Additive versus multiplicative trade costs and the gains from trade

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Abstract

This paper addresses welfare effects from trade liberalization in a heterogeneous-firms trade model including the empirically important per-unit (i.e. additive) trade costs in addition to the conventional iceberg (i.e. multiplicative) and fixed trade costs. The novel contribution of the paper is the result that the welfare gain for a given increase in trade openness is higher for reductions in per-unit (additive) trade costs than for reductions in iceberg (multiplicative) trade costs. The ranking derives from differences in intra-industry reallocations and in particular from dissimilar impacts on the number of exporters (i.e., the extensive margin of trade).

JEL: F12, F13, F15

Key Words: iceberg trade costs, per-unit trade costs, heterogeneous firms, trade liberalization, welfare.

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1 Introduction

Costs of international trade are economically significant and of various types (Anderson and Wincoop, 2004). However, since Samuelson (1954) the so-called iceberg trade costs, with the property that trade costs are proportional to the value of the goods traded, have been widely applied as a catch-all formulation of variable trade costs within trade theory. Such proportionality may be reasonable for some variable trade costs such as insurance, trade finance, and hedging of exchange rate and credit risk. Conversely for other trade costs (e.g., transport costs) proportionality seems unreasonable unless value is strongly correlated with size and/or weight.

The heterogeneous-firms trade literature following Melitz (2003) analyzes intra-industry trade in settings where firms produce firm-specific varieties of a differentiated good. Firms within narrowly defined industries differ in productivity and thus costs of production. There is no reason to expect that transport costs of these varieties vary proportionally or even systematically with the value of the varieties. A pure multiplicative iceberg specification actually implies that firms with higher productivities have proportionally lower transport costs. Assuming that all firms have access to the same shipping companies, this implication is problematic, at best. Indeed in a structural estimation of a heterogeneous-firms trade model with both per-unit (additive) and iceberg trade costs, Irarrazabal et al. (2011) find that per-unit trade costs are large and quantitatively important. Hence, a trade-cost structure including industry-specific per-unit transport costs - such as the one proposed by the present model - is highly appropriate; particularly in heterogeneous-firms trade models.

The present paper thus contributes to the literature by analyzing welfare effects from trade liberalization in a heterogeneous-firms trade framework including additive variable trade costs. Formally, a per-unit trade cost is added to the seminal Melitz (2003) framework while maintaining the customary iceberg and fixed export costs. First, it is shown that the well-known result of welfare gains from trade liberalization is robust to the inclusion of per-unit trade costs. Any mode of trade liberalization (reduction in any form of trade cost) increases welfare through intra-industry reallocations. Secondly, and a novel contribution of the paper, it is shown that the increase in welfare following an increase in trade openness is larger when the increase in openness is driven by a reduction in (additive) per-unit trade costs than by a reduction in (multiplicative) iceberg trade costs. Notably this ranking relies neither on the initial equilibrium considered nor on a specific functional form for the distribution of marginal productivities. This ranking is obviously important: when evaluating welfare gains from an observed increase in trade openness; the source of trade liberalization matters. Arkolakis et al. (2012) show that welfare gains from trade liberalization in a broad class of trade models including the Melitz (2003) model (under the assumption of Pareto distributed productivities) may be summarized by the elasticity of imports with respect to variable trade costs and the effect on trade openness. Thus a corollary to the welfare ranking established in the present paper is thus that the result of Arkolakis et al. (2012) is not robust to the inclusion of additive per-unit trade costs.

The novel welfare ranking arises due to differences in the intra-industry reallocations which are central to the heterogeneous-firms trade literature. Lower per-unit trade costs (contrary to lower iceberg trade costs) reduce the export market prices of the most productive exporters relative to the less productive exporters and thus shift relative market shares among exporters towards the more productive exporters. Given the equal openness criterion, total export sales must be the same across the liberalization modes and the lower market shares of the least productive exporters imply fewer exporting firms and thus fewer fixed/sunk costs of exporting. With identical export sales but fewer fixed export costs, the expected export profits increase by more in the case of lower per-unit costs. This larger increase in expected profits drives the welfare ranking since it is the increased incentive to enter the industry (due to higher expected export market profits) that causes the welfare enhancing intra-industry reallocations well known from the Melitz (2003) framework.

Distinguishing between per-unit and iceberg trade costs and their potential differential impacts on trade is not new to the trade literature. Alchian and Allen (1964) hypothesized that per-unit trade costs reduce the relative price and thus increase relative sales of high price/quality goods on the export market ("shipping the good apples out"). Forty years later, Hummels and Skiba (2004) found strong empirical support for this hypothesis. More recently, Berman et al. (2012) introduced a model with additive (per-unit) local distribution costs and heterogeneous firms. However, their focus is on pricing to market and heterogeneous price reactions to changes in the exchange rate. As mentioned above, Irarrazabal et al. (2011) add per-unit trade costs to a heterogeneous-firms trade model. They structurally estimate the per-unit trade costs to be substantial with an average value, expressed relative to the median price, of 33 % and reject the pure iceberg specification. They find that per-unit trade costs alone can explain between 40 and 70 percent of the elasticity of aggregate trade to distance. Hence, empirical evidence indicates that it is important to distinguish between per-unit and iceberg trade costs and that per-unit trade costs are important quantitatively.[1] In a paper closely related to the present paper, Schröder and Sørensen (2011) show, by use of numerical analysis, that welfare under the conventional iceberg specification exceeds welfare in the case where the iceberg costs have been replaced by per-unit costs yielding the same level of trade openness.[2] However, seen in the light of the diverse nature of real world trade costs and the empirical importance of per-unit trade costs, cf. Irarrazabal et al. (2011), the integrated framework of this paper with simultaneous presence of per-unit and iceberg trade costs is preferable.

The rest of the paper is organized in the following way. Section 2 describes the extended Melitz (2003) model including both iceberg and per-unit variable trade costs. Section 3 analyzes and compares the welfare effects from reducing iceberg and per-unit trade costs. Finally, Section 4 concludes.

[1] Another related work is Martin (2010). He shows in a setting of exogenous quality and CES preferences that his and other papers' empirical findings of higher f.o.b. prices to more distant markets at the firm level can be explained by per-unit trade costs.

[2] Irarrazabal et al. (2010) reach the same conclusion as Schröder and Sørensen (2011) although under different assumptions.

2 The Model

The seminal heterogeneous firms trade model of Melitz (2003) is extended to include per-unit trade costs on top of the conventional iceberg trade costs and the fixed/sunk costs of exporting. To highlight the differences between additive per-unit and multiplicative iceberg trade costs the model is kept identical to the one in Melitz (2003) in all other dimensions. In line with most of the literature only steady states are considered.

Households

The representative household in each country exogenously supplies L units of labor and maximizes utility $U = \left[\int_{\omega\in\Omega} \left[c\left(\omega\right)\right]^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$ by choosing consumption $c\left(\omega\right)$ of each variety ω subject to the budget constraint $\int_{\omega\in\Omega} p\left(\omega\right) c\left(\omega\right) d\omega \leq E$, where E is nominal expenditures, Ω is the set of varieties available to the household and $p\left(\omega\right)$ is the price of variety ω. Demand functions read $c\left(\omega\right) = EP^{\sigma-1}\left(p\left(\omega\right)\right)^{-\sigma}$ for all $\omega \in \Omega$, where $P = \left[\int_{\omega\in\Omega} \left[p\left(\omega\right)\right]^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ is the price index.

Firms and firm behavior

In the monopolistic industry each firm produces a single and unique variety using labor as the only factor of production. The wage is set as the numeraire and the following costs are all specified in terms of labor. At entry, a firm pays sunk entry costs, f_e, and subsequently draws a firm-specific marginal productivity, φ, from a known distribution, $G\left(\varphi\right)$. To produce q units, a firm employs $l\left(q\left|\varphi\right.\right) = f + \frac{q}{\varphi}$ units of labor, where f is fixed costs of production. Exporting to the n export markets is subject to fixed cost of f_x per export market served, to per-unit trade costs $t \geq 0$ and to iceberg trade costs, $\tau \geq 1$.[3] At each point in time, a firm is hit by an exogenous and idiosyncratic death shock with probability $\delta > 0$.

The CES preferences of the households imply a constant elasticity of demand and firms accordingly set prices as a constant mark-up on marginal costs. Thus prices on the domestic market and on export markets respectively read

$$
\begin{aligned}
p_d(\varphi) &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi} && (1)\\
p_x(\varphi) &= \frac{\sigma}{\sigma-1}\left(\frac{\tau}{\varphi}+t\right) && (2)
\end{aligned}
$$

Reduced form profits per market (domestic π_d and foreign π_x) in turn become

$$
\begin{aligned}
\pi_d(\varphi) &= \frac{r_d(\varphi)}{\sigma} - f_d = \frac{E\left(\frac{p_d(\varphi)}{P}\right)^{1-\sigma}}{\sigma} - f = B\varphi^{\sigma-1} - f && (3)\\
\pi_x(\varphi) &= \frac{r_x(\varphi)}{\sigma} - f_x = \frac{E\left(\frac{p_x(\varphi)}{P}\right)^{1-\sigma}}{\sigma} - f_x = B\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} - f_x, && (4)
\end{aligned}
$$

where r_d (r_x) denotes domestic (export) market revenue and $B \equiv EP^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{1}{\sigma-1}$. Profits increase with productivity, φ, and only sufficiently productive firms are able to cover the fixed costs of operating in a given market. Hence firms with $\varphi > \varphi^*$ supply the domestic market and firms with $\varphi > \varphi_x^*$ supply both

[3] Per-unit trade costs are only paid for the goods actually arriving at the export destination.

the domestic market and export markets, where

$$\pi_d(\varphi^*) = 0 \tag{5}$$

$$\pi_x(\varphi_x^*) = 0 \tag{6}$$

define the exit (φ^*) and export (φ_x^*) productivity thresholds. The parameter space is restricted to ensure that firms in line with empirical evidence partition into pure domestic firms and exporters, i.e. to ensure that $\varphi_x^* > \varphi^*$.

There is free entry into the monopolistic industry and accordingly the expected net present value of entry is driven to zero.[4] This free entry condition reads

$$\sum_{t=0}^{\infty} (1-\delta)^t \left(\int_{\varphi^*}^{\infty} \pi_d(\varphi)\, dG(\varphi) + n \int_{\varphi_x^*}^{\infty} \pi_x(\varphi)\, dG(\varphi) \right) = f_e \tag{7}$$

The free entry condition (7) jointly with the two threshold conditions (5) and (6) determine the industry structure (and the variables B, φ^* and φ_x^*).

Prices, market shares and trade costs

Next, the differential impact of per-unit and iceberg trade costs on relative prices and relative market shares of exporters is considered. From (2) it follows that the relative export price of a low cost exporter (productivity φ_H) to a high cost exporter (productivity φ_L with $\varphi_L < \varphi_H$) reads

$$\frac{p_x(\varphi_H)}{p_x(\varphi_L)} = \frac{\frac{\sigma}{\sigma-1}\left(\frac{\tau}{\varphi_H} + t\right)}{\frac{\sigma}{\sigma-1}\left(\frac{\tau}{\varphi_L} + t\right)} = \frac{\frac{\tau}{\varphi_H} + t}{\frac{\tau}{\varphi_L} + t} \in \left[\frac{\varphi_L}{\varphi_H}, 1\right] \text{ as } \varphi_L < \varphi_H \tag{8}$$

[4] In line with the literature we assume zero discounting and as a consequence the equilibrium interest rate equals zero.

and from (4) the relative market share (and thus relative variable profits) reads

$$\frac{s_x(\varphi_H)}{s_x(\varphi_L)} = \frac{r_x(\varphi_H)}{r_x(\varphi_L)} = \left(\frac{p_x(\varphi_H)}{p_x(\varphi_L)}\right)^{1-\sigma} \tag{9}$$

Lemma 1 *Trade liberalizations have the following effects on the relative price and the relative market share of exporters*

1. *A decrease in the per-unit trade cost decreases the price and increases the market share of low price (cost) exporters relative to high price (cost) exporters.*

2. *A decrease in the iceberg trade cost increases the price and reduces the market share of low price (cost) exporters relative to high price (cost) exporters*[5]

Proof. Follows directly from (9) and the partial derivatives of (8) with respect to t and τ. ∎

Welfare

As aggregate expenditures equal[6] L, it follows from the domestic profit expression (3) and the exit threshold condition (5) that welfare (indirect

[5] In the special case of no per-unit trade cost, i.e. $t = 0$, the iceberg trade cost has no effect on the price ratio as

$$\frac{p_x(\varphi_H)}{p_x(\varphi_L)} = \frac{\frac{\tau}{\varphi_H} + t}{\frac{\tau}{\varphi_L} + t} = \frac{\frac{1}{\varphi}}{\frac{1}{\varphi_L}} = \frac{\varphi_L}{\varphi_H}$$

[6] In line with the literature on heterogenous firms in international trade the subjective discount rate is assumed to equal zero. This implies an interest rate of zero and investments in firm entry accordingly earn no interests. Aggregate profits of all firms equal zero due to free entry. Accordingly labor income is the only source of income and as the wage is normalized to unity, it follows that $E = L$.

utility) reads

$$W = \frac{L}{P} = L\frac{\sigma - 1}{\sigma}\left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}}\varphi^* \tag{10}$$

It follows that any form of trade liberalization only affects welfare through the exit threshold, φ^*, i.e. trade liberalizations affect welfare through effects on intra-industry reallocations.

3 Welfare comparison of trade liberalizations

Before turning to the comparison of trade liberalization through reductions in per-unit and iceberg costs, it is established that any mode of trade liberalization increases welfare.

Proposition 2 *Trade liberalization through a reduction in either fixed trade costs, per-unit trade costs or iceberg trade costs increases welfare.*

Proof. *Follows from (10) and from the total derivative of the free entry condition (7) which reads*

$$d\varphi^* = -n\frac{(\sigma-1)(\varphi^*)^{1-\sigma}\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\left(\frac{1}{\varphi}d\tau + dt\right)dG(\varphi) + \int_{\varphi_x^*}^{\infty}dG(\varphi)\frac{1}{f}df_x}{(\sigma-1)(\varphi^*)^{-\sigma}\left[\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1}dG(\varphi) + n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma}dG(\varphi)\right]} \tag{11}$$

■

The increased attractiveness of exporting due to trade liberalization reduces market shares in the domestic market for all domestic firms due to increased presence of foreign firms, but also due to the increased entry of domestic firms induced by the increased expected profits from exporting prior to entry. Consequently, the least productive firms become unable to cover the fixed costs of production and exit the industry. Hence, the exit threshold increases and so does welfare due to the well-known intra-industry reallocations (see Melitz, 2003). It has thus been shown that the novel welfare gains from trade liberalization driven intra-industry reallocations of Melitz (2003) are robust to the addition of the additive per-unit trade costs.

Trade liberalizations through reductions in iceberg and per-unit trade costs are not directly comparable in settings with heterogenous firms as there is no iceberg equivalent to a per-unit trade cost. In order to compare trade liberalizations of various natures, I follow Venables (1994) and compare the two types of trade liberalizations by imposing a criterion of equal impact on trade openness. An advantage of this measure is that it is empirically observable. Trade openness (O) is defined as the share of imports in total domestic expenditures (evaluated at market prices) and reads

$$\begin{aligned} O &= \frac{nM\int_{\varphi_x^*}^{\infty} EP^{\sigma-1}\left(p_x\left(\varphi\right)\right)^{1-\sigma} dG\left(\varphi\right)}{M\int_{\varphi^*}^{\infty} EP^{\sigma-1}\left(p_d\left(\varphi\right)\right)^{1-\sigma} dG\left(\varphi\right) + nM\int_{\varphi_x^*}^{\infty} EP^{\sigma-1}\left(p_x\left(\varphi\right)\right)^{1-\sigma} dG\left(\varphi\right)} \\ &= \frac{n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} dG\left(\varphi\right)}{\int_{\varphi^*}^{\infty}\left(\varphi\right)^{\sigma-1} dG\left(\varphi\right) + n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} dG\left(\varphi\right)} \end{aligned} \tag{12}$$

The main result can now be established

Proposition 3 *The welfare gain from a reduction in per-unit trade costs exceeds that from a reduction in iceberg trade costs with an equal impact on trade openness*

Proof. *See the Appendix* ■

Hence, for trade liberalizations with an equal impact on openness a reduction in per-unit trade costs is preferable to a reduction in iceberg trade costs.

To better understand the differences between the two modes of trade liberalization, it is instructive to have an initial look at the similarities. First, aggregate expenditures (L) are constant and thus identical in the two scenarios. Second, the total market share of foreign firms in the domestic market as well as total market shares of domestic firms in the export markets are also identical under the two modes of trade liberalization due to the criterion of equal trade openness. This in turn implies that the effect on expected profits in the domestic market prior to entry is the same for a given exit threshold under the two modes of liberalization. Another and very important implication is that aggregate export market producer surplus is identical in the two scenarios as producer surplus equals total revenue (openness times expenditures (L)) divided by σ cf. (4).

Now turn to differences stemming from the differential impacts on the intra-industry reallocations. A reduction in per-unit trade costs reduces the prices of low cost exporters (relative to high cost exporters) and accordingly shifts relative market shares and export market profits towards the more efficient exporters, cf. Lemma 1. As total producer surplus on the export markets is identical for the liberalization modes, a reduction in per-unit costs is less attractive for low productivity/high cost exporters and thus implies entry of fewer firms into the export market than for the reduction in iceberg costs.[7] Therefore, total export market profits - equal to total export market

[7] A proof is available from the author upon request.

producer surplus minus total fixed costs of exporting - increase more in case of a reduction in per-unit trade costs due to fewer exporting firms and thus fewer fixed costs of exporting. The larger increase in expected export market profits from lower per-unit trade costs induces more market entry and thus a stronger selection effect (inter-industry reallocations). As the gains from trade liberalization come through this selection channel, cf. (10), the welfare ranking of Proposition 3 follows.

4 Conclusion

Arkolakis et al. (2012) show that welfare gains from trade liberalization in a broad class of trade models including the Melitz (2003) model (under the assumption of Pareto distributed productivities) may be summarized by the effect on trade openness, and the elasticity of imports with respect to variable trade costs. The present paper shows in a Melitz (2003) type framework extended to include per-unit trade costs that the nature of trade barriers matters when evaluating the gains from trade openness. In particular, it has been shown that a reduction in per-unit (additive) trade costs is preferable to a reduction in iceberg (multiplicative) trade costs for a given impact on trade openness. This result stems from differences in the intra-industry reallocations implied by the two modes of trade liberalization. Accordingly, the result of Arkolakis et al. (2012) is not robust to the inclusion of the empirically relevant per-unit (additive) trade costs.

Interesting future research along the lines of this paper includes a comparison of tariffs. Will the main result of the present paper, namely that a reduction in the additive trade costs (per-unit) is preferred to a reduction of similar size in the multiplicative trade cost (iceberg), extend to the case of trade liberalization through trade policies, i.e. is a reduction in per-unit tariffs preferred to a reduction of similar size in ad-valorem tariffs? Given the existing results in the literature of the ad-valorem tariff being more efficient in generating tariff revenue in a Krugman (1980) framework, see Jørgensen and Schröder (2005), and that an ad-valorem tax is more efficient in raising tax revenue than a per-unit tax in a closed economy version of the Melitz (2003) model, see Schröder and Sørensen (2010), the answer is expected to be affirmative. However, the results of Cole (2011a, 2011b) suggest that predictions derived for real trade costs do not trivially extend to settings of tariff trade costs and a formal analysis is needed to conclude on this important policy question.

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Appendix

The appendix provides the proof of Proposition 3. Use the definition of openness (12), (11) and that $\varphi_x^* = \frac{\tau}{\frac{1}{\varphi^*}\left(\frac{f_x}{f}\right)^{\frac{1}{1-\sigma}} - t}$ cf. (3)-(6) to compute the derivatives

$$
\begin{aligned}
\frac{dO}{dt} &= A\left(-C\varphi_x^* + D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma} dG(\varphi)\right) \\
\frac{dO}{d\tau} &= A\left(-C + D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma} \frac{1}{\varphi} dG(\varphi)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
A &\equiv \frac{n\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1} dG(\varphi)}{\left(\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1} dG(\varphi) + n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} dG(\varphi)\right)^2} > 0 \\
C &\equiv \left(\frac{\tau}{\varphi_x^*}+t\right)^{1-\sigma} g(\varphi_x^*)\frac{\varphi_x^*}{\tau} > 0 \\
D &\equiv (1-\sigma) + n\frac{\left(\frac{\tau}{\varphi_x^*}+t\right)^{1-\sigma} g(\varphi_x^*)\varphi_x^*\frac{\varphi_x^*}{\tau}\left(\frac{\tau}{\varphi_x^*}+t\right) - (\varphi^*)^{\sigma} g(\varphi^*)\frac{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} dG(\varphi)}{\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1} dG(\varphi)}}{\int_{\varphi^*}^{\infty}(\varphi)^{\sigma-1} dG(\varphi) + n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma} dG(\varphi)} \lesseqgtr 0.
\end{aligned}
$$

The openness equivalent per-unit cost change to an iceberg costs change becomes

$$
dt = \frac{-C + D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi} dG(\varphi)}{-C\varphi_x^* + D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma} dG(\varphi)} d\tau \tag{13}
$$

Using (10) and (11), the welfare effect from trade liberalization through lower τ reads

$$dW|_{\tau} = -WO\frac{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right)}{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma}dG\left(\varphi\right)}d\tau$$

From (10), (11) and (13) it follows that the openness equivalent reduction in per-unit trade costs has the following welfare impact

$$\begin{aligned} dW|_{t} &= -WO\frac{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)}{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma}dG\left(\varphi\right)}dt \\ &= dW|_{\tau}\frac{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)}{\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right)}\frac{-C+D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right)}{-C\varphi_x^*+D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)} \end{aligned}$$

Using $C>0$ combined with $\frac{dO}{d\tau}<0$ and $\frac{dO}{d\tau}=\frac{dO}{dt}$ which in turn implies that $-C\varphi_x^*+D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)<0$, it follows that

$$\frac{dW|_{t}}{dW|_{\tau}}>1\Leftrightarrow\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)>\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{\varphi_x^*}{\varphi}dG\left(\varphi\right)$$

which holds as $\frac{\varphi_x^*}{\varphi}<1$ since the integrals run for $\varphi>\varphi_x^*$. That $\frac{dO}{d\tau}<0$ can be seen from

$$\frac{dO}{d\tau}<0\Leftrightarrow -C+D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right)<0$$

$$
\begin{aligned}
& -C+D\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right) \\
< \; & -\left(\frac{\tau}{\varphi_x^*}+t\right)^{1-\sigma} g\left(\varphi_x^*\right)\frac{\varphi_x^*}{\tau}+\frac{n\left(\frac{\tau}{\varphi_x^*}+t\right)^{1-\sigma} g\left(\varphi_x^*\right)\varphi_x^*\frac{\varphi_x^*}{\tau}\left(\frac{\tau}{\varphi_x^*}+t\right)\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}\frac{1}{\varphi}dG\left(\varphi\right)}{\int_{\varphi^*}^{\infty}\left(\varphi\right)^{\sigma-1}dG\left(\varphi\right)+n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma}dG\left(\varphi\right)} \\
= \; & -\frac{\int_{\varphi^*}^{\infty}\left(\varphi\right)^{\sigma-1}dG\left(\varphi\right)+tn\int_{\varphi_x^*}^{\infty}\left(1-\frac{\varphi_x^*}{\varphi}\right)\left(\frac{\tau}{\varphi}+t\right)^{-\sigma}dG\left(\varphi\right)}{\int_{\varphi^*}^{\infty}\left(\varphi\right)^{\sigma-1}dG\left(\varphi\right)+n\int_{\varphi_x^*}^{\infty}\left(\frac{\tau}{\varphi}+t\right)^{1-\sigma}dG\left(\varphi\right)}\left(\frac{\tau}{\varphi_x^*}+t\right)^{1-\sigma}\frac{\varphi_x^*}{\tau}g\left(\varphi_x^*\right)<0
\end{aligned}
$$

as $\frac{\varphi_x^*}{\varphi} < 1$ since the integrals run for $\varphi > \varphi_x^*$

which completes the proof.