Asymmetric Trade Integration and Growth

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Abstract

We consider a many country endogenous growth model with complementary capital goods and analyze the impact of asymmetric trade integration on the world long run steady state. The model generates endogenous long run income clubs, i.e., country groups in which levels of endogenous variables (e.g., technology and output) differ even as the rate of long run growth is common. Results include the demonstration that customs unions reduce the long run rate of growth.

Key words: Endogenous growth, complementary capital goods, preferential trade agreements.

JEL codes: F43, F15.
1 Introduction:

The impact of international trade on economic growth is a topic of empirical and policy importance that has attracted much attention in economic literature. The proliferation of preferential trade agreements (PTAs) over the last fifteen years has motivated new interest in asymmetric trade integration but less attention has been directed toward its dynamic impact.\(^1\) In this paper, we present an endogenous growth model that we then apply to analyze these dynamic effects.\(^2\)

We expand the closed economy growth model of Evans, Honkapohja and Romer (EHR) (1998) and the two country symmetric trade model of Honkapohja and Turunen-Red (HTR) (2002). HTR showed that international trade in capital goods is capable of creating sudden growth jumps (bifurcations) in which countries quickly find their way to a new high growth equilibrium while previous low growth equilibria are eliminated. Such favorable growth jumps offer one possible explanation for the observed correlation of periods of exceptionally high growth and expanding international trade (Hausmann, Pritchett and Rodrik (2005)).

The present model includes several potentially asymmetric countries that may differ from each other in trade policy, size, cost of innovation, and overall productivity of resources. There are several aspects to the growth process that we consider. First, we characterize the steady states of the model. These define the pace of technological advance in the world and thus yield a common rate of long run growth for all countries. This common rate of long run growth is affected by all symmetries and asymmetries between countries, including asymmetries in trade policy (PTAs). The effects of policy and other asymmetries are determined through a growth multiplier that can be separately evaluated. We obtain that, if all countries are constrained to symmetry, any increase in tariffs slows down long run growth, assuming that there are no other distortions. However, the impact of asymmetries...the effect of an expanding customs union on growth is ambiguous because the entry of new members into a customs union creates both growth creating and growth diverting effects. Retaliation by countries that do not belong to a customs union generally lowers long run growth but may benefit the nonmembers.

Secondly, within our model, levels of endogenous variables generally differ across nations even if the long run rate of growth is common. Asymmetries in trade policy (PTAs) and other exogenous differences between countries generate endogenous long run "income clubs"; countries in such clubs exhibit common long run growth but persistently differ in levels of technology and output. A trade bloc that raises its trade barrier against the rest of the world has a negative effect on the technology levels of the outsiders, while retaliation by the nonmembers of an PTA alleviates losses. Asymmetries of trade policy can exacerbate or mitigate level differences that are due to other exogenous asymmetries. A customs union of countries that is larger, more productive or has lower innovation costs gains more in its relative technology position.\(^3\)

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\(^1\)Asymmetric trade integration here includes PTAs involving trade policy (such as free trade areas and customs unions). We do not consider economic unions that include joint decision making regarding monetary policy, factor movements, or other institutional arrangements.

\(^2\)Comprehensive discussions of static effects of PTAs can be found in Baldwin and Venables (1995), Bhagwati, Greenaway and Panagariya (1998) and World Bank (2005).

\(^3\)Since we focus on the impact of asymmetries in trade policy we do not model the foun-
Related Literature: The theoretical and empirical literature that directly addresses dynamic effects of PTAs is sparse. Walz’s (1997, 1999) analysis of asymmetric trade integration is based on the Grossman and Helpman (1991) approach and differs from ours in several respects. First, Walz defines growth dynamics using a state variable (the level of world technology) that evolves along an explicit trajectory under perfect foresight; this knowledge-driven formulation of growth includes complete international spillovers of knowledge. Our lab-equipment model of innovation does not include knowledge externalities and new innovations are shared through international trade in capital goods. We do not include transition dynamics based on a state variable but assume that individuals form expectations and learn about growth in a long run equilibrium. Steady states that are approached through learning dynamics are treated as stable and our comparative statics center around these equilibria.\footnote{See Evans and Honkapohja (2001) for a thorough discussion of learning dynamics. A companion paper (Honkapohja, Turunen-Red and Woodland (2009)) discusses the effects of trade policy and other country-specific asymmetries on short and long run learning dynamics.}

Second, Walz focuses on a North-South world in which only one country (country A) innovates; two other countries either produce a traditional commodity that does not require intermediate capital goods in its production (country C) or produce both the traditional commodity and an industrial final product (country B) that does require intermediate capital goods. Walz obtains that a reduction in trade barriers between the more advanced economies A and B can either speed up or slow down the rate of technological progress depending on whether the trade liberalization has a trade creating or trade diverting flavor (if A and B raise trade barriers against C, growth slows down (trade diversion); if A and B liberalize their mutual trade in intermediate capital goods, growth speeds up (trade creation)).

The significant recent empirical study of the growth effects of PTAs is Berthelon (2004). In contrast to earlier cross-country regression studies (for references, see Berthelon (2004)) that used dummy variables to indicate membership in PTAs and obtained mixed results, Berthelon constructs an explanatory variable that takes into account not only a country’s membership in an PTA but also the extent of the new market that is made accessible through the PTA. This allows Berthelon to differentiate between PTAs between small and large countries, and he further distinguishes PTAs according to the degree of development of the members (North-North, North-South and South-South agreements). Berthelon obtains that the market size of the partner countries matters and that PTAs contribute positively to growth. North-North type agreements are found to be most potent in improving growth, while the evidence for South agreements suggests that the growth impact may be negative.

Other related literature includes recent work on international income and growth differences. Waugh (2007) estimates that asymmetries in trade costs have a significant impact on relative incomes. Jones (2007) presents a theoretical model in which complementarity of intermediate inputs play a central role in explaining large differences in incomes across economies. Pritchett (2000, 2006) has argued that differences in incomes and growth can be understood in terms of growth regimes of countries that experience different steady states, each with its own transition dynamics.
2 Model:

In this section, we describe the model and its long run equilibrium solutions. Consumer preferences are taken to be identical everywhere and production in all countries is subject to a three sector structure. Exogenous asymmetries roughly reflect factors that have been identified as contributors toward income and growth divergence.\(^5\)

2.1 Basic Assumptions:

We assume that there are \(N (\geq 3)\) countries, indexed by \(i = 1, \ldots, N\). The aggregate consumer in each country maximizes the discounted utility expression

\[
U_i = \sum_{j=0}^{\infty} \frac{\beta^{t+j}C_{i,t+j}^{1-\sigma}}{1-\sigma}, \quad 0 < \sigma < 1,
\]

where \(C_{i,t+j}\) denotes final consumption in country \(i\) in period \((t + j)\). Given a constant interest rate, \(r\), each aggregate consumer’s preferred rate of consumption growth, \(g_c\), is obtained from the Euler equation

\[
\frac{C_{i,t+1}}{C_{it}} \equiv g_c = \left[\beta(1 + r)\right]^{1/\sigma}.
\]

Financial capital is taken to be freely mobile, so that the interest rate equals worldwide.

In each country \((i = 1, \ldots, N)\), final consumption is produced by a competitive production sector according to the production function

\[
Y_{it} = \tilde{L}_i^{1-\alpha} \left( \sum_{k=1}^{N} \int_{0}^{A_{kt}} x_{ikt}(j_k)\gamma dj_k \right)^{\phi}, \quad \tilde{L}_i \equiv \psi^{-1/\alpha}_i L_i.
\]

In (3), the quantity of intermediate capital goods imported from country \(k\) is indicated by \(x_{ikt}(j_k)\), where \(j_k\) indexes varieties of capital goods supplied by producers in country \(k\). The number of different capital goods produced in a country at time \(t\), denoted by \(A_{kt}\), defines the technology level in each location at a point in time. Parameters \(L_i\) represent the (fixed) endowments of immobile resources (country size) and the \(\psi_i\) reflect productivity differences in the production of consumer goods. Parameter \(\phi\) determines the degree of technological substitutability among capital inputs; we assume that \(\phi > 1\) so that all capital varieties are complements in production. The restriction \(\alpha = \gamma\phi\) is imposed in order to preserve linear homogeneity of the production process. The source of growth is the invention of new capital goods; in (3), aggregate output grows as new capital varieties are developed and, owing to the complementarity of capital goods, marginal productivity of each intermediate capital variety improves as each \(A_{kt}\) increases.

We assume that countries may impose trade barriers, denoted by \( \tau_{ik} \geq 1 \), against imported capital goods (to country \( i \) from country \( k \)); for domestic production trade barriers are set to zero (\( \tau_{ii} = 1 \)). All tariff revenues are distributed to the consumption sectors as lump sum income.

Competitive final production sectors take domestic prices as given (all prices are measured with respect to the world market price of final consumption) and maximize profit given technology (3). This yields the demand for all capital goods, obtained from equations

\[
\tau_{ik} R_{ikt}(j_i) = \bar{L}_1^{\phi - \alpha} \left( \sum_{l=1}^{\infty} x_{ilt}(j_l) \gamma dj_l \right)^{\phi - 1} \alpha x_{ikt}(j_k)^{\gamma - 1},
\]

where the \( R_{ikt}(j_k) \) denote the rental prices of capital varieties originating in country \( k \) and imported to country \( i \).

Intermediate capital goods are supplied by monopolistic competitive inventors; patent protection is complete so that there is no replicative innovation. A unit of each capital good is produced by converting one unit of aggregate capital (foregone consumption), denoted by \( Z \), into a specific capital variety. Production is realized at the end of a time period so that, at the end of a period, a capital goods producer in country \( i \) receives revenue \( R_{kit}(j_i) x_{kit}(j_i) \) from sales in country \( k \). In the beginning of a time period, \( x_{kit}(j_i) \) units of \( Z \) are needed to produce the capital units that are rented out. The rental cost for these units of aggregate capital over the time period is \( r_t p^2_t x_{kit}(j_i) \), where \( p^2_t \) is the opportunity cost of aggregate capital in final consumption. Thus, each inventor maximizes the end of the time period profit expression (\( i = 1, \ldots, N \))

\[
\pi_{it}(j_i) = \sum_{k=1}^{N} R_{kit}(j_i) x_{kit}(j_i) - r_t p^2_t \left[ \sum_{k=1}^{N} x_{kit}(j_i) \right], \quad j_i \in [0, A_{it}],
\]

where the \( x_{kit}(j_i) \) are obtained from (4). The resulting mark-up rules for capital goods in all markets are

\[
R_{kit} = \frac{r_t p^2_t}{\gamma} R_t(r_t, p^2_t), \quad i, k = 1, \ldots, N,
\]

and since all varieties of capital goods are priced equally, the index \( j_i \) for capital goods is subsequently dropped.

While the rate of technological progress will be the same in all countries in the long run, technology levels do not necessarily equate. We define the country-specific technological proportionality factors, \( \theta_i \), by setting

\[
A_{it} = \theta_i A_{it}, \quad i = 2, \ldots, N, \quad (\theta_1 = 1).
\]

Accordingly, \( A_{it} \) serves as an index for the world technology level at a point in time. Long run equilibrium values of \( \theta_i \) are to be solved from the model.

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6 The \( \tau_{ik} \) represent ad valorem tariffs (usually expressed as \( 1 + \tau_{ik} \)) but other non-policy trade costs may be included as well. In HTR (2002), symmetric tariffs were represented by a multiplier \( c \in [0, 1] \) yielding the fraction of export revenue that accrues to an exporter.

7 Intermediate capital goods are treated as service flows from durable (not depreciating) capital goods owned by their inventor producers.
Using equations (4) together with (6) and (7) we obtain the provision of each capital variety in all markets \((i = 1, \ldots, N, k \neq i)\):

\[
x_{iit} = \hat{L}_i(A_{1t}S_i)^\xi \left( \frac{R_i}{\alpha} \right)^{\frac{1}{1-\alpha}}, \quad x_{ikt} = x_{iit}r_{ik}^{\tau_{ik}}, \quad \xi \equiv \frac{\phi - 1}{1 - \alpha},
\]

(8)

where

\[
S_i \equiv \sum_k \theta_k \tau_{ik}^{\gamma} = \theta_i + \sum_{k \neq i} \theta_k \tau_{ik}^\gamma.
\]

(9)

Aggregate output in country \(i\) at time \(t\) equals

\[
Y_{it} = \hat{L}_i^{1-\alpha} x_{iit}^\alpha (A_{1t}S_i)^\phi = \hat{L}_i (A_{1t}S_i)^{1+\xi} \left( \frac{R_i}{\alpha} \right)^{\frac{\phi}{1-\alpha}}.
\]

(10)

By (8), imports of capital goods \((x_{ikt})\) decrease with trade barriers \((\tau_{ik})\) and increase with country size and total factor productivity \((\hat{L}_i)\) and the country's technology level, here represented by \((A_{1t}S_i)\), *ceteris paribus*.

The multiplicative terms \(S_i\) defined in (9) give an import tariff- deflated sum of the relative technology levels of a country’s trade partners and therefore reflect the accessibility of technology to the aggregate production sector of a country (openness factor of a country). In (8), these \(S_i\) yield the impact of a country’s trade policy and trade pattern on domestic capital goods production \((x_{iit})\) and imports of intermediate capital, *ceteris paribus*.

The openness factors \(S_i\) decrease (thus reducing \(x_{iit}\) and \(x_{ikt}\)) as trade barriers \((\tau_{ik})\) rise and this effect is the larger the higher the technology levels of the trade partners \((\theta_k)\) and the larger the contribution of capital goods in aggregate production \((\gamma)\). Keeping trade policies fixed, each \(S_i\) increases if a country itself or its trade partners become more developed \((\theta_k\) increase). If there are no trade distortions \((\tau_{ik} = 1\) for all \(i, k))\), then all \(S_i\) are equal and reflect the impact of other country-specific asymmetries in (8) and (10).

The aggregate output solution (10) shows that, in addition to institutional and other exogenous factors \((\psi_i)\), a country’s total factor productivity depends on its openness to technology imports \((S_i)\) and the world technology level \((A_1)\); as the degree of complementarity among capital goods \((\phi)\) increases, the contribution of openness becomes ever more important.

Analogously to EHR (1998) and HTR (2002), we assume that the development of the \((j_i)\)th capital commodity in country \(i\) costs \(v_i^2 p_i^*j_i^*\) \((\zeta, \xi > 0)\) units of aggregate consumption. According to this specification, later innovations are more costly but, owing to capital complementarity, they are also more valuable. Parameters \(v_i\) reflect any country-specific factors that impact research productivity across an economy (variation in human capital, institutions, and policies toward innovative activities); \(\zeta\) determines the dispersion of the innovation costs across nations.

The extent of innovation per time period is determined by the zero profit condition of the monopolistically competitive producers (equating the discounted monopoly rents from the last capital variety (obtained using (5), (6) and (8)) to the cost of invention):

\[
v_i^2 p_i^* j_i^*/\theta_i^* A_{1t}^\zeta = \sum_{s=0}^{\infty} (1 + r_{t+s})^{-(s+1)} \pi_{i,t+s},
\]

(11)
where

\[ \pi_{i,t} = (1 - \gamma) R_t \left[ \sum_{k=1}^{N} x_{ki} \right] = k_i \Omega A^c_t (r_t p^*_{it})^{-\alpha}, \quad (12) \]

\[ k_i(\theta; \tau, \hat{L}) \equiv \sum_{j=1}^{N} \hat{L}_j S^c_j \tau^{-\xi} \gamma_{ji}, \quad (13) \]

and \( \Omega \equiv (1 - \gamma)^{\gamma \alpha - \alpha}. \) By (12), profitability of invention in all countries is symmetrically affected by the interest rate \((r_t)\), the opportunity cost of general purpose capital \((p^*)\), and the world technology level \((A_1)\). Asymmetries appear through multipliers \(k_i\) that take into account the negative impact of tariff barriers against a country’s exports \((\tau_{ji})\) but increase in the productive size \((\theta_j)\) and openness \((S_j)\) of each export market. If all trade is free \((\tau_{ji} \equiv 1 \text{ for all } i, j)\), then

\[ k_i(\theta; \hat{L}) \equiv \left( \sum_{j=1}^{N} \hat{L}_j \right) \left( \sum_{j=1}^{N} \theta_j \right)^{\xi}, \quad i = 1, ..., N, \quad (14) \]

i.e., profitability of innovation depends on the productive size of the world market (the first term on the right-hand side of (14)) and other asymmetries that affect the levels of technological development in the world (the second term in (14)).

We obtain the opportunity cost of general purpose capital from the world trade-off between final consumption and aggregate capital. Taking aggregate consumption and foregone consumption to be freely traded, we express a world production possibility frontier that represents the desired trade-off as follows:

\[ Y^w_t = C^w_t + Z^w_t \Gamma \left( Z^w_{t+1} - Z^w_t \over Z^w_t \right). \quad (15) \]

According to (15), the total output of final consumption in the world \((Y^w_t)\) is either consumed \((C^w_t)\) or invested in aggregate capital \((Z^w_t \cdot Z^w_{t+1})\), the total investment in general purpose capital being \(Z^w_t\) in time period \(t\). The convex cost function \(\Gamma\) for new investment appears in (15) because of the sectoral structure of production in the model. Technology for the production of final consumption has been specified in (3) but production functions for capital goods production and innovation have not been explicitly stated (these technologies are assumed to differ from (3)); instead of writing down the latter two production functions, we apply, as a short cut, the cost function \(\Gamma\) which yields

\[ p^*_t = -dC^w_t / dZ^w_{t+1} = \Gamma' \left( Z^w_{t+1} - Z^w_t \over Z^w_t \right). \quad (16) \]

Assuming that the technologies for capital goods production and innovation are the same (so that there is a linear production possibility frontier between these activities), the total investment in aggregate capital is obtained as the sum of capital goods production \((K^w_t)\) and innovation \((I^w_t)\):\(^8\)

\[ Z^w_t = K^w_t + I^w_t = \quad (17) \]

\(^8\)If technologies in all three production sectors are the same, then the production possibility frontier (15) is replaced by the accumulation equation \(Y^w_t = C^w_t + (Z^w_{t+1} - Z^w_t)\). In this case, \(p^*_t \equiv 1\) for all \(t.\)
\[
\sum_{i=1}^{N} \left( \sum_{k=1}^{N} A_{kt} x_{kit}(jk) d_jk \right) + \left( \int_{0}^{A_{1t}} \sum_{i \neq 1} \frac{v_{1j}^i \delta_{ij}}{\sum_{i=1}^{N} \theta_{ij}} \right) = A_{1t+1}^{1+\xi} \left[ \left( \frac{R_t}{\alpha} \right) \sum_{i=1}^{N} \theta_{ij} k_i \right] + \frac{1}{1+\xi} \left( \sum_{i=1}^{N} v_{1j}^i \theta_{1i+1}^{1+\xi} \right).
\]

### 2.2 Balanced Growth:

In a long run equilibrium, the interest rate and the opportunity cost of aggregate capital remain constant while technology, output, aggregate capital and consumption grow at a constant rate. For technology, the growth rate is defined as

\[ g_A \equiv \frac{A_{1,t+1}}{A_{1t}}. \]  

Then, by (17), aggregate capital grows at the rate

\[ g_z = (g_A)^{1+\xi}, \]  

and, given (10), aggregate output must grow at this same rate as well. Due to (2) and (15), consumption also grows at rate \( g_z \).

Applying (18) in (11) and (12) yields equations

\[ g_A = \left[ 1 + r - \tilde{k}_i \Omega_r \frac{1}{\pi^+} (p^+)^{\frac{1}{1+\xi}} \right]^\frac{1}{1+\xi}, \]  

where

\[ \tilde{k}_i(\theta; \tau, \tilde{L}, v) \equiv \frac{k_i(\theta; \tau, \tilde{L})}{v_i^k \theta_{i+1}^k}, \quad i = 1, \ldots, N. \]  

Assuming that all countries innovate and reflecting the common rate of technology growth, multipliers \( \tilde{k}_i \) in (20) must satisfy the equilibrium conditions

\[ \tilde{k}_1(\theta; \tau, \tilde{L}, v) = \tilde{k}_i(\theta; \tau, \tilde{L}, v), \quad i = 2, \ldots, N. \]  

Further, substituting (19) into (20) and (2) and taking into account (16) yields equations

\[ g_z = \left[ \frac{1}{\pi^+} (p^+)^{\frac{1}{1+\xi}} \right]^{1+\xi}, \]  

Equations (22) - (25) jointly determine the steady state values of \( (g_z, r, p^+) \) and \( (\theta_2, \ldots, \theta_N) \), given the exogenous parameters \( (\tau, \tilde{L}, v) \). The equilibrium rate of technological advance \( (g_A) \) is obtained using (19).

Technological attainment can differ country by country in the long run as determined by the \( \theta_i \) solutions to (22). By (20) and (22), levels of innovation activity adjust, depending on all policy and other asymmetries, so as to maintain equal (zero) profitability of innovation everywhere.\(^9\)

\(^9\)If countries are symmetric, (22) gives \( \theta_i = 1 (= \theta_1) \) for all \( i = 2, \ldots, N \).
Given the steady state values of all \( \theta_i \), equations (21) determine the value of the common growth multiplier \( \tilde{k}_1 \) that appears in (25); this multiplier transmits the impact on growth of any asymmetries in trade policy (\( \tau \)), country size (\( L \)), total factor productivity (\( \psi \)), or cost of innovation (\( v \)). Due to the separability of equations (22) from the rest of the equilibrium conditions, the value of the growth multiplier can be solved using equations (22) and (21) only.\(^1\) Given (25), equations (24) and (23) determine the impact of asymmetries on the equilibrium values of \( p^z \) and \( r \).

Differences in \( \theta_i \) partly determine equilibrium levels of aggregate output since, by (10),

\[
y_i = \frac{Y_i}{L_i} \Rightarrow y_i = \left( \frac{S_i(\theta, \tau)}{S_j(\theta, \tau)} \right)^{1+\xi}, i, j = 1, ..., N. \tag{26}
\]

Thus, a country’s relative aggregate output per effective unit of labor \( (y_i/y_j) \) is the larger the more open the country’s trade policy and the more advanced its trade partners (as indicated by the relative openness factor \( S_i(.)/S_j(.) \)). Further, by (26), for any \( \tau \) and \( \theta \), the higher the degree of complementarity of capital goods, the larger the dispersion in aggregate output per effective unit of labor across countries.

Total expenditures and revenues measured in aggregate consumption equalize worldwide in every time period (equation (15)) so that

\[
C^w = Y^w - p^w Z^w(g - 1). \tag{27}
\]

Country by country balances of payments are maintained by allowing consumption expenditures to adjust to reflect trade in capital goods, i.e.,

\[
C_{it} = Y_{it} - p^w_i Z_{it}(g - 1) + A_{1i}R_t \left( \sum_{k \neq i} x_{ki} - \sum_{k \neq i} \theta_k x_{ik} \right), \tag{28}
\]

where capital investment \( Z_{it} \) is defined using the expression for \( Z^w = \sum_{i=1}^N Z_{it} \) in (17). In a balanced growth equilibrium, equations (28) remain satisfied over time.

Lifetime utility at a steady state \( (U_i) \) is determined by the rate of long run growth and the level of consumption at the initial time period \( (C_{i0}) \). In particular, using equation (1),

\[
U_i = \frac{C_{i0}^{1-\sigma} (1 - \beta g^1_{i-\sigma})^{-1}}{1-\sigma}, i = 1, ..., N. \tag{29}
\]

Figure 8 illustrates a configuration of steady states corresponding to equilibrium conditions (22) - (25). Curve CC graphs the \( r \) and \( g_z \) combinations that satisfy (23), and curve TT represents the \((r, g_z)\) at which (25) is maintained, given \( p^z \) from (24) and \( k_1(\theta) \) from (21) and (22). The slope of the TT curve reflects the complementarity parameter \( (\phi) \) and the sensitivity of the opportunity cost of capital to changes in aggregate investment \( (\Gamma^n) \); see the

\(^1\)Equations (22) are separable from other equilibrium conditions because depreciation of capital goods has been excluded from the model. If depreciation were included, the \( \theta_i \) would appear in (24).
Appendix. Long run equilibria are found at the intersections of the CC and TT curves (e.g., $E_1$, $E_2$ and $E_3$ in Figure 8). At each steady state, depending on structural heterogeneities, several long run income clubs may exist. By (22) and (26), dispersion in innovation and output across such clubs is influenced by the relative profitability of innovation and the relative openness of each country group.\footnote{\textit{\textsuperscript{11}}}.

\subsection*{2.3 Stability:}

While the model, as stated, does not yield explicit trajectories for quantity variables over time, we can apply learning dynamics so as to determine the stability properties of its solutions. In such dynamics, decision makers hold expectations about future growth, make consumption and investment plans based on expectations and, having observed the actual rate of growth, adjust expectations for the subsequent time periods. Through dynamic adjustments steady states that are stable under adaptive learning are approached over time. Without discussing the specifics of learning dynamics but assuming that all decision makers hold same expectations and employ identical adjustment rules (homogenous learning), we state the following Lemma:\footnote{\textit{\textsuperscript{12}}}

Lemma 1 A solution $(r^*, g^*_z)$ for equations (22) - (25) is a stable equilibrium under adaptive learning if

$$\frac{\partial r_z(g^*)}{\partial g} - \frac{\partial r_c(g^*)}{\partial g} < 0,$$

where $r_z(g^*)$ and $r_c(g^*)$ are the implicit solutions for the interest rate obtained using (22) and (24) - (25) (the TT curve) and (23) (the CC curve), respectively. A solution to equations (22) - (25) is unstable if expression (30) is positive.

In Figure 8, a stable steady state is characterized by a TT curve that cuts the CC curve from above (equilibria $E_1$ and $E_3$). Subsequent results are developed assuming stability under adaptive learning.

\subsection*{3 A Special Case: Free Trade}

Removing all trade distortions ($\tau_{ij} = 1$ for all $i, j = 1, ..., N$) yields an illustrative example. If, in particular, $n_1$ nations (countries 1, 2, ..., $n_1$) are symmetric with parameters $\left(\hat{L}_1, v_1\right)$, while the remaining $n_2$ ($\equiv N - n_1$) countries each possess the parameters $\left(\hat{L}_2, v_2\right)$, then, due to symmetry, $\theta_i = 1$ for 1, 2, ..., $n_1$ and $\theta_j = \theta$ for $j = n_1 + 1, ..., N$. By (22)

$$\theta(v) = \left(\frac{v_1}{v_2}\right)^\frac{1}{\theta},$$

and so the relative technology level of the two country groups is determined by the ratio of the innovation costs; aggregate productivity and the size of markets ($\hat{L}$) have no impact because all innovators have equal access to all markets.

\footnote{Growth bifurcations may also occur when multiple equilibria exist, see HTR (2002).} \footnote{Heterogeneous dynamics are discussed in Honkapohja, Turunen-Red and Woodland (2009). Lemma 1 of HTR (2002) (all countries symmetric) is a special case of the present Lemma 1.
The lower the cost ratio \((v_1/v_2)\), the less countries \((n_1 + 1), ..., N\) innovate. If \((v_1/v_2) < 1\), the position of the high cost countries is the weaker the larger the dispersion of the innovation costs \((\zeta)\) and the lower the degree of complementarity among capital goods \((\phi)\). The latter effect can be understood by considering profitability of innovation: when capital goods are close complements \((\phi and \xi high)\), the value of each innovation is supported by all other capital varieties and asymmetries in innovation costs are less damaging.

Differences in total factor productivity and the size of each country \((\tilde{L})\) determine the relative output between the country groups:

\[
\frac{Y_i}{Y_j} = \frac{\tilde{L}_1}{\tilde{L}_2}, \quad i = 1, ..., n_1, \quad j = n_1 + 1, ..., N.
\]  

(32)

Technology levels do not matter because, given free trade, each country has equal access to all capital goods \((S_1(\theta) = S_2(\theta)\) in (26)).

Given (14) and (31), the growth multiplier equals

\[
\hat{k}_1(\tilde{L}, v; n_1, n_2) = \left( n_1 \tilde{L}_1 + n_2 \tilde{L}_2 \right) \left[ \frac{n_1}{v_1^I} + \frac{n_2}{v_2^I} \right]^{\xi}.
\]  

(33)

The productive size of the world market defines the first term on the right-hand side of (33) while innovation costs, size of each country group, and the degree of complementarity among capital goods are reflected in the second term.\(^\text{13}\)

**Proposition 2** At a stable free trade equilibrium, growth speeds up if the growth multiplier increases.

Thus, using (33), expected conclusions are obtained: without trade distortions, long run growth is the faster the larger the world market and the lower the innovation costs everywhere. When there are asymmetries in innovation costs, complementarity of capital goods reduces the adverse growth impact of high costs whereas larger dispersion of such costs lowers the equilibrium rate of growth.

In Figure 8, an increase in the growth multiplier corresponds to an upward shift in the TT curve (dashed curve) whereby growth speeds up near stable equilibria \((E1 and E3)\). The following numerical example further illustrates the relative positions of the two country groups.

**Example 1:** Assume that \(\alpha = 0.39, \phi = 1.12, \beta = 0.90, \sigma = 0.22, \tilde{L}_1 = 0.0092, v_1 = 1, \zeta = 1.1; N = 3\) with \(n_1 = 2, n_2 = 1\); and \(\Gamma'(x) = \theta x^2 + 0.2\). The first row of Table 1A gives the symmetric free trade solution.

**TABLE 1A:** \(\tilde{L}_2 = 0.0092\)

<table>
<thead>
<tr>
<th>(v_2)</th>
<th>(\theta)</th>
<th>(Y_1/Y_2)</th>
<th>(Z_1/Z_2)</th>
<th>(C_1/C_2)</th>
<th>(r)</th>
<th>(g_z)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
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<td>0.132</td>
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<tr>
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<td>0.10</td>
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<td>9.65</td>
<td>1.35</td>
<td>0.129</td>
<td>1.075</td>
</tr>
</tbody>
</table>

\(^{13}\)All proofs can be found in the Appendix.
TABLE 1B:  \( \tilde{L}_2 = 0.007 \)

<table>
<thead>
<tr>
<th>( v_2 )</th>
<th>( \theta )</th>
<th>( Y_1/Y_2 )</th>
<th>( Z_1/Z_2 )</th>
<th>( C_1/C_2 )</th>
<th>( r )</th>
<th>( g_z )</th>
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</thead>
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</tr>
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<td>2.77</td>
<td>1.59</td>
<td>0.122</td>
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<td>1.31</td>
<td>9.65</td>
<td>1.97</td>
<td>0.119</td>
<td>1.033</td>
</tr>
</tbody>
</table>

Variations in technology levels (\( \theta \)), output ratios (\( Y_1/Y_2 \)), and growth (\( g_z \)) in Tables 1 reflect solutions (31) and (32) and Proposition 2, respectively. Aggregate investment ratios (\( Z_2/Z_1 \)) show a strong negative effect on investment from high innovation costs (\( v_2 \)). Relative consumption too, decreases when innovation costs are high and more so when the domestic economy is less productive (\( C_2/C_1 \) when \( \bar{L}_1 = \tilde{L}_2 \) (Table 1A) and when \( \tilde{L}_2 < \bar{L}_1 \) (Table 1B)). This occurs because the high cost economy (\( v_2 \)) produces less aggregate consumption and becomes a net importer of capital goods in exchange for net exports of aggregate consumption; the domestic cost of the capital imports increases as productivity in the \( Y \)-sector decreases.\(^{14}\) By (29), the utility ratio \( U_2/U_1 \) reflects variations in the consumption ratio \( C_2/C_1 \); all countries are best off at the symmetric free trade solution (first row of Table 1A) at which output and consumption are the highest.

4 Effects of Asymmetric Trade Policy:

If all international exchanges are free from distortions, country-specific asymmetries in technology costs, total factor productivity, and market size determine relative levels of technology, output, and consumption between countries and also impact the common rate of long run growth. Effects of trade policy can be determined by first analyzing a simple customs union (\( U \)) that includes \( n_1 \) nations (countries \( i = 1, 2, \ldots, n_1 \)) while the remaining \( n_2 \) (\( \equiv N - n_1 \)) countries (group \( R \)) form the rest of the world. All countries in \( U \) are assumed symmetric with parameters (\( \bar{L}_1, v_1 \)); these countries all impose a uniform tariff \( \tau_U > 1 \) against imported capital goods from \( R \) but maintain free trade within the union. Countries in \( R \) are taken to be mutually symmetric as well, each possessing the parameters (\( \tilde{L}_2, v_2 \)), and they do not impose any tariffs.

4.1 Levels:

Due to symmetry, we may set \( \theta_i = 1 \) for all \( i \in U \) and \( \theta_j \equiv \theta(\tau_U, \bar{L}, v) \) for all \( j \in R \).

Proposition 3 The long run relative technology level of countries \( R \) is an decreasing function of the union tariff (\( \tau_U \)), the ratio of innovation costs (\( v_2/v_1 \)), and the relative productivity and size of the customs union (\( n_1\bar{L}_1/n_2\tilde{L}_2 \)).

By Proposition 3, the customs union share of all innovation is the larger the higher the union tariff against imported capital goods. This is because

\(^{14}\)Hsieh and Klenow (2007) emphasize the role of low productivity in consumer goods production as an explanation for low capital investment in poor countries.
relative profitability of inventions improves as the customs union becomes more protective.

As under free trade, the relative innovation shares of the country groups depend on innovation costs, with less new technology being developed where these costs are higher. However, given a positive union tariff ($\tau_U > 1$), the $\theta$ solution is not solely a function of innovation costs as in (31); aggregate productivity and size of each country ($\tilde{L}_1/\tilde{L}_2$) also matter because, given the union tariff, access to all markets is not symmetric. Analogously, as the membership in the customs union ($n_1/n_2$) increases, stimulating innovation in $U$, technology share of the rest of the world declines.

Further, according to Proposition 3, all disadvantages of the rest of the world (higher innovation costs, less productive and smaller economies, fewer countries in this group) exacerbate each other and work in the same direction as the union tariff. Thus, trade barriers against imports of capital goods from $R$ tend to maintain relatively lower technological attainment in this group even as its other disadvantages may improve. However, if the customs union is the country group that suffers from the other asymmetries, by protecting its innovative sector countries $U$ can raise their relative share of all innovation.

The presence of the trade distortion does not alter the effect of complementarity ($\phi$) or the dispersion of innovation costs ($\varsigma$) on the $\theta$ solution: given all other parameters and tariff $\tau_U$, higher complementarity among capital goods and a lower degree of cost dispersion raise the relative technology level of countries $R$ (see the Appendix).

The relative aggregate output of countries in the customs union equals

$$\frac{Y_i}{Y_j} = \left(\frac{\tilde{L}_1}{\tilde{L}_2}\right) \left(\frac{n_1 + n_2\theta \tau_U}{n_1 + n_2\theta}\right)^{1+\xi}, \quad i \in U, \quad j \in R. \tag{34}$$

This ratio differs from the free trade expression (32) by the second multiplicative term that takes into account the relative openness of the two country groups. While an increase in tariff $\tau_U$ tends to reduce the output ratio, an increase favoring the customs union may also occur; this is because the technology share of the rest of the world ($\theta$) declines as $\tau_U$ is raised, thus reducing the negative impact of smaller capital imports within the union. However, a higher tariff wall unambiguously increases the total investment in capital and innovation in countries $U$, i.e., ratio $Z_i/Z_j$ must increase as $\tau_U$ becomes larger.

**Proposition 4** i) When tariff $\tau_U$ is sufficiently small (close to one), the output ratio $Y_i/Y_j$ is a decreasing function of tariff $\tau_U$. ii) Keeping $\tau_U$ fixed, the ratio $Y_i/Y_j$ increases if the customs union expands ($n_1/n_2$ increases), the productive size of union members ($\tilde{L}_1/\tilde{L}_2$) grows, or the relative innovation cost in the rest of the world ($\nu_2/\nu_1$) increases.

**Proposition 5** The total investment ratio $Z_i/Z_j$ is an increasing function of tariff $\tau_U$.

While Proposition 4 makes clear the negative effect of the tariff on the relative output of the customs union (at least when the tariff is sufficiently low), changes in the opposite direction result if the union becomes more productive or experiences a reduction in its relative innovation cost; entry of additional
members in the customs union (expanding free access to innovations from elsewhere) also yields a positive effect on the output ratio. In the latter case, the customs union may attain a sizable increase in its innovation share with small (if any) cost in relative aggregate output.

Example 2: Assume that $\alpha = 0.39$ and let $\xi$ and $\gamma$ take the values stated in the tables. The customs union is formed by two countries in Tables 2A and 2B ($n_1 = 2, n_2 = 1$); Tables 2C and 2D show the effect of a unilateral tariff ($n_1 = 1, n_2 = 2$).

TABLE 2A: $\xi = 0.16, \gamma = 0.35$  TABLE 2B: $\xi = 0.10, \gamma = 0.37$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\theta$</th>
<th>$Y_1/Y_3$</th>
<th>$\tau$</th>
<th>$\theta$</th>
<th>$Y_1/Y_3$</th>
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<tbody>
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<td>1.1</td>
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<td>1.2</td>
<td>0.33</td>
<td>0.98</td>
<td>1.2</td>
<td>0.16</td>
<td>0.99</td>
</tr>
<tr>
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<td>0.14</td>
<td>0.99</td>
<td>1.4</td>
<td>0.04</td>
<td>0.99</td>
</tr>
</tbody>
</table>

TABLE 2C: $\xi = 0.16, \gamma = 0.35$  TABLE 2D: $\xi = 0.10, \gamma = 0.37$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\theta$</th>
<th>$Y_1/Y_i$</th>
<th>$\tau$</th>
<th>$\theta$</th>
<th>$Y_1/Y_i$</th>
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<td>0.96</td>
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<td>0.62</td>
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<td>0.91</td>
<td>1.4</td>
<td>0.23</td>
<td>0.94</td>
</tr>
</tbody>
</table>

4.1.1 Growth:

In addition to changes in the level variables, the establishment of a customs union also alters the world long run growth rate ($g_Z$). As in Proposition 2, the direction of this change is determined by the growth multiplier $\tilde{k}_1$ in (25). The growth multiplier is a decreasing function of the tariff, which yields

Proposition 6 An increase in the external tariff imposed by the customs union is followed by a reduction in the world rate of growth. The reduction in growth is the larger the higher the union tariff.

According to Proposition 6, compared to symmetric free trade, the world economy will grow more slowly if a customs union forms, and the negative growth effect is monotonically increasing in the size of the union tariff.

Figure 8 again illustrates. The downward shift of the TT curve from its upper position (the dashed curve) to the position denoted by TT (the solid curve) depicts the negative impact of the union tariff on the profitability of innovation in the long run. The corresponding shift in (stable) equilibria $E_1$ and $E_3$ toward lower values of $g_Z$ in the figure shows the effect on growth. If there are multiple equilibria, a downward bifurcation may occur. In that case, the reduction in growth could be large.

Example 3 illustrates with some numerical calculations.

Example 3: Assume that $\alpha = 0.39, \beta = 0.90, \sigma = 0.22, \tilde{L} = 0.01, v_1 = v_2 = 1$ and $N = 3$. The customs union is formed by two countries in Tables

\[^{15}\text{The proof of Proposition 7 is in Appendix 2.}\]
3A and 3B \((n_1 = 2, n_2 = 1)\); Tables 3C and 3D show the effect of a unilateral tariff \((n_1 = 1, n_2 = 2)\).

**TABLE 3A:** \(\xi = 0.16, \gamma = 0.35\)  **TABLE 3B:** \(\xi = 0.10, \gamma = 0.37\)

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>(\hat{k}_1)</th>
<th>(gz)</th>
<th>(\tau)</th>
<th>(\hat{k}_1)</th>
<th>(gz)</th>
</tr>
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<td>1.1</td>
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<td>1.08</td>
<td>1.4</td>
<td>0.0322</td>
<td>1.05</td>
</tr>
</tbody>
</table>

**TABLE 3C:** \(\xi = 0.16, \gamma = 0.35\)  **TABLE 3D:** \(\xi = 0.10, \gamma = 0.37\)

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>(\hat{k}_1)</th>
<th>(gz)</th>
<th>(\tau)</th>
<th>(\hat{k}_1)</th>
<th>(gz)</th>
</tr>
</thead>
<tbody>
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<td>1.1</td>
<td>0.033</td>
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</tr>
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<td>1.2</td>
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<td>1.09</td>
<td>1.2</td>
<td>0.032</td>
<td>1.06</td>
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<td>1.4</td>
<td>0.033</td>
<td>1.07</td>
<td>1.4</td>
<td>0.031</td>
<td>1.05</td>
</tr>
</tbody>
</table>

All Tables 3 show the increasingly negative effect of a customs union on long run growth as the union becomes more protectionist. The reduction in growth is smaller (albeit from a lower initial level of growth) when capital goods are less complementary to each other (Tables 3B and 3D versus Tables 3A and 3C). This is interesting because Tables 2B and 2D above show that the reduction in the relative technology level of the rest of the world is likely to be more severe (with the customs union correspondingly gaining more in its technology share) when the degree of complementarity is smaller.

### 4.2 Expanding Customs Union

The size of the customs union appears to have little effect on growth in Tables 3 (Tables 3A and 3B versus Tables 3C and 3D) although one may expect, in the light of Proposition 6, that the expansion of a customs union should slow down growth. However, in general, the conclusion cannot be clear-cut. This is because the inclusion of additional members in the customs union is followed by two types of growth effects. First, by joining a customs union, a new member attains free access to markets where its exports previously were subject to a tariff. This new market access expands trade (trade creation) and, as a result, innovation becomes more profitable. Therefore, the expansion of the customs union partly speeds up growth (growth creation). On the other hand, a trade diversion effect also exists because the new member country must raise tariffs against the rest of the world. This limits access to innovations developed in the rest of the world and so the profitability of innovation within the customs union is reduced; growth must therefore slow down (growth diversion). The relative size of the two opposite effects determines whether the expansion of a customs union raises or lowers growth in the long run.

The growth creating effect of an expanding customs union is likely to be the larger the higher the union tariff (because of the larger significance of new market access) and the lower the level of innovation elsewhere (because then innovation already largely takes place within the customs union). Thus, because by Proposition 3 an increase in the union tariff lowers the relative technology share of the rest of the world, a customs union that raises its tariff wall as it
expands is more likely to enhance long term growth. On the other hand, the negative growth diversion effect is likely to be the larger the more the expanding customs union reallocates innovation toward itself (the more the value of $\theta$ decreases as the union becomes larger) and the larger the rest of the world ($n_2$) that is subject to the relocation of innovation. The growth diversion effect is likely to be decreasing in the tariff imposed by the customs union (because when the union tariff is high most of innovation already takes place within the customs union). Growth diversion may also occur more frequently if the degree of complementarity among capital goods is low because such complementarity increases the value of each invention and thus tends to stimulate innovation.

An example of the growth diversion effect appears in Tables 3B and 3D when $\tau = 1.1$ (relatively low union tariff) and $\phi = 1.06$ (low degree of complementarity). As the customs union expands (from $n_1 = 1$ in Table 3D to $n_1 = 2$ in Table 3B), growth rate ($g_Z$) decreases from 1.07 to 1.06. Tables 2B and 2D indicate that the corresponding reduction in the relative technology level of the rest of the world is rather large (from 0.62 in Table 2D when $n_1 = 1$ to 0.37 in Table 2B when $n_1 = 2$); furthermore, since $n_2 = 2$ in Table 2D, the rest of the world that is subject to the reallocation of innovation is also relatively large ($N = 3$). Tables 4 give an example in which the growth creation effect dominates; here, the customs union...
total factor productivity) and the potential for such gains may further motivate trade distortions. Along these lines, Propositions 3-7 suggest that imports of capital goods are more likely to be restricted in countries that are large and highly productive; such restrictions are also likely to be directed against smaller and less productive nations.

The above Proposition 6 (a higher tariff in a customs union lowers growth) and the contrasting ambiguity of the growth and welfare consequences of changes in the size of customs unions are consistent with the mixed empirical evidence of Berthelon (2004) and others regarding the growth implications of PTAs. While we can expect that the formation of a customs union reduces long term growth when compared to the free trade outcome (Proposition 6), there is little reason to believe that the growth consequences of an additional country’s joining an existing PTA should be uniformly positive or negative for any country in the short run or the world in the long run. The overall impact depends on the relative magnitude of growth creating and growth diverting effects and these, in turn, depend on the size of the markets involved, the tariff barrier imposed by the PTA, and the reallocation of innovation that follows the adjustment in the PTA membership.

Notably, Berthelon (2004) found strong evidence supporting the importance of new market access as fostering growth (growth creation). Berthelon’s observation that "North-North" PTAs are more likely to speed up growth and "South-South" PTAs may even lower growth are also roughly consistent with the above discussion (growth creation is more likely when highly innovating countries remove mutual barriers and the relative level of innovation in the rest of the world is low; a customs union of countries in which the level of innovation is low is more likely to reduce growth since access to capital goods from the more advanced rest of the world is further limited).

5 Retaliation: (Another Customs Union)

Countries outside the customs union may retaliate by raising a tariff wall of their own. Suppose, as above, that countries $1, 2, ..., n_1$ form customs union $U$ with an external tariff equal to $\tau_U (> 1)$ and let countries $(n_1 + 1), ..., N$ retaliate by forming a competing customs union $R$ which imposes the external tariff $\tau_R (> 1)$.

The presence of the retaliatory tariff does not alter the content of Propositions 4 and 6-7 above, i.e., a higher union tariff, $\tau_U$, implies slower growth in the long run, the relative technology level of countries in $R$ and the output ratio $Y_j/Y_1$ decrease if union $U$ expands, and the output ratio $Y_1/Y_j$ is a decreasing function of tariff $\tau_U$ near free trade. As for Proposition 3, the relative technology level of countries $R$ remains a decreasing function of $\tau_U$ even when retaliation occurs ($\tau_R > 1$) but condition (??) must be replaced by the weaker restriction

$$\theta \in \left( \frac{1}{\tau_U}, \frac{1}{\tau_R} \right)$$

that does not take into account the relative size of the markets in the two country groups (Appendix 3). However, condition (35) is still useful in that provides an upper limit for the technology ratio, $\theta$, that is larger than one. That is, tariff
retaliation by countries $R$ may raise the technology ratio above its free trade value ($\theta = 1$) even when $\tau_U > 1$.

The following proposition demonstrates that retaliation always improves the relative technology position of countries $R$ and that the aggregate output ratio $Y_j/Y_1$ and the long run growth rate will decline at least when the tariffs are not too high.\footnote{The proof of Proposition 8 is in Appendix 3.}

**Proposition 7** Let customs union $U$ impose an external tariff $\tau_U \geq 1$, and let $\tau_R \geq 1$ be the external tariff in countries $R$. Then: i) At any $\tau_U$, the long run technology level of countries $R$ is an increasing function of the retaliation external tariff, $\tau_R$. ii) Near free trade, the output ratio $Y_1/Y_j$, $j \in R$, increases in $\tau_R$ but, given $\xi < 1$ (equivalently, $\phi < 2 - \alpha$), the rate of long run growth decreases as $\tau_R$ becomes larger.

That the competing customs union $R$ gains a larger relative technology share by retaliating is not surprising since the retaliatory tariff raises profitability of innovation in countries $R$ and therefore attracts a larger proportion of future innovation there. Assuming that the tariffs remain low, Proposition 7 shows that the aggregate output ratio and the rate of steady state growth respond as may be expected. The condition on the complementarity parameter ($\phi < 2 - \alpha$) allows for a very high degree of complementarity between capital goods and is only a sufficient (but not necessary) condition.

Example 5 offers some numerical experiments.

**Example 5:** Assume that $\alpha = 0.39$, $\xi = 0.16$, $\gamma = 0.35$, $\beta = 0.90$, $\sigma = 0.22$, $\hat{L}_1 = \hat{L}_2 = 0.01$, $\nu_1 = \nu_2 = 1$ and $N = 3$. Customs union $U$ comprises two countries in Tables 5A and 5B ($n_1 = 2$, $n_2 = 1$); in Tables 5C and 5D, customs union $U$ includes only one country ($n_1 = 1$, $n_2 = 2$).

<table>
<thead>
<tr>
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<table>
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</table>

Example 5 expands the experiments of Tables 2 and 3 in which only one tariff was present: in Tables 5, the countries that do not belong to customs union $U$ retaliate. In Table 5A, countries $U$ choose the tariff $\tau_U \equiv 1.2$ and in Table 5B the tariff is higher ($\tau_U \equiv 1.4$). For completeness, the first row of both tables states the long run steady state values of $\theta$, $Y_1/Y_3$, and $g_Z$, without retaliation (this row is obtained from Tables 2 and 3).
When the retaliatory tariff ($\tau_R$) increases, the equilibrium level of innovation outside union $U$ increases in both Tables 5A and 5B (Proposition 7 (i)). Furthermore, illustrating a wider scope for Proposition 7 (ii), the output ratio $Y_1/Y_3$ increases with $\tau_R$ and the long run growth rate decreases for all tariffs considered in Tables 5A and 5B (not only near free trade). In this numerical example, therefore, the distortionary impact of the retaliatory tariff that works toward raising the output ratio and lowering the growth rate remains stronger than the opposite effect that appears when the allocation of innovation adjusts ($\theta$ increases when $\tau_R$ increases) for all tariffs considered and not only near free trade. Table 5B shows that a higher union tariff can have a considerable impact on the allocation of innovation (the relative technology ratio $\theta$ is much lower in Table 5B than in Table 5A) but the effects of the higher tariff on the output ratio and long term growth appear much smaller.

Tables 5C and 5D offer a comparison in which the customs union $U$ is smaller (unilateral tariff). According to Propositions 4 and 6, the equilibrium solutions for the technology level should be higher than in the corresponding Tables 5A and 5B and the output ratio $Y_1/Y_3$ in Tables 5C and 5D should be lower, and these effects are confirmed. Tables 5B and 5D also demonstrate the growth creating effect of a larger customs union: for each $\tau_R$, the value of $g_Z$ is larger in Table 5B (union $U$ includes two thirds of the world market) than in Table 5D (union $U$ contains one third of all markets). As predicted earlier, this effect is stronger in Tables 5B and 5D where the union tariff ($\tau_U$) is larger; in Tables 5A and 5C, growth creation dominates only when $\tau_R = 1$.

Tables 5 further emphasize the important role that the size of a customs union and its markets play in determining the impact of a PTA. Since union $U$ is smaller in Tables 5C and 5D than in Tables 5A and 5B, its tariff wall has much less effect on innovation in the rest of the world (the initial decline in $\theta$ in Tables 5C and 5D is smaller than in Tables 5A and 5B), and even a modest retaliation results in a significant correction in the equilibrium value of $\theta$. A small customs union is therefore much less likely to significantly gain in new innovation and, according to Tables 5C and 5D, the cost in reduced relative output for the union remains even as the rest of the world retaliates ($Y_1/Y_3 < 1$ in Tables 5C and 5D whereas $Y_1/Y_3$ significantly increases with retaliation in Tables 5A and 5B).

Overall, Propositions 3-8 indicate that, compared to a symmetric free trade equilibrium in which all (symmetric) countries innovate at the same rate and produce an equal amount of aggregate output, the establishment of a customs union will lead, in the long run, to the appearance of income clubs whose membership is defined by trade policy: members of the customs union will produce a relatively larger share of innovations in the world (with the effect being the larger the larger the customs union, the more productive its members, the larger the union markets and the less complementarity there is among capital goods) while supplying a relatively smaller share of the aggregate output (with this reduction in output being the smaller the larger the customs union). Countries that are not members of the customs union will see a reduction in innovation activity with the reduction being potentially large if the markets of these countries are small and total factor productivity low. While retaliation can increase the relative technology share of the nonmembers, it is likely to lower the rate of long run growth which affects all countries (not only the members of the customs union).
Reversing our viewpoint, we may reinterpret our results so as to consider the impact of unilateral and multilateral reductions of tariffs. By Propositions 7 and 8, unilateral tariffs and tariffs imposed by customs unions slow down growth. Conversely, therefore, reductions of unilateral or common tariffs in a customs union will speed up growth, assuming that no other distortions are present, and this is observed in Examples 3 and 5. In Tables 3 (with no retaliation), there is little difference in the improvement of the long term growth rate as the union tariff is reduced so that the number of countries participating in the tariff reduction does not appear to be significant. In Tables 5 in which a customs union faces retaliation by the rest of the world the increase in the steady state rate of growth is larger when both the union and retaliatory tariffs decline, i.e., here, a multilateral reduction of trade restrictions yields a larger boost in growth than a partial reduction. This conclusion is in the spirit of Rivera-Batiz and Xie (1992) who argued, using a knowledge-driven model of innovation, that an increase in the number of countries participating in a tariff reduction increases the likelihood of the effect on growth being positive.

6 Conclusions:

While there is a considerable body of literature on the analysis of customs unions and other forms of preferential trading agreements within a static framework, there has been surprisingly little written on their analysis within a dynamic, growth framework. This paper has the aim of contributing to the latter literature. Specifically, the paper has attempted to construct a model of growth that can then be used to consider the question of what happens to the rate of growth if a customs union is formed. Our model involves trade in machines produced by monopolistically competitive firms that innovate in the production of new machine varieties over time, thus generating growth. The formation of a customs union encourages innovation within the union due to the absence of tariffs on intra-union trade in machine services. On the other hand the common external tariff discourages innovation outside the union. The primary result is that the formation of a customs union reduced the world growth rate.

There are many more questions that need to be addressed, but these are left to another occasion. One of these concerns the effect of the formation of a union upon countries’ welfare. Another concerns the optimal formation of customs unions within a game-theoretic context. Finally, but not exhaustively, alternative innovation and trading contexts need to be formulated and analyzed.

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17Rivera-Batiz and Romer (1991) and Rivera-Batiz and Xie (1992) have shown that, in a knowledge driven model of innovation, the long run rate of growth is not necessarily a monotonic function of a tariff distortion.
7 Appendix:

The slope of the TT curve: The slope of the TT curve is obtained by differentiating equations (24) and (25):

\[
\frac{dr(g_z)}{dg_z} = \frac{1 + CD\Gamma''}{C(1 - E)},
\]

(36)

where

\[
C \equiv \left(1 + \frac{\xi}{\eta}\right) \left[1 + r - \Omega \hat{k}_1 r \frac{\varpi}{\varpi} (p^*) \frac{\varpi}{\varpi}\right]^{\frac{1}{2}} (\eta > 0), \quad \frac{1 + \xi}{\eta} \equiv \phi - \frac{\alpha}{\phi - 1},
\]

(37)

\[
E \equiv \frac{\hat{k}_1 \alpha \Omega}{\alpha - 1} r \frac{\varpi}{\varpi} (p^*) \frac{\varpi}{\varpi} (\eta < 0),
\]

(38)

\[
D \equiv \frac{\Omega \hat{k}_1}{\alpha - 1} r \frac{\varpi}{\varpi} (p^*) \frac{\varpi}{\varpi} (\eta < 0),
\]

(39)

\[
F \equiv \Omega r \frac{\varpi}{\varpi} (p^*) \frac{\varpi}{\varpi} (\eta > 0).
\]

(40)

The TT curve can have a negative slope if the numerator in (36) is negative. HTR (2002: Appendix) further discuss the slope of the TT curve in a symmetric two country model.

Proof of Proposition 2: Keeping \( r \) fixed, total differentiation of equations (24) and (25) yields

\[
\frac{dg_z}{d\hat{k}_1} = -\frac{CF}{1 + CD\Gamma''},
\]

(41)

where the multiplier expressions are defined in (37), (39) and (40). Expression (36) implies that (41) is negative (positive) when the TT curve slopes up (down). In Figure 8, these signs of (41) correspond to a global upward shift of the TT curve following any \( d\hat{k}_1 > 0 \) (dashed curve). Thus, assuming that the free trade steady state is stable, \( dg_z/d\hat{k}_1 > 0 \).

Proof of Proposition 3: Equations (22) give

\[
\left(n_1 + n_2 \theta \frac{\varpi}{\varpi} \frac{\varpi}{\varpi} \right)^{\xi} \left(v^* \frac{\varpi}{\varpi} \frac{\varpi}{\varpi} - \theta^* \right) = \frac{n_2 \hat{l}}{n_1} (\theta^* - v^*) (n_1 + n_2 \theta)^\xi,
\]

(42)

where \( \hat{l} \equiv \hat{L}_2 / L_1 \) and \( v \equiv v_1 / v_2 \). The left-hand side of equation (42) is a decreasing (negative-valued) function of \( \theta \), whereas the right-hand side is increasing in \( \theta \) (and negative-valued); see Figure 8 where the solution for (42) is denoted by \( \theta^* \).

Since curve LHS in Figure 8 shifts down when \( \tau_U \) increases, the \( \theta \) solution to (42) decreases as \( \tau_U \) increases. When \( l(n_2/n_1) \) increases, curve RHS shifts down and \( \theta^* \) increases. When \( v \) increases, curve LHS shifts up and RHS shifts down; thus \( \theta^* \) increases.

The \( \theta \) solution as a function of \( \xi \) and \( \zeta \): Equation (42) is equivalent to

\[
\frac{n_1 + n_2 \theta \frac{\varpi}{\varpi} \frac{\varpi}{\varpi} \frac{\varpi}{\varpi} \frac{\varpi}{\varpi}}{n_1 + n_2 \theta} = \left(\frac{n_2 \hat{l}}{n_1} \left(\frac{\theta^* - v^*}{v^* \frac{\varpi}{\varpi} \frac{\varpi}{\varpi} - \theta^*} \right)\right)^\frac{\xi}{\eta}.
\]

(43)
Given \( \theta \) and other parameters, if \( \xi \) increases, the right-hand side of (43) decreases and thus the left-hand side of (43) must also decrease. Accordingly, the \( \theta \) solution to (43) must be increasing in \( \xi \). (Because \( \gamma \phi = \alpha \), for a given value of \( \alpha \), an increase in \( \phi \) corresponds to a reduction in \( \gamma \). This change in the value of \( \gamma \) further increases the right-hand side of (43) as \( \xi \) increases so that the effect of a change in \( \xi \) is not altered.)

When \( \zeta \) increases, the right-hand side of (43) increases, and so the left-hand side must also increase; thus, the \( \theta \) solution decreases.\[ \]

**Proof of Proposition 4:** i) The derivative of the second multiplicative term in (34) equals (ignoring the power \((1 + \xi)\))

\[
\frac{\partial (\cdot)}{\partial \tau_U} = (n_2 \tau_U \frac{d\theta}{d\tau} + \frac{n_2 \theta_\gamma}{\gamma - 1} \frac{d\tau}{\tau_U}) (n_1 + n_1 \theta) - n_2 \frac{d\theta}{d\tau} (n_1 + n_2 \theta_\gamma \frac{d\tau}{\tau_U})
\]

\[
= (n_1 n_2 + n_2^2 \theta \frac{d\theta}{d\tau} (\tau_U^{-1} - 1) + n_2^2 \theta_\gamma \frac{d\tau}{\tau_U} + n_2^2 \theta^2 \gamma_\gamma \frac{d\gamma}{\gamma - 1} \frac{d\tau}{\tau_U})
\]

The first term in (45) is positive because \( d\theta / d\tau_U < 0 \) but the other terms are negative. Thus, when \( \tau_U = 1 \), or sufficiently near free trade, the derivative is negative.

ii) Using (34),

\[
\frac{Y_i}{Y_j} = \left( \frac{L_1}{L_2} \right)^{1+\xi} \left( 1 + \frac{\frac{n_1}{n_2} \theta_\gamma \frac{\tau_U^{-1} + 1}{1+\xi}}{1 + \frac{n_1}{n_2} \theta} \right), \quad i \in U, \ j \in R.
\]

Thus, if \( n_1/n_2 \) increases, the ratio \( Y_i/Y_j \) increases (by Proposition 3, \( \theta \) decreases when \( n_1/n_2 \) increases and \( \tau_U^{-1} \leq 1 \)). Similarly, when either \( L_1/L_2 \) or \( n_2/n_1 \) increases, \( Y_i/Y_j \) increases (since \( \theta \) decreases).\[ \]

**Proof of Proposition 5:** By (17),

\[
\frac{Z_i}{Z_j} = \left( \frac{K_i}{K_j} \right)^{-\frac{\tau_U^{-1} + 1}{1+\xi}} \frac{k_1 + \frac{n_1}{1+\xi} \theta k_2 + \frac{n_1^2 \theta_\gamma \tau_U^{-1}}{1+\xi} \theta k_2}{k_1 + \frac{n_1}{1+\xi} \theta k_2}, \quad i \in U, \ j \in R,
\]

where

\[
k_1 = n_1 L_1 S_1^\xi + n_2 S_2^\xi, \quad k_2 = n_1 L_1 S_1^\frac{1}{\tau_U^{-1}} + n_2 L_2 S_2^\xi,
\]

\[
S_1 = n_1 + n_2 \theta_\gamma \frac{1}{\tau_U^{-1}}, \quad S_2 = n_1 + n_2 \theta.
\]

When \( \tau_U \) is raised, \( S_1, S_2, k_1 \) and \( k_2 \) all decrease because \( \theta \) decreases. But, the reduction in \( k_2 \) is larger than the reduction in \( k_1 \) because \( \frac{1}{\tau_U^{-1}} \) decreases as well. The value of \( R_i \) also decreases as \( \tau_U \) increases; this occurs because the long run growth rate \( g_Z \) is reduced and thus the opportunity cost of capital, \( p_i^\xi \), is reduced (see subsequent propositions). Taking into account all changes, the denominator in (47) decreases more than the numerator as \( \tau_U \) increases and thus \( Z_i/Z_j \) must increase.\[ \]
Proof of Proposition 6: Using arguments analogous to the proof of Proposition 2 above, it can be shown that the TT curve in Figure 8 shifts up if the growth multiplier \( \hat{k}_1 \) increases. Further, because \( \hat{k}_1 = \frac{k_1}{\nu_1^k} \) and \( k_1 \) is defined in (?), we have \( \hat{k}_1/d\tau < 0 \) for all \( \tau \geq 1 \). Thus, the growth multiplier monotonically decreases as the union tariff increases.

According to results in Section 5, world steady states at which the TT curve cuts the CC curve from above are stable under adaptive learning. Then, following an increase in the union tariff and the corresponding downward shift of the TT curve, the \( g_Z \) solution at the new stable steady state must be lower than at the initial steady state. The reduction in the growth solution is the larger the higher the tariff.

Derivation of condition (35): Equation (50) in the proof of Proposition 3 is replaced by equation

\[
\left( n_1 + n_2\theta \tau_U^\frac{\hat{v}}{\nu} \right) \xi \left( \tau_U^\frac{\hat{v}}{\nu} - \theta^k \right) = \frac{n_2}{n_1} \left( \frac{\theta^k \tau_R^\frac{\hat{v}}{\nu} - 1}{\tau_U^\frac{\hat{v}}{\nu} - \theta^k} \right) \left( n_1 \tau_R^\frac{\hat{v}}{\nu} + n_2\theta \right), \quad \theta^k \neq \tau_U^\frac{\hat{v}}{\nu}. \tag{50}
\]

We can no longer conclude that the left-hand side of (50) is less than one. However, the left-hand side is positive and, therefore, the right-hand side of (50) is also positive. This gives condition (35).

Proof of Proposition 7: i) Equation (50) is equivalent to

\[
\left( n_1 + n_2\theta \tau_U^\frac{\hat{v}}{\nu} \right) \xi \left( \tau_U^\frac{\hat{v}}{\nu} - \theta^k \right) = \left( \frac{n_2}{n_1} \right) \left( \frac{\theta^k \tau_R^\frac{\hat{v}}{\nu} - 1}{\tau_U^\frac{\hat{v}}{\nu} - \theta^k} \right) \left( n_1 \tau_R^\frac{\hat{v}}{\nu} + n_2\theta \right). \tag{51}
\]

Given a \( \theta \) value that solves (51), an increase in the retaliatory tariff \( \tau_R \) reduces the right-hand side of (51). Therefore, the left-hand side of equation (51) must also decrease and, given a fixed value of \( \tau_U \), \( \theta \) must adjust. To determine the direction of change in \( \theta \), we obtain the derivative of the left-hand side in (51):

\[
\frac{d}{d\theta} \left( n_1 + n_2\theta \tau_U^\frac{\hat{v}}{\nu} \right) \xi \left( \tau_U^\frac{\hat{v}}{\nu} - \theta^k \right) = -2n_2\theta^k \tau_U^\frac{\hat{v}}{\nu} - n_1\theta^{k-1} + n_2\tau_U^\frac{\hat{v}}{\nu+1} \tag{52}
\]

The inequality follows because \(-2\theta^k + \tau_U^\frac{\hat{v}}{\nu+1} < 0\), and this follows using (35). Since the derivative (52) is negative, the value of \( \theta \) that solves (51) must increase as \( \tau_R \) increases.

ii) Using (26),

\[
\frac{\partial (Y_1/Y_j)}{\partial \tau_R} > 0 \iff \hat{S}_1 > \hat{S}_j, \tag{53}
\]

where \( \hat{S}_1 \equiv (dS_1/d\tau_R)/S_1 \) and \( \hat{S}_j \equiv (dS_j/d\tau_R)/S_j \), respectively. Since \( S_1 \) and \( S_j \) equal

\[
S_1 = n_1 + n_2\theta \tau_U^\frac{\hat{v}}{\nu}, \quad S_j = n_1 \tau_R^\frac{\hat{v}}{\nu} + n_2\theta, \tag{54}
\]

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condition (53) is satisfied if and only if

\[ n_2(\tau_R \tau_U - 1) \frac{\partial \theta}{\partial \tau_R} + \frac{\gamma \tau_R}{1 - \gamma} S_1 > 0. \]  

(55)

Thus, at free trade \((\tau_U = \tau_R = 1)\) and sufficiently near it, expression (55) is positive.

The growth multiplier equals \(\hat{k}_1 = \hat{L}(n_1 S_1^\xi + n_2 S_j^\xi \tau_R^{-1})/\nu^\xi_1\) which yields

\[ \frac{\partial \hat{k}_1}{\partial \tau_R} = \xi \left[ n_1 S_1^{\xi-1} \frac{\partial S_1}{\partial \tau_R} + n_2 S_j^{\xi-1} \frac{\partial S_j}{\partial \tau_R} \frac{\tau_R}{\gamma - 1} \right] + \frac{n_2 S_j^\xi \tau_R^{\gamma-1}}{\gamma - 1}. \]  

(56)

Given (54),

\[ \frac{\partial S_1}{\partial \tau_R} = n_2 \frac{\partial \theta}{\partial \tau_R} > 0, \quad \frac{\partial S_j}{\partial \tau_R} = \frac{n_1 \gamma}{\gamma - 1} \frac{\tau_R}{\gamma - 1} \]  

(57)

At free trade, \(S_1 = S_j = N\) and so (56) yields

\[ \frac{\partial \hat{k}_1}{\partial \tau_R} = \xi N^{\xi-1} n_2 \left[ N \frac{\partial \theta}{\partial \tau_R} + \frac{n_1 \gamma}{\gamma - 1} \right] + \frac{n_2 N^\xi}{\gamma - 1}. \]  

(58)

In order to obtain the sign of (58), we must evaluate \(\partial \theta/\partial \tau_R\) at free trade. Differentiation of equation (50) at free trade yields

\[ \frac{\partial \theta}{\partial \tau_R} = \frac{n_2}{N^\xi (1 - \gamma)} > 0 \]  

(59)

and substituting (59) into (58) gives

\[ \frac{\partial \hat{k}_1}{\partial \tau_R} = \frac{n_2}{\gamma - 1} (N^\xi - \xi n_2) + \frac{n_1 n_2 \xi N^\xi - 1}{\gamma - 1} < 0, \]  

(60)

if \(\xi < 1\).
8 Figures:

Balanced Growth States (Long Run Growth Equilibria)

Used in the Proof of Proposition 3
References


