1 Introduction

For a long time international trade was thought of as trade in final goods. However, with recent advances in information technology this changed up to date to an increase in trade in intermediate goods allowing economies to reap additional gains from trade. A question of importance for economies engaging in offshoring is how it affects the future prospects of the economy. This paper investigates the impact of offshoring on growth. I therefore show that offshoring enhances growth.

Jones and Kierzkowski (1990) argue that due to improvements in technology, the fragmentation of intermediate production has increased and economies are able to realize additional gains from trade by deepening their comparative advantage. They show that if offshoring is possible, firms are able to reduce their costs by importing a share of the inputs used. The economy as a whole will specialize in its comparative advantage which implies a more efficient usage of its factors of production. Jones and Kierzkowski (1998) remark that this effect is similar to technological progress.
The way I conceptualize offshoring is as trade in tasks, following Grossman and Rossi-Hansberg (2008). The idea is that firms have to perform a continuum of tasks to produce a final good. All tasks are offshorable, however, each task faces a specific trade cost if performed abroad. Firms, minimizing costs, offshore a task if the import costs of the task is lower then the cost of domestic production of the task. Therefore, firms can become more productive through offshoring.

The paper contributes to the literature by showing that efficiency gains of offshoring translate into a higher growth rate in an economy. The reason for this is twofold. Firstly, offshoring reduces the production cost of firms by replacing domestic high cost labour with foreign low cost labour. This increases profitability of firm and results in a higher investment in research and development. Secondly, the effective labour force is increased because jobs that, prior to offshoring, were done at home are now done by foreign workers in the foreign country\(^1\). Thus broadening the effective labour force increases growth.

The structure of the economy in the paper is depicted in figure 1. There are two final good sectors \(X\) and \(Y\). Both final sector use an

\(^1\)Baldwin and Robert-Nicoud (2007) call this shadow migration.
intermediate input and either skilled or unskilled labour. The X sector is assumed to be indirectly relative skilled labour intensive compared to the Y sector which is unskilled labour intensive. Both final goods are traded. The intermediate sector uses skilled and unskilled labour to produce the input. Offshoring takes place in the intermediate sector and only unskilled tasks are offshored. Skilled labour is the only input used in research, where it invents new varieties of the intermediate input. Growth is modelled as an increase in the number of intermediate varieties. The economy as a whole becomes more productive due to an increase in the number of varieties which is similar to the idea of specialization due to Adam Smith.

Glass and Saggi (2001) develop a one-sector growth model of quality ladders. They model offshoring by a fixed share of the production stage that is offshored. They show that an increase in the extent of offshoring increases the growth rate. In the present paper, the extent of offshoring is endogenous. Although the basic conclusion of higher growth with more offshoring is the same, the present model allows for a wider range of comparative statics for example the effect of a change of relative prices on the extent of offshoring and thus growth. Further, this paper differs from Glass and Saggi (2001) that I analyse an economy with two sectors and two primary factors of production.

2 A Two Sector Economy

2.1 Households

I assume an infinitely living representative household which consumes two final goods, X and Y, at each period t. The intertemporal utility function of the household is

\[ U = \int_0^\infty e^{-\rho t} \ln u(c_{xt}, c_{yt}) dt, \]

where the instantaneous utility function \( u(c_{xt}, c_{yt}) \) is non-decreasing, quasi-concave and homogeneous of degree one in consumption. The household faces
both a static- and a dynamic-optimization problems. Firstly, the household maximizes instantaneous utility in each period by optimizing expenditures $E_t$ on the two final goods. This yields the indirect per period utility $v_t = \ln \nu(p_x, p_y) + \ln E_t$, where $p_i$ is the price of the final good $i = X, Y$. Secondly, a household optimizes its pattern of expenditures over time such that life-time utility is maximized. Substituting the indirect per-period utility, the formal dynamics problem of the household is

$$V = \int_0^\infty e^{-\rho t} (\ln \nu(p_x, p_y) + \ln E_t) dt$$

subject to

$$\int_0^\infty e^{-R(t)} E_t dt \leq \int_0^\infty I_t dt + W_0$$

where $I_t$ is the income of a household in period $t$ and $\rho$ is the subjective discount rate. $R(t) = \int_0^t r(s) ds$ is the cumulative discount rate depending on the interest rate $r_t$. Wealth in the initial period is denoted by $W_0$. From the maximization follows the optimal path of expenditures is

$$\dot{E} = r_0 - \rho,$$

where the dot indicates a change over time.

2.2 Production

I assume that the economy is endowed with skilled workers $H$ and unskilled workers $L$. The composition of skills in the economy does not change over time in the sense that technological progress increases the effective endowments of both skilled and unskilled labour, leaving the effective relative endowment unchanged. Due to homothetic preferences I do not have to distinguish between the two types of workers but can consider a representative household. The endowed factors are perfectly mobile within the economy, but not internationally. Each of the final goods uses one of the factors and a continuum of intermediates in its
production process. I assume that the $X$ sector uses skilled workers and the $Y$ sector uses unskilled workers. The intermediates are assumed to be capital inputs that are used in both sectors. Let $Z_i$ denote the aggregate index of intermediates used in sector $i = X, Y$. I assume that both final goods are traded, whereas the capital goods are for domestic use only. The situation I think of here is a country or region that is highly developed but its technology is not implemented in the rest of the world. The final goods are produced with the production functions

\[ X = B Z^\beta X H_1^{1-\beta} \]
\[ Y = B Z^\beta Y L_1^{1-\beta}. \]

where $\beta$ is the input share of the intermediate input and $B$ is a constant. Both sectors are perfectly competitive. From minimizing cost in each final good sector I obtain

\[ p_X = w_H^{1-\beta} P_Z^\beta \]
\[ p_Y = w_L^{1-\beta} P_Z^\beta. \]

where $w_k$ is the wage of factor $k = H, L$ and $P_Z$ is the aggregate price index of the intermediate sector. Further, $B \equiv 1/[(1 - b\beta)^{(1-\beta)/\beta}]$. I assume that the economy is small compared to the rest of the world and thus takes prices of the traded goods as given. Additionally, without loss of generality, I assume that prices of the final goods are constant over time. Accordingly I divide the equations in (4) which results in

\[ \frac{p_x}{p_y} = \left(\frac{w_H}{w_L}\right)^{1-\beta}. \]

Hence, the relative factor price in the economy is fixed. Fixed relative prices allows us to write the model in terms of just one of the factor prices. This is similar to the traditional factor proportion theory, where relative final prices determine relative factor prices.

Each capital input, indexed by $\omega$, is manufactured by a different producer. The aggregate output index takes the functional form $Z_i =$
\[
\left( \int_{\omega \in \Omega} z_{\omega t}^\sigma d\omega \right)^{\frac{1}{\sigma}},
\]
where \( z_{\omega t} \) is the output of each individual producer. The intermediate inputs are substitutes with \( 0 < \sigma < 1 \) and an elasticity of substitution between any two varieties of \( \varepsilon = 1/1 - \sigma > 1 \). The number of potential varieties is infinite. I assume, however, that varieties have to be invented before they can be used in the production of a final good. I denote the set of existing varieties by \( \Omega_t \). As I will show, \( \Omega_t \) grows over time which implies productivity gains in the economy. For simplicity I skip the time subscript of the number of varieties. Let \( p_z \) be the price set by a particular intermediate producer. The implied aggregate price index of \( Z \) is
\[
P_Z = \left( \int_{\omega \in \Omega} p_z^{1 - \varepsilon} d\omega \right)^{\frac{1}{1 - \varepsilon}}. \tag{6}
\]
Each producer of a capital input maximizes profits, facing a downwards sloping demand curve,
\[
z_{\omega} = Z \left( \frac{p_z}{P_Z} \right)^{-\varepsilon}. \tag{7}
\]

The capital input is produced by using skilled and unskilled workers. I assume that unskilled workers have to perform a continuum of task in order to provide one unit of a labour input. The development of trade in tasks follows Grossman and Rossi-Hansberg (2006). I normalize the mass of tasks to be from zero to one. It is assumed that all tasks are offshorable. However, each task has a specific trade cost \( \tau_j > 1 \), where \( j \) indexes the task. I further assume that tasks are ordered such that trade costs are nondecreasing in \( j \), which orders the task according to their offshorability. A firm offshores a task as long as it is cheaper to import the task than produce it at home. This implies that
\[
w_L \geq w_L^* \tau_j, \tag{8}
\]
where the asterisk denotes the rest of the world. Each of the intermediate producers offshores up to the point where there are no more cost
savings possible. Let \( J \) denote the marginal tasks for which a firm is indifferent between offshoring or domestic production and equation 8 hold with equality. The marginal task is a function of the wages of unskilled workers at home and in the rest of the world, \( J \equiv J(w_L, w^*_L) \). For simplicity, however, I skip the arguments and denote the marginal task by \( J \).

The wage of unskilled workers in the rest of the world is assumed to be lower than in the domestic economy, \( w^*_L < w_L \). This assumption can be justified, for example, by a lower productivity in the rest of the world. Further, I assume that the wage of unskilled workers in the rest of the world grows at the same rate as the domestic wage. I make these assumptions in order to ensure that the marginal task exists over time. If wages domestic and foreign wages grow at different rates, corner solutions arise where either no offshoring takes place or the domestic economy fully specializes in research.\(^2\)

The production function of a capital input is assumed to be \( z(\omega) = \Lambda \psi^\alpha \omega^{1-\alpha} \), where \( \Lambda \) is a parameter and \( \psi \) is the aggregate index of tasks performed. Cost minimization yields a unit cost function of \( \bar{c}(w_L, w^*_L) \equiv \Theta(J)^\alpha c(w_L) \), where \( c = c(w_L) \) is the marginal cost of an intermediate producer. A detailed derivation of the cost function is given in the appendix. The second expression in the unit costs is

\[
\Theta(J) \equiv 1 - J + \frac{\int_0^J \tau(j) dj}{\tau(J)}. \quad (9)
\]

The intuition for \( \Theta(J) \) is that it is a cost savings parameter. If the economy is able to offshore a fraction of the tasks, domestic labour is replaced by lower cost labour from the rest of the world, which reduces the costs of production. This is similar to an increase of the productivity of domestic labour.

\(^2\)For further detail see discussion in chapter 5.
The behavior of each intermediate producer is characterized by a mark-up over marginal costs. The pricing rule is

\[ p(w_L, w^*_L) = \frac{\tilde{c}}{\sigma} \quad \forall \omega \in \Omega_t, \]  

(10)

where I have dropped the arguments on the right hand side. The mark-up is set over effective marginal costs. The price of a single variety falls in the effective marginal costs. The per period profits of an intermediate producer is

\[ \pi_\omega(w_L, w^*_L) = (1 - \sigma) p_z z(\omega) \quad \forall \omega \in \Omega_t. \]  

(11)

Due to symmetric producers, all capital input producers set a price equal to equation (10). Therefore, I can rewrite the aggregate price index in 6 as

\[ P_Z = n \frac{1}{\sigma} p_z, \]  

(12)

where \( n \) denotes the number of intermediate producers in period \( t \).\(^3\)

2.3 Research

Before entering the production stage in the intermediate sector a potential producer of a capital input must invest in research and development of a blueprint for a new capital input. With the invention of a blueprint the innovator receives a patent. I further assume that inventing around the patent is prohibitively costly and thus an incumbent intermediate producer faces no (direct) competition for her variety because the latter is protected by the patent. I assume that patents are non-contractible.\(^4\) Therefore, an intermediate producer that invests in research and development also becomes the producer of the capital input. I assume that research uses human capital as its sole input.

\(^3\)It holds that \( n_t = \Omega_t \) in each period. Otherwise some firms would invent blueprints which are then not use which I show in the next section.

\(^4\)This assumption is made analogous to Grossman and Helpman (1989)
A potential entrant makes an investment if the cost of the investment is not larger than the present discounted profits it earns from its investment. Let $v$ be the present discounted value of an investment and $aw_H/K$ the investment cost. The investment cost is composed of the input requirements of human capital $a$, the wage the producer has to pay to employ one unit of human capital in research and the capital stock $K$. The capital stock $K$ represents the existing experience in the economy in research. With each new variety the capital stock increases. Therefore, successful research has a positive externality on the investment costs. I make the assumption that the capital stock equals the number of already invented varieties, $K = n$. Investment costs, therefore, decline over time which permits more entry into the intermediate market. I assume free entry into the intermediate market. Accordingly, all the blueprints are marketed. If that were not the case, some R&D investment would be wasted. The free entry condition is

$$\int_0^\infty e^{-R(t)} \pi(t) dt = \frac{aw_H}{n}. \tag{13}$$

The discounted profits of successful innovation must equal the costs of developing a variety. If the costs are lower than the intertemporal profits then profitable opportunities exist in innovation. Differentiating equation (13) with respect to the initial period yields

$$r = \frac{n \pi}{aw_H} + \frac{\dot{w}_H}{w_H} - \frac{\dot{n}}{n} \tag{14}$$

which is a no-arbitrage condition. The intuition is similar to the one of the free-entry condition. Potential investors are able to issue a bond on the financial market to finance research. The issuer of the bond has to pay interest $r$ per period. The return from inventing a blueprint is the pure profits in the period of invention and the evolution of the future profits. An investor would issue a bond as long as rent payment

\[5\text{See Grossman and Helpman (1991b) for further discussion} \]
of the bond are not more than the return of investment, with equality in equilibrium.

2.4 Equilibrium Conditions

An equilibrium in the economy is characterized by a steady state, where all variables grow at a constant rate. I define \( g \equiv \dot{n}/n \) to be the growth rate of new varieties.\(^6\) For the economy to be in equilibrium the no-arbitrage condition in 14 must be satisfied and the factor markets have to clear. Each final good sector indirectly uses both factors. For example, the \( X \) sector uses skilled worker directly in its production and unskilled worker indirectly in the form of the capital input. Let \( a_{ki} \) denote the unit-input coefficients of input \( k = H, L \) used in sector \( i = X, Y, Z \). Further, let \( a_{ZX} \) and \( a_{ZY} \) be the unit input coefficients of the capital good in the respective final good sector. The input coefficients are derived from the unit-cost functions of the final good sectors in equation (4) and using Shepard’s lemma. The detailed derivation of the unit input coefficients is found in the appendix. The demand for skilled workers from the research sector is its input requirements \( a/n \) multiplied by the number of new entrants \( \dot{n} \). Therefore, I write the factor market clearing conditions as

\[
\begin{align*}
H &= a_{Hz}(a_{ZX}X + a_{ZY}Y) + a_{Hx}X + ag \\
L &= (1 - J)a_{Lz}(a_{ZX}X + a_{ZY}Y) + a_{Ly}Y.
\end{align*}
\]

(15)

As has been assumed, offshoring affect the labour market clearing of unskilled labour only. An increase in marginal task \( J \) reduces the demand for unskilled labour from the intermediate sector. The input coefficients are affected by the introduction of new varieties. Rewriting the factor prices in their productivity-adjusted form enables me to solve for the equilibrium growth rate. Let the productivity adjusted wage be \( \bar{w}_k \equiv w_k A^2 \), where \( A \equiv n^{1-\epsilon} \). I therefore rewrite the pricing equations

\(^6\)Solving for the equilibrium follows Grossman and Helpman (1991a)
in (10) as

\[ p_X = c_X(\bar{w}_H, \bar{w}_L) \]
\[ p_Y = c_Y(\bar{w}_L, \bar{w}_H). \]  

(16)

I define the coefficients \( b_{HX} = a_{HX} + a_{HZ}a_{ZX} \), \( b_{HY} = a_{HY}a_{ZY} \), \( b_{LX} = a_{LX}a_{ZX} \) and \( b_{LY} = a_{LY} + a_{LZ}a_{ZY} \). Given those definitions we can rewrite the factor market clearing as

\[
\begin{align*}
H &= b_{HX}\bar{X} + b_{HY}\bar{Y} + ag \\
L &= b_{LX}\bar{X} + b_{LY}\bar{Y} - J a_{LZ}(a_{ZX}X + a_{ZY}Y)
\end{align*}
\]

(17)

where \( \bar{X} = X A^\beta \) and \( \bar{Y} = Y A^\beta \) are the productivity-adjusted final outputs. Multiplying both equations in (17) with the respective effective wage \( \bar{w}_k \), adding them together and using the appropriate definitions of the unit input coefficients yields

\[
\bar{w}_L L + \bar{w}_H H = \{1 - [1 - \sigma(1 - J\alpha)]\beta\} (p_X\bar{X} + p_Y\bar{Y}) + w_Hag.
\]

(18)

I now turn to the evolution of the expenditures in the economy. Due to the presence of offshoring domestic expenditures correspond to the gross national product. Thus, the per period expenditures in the economy are the value of the production less the cost of the imported tasks, \( E = p_X X + p_Y Y - nzw_L^* \int_0^J \tau(j) dj \). The growth rate of the expenditures is

\[
\frac{\dot{E}}{E} = \beta \frac{1}{1 - \varepsilon} g,
\]

(19)

which is derived in the appendix in more detail. (19) implies that the growth of expenditures is proportional to the growth of varieties.

Finally, I rewrite the no-arbitrage condition in (14) using (19). Therefore,

\[
\beta \frac{1 - \sigma}{\bar{w}_H}(p_X\bar{X} + p_Y\bar{Y}) = g + \rho,
\]

(20)

which links current output to the growth rate, showing the trade-off the economy faces. For example, if more resources are invested in re-
search, current output reduces, but, future consumption possibilities are enhanced.

### 2.5 Equilibrium

I am now able to discuss a steady-state growth rate in the economy. In an equilibrium, the economy must satisfy the resource constraints in (18) and the no-arbitrage condition in equation (20). I consider an economy that is diversified in the production of both final goods, which is the case if the economy grows at a moderate rate. However, if the growth rate is too high in the economy all skilled labour is employed in research and the production of the capital input.

I proceed with a graphical presentation of the equilibrium and the trade-off between current output and growth. The resource constraint and the no-arbitrage condition are drawn in figure 2.5 with the effective per period output $Q$ on the vertical axis and the growth rate $g$ on the horizontal axis. The $RR$ line represents the resource constraint. The negative slope of the resource constraint reflects the trade-off between growth and current output. For instance if more skilled workers are employed in research, less skilled workers are available for production. The $AA$ line represents the no-arbitrage condition. This line is upwards sloping because a higher current output implies higher profits and therefore a bigger incentive to invest in new blueprints, thereby increasing the growth rate. The equilibrium is found at the intersection of the resource constraint and the no-arbitrage condition and is denoted by $E$.

For the analytical solution substitute the resource constraint into the no-arbitrage condition (14) yields

$$g = (1 - \eta) \frac{w_l}{w_H} L + H \frac{a}{a} - \eta \rho, \quad (21)$$
where \( \eta \equiv \frac{1-\beta(1+\sigma J\alpha)+\beta\sigma}{1-\sigma\alpha J} \). The basic structure of the growth rate is that growth is increased if either or both of the endowments increase. Growth decreases with an increase in the discounting rate, as consumers become more impatient and invest less. Further, growth increases with an increase in the relative wage \( \frac{wL}{wH} \).\(^7\)

### 2.6 Offshoring and Growth

How is the growth rate affected by the extent of offshoring? If offshoring affects the allocation of labour, growth is affected as well. On the one hand offshoring unskilled workers lose their jobs in the intermediate sector as offshoring enables intermediate producers to reduce their costs by moving tasks overseas. On the other hand, due to the reduced costs, the intermediate sector might expand its output, which might absorb the job losses and the two effect might cancel out. In this section I show that offshoring increases growth because factor of production are allocated more efficiently. However, before I investigate the link of offshoring and growth, I develop some results that are helpful

\(^7\)The relative wage is by assumption fixed by the relative final price.
to built an intuitive understanding for the underlying mechanisms.

Throughout this section I consider an increase in the extent of offshoring. The extent of offshoring is measured by the marginal task $J$. If $J$ increases, a larger fraction of tasks is offshored. Two reasons for a shift in the marginal task exist. Firstly, the wage in the rest of the world falls. By inspection of the condition for the marginal task in equation (8) we see that $J$ has to increase given the constance of the domestic wage. Secondly, the transportation costs of the task falls. For example, communication links to the rest of the world improve which reduces the costs $\tau$. If the transport cost for each individual task fall, then, by equation (8), $J$ must increase. In terms of their effect on the marginal task both reasons are equivalent. However, I will restrict the analysis in this section to a fall in trade costs in order to be able to track the effect of the change of cost savings parameter $\Omega(J)$. In my discussion I follow Grossman and Rossi-Hansberg (2008) and assume a uniform fall in the trade costs of all tasks. Formally I assume that the trade costs fall by $\nu < 1 \ \forall j$, where $(1 - \nu)\tau_j$ is the new level of trade costs of task $j$. Inspecting the definition of $\Theta$ in (9) reveals that it is only affected by a change in $J$ and not affected by change in the trade costs itself.

**Proposition 1.** Let the extent of offshoring, $J$, in the economy increase. Then the productivity adjusted wage $\tilde{w}_i$ of each factor $i = L, H$ increases.

**Proof.** Totally differentiating the log of the pricing equation (26) for either final good sector, yields $\hat{p}_i = \alpha\hat{\beta} + \alpha\beta\hat{w}_k + (1 + \beta - \alpha(1 + \beta))\hat{w}_k$, where the hat indicates a percentage change. Note that the change in both wages must be equal, $\hat{w}_H = \hat{w}_L$, because the relative factor prices

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8 A fall in the trade costs is similar to a cost reduction in service links in Jones and Kierzkowski (1990).

9 For a discussion of a proportional fall in trade costs see Grossman and Rossi-Hansberg (2008)
are determined by the relative final price which is unchanged. Taking this into account results in

\[ \hat{w}_k = -\alpha \beta \hat{\Theta} \]

This establishes a positive correlation between wages and offshoring.

Domestic unskilled labour is mixed with cheaper foreign unskilled labour which raises the effective productivity of domestic labour. As with Hicks-neutral technological progress the marginal product of labour increases which increases the return for labour. This is the productive effect in \( ? \). Offshoring reduces the costs in the intermediate sector. These cost savings are passed on to the final good sectors which become relatively more intensive in their use of capital inputs.\(^{10}\) The substitution effect raises the marginal product of the respective factor. In combination with constant final good prices the factor prices must increase.

Is the result in proposition 1 a result of the structure of the model? If only tasks of unskilled labour are offshored, \( ? \) find that the wages of unskilled labour in a Heckscher-Ohlin-Samuelson (HOS) framework increases with the extent of offshoring due to a productivity effect. Therefore the result for unskilled labour seems general. Considering the effect of offshoring unskilled labour tasks on the wage of skilled labour they conclude that it is possible to increase in the extent of offshoring. The condition is, that the skilled wage increase if the productivity effect in the skilled intensive sector is larger than in the unskilled intensive sector. This condition is satisfied in the present model because both final good sectors are affected by offshoring in the same way. In that respect, the proposition is well within the effect reported in Grossman\(^{10}\)

\(^{10}\)The aggregate price index in the intermediate sector is \( P_Z = \Theta^\alpha \hat{w}_L \hat{w}_H^1 \). Rewriting the latter expression in percentage changes and using the extent of the wage changes given in the above proof yields \( \hat{P}_Z = \alpha (1 - \beta) \hat{\Theta}, \) which is smaller one but positive if \( J \) increases.
Will all factors in this set-up support offshoring? In a standard HOS model, the Stolper-Samuelson theorem indicates that not all factors of production gain from liberalizing final goods trade. In this model none of factors has an incentive to oppose offshoring because they all gain from higher wages, as shown in proposition 1.

**Proposition 2.** Let the extent of offshoring, $J$, in the economy increase. Then the growth rate in the economy increases.

*Proof.* Differentiating the growth rate in equation (??) with respect to the marginal task which yields

$$\frac{\partial g}{\partial J} = \frac{\partial g}{\partial \eta} \frac{\partial \eta}{\partial J} > 0,$$

as $\partial g/\partial \eta < 0$ and

$$\frac{\partial \eta}{\partial J} = -\frac{\alpha \beta^2 \sigma (1 - \sigma)}{(1 - \alpha \beta \sigma J)^2} < 0.$$

The growth rate increases only if more skilled labour is allocated to research. In figure 2 we depict the the effect of a fall in $\Theta$ on the equilibrium conditions (14) and (18), and the equilibrium growth and output. Inspection of the no-arbitrage condition (14) reveals that it is not affected by a change in the extent of offshoring. Although intermediate producers are able to reduce costs, entry costs into research increase as well, due to a change in wages, and both effect cancel each other out. The resource constraint, however, shifts out with an increase in $J$. The new equilibrium is found at $E'$, with a higher output and growth rate. The reason for the shift of the resource constraint is as follows. Offshoring increases the effective endowment of the economy. In the production of the capital input, domestic unskilled labour is substituted with foreign unskilled labour. This substitution increases the
effective domestic unskilled labour force. Offshoring of unskilled labour spills over on skilled labour because more skilled labour is employed in research. The reason is that due to offshoring, intermediate firms become more profitable which positively affects entry into research. More research without diminishing the current output is possible because skilled labour can be substituted by unskilled labour, whose effective endowment increased due to offshoring.

2.7 Relative Prices and Growth

I investigate how a change in the final prices affect the wages of skilled and unskilled workers to determine an “augmented Stolper-Samuelson Theorem”. The change of the wages is derived in the appendix and yields

\[
\begin{align*}
\hat{w}_H &= \left(1 - \frac{\beta(1-a)}{1+\alpha \Lambda} \right) \hat{p}_X - \frac{1}{1-\beta} \left(1 - \frac{1-a}{1+\alpha \beta \Lambda} \right) \hat{p}_Y \\
\hat{w}_L &= \left(1 - \frac{\beta(1-a)}{1+\alpha \Lambda} \right) \hat{p}_Y - \frac{1}{1-\beta} \left(1 - \frac{1-a}{1+\alpha \beta \Lambda} \right) \hat{p}_X,
\end{align*}
\]

where \(\Lambda \equiv \varepsilon_{\Theta, J, \varepsilon, \omega_L} < 0\), which can be interpreted as the elasticity of the cost saving. For simplicity I assume for the rest of the section that \(\hat{p}_X > 0\) and \(\hat{p}_Y = 0\). The change of wages is summarized in figure 2.7. On the horizontal axis I depicted the negative of the cost savings elasticity, \(-\Lambda\). If no offshoring takes place, \(\Lambda = 0\) and I obtain the standard Stolper-Samuelson effect. There exist three sets of wage effects due to a change in final good prices. Firstly, the set with \(\hat{w}_H > 0\) and \(\hat{w}_L < 0\), which corresponds to the standard Stolper-Samuelson effect with offshoring. Secondly, the set with \(\hat{w}_H < 0\) and \(\hat{w}_L < 0\). Thirdly, the set with \(\hat{w}_H < 0\) and \(\hat{w}_L > 0\).

**Proposition 3.** Let the relative price of the final goods \(p_X/p_Y\) change. Then an increase in the relative final price decreases the extent of offshoring if the wage of the unskilled labour increases. In this case the growth rate falls. If the wage of the unskilled labour decreases the growth effect is ambiguous.
I rewrite the growth rate as a function of the marginal task and the relative final price, \( g = g(J, \frac{p_X}{p_Y}) \). The effect of a change in the relative final price can therefore be decomposed as follows

\[
dg = \frac{\partial g}{\partial J} dJ + \frac{\partial g}{\partial \frac{p_X}{p_Y}} \frac{p_X}{p_Y} dp_X
\]  

(23)

The first term on the right-hand side is the effect of offshoring whereas the second term refers to the impact of relative prices on growth. I assume that \( p_Y \) remains constant whereas the \( p_X \) increases which implies \( d(\frac{p_X}{p_Y}) > 0 \). From the growth rate I know that \( \frac{\partial g}{\partial \frac{p_X}{p_Y}} < 0 \). Thus the sign of the second term is negative. The reason is that the present value of the endowment in terms of skilled labour has decreased and consumers are less willing to invest in research. If the wage of the unskilled workers increases due to the change in final prices, less tasks are offshored and \( J \) must fall. As I showed previously, \( \frac{\partial g}{\partial J} > 0 \). Therefore, the first term is unambiguously negative and thus, taking both terms together, \( dg < 0 \). However, if the wage of the unskilled workers falls due to the final price effect, \( J \) must increase. Accordingly the first term is positive and, taking both terms together, the impact of the price change is ambiguous. The growth rate increases if the effect of offshoring (i.e. the first term) dominates the negative relative price effect.
3 Conclusion

In this paper I showed that offshoring increases growth in an economy. I further showed that the effect of a change in relative final prices has an ambiguous effect on growth. If the standard Stolper-Samuelson effects take place, growth decreases because research become more expansive. However, if the Stolper-Samuelson Theorem is does not hold anymore, growth might increase if the negative terms of trade effect id dominated by the efficiency gains of offshoring.

References


Appendix

Unit Input Coefficients

In this section we derive the unit input coefficients. We start with the pricing equations

\begin{align}
  p_X &= A_Z^{\beta} p_Z \frac{\beta}{\sigma} \bar{w}_H^{1-\beta} \\
  p_Y &= A_Z^{\beta} p_Z \frac{\beta}{\sigma} \bar{w}_L^{1-\beta},
\end{align}

(24)

where \( A_Z \equiv n \frac{\epsilon}{\mu} \). The pricing rule for the intermediate input is

\[ p_Z = \frac{\tilde{c}(.)}{\sigma}, \]

(25)

where \( \tilde{c}(.) = \Theta(J)^{\eta} c(.) \). Let the productivity adjusted factor prices be \( \bar{w}_i \equiv w_i A_Z^{\beta} \) for \( i = H, L \). We can therefore rewrite the pricing equations as

\begin{align}
  p_X &= \left( \frac{c_i(\bar{w}_L, \bar{w}_H)}{\sigma} \right)^{\beta} \bar{w}_H^{1-\beta} \\
  p_Y &= \left( \frac{c_i(\bar{w}_L, \bar{w}_H)}{\sigma} \right)^{\beta} \bar{w}_L^{1-\beta} \\
  P_Z &= \frac{\tilde{c}_i(\bar{w}_L, \bar{w}_H)}{\sigma} A_Z^{\beta}.
\end{align}

(26)

We define \( p_X \equiv \tilde{c}_X(\bar{w}_L, \bar{w}_H) \) and \( p_Y \equiv \tilde{c}_Y(\bar{w}_L, \bar{w}_H) \) to be the productivity adjusted cost functions of firms in the \( X \) and \( Y \) sector. The unit
input coefficients are defined as the derivative of the cost function with respect to the input price. We therefore have that

\begin{align*}
    a_{HX} &= (1 - \beta) \frac{p_X}{w_H} \\
    a_{LY} &= (1 - \beta) \frac{p_Y}{w_L} \\
    a_{ZX} &= \beta \frac{p_X}{w_L} \\
    a_{ZY} &= \beta \frac{p_Y}{w_L} \\
\end{align*}

(27)

We can now write the a’s. \( b_{HX} = a_{HX} + a_{HZ}a_{ZX} \)

\begin{align*}
    b_{HX} &= p_X \left( \frac{(1-\beta)}{w_H} + \frac{\beta \sigma a_{HZ}}{e} \right) \\
    b_{LY} &= p_Y \left( \frac{(1-\beta)}{w_L} + \frac{\beta \sigma a_{LZ}}{e} \right) \\
    b_{LX} &= p_X \frac{\beta \sigma a_{HZ}}{e} \\
    b_{HY} &= p_Y \frac{\beta \sigma a_{H}}{e} \\
\end{align*}

(28)

Derivation of Expenditures

We can rewrite \( nz = n_{ZX} + n_{ZY} \), where \( z_k \) is the demand for a capital input in sector \( k = X, Y \). Accordingly, the demand from a final sector for all varieties of the capital input is \( n_{ZX} = a_{ZX}X \) and \( n_{ZY} = a_{ZY}X \). Further, we can use the marginal task to substitute for the foreign unskilled wage. We rewrite the expenditures as

\[ E = A_Z^{-\beta} \left[ (p_X \bar{X} + p_Y \bar{Y}) - (a_{ZX}w_L \bar{X} + a_{ZY}w_L \bar{Y}) \frac{\int_0^T \tau(j) dj}{\tau(J)} \right]. \]

We can now determine the growth rate of the expenditures \( \frac{\dot{E}}{E} \) by taking logs of the above expression and differentiating with respect to time. Because prices and productivity adjusted outputs are constant, the expression in the square brackets is constant. The reason is that the unit input coefficients fall at the same rate as unskilled wages increases. Expenditures grow at the same rate as wages \( \frac{\dot{E}}{E} = \beta \frac{1}{2 - \gamma} g \).

Proof Proposition 3

This is the formal proof of the proposition. We can rewrite the growth rate as a function of the marginal task and the relative final price,
\[ g = g(J, \frac{p_Y}{p_X}). \] Totally differentiating yields
\[ dg = \frac{\partial g}{\partial J} dJ + \frac{\partial g}{\partial p_Y} d\left(\frac{p_Y}{p_X}\right). \] Accordingly we have to determine the signs of the derivatives to be able to proof the proposition. We know from proposition ?? that \( \frac{\partial g}{\partial J} \) is positive. Calculating
\[ \frac{\partial g}{\partial p_Y} = \frac{1 - n}{a} (1 - \beta) L \left( \frac{p_Y}{p_X} \right)^{-\beta} \] which is larger than zero.

If we assume that \( p_X \) is constant whereas \( p_Y \) increases, it follows that \( d\frac{p_Y}{p_X} > 0 \). Further, from the extended Stolper-Samuelson Theorem we know that with \( dp_Y > 0 \) the wage of unskilled labour increase, which, by the equation for the marginal task, implies an increase in the extent of offshoring, \( dJ > 0 \). Accordingly, \( dg > 0 \) if \( d\frac{p_Y}{p_X} > 0 \). The proof of a fall in the relative final price follows from analogous reasoning.

**Derivation of Equation (20)**

In this appendix we derive the no-arbitrage condition in equation (20) in more detail. We start by considering the basic no-arbitrage condition in equation (14). If this condition holds, the investment sector is in equilibrium as no firms have an incentive to enter or exit research. Consumer optimization yields the condition (19), which we substitute in the no-arbitrage condition. The evolution of expenditures is determined by the prevailing interest rate \( r_t \) and the discount factor \( \rho \). The correlation between change in income and the interest rate is positive. For instance, if the interest rate is high, consumers are willing to save more. Because savings must equal investments, income grows faster.

We substitute the definition of the growth rate, \( g \equiv \frac{\dot{n}}{n} \), and the growth of the high skilled wage, \( \frac{\dot{w}_H}{w_H} = \frac{\beta}{\varepsilon - 1} g \). The growth of the skilled wage is derived from the definition of the effective wage, \( \bar{w}_H = A_w w_h \), which constant. As we argue in the text the growth rate of the expenditures is \( \frac{\dot{E}}{E} = \). These substitution yield the modified no-arbitrage condition

\[ \frac{\beta}{\varepsilon - 1} g + \rho = \frac{n\pi}{aw_H} + \frac{\beta}{\varepsilon - 1} g - g. \] (29)

The profits of an intermediate producer is given in equation (11). We rewrite the profit function by using \( nz = nz_X + nz_Y \) which are the
demands for the capital input from each of the final input sectors. The demands are $a_{ZX} \bar{X}$ and $a_{ZV} \bar{Y}$. Multiplying both demands with $p_x$ yields $\beta(p_x \bar{X} + p_Y \bar{Y})$. Substituting this and making the appropriate cancellations yields the no-arbitrage condition in equation (20).

**Derivation of the augmented Stolper-Samuelson Theorem**

Taking pricing equations in 4, taking logs on both sides and totally differentiating both equations yields

$$
\hat{p}_X = \alpha \hat{\beta} + \alpha \beta \hat{w}_L + (1 - \alpha \beta) \hat{w}_H
$$

$$
\hat{p}_Y = \alpha \beta \hat{\theta} + (1 - (1 - \alpha) \beta) \hat{w}_L + (1 - \beta) \hat{w}_H
$$

(30)

I write $\Theta = \Theta(J)$ which totally differentiated yields $d\theta = (\partial \Theta / \partial J) dJ$. This can be rewritten as

$$
d\Theta = \frac{\partial \Theta}{\partial J} J \frac{dJ}{J} = \frac{\partial \Theta}{\partial w_L} \frac{d w_L}{J} \frac{d J}{w_L} = \hat{\Theta} = \varepsilon_{\Theta, J, \varepsilon_J, \varepsilon_{w_L}} \hat{w}_L,
$$

(31)

which I substitute in (30). I define $\Lambda \equiv \varepsilon_{\Theta, J, \varepsilon_J, \varepsilon_{w_L}}$. Rewriting the system of equation I obtain

$$
\begin{pmatrix}
\hat{w}_H \\
\hat{w}_L
\end{pmatrix} = \chi
\begin{pmatrix}
1 - \beta + \alpha \beta (1 - \Lambda) & -\alpha \beta(1 + \Lambda) \\
-(1 - \alpha) \beta & (1 - \alpha \beta)
\end{pmatrix}
\begin{pmatrix}
\hat{p}_x \\
\hat{p}_y
\end{pmatrix},
$$

(32)

where $\chi \equiv \frac{1}{(1 - \beta)(1 + \alpha \beta \Lambda)}$. Solving this system of equation for the changes in wages yields

$$
\hat{w}_H = \frac{[1 - \beta + \alpha \beta (1 - \Lambda)] \hat{p}_x - \alpha \beta(1 + \Lambda) \hat{p}_y}{(1 - \beta)(1 + \alpha \beta \Lambda)},
$$

$$
\hat{w}_L = \frac{(1 - \alpha \beta) \hat{p}_x - (1 - \alpha) \beta \hat{p}_y}{(1 - \beta)(1 + \alpha \beta \Lambda)},
$$

(33)

which, after some rearranging, yields the equations in 22.