Profit taxation of heterogeneous firms with provision of public infrastructure for differentiated goods

Abstract

This paper analyses the governments' optimal tax setting if firms producing in imperfect competition are heterogeneous and mobile between countries. A model of endogenous integration strategies of these firms depending on transport costs, tax rates levied, relative sizes of fixed costs, factor price differences, relative market shares and a firm's productivity is employed to derive the governments' optimal profit tax rates, whereas a government acts as a benevolent planner. Governments tax profits to be able to finance public infrastructure, which lets transport costs decline endogenously, whereas each government does not let its national welfare decline, because of taxation. Moreover the impact of profit taxation on economic outcome and firms' integration strategies depends on the state of trade and on investments.

JEL classification: F12, F23, H21, H49, I38, L22
Keywords: Heterogeneous Firms, Multinational Firms, Corporate Taxation, Public Infrastructure

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1 Introduction

Corporate expansion to foreign markets is a phenomenon which is known for at least two centuries. Early on, economic motives for cross-border activities of enterprises were mostly associated with the organization of supply networks to serve European markets from abroad. Since then, multinational enterprises (MNEs) have gained importance quite dramatically and the economic motives for corporate expansion to foreign markets have become way more complex than they were in old times. Firms now exploit comparative advantages across host countries and locations of plants to organize production networks across borders. New markets have emerged and economies of scale materialize so that manifold motives for cross-border firm organization can be distinguished.

New trade theory on multinational firms\(^1\) makes the case of the importance of the jointness of inputs in firm setup and transport cost savings which are associated with multi-plant production within MNEs. Such firms have gained importance quite obviously. For instance, between 1990 and 2001 the sales of foreign affiliates of MNEs grew a lot faster than goods exports. In 2001, foreign affiliates of MNEs earned 11% of world GDP and contributed 35% to world trade of goods.\(^2\)

As mentioned before, MNEs’ motives for foreign direct investments (FDI) and their modes of organization are quite diverse.\(^3\) Low-cost seeking versus market-seeking motives are important, but they rarely generate purely vertical or purely horizontal integration strategies of MNEs empirically.\(^4\) Recent theoretical work acknowledges complex integration strategies which combine vertical integration strategies as, e.g. associated with intermediate goods production and vertical slicing of production chains across international borders\(^5\) with horizontal ones such as export platform set-up.\(^6\)

Furthermore, a recent innovation to the literature is the incorporation of heterogeneous firms into the models of MNE activities. Due to international trade and

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\(^1\) Surveys of the literature include Markusen (2002) and Barba Navaretti and Venables (2004).
\(^2\) As in UNCTAD (2002).
\(^3\) Evidence herefore also is found in UNCTAD (1998).
\(^4\) Literature about vertical integration strategies includes Helpman (1984, 1985). Literature about horizontal integration strategies includes Markusen (1984), Markusen and Venables (1998) and empirical evidence that integration strategies are more complex include Hanson, Mataloni and Slaughter (2005) and Feinberg and Keane (2006).
\(^6\) As shown by Ekholm, Forslid and Markusen (2007).
FDI recent work introduces firms of heterogeneous productivity in models of imperfect competition. In previous models, firms were assumed to be identical. Allowing for heterogeneous firms gives rise to productivity differences as a reason of (ex-post) sorting of high-productivity firms into setting up foreign plants. Different integration strategies can arise to be optimal, whereas identical firms have the same preferences concerning their production locations. Some models concerning taxation with imperfect competition surely already exist, whereas most of them analyse governments’ taxation with identical firms. Recently also theoretical work concerning taxation with heterogeneous firms surfaced, claiming inefficiently low tax rates, which leads an underprovision of public goods, because of tax competition.

A single government is only interested in maximizing its own national welfare, but it does not take into account that its lower tax rate, which attracts the more productive firms, lowers tax revenues of countries with higher tax rates. For this reason also other governments can react with a lower tax rate and consequently, inefficiently low tax rates from a world welfare perspective emerge. This tax setting behavior with tax competition actually can be seen as a race-to-the-bottom scenario, due to profit-shifting activities of MNEs.

So, not only has the economic importance of MNEs risen in the last decades, but this development also influences governments’ decisions about corporate taxation. Surely, also other factors than taxation are important to firms concerning their location of corporate investments e.g. wage differences between countries, which suggests that governments have to be careful about their use. Nevertheless noting empirical studies corporate taxes are to be relevant for firms.

This chapter outlines a model of corporate taxation in the presence of MNEs, where it may be efficient for governments to use positive corporate taxation. Firms are heterogeneous in their productivity and engage in imperfect competition. They produce intermediate and final goods so that various simple as well as complex

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7 Examples are Melitz (2003), Ghironi and Melitz (2005) and Davies and Eckel (2007).
8 Examples in literature include Baldwin and Krugman (2004).
9 Davies and Eckel (2007) provide this approach. Further work to inefficiently low capital tax rates and underprovision of public goods due to tax competition is Haufler (2001).
10 As in Hines (1999).
11 As in Mintz (1999).
12 As in Hines (1999).
integration strategies may arise here.\textsuperscript{13} This is supported by empirical evidence mentioned above.

The model deals with two countries. The North is the home country of heterogeneous firms and the South is a country with a relatively lower wage level. Every single production activity of heterogeneous firms can be run in both countries.\textsuperscript{14} Also a simple setting with all production activities located in the North is possible. For this reason a heterogeneous firm must not necessarily be a multinational.

By assumption the wage level in the South is lower than in the North. For this reason, production costs for intermediate and final goods are lower there, too. However, to produce at cheaper costs in the South, firms have to invest to set-up a production plant there. These fixed costs do not occur when producing in the North and their size differs depending on shifting intermediate or final good production to the South. Yet, transportation can induce additional costs to a firm so that it may be optimal to avoid them at extra fixed costs associated with foreign plant set-up in the South. For this reason, the key parameters of the basic setting of this model are the size of transport costs, factor price differences, the relative size of fixed costs for MNE activities, the firms’ productivities and the market shares of final good consumption.\textsuperscript{15}

A host of factors influencing the choice of firms concerning their location of corporate investments is introduced into this model, which were already said to be important in general. The key factors to us are corporate taxes which influence firms’ decisions about their integration strategies.

Governments act as benevolent planners and can anticipate the impact of profit taxes on firms’ integration strategies. According, Northern and Southern tax rates are determined endogenously in this model and their impact on the integration strategies and welfare are discussed. Double taxation of profits is not possible in this setting.\textsuperscript{16}

By levying taxes, a government is only interested in its own national welfare, but combined tax revenue from both governments is used to finance public infrastructure,\textsuperscript{17} so that transport costs for goods produced by the heterogeneous firms decrease endogenously in the model, because of taxation. Even though tax

\textsuperscript{13} Concerning possible integration strategies of firms this model is very much based on Grossman, Helpman and Szeidl (2006).
\textsuperscript{14} As in Grossman, Helpman and Szeidl (2006).
\textsuperscript{15} As in Grossman, Helpman and Szeidl (2006).
\textsuperscript{16} An example here fore in literature is Egger et al. (2006).
\textsuperscript{17} An example for a model with public infrastructure analyzing its impact on trade patterns, industrial location and welfare is Martin and Rogers (1995), whereas their setting is totally different than this one and it does not deliver according optimal tax rates.
rate differentials can occur in this model, inefficiently low tax rates do not arise, because governments take the impact of their own tax rate on the foreign tax base into account.

This theoretical analysis is structured in the following way:
Section 2 outlines the model and derives optimal integration strategies of firms in the differentiated sector are derived. These depend on relative size of fixed costs for plant set-up, factor price differences, relative market size, firms' productivities and transport costs.
Subsequently, we introduce profit taxation and study its impact on the firms’ optimal integration strategies.
Section 3 concludes the analysis and illustrates impact of profit taxation on integration strategies and welfare.

2 Optimal profit taxation of heterogeneous firms with provision of public infrastructure

2.1 Some general features of the model set-up
The following partial analysis describes the optimal integration strategies of heterogeneous firms based on the theoretical framework of Grossman, Helpman and Szeidl (2006) with particular emphasis on the role of profit taxation.
This analysis studies the optimal tax policy of governments when providing public infrastructure for differentiated goods depending on the integration strategies chosen by heterogeneous firms.
We set-up a simple model with two countries, North (N) and South (S), where the former is developed and the latter is less developed. While factors are assumed to be immobile across national borders, goods are not. However, factor price equalization does not emerge due to the presence of transport costs. With regard to integration strategies, firms choose between two options: concentrating final production in one country and serving consumers world-wide from there (exporting) or engaging in multi-plant final production and serving consumers locally through domestic and foreign subsidiaries (multinational activity). In the parsimonious framework chosen, labor is the only factor used in production and in firm or plant set-up.
One industry produces a homogeneous good $x_0$ whereas the other industry produces differentiated goods.

The homogeneous good is supplied under perfect competition and N is more productive in this sector than S. For this reason there exists a gap between wages ($w$) in N and S. It is assumed that one unit of labor is needed to fabricate one of these goods in N. However, $\frac{1}{w} > 1$ units of labor are needed in S to produce one unit of the homogeneous good. We focus on parameter configurations which ensure that the homogeneous good is produced in both countries in equilibrium and traded across national boarders. The price of the homogeneous good is chosen as the numéraire. Consequently, $w^N = 1 > w^S = w$ arises, whereas $w^i$ is the wage rate in country i, $i \in \{N, S\}$ and transport costs for the homogeneous good exist.\(^{18}\)

The differentiated good is supplied under monopolistic competition. Each firm acts as a monopolist in supplying its variety. The price elasticity of demand between varieties is $\varepsilon > 1$ so that firms will charge a fixed mark-up over marginal costs and, due to monopolistic competition, the price will be lower than the monopoly price, because otherwise the firms would loose an over proportional amount of demand for their variety.

We assume that these firms are heterogeneous in productivity that they only can be founded in N and firms' headquarters are not internationally mobile. For this reason all headquarters are located in N, e.g. because of the unfavorable institutional environment in S, and they all are owned by N.

To enter the differentiated industry, an amount of h Northern units of labor, which are sunk costs, has to be invested. These are firm set-up costs. With this investment, a firm in the heterogeneous sector gets to know its own potential productivity level ($\theta$). Then, the firm can decide if it wants to enter the market, whereas firms in the differentiated sector can be diverse in their productivity so if they decide to enter the market after investing h they can make positive profits.\(^{19}\) But before investing h all heterogeneous firms have the same expectations about their profits; for this reason

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\(^{18}\) As in Grossman, Helpman and Szeidl (2006).

\(^{19}\) As in Helpman, Melitz and Yeaple (2004).
all heterogeneous firms are the same ex ante. If ex ante zero profits are expected, no more firms are willing to enter the differentiated sector. The productivity level drawn by a firm is a random variable.

In addition it is assumed that not only production costs in S are less than in N, but also that its market for differentiated goods (M^S) is a lot smaller than the differentiated good market in N (M^N).

### 2.2 A household’s utility function

All households have the same preferences and their utility function depends on a homogeneous good x₀ and the sub-utility of consumption of differentiated goods X. Each household consumes goods of either sector and, in formal accounts, its utility function may be written as:

\[
U_i = x_0 + X, \quad i \in \{N,S\}
\]

where

\[
X = \frac{1}{\mu \alpha} \left[ \int_0^{\theta^{\mu \alpha \gamma}} x^i(j)^{\alpha} dj \right], \quad 0 < \alpha < 1, 0 < \mu < 1 \text{ and } \mu < \alpha
\]

Consequently a household’s utility increases if more varieties of the differentiated product are available. These love for variety preferences exist for consumption of the differentiated goods.\(^{20}\) The elasticity of substitution between two of these varieties is constant \(\frac{1}{(1-\alpha)}\). All varieties in the differentiated sector are non-perfect substitutes for one another, as \(\alpha < 1\). But as \(\alpha > 0\) they are somehow substitutable. \(\mu\) is a constant with \(0 < \mu < \alpha < 1\) and reflects the differentiated industry to be worshiped more than the homogenous industry in the utility function of the representative household. X shows the sub-utility of consumption of the differentiated output, where \(x^i(j)\) features the consumption in i of the j-th variety in this industry.\(^{21}\)

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\(^{20}\) As in Krugman (1979) and Dixit and Stigliz (1977).  
\(^{21}\) As in Grossman, Helpman and Szeid (2006).
Thus the utility function of a representative household is linear in $x_0$ and non-linear in the differentiated goods. This implies that the demand for differentiated products depends on prices of differentiated goods, but not on earnings.

2.3 The heterogeneous firms

As mentioned before, heterogeneous firms arise in the differentiated industry and locate their immobile headquarters in the North. Irrespective of their integration strategies, these firms sell their products to each market.

This analysis discusses the optimal integration strategies of these firms, which depend on the firms’ productivity levels. An integration strategy is defined by a firm’s choice of the location of intermediate and final goods production. In the following chapters a firm’s choice for an integration strategy not only is influenced by transport costs but also by taxation on profits.

To begin with, in this analysis a firm in the differentiated industry with productivity $\theta$ produces final goods according to the production function $\theta F(m, a)$. The amount of intermediate input used is denoted by $m$ and $a$ is the level of final good activity. Both are measured in units of labor input. $F(m, a)$ is an increasing and concave function with constant returns to scale. Furthermore, the elasticity of substitution between $m$ and $a$ is not greater than 1. $c(p_m, p_a)$ describes the unit cost function referring to $F(m, a)$ whereas the price of input $i$ at the location of final goods production is denoted by $p_i$, $i \in \{m,a\}$. Taking stock, $c(p_m, p_a)/\theta$ describes the per-unit variable costs of production of a firm with productivity $\theta$ at the particular location.22

Households only consume final goods.

A firm producing intermediate goods in the South has to bear extra fixed costs of $g$ for communication and supervision, because the multinational firms’ headquarters are located in the North. Likewise multinational firms incur additional fixed costs of $f$ if they produce final goods in the South. These fixed costs are measured in labor units of the home country. Thereby it is assumed that fixed costs do not exist in the North.23

However, as already discussed, wages in the South are lower than in the North. It is assumed that production of one unit of the intermediate or final good requires one unit of local labor at the place of production, but that the intermediate good has to be

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transported to the location of final production, if the latter is manufactured elsewhere. However, per-unit variable costs of manufacturing differentiated final or intermediate goods in the South are lower than those of manufacturing these goods in the North. Since \( w^S = w < 1 = w^N \), the South has a comparative advantage in manufacturing differentiated goods.

**Table 1:**

<table>
<thead>
<tr>
<th>Intermediate production m</th>
<th>Final good production a</th>
<th>Fixed costs</th>
<th>Per-unit variable costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>in N</td>
<td>in N</td>
<td>0</td>
<td>( c(1, 1) / \theta )</td>
</tr>
<tr>
<td>in N</td>
<td>in S</td>
<td>( f )</td>
<td>( c(1, w) / \theta )</td>
</tr>
<tr>
<td>in S</td>
<td>in N</td>
<td>( g )</td>
<td>( c(w, 1) / \theta )</td>
</tr>
<tr>
<td>in S</td>
<td>in S</td>
<td>( f+g )</td>
<td>( c(w, w) / \theta )</td>
</tr>
</tbody>
</table>

As can be seen from table 1, optimality of integration strategies depends on per-unit variable costs \( c(p_m, p_a) \) and the fixed costs of a firm in the differentiated industry. Furthermore, a firm's total factor productivity is elemental for a firm's optimal integration strategy. Additionally these strategies also depend on market size, the level of transport costs and taxes.

In the following, strategies depend on the location of intermediate and final goods production, where both of them can be produced in S as well as in N.

### 2.4 Iceberg transport costs

Now, iceberg transport costs \( (d) \) are introduced. This implies that an exporting firm has to ship more than one unit of the final good so that one unit of the good arrives at the location of foreign consumers. Transport costs for final goods are identical across countries and differentiated products and they are proportional to the extent of shipments.\(^{25}\)

In the following section the size of transport costs is relatively high \( (d = d_H) \).

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\(^{25}\) As in Grossman, Helpman and Szeidl (2006).
2.5 An analysis with high transport costs and without taxation

Let us first describe the case when transport costs for final goods are high and taxes are zero. By assumption, transport of intermediate goods is for free.\(^{26}\) Compared to production in N, setting up foreign production plants in S induces additional fixed costs for a firm, but on the other hand its per-unit variable costs can be reduced when shifting production activities to S.\(^{27}\) Subsequently, the variation of a firm’s productivity level \(\theta\) in the differentiated industry will be observed more precisely.\(^{28}\)

A firm would never conduct activities in more than one plant per country. Such a strategy would unnecessarily incur additional costs.

To illustrate optimal integration strategies depending on a firm’s productivity level, we compare profits across alternative integration strategies. Consistent with the preferences depicted in (1) and (2), every manufacturing firm of this industry faces following demand function in each country \(i, i \in \{N, S\}\):\(^{29}\)

\[
x^i(j) = M^i \alpha [p_i(j) \mu]^{\alpha/(\alpha - 1)}
\]

(3)

\(x^i(j)\) describes total demand in one country for a single firm’s differentiated good. This demand depends on market size \(M^i, i \in \{N, S\}\). Furthermore, it depends on the substitutability of differentiated products among each other, \(\alpha\), on \(\mu\) which reflects the differentiated industry to be worshiped more than the homogenous industry by the representative household and on \(p_i(j)\) which is the effect of the individual firm’s own price on \(x^i\). Hence a single household’s demand for differentiated goods is independent of income.

Each firm therefore maximizes its profits according to:

\[
\pi = p_N(j)x^N(j) + p_S(j)x^S(j) - x^N(j) \frac{C_N}{\theta} - x^S(j) \frac{C_S}{\theta} - k
\]

(4)

\(^{26}\) As in Grossman, Helpman and Szeidl (2006).
\(^{27}\) As also in Yeaple (2003) and Helpman, Melitz and Yeaple (2004).
\(^{28}\) As in Grossman, Helpman and Yeaple (2004).
\(^{29}\) See derivation I in the Appendix.
Here \( p_N(j)x^N(j) + p_S(j)x^S(j) \) denotes total sales, \( (x^N(j)\frac{c_N}{\theta} + x^S(j)\frac{c_S}{\theta}) \) reflects total costs in both countries and \( k \) are the fixed costs. \( \frac{c_N}{\theta} \) and \( \frac{c_S}{\theta} \) are per-unit variable costs in N and S, respectively. 

This leads to the identification of the optimal prices of a firm:\(^{30}\)

\[
p_{N(j)}^{\text{opt}} = \frac{1}{\alpha} \frac{c_N}{\theta}, \quad p_{S(j)}^{\text{opt}} = \frac{1}{\alpha} \frac{c_S}{\theta}
\]

(5)

As shown here, a firm’s optimal price is independent of demand, respectively on market size in N and S. Each market’s price is defined as \( \frac{1}{\alpha} \) times the per-unit variable costs of a firm serving the specific market. Hence, prices entail a fixed markup over marginal costs.

In the following, we allow for transport costs and introduce terms which capture transport costs as well as variable (marginal) production costs. Generally, profits may be formulated as follows:\(^{31}\)

\[
\pi_{a,b} = \frac{X^N\Theta}{d_bC_{a,b}(j, r)} + \frac{X^S\Theta}{d_bC_{a,b}(j, r)} - k_{a,b}, \text{ with } a \in \{N, S\} \text{ and } b \in \{N, S\}
\]

(6)

We consider that a firm’s profits depend on its integration strategy, whereas a determines the location of intermediate goods production and b of final goods production. Hence, intermediate as well as final differentiated goods can be produced either in N, or in S, or in both countries. Depending on the location of production of a and b, different possible fixed costs \( (k_{a,b}) \) are taken into account in a firm’s profit function. If intermediate goods are produced in S, \( k_{a,b} \) has size \( g \), if final goods are produced in S, \( k_{a,b} \) has size \( f \), and if intermediate or final goods are produced in N fixed costs are zero. Also per-unit variable costs \( d_bC_{a,b}(j, r) \) depend on the locations of intermediate and final production, a and b. As denoted in table 1 j determines the dependency of total per-unit variable costs on per-unit variable costs for intermediate goods which is measured in units of labor at the location of production. Hence, j is

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\(^{30}\) See derivation II in the Appendix.

\(^{31}\) See derivation III in the Appendix.

Furthermore: \( \Theta = \theta^{\alpha/(1-\alpha)} \), \( dC = c^{\alpha/(1-\alpha)} \), \( X = (1-\alpha)\left(\frac{\alpha}{\mu}\right)^{\gamma/(1-\alpha)} \) and \( M^N = X^N \) and \( M^S = X^S \), whereas \( X^S < X^N \), because the market share of the South is smaller than that of the North.
\( j \in \{1, w\}, 1 \) occurs if intermediate goods are produced in N and \( w < 1 \) denotes for these costs if they are produced in S. Additionally, \( r \) determines the dependency of total per-unit variable costs on per-unit variable costs for final goods which is measured in units of labor at the location of production. Hence, \( r \in \{1, w\}, 1 \) occurs if final goods are produced in N and \( w < 1 \) denotes for these costs if they are produced in S. Finally, also transport costs depend on the location of final production \( (b), d_b \). If final goods have to be shipped to serve a market, \( d_b > 1 \) occurs and if final goods are produced at the location of consumption, \( d_b = 1 \) (no transportation of final goods) is taken into account in a firm’s profit function.

Furthermore, high transport costs exist if:

\[
\frac{C(w, 1)}{C(w, w)} < d_{hi} \quad \text{or} \quad \frac{C(1)}{C(1, w)} < d_{hi}
\]

For this reason, the following possible profit functions arise for a firm depending on its productivity level in a two-country setting with high transport costs:

\[
\pi_{NN} = \frac{X_N^{\Theta}}{C(1, 1)} + \frac{X_S^{\Theta}}{d_{hi} C(1, 1)}
\]

(II)

This strategy \( N, N \) with profit function \( \pi_{NN} \) describes concentration of intermediate and final good production in the home country, which is the North. A firm operates on the Southern market by exporting the differentiated products. For this reason supplying the Southern market is more expensive. This strategy minimizes fixed costs, but produces with relative high per-unit variable costs \( C(1, 1) \), because factor prices in N are higher than in S.

\[
\pi_{NNS} = \frac{X_N^{\Theta}}{C(1, 1)} + \frac{X_S^{\Theta}}{C(1, w)} - f
\]

Firms choosing strategy \( N, NS \) supply the Northern market by producing intermediate and final goods in N. Intermediate goods from N are shipped to S, where final goods production takes place to serve consumers in S locally. In this case, MNE activity in final goods eliminates all trade in final goods. With this strategy medium high fixed costs of \( f \) are incurred, but on the other hand the firm can save per-unit variable costs when supplying the South compared to (I).

A strategy N,S with the profit function $\pi_{N,S}$ is no alternative to N,NS in the case with high transport costs.

$$\pi_{N,S} = \frac{\bar{X}^N_-}{d_tC(1,w)} + \frac{\bar{X}^S_-}{C(1,w)} - f$$  \hfill (II')

Even though the fixed costs and the costs of supplying S are identical when selecting one of these two strategies, this strategy can be eliminated, because of the costs arising by supplying N. Supplying to N under strategy N,NS, a firm produces intermediate as well as final goods in N. In consequence the only difference to strategy N,S is that the per-unit variable costs of supplying the North are lower. This is due to the fact that high transport costs occur to supply the Northern market when selecting N,S instead of N,NS. For this reason this strategy never is reasonable if high transport costs exist. If transport costs vary, it also is possible that N,S is the better strategy, but one of these strategies always dominates the other.

$$\pi_{S,N} = \frac{\bar{X}^N_-}{C(w,1)} + \frac{\bar{X}^S_-}{d_tC(w,1)} - g$$  \hfill (III)

In this case intermediate goods are produced in S and final goods in N. For this reason this strategy can also be seen as “partial globalization”.\(^{33}\) Intra-firm trade exists. Transport costs arise when supplying the South, because final goods are produced in N.

Similarly to strategy N,NS a firm in this case has to bear medium high fixed costs here denoted by g, because intermediate goods are manufactured in S and final goods in N. However strategies N,NS and S,N cannot be brought to an adequate ranking without an exact identification of the level of the different fixed costs f and g and of the per-unit variable costs.

$$\pi_{S,NS} = \frac{\bar{X}^N_-}{C(w,1)} + \frac{\bar{X}^S_-}{C(w,w)} - f - g$$  \hfill (IV)

Firms choosing strategy S,NS supply to S by producing intermediate and final goods there. To satisfy Northern demand, these firms produce intermediate goods in S, ship them to N at zero transport costs, produce final goods in N and sell them there. In this case international trade in final goods does not occur. Strategy S,NS is associated with fixed costs of f and g. Hence these firms save high per-unit variable costs including trade costs. Because of the higher fixed costs associated with this

strategy, S,NS only is reasonable for highly productive firms, which face a high demand.

Table 2 summarizes facts for described profit functions:

<table>
<thead>
<tr>
<th>strategy</th>
<th>meaning</th>
<th>marginal costs of serving consumers in N</th>
<th>marginal costs of serving consumers in S</th>
<th>fixed costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>N,N</td>
<td>Intermediate good production in N; final good production in N</td>
<td>C(1,1)</td>
<td>d_H C(1,1)</td>
<td>0</td>
</tr>
<tr>
<td>N,S</td>
<td>Intermediate good production in N; final good production in S</td>
<td>d_H C(1,w)</td>
<td>C(1,w)</td>
<td>f</td>
</tr>
<tr>
<td>N,NS</td>
<td>Intermediate good production in N; final good production in N and S</td>
<td>C(1,1)</td>
<td>C(1,w)</td>
<td>f</td>
</tr>
<tr>
<td>S,N</td>
<td>Intermediate good production in S; final good production in N</td>
<td>C(w,1)</td>
<td>d_H C(w,1)</td>
<td>g</td>
</tr>
<tr>
<td>S,NS</td>
<td>Intermediate good production in S; final good production in N and S</td>
<td>C(w,1)</td>
<td>C(w,w)</td>
<td>f and g</td>
</tr>
</tbody>
</table>

Alternative strategies to S,NS would be S,S or NS,NS. Their costs of supplying to S and the fixed costs of these strategies match those of π_{S,NS}, but the per-unit variable costs for supplying the Northern market are higher. Since, in this scenario with high transport costs per-unit variable costs are lower with S,NS than with S,S or NS,NS, but fixed costs are the same, firms will never choose S,S or NS,NS.

Theoretically, two further integration strategies, namely NS,N and NS,S are possible. But these strategies never are reasonable either. A firm would only produce intermediate goods in both N and S (associated with extra fixed costs for intermediate production) if high transport costs for intermediate goods existed. But if
this was the case, it would only make sense to produce final products in both countries as well to save on transport costs.\footnote{All seven possible strategies with transport costs for final goods are shown in IV in the Appendix, whereas the size of transport costs determines the set of optimal strategies.}

Hence (I), (II), (III) and (IV) are the only relevant strategies if transport costs are high.\footnote{If and only if high transport costs exist this set of optimal integration strategies is chosen. This can be seen in derivation V in the Appendix.}

In this case a pure trade-off between fixed costs and per-unit variable costs exists. Transportation of final goods to the North is never optimal, because of the size of $d_H$. This can be seen from:

$$\frac{C(w,1)}{C(w,w)} < d_H.$$ The lowest per-unit variable costs to satisfy each market’s demand therefore can be achieved with local production of final goods. Strategies that are chosen in this setting where final goods are not only produced locally are S,N and N,N. If $g$ is very small, S,N can be a possible optimal strategy although high transport costs exist. Also N,N belongs to the set of optimal strategies. But both of these strategies are chosen from more unproductive firms, because these cannot afford high fixed costs.

A firm produces intermediate goods at only one location, because their transport is for free.\footnote{As in Grossman, Helpman and Szeidl (2006).} A firm producing intermediate goods in N also produces final goods there to satisfy Northern demand, because transportation of final goods from S is relatively costly, and for this reason it is not attractive for a firm.

This can be considered by: $$\frac{C(1,1)}{C(1,w)} < d_H.$$ A firm producing intermediate goods in S has two possibilities. Either it only produces final goods in N, or it ships some of them to N to serve the North and the intermediate goods for S stay in S so that final goods for every market are produced locally. The optimal strategy for a firm depends on its productivity and the fixed costs. So the following integration strategies are possibly relevant: N,N; N,NS; S,N; S,NS.

Graphic 1 shows $\pi_{NN}$ and $\pi_{SNS}$ depending on productivity $\Theta$. 

\footnote{As in Grossman, Helpman and Szeidl (2006).}
Firms always choose the integration strategy with the highest attainable positive profit at a given level of $\Theta$. $\pi_{S,NS}$ is associated with fixed costs $f$ and $g$ while $\pi_{NN}$ is associated with zero fixed costs. However, the variable production costs under $N,N$ are higher than those under $S,NS$. Therefore $\pi_{NN}$ is higher than $\pi_{S,NS}$ if productivity is lower than a critical level $\Theta(N,N; S,NS)$ and it is lower than $\pi_{S,NS}$ at $\Theta > \Theta(N,N; S,NS)$. $\pi_{NN}$ and $\pi_{S,NS}$ intercept at $\Theta(N,N; S,NS)$.

The profit function $\pi_{N,NS}$ now can be added to this analysis. $\pi_{N,NS}$ is associated with fixed costs $f$, and the variable production costs under $N,NS$ are higher than those under $S,NS$ and lower than those under $N,N$. Graphic 2 shows alternative possibilities for this strategy. Therefore $\pi_{NN}$ is higher than $\pi_{N,NS}$ if productivity is lower than a critical level $A$ and it is lower than $\pi_{N,NS}$ at $\Theta > A$. $\pi_{NN}$ and $\pi_{N,NS}$ intercept at $A$. $\pi_{S,NS}$ is lower than $\pi_{N,NS}$ if productivity is lower than a critical level $C$ and it is higher than $\pi_{N,NS}$ at $\Theta > C$. $\pi_{S,NS}$ and $\pi_{N,NS}$ intercept at $C$. Another possibility for the profit function corresponding to strategy $N,NS$ would be $\pi_{N,NS}''$. In this case $\pi_{N,NS}''$ is lower than $\pi_{NN}$ and $\pi_{S,NS}$. Only $\pi_{NN}$ and $\pi_{S,NS}$ then intercept at $B$ in the graphic.

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For $N,NS$ to be an optimal strategy, it is necessary that the intersection of $\pi_{NNS}$ and $\pi_{SNS}$ lies above the intersection of $\pi_{NN}$ and $\pi_{SNS}$, at $\Theta(N,N;S,NS)$.

It results:

$$g \geq \frac{X^N}{C(w,1)} - \frac{X^N}{C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{C(1,w)} = \gamma_H$$

(7)

This condition has to hold so that $N,NS$ is an optimal strategy for a firm. As shown in graphic 2, low productive firms locate all production activities at home, high productive firms produce intermediate goods only in S and final goods in N and S and if $\pi_{NNS}$ runs like $\pi_{NNS}'$, firms with intermediate productivity level manufacture intermediate goods in N and final goods in N and S.

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39 See derivation VI in the Appendix.
Just as $\pi_{NNS}$ can be added to the analysis in graphic 1, this is also possible with $\pi_{SN}$. $\pi_{SN}$ is associated with fixed costs $g$ and the variable production costs under S,N are higher than those under S,NS and lower than those under N,N. For S,N to become an optimal strategy, it is necessary that the intersection of $\pi_{SN}$ and $\pi_{SNS}$ lies above the intersection of $\pi_{NN}$ and $\pi_{SNS}$. Using the same approach as in the analysis for $\pi_{NNS}$ above, the following condition must hold for S,N to become an optimal strategy:

$$\gamma_L = \frac{dHC(w,1)}{X^S} - \frac{dHC(w,1)}{X^S} = \frac{C(w,1)}{dHC(w,1)} \leq \frac{C(1,1)}{dHC(w,1)}$$

If (8) holds, firms with a low productivity level locate all production activities at home, highly productive firms produce intermediate goods in S and final goods in N and S, and firms with intermediate levels of productivity manufacture intermediates in S and final goods in N.

From (7) and (8) it can be seen if $\gamma_L < \frac{g}{f} < \gamma_H$ is true, only two optimal strategies exist, namely all firms either only produce in N or intermediates are produced in S and final goods in both locations. The assumption that the elasticity of substitution between intermediate and final good production is not greater than 1, ensures that $\gamma_L < \gamma_H$ holds.

For $\pi_{NNS}$ or $\pi_{SN}$ to be a dominate integration strategy either $\frac{g}{f} \leq \gamma_L$ or $\frac{g}{f} \geq \gamma_H$ has to hold true. As this is not possible at the same time, only one of the strategies, S,N or N,NS, can be optimal depending on the size of the fixed costs relation. In graphic 2 $\pi_{NNS}$ is a possible optimal strategy for a firm depending on its productivity. This means that $\frac{g}{f} \geq \gamma_H$ must be true. Then, the fixed costs for a final good producing plant in S are relatively lower than the fixed costs for an intermediate good producing plant there.

40 See derivation VII in the Appendix.

41 $\gamma_L < \gamma_H$ is considered if and only if $\frac{1}{C(w,w)} + \frac{1}{dHC(1,1)} > \frac{1}{dHC(w,1)} + \frac{1}{C(1,w)}$. 

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In case that S,N and not N,NS is optimal \( \frac{g}{f} \leq \gamma_L \) holds. This means that the fixed costs for a final good producing plant in S are relatively higher than the fixed costs for an intermediate good producing plant in S. According, whether N,NS or S,N is optimal depends on the fixed costs.\(^{42}\)

For this reason another graphical description is adequate to show all areas of optimal strategies in one diagram.

**Graphic 3.**\(^{43}\)

\[ A = \Theta(S,N;S,NS) \]
\[ B = \Theta(N,NS;S,NS) \]
\[ C = \Theta(N,N;N,NS) \]

Graphic 3 depicts combinations of fixed costs \( g \) for intermediate goods and of \( \Theta \) that generate different strategies of integration. In this connection the level of fixed costs for final goods \( f \) is held constant. If \( f \) changed, the bold broken lines would change.\(^{44}\)

In the section N,N all activities of a firm are located in N, in section S,N intermediate goods are produced in S and final goods in N. Section N,NS shows firms that manufacture intermediate goods in N and final goods in N and S. Finally, if a firms productivity lies in region S,NS intermediate good producing activities are shifted to S and final goods are produced in both countries.\(^{45}\)

Hence, if depending on firms’ productivity levels strategies N,N; N,NS or S,NS are reasonable fixed costs \( g \) are high relative to the given value of \( f \). If depending on

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\(^{42}\) As in Grossman, Helpman and Szeidl (2006).


\(^{44}\) As in Grossman, Helpman and Szeidl (2006).

\(^{45}\) As in Grossman, Helpman and Szeidl (2006).
firms’ productivity levels strategies N,N; S,N or S,NS are reasonable fixed costs g are small relative to the given value of f and if depending on firms’ productivity levels only strategies N,N or S,NS are reasonable the fixed costs relation between g and f is medium high.

The five graphical analogues to the analytical cut-off levels separating the optimal integration strategies in graphic 3 result from the following:
The cut-offs are calculated by equating one strategy’s profits with another strategy’s profits and solving this for \( \Theta \). Four different possibly optimal \( \pi \) exist: \( \pi_{N,N} \), \( \pi_{N,NS} \), \( \pi_{S,N} \) and \( \pi_{S,NS} \) and five different cut-off levels arise. The sixth theoretically possible comparison is \( \pi_{N,NS} \) with \( \pi_{S,N} \). But, as already discussed, these two strategies cannot be equated, because they are never optimal at the same time if only firms’ productivities differ, because they arise when different fixed costs relations exist.

Hence the different cut-off levels are given by: \(^{46}\)

\[
\Theta(N,N;N,NS) = \frac{f}{X^S \left( \frac{1}{C(1,w)} - \frac{1}{d_hC(1,1)} \right)}
\]

This cut-off level between sections N,N and N,NS is independent of g. For this reason it is represented by a vertical line in graphic 3.

From graphic 2 it is known that N,NS is the optimal strategy for firms with an intermediate level of productivity if \( \frac{g}{f} \geq \gamma_h \) holds. If \( \Theta \) is smaller than \( \Theta(N,N;N,NS) \), N,N is optimal. If \( \Theta \) is greater, then N,NS is the optimal strategy. Firms with the exact productivity of \( \Theta(N,N;N,NS) \) are just indifferent between the production of all goods at home and the production of intermediate goods in N and of final goods in N and S, because their profits are the same in both cases.

\[
\Theta(N,N;S,N) = \frac{g}{X^N \left( \frac{1}{C(w,1)} - \frac{1}{C(1,1)} \right) + X^S \left( \frac{1}{d_hC(w,1)} - \frac{1}{d_hC(1,1)} \right)}
\]

This cut-off level between sections N,N and S,N depends on g and is represented by a line through the origin. At levels of \( \Theta \) which exceed \( \Theta(N,N;S,N) \) S,N is optimal, for lower levels N,N is the optimal strategy.

\(^{46}\) See derivation VIII in the Appendix.
\[ \Theta(N,N;S,NS) = \frac{f + g}{X^N \left[ \frac{1}{C(w,1)} - \frac{1}{C(1,1)} \right] + X^S \left[ \frac{1}{C(w,w)} - \frac{1}{d_n C(1,1)} \right]} \] (c)

This cut-off level between sections N,N and S,NS depends on g and it starts from a negative intercept. Because of the per-unit variable costs it is steeper than \( \Theta(N,N;S,N) \). At levels of \( \Theta \) which exceed \( \Theta(N,N;S,NS) \), S,NS is optimal, for lower levels N,N is the optimal strategy.

\[ \Theta(N,NS;S,NS) = \frac{g}{X^N \left[ \frac{1}{C(w,1)} - \frac{1}{C(1,1)} \right] + X^S \left[ \frac{1}{C(w,w)} - \frac{1}{C(1,w)} \right]} \] (d)

This cut-off level between sections N,NS and S,NS again depends on g and is represented by a line through the origin. Because of the per-unit variable costs it is steeper than \( \Theta(N,N;S,N) \). At levels of \( \Theta \) which exceed \( \Theta(N,NS;S,NS) \), S,NS is optimal, for lower levels N,NS is the better strategy for a firm. The higher fixed costs g are, the higher must also a firm’s productivity be for intermediate good production in S to be profitable.

\[ \Theta(S,N;S,NS) = \frac{f}{X^S \left[ \frac{1}{C(w,w)} - \frac{1}{d_n C(w,1)} \right]} \] (e)

This cut-off level between sections S,N and S,NS is independent of g and for this reason it is represented by a vertical line in graphic 3. If the level of \( \Theta \) is lower than \( \Theta(S,N;S,NS) \), then S,N is optimal. If the level of \( \Theta \) exceeds \( \Theta(S,N;S,NS) \), then S,NS is the optimal strategy for a firm.

Because of the high transport costs, firms choose this strategy S,NS to produce final goods locally. Hence in final good production these are horizontal firms.\(^{47}\) These highly productive firms shift most production activities to S, where per-unit costs are lower, to generate the highest possible reduction of variable per-unit costs.\(^{48}\) Due to the high transport costs, final goods for the Northern market are produced in N to maximize the firms’ profits. The fraction of firms choosing this strategy depends on the firms’ productivity. It rises if g falls and then becomes independent of g.

MNE activities for final good production only arise to the right of the bold, broken line in graphic 3. The smaller g, the greater this fraction of firms will be.

\(^{47}\) As in Markusen and Venables (1998).
\(^{48}\) As in Grossman, Helpman and Szeidl (2006).
If \( g \) is very high, the fraction of firms producing final goods in N and S is independent of \( g \), whereas the fraction of them choosing strategy N,NS or S,NS depends on the size of \( g \). The fraction of firms choosing strategy S,N rises if \( g \) is small and a firm’s productivity is \( > 0 \). At the graphical analogue to the cut-off level \( \Theta(S,N;S,NS) \) in graphic 3, this fraction becomes independent of \( g \).\(^{49}\)

If firms have an intermediate productivity level \( \Theta \), namely that \( \Theta(S,N;S,NS) < \Theta < \Theta(N,N;N,NS) \) then the fixed costs for a production plant in S for final goods are only borne if the fixed costs for intermediate good producing plants \( g \) are small. Accordingly, these firms either shift intermediate production activities to S and produce final goods in both countries, due to high transport costs, or they produce intermediate and final goods only in N, if \( \gamma_L < \frac{g}{f} < \gamma_H \) holds.\(^{50}\) For positive values of given fixed costs for intermediate goods \( g \), the most unproductive firms locate all production activities in the North and export their final good to deliver to the Southern market, whereas transport costs are high.

Consequently, a reduction of fixed costs such as of barriers to trade or transport costs influences the firms’ optimal integration strategies and encourages their economic outcome.

2.6 An analysis with high transport costs, profit taxation and provision of public infrastructure for differentiated goods

In the following, the governments set profit taxes in a first stage and cannot rescind their offers by assumption. Then, firms decide upon their optimal integration strategies, which is taken into account by the governments when setting tax rates. A government chooses a tax rate \( t_i \in \{N,S\} \) to tax firms’ profits. The achieved tax revenue is spent for public infrastructure for differentiated goods. Governments take the representative household’s utility in their jurisdiction into account, when deciding to levy profit taxes or not, i.e. the representative household’s utility may not decline when selecting \( t_i > 0 \) compared to its utility without profit taxation. The provision of this public infrastructure at least induces fixed costs of \( R \), whereas \( d \) depends on \( R \). If public infrastructure for differentiated goods is provided, \( d \) declines and therefore households’ utility is influenced.

\(^{49}\) As in Grossman, Helpman and Szeidl (2006).
\(^{50}\) As in Yeaple (2003).
By assumption, without investment of tax revenue in public infrastructure for differentiated goods, transport costs are high if \( R \) is zero. Furthermore, taxes reduce a firm’s profits. The set of optimal integration strategies is influenced by transport. They influence the location of the graphical analogues to the cut-off levels shown in the previous graphic. The set of optimal strategies differs depending on the height of \( d \), because the higher \( d \) gets the more local production is reasonable. Also profit taxes influence the location of these graphical analogues to the cut-off levels in the previous graphic, since they determine the profitability of alternative modes of firm integration. If tax revenue finances provision of public infrastructure for firms in the differentiated sector, this influences \( d \) and therefore the set of optimal integration strategies, if \( R \) is invested. Tax rates are set in such way that the set of optimal integration strategies from the governments’ and the firms’ point of view stays the same. Furthermore, households don’t gain utility by consuming the public good directly, but prices for differentiated goods depend on \( d \), which is influenced by \( R \).

### 2.6.1 The governments’ problem

In this section, cases are analyzed in which the governments of both countries N and S can levy taxes \( t_N \) and \( t_S \), which are taken into account in the firms’ profit functions. In this setting taxes are paid on a firm’s profits either in N or in S. The location of tax payment depends on a firm’s location of final good production. Intermediate good production is not taxed. As can be seen in this setting double taxation is not the problem of the analysis.\(^{51}\)

Governments select profit taxes to finance public infrastructure for differentiated goods, but in consequence social welfare in their jurisdictions may not decline. Hence, social welfare is defined by the representative household’s utility in a government’s jurisdiction as described in section 2.2. Furthermore, by assumption the highest possible productivity level \( \Theta_{\text{max}} = 1 \) always is part of the integration strategy with most of its total production in S. If public infrastructure for differentiated goods is provided this influences \( d \) and herefore the utility of households resident in a government’s jurisdiction rises, whereas not only \( d \) influences the households’ utility.

---

\(^{51}\) As in Egger et al. (2006).
Here, the price for the homogenous product is \( p_0 = 1 \) and prices for differentiated products are denoted by \( p_i(j) \), whereas \( p \) is the price for variety \( j \) in country \( i \), \( i \in \{N; S\} \).

As already shown in section 2.2, a representative household’s utility in country \( i \), \( i \in \{N; S\} \), is given by:

\[
U_i = x_0 + X
\]

where

\[
X = \frac{1}{\mu\alpha} \left[ \int_0^{\Theta_{\text{max}}} x(j)^\alpha dj \right]^{\alpha - 1} \quad 0 < \alpha < 1, 0 < \mu < 1 \text{ and } \mu < \alpha
\]

This alternatively can be denoted by:

\[
V_i = m_i + (1 - \alpha) \cdot \mu^{\alpha - 1} \int_0^{\Theta_{\text{max}}} p_i(j)^{\alpha - 1} dj
\]

A representative household’s utility rises in \( m_i \) and declines in \( p_i(j) \).

For a government to decide to provide a positive amount of public infrastructure, and therefore to select \( t_i > 0 \), the above shown utility of households in its country may not decline. Also the size of both governments’ combined tax revenue has to be high enough to afford \( R > 0 \).

To be able to show welfare implications, optimal integration strategies with taxation have to be looked at.

### 2.6.2 Integration strategies with profit taxation

If \( t_i > 0 \) is selected by the governments, firms can anticipate that \( d \) declines, because the governments only select \( t_i > 0 \) if tax revenue is high enough so that \( R \) can be afforded. If \( R \) is invested, \( d \) declines from high (\( d_H \)) to low (\( d_L \)).

For this reason, firms select other integration strategies to be reasonable than with high transport costs.

Low transport costs can be described in the following way:

\[
1 < d_L < \frac{C(1,1)}{C(1,w)}
\]

---

52 See derivation IX in the Appendix.
53 See derivation X in the Appendix.
In this case, the optimal profit functions of the firms depending on their productivity are: \( \pi_{NN}, \pi_{NS}, \pi_{SN} \) and \( \pi_{SS} \). A trade-off between fixed costs for production plants in \( S \) and the reduction of per-unit variable costs when shifting production activities to \( S \) is shown.\(^{54}\) These profit functions also depend on the size of fixed costs, market size, per-unit variable costs, and the degree of taxation:

\[
\pi_{NN} = \frac{X_N}{C(1,1)}(1-t_N) + \frac{X_S}{d_L C(1,1)}(1-t_N) \tag{I'}
\]

This strategy \( N,N \) is identical to the one with high transport costs. But the variable per-unit costs for products supplied on the Southern market are lower than with high transport costs since \( d_L < d_H \). Furthermore, the decline of these costs depends on the size of the Southern market, but in this setting \( M^N > M^S \). Referring to lower transport costs, the slope of \( \pi_{NN} \) is steeper than in the previous analysis with high transport costs, but a higher tax rate \( t_N \) reduces the slope of \( \pi_{NN} \). For this reason, the slope of \( \pi_{NN} \) depends on the exact parameter configurations and it cannot be compared with the slope of \( \pi_{NN} \) with high transport costs in general.

If a firm selects strategy \( N,N \), all its taxes on profits are paid in \( N \), because all final goods are produced there. \( t_N \) reduces a firm’s profits. The degree of the reduction depends on the tax rate chosen by the Northern government.

\[
\pi_{NS} = \frac{X_N}{d_L C(1,w)}(1-t_S) + \frac{X_S}{C(1,w)}(1-t_S) - f \tag{II''}
\]

The per-unit variable costs for products on the Northern market decline for firms selecting this as their optimal strategy in comparison to the analysis with high transport costs, when they selected \( \pi_{NNS} \). The decline of these costs is induced by a smaller \( d \) and therefore another optimal integration strategy concerning the Northern markets supply is chosen. Furthermore, the size of the decline of these costs depends on the size of the Northern market. Not only the size of \( d \) has decreased, this also has induced firms to select \( \pi_{NS} \) instead of \( \pi_{NNS} \) as an optimal profit function. Also the slope of \( \pi_{NS} \) instead of \( \pi_{NNS} \) is steeper than in the analysis with high transport costs referring to the per-unit variable costs, but the higher the degree of taxation, the less steep the slope of \( \pi_{NS} \) gets. For this reason it can be seen that

\(^{54}\) As in Helpman, Melitz and Yeaple (2004) and Yeaple (2003).
the slope of $\pi_{NS}$ depends on the exact parameter configurations and it cannot be compared with the slope of $\pi_{NNS}$ with high transport costs in general.

If a firm selects strategy $N,S$, taxes on profits are paid in $S$, because all final goods are produced there. When choosing strategy $N,S$, the degree of reduction of a firm’s profits depends on the tax rate chosen by the Southern government. Furthermore, hereby fixed costs $f$ are incurred in $S$.

A further profit function is:

$$\pi_{SN} = \frac{X^N\Theta}{C(w,1)}(1-t_N) + \frac{X^S\Theta}{d_C(w,1)}(1-t_N) - g$$

This strategy $S,N$ is identical to the one with high transport costs. But the variable per-unit costs for products supplied on the Southern market are lower than with high transport costs since $d<L<d_H$. Furthermore, the decline of these costs depends on the size of the Southern market, but in this setting $M^N > M^S$. Referring to the lower $d$, the slope of $\pi_{SN}$ is steeper than in the previous analysis with high transport costs, but a higher $t_N$ reduces the slope of $\pi_{SN}$. For this reason, the slope of $\pi_{SN}$ depends on the exact parameter configurations and it cannot be compared with the slope of $\pi_{SN}$ with high transport costs in general.

Again, if a firm selects strategy $S,N$, all its taxes on profits are paid in $N$, because all final goods are produced there. When choosing strategy $S,N$, the degree of reduction of a firm’s profits depends on the tax rate chosen by $N$.

$$\pi_{SS} = \frac{X^N\Theta}{d_LC(w,w)}(1-t_S) + \frac{X^S\Theta}{C(w,w)}(1-t_S) - f - g$$

The per-unit variable costs for products on the Northern market decline for firms selecting this as their optimal strategy in comparison to the analysis with high transport costs, when they selected $\pi_{SNS}$. The decline of these costs is induced by a smaller $d$, and therefore another optimal integration strategy concerning the Northern markets supply is chosen. Furthermore, the size of the decline of these costs depends on the size of the Northern market. Not only the size of $d$ has decreased, this also has induced firms to select $\pi_{SS}$ instead of $\pi_{SNS}$ as an optimal profit function. Also the slope of $\pi_{SS}$ instead of $\pi_{SNS}$ is steeper than in the analysis with high transport costs referring to the per-unit variable costs, but the higher the degree
of taxation, the less steep the slope of $\pi_{SS}$ gets. For this reason it can be seen that the slope of $\pi_{SS}$ depends on the exact parameter configurations and it cannot be compared with the slope of $\pi_{SS}$ with high transport costs in general.

If a firm selects strategy $SS$, taxes on profits are paid in $S$, because final goods are produced there. When choosing strategy $SS$, the degree of reduction of a firm’s profits depends on the tax rates chosen by the Southern government. Furthermore, hereby fixed costs $f$ and $g$ are incurred in $S$.

This set of strategies only is chosen by the firms if $d$ is low and if the firms select the welfare optimal set of integration strategies, as their choice also is influenced by $t_i$.

From the formal description of the profit functions it can be seen that they all run steeper than in the previous analysis with high transport costs, referring to $d$. Additionally, the higher $t_i$, the less steep are the profit functions.

According to the section with high transport costs, the cut-off levels between two strategies can be considered as already shown in previous analysis, whereas now $d$ is low, another set of integration strategies is reasonable and cut-off levels now additionally depend on $t_i$.\textsuperscript{55}

\[\Theta(N,N;N,S) = \frac{f}{d_iC(1,w) + \frac{X^N}{C(1,w)}(1-t_s) - \frac{X^S}{d_iC(1,1) + C(1,1)}(1-t_N)}\]  

(a’)

$\Theta(N,N;N,S)$ is independent of $g$ and for this reason represented by a vertical line in the graphical analysis.

In comparison to the analysis with high transport costs in graphic 3 this cut-off level changes from $\Theta(N,N;N,NS)$ to $\Theta(N,N;N,S)$. Only referring to lower transport costs the line representing this cut-off level shifts a bit inwards if the additional earnings on the Northern market by changing strategy $N,NS$ to $N,S$ are higher than the loss in savings on the Southern market, because of a lower $d$. The more inwards the line representing this cut-off level shifts, the lower a firm’s productivity has to be, to no longer select $N,N$ as its preferred strategy.

\textsuperscript{55} See derivation XI in the Appendix.
But the higher the tax rate selected by the Southern government the more the line representing this cut-off level shifts back outwards and the higher the tax rate selected by the Northern government the more the line representing this cut-off level shifts inwards.

\[ \Theta(N,N;S,N) = \frac{g}{\frac{X^N}{C(w,1)} - \frac{X^N}{C(1,1)} + \frac{X^S}{d_N C(w,1)} - \frac{X^S}{d_N C(1,1)}} (1 - t_N) \]  

(b' )

In comparison to the analysis with high transport costs in graphic 3 the line representing this cut-off level runs steeper induced by lower transport costs.

But the higher the tax rate selected by the Northern government the less steep the line representing this cut-off level runs.

\[ \Theta(N,N;S,S) = \frac{f + g}{\frac{X^N}{d_S C(w,w)} + \frac{X^S}{C(w,w)}} (1 - t_S) - \frac{\frac{X^N}{C(1,1)} + \frac{X^S}{d_S C(1,1)}} (1 - t_N) \]  

(c' )

In comparison to the analysis with high transport costs in graphic 3 this cut-off level changes from \( \Theta(N,N;S,NS) \) to \( \Theta(N,N;S,S) \). With lower transport costs the line representing this cut-off level runs steeper if the additional earnings on the Northern market by changing strategy S,NS to S,S are higher than the loss in savings on the Southern market, because of a lower \( d \). The steeper the line representing this cut-off level runs, the lower a firm’s productivity has to be, to no longer select N,N as its preferred strategy.

But the higher the tax rate selected by the Southern government the less steep the line representing this cut-off level runs and the higher the tax rate selected by the Northern government the more steep it runs.

\[ \Theta(N,S;S,S) = \frac{g}{\frac{X^N}{d_S C(w,w)} - \frac{X^N}{d_S C(1,w)} + \frac{X^S}{C(w,w)} - \frac{X^S}{C(1,w)}} (1 - t_S) \]  

(d' )

Because of the per-unit variable costs this cut-off level is steeper than \( \Theta(N,N;S,N) \).

In comparison to the analysis with high transport costs in graphic 3 this cut-off level changes from \( \Theta(N,NS;S,NS) \) to \( \Theta(N,S;S,S) \). If due to lower transport costs the additional earnings on the Northern market are higher than the loss in savings,
because of the by d induced change of optimal strategies, the line representing this cut-off level runs steeper. The steeper the line representing this cut-off level runs, the lower a firm’s productivity has to be, to select S,S as its preferred strategy. But the higher the tax rate selected by the Southern government the less steep the line representing this cut-off level runs.

\[
\Theta(S,N;S,S) = \frac{f}{\left[\frac{X^N}{d(C(w,w))} + \frac{X^S}{C(w,w)}\right](1-t_s) - \left[\frac{X^N}{C(w,1)} + \frac{X^S}{d(C(w,1))}\right](1-t_N)} \tag{e’}
\]

\(\Theta(S,N;S,S)\) is independent of g and for this reason represented by a vertical line in the graphical analysis.

In comparison to the analysis with high transport costs in graphic 3 this cut-off level changes from \(\Theta(S,N;S,NS)\) to \(\Theta(S,N;S,S)\). With lower transport costs the line representing this cut-off level shifts a bit inwards if the additional earnings on the Northern market by changing strategy S,NS to S,S are higher than the loss in savings on the Southern market, because of a lower d. The more inwards the line representing this cut-off level shifts, the lower a firm’s productivity has to be, to select S,S as its preferred strategy. But the higher the tax rate selected by the Southern government the more the line representing this cut-off level shifts back outwards and the higher the tax rate selected by the Northern government the more the line representing this cut-off level shifts inwards.

By combining all five graphical analogues to the aforementioned cut-off levels, the representation of optimal firm integration strategies as in graphic 4 is obtained. In comparison to graphic 3 with high transport costs the set of optimal integration strategies changes. If the vertical lines representing cut-off levels shift somewhat inwards or outwards and the other cut-off level representing lines run steeper or less steep than in graphic 3, cannot be said in general, but it depends on the exact parameter configurations.
A = Θ(S,N;S,S); B = Θ(N,S;S,S); C = Θ(N,N;N,S)

This graphical description is an example for the case, when the vertical lines shift somewhat inwards and the other cut-off level representing lines run steeper than in graphic 3. Firms change their optimal integration strategy in response to the size of transport costs and the set of optimal integration strategies selected now, delivers a higher economic outcome than the previous set with high transport costs only referring to d.

Referring to graphic 4 an increasing fraction of firms invests in MNE activities for final good production if fixed costs for intermediate good production sink. The lower transport costs are, the less productive firms must be, to manufacture final goods in S, because the per-unit variable costs decline if high transport costs decline. Transport costs in sum are higher to supply the Northern market than to supply the Southern market, because M^N>M^S. For this reason it is true that the smaller transport costs for final goods are, the more economic integrated strategies are chosen by the more productive firms.

If a firm invests in any activity in S, per-unit costs decrease. This raises the demand for output and consequently the willingness to also produce the other good there. This is also identified as “unit-cost complementarity”. Consequently, if g or f decline, the fraction of firms that invest in production plants in S for intermediate and final goods raises, because of the “unit-cost complementarity”.

---

This graphical description is only an example of how the lines representing the cut-off levels could possibly run. Theoretically a utility loss from profit taxation and its impacts also does not arise if the lines representing the cut-off levels in graphic 4 run further on the right hand side than those in graphic 3. This depends on the exact values of the following variables:

- the decreased \( d_L \) relative to \( d_H \) and the corresponding tax rates \( t_N \) and \( t_S \)
- the decreased \( C \), which only is relevant for the Northern jurisdiction when selecting strategy \( N,S \) or \( S,S \)
- the distribution of the firms over the integration strategies

Furthermore, the following question has to be answered:

Which tax rates are selected by \( N \) and \( S \) if they can anticipate the firms’ reaction?

2.6.3 The governments’ decision with combined provision of public infrastructure for differentiated goods

First of all that the governments decide to select \( t_i > 0 \), combined tax revenue at least has to be high enough to finance \( R \).

Furthermore, for a government to really decide to provide public infrastructure, it has to consider the representative household’s utility. In each government’s jurisdiction the representative household’s utility with provision of public infrastructure for differentiated goods at least has to be as high as its utility in the \( t_i = 0 \) scenario. As shown before each household’s utility is described by:

\[
V_i = m_i + (1 - \alpha) \cdot \mu^\int_{0}^{\Theta} p_j(i(\mu(\alpha - 1) dj
\]

(9)

If taxes are levied, \( \alpha \) and \( \mu \) do not change.

The impact of \( t_i \) on \( m_i \) is different for \( N \) and \( S \). All firms enter the market in \( N \), and by assumption they belong to households in \( N \). Accordingly, levying \( t_i \) lets \( m_N \) decline, because the firms’ profits are part of the income of Northern households. Additionally introducing \( t_i > 0 \) also induces a change of optimal integration strategies. This does not lower the labor income of the households in the original production location, because they can work in the homogenous sector. For this reason taxation in either country does not influence income in \( S \).

Furthermore, in the setting of this model, effects on income – which are only induced by \( t_i \) for households in \( N \) – only influence the consumption of the homogeneous good. Consumption of differentiated goods only depends on prices. This negative impact on
the representative Northern household's utility has to be compensated by positive impacts from consumption of differentiated goods. Furthermore, the degree of taxation does not influence the mass of firms entering the market in the North, because of profit taxation if $0 < t_N < 1$. Firms that decide to enter the market at least make zero profits and these don't become negative, if governments tax firms' profits in the described way. The only restriction here fore is that $t_N < 1$ always holds true, because firms invest before entering the market. With this investment a firm gets to know its productivity level and therefore it decides to enter the market or not. If a firm makes positive profits, it enters the market, but if $t_N = 1$ would hold true, then any firm would always make zero profits, would never invest to get to know its productivity and therefore never enter the differentiated market. This would not be in the government's interest. The tax rate in S does not influence market entry either, because the least productive firms entering the market choose strategy N,N which is not influenced by $t_S$ anyhow.

Furthermore, it has to be known that a single firm’s prices do not change because of the taxes levied. These are only influenced by the transport costs. For this reason it is clear that:\(^{59}\)

$$p_{i(j)}^{\text{opt}} = \alpha - \left( \frac{\Theta}{dC_1} \right)^{(a-1)/\alpha}$$

(10)

Transport costs are passed on to the households, whereas taxes have to be paid by the firms.

As already said, taxation itself is not passed on to the households by raising prices. But as taxation induces low transport costs, because tax revenue is spent for public infrastructure for differentiated goods, prices are influenced because of the lower $d$ and because other integration strategies become reasonable for the firms. Here fore also the single firms’ outputs are influenced, because demand for differentiated goods only depends on prices.

Optimal integration strategies now are: N,N; S,N; N,S and S,S instead of N,N; S,N; N,NS and S,NS if transport costs are high. This induces the following changes in prices for differentiated goods on both markets:

---

\(^{59}\) See derivation XII in the Appendix.
1.) If firms select strategy N,N, their optimal prices for differentiated goods on the Northern market are defined by: \( \frac{1}{\alpha} \left( \frac{C(1,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \). These prices stay the same as without public good provision.

If firms select strategy N,N, their optimal prices for differentiated goods on the Southern market are defined by: \( \frac{1}{\alpha} \left( \frac{d \cdot C(1,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \), whereas these firms’ optimal prices for differentiated goods on the Southern market with high transport costs were defined by: \( \frac{1}{\alpha} \left( \frac{d_h \cdot C(1,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \). As \( d_L < d_H \), it can be seen that prices with public good provision on the Southern market are lower than without.

2.) If firms select strategy S,N their optimal prices for differentiated goods on the Northern market are defined by: \( \frac{1}{\alpha} \left( \frac{C(w,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \). These prices stay the same as without public good provision.

If firms select strategy S,N their optimal prices for differentiated goods on the Southern market are defined by: \( \frac{1}{\alpha} \left( \frac{d_L \cdot C(w,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \), whereas these firms’ optimal prices for differentiated goods on the Southern market with high transport costs were defined by: \( \frac{1}{\alpha} \left( \frac{d_h \cdot C(w,1)}{\Theta} \right)^{(\alpha-1)/\alpha} \). As \( d_L < d_H \), it can be seen that prices with public good provision on the Southern market are lower than without.

3.) If firms select strategy N,S their optimal prices for differentiated goods on the Northern market are defined by: \( \frac{1}{\alpha} \left( \frac{d \cdot C(1,w)}{\Theta} \right)^{(\alpha-1)/\alpha} \), whereas these firms’ optimal prices for differentiated goods on the Northern market with high transport costs were defined by: \( \frac{1}{\alpha} \left( \frac{C(1,w)}{\Theta} \right)^{(\alpha-1)/\alpha} \), when selecting N,NS as their optimal integration strategy. Because transport costs decrease with provision of public infrastructure,
\[
\frac{1}{\alpha} \left( \frac{d_i C(1,w)}{\Theta} \right)^{(\alpha-1)/\alpha} < \frac{1}{\alpha} \left( \frac{C(1)}{\Theta} \right)^{(\alpha-1)/\alpha}
\]
holds true. For this reason, these firms’ prices are lower for consumers on the Northern market than in the analysis with high transport costs.

If firms select strategy N,S their optimal prices for differentiated goods on the Southern market are defined by:
\[
\frac{1}{\alpha} \left( \frac{C(1,w)}{\Theta} \right)^{(\alpha-1)/\alpha}.
\]
These prices stay the same as without public good provision, because the Southern market’s demand already was supplied with these prices when firms selected strategy N,NS with high transport costs.

4.) If firms select strategy S,S their optimal prices for differentiated goods on the Northern market are defined by:
\[
\frac{1}{\alpha} \left( \frac{d_i C(w,w)}{\Theta} \right)^{(\alpha-1)/\alpha},
\]
whereas these firms’ optimal prices for differentiated goods on the Northern market with high transport costs were defined by:
\[
\frac{1}{\alpha} \left( \frac{C(w,1)}{\Theta} \right)^{(\alpha-1)/\alpha},
\]
when selecting S,NS as their optimal integration strategy. Because transport costs decrease with provision of public infrastructure,
\[
\frac{1}{\alpha} \left( \frac{d_i C(w,w)}{\Theta} \right)^{(\alpha-1)/\alpha} < \frac{1}{\alpha} \left( \frac{C(w,1)}{\Theta} \right)^{(\alpha-1)/\alpha}
\]
holds true. For this reason, these firms’ prices are lower for consumers on the Northern market than in the analysis with high transport costs.

If firms select strategy S,S their optimal prices for differentiated goods on the Southern market are defined by:
\[
\frac{1}{\alpha} \left( \frac{C(w, w)}{\Theta} \right)^{(\alpha-1)/\alpha}.
\]
These prices stay the same as without public good provision, because the Southern market’s demand already was supplied with these prices when firms selected strategy S,NS with high transport costs.
Summarizing, firms selecting strategies N,N or S,N in both analyses, either with low or with high d, supply the Northern market at the same and the Southern market at lower prices if tax revenue is invested in public good provision for differentiated goods. Firms selecting strategies N,S and S,S instead of N,NS and S,NS in previous analysis supply the Northern market’s demand to lower prices, whereas the prices in the South stay the same as in the analysis without taxation and with d_H.

This section describes the impact of taxation on the firms’ prices if a firm still selects the corresponding strategy to previous analysis without taxation and with d_H, i.e.

- selecting N,N with low transport costs if they selected N,N with high transport costs,
- selecting S,N with low transport costs if they selected S,N with high transport costs,
- selecting N,S with low transport costs if they selected N,NS with high transport costs, and
- selecting S,S with low transport costs if they selected S,NS with high transport costs.

But optimal taxation depends on the impact of t_i on the representative household’s utility in a government’s jurisdiction. For example if the fixed costs relation is medium high some firms selecting S,NS in the analysis with high transport costs could possibly also find it reasonable to select N,N if its choice is influenced by taxation, whereas this decision again depends on a firm’s productivity level. In this case these firms’ prices for its differentiated good would rise in both jurisdictions. As can be seen, this possibility depends on the exact tax rates in both countries, which induce a shift of the graphical analogues to the cut-off levels in the previous graphical analysis either into one direction or into the other. For this reason the impact of taxation on a representative household’s utility from differentiated goods depends on the distribution of the firms over the integration strategies. If these firms’ prices do not rise either, the impact of taxation on a representative household’s utility from differentiated goods could not decline in this model, whereas both governments know the firms’ distribution over the integration strategies when selecting tax rates. If the total impact of taxation at least does not let utility decline in both jurisdictions both governments select t_i>0, as long as total tax revenue is high enough that R can be afforded and public infrastructure is provided.

This description shows all effects of taxation on the households’ utility depending on the parameter configurations in general.

Precisely this is shown in the case if the fixed costs relation is medium high.
In other words, if \( \gamma_L < \frac{g}{f} < \gamma_H \) holds true so that only strategies N,N or S,S are optimal. Because transport costs have decreased, taxation is introduced into the model and the set of optimal integration strategies has changed, the range of parameters for which \( \gamma_L < \frac{g}{f} < \gamma_H \) holds true has changed in comparison to the analysis without taxation and with high transport costs.

Now, only if the fixed costs relation is given by:60

\[
\gamma_L = \frac{X^N}{d_L C(w,1)} + \frac{X^S}{C(w,1)} - \frac{X^N}{d_L C(1,1)} - \frac{X^S}{d_L C(1,1)} (1-t_N) \leq \frac{g}{f} \quad \text{and} \quad (11)
\]

\[
g < \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} (1-t_S) - \frac{X^N}{d_L C(1, w)} - \frac{X^S}{C(1, w)} (1-t_N) = \gamma_H \quad (12)
\]

N,N and S,S are the only optimal strategies in this setting. Given this situation, at least \( V_{\text{new}} = V_{\text{old}} \) has to hold true for that \( t > 0 \) is selected, as already described above.

For this reason the Southern government is confronted with following utility functions:

\[
V_{S,new} = m_s + (1-\alpha)\mu \gamma(1-a) \left[ \int_0^{\theta_{(N,S,S)}} p_S(j)^\alpha(1-a) dj + \int_{\theta_{(N,N,S,S)}}^{\theta_{max}} p_S(j)^\alpha(1-a) dj \right] \quad (9a)
\]

\[
V_{S,old} = m_s + (1-\alpha)\mu \gamma(1-a) \left[ \int_0^{\theta_{(N,N,S,S)}} p_S(j)^\alpha(1-a) dj + \int_{\theta_{(N,N,S,S)}}^{\theta_{max}} p_S(j)^\alpha(1-a) dj \right] \quad (9b)
\]

This leads to:

60 See derivation XIII and XIV in the Appendix.
Only if parameter configurations are ensured for which 0<(1-\(t_S\))<1 holds true, 0<\(t_S\)<1 can be selected by the Southern government.\(^{61}\) \(E\) is positive and only then anyhow 0<\(t_S\)<1 can possibly be reasonable, whereas optimal taxation depends on the exact parameter configurations and not every configuration lets equation (13) become reasonable for 0<\(t_S\)<1. If 0<\((1-\alpha)\mu\gamma(\alpha-1)(\frac{1}{\alpha})\gamma(\alpha-1)\left(\frac{1}{2d_L C(1,1)} - \frac{1}{2C(w, w)}\right)\left(\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}\right)^2\) does not hold true, the optimal tax rate in S is zero.

\(^{61}\) See derivation XV in the Appendix. Furthermore:

\[ E = -\left(\frac{1}{2d_L C(1,1)} - \frac{1}{2C(w, w)}\right)\left(\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}\right)^2\]

and

\[ D = \left(1 - \alpha\right)\mu\gamma(\alpha-1)(\frac{1}{\alpha})\gamma(\alpha-1)\].

\(^{62}\) See derivation XVI in the Appendix.
As can be seen from above, the optimal Southern tax rate depends on the Northern tax rate. For this reason also the reaction of $t_S$ on $t_N$ is to be looked at. It has to be considered that in contrast to $m_N$ income $m_S$ is independent of $t_N$ and $t_S$.

$$\frac{\partial (1 - t_S)}{\partial (1 - t_N)} = \frac{\frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)}}{\frac{d_L C(w, w)}{C(w, w)}}$$

(14)

Accordingly, $\frac{\partial (1 - t_S)}{\partial (1 - t_N)}$ is never negative.

So if taxation itself is reasonable, $\frac{\partial (1 - t_S)}{\partial (1 - t_N)}$ is positive, i.e. if the tax rate in $N$ rises, the tax rate in $S$ rises, and if the tax rate in $N$ declines, the tax rate in $S$ declines, too. Because of this correlation the governments can set tax rates in such a way that combined tax revenue is high enough to finance $R$.

As can be seen from the derivation above, whether profit taxation in $S$ is reasonable, depends on the parameter configurations, and the optimal tax rate $0 < t_S < 1$, in the case that it exists, is positively correlated with $t_N$.

The same analysis can be done from the Northern government's perspective, which is confronted with the following utility functions:

$$V_{N\text{new}} = m_{N\text{new}} + (1 - \alpha)\mu \left[\int_0^{\Theta(N,N,S,S)} p_N(j)^{\alpha/(\alpha - 1)} dj + \int_{\Theta(N,N,N)}^{\Theta_{\text{max}}} p_N(j)^{\alpha/(\alpha - 1)} dj\right]$$

(9c)

$$V_{N\text{old}} = m_{N\text{old}} + (1 - \alpha)\mu \left[\int_0^{\Theta(N,N,S,N)} p_N(j)^{\alpha/(\alpha - 1)} dj + \int_{\Theta(N,N,S)}^{\Theta_{\text{max}}} p_N(j)^{\alpha/(\alpha - 1)} dj\right]$$

(9d)

This leads to:
Only if parameter configurations are ensured for which 0<(1-t_N)<1 holds true, 0<t_N<1 can be selected by the Northern government.\footnote{See derivation XVII in the Appendix. Furthermore:} In contrast to the analysis for the South, where E was positive anyhow, K can be positive or negative. Only if K is positive, 0<t_N<1 can possibly be reasonable, whereas optimal taxation depends on the exact parameter configurations and not every configuration lets equation (15) become reasonable for 0<t_N<1. If 0<\(\frac{\overline{X}^N}{d_L C(w,w)} + \frac{\overline{X}^S}{C(w,w)}\)\((1-t_s)\)\[\pm [K]\frac{1}{2} = (1-t_N)_{1/2}\] (15)

optimal tax rate 0<t_N<1 can be derived from equation (15).\footnote{See derivation XVIII in the Appendix.}

If 0<\(\frac{\overline{X}^N}{d_L C(w,w)} + \frac{\overline{X}^S}{C(w,w)}\)\((1-t_s)\)\[\pm [K]\frac{1}{2}<1\] does not hold true, the optimal tax rate in N is zero.

\[K = -\frac{(1-\alpha)_\mu}{(\alpha-1)\left[\frac{1}{\alpha}\right]}\frac{1}{\overline{X}^N C(1,1)} - \frac{1}{2d_L C(w,w)}\ \text{and}\ \frac{\overline{X}^N}{C(1,1)} + \frac{\overline{X}^S}{d_L C(1,1)}\]

\[B = \left(m_{\text{new}} - m_{\text{old}} + (1-\alpha)_\mu\left(\frac{1}{\alpha}\right)^{(\alpha-1)} \right)^{\gamma(\alpha-1)}\]

\[
\left[ \frac{1}{2d_L C(w,w)} \right] - \left[ \frac{1}{2C(1,1)} + \frac{1}{2C(w,1)} - \frac{1}{2C(w,1)} \right] \left[ \frac{f+g}{\left(\frac{\overline{X}^N}{C(w,1)} - \frac{\overline{X}^N}{C(1,1)} - \frac{\overline{X}^S}{C(w,w)} - \frac{\overline{X}^S}{d_L C(1,1)} \right)^2} \right]
\]
As can be seen from equation (15) the optimal Northern tax rate depends on the Southern tax rate. For this reason the reaction of \( t_N \) on \( t_S \) is to be looked at, whereas it has to be considered that income \( m_{N\text{new}} \) depends on \( t_N \) and \( t_S \) and additionally \( t_N \) and \( t_S \) depend on one another.\(^{65}\)

\[
\frac{\partial (1 - t_N)}{\partial (1 - t_S)} = \pm (\nu) \cdot \left[ \frac{\bar{X}^N}{d_L C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right] \left[ \frac{\bar{X}^N}{C(11) + d_L C(11)} + \frac{\bar{X}^S}{C(11) + d_L C(11)} \right] \tag{16}
\]

Whereas \( \nu \) is defined by:

\[
\nu = -\frac{1}{2} \left[ \frac{(1 - \alpha) \mu \left[ \frac{\gamma}{\gamma - 1} \left( \frac{1}{\alpha} \right) \right]^{(\alpha - 1)}}{2C(11) - 2d_L C(w,w)} \left[ \frac{\bar{X}^N}{C(11) + d_L C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] \right]^{\frac{1}{2}} \cdot (B)^{-\frac{3}{2}}\]

\[
\frac{\partial m_{N\text{new}}}{\partial (1 - t_S)} \right) \left( \frac{\partial m_{N\text{new}}}{\partial (1 - t_N)} \right) \left( \frac{\partial (1 - t_N)}{\partial (1 - t_S)} \right)
\]

The effect of \( \nu \) is new compared to the analysis for \( S \), because income in \( N \) is influenced by taxation.

1.) Income of households in \( N \) depends on taxation in \( N \) and \( S \).

Thereby

\[
\frac{(1 - \alpha) \mu \left[ \frac{\gamma}{\gamma - 1} \left( \frac{1}{\alpha} \right) \right]^{(\alpha - 1)}}{2C(11) - 2d_L C(w,w)} \left[ \frac{\bar{X}^N}{C(11) + d_L C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] \left( \frac{\partial m_{N\text{new}}}{\partial (1 - t_S)} \right) \left( \frac{\partial m_{N\text{new}}}{\partial (1 - t_N)} \right) \left( \frac{\partial (1 - t_N)}{\partial (1 - t_S)} \right)
\]

always is positive. If income in \( N \) is so far negatively influenced by taxation that \( B \) becomes negative,

\(^{65}\) See derivation XIX in the Appendix.
taxation no longer is reasonable from the Northern government’s point of view. For finding an optimal $0 < t_N < 1$, this case already is neglected above in equation (15) with the condition that $K$ may not be negative. Otherwise a negative impact of taxation on income cannot be compensated by the positive impact on prices.

2.) If B is positive, taxation can be reasonable, but again this depends on exact parameter configurations.

Solving (16) with a positive B following conditions can be derived:

\[
\frac{\partial (1-t_N)}{\partial (1-t_S)} = \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] - \left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] \left( \frac{1}{2} \right) \left( 1 - \frac{1}{2} \frac{1}{\alpha \mu (1-\alpha)} \left( \frac{1}{2} \right)^{(\alpha-1)} \left( \frac{1}{\alpha} \right)^{(\alpha-1)} \left( \frac{1}{2C(1,1)} - \frac{1}{2d_L C(w, w)} \right) \right) \left( \frac{1}{2} \right) \left( B \right)^{3/2} \left. \frac{\partial m_{N \text{new}}}{\partial (1-t_S)} \right|_{t_N = 0}^1 \left( B \right)^{-3/2} \left. \frac{\partial m_{N \text{new}}}{\partial (1-t_N)} \right|_{t_S = 0}^{-1}.
\]
Or

\[
\frac{\partial (1-t_N)}{\partial (1-t_S)} = \left[ \frac{X^N}{d_N C(w,w)} + \frac{X^S}{C(1,1) + d_L C(1,1)} \right] + \left[ \frac{X^N}{d_N C(w,w)} + \frac{X^S}{C(1,1) + d_L C(1,1)} \right]
\]

\[
\frac{1}{2} \left\{ 1 - \frac{1}{2} \right\}
\left[ \left( 1 - \alpha \right) \mu \left( u_{(a-1)} \frac{1}{\alpha} \right)^{(a-1)} \left( \frac{1}{2} - \frac{1}{2d_L C(w,w)} \right) \right. \\
\left. - \frac{1}{2} \left( 1 - \alpha \right) \mu \left( u_{(a-1)} \frac{1}{\alpha} \right)^{(a-1)} \left( \frac{1}{2} - \frac{1}{2d_L C(w,w)} \right) \right] \\
\left( f + g \right)^2
\]

(B) \left[ \frac{\partial m_{\text{new}}}{\partial (1-t_S)} \right]^{\frac{1}{2}}

(B) \left[ \frac{\partial m_{\text{new}}}{\partial (1-t_N)} \right]^{-1}

The tax rates \( t_N \) and \( t_S \) are always positively correlated so that \( 0 < t_S < 1 \) is selected by the government in \( S \). For this reason the governments in \( S \) and \( N \) will only find a solution for the provision of public infrastructure if \( \frac{\partial (1-t_N)}{\partial (1-t_S)} \) is positive too.

Additionally to the above mentioned restrictions for a tax rate \( 0 < t_N < 1 \), this can only be achieved if parameters are configured in such way that either (17) or (18) yields a positive \( \frac{\partial (1-t_N)}{\partial (1-t_S)} \).

For this reason following conditions can be derived from (17) and (18):
a) Condition from (17):
\[
\left[ \frac{\dot{X}^N}{d_L C(w,w)} + \frac{\dot{X}^S}{C(w,w)} \right] > \left[ \frac{\dot{X}^N}{C(1,1)} + \frac{\dot{X}^S}{d_L C(1,1)} \right]
\]
\[
\begin{pmatrix}
\frac{1}{2} & -(1-\alpha)\mu^{\frac{1}{\alpha-1}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha-1}} \left[ \frac{1}{2C(1,1)} - \frac{1}{2d_L C(w,w)} \right] \\
\left( \frac{\dot{X}^N}{C(1,1)} + \frac{\dot{X}^S}{d_L C(1,1)} \right)^2 & (f + g)^2 \\
\end{pmatrix}^{\frac{1}{2}} \left( B \right)^{-\frac{3}{2}} \cdot \frac{\hat{m}_{N\text{new}}}{\partial(1-t_S)}
\]

Otherwise \( \frac{\partial (1-t_N)}{\partial (1-t_S)} > 0 \) could not be derived and public infrastructure would not be provided. Then only (18) could possibly deliver a solution.

b) Condition from (18):
\[
\begin{pmatrix}
\frac{1}{2} & -(1-\alpha)\mu^{\frac{1}{\alpha-1}} \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha-1}} \left[ \frac{1}{2C(1,1)} - \frac{1}{2d_L C(w,w)} \right] \\
\left( \frac{\dot{X}^N}{C(1,1)} + \frac{\dot{X}^S}{d_L C(1,1)} \right)^2 & (f + g)^2 \\
\end{pmatrix}^{\frac{1}{2}} \left( B \right)^{-\frac{3}{2}} \cdot \frac{\hat{m}_{N\text{new}}}{\partial(1-t_N)}
\]

Otherwise \( \frac{\partial (1-t_N)}{\partial (1-t_S)} \) would be negative or not defined and public infrastructure would not be provided. Then only (17) could possibly deliver a solution.

Following these conditions, if taxation is reasonable at all, all parameter configurations serve a positive \( \frac{\partial (1-t_N)}{\partial (1-t_S)} \). In this case the governments can set tax rates in such a way that combined tax revenue is high enough to finance R. Theoretically a further argument against tax rates being correlated negatively exists. Optimal integration strategies from the firms’ point of view are defined endogenously.
This implies that taxation is considered by the firms when selecting their optimal integration strategies. E.g. if public infrastructure was provided but \( t_N \) was very low and \( t_S \) was very high, it could be optimal to select strategy \( S,NS \) instead of strategy \( S,S \) from a firms perspective even though transport costs were low. This would not be in the interest of a benevolent planner, because taxes are not passed on to households, only transport costs. For this reason the governments have to take into account that optimal integration strategies from the firms’ point of view stay the same as the welfare optimal ones when setting tax rates.

Furthermore, it can be seen that the utility functions of the representative households in both countries are different as income and prices are not the same in both jurisdictions. For this reason only by chance the chosen tax rates of both governments would correspond and a government would not have an incentive to deviate. Instead, if \( N \) selected a tax rate given \( t_S \), \( S \) would react on \( t_N \) again and select another \( t_S \) to be optimal. Then again \( t_N \) could select a higher tax rate without inducing a utility loss of the representative household in its jurisdiction. This would become a dynamic process. For this reason, both governments cooperate with each other. They select tax rates \( t_N \) and \( t_S \) for which combined tax revenue is high enough that \( R \) can be afforded. A further reaction selecting an even higher \( t_i \) than to finance \( R \) is not induced. So a possibly resulting utility gain can be given to all households by setting \( t_N \) and \( t_S \). Then neither jurisdiction has an incentive to deviate.

So, to achieve these tax rates further the required tax revenue to finance \( R \) has to be illustrated. If because of the analysis above positive values of \( t_i \) are reasonable, the governments have to achieve combined tax revenue that is high enough to cover \( R \).

Without the governments’ actions \( \frac{C(1,1)}{C(1,w)} < d_i \) holds true, but because of the governments investing \( R \), \( d \) declines and therefore \( \frac{C(1,1)}{C(1,w)} > d_L \) can be achieved. Up to now, only optimal tax rates \( t_N \) and \( t_S \) have been illustrated, saying that tax revenue is high enough to cover \( R \) and because firms know that \( d \) declines if tax rates are introduced, they change their optimal integration strategies from \( N,N, N,NS, S,N \) and \( S,NS \) to \( N,N, N,S, S,N \) and \( S,S \). Because a positive correlation of \( t_N \) and \( t_S \) holds true, if \( 0 < t_i < 1 \) is selected by the governments, the required total tax revenue can then always be achieved to finance \( R \).
Again this analysis exemplarily describes the situation when the fixed costs relation for production plants in S is medium high.

For this reason tax revenue in N is defined in the following way:

\[ t_N \left[ \int_0^{\epsilon_{(N,N,S,S)}} \left( \frac{\bar{X}^N\Theta(j)}{C(1,1)} + \frac{\bar{X}^S\Theta(j)}{d_j C(1,1)} \right) dj \right] \]

The formal description of the Southern government’s tax revenue then is described by:

\[ t_S \left[ \int_{\theta_{(N,N,S,S)}}^{\epsilon_{\max}} \left( \frac{\bar{X}^N\Theta(j)}{d_j C(w,w)} + \frac{\bar{X}^S\Theta(j)}{C(w,w)} \right) dj \right] \]

Hence, if the fixed costs relation for production plants in S is medium high, and it is ensured that utility of the representative households in both countries does not decline by levying taxes the optimal tax rates of both governments additionally have to ensure combined tax revenue of:

\[ t_N \left[ \int_0^{\epsilon_{(N,N,S,S)}} \left( \frac{\bar{X}^N\Theta(j)}{C(1,1)} + \frac{\bar{X}^S\Theta(j)}{d_j C(1,1)} \right) dj \right] + t_S \left[ \int_{\theta_{(N,N,S,S)}}^{\epsilon_{\max}} \left( \frac{\bar{X}^N\Theta(j)}{d_j C(w,w)} + \frac{\bar{X}^S\Theta(j)}{C(w,w)} \right) dj \right] = R \]

The illustrated tax rates are optimal from both governments’ perspectives for that the households’ utility at least does not decline when levying \( t_i \) and for that their tax revenues are high enough to be able to provide public infrastructure for differentiated goods, because tax revenue is as high as R.

If the described equations hold true and \( 0<ti<1 \) is selected by the governments the representative household’s utility in both jurisdictions rises in comparison to the scenario without taxation. For this reason, depending on parameter configurations, profit taxation by the social planner for N and S to finance combined public infrastructure, can be a reasonable political instrument.

If the described equations do not hold true and \( 0<ti<1 \) cannot be selected, because of the parameter configurations, then profit taxation to finance public good provision is not optimal from a social planners perspective. As combined tax revenue is needed to provide public infrastructure anyhow taxation by a single government never is optimal in this analysis. It only leads to optimal profit taxation from a social planners perspective.
3 Conclusion

In this analysis a trade-off between fixed costs and high per-unit variable costs is identified. Here firms can choose between different integration strategies. Their headquarters are located in the North and they serve the Northern and the Southern market with differentiated products. Every single firm has to produce intermediate and final goods for one’s self, whereas they can choose the North, the South or both places as production location for both production activities. Resulting, many different integration strategies can be identified. Their optimality depends on the relative size of fixed costs for MNE activities, the size of transport costs for final goods, both markets’ fraction of demand, where the wages in the South are relatively low, a single firm’s productivity and both governments’ degrees of profit taxation.

First the case with high transport costs is analyzed and taxes are excluded. Transport of intermediate goods by assumption is for free.

Low productive firms choose a strategy that minimizes fixed costs, whereas high productive firms minimize the per-unit variable costs to supply both markets, whereas the transport costs are considered.

If transport costs for final goods are high some firms depending on their productivity can find it optimal to produce intermediate goods in one country and final goods in both countries locally.

As a result of the analyses with high transport costs for final goods, its impact on the economic outcome becomes apparent. The higher transport costs are, the more local production is preferred by firms. Namely transport costs affect the per-unit variable costs.

Then, we analyze cases in which governments additionally levy profit taxes on final good production of firms in the differentiated sector, where they invest tax revenue in public infrastructure for differentiated goods. If tax revenue is high enough to cover costs associated with the provision of public infrastructure for differentiated goods, transport costs decline from high to low endogenously. Therefore the set of optimal integration strategies of the heterogeneous firms changes. Now, when transport costs for final goods are small no single activity is located at several places, rather every single activity, either intermediate or final production, is located in the North or in the South. Thereby, profit functions now additionally also depend on tax rates.
So, this analysis can economically explain optimal tax policies of governments when firms’ integration strategies are endogenous with comparative advantages on the one hand as well as with elements of the “New Trade Theory” on the other. Governments select tax rates in such a way that the representative household’s utility in its jurisdiction at least does not decline, as they are benevolent planners. If parameters are configured in such a way that this condition cannot hold optimal tax rates are zero. Levying taxes has several impacts on the utility of households in both countries: They influence prices paid for differentiated goods and income in N, but households don’t gain utility by consuming public infrastructure directly. If 0<\(t_i<1\) is selected from the governments, because a utility loss does not arise in either jurisdiction, the tax rates always are positively correlated. For this reason then total tax revenue always can be set high enough that public infrastructure will be provided. So the level of costs for public good provision is no restriction for the decision to levy taxes or not. If taxation is reasonable from both governments point of view, costs to finance public infrastructure only influence the exact level of 0<\(t_i<1\).

As higher tax revenue than to finance costs for public good provision is not required, if 0<\(t_i<1\) is selected, tax rates can be set in such way that the representative household’s utility can even rise. So because of the by taxation induced lower transport costs the economic outcome can even rise. The always more proceeding economic integration is positive from both governments perspective and in this model taxation also is reasonable from a world welfare perspective.

But using the derived model to achieve optimal tax rates, zero taxation can be the best choice of governments acting as benevolent planners, which depends on the exact parameter configurations. Whereas in these cases zero taxation is optimal from the welfare perspective of both governments, but not as shown in other literature, due to a race-to-the-bottom scenario under tax competition.

\[66\] The first approach in this direction was derived by Krugman (1979).
References


Appendix

Derivation I:

\[ U_i = x_0 + x_i, \ i \in \{A,B\} \]

\[ X = \frac{1}{\mu \alpha^\alpha} \left[ \sum_{j=0}^{\Theta_{\max}} x^i(j)^\alpha dj \right] \]

\[ L = x_0 + \frac{1}{\mu \alpha^\alpha} \left[ \sum_{j=0}^{\Theta_{\max}} (x^i(j) + \gamma z(j))^\alpha dj \right] + \lambda \left[ m_i - x_0 - \sum_{j=0}^{\Theta_{\max}} p_i(j)(x^i(j) + \gamma z(j))dj \right] \]

\[ m = \text{earnings} \]

\[ \frac{\partial L}{\partial x_0} = 1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \]

\[ \frac{\partial L}{\partial \lambda} = m_i - x_0 - \sum_{j=0}^{\Theta_{\max}} p_i(j)(x^i(j) + \gamma z(j))dj = 0 \quad \Rightarrow x_0 = m_i - \sum_{j=0}^{\Theta_{\max}} p_i(j)(x^i(j) + \gamma z(j))dj \]

\[ \frac{\partial L}{\partial \gamma} = \frac{\alpha}{\mu \alpha^\alpha} \left[ \sum_{j=0}^{\Theta_{\max}} (x^i(j) + \gamma z(j))^{(\alpha-1)} z(j) dj \right] - \sum_{j=0}^{\Theta_{\max}} p_i(j)z(j) dj = 0 \]

If \( \gamma = 0 \):

\[ \frac{\partial L}{\partial \gamma} = \frac{\alpha}{\mu \alpha^\alpha} \left[ \sum_{j=0}^{\Theta_{\max}} x^i(j)^{(\alpha-1)} - p_i(j) \right] z(j) dj = 0 \]

\[ \Rightarrow \sum_{j=0}^{\Theta_{\max}} \left[ \frac{\alpha}{\mu \alpha^\alpha} x^i(j)^{(\alpha-1)} - p_i(j) \right] z(j) dj = 0 \]

\[ \Rightarrow \left[ \frac{\alpha}{\mu \alpha^\alpha} x^i(j)^{(\alpha-1)} - p_i(j) \right] = 0 \]

\[ \Rightarrow x^i(j) = \alpha [p_i(j) \mu]^{(\alpha-1)} \]

This \( x^i(j) \) is the demand of one household for a firm's variety in country \( i \). Total demand for a firm's variety in country \( i \) therefore is given by:

\[ x^i(j) = M^i [p_i(j) \mu]^{(\alpha-1)} \]

Derivation II:

Inserting (3) in (4) following condition arises:

\[ \pi = \frac{\alpha}{(\alpha-1) \mu} y^{(\alpha-1) \alpha \mu} M_N + p_S j^{(\alpha-1) \alpha \mu} y^{(\alpha-1) \alpha M^S} - p_N j^{(\alpha-1) \mu} y^{(\alpha-1) \alpha M^N} c_N \]

\[ - p_S j^{(\alpha-1) \mu} y^{(\alpha-1) \alpha M^S} c_S - k \]  

(4')
\[ X = \mu \frac{\gamma}{\alpha-\gamma} \alpha \]

According:

\[
\frac{\partial \pi}{\partial p} = \frac{\alpha}{1 - \alpha} p_N(j)^{\gamma - (1 - \alpha) \alpha} 
+ \frac{\alpha}{1 - \alpha} p_S(j)^{\gamma - (1 - \alpha) \alpha} 
- \frac{1}{1 - \alpha} p_N(j)^{\gamma - (1 - \alpha) \alpha} \frac{c_N}{\theta} 
- \frac{1}{1 - \alpha} p_S(j)^{\gamma - (1 - \alpha) \alpha} \frac{c_S}{\theta} = 0
\]

Solving this for \( p_N \) and \( p_S \):

\[
p_N(j)_{\text{opt}} = \frac{1}{\alpha} \frac{c_N}{\theta}, \quad p_S(j)_{\text{opt}} = \frac{1}{\alpha} \frac{c_S}{\theta}
\]

**Derivation III:**

By inserting \( p_N(j)_{\text{opt}} \) in

\[
\sum p_N(j)^{\gamma - (1 - \alpha) M^N X} = \sum M^S X p_S(j)^{\gamma - (1 - \alpha) \alpha} \frac{c_S}{\theta} - k
\]

following profit condition for each firm derives:

\[
\pi^* = M^N X \left( \frac{c_N}{\theta} \right)^{\gamma - (1 - \alpha) \alpha} \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right) - k
\]

or

\[
\pi^* = M^N X \left( \frac{c_N}{\theta} \right)^{\gamma - (1 - \alpha) \alpha} \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right) + M^S X \left( \frac{c_S}{\theta} \right)^{\gamma - (1 - \alpha) \alpha} \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right) - k
\]

Whereas \( \Theta = \gamma^{\gamma - (1 - \alpha) \alpha} \), \( dC = \gamma^{\gamma - (1 - \alpha) \alpha} \) and

\[
\frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha = \mu \frac{\gamma}{\alpha - \gamma} \alpha \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right)
\]

is a country neutral size. Furthermore:

\[
M^N X \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right) = X^N \text{ and } M^S X \left( \frac{1}{\alpha} \gamma - (1 - \alpha) \alpha \frac{1}{\alpha} \right) = X^S
\]

whereas \( X^S < X^N \), because the market share of the South is smaller than of the North.

For this reason the profit function is:

\[
\pi_{ab}^* = \frac{X^N}{dC(j, r)} + \frac{X^S}{dC(j, r)} - k
\]

**IV: All seven possible profit functions with transport costs for final goods**

\[
\pi_{NN} = \frac{X^N}{C(1,1)} + \frac{X^S}{C(1,1)}
\]

\[
\pi_{NS} = \frac{X^N}{C(1, w)} + \frac{X^S}{C(1, w)} - f
\]
Derivation V:

For that this set of integration strategies is optimal, following is necessary:

\[ \pi_{NS} \succ \pi_{NS} \]

\[ \pi_{SN} \succ \pi_{SN} \]

\[ \pi_{SN} \succ \pi_{NS,NS} \]

This has to be the case, because each of these profit function pairs contains the same fixed costs and only differ by the components of per-unit variable costs and transport costs.

\[ \pi_{NNS} = \frac{\bar{X}^N}{dC(1,1)} + \frac{\bar{X}^S}{C(1,1)} - f \succ \frac{\bar{X}^N}{d_{11}C(1,1)} + \frac{\bar{X}^S}{C(1,1)} - f = \pi_{NS} \]

\[ \pi_{SN} = \frac{\bar{X}^N}{dC(w,w)} + \frac{\bar{X}^S}{C(w,w)} - f - g \succ \frac{\bar{X}^N}{d_{11}C(w,w)} + \frac{\bar{X}^S}{C(w,w)} - f - g = \pi_{SS} \]

\[ \pi_{NS} = \frac{\bar{X}^N}{dC(w,w)} + \frac{\bar{X}^S}{C(w,w)} - f - g \succ \frac{\bar{X}^N}{d_{11}C(w,w)} + \frac{\bar{X}^S}{C(w,w)} - f - g = \pi_{NS,NS} \]

If \( \pi_{SN,NS} > \pi_{SS} \) also \( \pi_{SN,NS} > \pi_{NS,NS} \) always holds, because \( C(w,1) < C(1,1) \).

If this is true it can also be followed from the equations that:

\[ C(1,1) < d_{11} C(1,1) \]  
\[ C(w,1) < d_{11} C(w,w) \]

holds.

Transformation delivers:

\[ \frac{C(1,1)}{C(1,1)} < d_{11} \text{ and } \frac{C(w,1)}{C(w,w)} < d_{11} \]

This proves that this set of optimal integration strategies only is optimal, if high transport costs exist.
Derivation VI:
To calculate this $\pi_{NN}$ is equated with $\pi_{S,NS}$. Then $\pi_{NNS}$ and $\pi_{NN}$ are compared at this location and for N,NS to be an optimal strategy it is necessary that: $\pi_{NNS} \geq \pi_{NN}$.

$\pi_{NN} = \pi_{S,NS}$

$$\frac{X^N}{C(1,1)} + \frac{X^S}{d_H C(1,1)} = \frac{X^N}{C(w,1)} + \frac{X^S}{C(w, w)} - f - g$$

$$\Theta(N,N;S,NS) = \frac{(f + g)}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}$$

This is the x-axis coordinate. At this point $\pi_{NNS}$ has to be greater than $\pi_{NN}$.

For this reason the y-axis coordinate of $\pi_{NN}$ is required:

$$y_{NN} = \left(\frac{X^N}{C(1,1)} + \frac{X^S}{d_H C(1,1)}\right) \left(\frac{f + g)}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right)$$

y-axis coordinate of $\pi_{NNS}$:

$$y_{NNS} = \left(\frac{X^N}{C(1,1)} + \frac{X^S}{C(1,w)}\right) \left(\frac{f + g)}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right) - f$$

It is necessary that:

$$y_{NNS} \geq y_{NN}$$

$$\left(\frac{X^N}{C(1,1)} + \frac{X^S}{C(1,w)}\right) \left(\frac{f + g)}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right) - f \geq$$

$$\left(\frac{X^N}{C(1,1)} + \frac{X^S}{d_H C(1,1)}\right) \left(\frac{f + g)}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right)$$

$$\left(\frac{X^N}{C(1,1)} + \frac{X^S}{C(1,w)}\right) \left(\frac{1 + \frac{g}{f}}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right) - 1 \geq$$

$$\left(\frac{X^N}{C(1,1)} + \frac{X^S}{d_H C(1,1)}\right) \left(\frac{1 + \frac{g}{f}}{X^N(\frac{1}{C(w,1)} - \frac{1}{C(1,1)}) + X^S(\frac{1}{C(w, w) - \frac{1}{d_H C(1,1)}}}\right)$$
\[
1 + \frac{g}{f} \geq \frac{X_N^{1,1}(1 - 1)}{C(w,1) - C(1)} + \frac{X_S^{1,1}(1 - 1)}{C(w,w) - d_{\mu}C(1)}
\]

\[
g \geq \frac{X_N^{1,1}(1 - 1)}{C(w,1) - C(1)} + \frac{X_S^{1,1}(1 - 1)}{C(w,w) - d_{\mu}C(1)} - 1
\]

\[
g \left( \frac{X_S^{1,1}}{C(1) - d_{\mu}C(1)} - \frac{X_N^{1,1}}{C(1) - C(w,1)} \right) \geq \left( \frac{X_N^{1,1}}{C(w,1) - C(1)} - \frac{X_N^{1,1}}{C(w,w) - d_{\mu}C(1)} \right) - \frac{X_S^{1,1}}{C(1) - C(w,w)} + \frac{X_S^{1,1}}{C(1) - d_{\mu}C(1)}
\]

\[
g \geq \frac{X_N^{1,1} - X_N^{1,1} + X_S^{1,1} - X_S^{1,1}}{C(1) - d_{\mu}C(1)} = \gamma_H
\]

**Derivation VII:**

\[
\pi_{NN} = \pi_{SNS}
\]

\[
\frac{X_N^{\Theta}}{C(1)} + \frac{X_S^{\Theta}}{d_{\mu}C(1)} = \frac{X_N^{\Theta}}{C(w,1)} + \frac{X_S^{\Theta}}{C(w,w)} - f - g
\]

\[
\Theta(N,N;S,NS) = \frac{(f + g)}{X_N^{1,1}(1 - 1)} + \frac{(f + g)}{X_S^{1,1}(1 - 1)} - 1
\]

\[
\text{This is the } x\text{-axis coordinate. At this point } \pi_{SN} \text{ has to be greater than } \pi_{NN}.
\]

For this reason the y-axis coordinate of \( \pi_{NN} \) is required:

\[
y_{NN} = \left( \frac{X_N^{1,1} + X_S^{1,1}}{d_{\mu}C(1)} \right) - \frac{(f + g)}{X_N^{1,1}(1 - 1) + \frac{X_N^{1,1}(1 - 1)}{C(w,1) - C(1)} + \frac{X_S^{1,1}(1 - 1)}{C(w,w) - d_{\mu}C(1)}}
\]

\[
y\text{-axis coordinate of } \pi_{SN}:
\]

\[
y_{SN} = \left( \frac{X_N^{1,1} + X_S^{1,1}}{d_{\mu}C(1)} \right) - \frac{(f + g)}{X_N^{1,1}(1 - 1) + \frac{X_N^{1,1}(1 - 1)}{C(w,1) - C(1)} + \frac{X_S^{1,1}(1 - 1)}{C(w,w) - d_{\mu}C(1)}}
\]

\[
\text{It is necessary that:}
\]

\[
y_{SN} \geq y_{NN}
\]
\[
\begin{align*}
\left( \frac{\chi_N}{C(w,1) + d_H C(w,1)} + \frac{\chi_S}{d_H C(w,1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1)} + \frac{1}{C(1)} \right) - \chi_S \left( \frac{1}{C(w,1)} - \frac{1}{C(1)} \right) - g} \\
\left( \frac{\chi_N}{C(1)} + \frac{\chi_S}{d_H C(1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1) - C(1)} \right) + \chi_S \left( \frac{1}{C(1)} - \frac{1}{C(w,1)} \right)} \\
\left( \frac{\chi_N}{C(1)} + \frac{\chi_S}{d_H C(1)} \right) & \geq \frac{(f + 1)}{g} \\
\left( \frac{\chi_N}{C(w,1) + d_H C(w,1)} + \frac{\chi_S}{d_H C(w,1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1)} - \frac{1}{C(1)} \right) + \chi_S \left( \frac{1}{C(w,1)} - \frac{1}{C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) - C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) - C(1)} \right)} \\
\left( \frac{\chi_N}{C(w,1) + d_H C(w,1)} - \frac{\chi_N}{C(1)} - \frac{\chi_S}{d_H C(1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1)} - \frac{1}{C(1)} \right) + \chi_S \left( \frac{1}{C(w,1) - C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) - C(1)} \right)} \\
\left( \frac{\chi_N}{C(w,1)} - \frac{\chi_N}{C(1)} - \frac{\chi_S}{d_H C(1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1)} - \frac{1}{C(1)} \right) + \chi_S \left( \frac{1}{C(w,1) - C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) - C(1)} \right)} \\
\left( \frac{\chi_N}{C(w,1)} + \frac{\chi_S}{d_H C(w,1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1) + d_H C(w,1)} \right) - \chi_S \left( \frac{1}{C(w,1) + d_H C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) + d_H C(1)} \right)} \\
\left( \frac{\chi_N}{C(w,1)} + \frac{\chi_S}{d_H C(w,1)} \right) & \geq \frac{(f + g)}{\chi_N \left( \frac{1}{C(w,1) + d_H C(w,1)} \right) - \chi_S \left( \frac{1}{C(w,1) + d_H C(1)} \right) - \chi_S \left( \frac{1}{C(w,1) + d_H C(1)} \right)} \\
g \leq \frac{\chi_N}{C(w,1)} + \frac{\chi_S}{d_H C(w,1)} - \frac{\chi_N}{C(1)} - \frac{\chi_S}{d_H C(1)} = \gamma_L
\end{align*}
\]
Derivation VIII:

\[ \pi_{NN} = \pi_{NNS} \]

\[ \frac{X^{N_\Theta}}{C(11)} + \frac{X^{S_\Theta}}{d_\mu C(11)} = \frac{X^{N_\Theta}}{C(11)} + \frac{X^{S_\Theta}}{C(1, w)} - f \]

\[ \Theta \left( \frac{X^N}{C(11)} - \frac{X^N}{C(11)} + \frac{X^S}{d_\mu C(11)} - \frac{X^S}{C(1, w)} \right) = -f \]

\[ \Theta(N, N; N, NS) = \frac{f}{X^S - \left( \frac{1}{C(1, w)} - \frac{1}{d_\mu C(11)} \right)} \]

\[ \pi_{NN} = \pi_{SNS} \]

\[ \frac{X^{N_\Theta}}{C(11)} + \frac{X^{S_\Theta}}{d_\mu C(11)} = \frac{X^{N_\Theta}}{C(1, w)} + \frac{X^{S_\Theta}}{C(w, w)} - f - g \]

\[ \Theta \left( \frac{X^N}{C(11)} - \frac{X^N}{C(1, w)} + \frac{X^S}{d_\mu C(11)} - \frac{X^S}{d_\mu C(w, 1)} \right) = f + g \]

\[ \Theta(N, N; S, NS) = \frac{f + g}{X^N - \left( \frac{1}{C(1, w)} - \frac{1}{C(11)} \right) + X^S - \left( \frac{1}{C(w, w)} - \frac{1}{d_\mu C(11)} \right)} \]

\[ \pi_{NN} = \pi_{SN} \]

\[ \frac{X^{N_\Theta}}{C(11)} + \frac{X^{S_\Theta}}{d_\mu C(11)} = \frac{X^{N_\Theta}}{C(1, w)} + \frac{X^{S_\Theta}}{d_\mu C(w, 1)} - g \]

\[ \Theta \left( \frac{X^N}{C(11)} - \frac{X^N}{C(1, w)} + \frac{X^S}{d_\mu C(11)} - \frac{X^S}{d_\mu C(w, 1)} \right) = -g \]

\[ \Theta(N, N; S, N) = \frac{g}{X^N - \left( \frac{1}{C(1, w)} - \frac{1}{C(11)} \right) + X^S - \left( \frac{1}{d_\mu C(w, 1)} - \frac{1}{d_\mu C(11)} \right)} \]

\[ \pi_{NNS} = \pi_{SNS} \]

\[ \frac{X^{N_\Theta}}{C(11)} + \frac{X^{S_\Theta}}{C(1, w)} - f = \frac{X^{N_\Theta}}{C(1, w)} + \frac{X^{S_\Theta}}{C(w, w)} - f - g \]

\[ \Theta \left( \frac{X^N}{C(11)} - \frac{X^N}{C(1, w)} + \frac{X^S}{C(w, w)} - \frac{X^S}{C(1, w)} \right) = g \]

\[ \Theta(N, NS; S, NS) = \frac{g}{X^N - \left( \frac{1}{C(1, w)} - \frac{1}{C(11)} \right) + X^S - \left( \frac{1}{C(w, w)} - \frac{1}{C(1, w)} \right)} \]

\[ \pi_{SN} = \pi_{SNS} \]

\[ \frac{X^{N_\Theta}}{C(w, 1)} + \frac{X^{S_\Theta}}{d_\mu C(w, 1)} - g = \frac{X^{N_\Theta}}{C(w, 1)} + \frac{X^{S_\Theta}}{C(w, w)} - f - g \]
\[ \Theta \left( \frac{X^N}{C(w,1)} - \frac{X^N}{C(w,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{d_p C(w,1)} \right) = f \]

\[ \Theta(S,N;S,NS) = f \frac{1}{X^S - \frac{1}{d_p C(w,1)}} \]

**Derivation IX:**

\[ U_i = x_0 + X \quad \Rightarrow \quad X = \frac{1}{\mu \alpha^\alpha} \left[ \Theta_{\text{max}} \int x_i(j)^\alpha \, dj \right] \]

\[ L = x_0 + \frac{1}{\mu \alpha^\alpha} \left[ \Theta_{\text{max}} \int (x_i(j) + \gamma \zeta(j))^\alpha \, dj \right] + \lambda \left[ m_i - x_0 - \Theta_{\text{max}} \int p_i(j)(x_i(j) + \gamma \zeta(j)) \, dj \right] \]

\[ \frac{\partial L}{\partial x_0} = 1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \]

\[ \frac{\partial L}{\partial \gamma} = \frac{\alpha}{\mu \alpha^\alpha} \left[ \Theta_{\text{max}} \int x_i(j)^{(\alpha-\gamma)} \zeta(j) \, dj \right] - \Theta_{\text{max}} \int p_i(j) \zeta(j) \, dj = 0 \]

If \( \gamma = 0 \):

\[ \frac{\partial L}{\partial \gamma} = \frac{\alpha}{\mu \alpha^\alpha} \left[ \Theta_{\text{max}} \int x_i(j)^{(\alpha-\gamma)} \zeta(j) \, dj \right] - \Theta_{\text{max}} \int p_i(j) \zeta(j) \, dj = 0 \]

\[ \Rightarrow \int \left[ \frac{\alpha}{\mu \alpha^\alpha} x_i(j)^{(\alpha-\gamma)} - p_i(j) \right] \zeta(j) \, dj = 0 \]

\[ \Rightarrow \left[ \frac{\alpha}{\mu \alpha^\alpha} x_i(j)^{(\alpha-\gamma)} - p_i(j) \right] = 0 \]

\[ \Rightarrow x_i(j) = \alpha \left[ p_i(j) \mu \right]^{(\alpha-\gamma)} \]

\[ \frac{\partial L}{\partial \lambda} = m_i - x_0 - \Theta_{\text{max}} \int p_i(j)(x_i(j) + \gamma \zeta(j)) \, dj = 0 \quad \Rightarrow \quad x_0 = m_i - \Theta_{\text{max}} \int p_i(j)(x_i(j) + \gamma \zeta(j)) \, dj \]

\[ \Rightarrow x_0 = m_i - \Theta_{\text{max}} \int p_i(j) \alpha \left[ p_i(j) \mu \right]^{(\alpha-\gamma)} \, dj \]

\[ V = m_i - \Theta_{\text{max}} \int p_i(j)^{(\alpha-\gamma)} \alpha \left[ p_i(j) \mu \right]^{(\alpha-\gamma)} \, dj + \frac{1}{\mu \alpha} \Theta_{\text{max}} \int p_i(j)^{\gamma} \alpha^\alpha \, dj \]

\[ V = m_i + (1 - \alpha) \mu \left[ p_i(j)^{\gamma} \right] \]

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Derivation X:
To show the set of optimal integration strategies depending on the size of d, profit functions independent of t_i have to be looked at. For that this set of integration strategies is optimal from the firms’ point of view, following conditions are necessary:

\[ \pi_{NS} > \pi_{NNS} \cdot \]
\[ \pi_{SS} > \pi_{SNS} \cdot \]
\[ \pi_{SS} > \pi_{NSNS} \cdot \]

The inequalities above have to hold, because these strategies are compared directly.

As a result, a further set of inequalities surfaces in our analysis:

\[ d_L C(1,w) < C(1,1), \text{ as } \pi_{NS} > \pi_{NNS}. \quad (i) \]
\[ d_L C(w,w) < C(w,1), \text{ as } \pi_{SS} > \pi_{SNS}. \quad (ii) \]
\[ d_L C(w,w) < C(1,1), \text{ as } \pi_{NS} > \pi_{NNS}. \quad (iii) \]

Whereas (iii) always holds if (ii) is true, because \( C(1,1) > C(w,1) \).

Transformation delivers:

\[ L_d \]
\[ \frac{C(1) - C(w,1)}{C(1,1) - d_L} \]

This proves that this set of integration strategies only is optimal, if low transport costs exist.

Derivation XI:

\[ \pi_{NN} = \pi_{NS} \]
\[ = \frac{X^N + X^S}{d_L C(1,1)} \]
\[ \left(1 - t_N\right) \]
\[ f \]
\[ \theta(N,N,N,S) = \frac{X^N}{d_L C(1,1)} + \frac{X^S}{C(1,1)} \left(1 - t_N\right) - \frac{X^N}{d_L C(w,1)} + \frac{X^S}{C(w,1)} \left(1 - t_N\right) \]

\[ \pi_{NN} = \pi_{SN} \]
\[ = \frac{X^N + X^S}{d_L C(1,1)} \]
\[ \left(1 - t_N\right) \]
\[ g \]
\[ \theta(N,N,S,N) = \frac{X^N}{d_L C(w,1)} - \frac{X^S}{C(w,1)} \left(1 - t_N\right) \]
\[ \pi_{NN} = \pi_{SS} \]
\[ = \frac{X^N + X^S}{d_L C(1,1)} \]
\[ \left(1 - t_N\right) \]
\[ f - g \]
\[ \omega(N,N;S,S) = \frac{f + g}{\left(\frac{X^N}{d_L C(w,w)} + \frac{X^S}{C(w,w)}\right)(1 - t_S) - \left(\frac{X^N}{d_L C(1,1)} + \frac{X^S}{C(1,1)}\right)(1 - t_N)} \]

\[ \pi_{NS} = \pi_{S,S} \]

\[ \left[ \frac{X^N_\Theta}{d_L C(w,w)} + \frac{X^S_\Theta}{C(w,w)} \right](1 - t_S) - f = \left[ \frac{X^N_\Theta}{d_L C(w,w)} + \frac{X^S_\Theta}{C(w,w)} \right](1 - t_S) - f - g \]

\[ \omega(N,S;S,S) = \frac{g}{\left(\frac{X^N}{d_L C(w,w)} - \frac{X^N}{d_L C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{C(1,1)}\right)(1 - t_S)} \]

\[ \pi_{SN} = \pi_{S,S} \]

\[ \left[ \frac{X^N_\Theta}{C(w,1)} + \frac{X^S_\Theta}{d_L C(w,1)} \right](1 - t_N) - g = \left[ \frac{X^N_\Theta}{d_L C(w,w)} + \frac{X^S_\Theta}{C(w,w)} \right](1 - t_S) - f - g \]

\[ \omega(S,N;S,S) = \frac{f}{\left(\frac{X^N}{d_L C(w,w)} + \frac{X^S}{C(w,w)}\right)(1 - t_S) - \left(\frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)}\right)(1 - t_N)} \]

**Derivation XII:**

since \( \Theta = \theta^{(1-\alpha)} \) and \( dC = c^{(1-\alpha)} \):

\[ \pi = \left[ p_N(j)\frac{\alpha}{(\alpha-1)}M^N X + p_S(j)\frac{\alpha}{(\alpha-1)}M^S X - p_N(j)\frac{1}{(\alpha-1)}M^N X \frac{\Theta}{dC_N} - p_S(j)\frac{1}{(\alpha-1)}M^S X \frac{\Theta}{dC_S} \right][1 - t_i] - k \]

\[ \overline{X} = \mu^{(1-\eta)} \]

According:

\[ \frac{\partial \pi}{\partial p_N} = \frac{\alpha}{(\alpha-1)}p_N(j)^{\frac{1}{(\alpha-1)}}M^N X(1 - t_i) - \frac{1}{(\alpha-1)}p_N(j)^{\frac{1}{(\alpha-1)}}M^N X \frac{\Theta}{dC_N}\] \[\left(\frac{1}{(\alpha-1)}\right)^{\frac{1}{(\alpha-1)}}(1 - t_i) = 0 \]

\[ \frac{\partial \pi}{\partial p_S} = \frac{\alpha}{(\alpha-1)}p_S(j)^{\frac{1}{(\alpha-1)}}M^S X(1 - t_i) - \frac{1}{(\alpha-1)}p_S(j)^{\frac{1}{(\alpha-1)}}M^S X \frac{\Theta}{dC_S}\] \[\left(\frac{1}{(\alpha-1)}\right)^{\frac{1}{(\alpha-1)}}(1 - t_i) = 0 \]

Solving this for \( p \):

\[ p_N(j)_{opt} = \frac{1}{\alpha} \left( \frac{dC_N}{\Theta} \right)^{\frac{1}{\alpha}} \]

\[ p_S(j)_{opt} = \frac{1}{\alpha} \left( \frac{dC_S}{\Theta} \right)^{\frac{1}{\alpha}} \]

if transport costs and taxes exist.

**Derivation XIII:**

To calculate this \( \pi_{NN} \) is equated with \( \pi_{S,S} \). Then \( \pi_{NS} \) and \( \pi_{NN} \) are compared at this location and for \( N,S \) to be an optimal strategy it is necessary that: \( \pi_{NS} > \pi_{NN} \).
\[ \pi_{NN} = \pi_{S,S} \]
\[
\left[ \frac{X^{N}_{\Theta}}{C(1,1)} + \frac{X^{S}_{\Theta}}{d_{L} C(1,1)} \right] (1 - t_{N}) = \left[ \frac{X^{N}_{\Theta}}{d_{L} C(w,w)} + \frac{X^{S}_{\Theta}}{C(w,w)} \right] (1 - t_{S}) - f - g
\]

\[ \Theta(N,N;S,S) = \frac{\left( f + g \right)}{\left( 1 - t_{S} \right) - \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right]} \left( 1 - t_{N} \right) \]

This is the \( x \)-axis coordinate. At this point \( \pi_{NS} \) has to be greater than \( \pi_{NN} \).

For this reason the \( y \)-axis coordinate of \( \pi_{NN} \) is required:

\[ y_{NN} = \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \]

\( y \)-axis coordinate of \( \pi_{NS} \):

\[ y_{NS} = \left[ \frac{X^{N}}{d_{L} C(1,w)} + \frac{X^{S}}{C(1,w)} \right] (1 - t_{S}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \]

It is necessary that:

\[ y_{NS} \geq y_{NN} \]

\[ \left[ \frac{X^{N}}{d_{L} C(1,w)} + \frac{X^{S}}{C(1,w)} \right] (1 - t_{S}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - f \geq \]

\[ \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \]

\[ \left[ \frac{X^{N}}{d_{L} C(1,w)} + \frac{X^{S}}{C(1,w)} \right] (1 - t_{S}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \]

\[ \left[ \frac{X^{N}}{d_{L} C(1,w)} + \frac{X^{S}}{C(1,w)} \right] (1 - t_{S}) \left[ \frac{X^{N}}{d_{L} C(w,w)} + \frac{X^{S}}{C(w,w)} \right] (1 - t_{S}) - \left[ \frac{X^{N}}{C(1,1)} + \frac{X^{S}}{d_{L} C(1,1)} \right] (1 - t_{N}) \]
1 + \frac{g}{f} \geq \left[ \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] (1 - t_N)

\frac{g}{f} \geq \left[ \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] (1 - t_N)

\frac{g}{f} \left[ \frac{\bar{X}^N}{d_L C(1, w)} + \frac{\bar{X}^S}{C(1, w)} \right] (1 - t_S) - \left[ \frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] (1 - t_N) \geq \frac{\frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} - \frac{\bar{X}^N}{d_L C(1, w)} - \frac{\bar{X}^S}{C(1, w)}}{f} \gamma_H

Derivation XIV:
To calculate this \( \pi_{NN} \) is equated with \( \pi_{SS} \). Then \( \pi_{SN} \) and \( \pi_{NN} \) are compared at this location and for S,N to be an optimal strategy it is necessary that: \( \pi_{SN} > \pi_{NN} \).

\( \pi_{NN} = \pi_{SS} \)

\left[ \frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] (1 - t_N) = \left[ \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_S) - f - g

\Theta(N,N;S,S) = \frac{f + g}{\left[ \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \right] (1 - t_N)}

This is the x-axis coordinate. At this point \( \pi_{SN} \) has to be greater than \( \pi_{NN} \).

For this reason the y-axis coordinate of \( \pi_{NN} \) is required:
\[ y_{NN} = \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) \left( f + g \right) \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) \]

y-axis coordinate of \( \pi_{SN} \):
\[ y_{SN} = \left[ \frac{X^N}{C(w, 1)} + \frac{X^S}{d_L C(w, 1)} \right] (1 - t_N) \left( f + g \right) \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) - g \]

It is necessary that:
\[ y_{SN} \geq y_{NN} \]
\[ \left[ \frac{X^N}{C(w, 1)} + \frac{X^S}{d_L C(w, 1)} \right] (1 - t_N) \left( f + g \right) \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) - g \geq 0 \]
\[ \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) \left( f + g \right) \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N) - g \geq 0 \]
\[ \left[ \frac{X^N}{C(w, 1)} + \frac{X^S}{d_L C(w, 1)} \right] (1 - t_N) \frac{1 + \frac{f}{g}}{\left( \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right) (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N)} - 1 \geq 0 \]
\[ \frac{1 + \frac{f}{g}}{\left( f + g \right) \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N)} - 1 \geq 0 \]
\[
\frac{f}{g} \left[ \frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)} - \frac{X^N}{d_L C(11)} \right] (1 - t_N) \geq \left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)} \right] (1 - t_N)
\]

\[
+ \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_L C(11)} \right] (1 - t_N)
\]

\[
\frac{f}{g} \geq \frac{\left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)} \right] (1 - t_N)}{\left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)} \right] (1 - t_N)} = \gamma_L
\]

Following it is defined:

\[
A = \left[ \frac{f + g}{\left[ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{X^N}{C(w,1)} + \frac{X^S}{d_L C(w,1)} \right] (1 - t_N)} \right]^2
\]
This leads to:

\[
\frac{1}{A} = (1 - t_s)^2 \left\{ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right\}^2 \left( f + g \right) - (1 - t_s)^2 \left\{ \frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)} \right\} \left( \frac{X^N}{C(1)} + \frac{X^S}{d_L C(1)} \right) \left( 1 - t_n \right)
\]

\[
+ \left\{ \frac{X^S}{C(1)} \right\} \left( 1 - t_n \right) \left\{ \frac{X^S}{d_L C(1)} \right\} \left( f + g \right)
\]

Then:

\[
V_{\text{new}} \frac{1}{A} = m_S \frac{1}{A} + (1 - \alpha) \mu \left( \frac{1}{\alpha} \right)^{\alpha/(\alpha-1)} \frac{1}{A} \left( \frac{1}{2C(w, w)} \right)
\]

\[
+ (1 - \alpha) \mu^{\alpha/(\alpha-1)} \left( \frac{1}{\alpha} \right)^{\alpha/(\alpha-1)} \left[ \frac{1}{2d_L C(1)} - \frac{1}{2C(w, w)} \right]
\]

Utility of the representative household in S without taxation:

\[
V_{\text{old}} = m_S + (1 - \alpha) \mu^{\alpha/(\alpha-1)} \left( \frac{1}{\alpha} \right)^{\alpha/(\alpha-1)} \left[ \int_0^{\Theta(N;S;N,S)} p_S(j)^{\alpha/(\alpha-1)} dj + \int_{\Theta(N;N;S)}^{\Theta_{\text{max}}} p_S(j)^{\alpha/(\alpha-1)} dj \right]
\]

\[
= m_S + (1 - \alpha) \mu^{\alpha/(\alpha-1)} \left( \frac{1}{\alpha} \right)^{\alpha/(\alpha-1)}
\]

\[
\left[ \frac{1}{2C(w, w)} + \left( \frac{1}{2d_L C(1)} - \frac{1}{2C(w, w)} \right) \left\{ \frac{f + g}{X^N} \frac{X^N}{C(w, 1)} + \frac{X^S}{C(1, 1)} + \frac{X^S}{C(w, w)} - \frac{X^S}{d_L C(1)} \left( f + g \right) \right\} \right]^2
\]

Then:

\[
V_{\text{old}} \frac{1}{A} = m_S \frac{1}{A} + (1 - \alpha) \mu^{\alpha/(\alpha-1)} \left( \frac{1}{\alpha} \right)^{\alpha/(\alpha-1)} \frac{1}{A}
\]

\[
\left[ \frac{1}{2C(w, w)} + \left( \frac{1}{2d_L C(1)} - \frac{1}{2C(w, w)} \right) \left\{ \frac{f + g}{X^N} \frac{X^N}{C(w, 1)} + \frac{X^S}{C(1, 1)} + \frac{X^S}{C(w, w)} - \frac{X^S}{d_L C(1)} \left( f + g \right) \right\} \right]^2
\]

For that \( V_{\text{new}} \frac{1}{A} = V_{\text{old}} \frac{1}{A} \) holds true, following condition is necessary:
\[ m_S \frac{1}{A} + (1 - \alpha) \mu^{(\alpha - 1)} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \frac{1}{A} \frac{1}{2C(w,w)} + (1 - \alpha) \mu^{(\alpha - 1)} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \left[ \frac{1}{2d_L C(1,1)} - \frac{1}{2C(w,w)} \right] = \]

\[ m_S \frac{1}{A} + (1 - \alpha) \mu^{(\alpha - 1)} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \frac{1}{A} \]

\[ \begin{bmatrix} \frac{1}{2C(w,w)} + \frac{1}{2d_L C(1,1)} \end{bmatrix} \begin{bmatrix} f + g \end{bmatrix} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \]

This leads to:

\[ (1 - t_S)^2 \begin{bmatrix} \frac{\chi^N}{d_L C(w,w)} + \frac{\chi^S}{C(w,w)} \end{bmatrix} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \]

\[ \begin{bmatrix} \frac{1}{2C(w,w)} - \frac{1}{2d_H C(1,1)} \end{bmatrix} \begin{bmatrix} f + g \end{bmatrix} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \]

\[ (1 - t_N)^2 \begin{bmatrix} \frac{\chi^N}{d_L C(w,w)} + \frac{\chi^S}{C(w,w)} \end{bmatrix} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \]

\[ \begin{bmatrix} \frac{\chi^N}{C(1,1)} + \frac{\chi^S}{d_L C(1,1)} \end{bmatrix} \left( \frac{1}{\alpha} \right)^{(\alpha - 1)} \]
\[
\begin{aligned}
&\left[\frac{1}{2C(w, w)} - \left(\frac{1}{2C(w, w)} + \frac{1}{2d_1C(1, 1)} - \frac{1}{2C(w, w)} \right) \right] \left(\frac{f + g}{X^N_X - \frac{\overline{X^N}}{C(w,1)} - \frac{\overline{X^S}}{C(1,1)} + \frac{\overline{X^S}}{C(w, w) - \frac{d_2C(1,1)}}}\right)^2 \right]
\end{aligned}
\]

\[
+ (1 - \alpha)\mu^{(\alpha-1)}(\frac{1}{\alpha})^{\alpha(\alpha-1)} \left[\frac{1}{2d_1C(1,1)} - \frac{1}{2C(w, w)} \right] = 0
\]

Following it is defined:

\[
D = \left(1 - \alpha\right)\mu^{(\alpha-1)}(\frac{1}{\alpha})^{\alpha(\alpha-1)}.
\]

\[
\begin{aligned}
&\left[\frac{1}{2C(w, w)} - \left(\frac{1}{2C(w, w)} + \frac{1}{2d_1C(1, 1)} - \frac{1}{2C(w, w)} \right) \right] \left(\frac{f + g}{X^N_X - \frac{\overline{X^N}}{C(w,1)} - \frac{\overline{X^S}}{C(1,1)} + \frac{\overline{X^S}}{C(w, w) - \frac{d_2C(1,1)}}}\right)^2 \right]
\end{aligned}
\]

Inserting D in \( V_n \text{new} \frac{1}{\alpha^\prime} = \frac{1}{2C_w(w, w)} \) and dividing by D and

\[
\begin{aligned}
(1 - t_s)^2 - (1 - t_s)^2 &\left[\frac{\overline{X^N}}{d_1C(1,1)} + \frac{\overline{X^S}}{C(1,1)}\right]^{(1 - t_N)} + \left[\frac{\overline{X^N}}{d_1C(1,1)} + \frac{\overline{X^S}}{C(1,1)}\right]^{(1 - t_N)^2}
\end{aligned}
\]

\[
\frac{(1 - \alpha)\mu^{(\alpha-1)}(\frac{1}{\alpha})^{\alpha(\alpha-1)} \left[\frac{1}{2d_1C(1,1)} - \frac{1}{2C(w, w)} \right]}{\left(\frac{\overline{X^N}}{d_1C(1,1)} + \frac{\overline{X^S}}{C(1,1)}\right)^2 D}
\]

Solving for \((1-t_s)\):
Following it is defined:

\[ E = - \frac{(1 - \alpha)\mu \gamma^{(a-1)}}{\gamma^{(a-1)}} \left\{ \frac{1}{\alpha} \left[ \frac{1}{2d_c C(11)} - \frac{1}{2C(w,w)} \right] \right\}^{1/2} \]

This leads to:

\[ \left[ \frac{X^N}{C(11)} + \frac{X^S}{d_c C(11)} \right] (1 - t_N) \pm \left[ E \right]^{1/2} = (1 - t_s)_{1/2} \]

Derivation XVI:

In any parameter configuration, E is positive:

2a.) \[ \frac{X^N}{d_c C(w,w) + C(w,w)} - \frac{X^S}{X^S} > \left[ E \right]^{1/2} \]
\[(1 - t_S)_1 = \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} + [E]^{1/2},\]

if \(0 < \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} < 1 \Rightarrow 0 < t_S < 1\)

or:

\[(1 - t_S)_2 = \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} - [E]^{1/2},\]

if \(0 < \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} < 1 \Rightarrow 0 < t_S < 1\)

\[2b.) \left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N) < [E]^{1/2}:\]

\[(1 - t_S)_3 = \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} + [E]^{1/2},\]

if \(0 < \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} < 1 \Rightarrow 0 < t_S < 1\)

or:

\[(1 - t_S)_4 = \frac{\left[ \frac{X^N}{C(1,1)} + \frac{X^S}{d_L C(1,1)} \right] (1 - t_N)}{\frac{X^N}{d_L C(w, w)} + \frac{X^S}{C(w, w)}} - [E]^{1/2},\]
but \[
\frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_N)
\] is negative. This lead to \( t_S > 1 \)
\[
\Rightarrow \text{no optimal } 0 < t_S < 1 \text{ exists}
\]

2c.) \[
\frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_N)
\] = \( [E]^{\frac{1}{2}} \):

\[
(1 - t_S)_2 = 2 \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_N)
\]

if \( 0 < 2 \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] < 1 \Rightarrow 0 < t_S < 1 \)

\[
\Rightarrow \text{no optimal } 0 < t_S < 1 \text{ exists}
\]

or:

\( (1 - t_S)_0 = 0 \), but this leads to \( t_S = 1 \)

\[
\Rightarrow \text{no optimal } 0 < t_S < 1 \text{ exists}
\]

Derivation XVII:

The fixed costs relation is medium high:

Utility of the representative household in \( N \) with taxation:

\[
V_{N, \text{new}} = m_{N, \text{new}} + (1 - \alpha) \mu \left( \begin{array}{c}
\int_0^{\theta(NN,S,S)} p_N(j) \gamma^{(a-1)} \, dj + \int_{\theta(NN,S,S)} \theta_{S,S} p_N(j) \gamma^{(a-1)} \, dj
\end{array} \right)
\]

\[
= m_{N, \text{new}} + (1 - \alpha) \mu \left( \frac{1}{\alpha} \right) \gamma^{(a-1)}
\]

\[
\left[ \frac{1}{2d_L C(w, w)} + \frac{1}{2C(1)} \right] \left[ \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} \right] (1 - t_S) - \left[ \frac{\bar{X}^N}{C(1)} + \frac{\bar{X}^S}{d_L C(1)} \right] (1 - t_N)
\]

Following it is defined:
\[
A = \left( \frac{\bar{X}_N^{N} d_L C(w, w) + \bar{X}_S^S}{d_L C(w, w)} (1 - t_s) - \frac{\bar{X}_N^{N} d_L C(1)}{C(1)} \frac{\bar{X}_S^S}{d_L C(1)} (1 - t_n) \right)^2
\]

This leads to:
\[
\frac{1}{A} = \left\{ \frac{\bar{X}_N^{N} d_L C(w, w) + \bar{X}_S^S}{d_L C(w, w)} (1 - t_s) \right\}^2 \frac{\bar{X}_N^{N} d_L C(1)}{C(1)} \frac{\bar{X}_S^S}{d_L C(1)} (1 - t_n)^2 - (1 - t_n) \left\{ \frac{\bar{X}_N^{N} d_L C(w, w) + \bar{X}_S^S}{d_L C(w, w)} (1 - t_s) \right\} \frac{\bar{X}_N^{N} d_L C(1)}{C(1)} \frac{\bar{X}_S^S}{d_L C(1)} (1 - t_n)
\]

\[
+ (1 - t_n)^2 \left\{ \frac{\bar{X}_N^{N} d_L C(1)}{C(1)} \frac{\bar{X}_S^S}{d_L C(1)} (f + g) \right\}^2
\]

Then:
\[
V_{N_{\text{new}}} = \frac{1}{A} V_{N_{\text{old}}} + (1 - \alpha) \mu_{(\alpha-1)} \left[ \frac{1}{\alpha} \right]^{\alpha_{(\alpha-1)}} \frac{1}{A} \frac{1}{2d_L C(w, w)}
\]

\[
+ (1 - \alpha) \mu_{(\alpha-1)} \left[ \frac{1}{\alpha} \right]^{\alpha_{(\alpha-1)}} \left[ \frac{1}{2C(1)} - \frac{1}{2d_L C(w, w)} \right]
\]

Utility of the representative household in N without taxation:
\[
V_{N_{\text{old}}} = m_{N_{\text{old}}} + (1 - \alpha) \mu_{(\alpha-1)} \left[ \int_0^{\Theta} p_N(j)^{\alpha_{(\alpha-1)}} dj \right. + \left. \int_0^{\Theta_{\text{max}}} p_N(j)^{\alpha_{(\alpha-1)}} dj \right]
\]

\[
= m_{N_{\text{old}}} + (1 - \alpha) \mu_{(\alpha-1)} \left[ \frac{1}{\alpha} \right]^{\alpha_{(\alpha-1)}}
\]

\[
\left[ \frac{1}{2C(w,1)} + \frac{1}{2C(1)} \right] \left[ \frac{f + g}{\bar{X}_N^{N} d_L C(w, w) + \bar{X}_S^S C(w, w) - \bar{X}_N^{N} d_L C(1) + \bar{X}_S^S C(1)} \right] \left[ \frac{f + g}{\bar{X}_N^{N} d_L C(w, w) + \bar{X}_S^S C(w, w) - \bar{X}_N^{N} d_L C(1) + \bar{X}_S^S C(1)} \right]^{2}
\]

Then:

\[
V_{\text{Nold}} \frac{1}{A} = m_{\text{Nold}} \frac{1}{A} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(w,l) + \left( \frac{1}{2C(1,1) - 2C(w,l)} \right) \left( f + g \right) \left( \frac{X^N}{C(w,l)} - \frac{X^S}{C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{d_m C(1,1)} \right) ^2}
\]

For that \( V_{\text{Nnew}} \frac{1}{A} = V_{\text{Nold}} \frac{1}{A} \) holds true, following condition is necessary:

\[
m_{\text{Nnew}} \frac{1}{A} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(1,1) - 2d_l C(w, w)} = m_{\text{Nold}} \frac{1}{A} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(w,l) + \left( \frac{1}{2C(1,1) - 2C(w,l)} \right) \left( f + g \right) \left( \frac{X^N}{C(w,l)} - \frac{X^S}{C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{d_m C(1,1)} \right) ^2}
\]

This leads to:

\[
(1 - t_N) ^2 \left( \frac{X^N}{C(1,1)} + \frac{X^S}{d_l C(1,1)} \right) \left( f + g \right) ^2 \left( m_{\text{Nnew}} - m_{\text{Nold}} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(1,1) - 2d_l C(w, w)} \right)
\]

\[
\left( \frac{1}{2d_l C(w, w)} - \left( \frac{1}{2C(1,1) - 2C(w,l)} \right) \left( f + g \right) \left( \frac{X^N}{C(w,l)} - \frac{X^S}{C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{d_m C(1,1)} \right) ^2 \right) \left( m_{\text{Nnew}} - m_{\text{Nold}} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(1,1) - 2d_l C(w, w)} \right)
\]

\[
- (1 - t_N) ^2 \left( \frac{X^N}{d_l C(w, w)} + \frac{X^S}{C(w,w)} \right) ^2 \left( f + g \right) ^2 \left( m_{\text{Nnew}} - m_{\text{Nold}} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(1,1) - 2d_l C(w, w)} \right)
\]

\[
\left( \frac{1}{2d_l C(w, w)} - \left( \frac{1}{2C(1,1) - 2C(w,l)} \right) \left( f + g \right) \left( \frac{X^N}{C(w,l)} - \frac{X^S}{C(1,1)} + \frac{X^S}{C(w,w)} - \frac{X^S}{d_m C(1,1)} \right) ^2 \right) \left( m_{\text{Nnew}} - m_{\text{Nold}} + (1 - \alpha) \mu \frac{\gamma(\alpha-\eta) \left( \frac{1}{\alpha} \right) ^{\alpha-\eta} \frac{1}{A}}{2C(1,1) - 2d_l C(w, w)} \right)
\]
Following it is defined:

\[ B = \left( m_{\text{new}} - m_{\text{old}} + (1 - \alpha)\mu^{(a-1)}\left( \frac{1}{\alpha} \right)^{a-1} \right). \]

Inserting \( B \) in \( v_{\text{new}} \) and dividing by \( B \) and

\[
\left( 1 - t_n \right)^2 - \left( 1 - t_n \right)2 \left( \frac{\overline{x}^N}{d_l C(w, w)} + \frac{\overline{x}^S}{C(w, w)} \right) (1 - t_s) \left( \frac{\overline{x}^N}{C(11)} + \frac{\overline{x}^S}{d_l C(11)} \right) = 0
\]

\[
\left( 1 - t_n \right)^2 - (1 - t_n)2 \left( \frac{\overline{x}^N}{d_l C(w, w)} + \frac{\overline{x}^S}{C(w, w)} \right) \left( \frac{\overline{x}^N}{C(11)} + \frac{\overline{x}^S}{d_l C(11)} \right) \left( \frac{\overline{x}^N}{C(11)} + \frac{\overline{x}^S}{d_l C(11)} \right) = 0
\]
Solving for \((1-t_N)\):

\[
\left[ \frac{\bar{X}^N}{d_L C(w,w) + C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right](1-t_S) \pm \left[ \frac{\bar{X}^N}{d_L C(w,w) + C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right](1-t_S)^2 = \left(1-t_N\right)^2
\]

\[
= (1-t_N)_{1/2}
\]

Following it is defined:

\[
K = -\frac{(1-\alpha)\gamma^{(\alpha-1)}(\frac{1}{\alpha})^{\gamma^{(\alpha-1)}}}{\left[ \frac{\bar{X}^N}{d_L C(w,w) + C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right]^2} B
\]

This leads to:

\[
\left[ \frac{\bar{X}^N}{d_L C(w,w) + C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right](1-t_S) \pm [K]^{1/2} = (1-t_N)_{1/2}
\]

**Derivation XVIII:**

1.) if \(K\) is negative \(\Rightarrow\) no optimal \(t_N\) exists

2.) if \(K\) is positive:

\[
\left[ \frac{\bar{X}^N}{d_L C(w,w) + C(w,w)} + \frac{\bar{X}^S}{C(w,w)} \right](1-t_S) \rightarrow \left[ K \right]^{1/2}
\]
\[(1 - t_N)_1 = \frac{\chi^N}{d_L C(w, w) + \chi^S C(w, w)} (1 - t_S)\] 
\[+ |K|^{1/2},\]

if 0 < \[
\frac{\chi^N}{d_L C(11) + \chi^S}\] 
\[\frac{\chi^S}{C(11)}\] 
\[< 1 \Rightarrow 0 < t_N < 1\]

or:

\[(1 - t_N)_2 = \frac{\chi^N}{d_L C(w, w) + \chi^S C(w, w)} (1 - t_S)\] 
\[+ |K|^{1/2},\]

if 0 < \[
\frac{\chi^N}{d_L C(11) + \chi^S}\] 
\[\frac{\chi^S}{C(11)}\] 
\[< 1 \Rightarrow 0 < t_N < 1\]

or:

\[(1 - t_N)_3 = \frac{\chi^N}{d_L C(w, w) + \chi^S C(w, w)} (1 - t_S)\] 
\[+ |K|^{1/2},\]

if 0 < \[
\frac{\chi^N}{d_L C(11) + \chi^S}\] 
\[\frac{\chi^S}{C(11)}\] 
\[< 1 \Rightarrow 0 < t_N < 1\]

or:

2b.) \[
\frac{\chi^N}{d_L C(w, w) + \chi^S C(w, w)} (1 - t_S)\] 
\[< |K|^{1/2}\]

leads to:

\[(1 - t_N)_3 = \frac{\chi^N}{d_L C(w, w) + \chi^S C(w, w)} (1 - t_S)\] 
\[+ |K|^{1/2},\]

if 0 < \[
\frac{\chi^N}{d_L C(11) + \chi^S}\] 
\[\frac{\chi^S}{C(11)}\] 
\[< 1 \Rightarrow 0 < t_N < 1\]

or:
\[
(1 - t_N)_4 = \frac{\bar{X}^N}{d_L C(w, w)} \frac{\bar{X}^S}{C(w, w)} (1 - t_S) - \left[ K \right]^{1/2}.
\]

but
\[
\frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} (1 - t_S) - \left[ K \right]^{1/2} \text{ is negative. This lead to } t_N > 1
\]

\Rightarrow \text{ no optimal } 0 < t_N < 1 \text{ exists}

\[
2c.) \quad \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} (1 - t_S) = \left[ K \right]^{1/2} \text{ leads to:}
\]

\[
(1 - t_N)_5 = 2 \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} (1 - t_S)
\]

\[
\frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)} \quad \text{if } 0 < 2 \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)} (1 - t_S) < 1 \Rightarrow 0 < t_N < 1
\]

\Rightarrow \text{ no optimal } 0 < t_N < 1 \text{ exists}

\text{Derivation XIX:}

Also \( m_{n, \text{new}} \) depends on taxation. For this reason:

\[
\frac{\partial (1 - t_N)}{\partial (1 - t_S)} = \pm \frac{\bar{X}^N}{d_L C(w, w)} + \frac{\bar{X}^S}{C(w, w)}
\]

\[
\frac{\bar{X}^N}{C(11)} + \frac{\bar{X}^S}{d_L C(11)}
\]
\[
\begin{align*}
\frac{1}{2} \left( (1 - \alpha)_{\mu} \gamma^{(a-1)} \left( \frac{1}{\alpha} \right) \left( \gamma^{(a-1)} \left( \frac{1}{\alpha} \right) \right)^2 \right) \\
\left( \frac{1}{2C(11)} - \frac{1}{2d_L C(w, w)} \right) \left( \frac{\chi^N}{C(11)} + \frac{\chi^S}{d_L C(11)} \right)^2 (f + g)^2 \\
\left( \frac{\beta m_{\text{new}}}{\partial (1 - t_N)} + \frac{\partial m_{\text{new}}}{\partial (1 - t_N)} \right) \left( \frac{\partial (1 - t_N)}{\partial (1 - t_N)} \right)^2 \\
\end{align*}
\]

Where:

\[
B = \left( m_{\text{new}} - m_{\text{old}} + (1 - \alpha)_{\mu} \gamma^{(a-1)} \left( \frac{1}{\alpha} \right) \right)^2.
\]

\[
\begin{align*}
\begin{bmatrix}
\frac{1}{2d_L C(w, w)} - \frac{1}{2C(w, 1)} + \frac{1}{2C(11)} - \frac{1}{2C(w, 1)} \\
\frac{f + g}{\chi^N} - \frac{\chi^N}{C(11)} + \frac{\chi^S}{d_L C(11)} - \frac{\chi^S}{C(w, w)} - \frac{\chi^S}{d_L C(11)}
\end{bmatrix}
\end{align*}
\]