the impact of reduced transport costs may be non-monotonic, by first increasing, and then reducing the volume of offshoring.

To drive home these points, Harms et al. (2012) rely on a simple partial equilibrium model, in which factor prices are exogenous. While such a framework is useful to describe

[1] A non-exhaustive list of important contributions to this literature includes Dixit and Grossman (1982), Jones and Kierzkowski (1990), Feenstra and Hanson (1996), Kohler (2004), and Grossman and Rossi-Hansberg (2008).

[2] See Baldwin and Venables (2010), Harms et al. (2012), Costinot et al. (2012a, b), Antras and Chor (2012), or Kim and Shin (2012).

individual firms' behavior, it does not lend itself to drawing conclusions about the entire economy. To arrive at such conclusions, we need to account for the influence of firms' offshoring decisions on factor prices at home and abroad, and we have to consider the repercussions of induced factor price changes on firms' optimal behavior – i.e., we need to model offshoring in a general-equilibrium framework. This is what the current paper does.

We present a two-country (North-South) model in which low-tech firms whose production is entirely domestic coexist with high-tech firms who decide on the international distribution of production steps. Production is based on a rigid sequence of individual steps, and the foreign cost advantage evolves in a non-monotonic fashion along the value-added chain: some steps are cheaper to perform in the South, some are cheaper to perform in the North, and so on. Finally, every step requires the presence of the unfinished intermediate good, and shifting the intermediate goods across borders is associated with transport costs. Wages and prices in both economies are endogenous, and the increasing demand for labor that is generated by accelerating North-South offshoring may eventually result in wage increases that make offshoring less attractive. Using this framework, we explore how changes in transport costs, relative productivities and properties of the production process affect the number of firms that choose not to offshore any step, the number of firms that decide to fragment their production process between North and South, and the number of firms that relocate (almost) the entire production process to the foreign country.

2 Related Literature

The idea that the "sequentiality" of many real-world production processes has to be taken into account has recently attracted considerable attention.[3]

Baldwin and Venables (2011) model firms' offshoring decisions for different configurations of production processes – "snakes" and "spiders" – with *spiders* reflecting a situation in which different intermediate inputs may be simultaneously produced in different countries to be eventually assembled at a central location, and *snakes* capturing the notion

[3] Important earlier contributions that spell out this idea are Yi (2003) and Barba Navaretti and Venables (2004). Fally (2012) provides an empirical characterization of production processes.

of *sequentiality* outlined in the introduction. Using these alternative frameworks, they analyze the consequences of exogenous variations in production costs and offshoring costs, which they interpret as the costs of shipping intermediate goods across borders. Highlighting "the tension between factor costs and co-location" (Baldwin and Venables 2012:31) that is also characteristic for our setup, they show that a decrease in shipping costs may result in an *overshooting* of the overall volume of offshoring, i.e., production stages may first be shifted abroad to take advantage of co-location and then return to the domestic country as shipping costs further decrease. However, the analysis of Baldwin and Venables (2010) takes production costs in the countries involved as given and thus neither captures the endogenous adjustment of wages nor the potential distributional effects of offshoring. Our model may thus be interpreted as general-equilibrium extension of a particular version of a Baldwin/Venables-type "snake".

In another related approach, Costinot et al. (2012a, b) use a general-equilibrium model with multiple countries to analyze how a countries' productivity – as reflected by its propensity to commit mistakes – determines the stages of production it attracts. These authors also emphasize the concept of *sequentiality*, i.e., the idea that the order in which tasks have to be performed is exogenously determined. Moreover, the transport costs in our model play a role that is somewhat similar to the "coordination costs" in Costinot et al. (2012a:20), with a decrease in these costs inducing a non-monontonic adjustment of the overall volume of offshoring. However, we deviate from the monotonicity assumption in Costinot et al. (2012a, b), which stipulates that countries can be ordered by their relative productivities. Conversely, we model a two-country world in which the relative cost advantage of the North fluctuates non-monotonically along the production chain. This enables us to analyze how specific properties of industry-specific production processes affect the volume of offshoring and relative factor wages across countries.

3 The Model

3.1 Preferences

There are two countries (or regions), North and South, with an asterisk denoting South-specific variables. Consumers in both countries have Cobb-Douglas preferences over two consumption goods, X and Y. The X sector produces a continuum of differentiated varieties, whereas goods from the Y sector are homogeneous. Household preferences are

$$Con = X^{\beta} Y^{1-\beta}, \qquad 0 < \beta < 1 \, , \quad \text{and} \tag{1}$$

$$X = \left[\int_{i \in N} x(i)^{\rho_X} di \right]^{\frac{1}{\rho_X}} , \qquad 0 < \rho_X < 1 \, , \tag{2}$$

where i indexes individual varieties and $\sigma_X = 1/(1-\rho_X)$ is the elasticity of substitution between these varieties. Maximizing utility for a given income level (Inc) yields the following demand system:

$$x(i) = \left(\frac{P_X}{p(i)} \right)^{\sigma_X} X \, , \tag{3}$$

$$P_X = \left[\int_{i \in N} p(i)^{1-\sigma_X} di \right]^{\frac{1}{1-\sigma_X}} , \tag{4}$$

$$P_X X = \beta Inc \, , \quad \text{and} \quad p_Y Y = (1-\beta) Inc \, . \tag{5}$$

3.2 Technologies

Each country is endowed with fixed quantities of $\bar{L}$ (unskilled labor) and $\bar{S}$ (skilled labor). As in Markusen and Venables (1998), we assume that one production factor ($\bar{S}$) can be employed in both sectors, whereas the other factor ($\bar{L}$) is used only in industry Y (see also Markusen, 2002). Good Y is produced in both countries by a competitive industry, and can be freely traded. Production of Y employs L and S according to a CES-technology

$$Q_Y = \left[\alpha_Y L_Y^{\rho_Y} + \beta_Y S_Y^{\rho_Y} \right]^{\frac{1}{\rho_Y}} , \qquad 0 < \rho_Y < 1 \, , \tag{6}$$

from which profit maximization of Y firms yields the following demand for the two factors of production:

$$L_Y = \alpha_Y^{\sigma_Y} \left(\frac{p_Y}{w_L} \right)^{\sigma_Y} Q_Y \, , \quad \text{and} \tag{7}$$

$$S_Y = \beta_Y^{\sigma_Y} \left(\frac{p_Y}{w_S} \right)^{\sigma_Y} Q_Y \ . \tag{8}$$

The term $\sigma_Y = 1/(1 - \rho_Y)$ stands for the elasticity of substitution between L_Y and S_Y, and w_L and w_S denote the wage rates of unskilled and skilled labor, respectively. If good Y is produced, perfect competition implies that

$$p_Y = \left[\alpha_Y^{\sigma_Y} w_L^{1-\sigma_Y} + \beta_Y^{\sigma_Y} w_S^{1-\sigma_Y} \right]^{\frac{1}{1-\sigma_Y}} \ . \tag{9}$$

We choose good Y as our numeraire throughout the paper, and from free trade in Y and product homogeneity we have $p_Y = p_Y^* = 1$.

We assume that varieties of good X can only be produced by firms whose headquarters are located in the North. While we do not explicitly model research and development, this assumption can be rationalized by arguing that only Northern firms are able to develop and use the blueprints necessary for production.

The production process of any variety $x(i)$ consists of a continuum of tasks, indexed by t, from 0 to t_{Max}. As in Harms et al. (2012), we assume that these tasks have to be performed following a strict sequence, i.e., they cannot be re-arranged at will. This property of a *sequential* production process is important since – as we will later argue – some of these tasks may be offshored to exploit international cost differences and since these cost differences may vary along the production process in a non-monotonic fashion. The labor coefficient $a(t)$ denotes the units of skilled labor that are necessary to perform task t.[4] For simplicity, we assume that $a(t) = a$ for all t, i.e., input coefficients do not differ across tasks. However, a firm producing a given variety of good X may choose between two available technologies: high-tech (H) and low-tech (L). There are three differences between these alternative modes of production: first, high-tech firms use a lower amount of (skilled) labor per task, i.e., $a_H < a_L$. Second, high-tech firms face higher fixed costs F to start production, i.e., $F_H > F_L$. Finally, while low-tech firms are constrained to perform the entire production process domestically, high-tech firms are free to offshore some of the tasks to the South. The goal of our analysis will be to determine the share of good-X

[4] By contrast to this assumption of fixed labor coefficients, Jung and Mercenier (2008), who analyze skill/technology upgrading effects of globalization, employ a heterogeneous workers framework in which a worker's productivity is determined endogenously by his/her own talent and the technology he/she uses.

producers that decides to take advantage of the possibility to produce internationally and to derive the amount of offshoring chosen by these firms.

When performing a task in the South, Northern high-tech firms employ skilled labor, using a linear technology whose input coefficient possibly differs from a_H. In addition, these firms have to use skilled labor to monitor the production process, to communicate with the headquarters in the North etc. The *effective* amount of labor used for task t is thus given by $a^*(t)$. Since the monitoring and communication requirements vary across tasks, there may be tasks which – given skilled labor wages w_S and w_S^* in the North and the South respectively – are cheaper to perform domestically and tasks which are cheaper to perform abroad. This is represented by Figure 1, which juxtaposes the (constant) costs per task $w_S a_L$ if these tasks are performed by a Northern low-tech firm, the (constant) costs $w_S a_H$ if they are performed domestically by a Northern high-tech firm, and the varying costs $w_S^* a^*(t)$ if these tasks are offshored to the South.

The first crucial assumption of our paper is that, unlike in Kohler (2004) or Grossman and Rossi-Hansberg (2008), the production chain cannot be re-arranged – with tasks for which the North has a cost-advantage being lumped together at the start of the chain and tasks for which it has a disadvantage clustered at the end (or vice versa). Instead, Northern high-tech firms have to adjust to the fact that, at given wages, some tasks that are cheaper to perform in the South may be surrounded by tasks that are cheaper to perform in the North (and vice versa).

Our second crucial assumption is that the first and the last task are tied to being performed in the North.

In Figure 1, the tasks $t \in [t_1, t_2]$ and $t \in [t_3, t_4]$ would be performed at lower cost if they were outsourced the South by Northern high-tech firms. Conversely, all tasks $t \in [0, t_1]$, $t \in [t_2, t_3]$ and $t \in [t_4, t_{max}]$ would be cheaper to perform domestically.

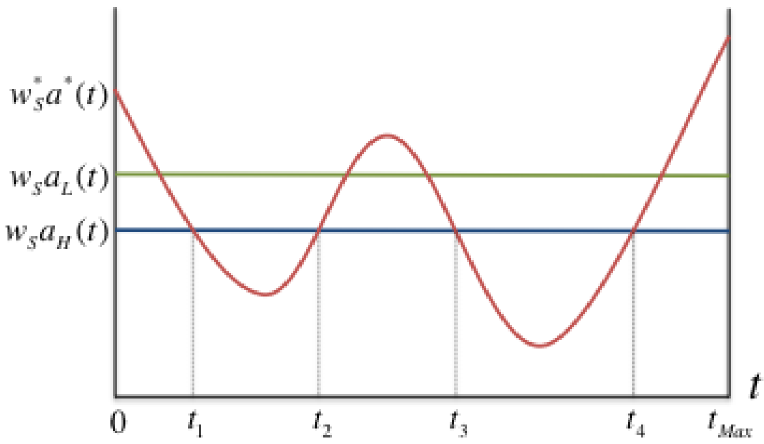

Figure 1: Costs along the Production Chain

In the rest of this paper we restrict our attention to a symmetric specification of the $a^*(t)$ curve:

$$a^*(t) = A\cos(t) + B\ . \tag{10}$$

For t^{max}, we assume $t^{max} = 2\pi n$. The components of this cosine-function have a straightforward interpretation: while the shift parameter B reflects the *average* costs of foreign production, the parameter A which determines the function's amplitude reflects the *heterogeneity* of task-specific input requirements in the South. Given the specification in (10), we use the variable n to characterize the number of cycles that $a^*(t)$ completes between $t = 0$ and $t = t_{max}$. We argue that production processes that are characterized by a higher number of cycles – i.e., a larger n – are more sophisticated, exhibiting more variability in terms of foreign cost advantages. To exclude the uninteresting setting in which one country has a cost advantage for the entire production chain, we focus on the cases in which the following inequalities hold:

$$\frac{B-A}{a_H} < \frac{w_S}{w_S^*} < \frac{B+A}{a_H}\ .$$

The third crucial assumption of this paper is that – as in Harms et al. (2012) – performing a task requires the presence of an unfinished intermediate good that incorporates all previous production steps, and that shifting these goods between countries is associated with a fixed trade cost. More specifically, we assume that any crossing of national borders requires T units of skilled labor. It is for this reason that Northern high-tech firms may find it profitable to stay with the entire production at home or to adopt a strategy of agglomerating (almost) the entire production process $t \in [t_1, t_4]$ abroad, rather than paying trade costs for each time the unfinished good is crossing borders. Hence, if foreign production costs fluctuate around domestic costs more than once – i.e., if $(n \geq 2)$ – we may distinguish three firm-types $j \in \{L, H^f, H^a\}$: domestic low tech firms (*domestic production*), high-tech firms that fragment their production chain several times (*fragmentation*) and firms that perform most tasks in the South (*production abroad*), respectively.[5]

Defining for future use the dummy term γ ($\gamma = 0$ if $j = L$, and $\gamma = 1$ if $j = H^f, H^a$), variable unit costs are given as follows:

$$C_j = w_S S_j + \gamma w_S^* S_j^* , \qquad j \in \left\{L, H^f, H^a\right\} , \tag{11}$$

where X-sector skilled labor demands for j-type firms are $(n \geq 2)$ given by:[6]

$$S_L = \int_0^{t_{Max}} a_L dt , \tag{12}$$

$$S_{H^f} = \left(\int_{t_{2n}}^{t_{Max}} a_H dt + \int_{t_{2n-2}}^{t_{2n-1}} a_H dt + \cdots + \int_0^{t_1} a_H dt \right) + nT , \tag{13}$$

$$S_{H^a} = \left(\int_{t_{2n}}^{t_{Max}} a_H dt + \int_0^{t_1} a_H dt \right) + T , \tag{14}$$

$$S_{H^f}^* = \left(\int_{t_{2n-1}}^{t_{2n}} a^*(t) dt + \cdots + \int_{t_1}^{t_2} a^*(t) dt \right) + nT , \quad \text{and} \tag{15}$$

$$S_{H^a}^* = \left(\int_{t_1}^{t_{2n}} a^*(t) dt \right) + T . \tag{16}$$

Firms in sector X compete on the output market. We assume monopolistic competition to prevail so that firms charge a constant mark-up rate over their variable unit costs:

$$p_j \leq \frac{\sigma_X}{\sigma_X - 1} C_j, \qquad j \in \left\{L, H^f, H^a\right\} , \qquad (x_j) \tag{17}$$

[5] These firm-types can also be interpreted in terms of their technologies: low-tech (L), high-tech-fragmenting (H^f), and high-tech-production abroad (H^a), respectively, with the corresponding subscripts. In general, additional firm types may be possible between these three ideal types. Our symmetric parametrization of $a^*(t)$, however, excludes these additional firm types.

[6] Note that with n cycles, partial-offshoring firms have $2n+1$ sequential production stages, and transport costs of n times T in each country, while full-offshoring firms have 3 sequential production stages and transport costs of T in each country.

and final X-goods are exported to the South with iceberg trade costs $\tau_X > 1$. Then, the X-good demand system as given by equations (3) and (4) can be written for $j \in \left\{L, H^f, H^a\right\}$ as

$$x_j^d = \left(\frac{P_X}{p_j}\right)^{\sigma_X} X \ , \tag{18}$$

$$x_j^{d*}\frac{1}{\tau_X} = \left(\frac{P_X^*}{p_j \tau_X}\right)^{\sigma_X} X^* \ , \tag{19}$$

$$P_X = \left[\sum_j N_j p_j^{1-\sigma_X}\right]^{\frac{1}{1-\sigma_X}} \ , \quad \text{and} \tag{20}$$

$$P_X^* = \left[\sum_j N_j \left(p_j \tau_X\right)^{1-\sigma_X}\right]^{\frac{1}{1-\sigma_X}} \tag{21}$$

where N_j stands for the mass of firms of each firm-type.

3.3 Equilibrium

In sector X, the cutoff tasks are determined by the following indifference conditions on costs:

$$w_S a_H = w_S^* a^*(t_m), \qquad m \in \{1, 2, \cdots, 2n\} \, . \tag{22}$$

Free entry ensures zero profits so that mark-up revenues cover fixed costs:

$$\frac{1}{\sigma_X} p_j x_j \leq F_j \ , \quad j \in \left\{L, H^f, H^a\right\} \ , \quad (N_j) \ , \tag{23}$$

where $N_j > 0$ if each condition holds with equality, and $N_j = 0$ otherwise. For convenience, we assume that fixed costs take the form of unsold final goods and consistent with evidence we assume $F_L < F_{H^f} < F_{H^a}$.[7]

[7] As is common in the literature, we assume that the fixed organizational and set-up costs are higher abroad than domestically. Consequently, they increase in the range of activities performed abroad, implying $F_{H^a} > F_{H^f}$.

From the factor market equilibrium, we have

$$\bar{S} \geq S_Y + S_L\left(x_L + F_L\right)N_L + S_{H^f}\left(x_{H^f} + F_{H^f}\right)N_{H^f} + S_{H^a}\left(x_{H^a} + F_{H^a}\right)N_{H^a} \ , \tag{24}$$

$$\bar{L} \geq L_Y \ , \tag{25}$$

$$\bar{S^*} \geq S_Y^* + S_{H^f}^*(x_{H^f} + F_{H^f})N_{H^f} + S_{H^a}^*(x_{H^a} + F_{H^a})N_{H^a} \ , \quad \text{and} \tag{26}$$

$$\bar{L^*} \geq L_Y^* \ , \tag{27}$$

$$\tag{28}$$

which determine w_S, w_L, w_S^* and w_L^*, respectively, and final-goods' market equilibrium imposes that

$$x_j \geq x_j^d + x_j^{d*} \ , \quad j \in \left\{L, H^f, H^a\right\} \ , \quad (p_j) \ , \quad \text{and} \tag{29}$$

$$Q_Y + Q_Y^* \geq Y + Y^* \ , \quad (p_Y) \ . \tag{30}$$

Finally, incomes follow from employment:

$$Inc = w_L\overline{L} + w_S\overline{S} \ , \quad \text{and} \quad Inc^* = w_L^*\overline{L^*} + w_S^*\overline{S^*} \ . \tag{31}$$

4 Qualitative Results

4.1 Setup and Cutoff Values

In this section, we analyze some comparative-static properties of our model. In particular, we are interested in the effect of reducing trade costs, changing the foreign cost (dis)advantage, and varying the number of cycles on the number of L, H^a and H^f firms in the market, and on the extent of offshoring chosen by these firms. To arrive at these results, we impose some further simplifying restrictions. First, we assume that there are at least 2 *cycles*, i.e., $n \geq 2$. Second, we assume that the varieties of good X are only consumed in the North, i.e., $\beta^* = 0$. Finally, we impose a Cobb-Douglas production technology in sector Y, i.e., $\sigma_Y \to 1$ and $\alpha_Y = \beta_Y = 1$ such that $Q_Y = S_Y^{1/2} L_Y^{1/2}$.

The main advantage of the symmetric specification in (10) is that it allows to determine all cut-off values t_m and thereby the length of all production segments for which production abroad is less (more) costly from the first cut-off t_1, since $t_2 = 2\pi - t_1$, $t_3 = 2\pi + t_1$, etc. Using (22), we can obtain the cut-off value t_1, which is

$$t_1 = \arccos\left(\frac{a_H w_S/w_S^* - B}{A}\right) . \tag{32}$$

That is, the range of offshored tasks depends on the domestic labor coefficient a_H, the average foreign production costs B, and on the wage w_S/w_S^* for skilled labor in the North relative to the South. Not surprisingly, an increase in w_S/w_S^* lowers the cut-off, i.e.,

$$\frac{\partial t_1}{\partial\left(w_S/w_S^*\right)} = -\frac{a_H}{A\sqrt{1-\left(\frac{a_H w_S/w_S^* - B}{A}\right)^2}} < 0 .$$

With our specification of the foreign input coefficients with cut-off (32), we obtain the following labor demands:

$$S_L = 2n\pi a_L , \tag{33}$$

$$S_{H^f} = 2n a_H t_1 + nT , \tag{34}$$

$$\begin{aligned} S_{H^f}^* &= n\int_{t_1}^{2\pi - t_1} \left[A\cos(t) + B\right] dt + nT \\ &= 2n\left[B\left(\pi - t_1\right) - A\sin\left(t_1\right)\right] + nT , \end{aligned} \tag{35}$$

$$S_{H^a} = 2a_H t_1 + T , \tag{36}$$

$$\begin{aligned} S_{H^a}^* &= n\int_0^{2\pi} \left(A\cos(t) + B\right) dt - 2\int_0^{t_1} \left(A\cos(t) + B\right) dt + T \\ &= 2\left[B\left(n\pi - t_1\right) - A\sin(t_1)\right] + T . \end{aligned} \tag{37}$$

From (33)–(37), we can infer the following: $S_L > S_{H^f} > S_{H^a}$ for $T < 2\left(\pi a_L - t_1 a_H\right)$ and $S_{H^a}^* > S_{H^f}^*$ for $T < 2\left[B t_1 + A\sin(t_1)\right]$. Thus, if transport costs T are sufficiently low and for a given cut-off t_1, from (11) a marginal increase of w_S or w_S^* raises costs of each firm type in the order of

$$\frac{\partial C_L}{\partial w_S} > \frac{\partial C_{H^f}}{\partial w_S} > \frac{\partial C_{H^a}}{\partial w_S} \quad \text{and} \quad \frac{\partial C_{H^a}}{\partial w_S^*} > \frac{\partial C_{H^f}}{\partial w_S^*} .$$

According to (5) aggregate demand in the North for differentiated goods can be written as

$$X = \frac{\beta\left(w_L \bar{L} + w_S \bar{S}\right)}{P_X} .$$

Since $w_L = 0.5 S_Y^{1/2} \bar{L}^{-1/2}$ and $w_S = 0.5 S_Y^{-1/2} \bar{L}^{1/2}$, we can write $w_L = w_S S_Y/\bar{L}$, such that

$$X = \frac{\beta w_S\left(S_Y + \bar{S}\right)}{P_X} .$$

Inserting the profit maximizing price (17) into the zero profit condition (23) yields

$$x_i = (\sigma_X - 1)F_i \ , \quad \text{and} \tag{38}$$

$$P_X^{1-\sigma_X} = \frac{p_i^{-\sigma_X} \beta w_S (S_Y + \bar{S})}{(\sigma_X - 1)F_i} \ . \tag{39}$$

4.2 Domestic and Fragmented Firms

Using the assumptions and results outlined in the previous paragraph, we proceed by analyzing a particular *regime*, in which Northern low-tech firms and Northern high-tech firms with fragmented production coexist.[8] We want to determine how variations in crucial model parameters – in particular, transport costs and the average cost-advantage of the South – influence both the number of firms that engage in offshoring (N_{Hf}) and the extent of offshoring chosen by these firms. Since both the adjustment at the extensive margin and the adjustment at the intensive margin have repercussions on countries' relative wages, a simultaneous consideration of all possible regimes is unlikely to deliver unambiguous results and is therefore reserved for the numerical analysis of the following section.

In a regime in which domestic firms and fragmented firms co-exist, equation (38) implies $x_{Hf}/x_L = F_{Hf}/F_L$. Since $x_{Hf}/x_L = (p_{Hf}/p_L)^{-\sigma_X}$, and $p_{Hf}/p_L = C_{Hf}/C_L$, we obtain

$$C_{Hf} = f_f C_L \ , \qquad \text{where} \qquad f_f = \left(\frac{F_{Hf}}{F_L}\right)^{-1/\sigma_X} \ .$$

Inserting for C_{Hf} and C_L yields $S_{Hf} w_S + S_{Hf}^* w_S^* = f_f S_L w_S$, or

$$\begin{aligned} \frac{w_S}{w_S^*} &= \frac{S_{Hf}^*}{f_f S_L - S_{Hf}} \\ &= \frac{2\left[B\left(\pi - t_1\right) - A\sin(t_1)\right] + T}{2 f_f a_L \pi - 2 a_H t_1 - T} \ . \end{aligned} \tag{40}$$

The combined zero-profit conditions for both firm types thus determine the wage for skilled labor in the North relative to the South as a function of the respective labor coefficients in both countries. A decline in the cut-off t_1, i.e., more offshoring by high-tech firms, raises S_{Hf}^* but lowers S_{Hf}. With the assumed cosine-specification of $a^*(t)$, both effects just cancel out, such that a change in t_1 does not influence the relative wage. This follows

[8] See Appendix B for an analysis of regimes in which good X varieties are produced by only one type of firms. Appendix C considers the case in which H^a and H^f firms coexist.

from taking the derivative of (40):

$$
\begin{aligned}
\frac{\partial(w_S/w_S^*)}{\partial t_1} &= \frac{-2a^*(t_1)\left(f_f S_L - S_{Hf}\right) + 2a_H S_{Hf}^*}{\left(f_f S_L - S_{Hf}^*\right)^2} \\
&= \frac{-2a_H w_S/w_S^*\left(f_f S_L - S_{Hf}\right) + 2a_H S_{Hf}^*}{\left(f_f S_L - S_{Hf}^*\right)^2} \\
&= \frac{-2a_H S_{Hf}^* + 2a_H S_{Hf}^*}{\left(f_f S_L - S_{Hf}^*\right)^2} \\
&= 0 \ .
\end{aligned}
$$

With N_L domestic firms and N_{Hf} firms that fragment their production, the aggregate price index can be written as $P_X = \left(N_L p_L^{1-\sigma_X} + N_{Hf} p_{Hf}^{1-\sigma_X}\right)^{1/(1-\sigma_X)}$. Inserting into (39) yields

$$
N_L + f_f^{1-\sigma_X} N_{Hf} = \frac{\beta\left(S_Y + \bar{S}\right)}{F_L \sigma_X S_L} \ .
$$

Inserting $S_Y = \bar{S} - \sigma_X F_L S_L N_L - \sigma_X F_L f_f^{-\sigma_X} S_{Hf} N_{Hf}$ yields a negative relationship between N_{Hf} and N_L:

$$
N_L = \frac{2\beta\bar{S}}{(1+\beta)\sigma_X F_L S_L} - \frac{f_f^{-\sigma_X}\left(\beta S_{Hf} + S_L f_f\right)}{S_L(1+\beta)} N_{Hf} \ . \tag{41}
$$

Profit maximization in sector Y implies $w_S = \left(S_Y\right)^{-1/2} \bar{L}^{1/2}$ and $w_S^* = \left(S_Y^*\right)^{-1/2}\left(\bar{L}^*\right)^{1/2}$. This leads to

$$
\left(\frac{S_Y}{S_Y^*}\right)^{-1} = \left(\frac{w_S}{w_S^*}\right)^2 \left(\frac{\bar{L}^*}{\bar{L}}\right) \ . \tag{42}
$$

Inserting from (40) yields

$$
\left(\frac{\bar{S} - \sigma_X F_L\left(N_L S_L + f_f^{-\sigma_X} N_{Hf} S_{Hf}\right)}{\bar{S}^* - \sigma_X f_f^{-\sigma_X} F_L N_{Hf} S_{Hf}^*}\right)^{-1} = \tilde{A}^2 \frac{\bar{L}^*}{\bar{L}} \ , \tag{43}
$$

where $\tilde{A} = S_{Hf}^*/(S_L f_f - S_{Hf})$. Equations (41) and (43) jointly determine N_L and N_{Hf}. For N_{Hf} we obtain

$$
N_{Hf} = \frac{\bar{S}^*(1+\beta) - \tilde{A}^2 \frac{\bar{L}^*}{\bar{L}} \bar{S}(1-\beta)}{\left[\tilde{A}^2 \frac{\bar{L}^*}{\bar{L}}\left(S_L f_f - S_{Hf}\right) + (1+\beta) S_{Hf}^*\right] \sigma_X F_L f_f^{-\sigma_X}} \ . \tag{44}
$$

We proceed by analyzing the effects of varying the average labor coefficient B in the South. Note that such a change may result from changes in relative labor productivities, but that it may also be interpreted as a consequence of changing information technologies, which affect the costs of monitoring and communication. Using (40), we can derive the impact of B on relative wages:

$$\frac{\partial(w_S/w_S^*)}{\partial B} = \frac{2n\,(\pi - t_1)}{f_f S_L - S_{Hf}} > 0\ .$$

The cut-off value t_1, which determines the intensive margin of offshoring, is affected as follows:

$$\frac{dt_1}{dB} = \frac{1 - a_H \frac{\partial(w_S/w_S^*)}{\partial B}}{A\sqrt{1 - \left(\frac{a_H w_S/w_S^* - B}{A}\right)^2}} \tag{45}$$

For the influence of B on the number of firms we obtain from (41) and (44)

$$\frac{\partial N_{Hf}}{\partial B} < 0\ , \qquad \text{and} \qquad \frac{\partial N_L}{\partial B} > 0\ . \tag{46}$$

A decline in B enhances the South's cost advantage, raises the number of firms that relocate parts of their production process abroad, and – for a given relative wage – induces fragmented firms to offshore a larger part of their production chain. As a result, the overall volume of offshoring increases, the demand for skilled labor in the North decreases while the demand for skilled labor in the South increases, lowering the relative wage w_S/w_S^*. This wage effect, however, has a negative influence on the extent of offshoring by fragmented firms (second term in the numerator of 46).

We next turn to the effect of varying the transport costs T on the model equilibrium. From (40) we see that a decline in T lowers the relative wage of the North, i.e.,

$$\frac{\partial(w_S/w_S^*)}{\partial T} = \frac{1 + \frac{w_S}{w_S^*}}{f_f S_L - S_{Hf}} > 0\ .$$

Lowering Tmakes offshoring more attractive, and therefore raises the demand for skilled labor in the South.

As for the intensive margin of offshoring, we can use (32) to derive

$$\frac{dt_1^*}{dT} = -\frac{a_H\left(1 + \frac{w_S}{w_S^*}\right)}{A\sqrt{1 - \left(\frac{a_H w_S/w_S^* - B}{A}\right)^2}\,(f_f S_L - S_{Hf})} < 0\ .$$

Globalization in the form of a decline in the transport costs for intermediates therefore lowers the wage for skilled workers in the home country relative to foreign workers. This results in a decline in the intensive margin of offshoring – i.e., firms decide to perform a larger share of the production process domestically. By contrast, the extensive margin of offshoring, i.e., the number of fragmented firms, increases. This can be seen after rearranging (43) to

$$N_{Hf} = \frac{\bar{S}^*(1+\beta) - \tilde{A}^2 \frac{\bar{L}^*}{\bar{L}} \bar{S}(1-\beta)}{\left[S^*_{Hf} \frac{\bar{L}^*}{\bar{L}} \tilde{A} + (1+\beta) S^*_{Hf} \right] \sigma_X F_L f_f^{-\sigma_X}} \ . \tag{47}$$

Since $\tilde{A}$ increases in T the numerator of (47) declines in T, whereas the denominator increases in T. Thus, and together with (41), we obtain

$$\frac{\partial N_{Hf}}{\partial T} < 0 \ , \quad \text{and} \quad \frac{\partial N_L}{\partial T} > 0 \ .$$

How a variation of T affects the *total* volume of offshoring depends on the relative strength of the adjustment at the intensive and the extensive margins. Instead of pursuing this analysis, we now turn to numerical simulations that allow us to consider how alternative parameter combinations give rise to constellations in which certain types prevail.

5 A Numerical Appraisal

5.1 Calibration

The two countries are scaled so that initially about half of the domestic consumption of the competitive good Y is imported from the South, while X goods are produced only by Northern firms and exported to the South. With preference parameter $\beta = 0.5$, the North is endowed with one third of the world unskilled labor (L) and two thirds of the world skilled labor (S), while the South is endowed with the rest. In industry X, we set $\sigma_X = 4$ and $n = 2$ as benchmark values for the substitution elasticity and the number of cycles of $a^*(t)$. Again, we assume a cosine function: $a^*(t) = A\cos(\alpha t) + B$, with $\alpha = 2n\pi$ and $t_{Max} = 1$. We choose somewhat arbitrarily – but within the ranges consistent with our theoretical constraints – the fixed costs and the trade cost of each crossing border of production stages: $F_L = 1.0$; $F_{Hf} = 1.3$; $F_{H^a} = 1.5$; $T = 0.2$. With this functional form and parameter values, we calibrate the key technological parameters – A, B, a_L and a_H –, so that initially about half of the total X is produced by L-type firms and the other half by H^f-type firms. Figure 2 displays the three calibrated technologies and the resulting cutoff tasks.[9]

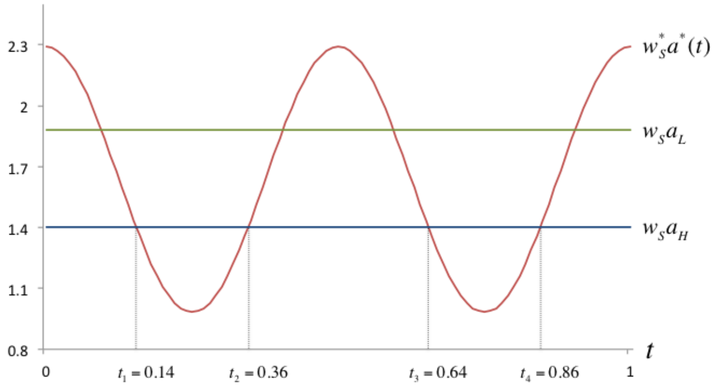

Figure 2: Technologies and Cutoff Tasks

5.2 Equilibrium Regimes

Given our benchmark parameter values, Figure 3 presents the equilibrium regimes over different allocations of production factors between the two countries. The vertical axis is the total world endowment of skilled labor and the horizontal axis is the total world endowment of unskilled labor, with the North measured from the southwest (SW) and the South from the northeast (NE).

[9] Appendix A reports the benchmark parameter and equilibrium variable values.

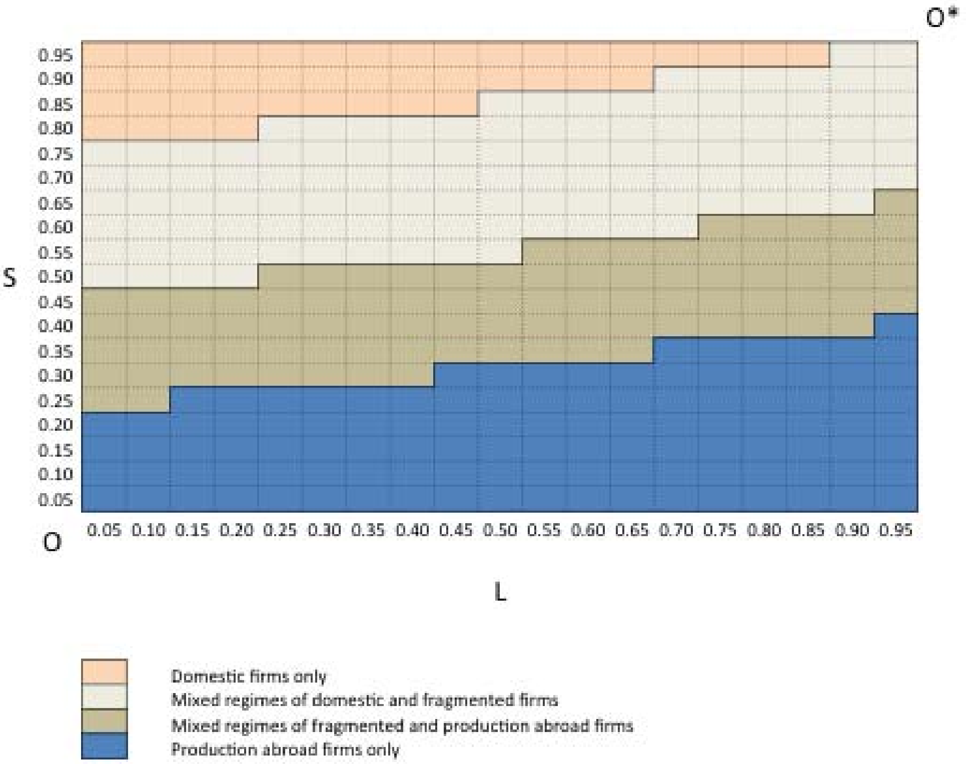

Figure 3: Equilibrium Regimes

We see that the equilibrium regimes are associated with differences in relative skilled labor endowments. Intuitively, if the North is highly abundant in skilled labor – such that this factor is relatively cheap – no offshoring occurs, while if the South is highly abundant in skilled labor firms producing abroad are dominant. In the middle, fragmented firms should dominate.[10] Figure 4 displays the equilibrium number of each firm-type along the NW-SE diagonal – the two countries differ in relative factor endowments – and along the SW-NE diagonal – the two countries have identical relative factor endowments but differ in size.

[10]Indeed, altering the distribution of the world endowment in much finer steps, we also have regimes in which only fragmented firms exist between the two regimes of $\{L, H^f\}$ and $\{H^f, H^a\}$.

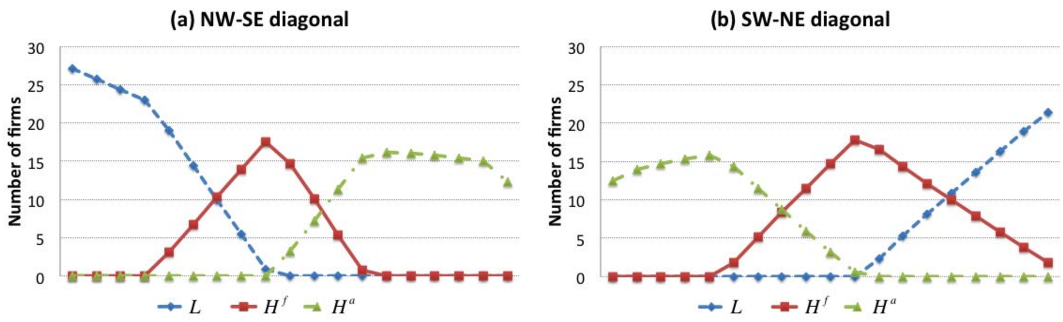

Figure 4: Endogenous Market Structure along the NW-SE and SW-NE Diagonals

Given our assumption concerning the X-sector technological difference between the two countries – the final X-goods are produced only in the North and exported to the South –, the market size difference does not directly affect the endogenous entry of each firm-type into the X-sector, but has an indirect influence via adjustments in the Y-sector and changes the world trade volume. Figure 5 displays the resulted trade pattern and volume along the two diagonals.

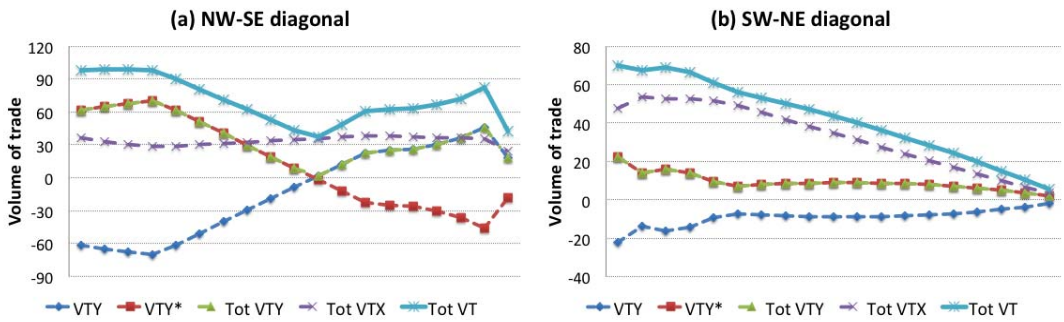

Figure 5: Trade Pattern/Volume along the NW-SE and SW-NE Diagonals

VTY and VTY* denote the volume of trade in Y – export/import of the North and the South, respectively –, and Tot VTY denotes the world total trade in Y. Equivalently, Tot VTX denotes total volume of trade in X – sum of exports of X-firms –, and Tot VT is the sum of Tot VTY and Tot VTX. Panel (a) shows that around the NW corner the North specializes in X-sector task production with high dominance of no-offshoring firms, while the South specializes relatively in Y-goods and exports to the North. Approaching the SE corner, this trade pattern in Y is reversed, and the South specializes relatively more in X-sector task production with a dominance of firms producing abroad. Along the SW-NE diagonal - panel (b), the overall world trade decreases from the market size effect.

5.3 The Effects of Globalization

Now we investigate the effects of globalization which we interpret as a decline in T.[11] Figure 6 reports the effects of a fall in T on unit production costs and entry of each firm-type (benchmark: $T = 0.2$). Note that the horizontal axis is *inverted*, moving from higher to lower values – i.e., the extent of globalization increases (with transport costs decreasing) from left to right.

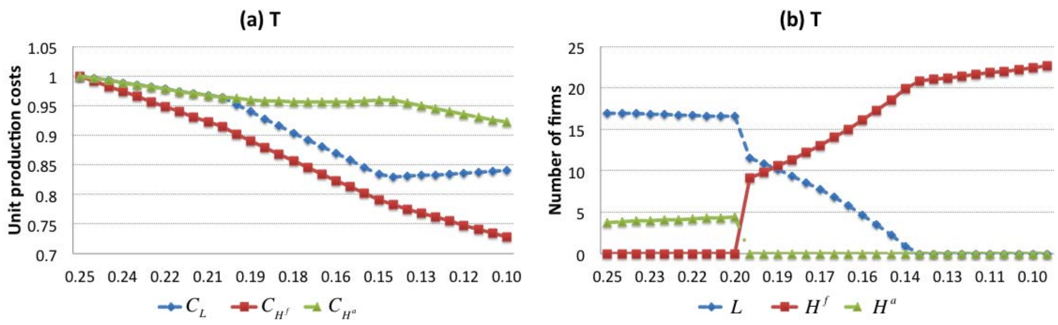

Figure 6: Globalization and C_j and N_j

We see that in general a fall in T reduces costs for all three types of firms, with fragmented firms benefitting most strongly. Conversely, the costs of firms producing abroad decrease much less: First, a decline in transport costs has a much lower impact for firms that produce abroad compared with fragmented firms. Second, the reduced transport costs are partly offset by increasing wages in the South. As for the adjustment of globalization at the extensive margin, domestic and foreign production firms prevail if transport costs are high. As T decreases, fragmentation becomes more attractive, and eventually, this becomes the dominant mode of production.

[11] Given that all final X-varieties are produced in the North and exported to the South, a fall in τ_X – the iceberg trade costs in final X-goods – only affects welfare of the South (positively).

Figure 7 presents the induced variations in cutoff tasks t_1 and t_2 (with symmetric effects on t_3 and t_4 from our cosine specification).

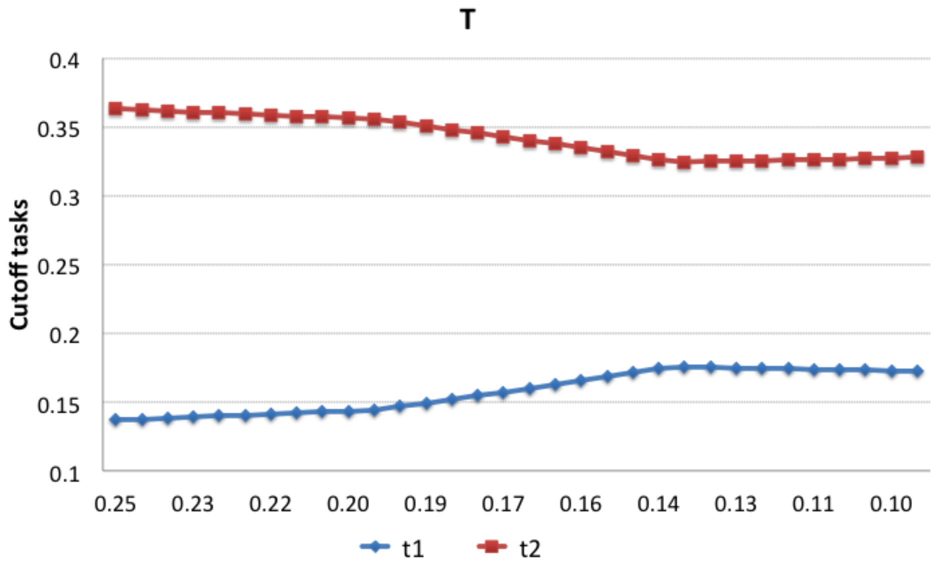

Figure 7: Globalization and Cutoff Tasks

Here we find, against common belief, that a reduction in trade costs first lowers the range of tasks offshored even though the total volume – measured in total skilled labor requirements in the South – increases as more firms adopt an offshoring strategy. This is a general equilibrium effect: a higher demand for skilled labor in the South, resulting from an increasing number of offshoring firms raises the relative skilled labor wage ratio (w_S^*/w_S). The indifference conditions on production costs (22) indicate that this makes it less profitable to offshore a large range of tasks. However, at the end part of Figure 7, if only fragmented firms exists, the effect of a decline in T on the cutoff values is reversed. This is due to the fact that lower transport costs release skilled labor in the South. This lowers w_S^*/w_S so that it is profitable to offshore a higher range of tasks to the South.

5.4 Technological Change

In this subsection, we explore how technological changes affect the relative importance of alternative production modes. Again, the horizontal axis is inverted, moving from higher to lower values – i.e., productivity increases are reflected by decreasing input coefficients from left to right.

Figure 8 shows the effects of domestic technological change on a_L and a_H (benchmark: $a_L = 2.50$, $a_H = 1.86$). Intuitively, domestic technological progress in low-tech – a fall in a_L – is beneficial to domestic firms, while it negatively affects offshoring firms. Between the two types of offshoring firms, the fragmented firms are affected most negatively. More entry by domestic low-tech firms raises the skilled wage ratio, which is more detrimental to fragmented firms given that they perform more tasks domestically than firms producing almost everything abroad. In contrast, domestic technological progress in high-tech – a fall in a_H – benefits high-tech offshoring firms, while it affects domestic low-tech firms negatively. Between the two types of offshoring firms, fragmented benefit more strongly by the same logic as above.

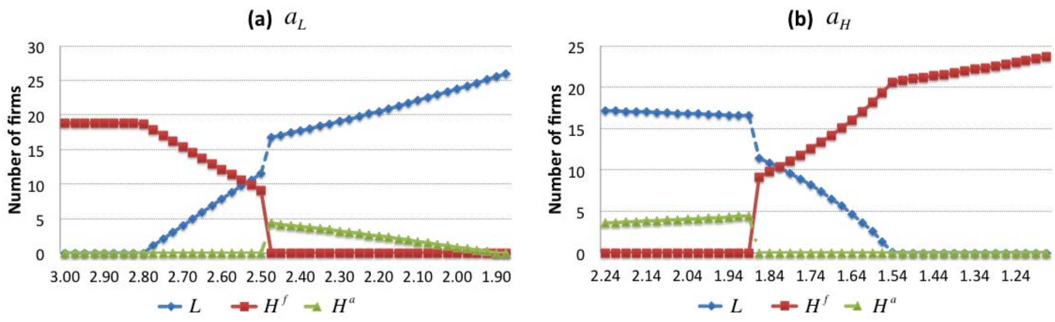

Figure 8: Technological Change on a_L and a_H

Figure 9 focuses on the South and presents the effects of varying the parameters A and B (benchmark: $A = 1.5$, $B = 2.5$). Recall that raising A increases the amplitude of the $a^*(t)$ function, representing greater cost differences between adjacent parts of the production chain. Conversely, reducing A implies a disappearance of the technological advantage in the South for the tasks $t \in [t_1, t_2]$ and $[t_3, t_4]$. This makes fragmentation less attractive, and eventually, only domestic firms and firms producing abroad prevail. A similar pattern emerges if B – i.e., the average costs associated with producing abroad – increases: in this case, the number of firms choosing any type of offshoring converges to zero.

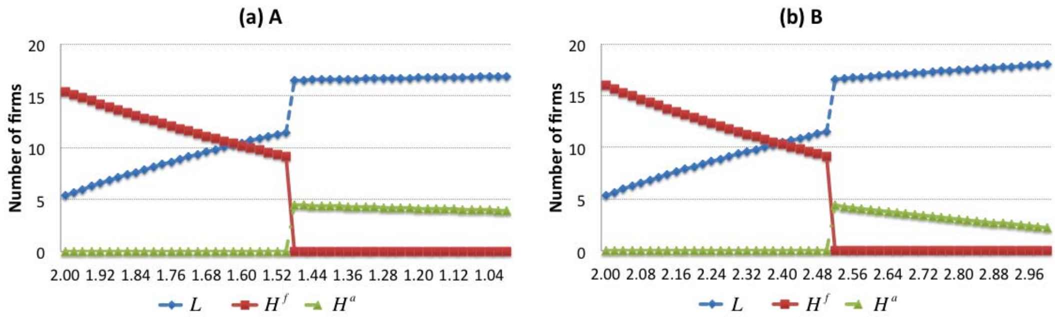

Figure 9: Technological Change on A and B

Finally, Figure 10 displays the effect of an increase in the number of cycles (n) on the relative importance of alternative production modes.

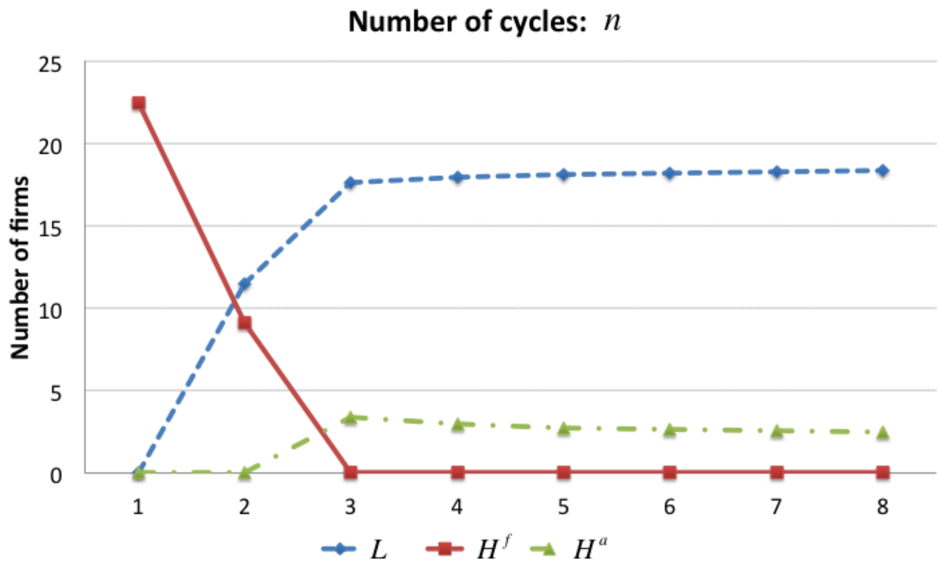

Figure 10: Increase in Sophistication of the Sequential Production Chain

More fluctuation of offshoring production costs around the domestic costs – an increase in the number of cycles n – has similar effects as an increase in T. Given that partial-offshoring option requires n-times T in each country, this would be the most detrimental to fragmented firms, while domestic firms are the the main beneficiary. Exit of fragmented firms and more entry of agglomerated firms (mostly domestic agglomeration) implies a reduction in firms' average number of production stages $\left(\frac{(1)N_L+(2n+1)N_{H^f}+(3)N_{H^a}}{N_L+N_{H^f}+N_{H^a}}\right)$.

6 Conclusion

In this paper, we have analyzed the extent of offshoring in a two-country general equilibrium model that is based on three crucial assumptions: First, firms production process follows a rigid structure that defines the sequence of production steps. Second, domestic and foreign relative productivities vary in a non-monotonic fashion along the production chain. Third, each task requires the presence of an unfinished intermediate good whose transport across borders is costly. We believe these assumptions to be quite plausible, characterizing, e.g., the production process in the automotive industry. As a consequence, firms may be reluctant to offshore individual production steps, even if performing them abroad would be associated with cost advantages: the reason is that adjacent tasks may be cheaper to perform in the domestic economy and that high transport costs do not justify shifting the unfinished good abroad and back home.

Given this basic structure, we have analyzed the influence of technological progress and globalization – interpreted as a variation in transport and communication costs – on the volume of offshoring at the extensive and the intensive margin. As relative production costs vary, a given number of firms adjusts the share of tasks it performs abroad (the intensive margin), and the number of firms that fragments its production process or produces entirely abroad changes (the extensive margin). Both adjustments affect relative wages at home and abroad, which may reinforce or dampen the initial impulse. We have shown that globalization in the form of declining transport costs may have different effects on offshoring at the extensive and intensive margin. Our analysis suggests a *decrease* of offshoring at the intensive margin – i.e., firms offshore a smaller part of the entire production process – but an *increase* in the number of firms that perform at least some tasks abroad.

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Appendix A: Benchmark Parameter and Variable Values

$\bar{L}$	$\bar{S}$	$\overline{L^*}$	$\overline{S^*}$	β	σ_X	a_L	a_H	n	A
100.00	200.00	200.00	100.00	0.50	4.00	2.50	1.86	2.00	1.50
B	t_{Min}	t_{Max}	τ_X	T	F_L	F_{H^f}	F_{H^*}	σ_Y	α_Y
2.50	0.00	1.00	1.30	0.20	1.00	1.30	1.50	3.00	0.50
β_Y	Inc	Con	P_{Con}	X	Y	x_L^d	$x_{H^f}^d$	$x_{H^*}^d$	P_X
0.50	190.48	101.14	1.88	107.41	95.24	1.68	2.19	2.52	0.89
Inc^*	Con^*	P_{Con}^*	X^*	Y^*	x_L^{d*}	$x_{H^f}^{d*}$	$x_{H^*}^{d*}$	P_X^*	x_L
149.49	69.62	2.15	64.84	74.75	1.32	1.71	1.98	1.15	3.00
x_{H^f}	x_{H^*}	C_L	C_{H^f}	C_{H^*}	p_L	p_{H^f}	p_{H^*}	N_L	N_{H^f}
3.90	4.50	1.88	1.76	1.70	2.51	2.35	2.27	11.48	9.13
N_{H^*}	t_1	t_2	t_3	t_4	Q_Y	Q_Y^*	p_Y	p_Y^*	S_Y
0.00	0.14	0.36	0.64	0.86	51.42	118.57	1.00	1.00	15.11
L_Y	S_Y^*	L_Y^*	w_S	w_L	w_S^*	w_L^*			
100.00	52.78	200.00	0.75	0.40	0.65	0.42			

Appendix B: Equilibrium Outcomes with One Firm Type

Domestic production only. If there is no offshoring, the aggregate price index can be written as $P_X = N_L^{1/(1-\sigma_X)} p_L$. Inserting into (39) and employing $C_L = S_L w_S$ this yields

$$N_L = \frac{\beta\left(S_Y + \bar{S}\right)}{F_L \sigma_X S_L} \, .$$

Inserting for $S_Y = \bar{S} - S_X = \bar{S} - \sigma_X F_L S_L N_L$ leads to

$$N_L = \frac{2\beta\bar{S}}{(1+\beta)\sigma_X F_L S_L} \quad \text{and} \quad S_Y = \frac{1-\beta}{1+\beta}\bar{S} \, .$$

Employing (42), we can derive the relative wage w_S/w_S^* as

$$\frac{w_S}{w_S^*} = \left(\frac{1-\beta}{1+\beta}\right)^{-1/2} \left(\frac{\bar{S}}{\bar{S}^*}\right)^{-1/2} \left(\frac{\bar{L}}{\bar{L}^*}\right)^{1/2} \, .$$

For X, we obtain

$$X = \left(\frac{2\beta\bar{S}}{(1+\beta)\sigma_X S_L}\right)^{\frac{\sigma_X}{\sigma_X - 1}} (\sigma_X - 1)\, F_L^{\frac{1}{1-\sigma_X}} \, .$$

Fragmented production only. If all firms choose to fragment production between North and South, the aggregate price index is $P_X = N_{H^f}^{1/(1-\sigma)} p_{H^f}$. Together with $C_{H^f} = S_{H^f} w_S + S_{H^f}^* w_S^*$ this yields

$$N_{H^f} = \frac{\beta\left(S_Y + \bar{S}\right)\frac{w_S}{w_S^*}}{f_f^{-\sigma_X} F_L \sigma_X \left(S_{H^f}\frac{w_S}{w_S^*} + S_{H^f}^*\right)} \, .$$

Inserting for $S_Y = \bar{S} - \sigma_X f_f^{-\sigma_X} F_L S_{H^f} N_{H^f}$ leads to

$$N_{H^f} = \frac{2\beta\bar{S}\frac{w_S}{w_S^*}}{\sigma_X f_f^{-\sigma_X} F_L \left[S_{H^f}(1+\beta)\frac{w_S}{w_S^*} + S_{H^f}^*\right]} \, . \tag{B.1}$$

Employing (42) yields the relative wage w_S/w_S^*

$$\frac{w_S}{w_S^*} = \left(\frac{\bar{S} - \sigma_X f_f^{-\sigma_X} F_L S_{H^f} N_{H^f}}{\bar{S}^* - \sigma_X f_f^{-\sigma_X} F_L S_{H^f}^* N_{H^f}}\right)^{-1/2} \left(\frac{\bar{L}}{\bar{L}^*}\right)^{1/2} \, . \tag{B.2}$$

Equations (B.1) and (B.2) jointly determine N_{H^f} and w_S/w_S^*.

Production Abroad. Similarly to the fragmented production setting, we can determine the equilibrium for the case in which only firms producing abroad exist by

$$N_{H^a} = \frac{2\beta\bar{S}\frac{w_S}{w_S^*}}{\sigma_X f_a^{-\sigma_X} F_L \left[S_{H^a}(1+\beta)\frac{w_S}{w_S^*} + S_{H^a}^*\right]} \, , \quad \text{and} \tag{B.3}$$

$$\frac{w_S}{w_S^*} = \left(\frac{\bar{S} - \sigma f_a^{-\sigma_X} F_L S_{H^a} N_{H^a}}{\bar{S}^* - \sigma_X f_a^{-\sigma_X} F_L S_{H^a}^* N_{H^a}}\right)^{-1/2} \left(\frac{\bar{L}}{\bar{L}^*}\right)^{1/2} \, , \quad f_a = \left(\frac{F_{H^a}}{F_L}\right)^{-1/\sigma_X} \, . \tag{B.4}$$

Appendix C: Fragmentation and Production Abroad

If fragmented and production abroad firms co-exist, the relative wage w_S/w_S^* is determined by $C_{H^f} f_a = C_{H^a} f_f$, which yields

$$\begin{aligned}\frac{w_S}{w_S^*} &= \frac{S^*_{H^a}\frac{f_f}{f_a} - S^*_{H^f}}{S_{H^f} - \frac{f_f}{f_a}S_{H^a}} \\ &= \frac{\frac{f_f}{f_a}\left[2\left[B\left(n\pi - t_1\right) - A\sin(t_1)\right] + T\right] - \left[2n\left[B\left(\pi - t_1\right) - \alpha\sin\left(t_1\right)\right] + nT\right]}{2na_H t_1 + nT - \frac{f_f}{f_a}\left[2a_H t_1 + T\right]} \ . \end{aligned} \tag{C.1}$$

Again, it can be shown that the cut-off t_1 does not affect the relative wage w_S/w_S^*:

$$\begin{aligned}\frac{\partial(w_S/w_S^*)}{\partial t_1} &= \frac{\left[-2a^*(t_1)\left(S_{H^f} - \frac{f_f}{f_a}S_{H^a}\right) + 2a_H\left(\frac{f_f}{f_a}S^*_{H^a} - S^*_{H^f}\right)\right]\left(\frac{f_f}{f_a} - n\right)}{\left(S_{H^f} - \frac{f_f}{f_a}S_{H^a}\right)^2} \\ &= \frac{\left[-2a_H\left(\frac{f_f}{f_a}S^*_{H^a} - S^*_{H^f}\right) + 2a_H\left(\frac{f_f}{f_a}S^*_{H^a} - S^*_{H^f}\right)\right]\left(\frac{f_f}{f_a} - n\right)}{\left(S_{H^f} - \frac{f_f}{f_a}S_{H^a}\right)^2} = 0\ .\end{aligned}$$

The aggregate price index is given by $P_X = \left(N_{H^f}p_{H^f}^{1-\sigma_X} + N_{H^a}p_{H^a}^{1-\sigma_X}\right)^{1/(1-\sigma_X)}$. With $p_{H^a} f_f = p_{H^f} f_a$ we obtain

$$\begin{aligned}N_{H^f} + \left(\frac{f_a}{f_f}\right)^{1-\sigma_X} N_{H^a} &= \frac{\beta\left(S_Y + \bar{S}\right)\frac{w_S}{w_S^*}}{F_{H^f}\sigma_X\left(S_{H^f}\frac{w_S}{w_S^*} + S^*_{H^f}\right)} \\ &= \frac{\beta\left(S_Y + \bar{S}\right)\left(S^*_{H^a}\frac{f_f}{f_a} - S^*_{H^f}\right)}{F_{H^f}\sigma_X\left(S_{H^f}S^*_{H^a} - S^*_{H^f}S_{H^a}\right)\frac{f_f}{f_a}}\ .\end{aligned}$$

As in section 4.2, we may proceed by inserting $S_Y = \bar{S} - \sigma_X F_{H^f} S_{H^f} N_{H^f} - \sigma_X F_{H^a} S_{H^a} N_{H^a}$ to obtain N_{H^f} as a function of N_{H^a}, with the property $dN_{H^f}/dN_{H^a} < 0$:

$$N_{H^f} = \frac{2\beta\bar{S}\tilde{B}}{F_{H^f}\sigma_X\left[1 + \beta S_{H^f}\tilde{B}\right]} - \frac{\left[1 + \beta S_{H^a}\tilde{B}\right]}{\left[1 + \beta S_{H^f}\tilde{B}\right]}\left(\frac{f_f}{f_a}\right)^{\sigma_X - 1} N_{H^a}\ , \tag{C.2}$$

$$\text{where}\hat{E} \qquad \tilde{B} = \frac{S^*_{H^a}\frac{f_f}{f_a} - S^*_{H^f}}{\left(S_{H^f}S^*_{H^a} - S^*_{H^f}S_{H^a}\right)\frac{f_f}{f_a}}\ .$$

Equations (42) and (C.1) yield

$$\frac{S^*_{H^a}\frac{f_f}{f_a} - S^*_{H^f}}{S_{H^f} - \frac{f_f}{f_a}S_{H^a}} = \left(\frac{\bar{S} - \sigma_X\left(F_{H^f}S_{H^f}N_{H^f} + F_{H^a}S_{H^a}N_{H^a}\right)}{\bar{S}^* - \sigma_X\left(F_{H^f}S^*_{H^f}N_{H^f} + F_{H^a}S^*_{H^a}N_{H^a}\right)}\right)^{-1/2}\left(\frac{\bar{L}}{\bar{L}^*}\right)^{1/2}\ . \tag{C.3}$$

Again, from these equations, N_{H^f} and N_{H^a} can be determined in closed form.