BUYER-SUPPLIER RELATIONSHIPS, EXPORTING AND INNOVATION

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Abstract

Starting from the empirical evidence on the causal effect of exporting on product innovation, in this paper we aim at exploring a potential channel through which the innovation-enhancing role of being involved in foreign markets may take place. We focus on a specific mechanism which acts between firms establishing buyer-supplier relationships related to production to order. After providing some evidence that suppliers involved in international matches are more likely to introduce product innovations than those involved in domestic matches only, we introduce a theoretical model rationalising this empirical fact. In a theoretical framework with imperfect information and incomplete contracts, we show under which conditions the supplier innovates, and which variables are likely to affect this choice. Our model provides a framework in which firms may implement different innovation and internationalization strategies depending on the characteristics of their products.

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1 Introduction and motivation

This paper aims at exploring some potential sources and pathways through which trade may induce product innovation by existing firms.\footnote{In our framework, we refer to ‘incremental’ product innovation by referring to all those changes/improvements that firms introduce on their existing products or the introduction of new products (by existing firms), in the spirit of Puga and Trefler (2010), as opposed to radical innovations that are usually related to processes of invention; in our framework, ‘innovation’ can also be considered as product differentiation.} There are several recent contributions showing a positive relationship between firms’ international activities and innovation performance. This outcome seems to be present over and above the common incentive of better firms both to enter foreign markets and to renew their products (i.e., the self-selection mechanism on which there is already a wide consensus in the empirical literature).

Some progress has been made since the literature review published by Wagner (2007), where just a minority of studies used to report evidence in favor of a positive causal relationship between exporting and productivity. Indeed, more recent studies have exploited longitudinal data or presumably exogenous ‘export shocks’ to identify the impact of a firm’s export status (or intensity) on productivity, showing that firm self-selection into foreign markets is not the only source of the positive association between exporting and firm performance observed in the data, and that there are genuine causal effects (Crespi et al., 2008; Serti and Tomasi, 2008; Fryges and Wagner, 2008; Lileeva, 2008; Yang and Mallick, 2010; Park et al., 2010; Lileeva and Trefler, 2010).

Exporting is also an important correlate of firm product innovation, even when trade is between similar countries and is not induced by technological/income differences. The positive association is robust to the inclusion of proxies of firm efficiency and quality, and of those covariates that are likely to mediate the effect of exporting on innovation in terms of higher ‘formal’ innovative effort, such as R&D investments or acquisition of foreign patents. Beyond this, recent research supports the existence of a causal effect of exporting also in this case. Salomon and Shaver (2005) find evidence of learning by exporting considering product innovation in a panel of Spanish manufacturing firms from 1990 to 1997. Liu and Buck (2007) using a panel of sub-sector level data for Chinese high-tech industries find that export sales have a positive effect on product innovation. Fafchamps et al. (2008) observe that product innovativeness is positively related to the length of exporting experience in a panel of Moroccan firms which they interpret as an instance of learning by exporting. Lileeva and Trefler (2010) use an instrumental variables (IV, herefater) approach with a plant-specific tariff-cut instrument and find that Canadian plants that were induced by the tariff cuts to start exporting or export more engaged in more product

What remains to be ascertained, however, are the precise channels of these ‘learning’ effects. There are studies giving some hints on the potential pathways. Crespi et al. (2008) show that past exporting is significantly associated with more learning from customers (either firms or consumers) relative to other sources, such as suppliers, competitors and trade associations and that firms which have an increase in learning from customers also have higher subsequent productivity growth. Baldwin and Gu (2004) show that exporters learn from foreign buyers through R&D agreements. Fafchamps et al. (2008) explain their evidence on learning-by-exporting as the need of Moroccan firms—mainly specialized in consumer items such as garment, textile, and leather—to design products that appeal to foreign consumers. Lileeva and Trefler (2010) interpret the positive effect of improved access to foreign markets on productivity and innovation as the result of an increased return of investing in innovation for exporters. By contrast Bratti and Felice (2010) shows that, at least for Italy, the positive effect of exporting in not completely mediated by a higher formal innovative effort, e.g., by higher R&D, and put forwards that the effect may be partly demand-induced and due to buyer-supplier relationships.

In this paper, we aim to go a step forward towards dissecting the mechanisms of the effect of exporting on product innovation and focus only on the specific channel going through firms’ interactions by means of buyer-supplier relationships. At this purpose, we focus on firms which are doing production to order, as this specific form of production entails complex buyer-supplier relationships and a non-negligible exchange of information between business partners, from which we may expect a substantial amount of ‘learning’. This happens for a variety of reasons, not last the fact that production to order is likely to prevail for highly differentiated goods (Casaburi and Minerva, 2011).

Production to order is widely spread among European firms. The European survey used in this paper shows that 86 percent of Manufacturing firms produce to order, with an average 85 percent of turnover produced by the latter. Moreover, about 53 percent of all firms produce exclusively (i.e., 100 percent of turnover) to order. Hence, we are considering a channel which is potentially relevant for a vast majority of manufacturing firms in Europe.

Production to order may represent a channel of knowledge transfer (Egan and Mody, 1992) both, but probably differently, in domestic and in international matches. In our paper, we assess the relationship between innovation and production to order in international versus domestic buyer-supplier matches, by using the information gathered in the EFIGE firm-level dataset (see the Section 2); we provide first evidence of a higher propensity to in-
roduce product innovations by suppliers engaged in a match with foreign firms. We develop a theoretical model in order to interpret the positive association between producing to order for foreign customers and product innovation, along the line of the contributions introducing incomplete contracts and imperfect information in international trade related to specific inputs provision (Rauch and Trindade, 2003; Grossman and Helpman, 2005; Puga and Trefler, 2010). We explore under which circumstances suppliers of specialized goods engaged in international matches with foreign buyers may show a higher propensity to introduce product innovations and adapt their goods to the buyer’s needs than suppliers engaged in domestic matches. We single out and discuss the conditions under which this happens, which are related to the interplay among the innovation cost parameters, a per-period fixed internationalization cost, the distance in the space of product characteristics in international matches, and the number of suppliers in both the supplier’s and the buyer’s countries. The model also shows that firms may have heterogeneous innovation and internationalization strategies due to the characteristics of their products.

Our work adds to the existing literature on the relationship between innovation and internationalization, both empirically and theoretically, by highlighting a specific channel through which trade may induce product innovation by existing firms. On the one side, from a policy perspective this work highlights that a reduction in trade barriers positively affects an economic system by improving the innovative performance of existing firms acting as suppliers and not only through the well known selection mechanism by which only ex-ante more innovative firms survive the international competition. On the other side, it suggests that policy interventions are called for in order to enforce an adequate institutional system capable to support firms’ activities in international markets. This is particularly relevant for those small-medium firms which do not have the scale to bear the high cost of R&D and for which the relationship with foreign buyers represent an important opportunity for innovation.

The structure of the paper is as follows. Section 2 briefly describes the dataset, and section 3 reports the empirical analysis. In section 4 we describe the setup of a simple theoretical model, which is then developed in section 5 to interpret the empirical evidence. Section 6 concludes.

2 The EFIGE data

In this paper, we use the EFIGE dataset which was collected within the project ‘EFIGE - European Firms in a Global Economy: internal policies for external competitiveness’. The EFIGE survey gathers firm-level data on Manufacturing firms in seven countries: around 3,000 firms for France, Germany, Italy and Spain, about 2,000 for the UK, and 500 for Austria
and Hungary. The survey questionnaire is mainly focused on 2008, with some questions on firm activities in 2009 and in previous years. The data set includes data on 14,911 firms. The survey gathers a wealth of information on firm international activities, innovation, and organization, which are complemented with balance sheet data from AMADEUS, a database of comparable financial information for public and private companies across Europe, collected by the Bureau van Dijk.

For the purpose of this paper, we select only firms which make some production to order for other firms, restricting the sample to 11,850 firms. Moreover, we drop all firms producing for other firms which belong to the same group, as buyer-supplier relationships may be very peculiar for this specific group of firms, and the sample falls to 10,222 firms, accounting for about 70 percent of the original sample size. These criteria select 60.35 percent of Austrian firms, 83.32 percent of French firms, 68.64 percent of German firms, 80.74 percent of Hungarian firms, 84.97 percent of Italian firms, 67.88 percent of Spanish firms and 79.02 percent of UK firms.

Our hypothesis is that producing to order for foreign customers may induce firms to introduce more product innovations. In Table 1 we split the sample of firms producing to order between those matched with a foreign customer and those which are producing to order only for domestic customers, and report some descriptive statistics. Firms matched internationally are about 49 percent of the sample. The raw statistics in Table 1 confirm that they have an advantage in the likelihood of introducing product innovations, market innovations and of applying for patents. In what follows, we go beyond the simple bivariate associations by controlling for observable heterogeneity between firms with different types of matching, and check whether this association survives.

3 Empirical evidence

We said that Crespi et al. (2008) report that exporting firms are relatively more likely to learn from customers with respect to non-exporters, and this was the only difference in the sources of learning between the two types of firms. Similarly, we want to assess in this section whether producers (to order) matched to foreign firms are relatively more likely to innovate their products. We start with a very simple empirical specification

\[ y_i = \alpha_0 + \alpha_1 FORCUST_i + \alpha_2 x_i + \epsilon_i \] (1)

where \( y_i \) is a dichotomous variable which takes on value one in case firm \( i \) introduced product innovations and zero otherwise, \( FORCUST_i \) is a dichotomous variable that is equal to one in case the firm produced to order for a foreign customer and zero otherwise, \( x_i \) a vector of control variables and \( \epsilon_i \) an error term. In what follows, we will omit the firm’s subscript \( i \) to simplify
notation. The $\alpha$’s are parameters to be estimated. Using cross-section data, we have no time variation.

At this stage of the analysis, we are simply interested in documenting statistically and economically significant associations, and we neglect any potential source of endogeneity, using OLS. We have already said that some studies interpret the fact that exporters are also more likely to innovate as a potential consequence of the exchange of knowledge with foreign customers, leading to product innovations. Compared to those studies, here we analyze a more specific type of interactions, those taking place between firms, and in particular those where there is a buyer which purchases an intermediate input from a supplier, who is producing to order. A positive association between $\text{FORCUST}$ and $y$ could be interpreted in loose sense as a higher ‘incentive’ for the supplier which sells abroad to innovate the product that is selling with respect to a supplier which only sells domestically. In what follows, we will use the term ‘supplier’ to indicate a firm that is producing to order.

We start the analysis by considering as dependent variable the answer to the following multiple question

\textbf{C14}. On average in the last three years (2007-2009), did the firm carry out any (multiple answers allowed):

- product innovation (i.e. introduction of a good which is either new or significantly improved with respect to its fundamental characteristics; the innovation should be new to your firm, not necessarily to the market)
- process innovation (i.e. the adoption of a production technology which is either new or significantly improved; the innovation should be new to your firm; your firm has not necessarily to be the first to introduce this process)
- none of the above.

in particular, we define a product innovation dummy which takes value one in case the firm answered positively to the first sub-question and zero otherwise.

Table 2 shows the OLS estimates. In column (1), we report the simple association between $\text{FORCUST}$ and product innovativeness from a regression without controls. $\text{FORCUST}$ is associated with a 0.21 increase in the probability of introducing product innovations. In column (2) we include country and 4-digit NACE fixed effects, which account for the potential greater diffusion of both product innovativeness and ‘trade openness’ in some countries/sectors, and observe a slight decrease in the coefficient of $\text{FORCUST}$ which becomes 0.18. In column (3) we introduce some measures of firm heterogeneity, in particular firm size, capital intensity, unit labor costs and the R&D employment ratio.\footnote{Firm size is controlled for using four categories (10–19, 20–49, 50–249, 250 or more);}
of \textit{FORCUST} drops to 0.12, suggesting that a great part of the previous positive association was due to other firm characteristics, but it remains nonetheless large and statistically significant.\(^3\)

In table 3, we investigate the ‘degree’ of product innovation. Indeed, the EFIGE questionnaire also asks

\begin{itemize}
  \item \textbf{C16.} Are the corresponding products innovative also with respect to the market?
    \begin{itemize}
      \item yes
      \item no
    \end{itemize}
\end{itemize}

and

\begin{itemize}
  \item \textbf{C17.} on average in the last three years (2007-2009) did your firm?\(^4\)
    \begin{itemize}
      \item apply for a patent
      \item register an industrial design
      \item REGISTER a trademark
      \item claim copyright
    \end{itemize}
\end{itemize}

we defined accordingly five additional dummies, the first that takes value one in case product innovations also represent innovation to the market, and zero otherwise, and the remaining four taking value one in case the firm applied for a patent, registered an industrial design, registered a trademark, claimed a copyright, respectively, and zero otherwise. With these additional information, we check whether \textit{FORCUST} is also positively associated with all these additional outcomes. Column (1) of Table 3 shows that firms matched internationally are 11 percent points (p.p.) more likely to introduce market innovations. \textit{FORCUST} is also strongly positively associated with all the other outcomes considered, except for the probability of claiming copyright. Thus, involvement in foreign markets under the form of being a supplier for a foreign firm appears to be associated with various innovation outcomes.

4 A theoretical explanation

In this section, we propose a theoretical model to explain why we observe a product innovation premium from producing to order for foreign customers,
and what are the variables that may strengthen or weaken such a positive association. In the framework of the literature introducing incomplete contracts and search due to imperfect information in international trade (in particular, Rauch and Trinidade (2003), Grossman and Helpman (2005) and Puga and Trefler (2010)), we develop a model to analyze alternative innovation strategies of the firm, while taking as given its boundaries.

4.1 Structure of the economy

In our setting, there are two types of agents engaged in production: downstream producers (i.e., Buyers, B) who purchase an input from upstream producers (i.e., Suppliers, S). Buyers and Suppliers are distributed over the product characteristics circle. We develop a partial equilibrium model, with two identical countries (i.e. neither income nor technology differences)—except for (possibly) the number of both Suppliers and Buyers—where, in order to produce, Buyers and Suppliers have to match. The price of the intermediate good \( p_x \), the price of the final good \( p_y \), wages and operating profits are given and equal in the two countries.

For a match to work, some product adaptation is needed, depending on the distance between the Buyer’s needs and the Supplier’s good characteristics. Since both the Buyer and the Supplier can adapt, there are two possible innovation strategies: the Buyer can purchase the Supplier’s input as it is, and then adapt it (changing either the input or his needs, i.e. Buyer Innovation mode—IB); or the Buyer can provide the Supplier with a ‘project’ according to which the Supplier adapts the input to fit his needs (i.e. Supplier Innovation mode—IS).

In both strategies, some of the innovation costs to be borne in order to fill the distance and match are related to the distance between the Buyer’s needs and the characteristics of the input produced by the Supplier. The Buyer and the Supplier can be located either in the same or in different countries and they can implement either of the two strategies, both in Domestic and in International matches. In this setting, the Buyer must make two decisions, one on the nature of the match (i.e. Domestic vs. International) and one on the innovation mode (IS, IB), as described above, under the Supplier participation constraint.

\( Z_{ij} \), with \( i, j \) countries where the Buyer (B) and the Supplier (S), respectively, are located, is the distance along the circle between B ‘needs’ and S good’s ‘characteristics’. \( Z_{ii} \) and \( Z_{jj} \) are the distances between B and S in a Domestic match (D), in B and S markets, respectively; \( Z_{ij} \) is the distance between S and B in an International match (I). \( Z_{ii} \sim U(0, 1/(2X_i)) \), \( Z_{ij} \sim U(0, 1/(2X_j)) \), where \( X_i, X_j \) are the number of Suppliers in the B’s country and in the S’s country, respectively. Information on \( Z_{ij} \) is imperfect (symmetrically) before matching (see section on timing below).
International matches differ from domestic ones for three reasons:

i Imperfect information on the location of Suppliers in the foreign market. B initially knows the locations of all Suppliers in his country and matches with the ‘closest’ Supplier; B does not know the locations of Suppliers in the foreign country, he only knows that Suppliers are symmetrically distributed at the same distance under $Z_{ij} \sim U(0, 1/(2X_j))$; they may be located at different points along the circle: a better match is potentially possible abroad, but this will be known only after ‘trying’. Sunk search costs have to be borne by B to know the distance $Z_{ij}$ in a random match with only one foreign Supplier. These costs are a determinant of B decision to look for an International match.

ii International matches differ from Domestic ones because they imply an additional cost: a per-period fixed ‘internationalization’ cost. Moreover, International matches differ from Domestic matches because in the former firms ending up in a bad match can go back home and match domestically, while in the latter, this outside option is not allowed for, and bad matches imply no production and zero profits for both B and S. The ‘internationalization’ costs together with the opportunity to go back home, do not only affect the profitability of an International match with respect to a Domestic one, but they also modify the relative profitability of the two innovation strategies in International vs. Domestic matches.

iii The distance-related adaptation costs for B in the IB strategy may differ in International and Domestic matches.

Depending on the distance between B’s needs and S’s characteristics in the Domestic match, there are heterogenous decisions across Buyers on whether to look for an International match or not; depending on the distance in an International match, there are heterogenous decisions across Buyers on whether to stay in an International match and to adapt or ask the Supplier to adapt the input. This, in turn, will imply some heterogeneity across Suppliers: some of them selling only domestically, some of them ‘exporting’, and, in both cases, some of them changing their good to match Buyers’ needs and others selling their existing input.

4.2 Timing

Buyers and Suppliers are initially involved in a Domestic match; they are producing, respectively, a final good and a customized intermediate good (what we deal with here is ‘innovation’ by existing firms).
B knows the actual distribution of the domestic Suppliers and he is matched with the closest one. B decides whether to go on with the existing relationship with his domestic Supplier or to look for a new Supplier abroad. Buyers who do not search abroad, stay in their current Domestic match. B has imperfect information on the location of Suppliers abroad: he only knows the number of Suppliers and that they are symmetrically spaced; so when searching in the foreign market B knows that it will match with an S at a random distance $Z_{ij} \sim U(0, 1/(2X_j))$. The ones who go abroad, pay a sunk cost to randomly match with one and only one foreign S (following Casella and Rauch (2003)). In this first meeting, they exchange the existing S good, and neither B nor S innovate. We assume that adaptation requires time and knowledge of the reciprocal characteristics (i.e., $Z_{ij}$). This is the reason why they engage in this first ‘meeting’. B may have a profit loss which adds to the cost of searching, due to the fact that in this intermediate period, since no adaptation takes place, the S good does not match his needs. After this intermediate match, $Z_{ij}$, the distance between B and the randomly matched S, is revealed. By exchanging the existing good (i.e. from the S point of view, by exporting in $t_0$), B and S meet, know each other and B decides whether to stay in the International match or not and, if so, under which type of innovation agreement. Only one attempt of International match is allowed for; so far, we have assumed that the costs of searching again for an International match are too high to bear them a second time.


6 There are several reasons why B may want to look for a new Supplier abroad, as pointed out by Egan and Mody (1992). B may want to preserve credibility in negotiating prices and/or to protect against S non-performance; B may be looking for a new Supplier for either current or future needs he forsees.

7 The role of a first exchange before engaging in an investment follows the intuition developed by Rauch and Watson (2003), in a slightly different context, and Egan and Mody (1992), who point out several reasons why buyer-seller relationships grow incrementally and start usually with a short-term agreement. Here, geographical proximity is necessary to reveal information about the location in the product space (i.e. on the relative distance between B needs and S characteristics). We do not contrast here the ‘learning by exporting’ vs. the ‘learning to export’ hypothesis, according to which firms (Suppliers in our framework) may carry out some innovation before entering the foreign market to meet some specific needs of the foreign buyers (Iacovone and Javorcik, 2010). We assume that the costs of gathering information on the needs of a specific foreign buyer, and the costs of implementing some adaptation before ‘meeting’ are too high and the expected profits too uncertain to do it. On the other side, engaging in an R&D investment in order to discover new products without specific characteristics (i.e. a new core line in order to enter the foreign market) coincides with firms entering the foreign market with their own product, which is what S does here in the first meeting. Moreover, empirically we observe many small firms not engaged in R&D which introduce product innovations.

8 It seems more reasonable to assume that only one match in a foreign country is not enough for B to acquire knowledge on the location along the circle of all the foreign Suppliers; nevertheless this is not a necessary assumption.
If B and S end up in a bad International match they can only go back home, and match with the, (possibly) new, closest domestic partner.\footnote{We follow Casella and Rauch (2003) and Rauch and Trinidade (2003) and allow firms ended up in a bad international match, to go back and match domestically, differently from Puga and Trefler (2010), where firms have to remain in the match. Since here the intuition is that firms may match in a first meeting without carrying out innovation, in order to know each other and see whether it is worth matching internationally and how, it would unreasonable not to allow them to go back home when ended up in a bad match.} We assume that in the time interval while B and S are involved in the International match, there might be some changes in the actual distribution of S in the domestic market (in a GE framework this would be endogenous), so the knowledge of the domestic distribution in \( t_1 \) is imperfect for who went abroad in \( t_0 \).\footnote{For S, it means matching with a new domestic B, replacing the foreign B which he was matching with in the bad international match (we are assuming that in \( t_0 \) S has a large enough productive capacity to add a B); for B it means that not necessarily his previous domestic S is still the closest one.} \( (B/S \) who in \( t_0 \) stayed in their domestic matches, still produce with their own domestic partner).

### 4.3 Innovation, costs and contract

Each supplier has a core production line, and must bear some fixed costs in order to specialize the input for each buyer. There is a sunk cost to enter the market and set up the core production line, and then, it is reasonable to imagine that there are also some fixed costs that supplier bear for each buyer, each time that the customized good has to be produced and not once and for all.

Generally speaking, we have in mind a world where two types of innovation efforts are possible:

I) ‘invention’: is a stochastic process (it may be the outcome of R&D investment); the output is the ‘ideation’ of a new good; it generates an order for step II. Both buyers and suppliers can engage in I): Buyers invent a new product for the final market and suppliers invent a new core line. In particular, in order to enter the (domestic) market they have to.

II) ‘implementation’: is a deterministic process; the output is a new product or a change/improvement in an existing one, ready for the market. The production of a final good always involves a specific supplier’s input, that may or may not already exist in the market. So the implementation stage for the buyer may generate changes in an existing intermediate good.

In this paper, we are not interested in the first type of innovation. We are not interested in the process of entry, nor we are in the (always possible) strategy for both S and B of engaging in a stochastic R&D process in order to change location along the circle after entering, or to add products (i.e. multiproduct firms).

We are interested in showing how B needs may induce a deterministic
process of product innovation for S (i.e. inducing S to adapt and specialize
his good to match B needs), through an order by B.\(^{11}\)

As a consequence of his ‘invention’ effort (which we do not model, as
pointed out above), there are two alternative implementation strategies
which B has two choose between:\(^{12}\)

- **IB strategy** (Buyer Implementation-IB): B buying an exisiting S good,
  and adapting either his process or the acquired S good to his needs,
  by bearing a distance-related fixed cost, \(b^{B_i} Z_{ii}\), in a Domestic match,
  and \(b^{B_j} Z_{ij}\) in an International match; in this case, S has to help B in
  adapting the input, by bearing a fixed cost \(F^S\) (for instance, the cost
  of technical assistance).

- **IS strategy** (Supplier Implementation-IS): B bearing the fixed cost \(F^B\)
  to solve the problem of figuring out what input exactly he needs to
  produce his good and asking S to produce it.\(^{13}\) In this case S bears the
distance-related fixed cost \(b^{S_i} Z_{jj}\) in a Domestic match and \(b^{S_j} Z_{ij}\) in
an International match.

There are two other costs:

- a **search cost**: \(\eta\) (sunk cost); B bears this cost when searching in the
  foreign market;

- a **per-period fixed ‘internationalization’ cost**: \(\gamma_{int}\), the sum of B and S
  costs.

The per-period ‘internationalization’ cost \(\gamma_{int}\) represents a collection of
(verifyable) costs: the costs of managing operations and of exchanging information
between different countries\(^{14}\) the costs of insurance against exchange rate fluctuations, and ‘burocratic’ costs.

\(^{11}\)Here, we borrow from the literature on incremental innovation, on the role of
the demand side of the market as sophisticated needs inducing innovation, on the role of
the interaction with users as a source of innovation, on the role of mutual learning in
buyer-seller relationships. See, for instance, Vernon (1966), Rosenberg (1982), Hippel
(1988), Egan and Mody (1992). It is worth noting that in our framework innovation
in the intermediate goods can emerge both through ‘invention’ (path I) and through
‘implementation’ induced by B ‘invention’, the latter being what we are interested in.

\(^{12}\)IS strategy is modeled borrowing from Grossman and Helpman (2005) and Puga and
Treffer (2010), while IB strategy follows the intuition in Helsley and Strange (2002).

\(^{13}\)If B is looking for a new S in the foreign market for the production of his current good
and if he is matched under an IS strategy in the domestic match, this cost would be close
to zero; we assume here that \(F^B\) is always positive and we do not consider that it may
differ across B depending on whether they are domestically matched under the IS strategy
or under the IB strategy; it may be interesting to consider this aspect in the future.

\(^{14}\)Since B is the downstream firm, the final product is assembled in his country; the S
good has to travel from country S to country B; this generates some costs due to managing
transport operations between different countries, not necessarily related to geographical
distance which could be relevant also within country for domestic matches.
B and S, matching either Domestically or Internationally bargain over the operating profit $\Pi$, from selling the final good to the market. In a bad match, the operating profit will be $\Pi = 0$, since no production takes place, while in a good match, the profits will be $\Pi^i = \Pi^j = \Pi^{ij} = \Pi$ (the two countries are identical with respect to prices of both final and intermediate goods, wages and profits).

Both in the Domestic and in the International matches, B offers a contract to S that can either be IS or IB depending on what maximizes his profits. S can accept or turn down the offer (i.e., B maximizes his profits under S participation constraint – PC, hereafter). Whenever B knows S will not accept his first best strategy, B offers S the alternative one, if it is still convenient to both.

The setting is one of incomplete contracts. We follow the literature on relation specific investment (see, in particular, in a similar framework, Grossman and Helpman (2005), Antràs and Helpman (2004), Puga and Treﬂer (2010)) in assuming that firms cannot sign ex ante enforceable contracts specifying the innovation effort (i.e., no commitment to no renegotiation and no third part verifiability are assumed): we rely on ex-post (Nash) bargaining sharing rule. The innovation costs do not enter the contract (B and S sign the contract and split profits ‘after’ having individually borne these costs). When they sign the contract $Z^{ij}$ is already revealed, but, due to the particular characteristics of the innovation effort, we assume that the contract is not contingent on $Z^{ij}$. The contract specifies what the payment will be contingent on production taking place, and on the type of innovation strategy (IS, IB). In the International matches, we assume that the per-period internationalization cost enters the contract, being verifiable.

After a decision is made, innovation costs are borne individually by B and S, and then the contract is signed; production takes place; profits are realized and all the payments are made depending on the strategy.

No bargaining takes place in the exchange of the existing good in $t_0$: B gets $\Pi$, the operating profit (net of the expenditure for the intermediate good bought from S), he bears a profit loss $\gamma_l$ and a sunk cost to search in the foreign market $\eta$. The profit loss is due to the fact that in this first exchange no adaptation occurs, so that the S good will not fit B needs unless the distance of the randomly drawn S is 0.

5 The Model

Before continuing, we introduce some simplifying assumptions. We assume that the unitary distance-related cost (i.e., the cost per unit of product distance) of adapting the good for S is the same in both International and Domestic matches ($b^{Sij} = b^{Sij} = b^S$). S receives an order from B with the exact specifics on what he needs, and S has to adapt following the order;
moreover, S knows his own input; then, the difference in the adaptation costs in International vs. Domestic matches should not be relevant for S. We express the unit of distance-related cost of adapting for B in terms of the cost for S and we allow it to be different in International and Domestic matches \((b^Bji = \alpha^Db^S, b^Bij = \alpha^I b^S)\). The cost for B to adapt his process or product to a foreign good will be probably different (and most probably higher) from the cost of adapting for a domestic S. B, who is already matched with a domestic S, is likely to be ‘less familiar’ with a foreign S intermediate good. With these assumptions, we want to underline here the role of the difference in the cost of adapting between B and S and how this difference may change in International matches. Moreover, we assume that the cost for S of assisting B in the IB strategy and the cost for B to provide a project for S in the IS strategy are the same \((F^B = F^S = F)\). The model is solved by backward induction; we look for the Nash Bargaining solution.

In what follows, we assume both \(\alpha^D \geq 1\) and \(\alpha^I \geq 1\). The cost of adapting for B is at least as big as the cost of adapting for S for given \(Z\). We think it is reasonable to assume to be more costly for B to adapt to an existing input (for any given distance) than for S to modify his own good following an order by B. Nevertheless, we also consider the case in which the cost of adapting per unit of distance is higher for S, \(\alpha^D < 1\) and \(\alpha^I < 1\). Table 4, summarizes the definition of variables and parameters.

### 5.1 Domestic Matches (D)

The outside options in a D match are represented by \(OUT^D_k = 0\), where \(k = S, B\), since no production will take place. By assuming an ex-post splitting rule, with \(\Pi = \Pi_B + \Pi_S\), we obtain \(\Pi_B = \Pi_S = \frac{\Pi}{2}\). The pay offs in the Domestic matches under IB strategy are given by:

\[
\pi^{IB,D}_B = \frac{\Pi}{2} - \alpha^Db^SZ_{ii} \tag{2}
\]

\[
\pi^{IB,D}_S = \frac{\Pi}{2} - F \tag{3}
\]

where \(\Pi\) is the total operational profit, \(\pi^{IB,D}_B\) and \(\pi^{IB,D}_S\) are the net total profits received by B and S, respectively, in a domestic match under IB. It is worth noting that General Equilibrium conditions must hold so as the
gains from trade (GFT) are non-negative and the participation constraints (PC) are satisfied:

\[
\frac{\Pi}{2} \geq 0
\]
\[
\pi_{IB,D}^{IB} \geq 0 : Z_{ii} \leq \left(\frac{\Pi}{2}\right)\left(\frac{1}{\alpha_{ib}}\right)
\]
\[
\pi_{S}^{IB,D} \geq 0 : \frac{\Pi}{2} \geq F
\]

The pay offs in the Domestic matches under IS strategy are given by

\[
\pi_{IS,D}^{B} = \frac{\Pi}{2} - F
\]

\[
\pi_{IS,D}^{S} = \frac{\Pi}{2} - b^{S} Z_{ii}
\]

where \(\Pi\), is the total operational profit, \(\pi_{B}^{IS,D}\) and \(\pi_{S}^{IS,D}\) are the net total profits received by B and S, respectively, in a domestic match under IS. General Equilibrium conditions must hold such that GFT are non-negative and the participation constraints (PC) are satisfied:

\[
\frac{\Pi}{2} \geq 0
\]
\[
\pi_{B}^{IS,D} \geq 0 : \frac{\Pi}{2} \geq F
\]
\[
\pi_{S}^{IS,D} \geq 0 : Z_{ii} \leq \left(\frac{\Pi}{2}\right)\left(\frac{1}{\alpha_{ib}}\right).
\]

In this framework, B chooses the IB strategy if

\[
\pi_{B}^{IB,D} > \pi_{B}^{IS,D}
\]

under the supplier’s participation constraint, PC

\[
\pi_{S}^{IB,D} \geq 0.
\]

B chooses instead the IS strategy if:

\[
\pi_{B}^{IS,D} > \pi_{B}^{IB,D}
\]

under the supplier’s participation constraint, PC

\[
\pi_{S}^{IS,D} \geq 0
\]
The solution of the B decision problem allows us to identify the following intervals where either the IB or the IS strategy are implemented, respectively, or no match take place.

**IB:**  \( Z_{ii} \in \{0, Z_{ii}^D\} \)

**IS:**  \( Z_{ii} \in \{Z_{ii}^I, Z_{ii}^S\} \)

no match:  \( Z_{ii} \in \{Z_{ii}^I, \frac{1}{2X_i}\} \)

(with  \( X_i \) number of S in B Domestic market)

and where

\[
Z_{ii}^I = \frac{F}{\alpha^D b^S} \tag{10}
\]

\[
Z_{ii}^S = \frac{1}{b^S} \left( \frac{\Pi}{2} \right) \tag{11}
\]

are the relevant distance thresholds in a domestic match.

\( Z_{ii} < Z_{ii}^I \) whenever  \( \Pi > \frac{2F}{\alpha^D} \) (when  \( \alpha^D > 1 \) this constraint is not binding while it is the S participation constraint in IB; the opposite occurs when  \( \alpha^D < 1 \)).

### 5.2 International Matches (I)

The outside options in an international match are  \( OUT_k^I = E(\pi_k^I) \), where  \( k = S, B \). When B and S end up in a bad International match, they can always go back home and look for a (possibly new) partner in the Domestic market. After the intermediate period in which they have been involved in the International match, information on the locations of the domestic suppliers and buyers, for B and S, respectively, is imperfect.

These outside options are given by:

\[
E(\pi_S^D) = \int_0^{Z_{jj}^I} \pi_S^{IB,D} \cdot g(Z_{jj}) dZ_{jj} + \int_{Z_{jj}^I}^{Z_{jj}^S} \pi_S^{IS,D} \cdot g(Z_{jj}) dZ_{jj} \equiv G(X_j, F, \Pi, \alpha^D, b^S)
\]

and

\[
E(\pi_B^D) = \int_0^{Z_{ii}^I} \pi_B^{IB,D} \cdot h(Z_{ii}) dZ_{ii} + \int_{Z_{ii}^I}^{Z_{ii}^S} \pi_B^{IS,D} \cdot h(Z_{ii}) dZ_{ii} \equiv H(X_i, F, \Pi, \alpha^D, b^S)
\]

where  \( E(\pi_S^D) \) and  \( E(\pi_B^D) \) are the expected profits of the Domestic matches for S and B, respectively;  \( g(Z_{jj}) = 2X_j \) and  \( h(Z_{ii}) = 2X_i \) are the densities of the distances in the S and B domestic markets, respectively.

**Pay offs in the International Matches under IB and IS strategy**

Assuming again an ex-post splitting rule, with the above specified outside options, from
\[ \Pi - \gamma_{\text{int}} = \Pi_B + \Pi_S \]

and

\[ V = (\Pi_B - E(\pi_D^B))(\Pi - \gamma_{\text{int}} - \Pi_B - E(\pi_S^D)) \]

we obtain the following net total profits, under the IB strategy:

\[ \pi_{IB, B}^I = \Pi_B - \alpha DBS Z_{ij} = \frac{1}{2} \left[ \Pi - \gamma_{\text{int}} - E(\pi_B^D) + E(\pi_B^D) - E(\pi_B^D) \right] + E(\pi_B^D) - \alpha DBS Z_{ij} \]

\[ \pi_{IB, S}^I = \Pi_B - \frac{1}{2} \left[ \Pi - \gamma_{\text{int}} - E(\pi_B^D) + E(\pi_B^D) \right] + E(\pi_B^D) - F. \]

The GE condition for the International match to be profitable must hold (non negative GFT):

\[ GFT_{IB, F}^I : [\Pi - \gamma_{\text{int}} - E(\pi_B^D) - E(\pi_S^D)] \geq 0 \]

\[ \pi_{IB, B}^I \text{ and } \pi_{IB, S}^I \] are the net total profits received by B and S, respectively, in an International match under IB.

We obtain the following net total profits, under the IS strategy:

\[ \pi_{IS, B}^I = \Pi_B - F = \frac{1}{2} \left[ \Pi - \gamma_{\text{int}} - E(\pi_B^D) + E(\pi_B^D) \right] + E(\pi_B^D) - F \]

\[ \pi_{IS, S}^I = \Pi_B - b^S Z_{ij} = \frac{1}{2} \left[ \Pi - \gamma_{\text{int}} - E(\pi_B^D) - E(\pi_B^D) \right] + E(\pi_B^D) - b^S Z_{ij} \]

(\text{the GE condition for the International match to be profitable which is the same as above})

\[ \pi_{IS, B}^I \text{ and } \pi_{IS, S}^I \] are the net total profits received by B and S, respectively, in an International match under IS.

### 5.3 Equilibrium

A Buyer who has decided to look for a better match (i.e., a closer Supplier) in the international markets will decide whether to stay or not in the randomly drawn match and under which innovation strategy, as opposed to go back to its domestic market only after that \( Z_{ij} \) is revealed.\(^{16}\)

\(^{16}\)At \( t_0 \), B decides whether to look for a International match or not. B knows his own \( Z_{ii} \), but not \( Z_{ij} \), so expected profits are considered. B goes and looks for an international match if \( E(\pi_{IB}^I) + \pi_{IB}^{I, 0} \geq 2\pi_{IB}^{I, 0, B} \) under IB in the initial domestic match, and if \( E(\pi_{IS}^I) + \pi_{IS}^{I, 0} \geq 2\pi_{IS}^{I, 0, B} \) under IS in the initial domestic match, where \( \pi_{IB}^{I, 0} = \Pi - \eta - \gamma_l \) are the total net profits that B receives during the first meeting (with \( \eta \) and \( \gamma_l \) respectively the sunk cost of searching and the profit loss). \( E(\pi_{IB}^I) \) are the expected profits of the international match.

As a result, we obtain B making the decision whether to search or not depending on the thresholds on the domestic distance, \( Z_{ii} \). (To BE DONE)
B chooses IB internationally if:

\[ \pi_{IB,I}^B > \pi_{IS,I}^B \]  

under

\[ \pi_{IS}^B \geq E(\pi_{IS}^D) \]  
\[ \pi_{IB}^B \geq E(\pi_{IB}^D). \]  

B chooses IS internationally if:

\[ \pi_{IS,I}^B > \pi_{IB,I}^B \]  

under

\[ \pi_{IS}^B \geq E(\pi_{IS}^D) \]  
\[ \pi_{IS,I}^B \geq E(\pi_{IS}^D). \]  

The outcome of the B decision process allow us to identify the following intervals where either of the two strategies are implemented or no international match takes place:

**IB:** \( Z_{ij} \in \{0, Z_{ij}\} \)

**IS:** \( Z_{ij} \in \{Z_{ij}, Z_{ij}\} \)

no International match: \( Z_{ij} \in \{Z_{ij}, \frac{1}{2X_j}\} \)

with \( X_j \) number of S in the foreign market where B has searched, and

where

\[ Z_{ij} = \frac{F}{\alpha I b^S} \]  
\[ \overline{Z}_{ij} = \frac{1}{2b^S} [\Pi - \gamma_{int} - E(\pi_B^D) - E(\pi_S^D)] \]

are the relevant thresholds in an international match.

\( Z_{ij} < \overline{Z}_{ij} \) whenever GE is such that \( \alpha I [\Pi - \gamma_{int} - E(\pi_B^D) - E(\pi_S^D)] > 2F \) (that is that when \( \alpha I > 1 \) is not binding, PCs holding under the IB strategy, while the opposite happens with \( \alpha I < 1 \)).

To analyse how the set of distances for which IS strategy is implemented differs between International and Domestic matches we consider the measure
of the relative share of the IS interval over the sum of (IS+IB) intervals:

\[ (IS)^D = (1 - \frac{Z_i}{Z_{ij}}) \] and \( (IS)^I = (1 - \frac{Z_i}{Z_{ij}}) \). We compare this measure in Domestic and International matches. One can easily show that the difference between \((IS)^I\) and \((IS)^D\) is:

\[ (IS)^I - (IS)^D \equiv \Delta(IS) = \frac{2F}{\alpha^D \Pi} - \frac{2F}{\alpha^I \Pi - \gamma_{int} - E(\pi_B^D) - E(\pi_S^D)}. \] (22)

When this difference is positive (negative) the share of the set of distances for which IS is implemented over total good matches is higher (lower) in an International match than in a Domestic one. One can easily see that this difference is negative for \( \alpha^I = \alpha^D \). \( \Delta(IS) > 0 \) whenever

\[ \alpha^I > \frac{\alpha^D \Pi}{\Pi - \gamma_{int} - E(\pi_B^D) - E(\pi_S^D)} \equiv \tilde{\alpha}^I \] (23)

which implies \( \alpha^I > \alpha^D \).

We can conclude from this part of the analysis, that in order for S to have a higher probability to adapt to B needs in an International match, the distance-related adaptation cost for B has to be higher in an International match than in a Domestic one for any given distance Z (\( \alpha^I > \alpha^D \)). This could be justified on the ground, for instance, that B comes from a Domestic match and has a better knowledge of domestic inputs. With \( \alpha^I = \alpha^D \) (even more with \( \alpha^I < \alpha^D \)), B would be more likely to buy the existing good provided by S and adapt it to his needs in an International match than in a Domestic one. This is due to the fact that the IS strategy is implemented for relatively large distances both in Domestic and International matches (when Z is ‘large’ B asks S to adapt) and since International matches are successful for ‘shorter’ distances (due to the effect of \( \gamma_{int} \) and the outside options), the ‘IS strategy set’ is smaller in this type of matches. By contrast, if \( \alpha^I > \alpha^D \), the higher cost of adapting for B in an International match can revert the previous result, causing a ‘shrinking’ of the IB strategy set, and increasing the relative weight of the IS strategy set. It is also worth noting that the IS set in an International match depends negatively on the number of suppliers, \( X_j \) and \( X_i \), in the country of origin of S and B, respectively, and on \( \gamma_{int} \), the internationalization cost.

6 Concluding remarks

In this paper, we are interested in reassessing the relationship between exporting and innovation, by focusing on pathways of knowledge transfers running between firms, and, in particular, on the information exchange between firms engaged in a ‘production to order buyer-supplier’ relationship.
By using the EFIGE dataset, a survey gathering firm-level data on manufacturing firms in seven European countries, we show that producing to order for foreign customers is positively associated to product innovativeness; the association is not only statistically, but also economically significant. We provide a theoretical model in order to give a potential interpretation of this empirical evidence, in the framework of the incomplete contracts and imperfect information literature related to specific input provision in international trade. In our setting, there are two types of agents engaged in production: downstream producers (i.e., Buyers) who purchase an input from upstream producers (i.e., Suppliers). Buyers and Suppliers are distributed over the product characteristics circle. We provide a set up in which for a Buyer-Supplier match to work, some adaptation is needed depending on the distance between the Buyer’s needs and the good produced by the Supplier. Buyers can either purchase the intermediate good as it is and then adapt it or they can give Suppliers a ‘project’ in order to adapt the input, bearing the distance-related costs. We show how, due to the interplay between the innovation cost parameters, a per-period fixed internationalization cost, the distance in the International match, and the number of suppliers in Suppliers’ and Buyers’ countries, respectively, the set of distances for which Suppliers adapt in International matches differs from the set of distances for which they adapt in the Domestic ones. We single out the conditions under which Suppliers are more likely to adapt their product for foreign customers than for domestic ones. In summary, our model thus provides a framework in which firms may implement different innovation and internationalization strategies depending on the characteristics of their products.
Table 1: Summary statistics on firm’s innovation by domestic and foreign matches

<table>
<thead>
<tr>
<th>Type of match</th>
<th>product innovation (share)</th>
<th>market innovation (share)</th>
<th>applied for patents (share)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic customer</td>
<td>0.388</td>
<td>0.219</td>
<td>0.067</td>
</tr>
<tr>
<td>Foreign customer</td>
<td>0.584</td>
<td>0.409</td>
<td>0.186</td>
</tr>
</tbody>
</table>

Note. (a) Produces for domestic customers only. (b) Produces for at least one foreign customer.

Table 2: Producing to order for foreign customers (FORCUST) and product innovations (OLS)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORCUST</td>
<td>0.213***</td>
<td>0.175***</td>
<td>0.115***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>R&amp;D employment ratio</td>
<td>0.307***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital intensity</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unit labor costs</td>
<td>-0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>firm size fixed effects (4 categories)</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>country fixed effects</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>NACE fixed effects</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>N. obs.</td>
<td>6980</td>
<td>6980</td>
<td>6980</td>
</tr>
<tr>
<td>R2</td>
<td>0.05</td>
<td>0.14</td>
<td>0.22</td>
</tr>
</tbody>
</table>

*,**,*** significant at 10, 5 and 1 percent, respectively. Standard errors robust to heteroskedasticity. All regressions use survey weights.
Table 3: Producing to order for foreign customers (FORCUST) and ‘degree’ of innovation (OLS)

<table>
<thead>
<tr>
<th></th>
<th>market innovations (1)</th>
<th>patent (2)</th>
<th>design (3)</th>
<th>trademark (4)</th>
<th>copyright (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORCUST</td>
<td>0.110***</td>
<td>0.056***</td>
<td>0.021***</td>
<td>0.039***</td>
<td>0.009*</td>
</tr>
<tr>
<td>R&amp;D employment ratio</td>
<td>0.235***</td>
<td>0.115***</td>
<td>0.043***</td>
<td>0.076***</td>
<td>0.019***</td>
</tr>
<tr>
<td>capital intensity</td>
<td>0.005</td>
<td>0.010</td>
<td>0.011**</td>
<td>0.014</td>
<td>0.002</td>
</tr>
<tr>
<td>unit labor costs</td>
<td>0.000</td>
<td>-0.000</td>
<td>-0.004*</td>
<td>-0.008</td>
<td>-0.005**</td>
</tr>
<tr>
<td>firm size fixed effects (4 categories)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>country fixed effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>NACE fixed effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>N. obs.</td>
<td>6980</td>
<td>6980</td>
<td>6980</td>
<td>6980</td>
<td>6980</td>
</tr>
<tr>
<td>R2</td>
<td>0.19</td>
<td>0.16</td>
<td>0.13</td>
<td>0.15</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*,**,*** significant at 10, 5 and 1 percent, respectively. Standard errors robust to heteroskedasticity.
Table 4: Legend of variables and parameters

<table>
<thead>
<tr>
<th>variable/parameter</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{ii}$</td>
<td>distance between B and S in domestic matches for B</td>
</tr>
<tr>
<td>$Z_{jj}$</td>
<td>distance between B and S in domestic matches for S</td>
</tr>
<tr>
<td>$Z_{ij}$</td>
<td>distance between B and S in international matches</td>
</tr>
<tr>
<td>$b_{B_{ij}}$</td>
<td>cost of adapting per unit of distance by B in Domestic matches</td>
</tr>
<tr>
<td>$b_{B_{ij}}$</td>
<td>cost of adapting per unit of distance by B in International matches</td>
</tr>
<tr>
<td>$b_{S_{ij}}$</td>
<td>cost of adapting per unit of distance by S in Domestic matches</td>
</tr>
<tr>
<td>$b_{S_{ij}}$</td>
<td>cost of adapting per unit of distance by S in International matches</td>
</tr>
<tr>
<td>$F_B$</td>
<td>B cost of providing a “project” to S</td>
</tr>
<tr>
<td>$F_S$</td>
<td>S cost of assisting B</td>
</tr>
<tr>
<td>$a_D$</td>
<td>ratio between costs in Domestic matches ($b_{B_{ij}} / b_{S_{ij}}$)</td>
</tr>
<tr>
<td>$a_I$</td>
<td>ratio between costs in International matches ($b_{B_{ij}} / b_{S_{ij}}$)</td>
</tr>
<tr>
<td>$\gamma_{int}$</td>
<td>sum of B and S cost of internationalization</td>
</tr>
<tr>
<td>$\eta$</td>
<td>B search cost</td>
</tr>
</tbody>
</table>

assumptions/further definitions

- $b_{S_{jj}} = b_{S_{ij}} = b^S$
- $b_{B_{ij}} = (a^D)b^S$
- $b_{B_{ij}} = (a^I)b^S$
- $F_B = F_S = F$

References


