Trade, Innovation and Productivity

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Abstract

Empirical evidence shows that trade liberalization improves productivity not just because of a selection effect but also because of productivity gains within firms. This paper proposes a trade model that allows for both channels, by adding the option to innovate. In contrast to the existing literature, the process innovation is modeled as a continuous variable and there is both a fixed and a variable cost to innovate. The interaction between the innovation and export choices is key to understand the different equilibria in the open economy and the outcomes following a trade liberalization. I calibrate the model to match the Spanish economy and explore the consequences of different trade policies. Simulations reveal that a fixed trade cost liberalization is more effective on innovation while a variable trade cost liberalization is more effective on average productivity.

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1 Introduction

The link between firm productivity and international trade has become increasingly important as an area of economic research over the last ten years. Evidence strongly supports the self selection of more productive firms into foreign market participation, but there is mixed evidence on the positive impact of export market participation on firm productivity. Motivated by these observations, I study the existence of within firm productivity gains through innovation in an open economy.

The literature has focused on models with heterogeneous firms because of their ability to match the export decisions of firms to characteristics such as productivity, size or ownership status. This literature, with early contributions by Bernard et al. (2003) and Melitz (2003), emphasizes the selection effect into export status and the reallocation of production factors and shares between firms as the source of aggregate productivity gains. However, empirical evidence points out that productivity gains occur not only between firms but also within firms(Pavcnik (2002), Trefler (2004), and De Loecker (2007)). The early literature did not identify within firm gains, because firm productivity was modeled by a random draw from a probability distribution.

The empirical literature has emphasized that the effect of trade liberalization on within firm productivity happens both along the extensive margin and the intensive margin. For example. Alvarez (2001) supports the hypothesis that exporters invest more intensively than non exporters. Likewise, Aw et al. (2008) show empirically that prior export market activity increases the probability of investing in R&D and that the interdependence between R&D activities and the exporting choice is critical to explain current investment decisions. Finally, Lileeva and Trefler (2010) show that new exporters innovate along different dimensions. Hence, evidence suggests that both the intensive and extensive margin are important in order to explain the productivity gains within firms and that the interaction of innovation, entry, exit and export decisions should be further explored.

This paper proposes a trade model with heterogeneous firms that have the option to invest in process innovation. The model follows the standard setup of Melitz (2003) with a basic difference: once a firm learns about its productivity it can decide to spend resources in process innovation to improve its technology. I am not the first to explore the effect of trade on within firm productivity gains, but I am the first to study this issue along both the intensive and the extensive margin. In contrast to Navas-Ruiz and Sala (2007) and Costantini and Melitz (2008) , I model process innovation as a continuous variable, and therefore can analyze the intensive margin. And, in contrast to Vannoorenberghe (2008), Bustos (2011) and Atkeson and Burstein (2010) innovation involves both fixed and variable costs. By having both a fixed and variable costs not all firms will be innovating and I can explore how trade liberalization affects the extensive margin and intensive margin of innovation at the same time.

The interaction between the innovation decision and the exporting decision will determine which one of the equilibria emerges. In all equilibria, firms at the high end of the distribution will export and innovate, while firms at the lower end of the distribution will not perform any of those activities. The behavior of middle productivity firms differs across equilibria. In the *low cost innovation equilibrium* trade costs are high in comparison with the cost-benefit ratio of innovation, so that middle productivity firms choose to innovate rather than enter new markets. In the *low cost trade equilibrium*, trade costs are low in relation to the cost-benefit to innovation, and firms that are productive enough choose to export rather than engage in innovation. In between these equilibria is the *intermediate equilibrium*, where firms are either very productive and can undertake both activities or do not perform any of them.

Analytically the main contribution of the paper is the ability to analyze the innovation decisions of firms through a tractable innovation policy function. A second contribution of the paper is understanding how a trade liberalization affects firms decision to innovate and export and to provide insight into the channels through which productivity gains occur. To my knowledge this is the first paper that studies along which margin within-firm productivity gains from innovation may happen.

The paper is organized as follows. In Section 2, I present the model of the economy where firms take decisions on innovation and exporting. In Section 3, I explore the equilibria determined by the interaction between the exporting and innovation choices creates. In Section 4, I calibrate the model to match the Spanish economy. In Section 5 I analyze the effects of two different trade policies on firms decisions, aggregate innovation and aggregate productivity. Section 6 concludes.

2 Model

The model is based on the monopolistic competition framework proposed by Melitz (2003). I consider a symmetric $n+1$ country world each of which use a single factor of production (labour L) to produce goods.The model is extended to allow these firms to have the opportunity to engage in process innovation.

2.1 Demand

I denote the source country by i and the destination country by j, where $i, j = 1, ..., n+1$. In each country j, there is a continuum of consumers of measure L_j. Given the set Ω of varieties supplied to the market, the consumer's preferences of country j are represented by the standard C.E.S. utility function

$$\left[\int_{\omega\in\Omega} q_{ij}^{\rho}(\omega)d\omega\right]^{\frac{1}{\rho}}$$

where $q_{ij}(\omega)$ denotes the quantity consumed of variety ω produced by firm i in country j and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution across varieties.

The market is subject to the expenditure-income constraint:

$$\int_{\omega\in\Omega} p_{ij}(\omega) q_{ij}(\omega) d\omega = R_j$$

where R_j is the total revenues obtained in country j.

Then standard utility maximization implies that the demand for each individual variety will be:

$$q_{ij}(\omega) = [p_{ij}(\omega)]^{-\sigma} \frac{R_j}{P_j^{1-\sigma}} \tag{1}$$

where $p_{ij}(\omega)$ is the price of each variety ω and $P_j = \left[\int_{\omega\in\Omega} p_{ij}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ denotes the price index of the economy.

2.2 Supply

There is a continuum of firms, each producing a different variety ω. Each firm draws its productivity φ from a distribution $G(\varphi)$ with support $(0, \infty)$ after paying a labor sunk cost of entry f_E. Since a firm is characterized by its productivity φ, it is equivalent to talk about variety ω or productivity φ.

Production requires only labor, which is inelastically supplied at its aggregate level L_j, and therefore can be taken as an index of country's j size.

In contrast to the Melitz model where firms use a constant returns to scale production technology, firms can affect their marginal cost through process innovation. To enter country j, firm i needs $f_{ij} > 0$ labour units and I make the standard iceberg cost assumption that $\tau_{ij} > 1$ units of the good have to be produced by firm i to deliver one unit to country j. Without loss of generality, I assume that $\tau_{ii} = 1$ and thus I denote $\tau_{ij} = \tau \ \forall i \neq j$.[1] Hence,

[1]Note that $\tau_{ij} = \tau_{ji}$ by symmetry and there is no possibility of transportation arbitrage

to produce an output $q_{ij}(\varphi)$, a firm requires $l_{ij}(\varphi)$ labor units

$$l_{ij}(\varphi) = f_{ij} + c(z_i) + \frac{q_{ij}(\varphi)}{\varphi} \frac{\tau_{ij}}{(1+z_i)^{\frac{1}{\sigma-1}}}$$

where z_i is a measure of the productivity increase from innovation that has an associated cost function $c(z_i)$.

The cost function of the innovation follows Klette and Kortum (2004), Lentz and Mortensen (2008) and Stähler et al. (2007). Firms pay a fixed cost, that can be attributed to the acquisition and implementation of the technology, plus a variable cost that depends directly on the process innovation performed by each firm. Hence the cost function $c(z_i)$ is defined as

$$c(z_i) = \begin{cases} z_i^{(\alpha+1)} + \kappa & \text{if } z > 0 \\ 0 & \text{if } z = 0 \end{cases}$$

where κ is the fixed cost required to implement the process innovation and $\alpha > 0$ measures the rate at which the marginal cost of the innovation increases, thus the higher the level of innovation the higher the cost associated with marginal increases.

Even though it can be argued that the cost of innovation can be simplified by imposing a linear variable cost, the existence of convex innovation costs is a standard feature in the literature and ensures that innovation is finite. Another simplification would be to have either a fixed cost or a variable cost but not both. Nevertheless maintaining a flexible cost function is important. For example, Vannoorenberghe (2008) assumes away a fixed innovation cost, which implies that all firms engage in process innovation. This eliminates the possibility of studying the interaction between the export and innovation decisions along the extensive margin, which is one of the purposes of this paper.

2.3 Firm's problem

Figure 1 represents the timing of the firm problem in the open economy. In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E, in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide which activities to undertake. Since both exporting and innovation require paying a labor fixed cost, f_X and κ respectively, there will be four types of firms in the open economy. Type D firms are only active in the domestic market and do not perform innovation; Type DI firms are those active only in the domestic market that innovate; Type X firms are those active in both the domestic and the foreign market that do not perform any innovation; and Type XI firms are active in the domestic and foreign markets that engage on innovation activities. Finally, in the third stage, firms decide prices.

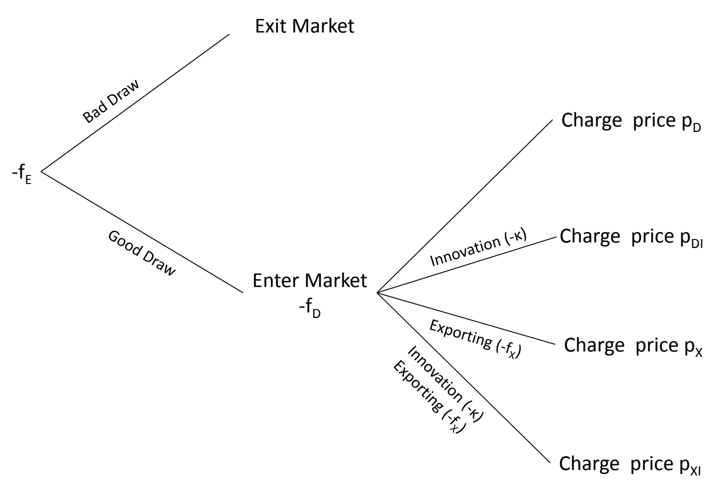

Figure 1: Timing

Given the timing, I solve the firms problem through backward induction.

Optimal Pricing Rule In the last stage of the problem the firm sets its optimal price given its innovation decision and the market conditions which are summarized by the price index P_j and R_j.

$$\max_{p_{ij}(\varphi)} p_{ij}(\varphi)\, q_{ij}(\varphi) - f_{ij} - \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi \left[(1+z_i)^{\frac{1}{\sigma-1}}\right]} - c(z_i)$$

The corresponding first order condition is

$$p_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\right) \frac{\tau_{ij}}{\varphi} \cdot \frac{1}{(1+z_i)^{\frac{1}{\sigma-1}}} \quad \forall\, z \tag{2}$$

Optimal Innovation Decision The returns of process innovation increase with the participation in more countries. Thus, the optimal innovation rule for firm i is obtained from the first order condition of the maximization of $\sum_j \pi_{ij}(\varphi) = \sum_j [p_{ij}(\varphi) q_{ij}(\varphi) - l_{ij}(\varphi)]$ with respect to z_i, provided that the firm makes higher profits by innovating than by choosing not to innovate.

$$z_i(\varphi) = \begin{cases} [1+n\tau^{1-\sigma}]^{\frac{1}{\alpha}} \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}} & \text{if } \sum_j \pi^{I}_{ij}(\varphi) \geq \sum_j \pi^{NI}_{ij}(\varphi) \\ 0 & \text{if } \sum_j \pi^{I}_{ij}(\varphi) < \sum_j \pi^{NI}_{ij}(\varphi) \end{cases} \tag{3}$$

where $\frac{1}{\alpha}$ is the parameter that shapes the optimal innovation function and tells us how innovation rises with size, where I take the productivity parameter $\varphi^{\sigma-1}$ to be the indicator of size. If the function is linear ($\alpha = 1$), then innovation rises proportionately with size, however, if the function is concave ($\alpha > 1$), then the amount of innovation performed will rise less than proportionally with size, and if the function is convex ($0 < \alpha < 1$) the amount of innovation performed will increase more than proportionally with the productivity.

To make the joint decision of whether to enter the foreign markets and whether to innovate or not, firms will choose the option that yields the highest profits. Since countries are symmetric we can drop the subscripts and classify firms in four types.

- Profits of a domestic non-innovator firm (Type D):

$$\pi_D = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - f_D$$

- Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1 + z_D\left(\varphi\right)\right) - f_D - c\left(z_D\left(\varphi\right)\right)$$

- Profits of an exporter non-innovator firm (Type X):

$$\pi_X = \left(1 + n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - nf_X - f_D$$

- Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1 + n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1 + z_X\left(\varphi\right)\right) - nf_X - f_D - c\left(z_X\left(\varphi\right)\right)$$

where $f_D = f_{ii}$, $f_X = f_{ij} = f_{ji}\ \forall j \neq i$, $z_D(\varphi) = \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$, and $z_X(\varphi) = \left[1 + n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}}\left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$.

3 Equilibrium

There will be three different equilibria that will cover the whole parameter space. First, the *low-cost innovation equilibrium*, where the activity of exporting is relatively costly in comparison to innovation and therefore only the most productive firms will carry on both activities, middle productivity firms will innovate but not export and the lower productivity firms will neither innovate nor export. Second, the *low-cost trade equilibrium,* where the activity of innovation is relatively costly in comparison to exporting and therefore only the most productive firms will carry on both activities, middle productivity firms will export but not engage in innovation and the lower productivity firms will neither innovate nor export. Thirdly, between these two equilibria there will be the *intermediate equilibrium* where firms are either very productive and can undertake both activities or do not perform any of them.

The existence of these three equilibria is consistent with the empirical evidence found both in the trade and the innovation literature. Costantini and Melitz (2008) suggest that exporting and innovation are performed by the most productive firms while domestic producers are typically less innovative and less productive, a feature common to all the equilibria. Vives (2008)provides intuition for the decisions taken by middle productivity firms in each equilibrium. If trade costs are relatively high, middle productivity firms are domestic innovators while being an exporter without innovating is not profitable. A decrease in trade costs attracts the most productive firms from the foreign country, discouraging middle productivity domestic firms to undertake innovation. The disappearance of domestic innovators as trade costs fall can be explained by this Schumpeterian effect and is also predicted by the dynamic model of Costantini and Melitz (2008). However, a fall in trade costs enables more firms to participate actively in both markets which explains the existence of exporter non-innovators when trade costs are low enough.

Different papers have identified these equilibria separately, but never all in a single model. Bustos (2011) identifies the equilibrium where there are no domestic innovators firms since it is an unprofitable choice. In Vannoorenberghe (2008) all firms innovate, therefore it is not possible to study the interaction between both decisions. Finally, Navas-Ruiz and Sala (2007) identify the two extreme equilibria, but fail to identify the intermediate equilibrium. The main contribution of the theoretical model is the identification of all the equilibria with the ability to study the transitions between them and the possible productivity gains that might occur through the intensive and extensive margins of innovation. In this section, I describe each of the two main equilibria, the effects that trade has on innovation in each case, the parameter restrictions that give rise to the different equilibriums and conclude by focusing on the interaction between exporting and innovation.

3.1 Low Cost Innovation Equilibrium

The *low cost innovation equilibrium* is characterized by exporting being less attractive than innovation. In Figure 2, I depict the profits of all types of firms as a function of productivity when trade costs are relatively high in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating is profitable, that is, the marginal exporter is an innovator as well.

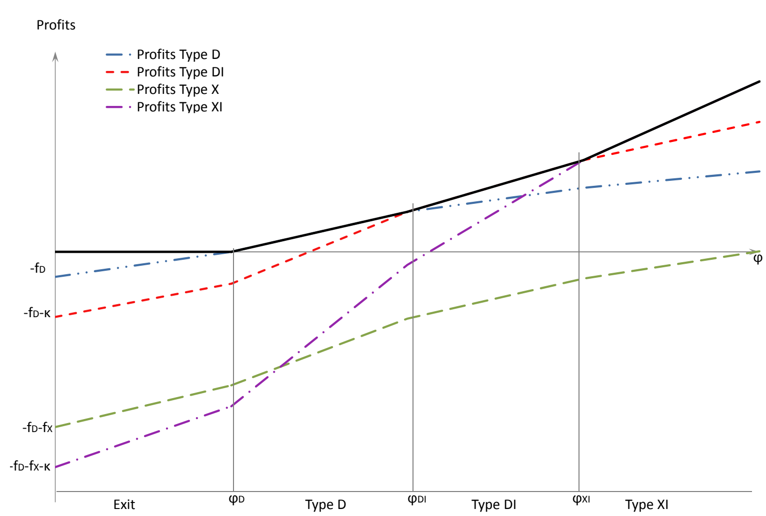

Figure 2: Low Cost Innovation Selection Path

The conditions of entry in the domestic and export markets plus the innovation condition allows to solve for the different productivity cutoffs in the *low cost innovation equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market is $\pi_D\left(\varphi_D^*\right)=0$, so that:

$$\left(\varphi_D^*\right)^{\sigma-1}=\frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{4}$$

The Innovation Profit Condition (IPC) determines the productivity cutoff φ_{DI}^* which is the productivity of the firm indifferent between innovating or not while operating only on the domestic market, i.e. $\pi_{DI}\left(\varphi_{DI}^*\right)=\pi_D\left(\varphi_{DI}^*\right)$, so that:

$$\left(\varphi_{DI}^*\right)^{\sigma-1}=\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{5}$$

The Innovation Export Profit Condition (IXPC) determines the exporting-innovation cutoff φ_{XI}^* which is the productivity of an innovating firm indifferent between participating also on the exporting market or not.

$$\pi_{XI}\left(\varphi_{XI}\right)-\pi_{DI}\left(\varphi_{XI}\right)=0 \tag{6}$$

Proposition 1.

The economy is in the low cost innovation equilibrium, $\varphi_{XI}^*>\varphi_{DI}^*>\varphi_D^*$, if the following parameter restrictions hold

1. $\tau^{\sigma-1}f_X\geq\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa+\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$
2. $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)\geq f_D$

Proof. The formal proof can be found in the Appendix A. The proof is divided in two parts. First I show that there exist a single solution to equation (6). The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ^*_{XI}. Nevertheless, I show that selection into exporting and innovation ($\varphi^*_{XI} > \varphi^*_{DI}$) requires that condition 1 of Proposition 1 holds, that is exporting costs should be high enough relative to innovation costs. Notice that condition 2 of Proposition 1 ensures that there is selection into innovation ($\varphi^*_{DI} > \varphi^*_{D}$). Secondly, I show that equations (4) to (6) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_{DI}} \pi_D(\varphi)\,dG(\varphi) + \int_{\varphi^*_{DI}}^{\varphi^*_{XI}} \pi_{DI}(\varphi)\,dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\,dG(\varphi)\right] = f_E \tag{7}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs $\varphi^*_D,\ \varphi^*_{DI}$ *and* φ^*_{XI}. □

3.2 Low Cost Trade Equilibrium

The *low cost trade equilibrium* is characterized by exporting being more attractive than innovation. In Figure 3, I depict the profits of all types of firms as a function of productivity when trade cost are relatively low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where innovation without exporting is profitable, that is, the marginal innovator is an exporter.

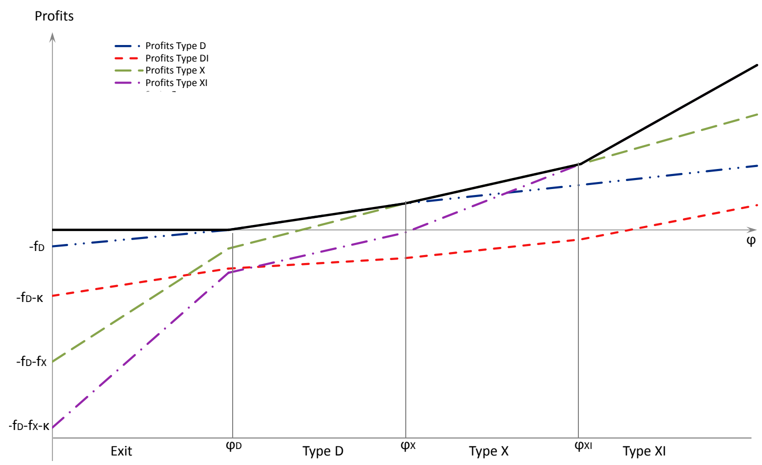

Figure 3: Low Cost Trade Selection Path

The conditions of entry in the domestic and export markets, plus the innovation conditions allows to solve the different productivity cutoffs in the *low cost trade equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[2] is $\pi_D\left(\varphi_D^*\right)=0$ so that:

$$\left(\varphi_D^*\right)^{\sigma-1}=\frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{8}$$

The Exporting Profit Condition (XPC) determines the exporting-entry productivity cutoff φ_X^* which is the productivity of the firm indifferent between staying in the domestic market and participating in the export market, i.e. $\pi_X\left(\varphi_X^*\right)=\pi_D\left(\varphi_X^*\right)$:

$$\left(\varphi_X^*\right)^{\sigma-1}=\frac{f_X}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\tau^{1-\sigma}} \tag{9}$$

[2] The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ^*_{XI}, which is the productivity of an exporting firm indifferent between innovating or not, i.e. $\pi_{XI}(\varphi^*_{XI}) = \pi_X(\varphi^*_{XI})$:

$$(\varphi^*_{XI})^{\sigma-1} = \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)(1+n\tau^{1-\sigma})} \tag{10}$$

Proposition 2.

The economy is in the low cost trade equilibrium, $\varphi^*_{XI} > \varphi^*_X > \varphi^*_D$, if the following parameter restrictions hold

$$\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1}f_X \geq f_D$$

Proof. Selection into exporting and innovation ($\varphi^*_{XI} > \varphi^*_X$) requires innovation costs to be high enough relative to trade costs and selection into exporting ($\varphi^*_X > \varphi^*_D$) requires trade costs to be high enough relative to production costs. Equations (8) *to* (10) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_X}\pi_D(\varphi)\,dG(\varphi) + \int_{\varphi^*_X}^{\varphi^*_{XI}}\pi_X(\varphi)\,dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty}\pi_{XI}(\varphi)\,dG(\varphi)\right] = f_E \tag{11}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_{XI}, φ^*_X φ^*_{XI}. See Appendix B for a formal proof. □

3.3 Discussion

The firm productivity distribution varies along the parameter space according to the relation between trade costs and the relative innovation costs. This is especially relevant for firms with an intermediate level of productivity, as their decisions will be most sensitive to these costs. In particular, in the *low cost innovation equilibrium*, when trade costs are high enough, they are domestic innovators. In the *low cost trade equilibrium*, when trade costs are low enough in relation to innovation costs, middle productivity firms will be exporters and the most productive of them will export and innovate. In between these two equilibria, there is the *intermediate equilibrium*, where trade costs are not relatively high enough for firms to be domestic innovators nor low enough for firms to be exporters non- innovators. That is, middle productivity firms are either exporter innovators or domestic firms. These choices are the ones that determine the parameter restrictions associated to each equilibrium. Furthermore, notice that the three equilibria cover the whole parameter space, and therefore the firm productivity distribution and the effects of opening up to trade of an economy can be determined always. Table 1 summarizes all the possible equilibria in the open economy and the parameter restrictions associated to each one.

Equilibrium	Conditions
Low Cost Innovation Equilibrium	$\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$ & $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$
Intermediate Equilibrium	$\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq \tau^{\sigma-1} f_X$ & $\tau^{\sigma-1} f_X \geq \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq f_D$
Low Cost Trade Equilibrium	$\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1} f_X \geq f_D$

Table 1: Equilibria in the Open Economy

Furthermore, the model has implications on the aggregate productivity level. Firstly, trade induces the exit of the less productive firms and the reallocation of market shares towards the more productive firms, rising the industry average productivity in the long run. This is the selection effect described in Melitz (2003). And secondly, trade has indirect effects on the average productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect trade has on innovation will be differentiated according to the level of transportation costs. On the one hand, there is an effect through the intensive margin of innovation. The innovation intensity increases with the participation in foreign markets and thus, the effect will be larger in the *low cost trade equilibrium* where the economy is more open. On the other hand, there is an effect through the extensive margin of innovation. In Crespo Rodríguez (2011), it is shown that the impact on average productivity through the extensive margin will be negative in the *low cost innovation equilibrium*, undetermined in the *intermediate equilibrium* and can be positive in the *low cost trade equilibrium*. In the empirical analysis we will decompose the change in productivity due to trade costs into these components and quantify their relevance.

4 Calibration

The calibration uses the final version of the firm level EFIGE dataset, a survey collected within the project 'EFIGE - European Firms in a Global Economy: internal policies for external competitiveness'. The sample includes around 3,000 firms for France, Germany, Italy and Spain, more than 2200 firms for UK and 500 firms for Austria and Hungary. This survey, conducted during the year 2009, contains both qualitative and quantitative information data on firms' characteristics and activities. All questions concern the year 2008, with some questions asking information in 2009 and previous years in order to have a picture of the crisis effects and the dynamic evolution of firms' activity. The model is calibrated to match five of the seven European economies of the dataset: France, Germany, Italy, Spain and the U.K. . The distribution by firm size for the sample and the reference population [3] [4] are shown for each country in Table 2.

	Between 10 and 49		Between 50 and 249		More than 250		Total	
Country	S	P	S	P	S	P	S	P
France	2,151	32,019	608	7,365	214	1,986	2973	41,370
Germany	1836	52,489	793	16,988	306	3,970	2,935	79,144
Italy	2,447	77,092	429	10,062	145	1,408	3,021	88,562
Spain	2,280	38,116	406	6,241	146	1,010	2,832	45,367
U. K.	1,515	27,187	529	7,794	112	1,758	2,156	36,739

Table 2: Distribution by size, sample (S)/reference population(P)

Parameters common to all countries are taken directly from the empirical literature so that the model displays key aggregate and firm level patterns of the data, while parameters specific to each country are calibrated such that particular moments in the model match those moments in the data.

Parameters common to all countries are the elasticity of substitution, the elasticity of innovation, the death shock and the sunk cost of entry. The elasticity of substitution is set to be consistent with empirical estimates provided by Broda and Weinstein (2006), who estimate over 30,000 import elasticities. The medians reported vary from 2.2 to 4.8 depending on the level of aggregation, thus I set $\sigma = 3$ which lies within the estimated values. The innovation parameter is $\alpha = 0.9$. This value is consistent with the estimate of Rubini (2009), who sets the elasticity of productivity to resources devoted to innovation to match a 5% gain in labor productivity in Canada due to

[3] The sample design over-represents large firms, therefore sampling weights have been constructed in terms of size-sector cells to make the sample representative of the underlying population.

[4] Calibration is based on the weighted sample.

the tariff reduction in the U.S.-Canada Free Trade Agreement between 1980 and 1996. The death shock and the sunk cost of entry determine the entry and exit of firms, following Bernard et al. (2007) I set them to $\delta = 0.025$ and $f_E = 2$.

Parameters specific to each country are the number of trading countries, the productivity distribution, innovation costs, export fixed costs , variable trade costs and domestic fixed costs. The last four are calibrated jointly to match the number of workers in innovation, the percentage of exporters innovators in the economy, the aggregate export volume and the percentage of skilled workers in the labour force. To match the productivity distribution, I target the slope of the firm distribution according to employees and similarly to Helpman et al. (2004) and Chaney (2008), I assume the productivity is distributed according to a Pareto with a probability density function

$$g(\varphi) = \frac{\theta}{\varphi^{\theta+1}}$$

where $\varphi \in [1, \infty)$ and θ is the curvature parameter. In accordance to the model considered, I estimate by maximum likelihood the curvature parameter associated to the distribution of firms, $\tilde{\theta} = \theta/(\sigma - 1)\left(\frac{\alpha+1}{\alpha}\right)$. Finally, the number of trading countries is determined by a country's size with respect to the other countries in the EFIGE dataset For a country's size, we take total employment in the EFIGE dataset. The targets are in Table 3.

Country	Slope	Employees	Executives	Trade Flow	Type XI	R&D Workers
France	1.06	2,903,820	17.4%	27.30%	22.22%	6.81%
Germany	1.10	5,565,414	9.3%	19.48%	27.59%	6.16%
Italy	1.43	3,555,052	7.6%	32.81%	27.73%	5.81%
Spain	1.27	2,010,424	9.5%	21.50%	19.89%	4.85%
U.K.	1.01	3,729,340	14.5%	25.84%	24.31%	7.38%

Table 3: Calibration Targets

The calibrated parameters along with the predicted equilibrium for each country are in Table 4.

Country	θ	n	f_D	τ	f_X	κ	Predicted Equilibrium
France	4.9	6	0.95	1.91	0.40	5.75	Low Cost Trade
Germany	5.1	2	2	1.14	8.4	10.6	Intermediate
Italy	6.6	4	1.5	1.19	5.5	6	Intermediate
Spain	5.9	8	2	1.93	4.3	2.55	Low Cost Innovation
U.K.	4.7	1.25	1.68	1.45	0.6	8.5	Low Cost Trade

Table 4: Calibrated Parameters

5 Empirical Analysis *(Coming Soon)*

6 Conclusions

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Appendix A - Low Cost Innovation Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, active innovators and active innovators and exporters prior to innovation.

$$\mu_D(\varphi)=\begin{cases}\frac{g(\varphi)}{G(\varphi_{DI})-G(\varphi_D)} & ,\varphi_{DI}>\varphi\geq\varphi_D\\ 0 & ,otherwise\end{cases}$$

$$\mu_{DI}(\varphi)=\begin{cases}\frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_{DI})} & ,\varphi_{XI}\geq\varphi\geq\varphi_{DI}\\ 0 & ,otherwise\end{cases}$$

$$\mu_{XI}(\varphi)=\begin{cases}\frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\varphi\geq\varphi_{XI}\\ 0 & ,otherwise\end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi}=\left[\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X=\left[\int_{\varphi_{XI}}^{\infty}\varphi^{\sigma-1}\mu_{XI}(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t=\left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1}+nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{DI}=\left[\int_{\varphi_{DI}}^{\infty}\left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}}\mu_{DI}(\varphi)\,d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_{XI}$ represent the average productivity the domestic innovators and exporter innovators get from innovation. Then the weighted productivity average that reflects the combined market share of innovation can be defined as

$$\widetilde{\varphi}_t^I=\left\{\frac{1}{M_t^I}\left[M_I\left(\widetilde{\varphi}_{DI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}+m_{XI}\left(\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right)\left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right]\right\}^{\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\sigma-1}\right)}$$

Aggregate Variables

Denote by m_{XI}, m_{DI} and m_D respectively the mass of active innovators and exporters, active innovators but non-exporters and non-innovators and non-exporters present in the economy,

$$\begin{aligned} m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M \\ m_{DI} &= \frac{G(\varphi_{XI})-G(\varphi_{DI})}{1-G(\varphi_D)}M \\ m_D &= \frac{G(\varphi_{DI})-G(\varphi_D)}{1-G(\varphi_D)}M \end{aligned}$$

with M being the mass of incumbent firms in the economy, $M_I = m_{DI} + m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$, and the total number of varieties coming from innovators will be $M_t^I = M_I + nM_X$. It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}$$

- Aggregate Production

$$Q^{\rho} = M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho}$$

- Aggregate Revenue

$$R = M_t r_D\left(\widetilde{\varphi}_t\right) + M_t^I\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)$$

- Aggregate Profits

$$\begin{aligned}\Pi &= M_t\frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} + M_t^I\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\frac{r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)}{\sigma} \\ &\quad - Mf_D - nM_X f_X - M_I\kappa - M_t^I\left(\frac{\kappa}{\alpha}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}}\left(\widetilde{\varphi}_t^I\right)^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}\end{aligned} \tag{A.1}$$

Low Cost Innovation Equilibrium

Proof of Proposition 1, part II

If there are sufficiently high fixed export cost, there exist a single cutoff φ_{XI}^* that solves equation (6)

Proof. The proof is divided in three sections

First, I show that the LHS of equation (6) is positive with respect to the productivity parameter. $\pi_{XI}\left(\varphi_{XI}\right) - \pi_{DI}\left(\varphi_{XI}\right) \geq 0$

$$\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\alpha\left(\tfrac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + n\tau^{1-\sigma}\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_x \geq 0$$

$$C_1\left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}} + C_2\varphi^{\sigma-1} - nf_x \geq 0$$

$$\frac{\partial LHS}{\partial\varphi} = C_1\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1} + C_2(\sigma-1)\varphi^{\sigma-2} > 0$$

Secondly, I show that $\pi_{XI}\left(\varphi_{DI}\right) - \pi_{DI}\left(\varphi_{DI}\right) < 0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\begin{aligned}&\pi_{XI}\left(\varphi_{DI}\right)-\pi_{DI}\left(\varphi_{DI}\right)<0\\&\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\kappa+n\tau^{1-\sigma}\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)-nf_X<0\end{aligned}$$

Thus, for f_X large enough, that is for

$$f_X>\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\frac{\kappa}{n}+\tau^{1-\sigma}\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$$

it holds that $\pi_{XI}\left(\varphi_{DI}\right)-\pi_{DI}\left(\varphi_{DI}\right)<0$

Finally, I show that the difference between the profits of the exporting and non-exporting strategies while innovation goes to infinite as the productivity of the firm is larger.

If $\varphi\to\infty$,then $\pi_{XI}\left(z\left(\varphi\right)\right)-\pi_{DI}\left(z\left(\varphi\right)\right)\to\infty$, since by definition $\pi_{XI}\left(z_x\left(\varphi\right)\right)>\pi_{XI}\left(z\left(\varphi\right)\right)$ then it must be that $\pi_{XI}\left(z_X\left(\varphi\right)\right)-\pi_{DI}\left(z\left(\varphi\right)\right)\to\infty$ as $\varphi\to\infty$

$$\begin{aligned}\pi_{XI}\left(j\left(\varphi\right)\right)-\pi_{DI}\left(j\left(\varphi\right)\right)&=n\tau^{1-\sigma}\left[1+z\right]\left(\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}-nf_X\\&=n\tau^{1-\sigma}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{\alpha}}\left[\left(\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\\&\quad+n\tau^{1-\sigma}\left(\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}-nf_X\end{aligned}$$

$$\begin{aligned}\lim_{\varphi\to\infty}\left[\pi_{XI}\left(j\left(\varphi\right)\right)-\pi_{DI}\left(j\left(\varphi\right)\right)\right]&=\lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}+C_5\varphi^{\sigma-1}-C_6\right]\\&=\lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\right]+\lim_{\varphi\to\infty}\left[C_5\varphi^{\sigma-1}\right]-\lim_{\varphi\to\infty}\left(C_6\right)\to\infty\end{aligned}$$

□

Proof of Proposition 1, part I

Equations (4) to (6) along with the Free Entry condition (7) completely determine the equilibrium and the productivity cutoffs can be uniquely determined and allow me to rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = \left[1 - G\left(\varphi_D^*\right)\right]\overline{\pi}$$

$$\begin{aligned} \delta f_E \quad = \quad & f_D j_1\left(\varphi_D^*\right) + n\tau^{1-\sigma} f_D j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) - \left[1 - G\left(\varphi_{XI}^*\right)\right] n f_X \\ & - \left[1 - G\left(\varphi_{DI}^*\right)\right]\kappa + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D^{\frac{\alpha+1}{\alpha}} j_3\left(\varphi_D^*\right) \\ & + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D{}^{\frac{\alpha+1}{\alpha}}\left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] j_4\left(\varphi_D^*\right) \end{aligned} \tag{A.2}$$

where $j_1\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1} - 1\right]}{\left[1-G\left(\varphi_D^*\right)\right]}$, $j_2\left(\varphi_D^*\right) = \frac{\left(\widetilde{\varphi}_x\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$,
$j_3\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{DI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}}{\left[1-G\left(\varphi_{DI}^*\right)\right]}$ and $j_4\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$

Proof. Assume the parameter restrictions $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$ and $\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$ hold, then the Low Cost Innovation Equilibrium exists and is unique. I shall proof that the RHS of equation ($A.2$) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.

Let $k_1\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1} - 1\right]$, then

$$k_1'\left(\varphi_D^*\right) = \frac{g\left(\varphi_D^*\right)}{1 - G\left(\varphi_D^*\right)} k_1\left(\varphi_D^*\right) - \frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right) + 1\right]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1-G(\varphi_I^*)} \left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$

Now, define $j_1(\varphi_D^*) = [1-G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1-G(\varphi_{DI}^*)]\, k_2(\varphi_D^*)$ which are non-negative.

Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)[k_1(\varphi_D^*)+1]}{\varphi_D^*}[1-G(\varphi_D^*)] < 0$$
$$\frac{j_1'(\varphi_D^*)\cdot\varphi_D^*}{j_1(\varphi_D^*)} = \underbrace{-(\sigma-1)\left[1+\frac{1}{k_1(\varphi_D^*)}\right]}_{<0\text{and bounded away of it}} < -(\sigma-1)$$

and

$$j_3'(\varphi_D^*) = -g(\varphi_{DI}^*)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1)\frac{k_3(\varphi_D^*)}{\varphi_D^*}[1-G(\varphi_{DI}^*)] < 0$$
$$\frac{j_3'(\varphi_D^*)\cdot\varphi_D^*}{j_3(\varphi_D^*)} = \underbrace{-\frac{g(\varphi_{DI}^*)}{[1-G(\varphi_{DI}^*)]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2(\varphi_D^*)}\varphi_D^* - \beta(\sigma-1)}_{<0\text{and bounded away of it}} < -\beta(\sigma-1)$$

Thus, $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1(\varphi_D^*) = \infty$.and $\lim_{\varphi_D^*\to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^*\to 0} k_3(\varphi_D^*) = \infty$

Since $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$, it follows that $j_2(\varphi_D^*)$ and $j_4(\varphi_D^*)$ do also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of ($A.2$) is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix B - Low Cost Trade Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, exporters and innovators exporters.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_X)-G(\varphi_D)} & , \varphi_X > \varphi \geq \varphi_D \\ 0 & , otherwise \end{cases}$$

$$\mu_X(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_X)} & , \varphi_{XI} \geq \varphi \geq \varphi_X \\ 0 & , otherwise \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & , \varphi \geq \varphi_{XI} \\ 0 & , otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{XI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity the innovators get from innovation.

Aggregate Variables

Denote by m_{XI}, m_X and m_D respectively the mass of active innovators and exporters, only exporters and non-innovators non-exporters present in the economy,

$$
\begin{aligned}
m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M \\
m_X &= \frac{G(\varphi_{XI})-G(\varphi_X)}{1-G(\varphi_D)}M \\
m_D &= \frac{G(\varphi_X)-G(\varphi_D)}{1-G(\varphi_D)}M
\end{aligned}
$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_X + m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.
It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$
P^{1-\sigma} = M_t \left[p_D(\widetilde{\varphi}_t)\right]^{1-\sigma} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{1-\sigma}
$$

- Aggregate Production

$$
Q^{\rho} = M_t \left[q_D(\widetilde{\varphi}_t)\right]^{\rho} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[q_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{\rho}
$$

- Aggregate Revenue

$$
R = M_t r_D(\widetilde{\varphi}_t) + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)
$$

- Aggregate Profits

$$\Pi = M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \frac{r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)}{\sigma} \tag{B.1}$$

$$-Mf_D - nM_Xf_X - m_{XI}\kappa - m_{XI}\left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}}\left(\widetilde{\varphi}_{XI}^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}}$$

Low Cost Trade Economy Equilibrium

Proof of Proposition 2

Equations (8) to (10) along with the Free Entry condition (11)completely determine the equilibrium and the productivity cutoffs can be uniquely determined and I can rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = \left[1-G\left(\varphi_D^*\right)\right]\overline{\pi}$$

$$\delta f_E = f_D l_1\left(\varphi_D^*\right) + n f_X l_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[f_D\left(1+\tau^{1-\sigma}\right)\right]^{\left(\frac{\alpha+1}{\alpha}\right)} l_3\left(\varphi_D^*\right) - \left[1-G\left(\varphi_{XI}^*\right)\right]\kappa \tag{B.2}$$

where $j_1\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]}{\left[1-G\left(\varphi_D^*\right)\right]}$, $j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_X^*\right)/\varphi_X^*\right)^{\sigma-1}-1\right]}{\left[1-G\left(\varphi_X^*\right)\right]}$ and $j_3\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\left(\frac{\alpha+1}{\alpha}\right)}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$

Proof. Assume the parameter restriction $\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1}f_X \geq f_D$ holds, then the Low Cost Trade Equilibrium exists and is unique. I shall proof that the RHS of equation (9) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.

Let $k_1(\varphi_D^*) = \left[\left(\widetilde{\varphi}(\varphi_D^*) / \varphi_D^* \right)^{\sigma-1} - 1 \right]$, then

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[\left(\widetilde{\varphi}_{DI}(\varphi_D^*) / \varphi_D^* \right)^{\sigma-1} \right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1 - G(\varphi_I^*)} \left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}} \right] - \left(\frac{\alpha+1}{\alpha} \right) (\sigma - 1) \frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)}{f_D} \right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$

Now, define $j_1(\varphi_D^*) = [1 - G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1 - G(\varphi_{DI}^*)]\, k_2(\varphi_D^*)$ which are non-negative.

Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$\begin{aligned} j_1'(\varphi_D^*) &= -\frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*} \left[1 - G(\varphi_D^*)\right] < 0 \\ \frac{j_1'(\varphi_D^*) \cdot \varphi_D^*}{j_1(\varphi_D^*)} &= \underbrace{-(\sigma-1)\left[1 + \frac{1}{k_1(\varphi_D^*)}\right]}_{<0 \text{and bounded away of it}} < -(\sigma - 1) \end{aligned}$$

and

$$\begin{aligned} j_3'(\varphi_D^*) &= -g(\varphi_{DI}^*)\, \Lambda^{\frac{1}{\sigma-1}} \Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1) \frac{k_3(\varphi_D^*)}{\varphi_D^*} \left[1 - G(\varphi_{DI}^*)\right] < 0 \\ \frac{j_3'(\varphi_D^*) \cdot \varphi_D^*}{j_3(\varphi_D^*)} &= \underbrace{-\frac{g(\varphi_{DI}^*)}{\left[1 - G(\varphi_{DI}^*)\right]} \frac{\Lambda^{\frac{1}{\sigma-1}} \Lambda^{\frac{\alpha+1}{\alpha}}}{k_2(\varphi_D^*)} \varphi_D^* - \beta(\sigma-1)}_{<0 \text{and bounded away of it}} < -\beta(\sigma - 1) \end{aligned}$$

Thus, $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^* \to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_1(\varphi_D^*) = \infty$.and $\lim_{\varphi_D^* \to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_3(\varphi_D^*) = \infty$

Since $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are decreasing from infinity to zero on $(0,\infty)$,from the closed economy case, it follows that $j_2(\varphi_X^*(\varphi_D^*))$ does also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of ($B.2$) is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix C - Aggregates

Aggregate Innovation

The general expression of aggregate innovation is given by equation (**??**), since the distribution of innovators changes in the open economy according to the relationship between export costs and innovation costs, for each case aggregate innovation can be expressed as follows

Low cost innovation equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{DI}}^{\varphi_{XI}} z_D(\varphi)\, m_{DI}\mu_{DI}\varphi d\varphi + \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\widetilde{\varphi}_t^I\right)^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.1}$$

Intermediate equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= m_{XI}\left[\left(1+n\tau^{1-\sigma}\right)\frac{f_D}{\alpha+1}\right]^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.2}$$

Low cost trade equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= m_{XI}\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}(\widetilde{\varphi}_{XI})^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.3}$$

Aggregate Productivity

In what follows I show that the output of the economy can be expressed as a function of the number of workers in the economy, their productivity and the elasticity of substitution and that equation (**??**) is the general form of such expression in the open economy. For the proof we use the facts that in equilibrium $L = R$, that the budget constraint is $PQ = R$ and the price rule given by equation (2).

Low cost innovation equilibrium:

$$\begin{aligned} R &= M_t r_D(\widetilde{\varphi}_t) + M_t^I\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{DI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\ &+ \left.\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\} \end{aligned}$$

Then,

$$
\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma} \int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{DI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\
&+ \left. \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}}
\end{aligned}
$$

And

$$
\begin{aligned}
Q &= \left(\frac{\sigma-1}{\sigma}\right) \left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\
\Psi_I &= \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\Psi_{DI} + \left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\Psi_{XI}\right)
\end{aligned}
$$

where $\Psi_D = \int_{\varphi_D}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_{DI} = \int_{\varphi_{CI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Intermediate equilibrium:

$$
\begin{aligned}
R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_{XI}\right)^{\frac{\alpha+1}{\alpha}}\right) \\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma} \int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left. \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}
\end{aligned}
$$

Then,

$$\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left. \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}}
\end{aligned}$$

And

$$\begin{aligned}
Q &= \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\
\Psi_I &= \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \Psi_{XI}
\end{aligned}$$

where $\Psi_D = \int_{\varphi_D}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Low cost trade equilibrium:

$$\begin{aligned}
R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right) \\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma} \left\{ \int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left. \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}
\end{aligned}$$

Then,

$$\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left. \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}}
\end{aligned}$$

And

$$
\begin{aligned}
Q &= \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\
\Psi_I &= \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\Psi_{XI}
\end{aligned}
$$

where $\Psi_D = \int_{\varphi_D}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$