General Equilibrium Theory and Competitive Trade Models

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1. Introduction

General Equilibrium Theory is a sub-field in economics which had its most active period of research in the 1950’s and 1960’s, with contributions by Lionel McKenzie, Kenneth Arrow, Gerard Debreu, Paul Samuelson, Hiroshi Atsumi, Hiroyumi Uzawa, Michio Morishima and many others, building on earlier work by Abraham Wald in the 1930’s and, of course, that giant among economists in the 19th century, Leon Walras. An even earlier field in economics is that of International Trade, building on the basic foundations laid down by David Ricardo in the early part of the 19th century, followed later by a veritable army of trade theorists with Alfred Marshall, Eli Heckscher, Bertil Ohlin, Abba Lerner and Paul Samuelson, as names that cannot avoid being singled out. Competitive models in the Theory of International Trade make abundant use of General Equilibrium Theory, but these two fields often address different questions and use somewhat different techniques. One way of saying this is that many theorists who have made contributions to General Equilibrium Theory have not paid much attention to the more applied questions dealt with in the theory of international trade, despite its pride in making good use of general equilibrium theory. A favorite example of mine that provides an exception to this rule concerns the famous article written by Paul Samuelson in 1953, Prices of Factors and Goods in General Equilibrium, an article whose title betrays no particular attention to international trade theory and yet an article whose approach to its

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1 I wish to thank Romans Pancs and Rolf Weder for useful discussions.
subject would be somewhat alien to any general equilibrium theorist not familiar with trade theory. Of course Paul Samuelson’s name probably appears more frequently in general equilibrium trade theory than that of any other economist. I postpone until later discussing the reason I select this article to help to define the difference between the questions that G.E. theorists focus on and those usually discussed by Trade theorists.

I recall an early conversation I had with Lionel McKenzie when he pointed out that there was a 4-part array of questions basic to G.E. theory: To prove the existence of competitive equilibrium, to establish the strong optimality properties of such an equilibrium, and to provide a set of conditions that would suffice to establish that such an equilibrium would be unique and stable. These are properties of competitive equilibrium often just assumed by trade theorists in their quest for answers to questions that are frequently those of comparative statics: asking about the effect of a change in some parameter on the competitive equilibrium in a context in which separate countries are engaged in trade with each other. I contrast this with the similar kind of question that might be asked in the context of a single economy not engaged in international trade, a context most likely to be in the minds of G.E. theorists. To the typical G.E. theorist the competitive equilibrium satisfies the four properties mentioned earlier. The setting is one in which \( n \) commodities are actively produced and consumed, leading to \( n \) equations describing market clearing. There are \( r \) inputs into the production processes, and it is assumed (to simplify) that the total endowment of each is given by a vector, \( V \), assumed constant. Technology (in the form of production functions for each commodity, assuming constant returns to scale) is assumed given, allowing for factor substitutability so that input-output coefficients, \( a_{ij} \), are all functions of the factor returns, \( w_i \). The given relationships of the general equilibrium include taste patterns on the demand side, technology (production functions or the matrix of the \( a_{ij} \) coefficients as functions of the factor price vector \( w \)), and the country’s endowment bundle (the \( V \) vector). A comparative statics exercise would suppose an arbitrary change in one or more of these parameters and investigate how this causes alterations in the equilibrium set of commodity prices \( (p) \), factor returns \( (w) \), the market-clearing set of outputs \( (x) \), and the matrix of factor assignments to all industries, \( V_{ij} \)^2

The approach taken by Samuelson (1953) was to break the analysis in this comparative statics exercise into two steps. The first step involved taking commodity prices \( (p) \) as given, and asking how the equilibrium values of outputs, factor prices, and input allocations would be changed if endowments changed parametrically or if one or more of the commodity prices changed arbitrarily. The second step involved using the information thus obtained.

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^2 Of course once the output vector, \( x \), is known, the more simple \( a_{ij} \) matrix can substitute for the \( V_{ij} \) matrix. This technique was used in Jones (1965) to simplify the algebra involved in obtaining equilibrium solutions.
about how outputs produced *(in each country)* depend upon commodity prices (as well as factor endowments) from the first step in order to match *(global)* output variations with *(global)* demand functions and thus to solve for the market equilibrium set of commodity prices. Of course the model cannot solve for the price level, and thus one commodity may be chosen as numeraire, with Walras’ law (and the budget constraints imbedded in demand functions) making one of the market-clearing equations redundant. Samuelson’s first step matched well the procedure often (but not always) taken in International Trade theory, of asking how a single economy adjusts to price changes determined in world markets that also involve additional countries. The typical assumptions found in these competitive trade theory models vary: Countries may (as in some simple Heckscher-Ohlin models) share the same technologies, although in other instances they may have different technologies. (This, of course, is basic in the earlier Ricardian models). For most questions it is also assumed that factor endowments are not mobile between countries, although some theory does allow foreign investment or international labor flows. These issues are, of course, avoided in a general equilibrium model in which there is only a single country. Thus trade theory asks questions involving more than one country, including cases in which it is assumed that the country under investigation is so relatively unimportant (or small) in world markets that it can take commodity prices as given in world markets because what that country might do will have no effect on such prices. Alternatively, such analysis suggests how any larger country would respond to changes in commodity prices that are triggered by originating changes that take place anywhere in the trading system. In such a manner does competitive trade theory make ample use of the basic setting provided in general equilibrium.

### 2. Trade and Factor Prices

One of the classic questions raised by Samuelson (and earlier by Abba Lerner) in his work in International Trade theory concerns not a comparative statics exercise querying the effect

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3 In early discussions of Activity Analysis, e.g. Tjalling Koopmans (1951), activities allowed *joint production* (i.e. production processes involving not only multiple inputs but, as well, multiple outputs). Here I simplify by assuming away the possibility of joint production. In trade theory the lack of joint production insures an asymmetry between inputs (many) and outputs (one) that is the basis for the Stolper/Samuelson theorem (1941). As to its effect on the Factor-Price Equalization theorem, opinions differ. For example, see Samuelson (1992) and Jones (1992).
of a price change on outputs or the distribution of income, but rather an investigation of how factor prices compare between two countries that share the same technology and face the same set of world prices. Samuelson’s answer: As long as their endowment bundles are sufficiently similar that they produce the same pair of commodities in a two-factor setting, their factor returns will be equalized in free trade. Such a result seemed to cause sufficient surprise among

the economics profession that Samuelson was asked to follow his original contribution (1948) in the Economic Journal with another article in the same journal a year later entitled “International Factor-Price Equalisation Once Again”. Surprise at this result stemmed, in large part, from the fairly obvious empirical observation that wage rates, say, seemed to differ widely among many countries. Samuelson took pains to be explicit that the necessary conditions leading to factor price equalization were severe: “technologies” assumed to be the same between countries included the assumption that labor skills and education were similar across countries and, furthermore, that countries produced the same pair of commodities. This latter proviso was important enough in a two-factor, two-commodity world, but in a world of many commodities (and perhaps more than two factors) it was easy to see that if factor endowments were not sufficiently similar countries might be producing a different set of commodities. Crucial here is the basic notion that with trade a country need not produce more commodities than it has factors of production. For example, in a Ricardian world a country need only produce a single commodity with trade, one in which its labor comparative advantage is greatest. A common, and extremely important, feature of a free-trade competitive equilibrium is that a country’s production pattern becomes more specialized. Thus in a setting in which the number of possible commodities (not including non-tradeables) exceeds the number of productive factors, the question of which commodities a country produces becomes centrally important. Factor-price equalization depends on countries producing the same set of commodities, and this entails that their factor endowment proportions not be too dissimilar.

These concerns with which commodities a country produces pose important questions in International Trade theory, but would not even arise in a general equilibrium setting in which there is only a single country: Production would match precisely, in equilibrium, the pattern of demand. The question of comparing the number of commodities that could be produced and the number of productive factors is thus a fundamental concern in a typical international trade setting, one in which factors of production are not completely mobile among all parts of the world economy or in which natural or man-made barriers to trade among different regions exist.
A precursor to Samuelson’s work on factor-price equalization is his earlier collaboration with Wolfgang Stolper (1941) on the possibility, and perhaps necessity, that a country that levies a tariff on imports of a commodity that is produced locally by labor-intensive techniques (in a two-commodity, two-factor setting) will succeed in an unambiguous fashion in raising the real wage rate at home. This is a question that might seem of little interest to a general-equilibrium scholar who is not conversant with International Trade theory. However, the Stolper-Samuelson result was considered sufficiently important to lift it out of the trade context in part because of the observation by Lloyd Metzler (1949) that a tariff on imports may not prove to be protective in the sense of raising the domestic price of the importable. Subsequent work in this area often posed the question in a more general setting: If an increase in a single commodity price, all others constant, takes place, does the return to the productive factor used most intensively in that commodity increase by a greater percentage amount (i.e. is there a magnification effect of commodity price changes on factor prices)? More will be said about this question below. Here I wish to emphasize the possibilities of changing what seems like a question embedded in Trade Theory into a more general question of interest to economists working in General Equilibrium theory.

3. Common Sense and Price Changes

In asking about general results found in International Trade Theory that may be of interest to G.E. theorists not conversant with the Trade literature, consider the following question: What results might one expect if only a single commodity price should increase? 4 It might be reasonable to expect that the output of this commodity would increase and, furthermore, that every other industry would lose some resources to the favored industry. Is there such a model in competitive trade theory that yields such a result regardless of how many productive sectors exist in the economy? Yes – the Specific Factors model.5 Suppose each of n productive sectors uses as an input in production some amount of a factor used nowhere else in the economy, as well as a single other factor (often taken to be labor) that is mobile among all sectors. A price rise in one sector (only) attracts the mobile factor from

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4 This exercise is restricted to Samuelson’s first stage, where commodity prices are treated as parameters.

5 This is what Samuelson calls the Ricardo-Viner model and it encompasses much of the expected results in partial equilibrium theory. The more formal exposition of this model can be found in Samuelson (1971) and Jones (1971, 1975).
all other sectors – they all have output reductions and the favored sector expands. As for factor returns, the factor specifically used in the favored sector is the big winner, the return to the mobile factor also increases, but relatively not as much as the price of the commodity (so that the change in that factor’s real return is left in doubt), while returns to all factors specific to other sectors must fall. If, instead of a commodity price increase, the endowment of any sector’s specific factor expands (and its return falls), that sector’s output also expands, and the resulting increase in the return to the mobile factor causes less of it to be used in all other sectors, and their outputs contract, along with reductions in the return to specific factors used in these other sectors.

Now consider the specific-factors model in a lower dimensional setting: Only two commodities, each using an input used specifically in that sector and not the other, along with a productive factor, say labor, which is mobile between sectors. Let \( \mathbf{w}_1 \) and \( \mathbf{w}_2 \) denote the returns to the two specific factors and \( \mathbf{w}_3 \) the wage rate, and suppose that the two commodity prices are denoted by \( \mathbf{p}_1 \) and \( \mathbf{p}_2 \). Furthermore let any variable, \( \dot{x} \), denote the relative change in variable \( x \), i.e. the expression for \( \frac{dx}{x} \). Then it is clear that if \( \mathbf{p}_1 \) increases and \( \mathbf{p}_2 \) remains constant, the inequalities shown in (1) must hold:

\[
(1) \quad \hat{w}_1 > \hat{p}_1 > \hat{w}_3 > \hat{p}_2 (= 0) > \hat{w}_2
\]

This set of inequalities captures an important feature of the specific-factors 3x2 setting when the relative commodity price changes, namely that the (relative) change in any commodity price must be matched by a positive weighted average of changes in returns to factors used to produce the commodity, the weights reflecting distributive factor shares. (Reminder: It is assumed that there is no joint production of outputs). Note that the relative change in the return to the mobile factor (say labor) must be trapped between the commodity price changes. This helps to account for the position of changes in the returns to specific factors as the highest and lowest terms in (1).

In this 3x2 version of the specific-factors model two aspects of technology are important in comparing the order in which factor returns change as commodity prices change, such as illustrated in (1), as well as the extent of such changes. Consider the effect of price changes on the return to the mobile factor: First is the ranking between industries as to their use of the mobile factor (the factor intensity of specific factors being given by definition). The factor-intensity ranking is determined by the comparison of distributive factor shares: The first industry is deemed to be the more labor-intensive of the pair if and only if labor’s distributive share is greater in the first sector than in the second. The second aspect of
technology refers to the degree of factor substitutability and its difference between sectors. This in general can refer to the elasticity of substitution (here between labor and the specific factor in each sector) or, more focused on the mobile factor, the elasticity of demand for labor in each sector (the elasticity of the curve showing how labor demand in a sector responds to lowering of the “real” wage rate, i.e. relative to the commodity price in that sector)\(^6\). In addition to this pair of asymmetries in technology between the two sectors is the relative importance in the national income (share of the commodity in the national product) of the commodity whose price has changed.

More generally, in the higher dimensional version of the specific-factors model (with labor the single mobile factor and all endowments given), equation (2) depicts the relative effect of an increase in \(p_j\) on the wage rate as a product of the share of industry \(j\) in the national income, \(θ_j\), and the two features of technology: the relative labor intensity of the \(j^{\text{th}}\) sector (i.e. its labor share compared with that for labor in the aggregate economy), \(i_j\), and the relative degree of substitutability of the demand for labor in the \(j^{\text{th}}\) commodity as compared with the economy-wide elasticity of demand for labor. Call this \(s_j\). Equation (2) draws these elements together: \(^7\)

\[
(2) \quad \dot{w} = θ_j \frac{1}{i_j} s_j \frac{1}{j} \hat{p}_j
\]

Once the change in the wage rate (for the mobile factor) is obtained, changes in all returns to specific factors (in the form of rents) follow from the result that the distributive share weighted average of changes in relative factor returns must, in each sector, equal the relative change in that sector’s commodity price.

4. A (2x2) Heckscher-Ohlin Setting

In competitive trade theory Paul Samuelson utilized a two-factor, two-commodity version of Heckscher-Ohlin Theory in his previously cited two articles on factor-price

\(^6\) If \(σ_j\) is the elasticity of substitution in the \(j^{\text{th}}\) industry, the elasticity of demand for labor in that industry, \(γ_j\) (assuming labor is the mobile factor), is \(σ_j/θ_{kj}\).

\(^7\) This breakdown of the determinants of a price change on the wage rate is explained in Caves, Frankel and Jones: World Trade and Payments (10\(^{\text{th}}\) ed.), pp. S-20 – S-21.
equalization in 1948 and 1949. This 2x2 model assumes the same number of commodities as factors, with both factors of production mobile between sectors. (This is also the model used in the 1941 Stolper/Samuelson paper). Assuming techniques are different between sectors, this model has the feature that if commodity prices are given, factor prices get determined if endowments allow both commodities to be produced. Furthermore, unlike the kind of factor-price solution given for the specific-factors model in (2), only the technology feature that is captured by the factor-intensity ranking of the two sectors serves as a guide to the effect of a commodity price change on factor prices. Thus if \( p_1 \) increases and \( p_2 \) remains constant, the wage rate increases if and only if the first sector is labor-intensive relative to the second. Why does the degree of factor substitutability in each sector not appear, as it does in the specific-factors model? If both commodities are produced, the relative cost change in each must be equal to the relative price change in each, and the cost change is the weighted average of all the input price changes. (And the weights are distributive factor shares, which reflect only factor intensities). Query: How about changes in the techniques used in each industry? Answer: With competitive firms minimizing costs, techniques may well change, but for small changes in commodity prices the weighted average of changes in the intensity with which each factor input is used is a second-order small. (Of course if the number of commodities matches the number of factors, as in the

2x2 model, there is no change in factor prices if commodity prices do not change but factor endowments do (slightly), so that there are no changes in techniques). If the number of factors exceeds the number of commodities produced, as in the specific-factors model, the competitive profit equations of change do not suffice to yield solutions for factor price changes – further information must be used, and that comes from the requirement that factors are fully employed. For a given set of factor endowments this requires information on the flexibility of technology in each sector, so this feature of technology will appear in the solutions for factor price changes when the number of factors exceeds the number of produced commodities (as in equation (2)).

5. Expanding Dimensionality: The (nxn) Setting

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8 The writings of Eli Heckscher (1919) and Bertil Ohlin (1933) most often referred to three factors of production, land, labor and capital. In those years it was more difficult to leave land out of consideration.
Many attempts have been made to expand dimensionality to a model with the same number of factors and commodities. Details of these efforts are discussed in Jones (2006). Here I wish to focus only on the kind of question typically asked in higher \((n \times n)\) dimensional models – what kind of structural details suffice to guarantee the following result, sometimes referred to as the “strong” form of the Stolper/Samuelson theorem: Every factor of production is used “intensively” in a unique industry such that an increase in the price of that industry guarantees that the specified factor’s return increases relatively more than the commodity price increase, and all other factors have their returns reduced. In 1969 Murray Kemp and Leon Wegge did provide conditions sufficient for this result for a \((3 \times 3)\) model, but the conditions proved insufficient to establish the result for the \((4 \times 4)\) model or higher dimensional cases. Sufficient conditions for the higher dimensional \((n \times n)\) case were provided in Jones, Marjit and Mitra (1993), making use of properties of dominant diagonal matrices investigated years previously by Lionel McKenzie (1960). The basic idea is simple: The strong form of the Stolper/Samuelson theorem should require the shares for all factors in an industry (except the favored factor) to be somewhat similar because (for the strong form of the theorem) all their returns have somewhat similar results once the commodity price increases – they all fall. The sufficient condition we proposed is that the extent of the relative factor intensity dominance of the favored factor exceed the sum of the absolute values of distributive share discrepancies for all other factors.

This line of argument betrays a mathematical interest that may stray far from a question that is of greater interest among economists: In general does there exist a set of changes in commodity prices (not just one) that will guarantee that any pre-selected productive factor can have its real return increased? The answer is in the affirmative – a strong existence result. However, it neither ensures that the nominal factor price increases relatively more than any commodity price nor that only a single commodity price is raised. In addition it does not require that all other factors lose.\(^9\) Why would an economist require that the “heavy lifting” be left to a single commodity? Mathematicians, on the other hand, ask about the effect of any single commodity price increase when they ask about details i.e. signs and sizes of every element of the inverse matrix of distributive factor shares. Letting \(\theta\) represent the matrix of distributive factor shares, the initial equilibrium between costs and prices leads to (3),

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\(^9\) Indeed the factor that gains may wish to “hide behind the coat-tails” of other factors who also may be winners.
(3) \[ \theta \hat{w} = \hat{p} \]

Assuming that the matrix is of full rank \( n \), inversion leads to (4):

(4) \[ \hat{w} = [\theta]^{-1} \hat{p} \]

The sufficient conditions cited in Jones, Marjit and Mitra (1993) are for the inverse share matrix to have a positive diagonal with values greater than unity and negative off-diagonal entries.

To prove the more general result for real factor returns, consider the following two statements: (i) If commodity prices change, the relative change in any factor's nominal return is a weighted average of the relative change in all commodity prices. These weights, of course, may be difficult to determine because they depend sensitively on the structure of production. However, any credible general equilibrium model requires the weights to add to unity (although some weights may easily be negative, as in the (2x2) model), otherwise relative commodity prices would change as the price level is altered.\(^{10}\) (ii) The change in real returns depends on that factor's cost-of-living index, a non-negative weighted average of all commodity price changes reflecting taste patterns for this factor.\(^{11}\) As long as the weights for these two averages differ, a change in relative prices of commodities exists that will guarantee a real income gain for the chosen factor.\(^{12}\) Furthermore, there is no need to have a balance between the number of commodities and the number of factors of production, nor, indeed, to rule out joint production. The Stolper/Samuelson theorem shows how a factor's real return can be increased without recourse to direct subsidy payments.\(^{13}\) Instead, an “indirect” (and less transparent) route of altering commodity prices can be used, one that helps to disguise the

\(^{10}\) Such a change would indicate that there is money illusion, a possibility ruled out in order to have an effective general equilibrium model.

\(^{11}\) This presumes that all agents representing the designated factor of production have identical (and homothetic) tastes.

\(^{12}\) It must be remembered that one of the marked features of the original Stolper/Samuelson article was the absence of any knowledge of taste patterns – the wage rate increased by more than any commodity price so that no knowledge of tastes was required.

\(^{13}\) More explicitly, labor was not allowed to partake in any part of the tariff revenue in the Stolper/Samuelson (1941) setting.
support for that factor. This existence result should be a basic theorem in the field of political economy. Of course the set of instruments to be used by government to change relative commodity prices, as well as how changes in these prices affect factor returns, depend on the details of technology and tastes in this economy. What is shown here is only an existence result, not a finely specified blueprint for policy actions.

6. General Equilibrium for Closed vs. Open Economies

There is no doubt as to the wide use of general equilibrium modeling for competitive open economies. There are two essential differences between such models and those of more interest to general equilibrium theorists who want to focus on economies closed to trade.

The first concerns demand. The existence of trade implies that quantities produced at home once a country trades with other countries differ from quantities consumed at home. The (2x2) Heckscher-Ohlin model as developed by Samuelson and others is still highlighted in most textbooks on international trade. The most typical graphical representation illustrates a bowed-out transformation curve with autarky shown by a point on the transformation schedule where a community indifference curve is tangent to that schedule at a relative price that clears local commodity markets. International trade, by contrast, is typically shown to lead to a move to a different point on the transformation schedule where there is a tangency between the schedule and a budget line whose slope reflects prices ruling on a world market, with, at a different point, an indifference curve tangent to the same budget line.

What is left out of such an account is an extremely important feature of the reality of most situations in international trade: Most trade situations allow and bring about an equilibrium in which a country’s production pattern is concentrated with fewer commodities produced than is the number consumed. Trade allows a great degree of specialization in production. This could be illustrated in the 2x2 model by a move to one corner or the other of the bowed out transformation schedule, but most users of the transformation schedule typically show for

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14 Of course it may be the case that international trade takes place only in parts and components used as inputs in production so that item by item quantities of goods consumed locally are matched by local production.

15 The two country, two commodity Ricardian model of trade emphasizes that at least one country produces fewer commodities in trade than in autarky - the commodity in which that country has a comparative advantage.
trade only a different balance in the production bundles along the transformation curve, with both commodities still being produced. By contrast, the many-commodity case easily shows a concentration in production as a consequence of trade. Furthermore, the many-commodity case opens up an extremely important question for a trading economy, viz, which commodities get produced with trade? For a developing open economy the answer to such a query typically changes in the course of development. As a country succeeds in raising its endowment ratio of capital to labor with development, it systematically shuts down production of its more labor-intensive commodities and pours resources into some more capital-intensive commodities that trade would not allow to be produced in earlier times. Such a churning activity with development is an important feature of trade.\(^{16}\) The (2x2) model is, of course, extremely useful, but it cannot be used to emphasize the way in which trading open economies differ from a more “balanced growth” characteristic found with growth in closed economies.

It is also the case that some general equilibrium theorists not familiar with international trade theory seem to object to Samuelson’s procedure (shared by most trade theorists) of asking how a single commodity price change alters production patterns if other prices are kept constant. Of course the answer to such a question depends upon whether the economy is open to trade. If it is a closed economy, a price change can nonetheless be treated parametrically just as, in exploring local preferences, a demand function could be probed in order to ascertain how any commodity price change by itself would alter quantities demanded. Such a search for demand elasticities in no way denies the fact that price changes are determined in a general equilibrium. A similar device can also be used in production. However, it is the case that in a trading context a particular country might be “small” in the sense that it is a “price-taker” in world markets. For this case the equilibrium effect on local production, say, of levying a tariff might well be answered by keeping constant all prices other than the domestic price of the protected item. By contrast, in the many-commodity case for a closed economy the search for the new equilibrium must enquire as to changes in other commodity prices as all markets could be disturbed. In any event Samuelson’s procedure of changing one commodity price at a time in order to register changes in production patterns is perfectly appropriate even if, in the new equilibrium with all markets clearing, all relative prices change.

\(^{16}\) For example, see Jones (2004).

Extensions of the production side of general equilibrium models in trade theory have been made to the many-commodity case, but typically not to cases in which there are more than two factors used in any industry. Why? The complications introduced by having more than a pair of inputs used in any particular sector stem in large part from the possibility of factor complementarity or of different degrees of factor substitutability. However, to see what new possibilities emerge, I consider two examples in which such issues have been studied in small-scale open-economy settings, and these illustrate how the phenomenon of complementarity is explained, even if the economy is closed to trade.

For a case of possible strong asymmetries in substitutability among factors of production, consider a variation of the simple (3x2) specific-factors model. Change the setting a bit by allowing the two specific factors merely to be “extreme” factors in the sense that factor 1 is now used to produce not only the first commodity but also the second and some of factor 2 is used to produce the first commodity as well as the second. That is, suppose all three factors are used in each of the two sectors, although distributive shares are extreme in the sense that $\theta_{1i} > \theta_{ki}$ for $i = 1, 2$, and $k = 1, 2, 3$, $k \neq i$. Factor 3 (previously assumed to be the only mobile factor) is now referred to as the “middle factor”. Suppose, now, that $p_1$ increases relative to $p_2$, just as in equation (1). This requires the cost of producing the first commodity to increase relative to the cost of producing the second, and, as in (1), this is most easily achieved when $w_1$ rises more than any other factor price and $w_2$ rises the least (or falls). However, suppose now that the “extreme” factors 1 and 2 happen to be very good substitutes for each other in each industry, compared with the degree of substitution between either extreme factor and “middle” factor, 3. This implies that the extremes in the ranking shown by equation (1) may no longer be possible because $\hat{w}_1$ and $\hat{w}_2$ lie closer together (although $\hat{w}_1$ still exceeds $\hat{w}_2$). In this new scenario an important role emerges for the comparison of the “middle” factor’s distributive share in the first industry with that in the second. Suppose $\theta_{31} > \theta_{32}$, so that an increase in the return to the middle factor would raise costs more in the first sector than in the second. With the two extreme factors close substitutes for each other, this three-factor, two-commodity model resembles more closely a two-factor, two-commodity model, and the middle factor would be required to support
the relative price increase for the first commodity by increasing. One possibility for the ranking when \( p_1 \) increases relative to \( p_2 \) is shown in (5):

\[
(5) \quad \hat{\hat{w}}_3 > \hat{p}_1 > \hat{\hat{w}}_1 > \hat{\hat{p}}_2 > \hat{w}_2
\]

The relative cost increase for the first commodity is now brought about by a relatively large increase in \( w_3 \), greater than the relative increase in \( p_1 \) so that \( \hat{\hat{w}}_1 \) comes closer to \( \hat{w}_2 \).

Maintaining the assumption that the middle factor is used more intensively in the first sector, other possibilities when \( p_1/p_2 \) increases and the extreme factors are better substitutes for each other than either is with the middle factor are that the return to factor 1 still manages to increase more than \( p_{11} \), or, at the opposite extreme, that it rises, if at all, less than \( p_2 \) but still more than the return to the second factor. By contrast, if \( \theta_{31} \) were to be smaller than \( \theta_{32} \), but extreme factors were still very substitutable, the return to factor 3 would fall relative to any other price and \( \hat{\hat{w}}_1 \) and \( \hat{\hat{w}}_2 \) could be even larger than \( \hat{p}_1 \), with \( \hat{\hat{w}}_3 \) bringing up the rear. With the substitutability factor reversed, i.e. if the two extreme factors are complements, the inequalities expressed in equation (1) would merely become amplified. For factor prices, substitutes attract and complements repel, and this is a behavior in small dimensional models that suggest forces at work in higher dimensions.17

An example of how, in a trade situation, an increase in a commodity price for only a single commodity might not only result in increased production of that commodity but in an expansion of another commodity as well (thus revealing complementarity in production) was provided by Fred Gruen and Max Corden (1970) in a setting that shows how a country engaged in trade might, by levying a tariff, actually worsen its terms of trade. They have a (3x3) setting in which any of the three commodities makes use only of two out of the three factors. With a country like Australia in mind, imagine a manufacturing sector that uses labor and capital, and it produces only textiles. By contrast, there is a pair of industries in the agricultural sector, wheat and wool, combining land and labor in production. Suppose that the country raises a tariff on imports of textiles, but is too small in the world textile industry to affect world prices. Nonetheless there is an increase in the domestic textile price because of the tariff. Such a price rise induces an inflow of labor into the textile industry from agriculture. How does this affect production separately of wheat and wool? Suppose that wool is the more land intensive sector in what is really the (2x2) setting in agriculture. Factor intensity rankings are important in Heckscher-Ohlin (2x2) models, and in

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17 Further details on weakening the (3x2) specific-factors model so that “specific” factors become “extreme” factors are found in Jones and Easton (1983).
this case if wool and wheat prices stay constant the outflow of labor towards textiles reduces output of the labor-intensive sector (wheat), but this serves to cause output of wool to increase. With Australia a large player in the world wool industry its price would tend to fall – an example of a country raising a tariff and worsening its terms of trade. For my purposes here the crucial point is that as one price increases (textiles), another industry (wool) might have an increase in its output as well as that of textiles, a complementary effect in production.

8. Comparative Advantage

Throughout my remarks I have suggested that some of the developments in the use of general equilibrium theory in international trade theory might be of some interest to G.E. theorists who do not work in the International area. Let me finish by asking a question about a fundamental concept in International Trade theory: the doctrine of Comparative Advantage. Does this concept arise in “general” G. E. theory? If the “general” version omits any markets in which trade takes place between residents at Home and in any other country, much as I have been interpreting the “general” form of the theory, the answer might seem to be negative. After all, Ricardo and successors in the Trade area were discussing trade among two or more countries in which the “classical” assumptions were made that labor and other factors of production were immobile between countries, whereas the commodities that a country did import or export could freely and costlessly be transferred from country to country (assuming no man-made interferences such as tariffs or quotas and ignoring costs of transport or transfer). In this setting, argued Ricardo, comparisons of absolute levels of productivity indeed would tend to matter in terms of wage or other factor-price comparisons among countries, but the determinants of which commodities get exported and imported would depend only on costs incurred in producing one commodity relative to local costs incurred in other commodities compared with the comparable ratio abroad. Thus a country might well export a commodity in which its productivity is low to another country in which local labor’s productivity was much higher. In that sense any country has a comparative advantage in producing some commodities, and might even export these to countries in which productivity is much higher, but in which wage rates are also higher so that such trade does not confound the traditional rules of the market place. This was crucial, and the doctrine of comparative advantage provided the

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18 Putting things in reverse, any increase in the labor supply in agriculture would favor labor-intensive wheat to the detriment of land-intensive wool – the kind of effect at constant prices reflected of the Rybczynski (1955) effect.
underpinning of one of the most surprising statements in all of economics, viz. that all countries stand to gain by free and open international trade compared with remaining in a state of autarky. Trade is not zero-sum in its welfare effects.

So, does the concept of comparative advantage have no role to play for countries that have no international economic relations with other countries? I would argue that this may not be the case, and will say a few words about the argument put forth by Michihiro Ohyama and myself (1995) in order to suggest that the doctrine of comparative advantage has applications within countries that may have no trading relationships with other countries. In brief, Ohyama and I discussed a scenario in the field of industrial organization in which we compare the situation in which two firms in an industry reflect heterogeneity (a not uncommon characteristic indeed of so-called “new, new trade theory”) based on the concept of learning by doing. The “learning” in this case has to do with acquiring ever greater technical knowledge in some field of endeavor. The early entrant has an absolute advantage in “know-how” in a type of technology labeled the θ-technology. Both firms are embedded in an economy with lots of industries, an economy in which new technological developments take place continuously. A question then arises – suppose a general technology that is developed elsewhere in this closed economy is of a different form than the θ-type – call it the β-type. This may not have much relevance to the two firms using the θ-type technology, and both choose to ignore it. By contrast, both firms may decide the β-type technology would be better for them, and both would therefore switch to it, with a period of learning that is less productive for both than the θ-technology, but subsequently better for both. The possibility we wished to emphasize lies in between: it may be the case that the new β-technology indeed on net proves of value to one of the firms, but not to the other. Query: If there is a mixed response, which firm is likely to make the switch? Answer: the firm that is further behind in the learning curve for the θ-technology. The rationale: the firm that is advanced (absolutely) in the θ-technology thereby tends to have a comparative disadvantage in the β-technology, and after a learning period the original laggard becomes the new leader. This switch, we argued, was a natural outcome of the workings of comparative advantage, and was undertaken even with full knowledge of the future by both firms.

9. Concluding Remarks

In this paper I have attempted to discuss a limited number of issues in which the International Trade literature discusses models that are comparatively simple to explain and
for which comparative static results may yield insights to those economists who have worked in the General Equilibrium field without much concentration on the economy’s stance in world markets. From an empirical point of view, international trade is becoming increasingly ubiquitous, so that the rewards for increasing the international content of a general equilibrium model seem to grow.19 In the examples I have selected, I have concentrated on the production side of models. For some issues demand behavior or disparities among countries are responsible for somewhat surprising results – e.g. the possibility that transfers among countries might damage recipient economies or that demand asymmetries working through income effects might endanger the stability of a market equilibrium. In trade issues, both blades of the scissors are important.

Finally, let me again stress the importance of Samuelson’s (1953) technique of asking about the effect on factor returns and outputs of a change in a single commodity price. This is a natural question to ask for a country that engages in international trade with other countries and in which prices are taken as given in world markets. Even if, for an economist engaged in analysis of a single economy not engaged in trade, commodity prices are not parameters, but are variables to be determined in the general equilibrium, it is legitimate to ask about the consequences of arbitrary commodity price changes, just as in demand one can ask separately about the elasticities of demand with respect to any individual price change.

10. A Postscript: The Reciprocity Relationship

This paper begins with a quote from Paul Samuelson. Another one that appeared early in his career is somewhat similar: “Historically the development of economic theory owes much to the theory of international trade.” (1938). This was written some years before the important work on trade theory that I have discussed here, especially his classic 1953 article. In this postscript I describe briefly his contribution often described as The Reciprocity Theorem. It emerges naturally from the way in which Trade Theory deals with the comparative static changes in an economy that accompany either changes in factor

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19 In travelling on leave to Australia in the mid-1970’s I spent a few days in New Zealand. An enquiry at a local university as to who was teaching courses in International Trade met with a puzzled response: no one. That is, all field courses were assumed to include ramifications introduced by the existence of world markets. In a sense I hope such an absence of a separate Trade field in academia does not spread too far.
endowments when commodity prices are assumed to be constant or changes in commodity prices when endowments are assumed not to change. The Reciprocity result lays bare the basic duality features of general equilibrium that are related to the way in which technology links factor and commodity prices, on the one hand, and factor endowments and outputs, on the other.\(^{20}\)

Consider the following two possible changes in an economy engaged in world trade, with the economy producing many commodities and using many factors. (i) the price of commodity \(j\) increases, with no change in any other commodity price. This commodity is produced before and after the (small) change in price. What can be said about the effect on the return to some factor, \(i\), used in its production assuming not only that other commodity prices are constant but that all factor endowments are also constant? This depends on other properties, and \(\partial w_j/\partial p_j\) may be positive, negative, or zero. (ii) another situation has the endowment of factor \(i\), \(V_i\) increasing, with all commodity prices and other factor endowments kept constant. What can be said about \(\partial x_j/\partial V_i\)? Answer: It has the same sign as that of \(\partial w_j/\partial p_j\). Furthermore, these two partial derivatives have the same value.

The reasoning is fairly simple. There are two ways of looking at national income, \(Y\). On the one hand it is the aggregate value of production, \(\sum_i p_i x_i\). On the other hand \(Y\) equals the total value of income earned, \(\sum w_s V_s\). Taking differentials and equating yields equation (6):

\[
(6) \quad \sum s V_s dw_s + \sum w_s dV_s = \sum x_i dp_i + \sum p_i dx_i
\]

From the competitive profit equations of change, the monetary changes on both sides are equal, leaving the real changes on both sides also equal. Now partially differentiate \(Y\) (expressed as the aggregate value of outputs) with respect to \(p_j\), holding all other commodity prices constant and all endowments constant. This leads to (7):

\[
(7) \quad \partial Y/\partial p_j = x_j + \sum p_i \partial x_i/\partial p_j
\]

The second term must vanish since the endowments are all held constant. By somewhat similar reasoning the partial derivative of \(Y\) (expressed as the aggregate value of incomes earned) with respect to \(V_i\) leads to (8):

\[
(8) \quad \partial Y/\partial V_i = w_i + \sum V_i \partial w_i/\partial V_i
\]

\(^{20}\) Fortunately, economists are said to be endowed with two hands.
The second term must vanish since all prices are held constant. The final step follows from noting that the second partial derivative of national income with respect to changes in $p_j$ and $V_i$ is independent of the order of differentiation. The reciprocity result follows, as in (9):

$$\frac{\partial w_i}{\partial p_j} = \frac{\partial x_j}{\partial V_i}$$

The Stolper/Samuelson theorem relates the effect of commodity price changes on factor returns, and by the reasoning leading to equation (9), this result is dual to the Rybczynski theorem, linking endowment changes to the response of outputs at given commodity prices. These two results in trade theory are duals of each other. It is ironic that the Samuelson (1953) article appeared a couple of years prior to the Rybczynski article (1955).\(^\text{21}\)

\(^{21}\) It is also ironic that the box diagram used by Rybczynski to prove (quite correctly) his theorem was incorrectly drawn, in such a fashion as to suggest that the Stolper/Samuelson result is violated. See Jones (2009).
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