Within firm productivity gains from trade?
Not always

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Abstract

Empirical evidence shows that trade liberalization improves productivity not just because of a selection effect but also because of productivity gains within firms. This paper proposes a model that allows for both channels. In contrast to the existing literature, the process innovation is modeled as a continuous variable and there is both a fixed and a variable cost to innovate. This leads to a rich set of testable predictions about the intensive and extensive margin of innovation and the strategic interaction between the innovation and export choices. In one equilibrium, only the most productive firms are able to export and increase their level of process innovation; in other equilibrium the possibility of accessing the export market gives an incentive for non-innovating firms to start. Which equilibrium prevails depends on the cost-benefit ratio of innovation and the exporting cost. This has implications for the effect of trade liberalization on exports in different countries and sectors.

1 Introduction

The link between firm productivity and international trade has become increasingly important as an area of economic research over the last ten years. Evidence strongly supports the self selection of more productive firms into foreign market participation, but there is mixed evidence on the positive impact of export market participation on firm productivity. Motivated by these observations, I study the existence of within firm productivity gains through innovation in an open economy.

The literature has focused on models with heterogeneous firms because of their ability to match the export decisions of firms to characteristics such as productivity, size or ownership status. This literature, with early contributions by Bernard et al. (2003) and Melitz (2003), emphasizes the selection effect into
export status and the reallocation of production factors and shares between firms as the source of aggregate productivity gains. However, empirical evidence points out that productivity gains occur not only between firms but also within firms (Pavnick 2002, Trefler 2004, and De Loecker 2007). The early literature did not identify within firm gains, because firm productivity was modeled by a random draw from a probability distribution.

The empirical literature has emphasized that the effect of trade liberalization on within firm productivity happens both along the extensive margin and the intensive margin. For example, Alvarez (2001) supports the hypothesis that exporters invest more intensively than non exporters. Likewise, Aw, Roberts & Xu (2008) show empirically that prior export market activity increases the probability of investing in R&D and that the interdependence between R&D activities and the exporting choice is critical to explain current investment decisions. Finally, Lileeva & Trefler (2009) show that new exporters innovate along different dimensions. Hence, evidence suggests that both the intensive and extensive margin are important in order to explain the productivity gains within firms and that the strategic interaction of innovation, entry, exit and export decisions should be further explored.

This paper proposes a trade model with heterogeneous firms that have the option to invest in process innovation. The model follows the standard setup of Melitz (2003) with a basic difference: once a firm learns about its productivity it can decide to spend resources in process innovation to improve its technology. I are not the first to explore the effect of trade on within firm productivity gains, but I are the first to study this issue along both the intensive and the extensive margin. In contrast to Navas-Ruiz & Sala (2007) and Constantini & Melitz (2008), I model process innovation as a continuous variable, and therefore are able to analyze the intensive margin. And, in contrast to Vannoorenbergh (2008), Bustos (2009) and Atkeson & Burstein (2010) innovation involves both fixed and variable costs. By having both a fixed and variable costs not all firms will be innovating and I can explore how trade liberalization affects the extensive margin and intensive margin of innovation at the same time.

The interaction between the innovation decision and the exporting decision will determine which one of the two equilibria emerges. In both equilibria, firms at the top of the distribution will export and innovate, while firms at the bottom of the distribution will not perform any of those activities. The behavior of middle productivity firms differs across equilibria. In the low cost innovation
equilibrium trade costs are high in comparison with the cost-benefit ratio of innovation, so that middle productivity firms choose to innovate rather than enter new markets. The selection effect reduces the extensive margin of innovation but productivity gains along the intensive margin are still possible if the gains from those who entered the export market offset the losses of those who stopped performing innovation activities. In the low cost trade equilibrium, trade costs are low in relation to the cost-benefit to innovation, and firms that are productive enough choose to export rather than engage in innovation. Trade generates within plant gains, increasing both the intensive and extensive margins of innovation beyond the levels of the low cost innovation equilibrium by overcoming the selection effect generated when an economy opens up to trade.

Analytically the main contribution of the paper is the ability to analyze the innovation decisions of firms through a tractable innovation policy function and how trade liberalization affects it through variations of firms’ decisions. A second contribution of the paper is to provide insight into the channels through which firms can have productivity gains from trade by performing innovation. To my knowledge this is the first paper that studies along which margin within-firm productivity gains from innovation may happen. Taking the model one step further, the ultimate aim is to understand how the interplay of the intensive and the extensive margin affects aggregate productivity gains and welfare.

The paper is organized as follows. In Section 2, I present the closed model of the economy where firms take decisions on innovation and domestic participation. In Section 3, I open up the economy to trade and explore the different scenarios that the interaction between the exporting and innovation choices creates. In Section 4, I take a look at how trade liberalization affects the distribution of firms, aggregate welfare and aggregate innovation. Section 5 concludes.

2 The Closed Economy

The model is based on the monopolistic competition framework proposed by Melitz (2003). Goods are differentiated and each good will be produced by one firm. The model is extended to allow these firms to have the opportunity to engage in process innovation.
2.1 Demand

There is a continuum of consumers of measure $L$. Given the set $\Omega$ of varieties supplied to the market, the consumer’s preferences are represented by the standard C.E.S. utility function

$$\left[ \int_{\omega \in \Omega} q_{\omega} d\omega \right]^\frac{1}{\sigma}$$

where $q_{\omega}$ denotes the quantity consumed of variety $\omega$ and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution across varieties. Since the market is subject to the expenditure-income constraint, that is the:

$$\int_{\omega \in \Omega} p_{\omega} q_{\omega} d\omega = R$$

where $R$ is the total revenues obtained in the economy.

Then standard utility maximization implies that the demand for each individual variety will be:

$$q_{\omega} = \left[ p_{\omega} \right]^{-\sigma} \frac{R}{P^{1-\sigma}}$$

(1)

where $p_{\omega}$ is the price of each variety $\omega$ and $P = \left[ \int_{\omega \in \Omega} p_{\omega}^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$ denotes the price index of the economy.

2.2 Supply

There is a continuum of firms, each producing a different variety $\omega$. Each firm draws its productivity $\varphi$ from a distribution $G(\varphi)$ after paying a labor sunk cost of entry $f_E$. Since a firm is going to be characterized by its productivity $\varphi$, it will be the same to talk about the variety $\omega$ than the productivity $\varphi$ with which it is produced.

Production requires only one factor, labor, which is inelastically supplied at its aggregate level $L$, and therefore can be taken as an index of the economy’s size.

In contrast to the Melitz model where firms use a constant returns to scale production technology with a fixed overhead cost, I introduce process innovation in the production technology so that firms can affect their marginal cost. Hence, to produce an output $q(\varphi)$, a firm requires $l(\varphi)$ labor units

$$l(\varphi) = f_D + c(\Gamma) + \frac{q_{\omega}}{\varphi} \frac{1}{(1+\Gamma)^{\frac{1}{\sigma}}}$$
where $\varphi$ is a productivity parameter that every firm draws from a distribution with pdf $g(\varphi)$ over support $(0, \infty)$, $f_D > 0$ is a fixed labor cost of production shared by every firm, and $\Gamma$ is the amount of innovation performed by the firm in order to enhance its productivity that has an associated cost function $c(\Gamma)$.

The cost function of the innovation follows the suggestions of Klette & Kortum (2004), Lentz & Mortensen(2005) and Long, Raff & Stähler (2007). Firms pay a fixed cost, that can be attributed to the acquisition and implementation of the technology, plus a continuous cost that depends directly on the process innovation performed by each firm. Hence the cost function $c(\Gamma)$ is defined as

$$c(\Gamma) = \begin{cases} \Gamma^{\alpha+1} + \kappa & \text{if } \Gamma > 0 \\ 0 & \text{if } \Gamma = 0 \end{cases}$$

where $\kappa$ is the fixed cost required to implement the process innovation and $\alpha > 0$ measures the rate at which the marginal cost of the innovation increases, thus the higher the level of innovation the higher the cost associated with marginal increases.

Even though it can be argued that the cost of innovation can be simplified by imposing a linear variable cost, the existence of convex innovation costs are a standard feature on the literature and ensure that innovation does not become explosive in large firms. Another simplification would be to have either a fixed cost or a variable cost but not both, nevertheless maintaining a flexible cost function is important. For example, Vannoorenberghue (2008) assumes away a fixed innovation cost, which implies that all firms engage in process innovation. This eliminates the possibility of studying the interaction between the export and innovation decisions along the extensive margin, which is one of the purposes of my research.

### 2.2.1 Timing

In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost $f_E$ in order to get a draw of the productivity parameter $\varphi$. In the second stage, with the knowledge of their own productivity firms decide how much to innovate. Since innovation requires paying a labor sunk cost $\kappa$ on top of the cost associated to the amount invested, there will be two types of firms in a closed economy. Firms active on the domestic market that do not perform innovation which I will name Type $D$ firms, and those active on the market that engage in innovation activities will be Type $DI$ firms. Finally, in the third
stage, the firms belonging to these types, with the technology choice on mind, decide upon an optimal price and production level.

![Figure 1: Timing](image)

### 2.2.2 Firm’s problem

Given the timing, I solve the firms problem through backward induction.

**Optimal Pricing Rule** In the last stage of the problem the firm sets its optimal price given its innovation decision and the market conditions which are summarized by the price index $P$.

\[
\max_{p(\varphi)} p(\varphi) q(\varphi) - f_D - \frac{q(\varphi)}{\varphi (1 + \Gamma)^{\frac{1}{\sigma} - 1}} - c(\Gamma)
\]

\[
\leftarrow p(\varphi) = \left( \frac{\sigma}{\sigma - 1} \right) \frac{1}{\varphi} \cdot \frac{1}{(1 + \Gamma)^{\frac{1}{\sigma} - 1}} \quad \forall \Gamma \quad (2)
\]

Since firm type $D$ chooses not to innovate ($\Gamma = 0$) , then the price rule is the usual mark up $p_D(\varphi) = \left( \frac{\sigma}{\sigma - 1} \right) \frac{1}{\varphi}$. Furthermore, notice that a firm type DI is able to impose a lower effective price to consumers than if he were not innovating since $p_{DI}(\varphi) = (1 + \Gamma)^{\frac{1}{\sigma} - 1} p_D(\varphi)$.

Finally, by substituting the optimal price (2) in (1), I obtain the output of a non-innovator $q_D(\varphi) = \left[ \frac{1}{\rho \varphi} \right]^{\frac{\sigma}{\rho - \sigma}} \frac{R}{p^{1-\sigma}}$ and likewise the innovators output will be $q_{DI}(\varphi) = (1 + \Gamma)^{\frac{1}{\sigma} - 1} q_D(\varphi)$.

And thus, the domestic profits of non-innovators and innovators are given by:

\[
\pi_D(\varphi) = \frac{r_D(\varphi)}{\sigma} - f_D \quad (3)
\]

\[
\pi_{DI}(\varphi) = (1 + \Gamma) \frac{r_D(\varphi)}{\sigma} - f_D - c(\Gamma) \quad (4)
\]
Optimal innovation Decision  The optimal innovation rule is obtained from the first order condition of the maximization of $\pi_{DI} (\varphi)$, given by equation (4), with respect to $\Gamma$ provided that the firm makes higher profits by innovating than by choosing not to innovate at all.

$$\Gamma (\varphi) = \begin{cases} \left[ \frac{1}{\alpha+1} \left( \frac{R(P\varphi)^{\sigma-1}}{\sigma} \right) \varphi^{\sigma-1} \right]^{\frac{1}{\sigma}} & \text{if } \pi_{DI} (\varphi) \geq \pi_{D} (\varphi) \\ 0 & \text{if } \pi_{DI} (\varphi) < \pi_{D} (\varphi) \end{cases} \quad (5)$$

Since $r_{D} (\varphi)$ is increasing in $\varphi$, it is obvious that there will be a cut-off productivity $\varphi_{I}^{*}$, such that all firms with a productivity $\varphi \geq \varphi_{I}^{*}$ will innovate. The innovation decision of all firms that innovate can be expressed in terms of the innovation decision of the cutoff firm:

$$\Gamma (\varphi) = \Gamma (\varphi_{I}^{*}) \left[ \frac{\varphi^{\sigma-1}}{(\varphi_{I}^{*})^{\sigma-1}} \right]^{\frac{1}{\sigma}} \quad (6)$$

where $\frac{1}{\sigma}$ is the parameter that shapes the optimal innovation function and tells us how will the innovation rises with size, where I take the productivity parameter $\varphi^{\sigma-1}$ to be the indicator of size. If the function is linear ($\alpha = 1$), then innovation rises proportionately with size, however, if the function is concave ($\alpha > 1$), then the amount of innovation performed will rise less than proportionally with size, and if the function is convex ($\alpha \in (0, 1)$) the amount of innovation performed will increase more than proportionally with the productivity.

2.3 Equilibrium in a Closed Economy

In this economy there are two relevant cutoffs I must determine in equilibrium. First, the entry productivity-cutoff $\varphi_{D}^{*}$ which satisfies the zero profit condition (ZPC):

$$\pi_{D} (\varphi_{D}) = 0$$

$$\leftrightarrow (\varphi_{D}^{*})^{\sigma-1} = \frac{\bar{f}_{D}}{\left( \frac{(R(P\varphi)^{\sigma-1})}{\sigma} \right)} \quad (8)$$

Secondly, the innovation productivity cutoff $\varphi_{I}^{*}$ corresponding to the firm that
is indifferent between innovating or not innovating:

$$\pi_{DI} (\psi_I) = \pi_D (\psi_I)$$

$$\leftrightarrow (\psi_I^*)^{\sigma - 1} = \frac{(\frac{\pi}{\alpha})^{\frac{\alpha}{\alpha + 1}} \cdot (\alpha + 1)}{(R(P)^{\sigma - 1})}$$

By combining equations (8) and (10) I have the relation between the innovation cutoff and the entry cutoff in terms of the fixed cost to produce, the fixed cost to innovate and the parameters of the innovation:

$$(\psi_I^*)^{\sigma - 1} = \frac{(\frac{\pi}{\alpha})^{\frac{\alpha}{\alpha + 1}} \cdot (\alpha + 1)}{f_D} \cdot (\psi_D^*)^{\sigma - 1} = \Lambda (\psi_D^*)^{\sigma - 1}$$

(11)

where $\frac{(\frac{\pi}{\alpha})^{\frac{\alpha}{\alpha + 1}} \cdot (\alpha + 1)}{f_D}$ can be interpreted as the ratio between the innovation costs and the operating costs. The numerator is composed by the sunk cost of innovation, whose effect is determined by the shape of the innovation and the elasticity $^1$ of the variable costs to innovate, while the denominator is simply the operating costs that every firm must incur into in order to actively participate in the market.

Obviously, $\Lambda > 1$, or $\frac{(\frac{\pi}{\alpha})^{\frac{\alpha}{\alpha + 1}} \cdot (\alpha + 1)}{f_D} > f_D$, is needed to have selection into the innovation status. The assumption is likely to hold since the empirical evidence suggest that only a subset of more productive firms undertakes process innovations.

In order to find an stationary equilibrium where the aggregate variables remain constant over time, I require a mass $M_e$ of new entrants in every period. As in Melitz 2003, the mass of successful entrants $M_e (1 - G (\psi_D^*))$ exactly replaces the mass $\delta M$ of incumbents who exit.

Free entry requires the expected profits to be equal to the sunk costs

$$\left[ \int_{\psi_D^*}^{\psi_I^*} \pi_D (\psi) dG (\psi) + \int_{\psi_D^*}^{\psi_I^*} \pi_{DI} (\psi) dG (\psi) \right] = \delta f_E$$

(12)

Given (11), (12) is a function of only $\psi_D^*$ which is determined by the ZPC condition. It can be shown that in the $(\varphi, \pi)$ space, the FE curve is increasing and is cut by the ZPC curve only once from above ensuring the existence and uniqueness of the equilibrium (See Appendix for proof).

$^1\varepsilon_C = \frac{\psi' (j)}{\psi (j)} = (\alpha + 1)$
Finally, notice that the cutoff productivity level in this economy is higher compared to the one found in Melitz (2003) where firms have no choice to invest in a productivity enhancing technology (See Appendix for proof). The reasoning behind this result is that the ability of some firms to invest in a cost reducing technology enables them to have more market shares than they would without the presence of innovation. Logically, those market shares are ‘stolen’ from the less productive firms of the economy.

3 Open Economy

3.1 The Set-Up

I consider a symmetric two-country world, home and foreign. The symmetry of both countries ensures that factor price equalization holds, countries have a common wage which can be still taken as a numerare and they share the same aggregate variables. Furthermore, exporters incur in two additional costs: a per period fixed cost to export \( f_X \) and a per-unit iceberg trade cost \( \tau \), \( \tau > 1 \) units of a good must be shipped in order for one unit to arrive at destination.

Since I assume that a firm that supplies to other countries is also supplying to the domestic market, then the total labor cost of an exporting firm is

\[
l_x (\varphi) = \frac{\tau q_x (\varphi)}{\varphi} \frac{1}{(1 + \Gamma) \pi - \tau} + \frac{q (\varphi)}{\varphi} \frac{1}{(1 + \Gamma) \pi - \tau} + f_D + f_X + c (\Gamma)
\]

where \( l_x (\varphi) = \frac{\tau q_x (\varphi)}{\varphi} + \frac{q (\varphi)}{\varphi} + f_D + f_X \) if the firm chooses not to innovate.

On the open economy, firms have two decisions to take, whether to innovate and whether to enter into the foreign market. In autarky, firms did not have to worry over entering foreign markets, only about innovation, therefore firms were classified in two types. However, in the open economy, the existence of the export decision implies that firms will be classified in four types. Type D firms are those only active on the domestic market that do not perform innovation and type DI firms are those active only on the domestic market that innovate; Type X firms are those active on both the domestic and the foreign market that do not perform any innovation and finally, type XI firms, active on the domestic and foreign markets that engage on innovation activities.
Optimal Prices  The optimal price rule set on the domestic market does not vary with respect to autarky and is given by equation (2). Firms active on foreign markets, due to the transportation costs, will decide to charge a higher price on the exports. As a result, the effective price of a particular variety in the foreign county is

\[ p^* (\varphi) = \tau p (\varphi) = \begin{cases} 
  p_X (\varphi) = \tau p_D (\varphi) & \text{if } \Gamma = \Gamma_X = 0 \\
  p_{XI} (\varphi) = [1 + \Gamma_X \frac{1}{\sigma}] \frac{1}{\sigma} p_X (\varphi) & \text{if } \Gamma_X \neq 0
\end{cases} \] (13)

where \( \Gamma_X \) is the amount of innovation performed by an exporter.

Optimal Innovation  The returns of process innovation increase with the participation in foreign markets. Hence, the optimal innovation rule of firms active on foreign markets (type XI) will differ from the one of those active only in the domestic market (type DI). The relation between both optimal rules will be

\[ \Gamma_X (\varphi) = [1 + \tau^{1-\sigma}] \frac{1}{\sigma} \Gamma (\varphi) \] (14)

where \( \Gamma (\varphi) \) is given by equation (5).

To make the joint decision of whether to enter the foreign market and whether to innovate or not, firms compare the total profits they would get with each decision that are described below.

Profits of a domestic non-innovator:

\[ \pi_D = \frac{R (P \rho)^{\sigma-1}}{\sigma} \varphi^{\sigma-1} - f_D \]

Profits of a domestic innovator:

\[ \pi_{DI} = \frac{R (P \rho)^{\sigma-1}}{\sigma} \varphi^{\sigma-1} (1 + \Gamma (\varphi)) - f_D - c (\Gamma (\varphi)) \]

Profits of an exporter non-innovator:

\[ \pi_X = (1 + \tau^{1-\sigma}) \frac{R (P \rho)^{\sigma-1}}{\sigma} \varphi^{\sigma-1} - f_X - f_D \]

Profits of an exporter innovator:

\[ \pi_{XI} = (1 + \tau^{1-\sigma}) \frac{R (P \rho)^{\sigma-1}}{\sigma} \varphi^{\sigma-1} (1 + \Gamma_X (\varphi)) - f_X - f_D - c (\Gamma_X (\varphi)) \]
I will show that along the parameter space there will be three different equilibriums. Empirical evidence suggests that exporting and innovation are performed by the most productive firms while domestic producers are typically smaller, less innovative and less productive. Therefore in any equilibrium, those with the lowest productivity will be domestic non-innovators, while those at the top will be exporters innovators. However, it is the decision of middle productivity firms the one that will differ from one equilibrium to another.

Middle productivity firms might be productive enough to either innovate or export but not both, and the choice among both activities is determined by the relation between the costs of innovation and the costs of trade. In an environment with high enough trade costs, firms who are just productive enough will choose to innovate first rather than go into the exporting market. As trade costs fall, the innovation option becomes relatively less attractive, until middle productivity firms do not choose to export or innovate. Finally, in an environment with low enough trade cost, the export option is relatively attractive and firms choose to enter the export market without innovating.

I will refer to the first situation as the Low cost Innovation equilibrium since exporting is relatively unattractive prior to innovation. This is because middle productivity firms will engage on innovation activities rather than enter new markets, and only the most productive firms will participate in foreign markets as well as perform innovation. To the last situation described, I refer to as the Low cost Trade equilibrium since exporting is relatively more attractive prior to innovation. The decrease on trade cost (or the increase on innovation costs) induces innovation to be less attractive to mid-level productivity firms, and those productive enough will choose to perform export activities and only the most productive will innovate as well.

The literature up till now has identified some of these equilibriums, but never all through a single model. Bustos (2009) identifies the last equilibrium while Vannorenbergue (2008) identifies partially the first equilibrium. Finally, Navas & Sala (2007) identify the two extreme equilibriums, but fail at identifying the intermediate equilibrium and therefore to explain the decisions of firms along all the parameter space. Therefore, the main contribution of the theoretical model is the identification of all the equilibriums with the ability to study the transitions between them and the possible productivity gains that might occur through the intensive and extensive margins of innovation.
3.2 Low Cost Innovation Equilibrium

The *Low Cost Innovation Equilibrium* is characterized by exporting being less attractive than innovation. In the figure below, I depict the profits as a function of productivity whenever being an exporter non innovator is dominated by some other choice. The least productive firms \((\varphi < \varphi_D)\) exit, the low productivity firms \((\varphi_D < \varphi < \varphi_{DI})\) are active on the domestic market but do not innovate or export, middle productivity firms \((\varphi_{DI} < \varphi < \varphi_{XI})\) are active only on the domestic market but innovate, and the most productive firms \((\varphi > \varphi_{XI})\) are active both in the domestic and export market, and innovate.

![Figure 2: Low Cost Innovation Selection Path](image)

The conditions of entry in the domestic and export markets plus the innovation condition solve the *Low Cost Innovation Equilibrium*.
Zero Profit Condition (ZPC) on the domestic market\(^2\):

\[
\pi_D (\varphi_D) = 0 \\
\Leftrightarrow (\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \quad (15)
\]

Innovation Profit Condition (IPC), the productivity cutoff \(\varphi_{DI}^*\) will be the productivity of the firm indifferent between innovating or not while operating only on the domestic market:

\[
\pi_{DI} (\varphi_{DI}) = \pi_D (\varphi_{DI}) \\
\Leftrightarrow (\varphi_{DI}^*)^{\sigma-1} = \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha+1}{\sigma}} (\alpha + 1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \quad (16)
\]

Innovation Export Profit Condition (IXPC), the exporting-innovation cutoff \(\varphi_{XI}^*\) will be the productivity of an innovating firm indifferent between participating also on the exporting market or not.

\[
\pi_{XI} (\varphi_{XI}) - \pi_{DI} (\varphi_{XI}) = 0 \quad (17)
\]

**Proposition 1** There exist a single cutoff \(\varphi_{XI}^* > \varphi_{DI}^* > \varphi_D^*\) as long as I have sufficiently high fixed export costs.

**Proof.** See Appendix II ■

The difference on the innovation rules between the firms that participate on the export market and those who do not, are the source of the complexity of finding a closed form for the cutoff \(\varphi_{XI}^*\). Nevertheless, as long as the following parameter restriction holds the single unique cutoff will exist:

\[
\tau^{\sigma-1} f_X \geq \frac{\left[(1 + \tau^{1-\sigma})^{\frac{\alpha+1}{\sigma}} - 1\right]}{\tau^{1-\sigma} \kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha+1}{\sigma}} (\alpha + 1)} \quad (18)
\]

\(^2\)The ZPC condition is defined theoretically in the same way in the closed economy, the pro-trade equilibrium and the pro-innovation equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.
The second parameter restriction that holds the equilibrium $\varphi_{XI} \geq \varphi_{DI} \geq \varphi_D$ is

$$\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha + 1}} (\alpha + 1) \geq f_D$$ (19)

Hence, I require the exporting activity to be more expensive than innovation that is more expensive than operating only on the domestic market. Obviously, the parameter restriction is in tune with an equilibrium where middle productivity firms would rather perform into innovation activities than into the export market.

As in the closed economy, the interest lays in finding a stationary equilibrium where all aggregate variables are constant over time. Equations (15) to (17) along with the Free Entry condition completely determine the equilibrium and the productivity cutoffs can be uniquely determined (see Appendix II for proof)

$$\left[\int_{\varphi_D}^{\varphi_{DI}} \pi_D (\varphi) dG (\varphi) + \int_{\varphi_{DI}}^{\varphi_{XI}} \pi_{DI} (\varphi) dG (\varphi) + \int_{\varphi_X}^{\infty} \pi_{XI} (\varphi) dG (\varphi)\right] = \delta f_E$$ (20)

**Trade Effects on Innovation** Combining equations (15) and (16), the relation between the cutoffs can be written explicitly as:

$$(\varphi_{DI})^{-1} = \left(\frac{\alpha}{\alpha + 1}\right)^{\frac{\alpha}{\alpha + 1}} (\alpha + 1) (\varphi_D)^{\sigma - 1} = \Lambda (\varphi_D)^{\sigma - 1}$$ (21)

The relationship between the cutoffs of domestic innovators and domestic non innovators is the same on autarky than on the low cost innovation equilibrium (see equation (11)) but I cannot forget the indirect effects via the input-factor market. In the open economy, survival becomes tougher and entry in the domestic market is harder than in autarky. The presence of exporters, innovators and potential entrants increase the competition pushing up real wages, and therefore, exporting and innovating are more costly activities. Thus, the proportion of incumbents is reduced (higher $\varphi_D$) along with the proportion of innovators in the economy (higher $\varphi_{DI}$).

Therefore, on the low cost innovation equilibrium the number of firms that perform innovation in the economy is reduced with respect to the autarky case. Nevertheless, I cannot say anything on the amount of process innovation, since it could be possible that the increase of firms who also export make up for the
loss of firms who do not innovate anymore. In other words, gains along the intensive margin might offset the loss along the extensive margin. This question is approached on section 4 where I explore the behavior of aggregate innovation, the extensive and intensive margins of innovation under a trade liberalization.

3.3 Low Cost Trade Equilibrium

The Low Cost Trade Equilibrium is characterized by exporting being more attractive than innovation. In the figure below, I depict the profits as a function of productivity whenever being a domestic innovator is dominated by some other choice. The least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_X$) are active on the domestic market but do not innovate or export, middle productivity firms ($\varphi_X < \varphi < \varphi_{XI}$) are active both on the domestic and export market but do not engage on innovation, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate.

![Figure 3: Low Cost Trade Selection Path](image)

The conditions of entry in the domestic and export markets, plus the innovation conditions solve the Low Cost Trade Equilibrium.
Zero Profit Condition (ZPC) on the domestic market\(^3\):

\[
\pi_D (\varphi_D) = 0 \iff (\varphi_D^*)^{\sigma-1} = \frac{f_D}{(R(P\rho)^{\sigma-1})} \tag{22}
\]

Exporting Profit Condition (XPC), the exporting-entry productivity cutoff \(\varphi_X^*\) will satisfy the zero profit condition on the export market, which translates into finding the firm which is indifferent between staying in the domestic market and participating in the export market:

\[
\pi_X (\varphi_X) = \pi_D (\varphi_X) \iff (\varphi_X^*)^{\sigma-1} = \frac{f_X}{(R(P\rho)^{\sigma-1})} \left(\frac{1}{\tau^{\sigma-\rho}}\right) \tag{23}
\]

Exporting Innovation Profit Condition (XIPC), the innovation exporting productivity cutoff \(\varphi_{XI}^*\) will be the productivity of an exporting firm indifferent between innovating or not:

\[
\pi_{XI} (\varphi_{XI}) = \pi_X (\varphi_{XI}) \iff (\varphi_{XI}^*)^{\sigma-1} = \frac{\left(\frac{\alpha}{\gamma}\right) \frac{\pi^{\gamma-\tau}}{(\alpha + 1)} (1 + \frac{1-\tau}{\alpha + 1})}{(R(P\rho)^{\sigma-1}) (1 + \tau^{1-\rho})} \tag{24}
\]

The parameter restriction that hold this equilibrium \((\varphi_{XI} \geq \varphi_X \geq \varphi_D)\) is

\[
\frac{\left(\frac{\alpha}{\gamma}\right) \frac{\pi^{\gamma-\tau}}{(\alpha + 1)} (1 + \frac{1-\tau}{\alpha + 1})}{(1 + \tau^{1-\sigma})} \geq \sigma^{\rho-1} f_X \geq f_D \tag{25}
\]

which requires innovation to be more expensive than exporting for this activity to be more attractive than innovation to middle productivity firms.

As in the closed economy, the interest lays in finding a stationary equilibrium where all aggregate variables are constant over time. Equations (22) to (24) along with the Free Entry condition completely determine the equilibrium and

\(^3\)The ZPC condition is defined theoretically in the same way in the closed economy, the pro-trade equilibrium and the pro-innovation equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.
the productivity cutoffs can be uniquely determined (see Appendix II for proof)

\[
\int_{\varphi_D}^{\varphi_I} \pi_D (\varphi) \, dG (\varphi) + \int_{\varphi_X}^{\varphi_{X_I}} \pi_X (\varphi) \, dG (\varphi) + \int_{\varphi_{X_I}}^{\infty} \pi_{X_I} (\varphi) \, dG (\varphi) = \delta f_E
\]

(26)

**Trade Effects on Innovation** Combining equations (22) to (24), the relation between the cutoffs can be written explicitly as:

\[
(\varphi_{XI})^{\sigma-1} = \frac{\left(\frac{\varphi_{XI}}{\alpha}\right)^{\frac{\sigma-1}{1+\tau^{-\sigma}}} (\alpha + 1) (\varphi_D)^{\sigma-1}}{f_D} = \Lambda^t (\varphi_D)^{\sigma-1}
\]

(27)

\[
(\varphi_{XI})^{\sigma-1} = \left[\left(\frac{\varphi_{XI}}{\alpha}\right)^{\frac{\sigma-1}{1+\tau^{-\sigma}}} (\alpha + 1)\right] \frac{\varphi_{XI}}{f_X} = \Lambda_X^t (\varphi_X)^{\sigma-1}
\]

(28)

\[
(\varphi_X)^{\sigma-1} = \frac{f_X \tau^{\sigma-1}}{f_D} (\varphi_D)^{\sigma-1} = \frac{\Lambda^t}{\Lambda_X^t} (\varphi_D)^{\sigma-1}
\]

(29)

where \(\Lambda_X^t = \frac{f_X \tau^{\sigma-1}}{f_D} \Lambda^t\), \(\Lambda^t = \frac{\Lambda}{1+\tau^{-\sigma}}\) and \(\Lambda\) is the relationship between the innovation and the entry cutoff in autarky (equation (11)) and in the low cost innovation equilibrium (equation (21)).

Two opposite forces take place in this equilibrium. First, the effect via general equilibrium. Survival becomes tougher and \((\varphi_D)^{\sigma-1}\) increases with respect to autarky. The presence of exporters, innovators and potential entrants increase the competition pushing up real wages, and therefore, exporting and innovating are more costly activities. Secondly, an effect through partial equilibrium on the opposite direction. The cost ratio is smaller in the Low Cost Trade Equilibrium than in autarky (or in the Low Cost Innovation equilibrium) since \(\Lambda \geq \Lambda^t\). This difference is numerically given by \(\frac{1}{1+\tau^{-\sigma}}\) whose denominator indicates the further revenue differential associated to innovation on the foreign market available through trade. Economically, since exporters expand their scales of operation, the variable cost and benefits of the productivity enhancing innovation performed are spread on more units while the up-front cost of innovation is unchanged, creating the difference I are talking about. The comparison of (11) and (27) shows that trade decreases (ceteris paribus) the innovation productivity cutoff boosting within-plant innovation.

Thus, although the proportion of incumbents is reduced (higher \(\varphi_D\)), the proportion of innovating firms among them will raise with respect to the situation in autarky if the adjustments through innovation dominate those happening
through trade, that is there is a larger extensive margin if the partial equilibrium effect dominates the general equilibrium effect.

**Proposition 2** On the Low Cost Trade equilibrium, if productivity draws are distributed according to a Pareto distribution, then \( \varphi_{D1}^{A} > \varphi_{X1} \). That is, the proportion of firms doing innovation activities on the open economy rises with respect to autarky.

**Proof.** See Appendix □

### 4 Trade & Innovation

The consequences of opening up to trade have been briefly discussed in the previous section. Nevertheless, a deeper understanding of the effects that a trade liberalization has on the firm decisions about exporting and innovation is needed.

The following section relies on comparisons of the steady state equilibria, comparative statistics and numerical simulations to study the reallocation of market shares after a trade liberalization, and the implications it has on the aggregate welfare and aggregate innovation of the economy.

#### 4.1 Parameters

I next describe the parameters of the model chosen to run the simulations and the reasoning behind choosing each one of them.

The productivity is distributed according to a Pareto with a probability density function \( g(\varphi) = \frac{\xi}{\varphi^{\xi+1}} \) with shape parameter \( \xi > (\sigma - 1) \left( \frac{\alpha + 1}{\alpha} \right) \) and support \([1, \infty)\). I set \( \xi = 5 \) in agreement with the estimations in Balistreri, Hillberry & Rutherford (2011).

The entry and exit of firms is determined by the death shock \( \delta \) and the sunk cost of entry \( f_E \). Following Bernard, Redding and Schott (2007) I set the death shock to \( \delta = 0.025 \) and the entry sunk cost to \( f_E = 2 \).

Broda & Weinstein (2006) provide the most detailed study of elasticities. The values reported vary from 2.1 to 4.8 depending on the level of aggregation. I set \( \sigma = 3 \), which lies within the estimated values.

Cohen & Keppler (1996) report that in most industries there is no systematic relationship between firm size and innovation, and that the hypothesis of linearity cannot be rejected. Therefore, I set the innovation parameter \( \alpha = 1 \),
which implies a linear relationship between innovation and size (see eq 5). Furthermore, the only requirement for the fix cost of innovation is that it should be positive and larger than $\alpha$, hence I set $\kappa = 3$.

The values of the fixed cost of production and exporting are taken from Balistreri, Hillberry & Rutherford (2011). They use a nonlinear structural estimation procedure to identify core parameters, among them the fixed costs of production, the fixed costs of exporting and the Pareto shape parameter that best fit the geographic pattern of trade. The values of the fix cost of production range from 0.299 ($f_D^{USA}$) to 1.3 ($f_D^{EER}$), therefore I set it to $f_D = 1$. This value is sensible with the estimates and represents a 50% of the fix costs of entry. The fixed costs of export range from 0.714 ($f_X^{USA}$) to 15.153 ($f_X^{CAN}$). Therefore, in the simulations I will let $f_X$ be between $[1, 12]$. Finally, for these parameter values I compute the equilibrium for trade costs ranging from $\tau = 1.5$ to $\tau = 1$ (free trade).

4.2 Autarky to Trade

The main impact of the transition from autarky to trade was the exit of the less productive firms and the reallocation of market shares from the less productive firms to the more productive firms.

I address now question concerning this reallocation given the heterogeneity of the firms and the existence of fixed costs to export and innovate: Do all firms benefit from trade? Are the benefits of trade larger in the low cost innovation equilibrium or in the low cost trade equilibrium?. The analysis will rely on comparisons of steady state equilibria: closed economy vs. open economy.

Reallocation of Markets Shares and Profits I contrast now the performance of a firm with productivity $\varphi \geq \varphi^A$ before and after the transition to trade.$^4$

In the left (figure4a) is the low cost innovation equilibrium case. The increased competition is reflected in the loss of domestic market shares for all firms and it has two opposite effects on innovators: the less productive innovators will stop being innovators since they can’t cover the innovation costs with the new (lower) profits while the most productive innovators enter the export market and will innovate more intensively in order to take as much advantage as possible from the reallocation.

$^4$The superindex $A$ will from now on indicate autarky
The impact on market shares is reflected in the following inequality\footnote{See the Appendix for a formal proof}.

\[ r_D(\varphi) < r_D^A(\varphi) < r_{DI}(\varphi) < r_{DI}^A(\varphi) < r_{XI}(\varphi) \]

Notice then that if firms do not export, they will have lower revenues since their market shares are smaller which translates into incurring in a profit loss. If the firm exports, revenues are higher because they also have market shares from the export market, but they must face exportation costs. The profit variation for firms with \( \varphi \geq \varphi_{XI} \) can be written as

\[
\Delta \pi(\varphi) = \pi_{XI}(\varphi) - \pi_{DI}^A(\varphi) \\
= \frac{1}{\sigma} \left\{ (1 + r^{1-\sigma}) r_D(\varphi) - r_D^A(\varphi) \right\} - f x + \\
+ \alpha \left( \frac{1}{\alpha + 1} \right)^{\frac{a+1}{\alpha}} \\
\left\{ \left[ 1 + (1-\sigma) \frac{r_D(\varphi)}{\sigma} \right]^{\frac{a+1}{\alpha}} - \left[ \frac{r_D^A(\varphi)}{\sigma} \right]^{\frac{a+1}{\alpha}} \right\}
\]

The terms in the braces are positive and the expression is an increasing function of the productivity level \( \varphi \). This variation will be negative for the firm with the cutoff productivity level \( \varphi_{XI} \), which follows from the zero profit
condition on the export market. Hence, only a subset of the more productive firms who export and innovate will have profit gains from trade.

The low cost trade equilibrium is depicted on the right side (figure 4b). As it happens in the low cost innovation equilibrium, there is a loss in domestic market shares due to competition. However, the favorable conditions to entry into the export market has a positive effect on the innovators and the choice to innovate that is reflected on their market shares. Not only every innovator on autarky will continue to be innovators and more intensively than in autarky, but they will all become exporters and some of the previously non-innovators will decide to become both exporters and innovators, or simply exporters which implies an increase on the market shares through the foreign markets.

On this equilibrium, the following inequality on market shares holds\(^6\):

\[
 r_D(\varphi) < r_D^A(\varphi) < r_X(\varphi) < r_{XI}(\varphi) < r_{DI}(\varphi)
\]

On the one hand, firms that do not export in the open economy will have lower revenues since their market shares are smaller, just as it happens in the low cost innovation equilibrium. On the other hand, trade has a vertical integration effect in this equilibrium, all the firms performing innovation will have profit gains from trade and only a subset of the exporting non-innovators will suffer a profit loss with respect to the autarky case due to opening the markets.

The reallocation of market shares due to opening up to trade has benefits only for the most productive firms. The more open the economy, higher the number of exporters and higher the number of firms that benefit from trade.

### 4.3 Partial Trade Liberalization

Nowadays, most countries trade with at least another country and often agreements are made in order to lower the costs related to the transaction. This section centers on understanding whether the effects that a partial trade liberalization has on the firm decisions induces the same effects than a transition of an economy from autarky to open trade.

Through comparative statistics of the open economy equilibrium with respect to the variable trade costs \((\tau)\) and numerical simulations, I will show that

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\(^6\)See Appendix for a formal proof
the exposure to trade also induces the least productive firms to exit reallocating market shares from less productive to more productive firms.

**Decrease in Trade Costs** A decrease in the variable trade costs $\tau$ will induce the least productive firms to exit and re-allocate market shares from less productive to more productive firms\(^7\).

A decrease from $\tau$ to $\tau' < \tau$ shifts up the ZPC and IPC curves, inducing an increase in the cutoff productivity levels $\varphi'_{D} > \varphi_{D}$ and $\varphi'_{DI} > \varphi_{DI}$. However, exporting is now easier and the new export cutoff $\varphi'_{X}$ along with the export-innovating cutoff $\varphi'_{XI}$ will be below their previous levels, $\varphi_{X}$ and $\varphi_{XI}$ respectively.

If I analyze carefully the equilibrium, this results should not be surprising at all. In the open economy, there are three possible equilibriums according to the relationship of the parameters. On the low cost innovation equilibrium, the trade costs are high, there are domestic innovators and there are no exporters that do not innovate. On the low cost trade equilibrium, the trade costs are low, there are no domestic innovators but there are exporters that do not innovate. The comparative statistics are reveling the same patterns in the firm decisions. As trade costs lower, it is less profitable to be a domestic innovator, $\varphi'_{DI} > \varphi_{DI}$ if $\tau' < \tau$, and more profitable to be a exporter non-innovator, $\varphi'_{X} < \varphi_{X}$ if $\tau' < \tau$.

### 4.4 Welfare

The welfare per worker is given by

$$W = P^{-1} = \rho \left( \frac{L}{\sigma fD} \right) \varphi_{D}$$

Welfare will be higher in a larger country ($L' > L$) or in a more competitive country ($\varphi'_{D} > \varphi_{D}$). The previous sections have highlighted how a trade liberalization forces the less productive firms to exit re-allocating market shares towards the more productive firms. The exit of this firms reflects the increased competition on the market, hence a transition from autarky to trade will generate welfare gains\(^8\).

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\(^7\)Formal derivations of the comparative statistics and numerical simulations can be seen in the Appendix.

\(^8\)See Appendix III for a formal proof
In the open economy, no matter in which equilibrium the economy is in, welfare is higher than in autarky. But, which equilibrium is better? The equilibrium with lower trade costs, that is the low cost trade equilibrium. The lower the trade cost, the higher the selection effect on the domestic entry cutoff, or in other words, higher the increase in competition in the domestic market.

4.5 Innovation

The decisions of exporting and innovation are endogenous, therefore any liberalization that induces the exit of some firms and a reallocation of market shares has an impact on the evolution of the aggregate innovation of the economy.

There are two dimension through which a trade liberalization can affect the aggregate innovation. First, through the amount of resources each firm dedicates to innovation, that is the intensive margin of innovation. And secondly, through the number of innovators, that is the extensive margin of innovation.

On the following section, I examine how the evolution of the extensive and intensive margin of innovation determine the evolution of the aggregate innovation of the economy under a trade liberalization and in every possible equilibrium of the open economy.

The extensive and intensive margin of innovation

Equation (5) gives the innovation rule in autarky while equation (14) relates the optimal rule of innovation of exporters with the one of non-exporters (the autarky rule).

The returns of process innovation increase with the participation in foreign markets, hence the amount of the innovation performed by the firms that enter the export market after a lowering of trade barriers is higher now. Whether it is from going from autarky to trade or due to a partial trade liberalization, the intensive margin of innovation increases regardless of the equilibrium the economy is in and will affect positively to the aggregate innovation.

The evolution of the number of innovators requires a more careful examination of the effects of a liberalization on the firm decisions. When trade cost are high and the economy is in the low cost innovation equilibrium a trade liberalization implies that more domestic innovators will become exporter innovators while it is harder to be a domestic innovator. Therefore, the mass of innovators $M'_I = m'_D + m'_X$ is reduced with respect to $M_I$, the number of innovators when $\tau > \tau'$, and keeps decreasing as the economy open.
On figure 5 can be seen the decrease of the extensive margin after a trade liberalization in the low cost innovation equilibrium.\footnote{High trade costs, we set $f_X = 12$}

![Figure 5](image1)

Since the cutoffs of domestic innovators ($\varphi_{DI}$) and exporter innovators ($\varphi_{XI}$) move in opposite directions under a trade liberalization, it is only reasonable to think that at some point $\varphi_{XI} < \varphi_{DI}$ and no firms will choose to be innovators without exporting as well. When this happens, the equilibrium of the economy changes from being the low cost innovation to be the intermediate equilibrium. If the trade liberalization continues to take place, the number of innovators would increase since the relevant cutoff now is $\varphi_{XI}$ and $\varphi'_{XI} < \varphi_{XI}$ whenever $\tau' < \tau$.

Figure 6 shows the evolution of the extensive margin when a trade liberalization induces the mass of domestic innovators to disappear completely.\footnote{Middle-high trade costs, we set $f_X = 8$}

![Figure 6](image2)
Once I am in an equilibrium where there are no domestic innovators, either the intermediate equilibrium or the low cost trade equilibrium, the number of innovators increases under a trade liberalization. Hence, in the open economy, the extensive margin of innovation has a U-shape where the minimum is reached as the mass of firms being domestic innovators disappears.

### 4.5.1 Aggregate Innovation

A trade liberalization has a positive impact on the intensive margin of innovation while the extensive margin follows a U-shape. This two dimensions determine the behavior of the aggregate innovation.

*Proposition 2* ensures that the number of innovators in the low cost trade equilibrium is larger than in autarky and it increases under a partial trade liberalization. In this equilibrium it is clear that trade has a positive impact on the aggregate innovation both through the intensive and extensive margin.

In the intermediate equilibrium, a trade liberalization induces an increase of the extensive margin, but in the starting point the extensive margin is lower than in autarky. Therefore even though the aggregate innovation will increase under a partial trade liberalization it will not be higher than in autarky unless trade costs are sufficiently low.

Figure 7 shows the evolution of both the extensive margin and the aggregate innovation in the intermediate equilibrium\(^{11}\). Remark that under a trade liberalization, aggregate innovation picks up over the autarky level earlier than the extensive margin thanks to the positive influence of the intensive margin. Nevertheless, even if a partial trade liberalization has a positive effect on the aggregate innovation (upward trend), a liberalization from autarky to trade will not automatically imply that aggregate innovation is higher on the open economy than in autarky.

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\(^{11}\)Middle trade costs, we set \(f_X = 4\)
Finally, on the low cost innovation equilibrium a partial trade liberalization induces a negative effect on the extensive margin while the intensive margin goes in the opposite direction. The numerical simulation reveals that the extensive margin dominates over the intensive margin, and therefore, trade has a negative impact on the aggregate innovation.

Figure 8 shows the evolution of the aggregate innovation in the low cost innovation equilibrium.\textsuperscript{12}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8.png}
\caption{Evolution of aggregate innovation}
\end{figure}

5 Conclusions

I have developed a model à la Melitz where firms can affect their productivity through process innovation in order to account not only for productivity gains due to the reallocation of shares between firms’ but also for productivity gains within the firm.

The interaction between the innovation and exporting decisions determine the shape of the firms’ distribution in the open economy. Which of the two equilibria emerges is determined by the relation between the cost-benefit ratio of innovation and the exporting costs. On one side, I have the low cost innovation equilibrium, where enough productive firms decide to innovate rather than enter the export market, since exporting is relatively more expensive than innovation and only the most productive firms undertake both activities. Nevertheless, opening the economy up to trade reduces not only the number of active firms from the country but also the number of firms that perform innovation activities due to the selection effect. On the other side is the low cost trade equilibrium, in which firms decide to export rather than innovate since exporting is

\textsuperscript{12}Middle trade costs, we set $f_X = 12$
relatively cheaper than innovation and only the most productive firms perform innovation and exporting activities. Only in this equilibrium, trade induces not only a selection effect but also a partial equilibrium effect through which there are within plant productivity gains. The productivity gains occur through the extensive margin since there is a larger number of incumbents performing innovation in the open economy, and through the intensive margin since the presence of foreign markets increase their innovation activities.

Finally, using comparative statistics and numerical simulations I explore the consequences of a partial trade liberalization on the firm decisions, aggregate welfare and aggregate innovation. A trade liberalization induces a reallocation of market shares from less productive firms (those that exit) to the more productive. the more open the economy, the higher the number of firms that benefit from this reallocation and higher the increase in welfare per worker. However, the aggregate innovation and trade liberalization have a U-shape relationship. The aggregate innovation of the economy falls under a liberalization as long as there are domestic innovators on the economy. Once they disappear it picks up and will rise above autarky levels once trade costs are low enough.

The aim of the paper was to provide insight into the productivity gains from trade through innovation activities. The paper shows how the existence of export market entry costs and innovation fixed costs affects the distribution of firms in the open economy and the impact of trade across different types of firms. On one hand, the paper shows that the introduction of innovation does not affect the main results of trade models: exposure to trade leads to welfare gains. On the other hand, the paper shows how trade drastically affects the aggregate innovation inducing only gains when trade costs are low enough.

Further work will be directed towards testing the predictions of the model regarding the aggregate innovation and the firm size distribution to better understand which innovation-trade policies are best for an economy.
References


Appendix I - Closed Economy

Productivity distribution and weighted averages  Let us denote by $\eta_D(\varphi)$ and $\eta_{DI}(\varphi)$ respectively, the productivity distribution of domestic producers and active innovators prior to innovation. The distribution of all firms is given by

$$\eta_D(\varphi) = \frac{g(\varphi)}{G(\varphi_D)}, \varphi > \varphi_D, \text{otherwise}$$

$$\eta_{DI}(\varphi) = \frac{g(\varphi)}{1-G(\varphi_{DI})}, \varphi \geq \varphi_{DI}, \text{otherwise}$$

$\eta_D(\varphi)$ and $\eta_{DI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$, where $\mu(\varphi) = \frac{g(\varphi)}{1-G(\varphi_D)}$.

I shall define two weighted averages, $\bar{\varphi}$ which represents the average productivity of all the firms in the economy prior to innovation and $\bar{\varphi}_I$ which represents the average productivity of the innovators after innovation.

$$\bar{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} \mu(\varphi) \, d\varphi\right]^{\frac{1}{1-\sigma}}$$  \hspace{1cm} (30)

$$\bar{\varphi}_I = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} \eta_{DI}(\varphi) \, d\varphi\right]^{\frac{1}{1-\sigma}}$$  \hspace{1cm} (31)

Aggregate Variables  Denote by $m_I$ and $m$ respectively the mass of active innovators and non-innovator producers, where

$$m_I = \frac{1-G(\varphi_{DI})}{1-G(\varphi_D)} M$$

$$m = \frac{G(\varphi_{DI}) - G(\varphi_D)}{1-G(\varphi_D)} M$$

with $M$ being the mass of incumbent firms in the economy.

Then, it can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = \int_{\varphi_D}^{\varphi_{DI}} [p_D(\varphi)]^{1-\sigma} m \eta_D(\varphi) \, d\varphi + \int_{\varphi_{DI}}^{\infty} [p_{DI}(\varphi)]^{1-\sigma} m_I \eta_{DI}(\varphi) \, d\varphi$$

$$= M [p_D(\bar{\varphi})]^{1-\sigma} + m_I \left[\frac{\kappa}{\alpha} \right]^{\frac{1}{1-\sigma}} \left(\frac{1}{\varphi_{DI}}\right)^{\frac{1}{\sigma-1}} \left[p_D\left(\bar{\varphi}_I(\frac{\alpha+1}{\alpha})\right)\right]^{1-\sigma}$$  \hspace{1cm} (32)
Notice that the first term coincides exactly with the aggregate price of the Melitz 2003 economy, therefore I can distinguish exactly the effect that having an innovation choice has on the aggregates of the economy, since this term will be distinguished in every one of the aggregates.

- **Aggregate Production**

\[
Q^\rho = \int_{\varphi_D}^{\varphi_D} [q_D (\varphi)]^\rho m\eta_D (\varphi) d\varphi + \int_{\varphi_D}^{\infty} [q_D (\varphi)]^\rho m\eta_D (\varphi) d\varphi
\]

\[
= M [q_D (\bar{\varphi})]^\rho + m_I \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D} \left( \frac{1}{\varphi_D^\sigma-1} \right)^\frac{\alpha}{\sigma} \left[ q_D (\bar{\varphi})^\frac{\alpha+1}{\alpha} \right]^\rho \quad (33)
\]

- **Aggregate Revenue**

\[
R = \int_{\varphi_D}^{\varphi_D} r_D (\varphi) m\eta_D (\varphi) d\varphi + \int_{\varphi_D}^{\infty} r_D (\varphi) m\eta_D (\varphi) d\varphi
\]

\[
= M \cdot r_D (\bar{\varphi}) + m_I \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D} \left( \frac{1}{\varphi_D^\sigma-1} \right)^\frac{\alpha}{\sigma} r_D (\bar{\varphi})^\frac{\alpha+1}{\alpha} \quad (34)
\]

- **Aggregate Profits**

\[
\Pi = \int_{\varphi_D}^{\varphi_D} \pi_D (\varphi) m\eta_D (\varphi) d\varphi + \int_{\varphi_D}^{\infty} \pi_I (\varphi) m\eta_D (\varphi) d\varphi
\]

\[
= M \pi_D (\bar{\varphi}) + m_I \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D} \left( \frac{1}{\varphi_D^\sigma-1} \right)^\frac{\alpha}{\sigma} \pi_D (\bar{\varphi})^\frac{\alpha+1}{\alpha} \\
\ldots - m_I \kappa - m_I \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D} \left( \frac{1}{\varphi_D^\sigma-1} \right)^\frac{\alpha+1}{\alpha} \varphi_D^{\frac{\alpha+1}{\alpha}(\sigma-1)} \quad (35)
\]

**Existence of Equilibrium** Using the ZPC condition on equation (7), the average profits, which are defined as \( \pi = \frac{\Pi}{M} \), can be expressed in terms of the aggregates as

\[
\pi = f_D k_1 (\varphi_D^*) + \left[ \frac{1 - G (\varphi_D^*)}{1 - G (\varphi_D^*)} \right] B k_2 (\varphi_D^*) - \left[ \frac{1 - G (\varphi_D^*)}{1 - G (\varphi_D^*)} \right] \kappa \quad (36)
\]

where \( k_1 (\varphi_D^*) = \left[ \left( \frac{\varphi_D^*}{\varphi_D^*} \right)^{\sigma-1} - 1 \right], k_2 (\varphi_D^*) = \left[ \left( \frac{\varphi_D^*}{\varphi_D^*} \right)^{(\sigma-1)} \right] \frac{\alpha+1}{\alpha} \) and

\[
B = \alpha \left( \frac{1}{\alpha+1} \right) \frac{\alpha+1}{\alpha} (f_D) \frac{\alpha+1}{\alpha} > 0
\]

**Proof.** I only need to use the expressions on equations (8), (10), and (35)

Then

\[
\pi = \pi_D (\bar{\varphi}) + \left( \frac{m_I}{M} \right) \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D} \left( \frac{1}{\varphi_D^\sigma-1} \right)^\frac{\alpha}{\sigma} \pi_D (\bar{\varphi})^\frac{\alpha+1}{\alpha} - \left( \frac{m_I}{M} \right) \kappa \\
- \left( \frac{m_I}{M} \right) \left( \frac{\kappa}{\alpha} \right) \frac{1}{\varphi_D^\sigma-1} \varphi_D^\frac{\alpha+1}{\alpha}(\sigma-1)
\]

32
Term 1
\[ \pi_D(\tilde{\varphi}) = \frac{r_D(\tilde{\varphi})}{\sigma} - f_D = \frac{r_D(\phi_D^*)}{\sigma} \frac{\sigma - 1}{\tilde{\varphi}_D} - f_D = f_D \left( \frac{\tilde{\varphi}_D}{\varphi_D^*} \right)^{\sigma - 1} - 1 \]

Term 2
\[ \left( \frac{\kappa}{\pi_D} \right) \frac{1}{\varphi_D^*} \left( \frac{1}{\varphi_D^*} \right)^{(\alpha+1)\sigma} = (\alpha + 1) \left( \frac{1}{\alpha + 1} \right) \frac{\sigma + 1}{\alpha} f_D^\alpha \left[ \frac{\tilde{\varphi}_f(\sigma_D)}{(\varphi_D^*)^{\sigma + 1}} \right]^{\sigma + 1} \]

Term 4
\[ \left( \frac{\kappa}{\pi_D} \right) \left( \frac{1}{\varphi_D^*} \right)^{(\alpha+1)}(\sigma - 1) = \left( \frac{1}{\alpha + 1} \right) \frac{\sigma + 1}{\alpha} f_D^\alpha \left[ \frac{\tilde{\varphi}_f(\sigma_D)}{(\varphi_D^*)^{\sigma + 1}} \right]^{\sigma + 1} \]

Then, by combining T2 and T4
\[ T2 - T4 = \alpha \left( \frac{1}{\alpha + 1} \right) \frac{\sigma + 1}{\alpha} f_D^\alpha \left[ \frac{\tilde{\varphi}_f(\sigma_D)}{(\varphi_D^*)^{\sigma + 1}} \right]^{\sigma + 1} \]

Therefore
\[ \pi = f_D \left( \frac{\tilde{\varphi}_D}{\varphi_D^*} \right)^{\sigma - 1} - 1 \]

Furthermore, notice that the FE condition on equation (12) can also be expressed in terms of the aggregates as
\[ [1 - G(\varphi_D^*)] \pi = \delta f_E \]

Since \( \pi = \frac{\delta f_E}{1 - G(\varphi_D^*)} \) is increasing I should show that the RHS of \( \pi = \frac{\pi_D}{\pi_M} \) is decreasing to prove that there exists an equilibrium, and it is unique.

**Proof.** I am going to prove that the RHS of (36) is decreasing in \( \varphi_D^* \) on the domain \( (\varphi_D^*, \infty) \), so that \( \varphi_D^* \) is uniquely determine by the intersection of the latter curve with the flat line \( \delta f_E \) in the \( (\varphi_D^*, \infty) \) space. The last term on equation (36) is constant, therefore I only need to show that the other two terms are decreasing.

Remember that \( k_1(\varphi_D^*) = \left[ \left( \frac{\tilde{\varphi}_D}{\varphi_D^*} \right)^{\sigma - 1} - 1 \right] \), then its derivative with respect to \( \varphi_D^* \) is
\[ k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma - 1)}{\varphi_D^*} [k_1(\varphi_D^*) + 1] \]
Similarly, \( k_2 (\varphi_D) = \left[ \frac{\varphi_I (\varphi_D)}{\varphi_D} \right]^{(\sigma - 1)h} \), and its derivative with respect to \( \varphi_D \) is

\[
k'_2 (\varphi_D) = \Lambda \frac{1}{1 - G(\varphi_I)} \left[ \frac{k_2 (\varphi_D) - \Lambda \frac{\varphi_I}{\alpha} (1 - \sigma) \frac{k_2 (\varphi_D)}{\varphi_D} \right]
\]

where \( \frac{\partial \varphi_D}{\partial \varphi_D} = \left[ \frac{(\frac{\varphi_I}{\alpha (\varphi_D)})}{1 - G(\varphi_I)} \right]^{\frac{1}{\alpha}} = \Lambda \frac{1}{1 - G(\varphi_I)} \). 

Now, define \( j_1 (\varphi_D) = [1 - G(\varphi_D)] k_1 (\varphi_D) \), and \( j_2 (\varphi_D) = [1 - G(\varphi_D)] k_2 (\varphi_D) \) which are non-negative.

Then the derivative and elasticity of each of the expressions are respectively

\[
j'_1 (\varphi_D) = -\frac{(\sigma - 1) [k_1 (\varphi_D) + 1]}{\varphi_D} [1 - G(\varphi_D)] < 0
\]

\[
j'_1 (\varphi_D) \cdot \varphi_D = -\frac{(\sigma - 1)}{1 + k_1 (\varphi_D)} < - (\sigma - 1)
\]

and

\[
j'_2 (\varphi_D) = -g (\varphi_D) \frac{\Lambda \frac{\varphi_I}{\alpha} (1 - \sigma) \frac{k_2 (\varphi_D)}{\varphi_D} [1 - G(\varphi_D)] < 0
\]

\[
j'_2 (\varphi_D) \cdot \varphi_D = -\frac{g (\varphi_D) \frac{\Lambda \frac{\varphi_I}{\alpha} (1 - \sigma) \frac{k_2 (\varphi_D)}{\varphi_D} [1 - G(\varphi_D)] < -\beta (\sigma - 1)}{k_2 (\varphi_D)}
\]

Therefore, \( j_1 (\varphi_D) \) and \( j_2 (\varphi_D) \) must be decreasing to zero as \( \varphi \) goes to infinite. Furthermore, it must be that \( \lim_{\varphi_D \to 0} j_1 (\varphi_D) = \infty \) since \( \lim_{\varphi_D \to 0} k_1 (\varphi_D) = \infty \).

Hence, \( j_1 (\varphi_D) \) and \( j_2 (\varphi_D) \) decrease from \( \infty \) to zero on the parameter space \((0, \infty)\), and thus the RHS of eq (36) is decreasing on the parameter space.

**Comparison of the entry cutoff with Melitz**'s (2003) Let’s denote the cutoff productivity level in a closed economy found in Melitz (2003) by \( \varphi_M^* \), then I have that \( \varphi_D^* > \varphi_M^* \)

**Proof.** Since (7) and R=L are common to both models, then

\[
\left( \frac{\varphi_D}{\varphi_M^*} \right)^{\frac{\sigma - 1}{\sigma}} = \frac{p_{1 - \sigma}}{P_{1 - \sigma}^M}
\]

where \( P_{1 - \sigma}^M = M [p_D (\varphi)]^{\frac{1 - \sigma}{\sigma}} + m_M \left( \frac{\varphi_I}{\alpha} \right) \left( \frac{1}{\varphi_I} \right) \left[ p_D (\varphi)^{\frac{\alpha + 1}{\alpha}} \right] \) and \( P_{1 - \sigma}^M = \)

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Let \( \varphi_M^* \geq \varphi^*_D \), then \( p_1^{1-\sigma} \geq \sigma_1^{1-\sigma} \), which implies that
\[
\int_{\varphi_1^0}^{\varphi_M^*} [p_D(\varphi)]^{1-\sigma} \mu(\varphi) d\varphi - \int_{\varphi_D^0}^{\varphi_M^*} [p_D(\varphi)]^{1-\sigma} \mu(\varphi) d\varphi \geq \frac{m_1}{M} \left( \frac{\kappa}{\alpha} \right)^{\frac{1}{1+\kappa}} \left( \frac{1}{\varphi_1^{\sigma-1}} \right)^{\frac{1}{1+\kappa}} \left[ p_D(\varphi_D^{-\sigma}) \right]^{1-\sigma}
\]
which is impossible since the RHS is positive and the LHS is negative.

Therefore, it must be that \( \varphi_D^* > \varphi_M^* \). □

The ability of some firms to invest in a cost reducing technology enables them to have more market shares than they would without the presence of innovation, logically, those market shares are "stolen" from the less productive firms of the economy, i.e. to enter in the market in this economy a firm must be more productive than in an economy without technology. Hence I have firms that are more efficient but less varieties on the economy.

\section{Appendix II - Open Economy}

\subsection{Low Cost Innovation Equilibrium}

\textbf{Proof of Proposition 1} \hspace{1em} \textit{If there are sufficiently high fixed export cost, there exist a single cutoff \( \varphi_{XI} \).}

\textbf{Proof}. The proof is divided in three sections.

First, I show that the LHS of equation (17) is positive with respect to the productivity parameter.
\[
\pi_{XI}(\varphi_{XI}) - \pi_{DI}(\varphi_{XI}) \geq 0
\]

\[
\left[ (1+\tau^{1-\sigma})^{\frac{\alpha+1}{\sigma}} - 1 \right] \alpha \left( \frac{1}{\alpha+1} \right)^{\frac{\alpha+1}{\sigma}} \left[ \left( \frac{R(\pi)^{\sigma-1}}{\varphi^{\sigma-1}} \right) \varphi^{\sigma-1} \right]^{\frac{\alpha+1}{\sigma}} + \tau^{1-\sigma} \left( \frac{R(\pi)^{\sigma-1}}{\varphi^{\sigma-1}} \right) \varphi^{\sigma-1} - f_x \geq 0
\]

\[
C_1 (\varphi^{\sigma-1})^{\frac{\alpha+1}{\sigma}} + C_2 \varphi^{\sigma-1} - f_x \geq 0
\]

Secondly, I show that \( \pi_{XI}(\varphi_{DI}) - \pi_{DI}(\varphi_{DI}) < 0 \), otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

\[
\pi_{XI}(\varphi_{DI}) - \pi_{DI}(\varphi_{DI}) < 0
\]

\[
\left[ (1+\tau^{1-\sigma})^{\frac{\alpha+1}{\sigma}} - 1 \right] \kappa + \tau^{1-\sigma} \left( \frac{\kappa}{\alpha} \right)^{\frac{1}{1+\kappa}} (\alpha+1) - f_x < 0
\]
Thus, for \( f_X \) large enough, that is for \( f_X > \left( 1 + \tau^{1-\sigma} \right)^{\alpha+1} - 1 \) it holds that \( \pi_{XI}(\varphi_{DI}) - \pi_{DI}(\varphi_{DI}) < 0 \).

Finally, I show that the difference between the profits of the exporting and non-exporting strategies while innovation goes to infinite as the productivity of the firm is larger.

If \( \varphi \to \infty \), then \( \pi_{XI}(\Gamma(\varphi)) - \pi_{DI}(\Gamma(\varphi)) \to -\infty \), since by definition \( \pi_{XI}(\Gamma_{E}(\varphi)) > \pi_{XI}(\Gamma(\varphi)) \) then it must be that \( \pi_{XI}(\Gamma_{X}(\varphi)) - \pi_{DI}(\Gamma(\varphi)) \to -\infty \) as \( \varphi \to \infty \).

\[
\pi_{XI}(j(\varphi)) - \pi_{DI}(j(\varphi)) = \tau^{1-\sigma} [1 + \Gamma] \left( \frac{R(P\rho)^{\sigma-1}}{\sigma} \right) \varphi^{\sigma-1} - f_X
\]

\[
= \tau^{1-\sigma} \left( \frac{1}{\alpha+1} \right)^{\frac{1}{\alpha+1}} \left[ \left( \frac{R(P\rho)^{\sigma-1}}{\sigma} \right) \varphi^{\sigma-1} \right]^{\alpha+1} + \tau^{1-\sigma} \left( \frac{R(P\rho)^{\sigma-1}}{\sigma} \right) \varphi^{\sigma-1} - f_X
\]

\[
\lim_{\varphi \to \infty} \left[ \pi_{XI}(j(\varphi)) - \pi_{DI}(j(\varphi)) \right] = \lim_{\varphi \to \infty} \left[ C_4 \left[ \varphi^{\sigma-1} \right]^{\frac{\alpha+1}{\alpha}} + C_5 \varphi^{\sigma-1} - C_6 \right]
\]

\[
= \lim_{\varphi \to \infty} \left[ C_4 \left[ \varphi^{\sigma-1} \right]^{\frac{\alpha+1}{\alpha}} \right] + \lim_{\varphi \to \infty} \left[ C_5 \varphi^{\sigma-1} \right] - \lim_{\varphi \to \infty} \left[ C_6 \right] \to \infty
\]

**Productivity distribution and weighted averages** Let us denote by \( \eta_D(\varphi) \), \( \eta_{DI}(\varphi) \) and \( \eta_{XI}(\varphi) \) respectively, the productivity distribution of domestic producers, active innovators and active innovators and exporters prior to innovation.

\[
\eta_D(\varphi) = \frac{g(\varphi)}{\mu(\varphi_D) - \mu(\varphi)} \quad ; \varphi_D > \varphi \geq \varphi_D
\]

\[
\eta_{DI}(\varphi) = \frac{g(\varphi)}{\mu(\varphi_{DI}) - \mu(\varphi)} \quad ; \varphi_{DI} \geq \varphi \geq \varphi_{DI}
\]

\[
\eta_{XI}(\varphi) = \frac{g(\varphi)}{1 - \mu(\varphi_{XI})} \quad ; \varphi \geq \varphi_{XI}
\]

\( \eta_D(\varphi), \eta_{DI}(\varphi) \) and \( \eta_{XI}(\varphi) \) are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution \( \mu(\varphi) \).

Using the weighted average function defined in (30), let \( \bar{\varphi} = \bar{\varphi}(\varphi_D) \) and \( \bar{\varphi}_X = \bar{\varphi}(\varphi_{XI}) \) denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity
average that reflects the combined market share of all firms can be defined as

\[
\bar{\varphi}_t = \left\{ \frac{1}{M_t} \left[ M \varphi_{\sigma}^{-1} + M_X \left( \tau^{-1} \varphi_X \right)^{\sigma^{-1}} \right] \right\}^{\frac{1}{\sigma-1}}
\]

Following equation (31), \( \bar{\varphi}_{DI} = \bar{\varphi}(\varphi_{DI}) \) and \( \bar{\varphi}_{XI} = \bar{\varphi}(\varphi_{XI}) \) will represent the average productivity of the domestic innovators and exporter innovators after innovation. Then the weighted productivity average that reflects the combined market share of all innovators after performing innovation can be defined as

\[
\bar{\varphi}_t^I = \left\{ \frac{1}{M_t} \left[ M_I \left( \varphi_{DI} \right)^{(\sigma^{-1})} + m_{XI} \left( (1 + \tau^{1-\sigma}) \varphi_{DI} \left( \varphi_{XI} \right)^{(\sigma^{-1})} \right) \right] \right\}^{\frac{1}{\sigma-1}}
\]

**Aggregate Variables** Denote by \( m_{XI}, m_{DI} \) and \( m_D \) respectively the mass of active innovators and exporters, active innovators but non-exporters and non-innovators and non-exporters present in the economy,

\[
m_{XI} = \frac{1 - G(\varphi_{XI})}{1 - G(\varphi_D)} M
\]

\[
m_{DI} = \frac{G(\varphi_{XI}) - G(\varphi_{DI})}{1 - G(\varphi_D)} M
\]

\[
m_D = \frac{G(\varphi_{DI}) - G(\varphi_D)}{1 - G(\varphi_D)} M
\]

with \( M \) being the mass of incumbent firms in the economy, \( M_I = m_{DI} + m_{XI} \) the number of firms that perform innovation activities and \( M_X = m_{XI} \) the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be \( M_t = M + M_X \), and the total number of varieties coming from innovators will be \( M_I^I = M_I + M_X \).

It can be shown that the aggregates will take the following expressions

- **Aggregate Price Index**

\[
P^{1-\sigma} = \int^{\varphi_{DI}} p_D(\varphi)^{1-\sigma} m_{DI} \eta_{DI}(\varphi) \ d\varphi + \int^{\varphi_{DI}} p_D(\Gamma(\varphi))^{1-\sigma} m_{DI} \eta_{DI}(\varphi) \ d\varphi + \int^{\varphi_{XI}} p_D(\varphi) \ d\varphi + \int^{\varphi_{XI}} p_D(\Gamma(\varphi)) \ d\varphi + \int^{\varphi_{XI}} p_D(\varphi) \ d\varphi
\]

\[
P^{1-\sigma} = M_I p_D(\bar{\varphi}_I)^{1-\sigma} + M_I^I \left( \frac{\kappa}{\alpha} \right)^{\frac{1}{\sigma-1}} \left[ p_D \left( \frac{\bar{\varphi}_I}{\varphi_{DI}} \right)^{\frac{\alpha+1}{\sigma-1}} \right]^{1-\sigma}
\]

Notice that the first term coincides exactly with the aggregate price of the Melitz 2003 economy, therefore I can distinguish exactly the effect that
having an innovation choice has on the aggregates of the economy, since this term will be distinguished in every one of the aggregates.

- Aggregate Production

$$Q^p = \int_{\varphi_D}^{\varphi_D} [q_D(\varphi)]^p m_D \eta_D(\varphi) \, d\varphi + \int_{\varphi_D}^{\varphi_{XI}} [q_{DI}(\varphi)]^p m_{DI} \eta_{DI}(\varphi) \, d\varphi$$

- Aggregate Revenue

$$R = \int_{\varphi_D}^{\varphi_D} r_D(\varphi) m_D \eta_D(\varphi) \, d\varphi + \int_{\varphi_D}^{\varphi_{XI}} r_{DI}(\varphi) m_{DI} \eta_{DI}(\varphi) \, d\varphi$$

- Aggregate Profits

$$\Pi = \int_{\varphi_D}^{\varphi_D} \pi_D(\varphi) m_D \eta_D(\varphi) \, d\varphi + \int_{\varphi_D}^{\varphi_{XI}} \pi_{DI}(\varphi) m_{DI} \eta_{DI}(\varphi) \, d\varphi$$
\[ \Pi = M_t \frac{r_D}{\sigma} \bar{e}_t + M_t^I \left( \frac{r}{\alpha} \right) \left( \frac{1}{\varphi_D^1} \right)^{\frac{1}{\alpha}} - M f_D - M_X f_X - M_t \kappa - M_t^I \left( \frac{1}{\varphi_D^1} \right)^{\frac{1}{\alpha}} \left( \varphi_i^1 \right)^{\frac{1}{\alpha}} \] (37)

**Existence of Equilibrium**  
Equations (15) to (17) along with the Free Entry condition (20) completely determine the equilibrium and the productivity cutoffs can be uniquely determined.

With the help of equations (21) and (37), I can rearrange the FE conveniently for the characterizing of the equilibrium as a function of \( \varphi_D^* \)

\[ \delta f_E = \left[ 1 - G \left( \varphi_D^* \right) \right] \pi \]

\[ \delta f_E = f_D l_1 \left( \varphi_D^* \right) + \tau^{1-\sigma} f_D l_2 \left( \varphi_X^* \left( \varphi_D^* \right) \right) - \left[ 1 - G \left( \varphi_X^* \right) \right] f_X - \left[ 1 - G \left( \varphi_D^* \right) \right] \kappa \]

\[ + \alpha \left( \frac{1}{\alpha + 1} \right)^{\frac{1}{\alpha + 1}} f_D \frac{\alpha + 1}{\alpha} l_3 \left( \varphi_D^* \right) \]

\[ + \alpha \left( \frac{1}{\alpha + 1} \right)^{\frac{1}{\alpha + 1}} f_D \frac{\alpha + 1}{\alpha} \left[ \left( 1 + \tau^{1-\sigma} \right)^{\frac{1}{\alpha}} - 1 \right] l_4 \left( \varphi_D^* \right) \] (38)

where

\[ l_1 \left( \varphi_D^* \right) = \left[ 1 - G \left( \varphi_D^* \right) \right] \left[ \left( \frac{\hat{z} \left( \frac{\varphi_D^*}{\varphi_D^1} \right)}{\varphi_D^1} \right)^{\sigma - 1} - 1 \right] \]

\[ l_2 \left( \varphi_D^* \right) = \left[ 1 - G \left( \varphi_X^* \right) \right] \left[ \left( \frac{\hat{z} \left( \frac{\varphi_D^*}{\varphi_D^1} \right)}{\varphi_D^1} \right)^{\sigma - 1} \right] \]

\[ l_3 \left( \varphi_D^* \right) = \left[ 1 - G \left( \varphi_D^* \right) \right] \left[ \left( \frac{\hat{z} \left( \frac{\varphi_D^*}{\varphi_D^1} \right)}{\varphi_D^1} \right)^{\sigma - 1} \right] \frac{\alpha + 1}{\alpha} \]

\[ l_4 \left( \varphi_D^* \right) = \left[ 1 - G \left( \varphi_X^* \right) \right] \left[ \left( \frac{\hat{z} \left( \frac{\varphi_D^*}{\varphi_D^1} \right)}{\varphi_D^1} \right)^{\sigma - 1} \right] \frac{\alpha + 1}{\alpha} \]

**Proof.** Assume the parameter restrictions (18) & (19) hold, then the Low Cost Innovation Equilibrium exists and is unique. I shall proof that the RHS of equation (38) is decreasing in \( \varphi_D^* \) on the domain \((\varphi_D^*, \infty)\), so that \( \varphi_D^* \) is uniquely determined by the intersection of the latter curve with the flat line \( \delta f_E \) in the \((\varphi_D^*, \infty)\) space.

Since \( l_1 \left( \varphi_D^* \right) \) and \( l_3 \left( \varphi_D^* \right) \) are decreasing from infinity to zero on \((0, \infty)\), from the closed economy case, it follows that \( l_2 \left( \varphi_D^* \right) \) and \( l_4 \left( \varphi_D^* \right) \) do also monotonically decrease from infinity to zero on the \((0, \infty)\) parameter space.

Therefore, the RHS of (40) is a monotonic decreasing function from infinity to zero on the space \((0, \infty)\) that cuts the FE flat line from above identifying a unique cutoff level \( \varphi_D^* \).
B.2 Low Cost Trade Equilibrium

Productivity distribution and weighted averages  Let us denote by $\eta_D(\varphi)$ and $\eta_{DI}(\varphi)$ respectively, the productivity distribution of domestic producers and active innovators prior to innovation. The distribution of all firms is given by

$$
\eta_D(\varphi) = \begin{cases} 
\frac{g(\varphi)}{G(\varphi_D) - G(\varphi)} & , \varphi > \varphi \geq \varphi_D \\
0 & , otherwise
\end{cases}
$$

$$
\eta_X(\varphi) = \begin{cases} 
\frac{g(\varphi)}{G(\varphi_X) - G(\varphi)} & , \varphi_X \geq \varphi \geq \varphi_X \\
0 & , otherwise
\end{cases}
$$

$$
\eta_{XI}(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1 - G(\varphi_{XI})} & , \varphi \geq \varphi_{XI} \\
0 & , otherwise
\end{cases}
$$

Using the weighted average function defined in (30), let $\bar{\varphi} = \bar{\varphi}(\varphi_D)$ and $\bar{\varphi}_x = \bar{\varphi}(\varphi_X)$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$
\bar{\varphi}_t = \left\{ \frac{1}{M_t} \left[ M \varphi^{\sigma-1} + M_X \left( r^{-1} \varphi_x \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}
$$

Following equation (31), $\bar{\varphi}_{XI}$ will represent the average productivity of the innovators after innovation.

$$
\bar{\varphi}_{XI} = \left[ \int_{\varphi_{XI}}^{\infty} \left( \varphi^{\sigma-1} \right)^{\frac{n+1}{n}} \eta_{XI}(\varphi) d\varphi \right]^{\frac{1}{n+1}}
$$

Aggregate Variables  Denote by $m_{XI}, m_X$ and $m_D$ respectively the mass of active innovators and exporters, only exporters and non-innovators non-exporters present in the economy,

$$
m_{XI} = \frac{1 - G(\varphi_{XI})}{1 - G(\varphi_D)} M
$$

$$
m_X = \frac{G(\varphi_{XI}) - G(\varphi_X)}{1 - G(\varphi_D)} M
$$

$$
m_D = \frac{G(\varphi_X) - G(\varphi_D)}{1 - G(\varphi_D)} M
$$

with $M$ being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_X + m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + M_X$

It can be shown that the aggregates will take the following expressions
• Aggregate Price Index

\[
P^{1-\sigma} = \int_{p_D}^{p_{XI}} [p_D (\varphi)]^{1-\sigma} m_D \eta_D (\varphi) d\varphi \\
+ \int_{p_X}^{p_{XI}} [p_X (\varphi)]^{1-\sigma} m_X \eta_X (\varphi) d\varphi + \int_{p_X}^{p_{XI}} [p_X (\varphi)]^{1-\sigma} m_X \eta_X (\varphi) d\varphi \\
+ \int_{p_{XI}}^{p_{XI}} [p_{XI} (\varphi)]^{1-\sigma} m_{XI} \eta_{XI} (\varphi) d\varphi + \int_{p_{XI}}^{p_{XI}} [p_{XI} (\varphi)]^{1-\sigma} m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
= \int_{p_D}^{p_{XI}} [p_D (\varphi)]^{1-\sigma} m_D \eta_D (\varphi) d\varphi + \int_{p_{XI}}^{p_{XI}} [p_{XI} (\varphi)]^{1-\sigma} M \eta (\varphi) d\varphi + \\
+ (1 + \tau^{1-\sigma}) \int_{p_{XI}}^{p_{XI}} X (\varphi) [p_{XI} (\varphi)]^{1-\sigma} m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
P^{1-\sigma} = M_t [p_D (\varphi)]^{1-\sigma} m_{XI} (1 + \tau^{1-\sigma}) \left[ \frac{\sigma}{\alpha} \left( \frac{1}{\varphi_{XI}^{\alpha \sigma - 1}} \right)^\frac{1}{\sigma - 1} \right]^{1-\sigma}
\]

Notice that the first term coincides exactly with the aggregate price of the Melitz 2003 economy, therefore I can distinguish exactly the effect that having an innovation choice has on the aggregates of the economy, since this term will be distinguished in every one of the aggregates. Modify

• Aggregate Production

\[
Q^p = \int_{q_D}^{q_{XI}} [q_D (\varphi)]^p m_D \eta_D (\varphi) d\varphi \\
+ \int_{q_{XI}}^{q_{XI}} [q_{XI} (\varphi)]^p m_{XI} \eta_{XI} (\varphi) d\varphi + \int_{q_{XI}}^{q_{XI}} [q_{XI} (\varphi)]^p m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
= \int_{q_D}^{q_{XI}} [q_D (\varphi)]^p M \eta (\varphi) d\varphi + \int_{q_{XI}}^{q_{XI}} [q_{XI} (\varphi)]^p M \eta (\varphi) d\varphi + \\
+ (1 + \tau^{1-\sigma}) \int_{q_{XI}}^{q_{XI}} X (\varphi) [q_{XI} (\varphi)]^p m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
Q^p = M_t [q_D (\varphi)]^p m_{XI} (1 + \tau^{1-\sigma}) \left[ \frac{\sigma}{\alpha} \left( \frac{1}{\varphi_{XI}^{\alpha \sigma - 1}} \right)^\frac{1}{\sigma - 1} \right]^p
\]

• Aggregate Revenue

\[
R = \int_{r_D}^{r_{XI}} r_D (\varphi) m_D \eta_D (\varphi) d\varphi \\
+ \int_{r_{XI}}^{r_{XI}} r_{XI} (\varphi) m_{XI} \eta_{XI} (\varphi) d\varphi + \int_{r_{XI}}^{r_{XI}} r_{XI} (\varphi) m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
= \int_{r_D}^{r_{XI}} r_D (\varphi) M \eta (\varphi) d\varphi + \int_{r_{XI}}^{r_{XI}} r_{XI} (\varphi) M \eta (\varphi) d\varphi + \\
+ (1 + \tau^{1-\sigma}) \int_{r_{XI}}^{r_{XI}} i (\varphi) \left[ \frac{\alpha}{\sigma - 1} \right] r_D (\varphi) m_{XI} \eta_{XI} (\varphi) d\varphi
\]

\[
R = M_t r_D (\varphi) m_{XI} (1 + \tau^{1-\sigma}) \left[ \frac{\sigma}{\alpha} \left( \frac{1}{\varphi_{XI}^{\alpha \sigma - 1}} \right)^\frac{1}{\sigma - 1} \right] r_D (\varphi)
\]
• Aggregate Profits

\[ \Pi = \int_{\varphi_D}^{\varphi_X} \pi_D (\varphi) m_D \eta_D (\varphi) d\varphi + \int_{\varphi_D}^{\varphi_X} \pi_X (\varphi) m_X \eta_X (\varphi) d\varphi + \int_{\varphi_X}^{\varphi_D} \pi_I (\varphi) m_I \eta_I (\varphi) d\varphi \]

\[ = \int_{\varphi_D}^{\varphi_X} \frac{r_D (\varphi)}{\sigma} M_\mu (\varphi) d\varphi + \int_{\varphi_X}^{\varphi_D} \frac{r_X (\varphi)}{\sigma} M_X \mu_X (\varphi) d\varphi + \]

\[ + (1 + \tau^{1-\sigma}) \int_{\varphi_X}^{\varphi_D} \left( j_x (\varphi) \right) \frac{\alpha + 1}{\sigma} \frac{r_D (\varphi)}{\sigma} m_X \eta_X (\varphi) d\varphi \]

\[ - M f_D - M X f_X - \int_{\varphi_X}^{\varphi_D} \left[ \kappa + (j_x (\varphi))^{\alpha + 1} \right] m_X \eta_X (\varphi) d\varphi \]

\[ \Pi = M_t \left( \frac{r_D (\varphi_D)}{\sigma} + m_X (1 + \tau^{1-\sigma}) \left[ \frac{\kappa}{\alpha + 1} \right] \frac{1}{\varphi_X^{\sigma - 1}} \left( \frac{1}{\varphi_X^{\sigma - 1}} \right) \right) \]

\[ - M f_D - M X f_X - m_X \kappa - m_X (1 + \tau^{1-\sigma}) \left( \frac{1}{\varphi_X^{\sigma - 1}} \right) \frac{\alpha + 1}{\varphi_X^{\sigma - 1}} \frac{r_D (\varphi_D)}{\sigma} \]

\[ \text{Existence of Equilibrium} \quad \text{Equations (22) to (24) along with the Free Entry condition (26) completely determine the equilibrium and the productivity cutoffs can be uniquely determined.} \]

With the help of equations (27) to (29) and (39), I can rearrange the FE conveniently for the characterizing of the equilibrium as a function of \( \varphi_D \)

\[ \delta f_E = [1 - G (\varphi_D)] \pi \]

\[ \delta f_E = f_D l_1 (\varphi_D) + f_X l_2 (\varphi_X (\varphi_D)) \]

\[ + \alpha \left( \frac{1}{\alpha + 1} \right) \left[ \frac{\alpha + 1}{\alpha} \right] l_3 (\varphi_D) - (1 - G (\varphi_X)) \kappa \]

where \( l_1 (\varphi_D) = [1 - G (\varphi_D)] \left[ \left( \frac{\varphi_D}{r_D} \right)^{\sigma - 1} - 1 \right] \)

\[ l_2 (\varphi_X (\varphi_D)) = [1 - G (\varphi_X)] \left[ \left( \frac{\varphi_X}{r_X} \right)^{\sigma - 1} - 1 \right] \]

\[ l_3 (\varphi_D) = (1 - G (\varphi_D)) \left[ \left( \frac{\varphi_D}{r_D} \right)^{\sigma - 1} \right] \]

\[ \text{Proof.} \quad \text{Assume the parameter restriction (25) holds, then the Low Cost Trade Equilibrium exists and is unique. I shall proof that the RHS of equation (40) is decreasing in } \varphi_D \text{ on the domain } (\varphi_D, \infty), \text{ so that } \varphi_D \text{ is uniquely determined by the intersection of the latter curve with the flat line } \delta f_E \text{ in the } (\varphi_D, \infty) \text{ space.} \]

The last term on equation (40) is constant, therefore I only need to show that the other three terms are decreasing. Since \( l_1 (\varphi_D) \) and \( l_3 (\varphi_D) \) are decreasing from infinity to zero on \((0, \infty)\), from the closed economy case, it follows that \( l_2 (\varphi_X (\varphi_D)) \) does also monotonically decrease from infinite to zero on the \((0, \infty)\) parameter space.
Therefore, the RHS of (40) is a monotonic decreasing function from infinity to zero on the space $(0, \infty)$ that cuts the FE flat line from above identifying a unique cutoff level $\varphi_D$.

**B.3 Trade Effects**

**Proof of Proposition 2.** If (25) holds, and that the productivity draws are distributed according to a Pareto distribution with low productivity bound $1$ and $\xi > \left(\frac{\alpha+1}{\sigma}\right) (\sigma - 1)$, then the innovation cutoff in the open economy is lower than in autarky.

**Proof.** Use (11) and (27) to get

$$\frac{\varphi_{DL}^A}{\varphi_{X1}^T} = \left(1 + \tau^{1-\sigma}\right) \frac{\varphi_D^A}{\varphi_D^T}$$

The FE conditions in autarky and free trade give us the following relationship between profits and cutoffs

$$\frac{\pi^A}{\pi^T} = \left(\frac{\varphi_D^A}{\varphi_D^T}\right)^{\xi}$$

Hence, I need to show that $\left(1 + \tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \pi^A > \pi^T$, from where it follows that $\varphi_{DL}^A > \varphi_{X1}^T$.

Using (36) and (39), I can express $\pi^A$, and $\pi^T$ as

$$\pi^A = Af_D + B \left(\frac{1}{A}\right)^{\frac{\xi}{\sigma-1}}$$

$$\pi^T = Af_D + B \left(\frac{1 + \tau^{1-\sigma}}{\Lambda}\right)^{\frac{\xi}{\sigma-1}} + Af_X \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\xi}{\sigma-1}}$$

where $A = \frac{\sigma-1}{\xi-(\sigma-1)}$ and $B = \left(\frac{\alpha+1}{\xi-(\sigma+1)}\right) \kappa$

Thus

$$\left(1 + \tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \pi^A > \pi^T$$

$$\xrightarrow{f_D} \left(1 + \tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1 > f_X \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\xi}{\sigma-1}}$$

$$\xrightarrow{f_D} \left[1 + \tau^{1-\sigma}\right]^{\frac{\xi}{\sigma-1}} - 1 > \tau^{-\xi} \left(\frac{f_D}{f_X}\right)^{\frac{\xi-(\sigma-1)}{\sigma-1}}$$

From the parameter restriction I know that $1 > \frac{f_D}{f_X \tau^{\sigma-1}}$, then it follows that $1 > \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\xi-(\sigma-1)}{\sigma-1}} \Rightarrow \tau^{-\xi} > \left(\frac{f_D}{f_X}\right)^{\frac{\xi-(\sigma-1)}{\sigma-1}} \Rightarrow \tau^{-\xi} > \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\xi}{\sigma-1}}$
\[ 1 + r^{1-\sigma} > 1 + \left( \frac{f_\delta}{f_\kappa} \right)^{\frac{\xi}{\tau}} \tau^{-\xi}. \]

C Appendix III - Trade & Innovation

Table 1.- Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{Pareto Distribution}</td>
<td>$\overline{\phi} = 1$</td>
<td>Bernard et al.(2007)</td>
</tr>
<tr>
<td>Shape</td>
<td>$\xi = 5$</td>
<td>Balistreri et al. (2010)</td>
</tr>
<tr>
<td>\textit{Entry &amp; Exit}</td>
<td>$f_E = 2$</td>
<td>Bernard et al.(2007)</td>
</tr>
<tr>
<td>Sunk cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Death shock</td>
<td>$\delta = 0.025$</td>
<td></td>
</tr>
<tr>
<td>\textit{Demand}</td>
<td></td>
<td>Broda &amp; Weinstein (2006)</td>
</tr>
<tr>
<td>Elasticity of Substitution</td>
<td>$\sigma = 3$</td>
<td></td>
</tr>
<tr>
<td>\textit{Investment}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable cost</td>
<td>$\alpha = 1$</td>
<td>*Linear innovation</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$\kappa = 3$</td>
<td>*Set so $\kappa &gt; \alpha$</td>
</tr>
<tr>
<td>\textit{Production}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic fixed cost</td>
<td>$f_D = 1$</td>
<td>Balistreri et al. (2010)</td>
</tr>
<tr>
<td>Export fixed cost</td>
<td>$f_X \in (1,12)$</td>
<td></td>
</tr>
<tr>
<td>Export variable costs</td>
<td>$\tau \in (1,1.5)$</td>
<td></td>
</tr>
</tbody>
</table>

C.1 Autarky to Trade

reallocation of market shares on the low cost innovation equilibrium

\[ r_D (\varphi) < r_D^A (\varphi) < r_{DI} (\varphi) < r_{DI}^A (\varphi) < r_{XI} (\varphi) \]

\textbf{Proof.} Recall that \( r_D^A (\varphi) = \left( \frac{\varphi}{r_D^A} \right)^{\sigma-1} \sigma f_D, r_D (\varphi) = \left( \frac{\varphi}{r_D} \right)^{\sigma-1} \sigma f_D \), and that

\[ r_D^A (\varphi) = r_D^A (\varphi) + \left[ \left( \frac{1}{\alpha+1} \right)^{\frac{1}{\alpha}} \right]^\alpha \frac{r_D^A (\varphi)^{\alpha+1}}{\alpha} + \frac{1}{\alpha+1} \right]^{\frac{1}{\alpha}} \left[ r_D (\varphi)^{\frac{\alpha+1}{\alpha}} \right]. \]

Finally, \( r_{XI} (\varphi) = (1 + \tau^{1-\sigma}) r_{DI} (\varphi) \).

Hence it follows that \( r_D^A (\varphi) > r_D (\varphi) \) since \( \varphi_T^D > \varphi_T^A \) and consequently \( r_{DI}^A (\varphi) > r_{DI} (\varphi) \).

The last inequality is a direct consequence of \( (1 + \tau^{1-\sigma}) r_{DI} (\varphi) \) decreasing as \( \tau \) increases. Since the autarky equilibrium is obtained as the limiting equilibrium as \( \tau \) increase to infinity, \( r_{DI}^A (\varphi) = \lim_{\tau \to \infty} r_{DI} (\varphi) = \lim_{\tau \to \infty} \left( 1 + \tau^{1-\sigma} \right) r_{DI} (\varphi) \).

Therefore, for any finite \( \tau \), it follows that \( r_{DI}^A (\varphi) < r_{XI} (\varphi) \).  

reallocation of market shares on the low cost trade equilibrium

\[ r_D (\varphi) < r_D^A (\varphi) < r_X^T (\varphi) < r_X^T (\varphi) < r_{DI}^A (\varphi) \]
Proof. Above I have shown that $r^A_D(\varphi) > r_D(\varphi)$, $r_{XI}(\varphi) > r_X(\varphi)$ is given by
the model and the last inequality is derived from $\varphi^A_{DI} > \varphi^A_{XI}$. Therefore, I only
need to show the second inequality, that is $r_X(\varphi) > r^A_D(\varphi)$
Recall that $r_X(\varphi) = (1 + \tau^{1-\sigma}) r_D(\varphi)$, therefore this inequality is a direct
consequence of $(1 + \tau^{1-\sigma})$ decreasing as $\tau$ increases. Since the autarky
equilibrium is obtained as the limiting equilibrium as $\tau$ increase to infinity,
$r^A_D(\varphi) = \lim_{\tau \to +\infty} r_D(\varphi) = \lim_{\tau \to +\infty} (1 + \tau^{1-\sigma}) r_D(\varphi)$ . Therefore, for any
finite $\tau$, it follows that $r^A_D(\varphi) < r_X(\varphi)$.

C.2 Partial Trade Liberalization

Decrease in Trade Costs These comparative statics are derived from the
equilibrium condition for the cutoff levels (eq 40), the assumption that $\varphi \sim
G(\varphi) = 1 - \frac{1}{\varphi^\xi}$, and the implicit definition of $\varphi_{XI}$ and $\varphi_X$ as function of $\varphi_D$
given in equations (27) and (29).

- $\varphi_D$
  
  \[ FE : \pi = \frac{\delta f_E}{1 - G(\varphi_D)} \]
  \[ ZPC : \pi = A \cdot f_D + A \cdot f_X \left( \frac{f_D}{f_X} \right)^{\frac{\xi}{\xi-1}} \cdot \tau^{-\xi} + B \cdot \kappa \cdot \left( \frac{1 + \tau^{1-\sigma}}{\Lambda} \right)^{\frac{\xi}{\xi-1}} \]
  
  where $A = \frac{\xi}{\xi-1}$ and $B = \frac{(\sigma-1)(\frac{\alpha+1}{\alpha})}{\xi-1}$(\frac{\alpha}{\alpha+1})$

  \[ \frac{\partial \varphi_D}{\partial \tau} = \frac{\partial(ZPC)}{\partial \varphi_D} = \frac{\partial(FE)}{\partial \varphi_D} = \frac{-A' \cdot \xi \tau^{-\xi-1} - B' \cdot \xi \tau^{-\sigma} \left( 1 + \tau^{1-\sigma} \right)^{\frac{\xi}{\xi-1}}}{\delta f_E \cdot \xi (\varphi_D)^{\xi-1}} < 0 \] (41)

  where $A' = A \cdot f_X \left( \frac{f_D}{f_X} \right)^{\frac{\xi}{\xi-1}}$ and $B' = B \cdot \kappa \cdot \left( \frac{1}{\Lambda} \right)^{\frac{\xi}{\xi-1}}$

- $\varphi_X$

  \[ \varphi^{-1}_X = \frac{f_X \tau^{\sigma-1}}{f_D} \varphi^{-1}_D \]

  \[ \frac{1}{1 - G(\varphi_X)} = \varphi^{-\xi}_X = \left( \frac{f_X \tau^{\sigma-1}}{f_D} \right)^{\frac{\xi}{\xi-1}} \varphi^{-\xi}_D = \left( \frac{f_X}{f_D} \right)^{\frac{\xi}{\xi-1}} \tau^{\xi} \frac{\pi}{\delta f_E} \]

  \[ \frac{\partial \varphi_X}{\partial \tau} = \frac{\partial(RHS)}{\partial \tau} = \frac{\partial(\tau^{-\xi}(\varphi_X))}{\partial \tau} = \frac{\xi D \cdot \left[ A'' \cdot \tau^{-\xi-1} + \tau^{\xi-1} (1 + \tau^{1-\sigma})^{\frac{\xi}{\xi-1}} \right]}{\xi (\varphi_X)^{\xi-1}} > 0 \] (42)
where $A'' = A \cdot f_D$ and $D = \left( \frac{L_x}{f_D} \right)^{\frac{\xi}{\tau}}$

- $\varphi_{XI}$

\[
\varphi_{XI}^{-1} = \left( \frac{\xi}{\alpha} \right)^{\frac{\sigma}{\tau}} \frac{\alpha + 1}{f_D} \frac{1}{1 + (\tau^{1-\sigma})} \varphi_{D}^{-1}
\]

\[
\frac{1}{1 - G(\varphi_{XI})} = \varphi_{XI}^{\xi} = \left( \frac{\xi}{\alpha} \right)^{\frac{\sigma}{\tau}} \frac{\alpha + 1}{f_D} \frac{1}{(1 + (\tau^{1-\sigma}))} \frac{\xi^{\eta-1}}{\delta f_E}
\]

\[
\frac{\partial \varphi_{XI}}{\partial \tau} = \frac{\partial (\text{RHS})}{\partial \varphi_{XI}} = \xi E \frac{A'' \cdot \tau^{-\sigma} - A' \cdot \tau^{-\xi-1}}{(1 + (\tau^{1-\sigma})^{\frac{\xi}{\tau}} + 1) \xi (\varphi_{XI})^{\xi-1}} > 0
\]

where $E = \left( \frac{\xi}{\alpha} \right)^{\frac{\sigma}{\tau}} \frac{\alpha + 1}{f_D}$ and $A'' \cdot \tau^{-\sigma} - A' \cdot \tau^{-\xi-1} > 0$ since $\frac{L_x}{f_D} > \tau^{1-\sigma}$ in equilibrium.

I have performed all the comparative statistics in the Low Cost Trade equilibrium since it is the one for which I have all the closed forms solutions. Since I do not have all the closed form solutions of the Low Cost Innovation equilibrium, I cannot show mathematically that $\frac{\partial \varphi_{DI}}{\partial \tau} < 0$ but the numerical simulation performed using the calibration of table 1, setting $f_X = 12$ to ensure the economy is on the low cost innovation equilibrium and shown below confirms the result.

Evolution of $\varphi_{DI}$ under a trade liberalization
C.3 Welfare Gains

\( W^{\text{Open Economy}} > W^{\text{Autarky}} \)

**Proof.** \( W = P^{-1} \)

Let the price be written as \( P = \left( \frac{\sigma}{\sigma - \tau} \right)^{1 - \sigma} \times \frac{R}{\sigma f_D} \) and since \( R = L \), the welfare in each situation can be written as:

\[
W^A = \frac{1}{P^A} = \rho \left( \frac{L}{\sigma f_D} \right)^{\frac{\alpha}{\alpha - 1}} \times \varphi_D^A
\]

\[
W^{PI} = \frac{1}{P^{PI}} = \rho \left( \frac{L}{\sigma f_D} \right)^{\frac{\alpha}{\alpha - 1}} \times \varphi_D^{PI}
\]

\[
W^{PT} = \frac{1}{P^{PT}} = \rho \left( \frac{L}{\sigma f_D} \right)^{\frac{\alpha}{\alpha - 1}} \times \varphi_D^{PT}
\]

In the open economy, the productivity needed to enter the domestic market is higher than in autarky \( \varphi_D^{PT} > \varphi_D^A \) and \( \varphi_D^{PI} > \varphi_D^A \).

It follows that \( W^{PT} > W^A \) and \( W^{PI} > W^A \). ■