Heterogeneous firms and homogenising standards in agri-food trade
– the Polish meat case

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Abstract
The paper develops a partial equilibrium trade model with heterogeneous firms. The model is applied to the issue of compliance with the EU food standards in the Polish meat sector. Estimates of the size and productivity distribution of Polish meat firms reflect firm heterogeneity. The model parameters are estimated using limited data and a full information least squares method to match observed patterns of trade. Kernel density estimates based on a Monte Carlo sample of parameter estimates show the asymmetric trade cost between Poland and the EU15. Simulation analysis finds that investment support programmes in the context of EU accession lower the firms’ productivity threshold to meet standards and to qualify for exporting. This can slow down structural changes in the industry in the receiving country since existing smaller and less productive firms can continue to exist. However when considering productivity upgrades, the simulation analysis points out that a modest increase of the minimum productivity level in the industry would be more than sufficient to compensate for the export drops following an increase in the fixed costs of compliance.

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1. Introduction

Technical regulations and standards have gained great importance in international agri-food trade. Governments of high-income countries have increasingly implemented tighter and mandatory standards for agri-food products and also demand that agri-food imports comply with them. Since food standards differ across countries, they can potentially restrict market access of exporters. In the context of free trade agreements, the facilitation of trade through dovetailing regulatory frameworks is increasingly important, next to the reduction of traditional trade policy measures such as tariffs and quota. While streamlining standards can be expected to facilitate bilateral trade, changing from existing practices to new production processes or new product specifications also involves costs. Producers that wish to participate in and benefit from the deeper integration of markets have to pay an “entrance fee” in the form of investments in compliance with new standards.

In this regard, the most recent wave of eastward enlargement of the European Union (EU) constitutes an interesting case. In the agri-food sector, establishing common EU standards harmonised the diverging standards requirements across the member states, and the common EU food standards also apply in the new member states of Central and Eastern Europe. In the new member states, adopting the EU food standards has required considerable efforts on the side of governments and producers alike. While EU membership opened trade opportunities, particularly the small and medium size agri-food firms struggled to meet the tight requirements and the capacity to comply continues to determine access to the EU single market.

Some studies on EU enlargement include the issue of deep integration and trade facilitation through harmonisation. Smith and Venables (1988) first introduced the now standard modelling approach of “iceberg tariffs” in the context of the EU single market. In this approach, non-tariff measures (NTMs) such as standards are regarded as variable trade costs that use up resources of exporting firms, and different methods are applied to obtain estimates of respective tariff-equivalents. They range from the direct measurement of compliance costs or careful price comparisons along a given supply chain to cross-country econometric estimations. Ferrantino (2005) provides an overview of recent studies. However, price wedges and “iceberg tariffs” cannot fully represent standards, in particular food standards, as they explicitly lead to fixed (through additional investments) and variable (through additional activities) compliance costs for exporting firms.

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1 Standards often refer to industry requirements that firms tend to meet on a voluntary basis, whereas technical regulations are always mandatory requirements imposed by governments. This paper uses the term “standard” in the sense of mandatory norms; industry standards are not considered.
Standards are more than just border measures, and their fixed and variable compliance costs become an integral part of the export decision of firms. In the agri-food sector, like in other sectors, firm size is not uniform but typically shows a skewed pattern, with some large firms and many small firms, and the different types of costs will weigh differently across firms. Larger, and possibly more productive, firms find it easier to cover a given amount of fixed compliance costs than their smaller competitors. If compliance with certain import standards is a precondition for exporting, foreign sales therefore tend to be more concentrated within the larger firms. The empirical trade literature also points out that those firms that engage in exporting typically only export a fraction of their produce; for a review of empirical studies on the characteristics of exporting firms see for example Bartelsman and Doms (2000) or Eaton et al. (2004). These empirical findings have led to advances in trade theory that account for the presence of firm heterogeneity. Heterogeneous firms models such as those developed by Melitz (2003) and Bernard et al. (2003) are particularly useful for analysing standards since they allow us to incorporate different types of compliance costs while accounting for size and productivity differences of firms. First, in heterogeneous firm models both the fixed and variable costs of complying with the standards can be reflected. Secondly, firms are considered to be heterogeneous in their productivity level. The most productive firms engage in exporting, while at the same time serving the domestic market. In the case of standards, only productive firms find it profitable to pay compliance costs so as to satisfy the specific requirements for exporting and gain access to foreign markets.

The goal of this paper is to explore the market and trade effects of food standards by accounting for their fixed and variable compliance costs in the presence of firm heterogeneity. For our analysis, we develop a partial equilibrium variant of a trade model with heterogeneous firms and apply it to the case of meat trade between Poland and the old member states (EU15). Using simulations, we specifically look at the trade and market structure effects of the financial support programme to assist firms in complying with the EU food standards. Rau and van Tongeren (2007) account for the fixed and variable cost of compliance in the Polish meat sector in an oligopolistic market framework where the partitioning of firms into exporters and non-exporters is fixed. The current paper extends their analysis by making the decision to export endogenous, and hence by providing more insights into the characteristics of firms that engage in exporting.

In the context of EU enlargement, the Polish meat case is particularly illuminating since 1) the Polish meat sector is very fragmented, with a large share of small and medium sized firms, 2) the EU standards are particularly strict for food products of animal origin that can present serious hazards to human health and 3) the EU standards in the Polish meat sector represent a prototypical instance of a small country adopting standards to gain access to a
larger market. While taking the Polish meat sector as a case study, the paper however
contributes to the broader discussion of trade integration that goes beyond reducing traditional
trade barriers. In the heterogeneous firm model, lowering trade costs has the additional benefit
of increasing the number of new varieties and firms, next to the usual expansion of trade by
existing firms.

The paper is structured as follows: We first derive the heterogeneous firm model. This is
followed by an introduction to the case study of Polish meat trade, with a focus on the EU
standards and EU support programmes to assist firms in complying. Next comes a section on
parameter estimation using full information least squares. This section also includes Monte
Carlo simulation to obtain insights into the reliability of parameter estimates. Finally, we
apply the heterogeneous firm model in a number of simulation scenarios to investigate the
impact of the stricter EU standards on meat trade between Poland and the EU15. The
simulations specifically explore the role of EU support programmes on lowering the market
entry cost for Polish meat firms.

2. The model

The heterogeneous firm model we derive is a partial equilibrium variant close to the general
equilibrium model by Balistreri et al. (2007). There is a set of countries/regions R with each
region \( r = 1, 2, \ldots, n_r \) exporting and importing. For pairs of trading regions, \( s \) denotes the
exporting or source region, and \( r \) denotes the importing or destination region; \( s, r \in R \). If
\( s = r \) products are sold domestically, whereas \( s \neq r \) implies sales on foreign markets.

Assuming Dixit-Stiglitz monopolistic competition, firms in each region produce one specific
variety of the differentiated product, and consumers demand a standard CES composite
bundle of the product varieties with the substitution elasticity \( \sigma \). Note that \( \sigma \) measures the
constant substitutability among the product varieties and consumers are assumed to equally
prefer respective varieties, no matter where they come from. The total demand for the
differentiated product in each region is given by

\[
Q_r = \mu_r \frac{Y_r}{P_r} \tag{I}
\]

where \( Y_r \) denotes the national income in region \( r \), and \( \mu_r \) denotes the expenditure share. \( P_r \)
refers to a (dual) price index that is defined over the prices of each supplying firm.

In the profit maximisation problem, firms consider the variable costs of supply to the
domestic and foreign markets, next to tariffs. Focusing on foreign markets, the variable costs
of exporting not only include transport costs but also contain the variable compliance costs
that firms incur to gain market access. As firms use resources to cover the variable export
costs, they increase firms’ marginal unit costs, and the model captures them in the standard fashion of “iceberg trade costs”.

Each firm sets an optimal price $p_{sr}$ to maximize profits $p_{sr} \cdot q_{sr} - \frac{c_s \cdot \tau_{sr}}{\theta_{sr}} \cdot q_{sr}$ on each of its potential markets, where $\tau_{sr}$ refers to the “iceberg trade costs”.\(^2\) Note that for exporting firms $\tau_{sr} \geq 1$, whereas for firms selling on the domestic market only $\tau_{sr} = \tau_{sr} = 1$. The marginal unit costs are denoted $c_s$, and $\theta_{sr}$ is the firms’ productivity, which will be greater than some minimum productivity level $\theta^*_{sr}$, whose function will become apparent below.

It is convenient to express all firm level variables in terms of the variety with average productivity $\theta_{sr} = \bar{\theta}_{sr}$. The profit-maximizing price of this average variety is given by:

$$\bar{p}_{sr} = \frac{c_s \cdot \tau_{sr}}{\bar{\theta}_{sr}} \cdot \frac{\sigma}{\sigma - 1}$$

The corresponding CES demand of the average variety is determined by:

$$\bar{q}_{sr} = Q_r \cdot \left[ \frac{\bar{p}_{sr}}{P_r} \right]^{-\sigma}$$

The price index $P_r$ can also be expressed in terms of average variety. Defining the average price as $P_r = \left[ \sum_s \int_{\theta_{sr}}^\infty p_s(\theta_s)^{1-\sigma} \, d\theta_s \right]^{1/\sigma}$ and, using a CES weighted average of productivity, the price index in each region can be approximated by:

$$P_r = \left[ \sum_s N_{sr} \cdot \bar{p}_{sr}^{1-\sigma} \right]^{1/\sigma}$$

where $N_{sr}$ is the number of supplying firms, equal to the number of varieties, and $\bar{p}_{sr}$ denotes the price in region $r$ for the variety sourced from $s$ produced at average productivity.

The equation for bilateral trade is derived by multiplying equation (III) with $\bar{p}_{rs}$ and substituting total demand $Q_r$ (equation I):

$$\bar{q}_{sr} \cdot \bar{p}_{sr} = \mu_s \cdot Y_s \cdot N_{sr} \cdot \left[ \frac{P_{sr}}{P_s} \right]^{1-\sigma}$$

Firms also face fixed costs and, like the variable costs, they are assumed to be equal across firms within each region. The model distinguishes between the firms’ fixed costs to set-up in

\(^2\) For expositional clarity, the model formulation presented here does not include tariffs. However these border instruments can easily be included in order to allow for the distinction between traditional trade barriers and NTMs.
the domestic market (s = r) and those to export (s ≠ r). We consider the fixed export costs to include the compliance costs of standards that exporting firms have to meet in order to supply foreign markets. Each firm’s market entry decision crucially depends on its distinct productivity level, and there are cut-off productivity thresholds that determine which markets firms supply. Let \( \theta_{sr} \) denote the firm’s productivity and \( \theta_{sr}^* \) refer to the productivity threshold. Firms with \( \theta_{sr} > \theta_{sr}^* \) serve the domestic market and those with \( \theta_{sr} > \theta_{sr}^* \) serve foreign markets. The condition \( \theta_{sr} > \theta_{sr}^* \) holds in order to ensure the partitioning of firms by the markets they supply.

Firm productivity is assumed to be distributed according to the Pareto distribution with the shape parameter \( \alpha \) and the location parameter \( \beta \): The probability distribution function is

\[
g(\theta_{sr}) = \frac{\alpha}{\beta} \left( \frac{\beta}{\theta_{sr}} \right)^{\alpha+1} \text{ and the cumulative density } G(\theta_{sr}) = 1 - \left( \frac{\beta}{\theta_{sr}} \right)^\alpha.
\]

Given the number of active firms, we can derive the productivity level of the marginal firm, i.e. the firm that just finds it profitable to operate. The proportion of firms trading from s to r equals the ratio \( \frac{N_{sr}}{M_s} \) where \( M_s \) refers to the mass of firms potentially operating in the respective region. \( M_s \) is assumed to be constant.³

In each region, the probability of finding a firm with productivity level greater than the threshold is \( Pr \{ \theta_{sr} \geq \theta_{sr}^* \} = 1 - G(\theta_{sr}^*) = \frac{N_{sr}}{M_s} \). With the Pareto distribution for firm productivity, the cut-off productivity threshold \( \theta_{sr}^* \) is thus given by

\[
\theta_{sr}^* = \beta_s \left[ \frac{N_{sr}}{M_s} \right]^{-\frac{1}{\alpha_s}} \tag{VI}
\]

To determine the number of firms (varieties), we need a zero-profit condition. Only sufficiently productive firms find it profitable to pay the respective fixed entry costs and engage in trading from s to r. At the cut-off productivity level \( \theta_{sr}^* \), the firms’ variable profit (gross of tariff) equals the fixed costs \( f_{sr} \) which firms face when supplying the respective market, and we derive the zero profit cut-off productivity accordingly:

³ Focusing on the medium term, we assume \( M_s \) to be given. However, the model could also endogenously determine \( M_s \) by defining an equation for the firms general decision to produce or not. Such an equation would involve specifying the firms expected profits; see Balistreri et al. (2007).
\[ q_{st} (\theta^*_{st}) \ast \left[ p_{st} (\theta^*_{st}) - \frac{c_s \ast \tau_{st}}{\theta^*_{st}} \right] = f_{st} = 0. \] Note, once a firm overcomes the entry costs to the domestic market, they are sunk and hence do not matter in the firm’s decision to supply foreign markets. A salient feature of the monopolistic competition framework is that the ratio of revenues between any two firms only depends on the ratio of productivities. We exploit this feature to retrieve the zero profit cut-off productivity in terms of averages. Using equations (II) and (III), we obtain

\[ \frac{r(\bar{\theta}_{st})}{r(\theta^*_{st})} = \left[ \frac{\bar{\theta}_{st}}{\theta^*_{st}} \right]^{\sigma-1} \] where \( r(\bar{\theta}_{st}) \) refers to the revenue of firms producing the average differentiated product with average productivity and \( r(\theta^*_{st}) \) denotes revenue at the respective productivity thresholds.

The relation between \( \bar{\theta}_{st} \) and \( \theta^*_{st} \) can be established by using a CES weighted average productivity. Define the average productivity as follows:

\[ \bar{\theta}_{st} = \frac{1}{1-G(\theta^*_{st})} \int_{\theta^*_{st}}^{\infty} \theta^\sigma_{st} g(\theta_{st}) d\theta_{st} \] . With the Pareto distribution, this yields a constant ratio between the cut-off level and the average productivity:

\[ \bar{\theta}_{st} = \theta^*_{st} \ast \left[ \frac{\alpha_s}{(\alpha_s + 1 - \sigma)} \right]^{\frac{1}{\sigma-1}} \] (VII)

Using this relation, the revenue at the productivity thresholds \( r(\theta^*_{st}) \) can be expressed in terms of \( r(\bar{\theta}_{st}) \) and yields the zero profit cut-off productivity in terms of averages as follows:

\[ \frac{\bar{q}_{st} \ast \bar{q}_{st}}{\sigma \ast (1 + t_{st})} \ast \left( \frac{\alpha_s + 1 - \sigma}{\alpha_s} \right) - f_{st} = 0 \] (VIII)

where \( f_{st} \) denotes the fixed entry cost into each market.
3. The Polish meat case

The integration of Poland into the EU started long before its accession in May 2004. Initial associations with the EU and subsequent amendments gradually liberalised agri-food trade between Poland and the EU15 in order to establish a free trade area with common border protection (custom union). While offering improved trading opportunities, EU membership was conditional on taking over the entirety of the EU legal rules and regulations (acquis communautaire). As such, Poland had to adopt the EU food standards, just like the other new member states, but transitional periods for the time after accession and special safeguard clauses to ensure the functioning of the EU common market were agreed upon; for details see Inglis (2004).

In order to place meat products on the EU single market, slaughterhouses, cutting plants and possessing firms in the member states have to comply with Directive 77/99/EEC and 64/433/EEC that include the EU meat standards and specify some additional provisions regarding product testing, transportation and administrative matters. For meeting the EU requirements, both product and process standards, Polish meat firms had to substantially modernise production facilities and processes, leading to fixed costs of compliance. While promoting the productivity of Polish meat firms, the necessary changes in processing methods and control systems for food safety/hygiene also increased variable production costs, mainly due to the employment of more skilled workers, more frequent checks as well as documentation requirements. Representative information about the fixed and variable compliance costs in the Polish meat sector is not available. However, Eurostat reports the investments undertaken in the sector, and they most likely contain the fixed costs of compliance.

Figure 1 plots the development of meat trade between Poland and the EU15 against the investments in the Polish meat sector. The volume of Polish meat exports steadily grew in the pre-accession period, and the largest increase is observed after Poland became EU member country in 2004. While obviously reflecting the liberalisation of meat trade between Poland and the EU15, trade growth shows a rather strong correlation with the investments undertaken. In the years shortly before accession, investments in the Polish meat sector increased dramatically, and in 2004 it made up for about 25% of the total investment in the

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agri-food industry. Investments were mainly made by larger firms with more than 50 persons employed, and in some cases foreign investors were involved. While favourable interest rates and credit guarantees for example were provided, the EU’s Special Accession Programme for Agriculture and Rural Development (SAPARD) was implemented to prepare the Polish agri-food firms for EU membership.

The actual start of the SAPARD programme scheduled for the time period 2000-2006 was delayed in Poland, and the first payments were made in 2002. Almost 40% of the total SAPARD funds in Poland were earmarked for supporting agri-food processing firms in their adjustment to the EU standards (Measure 1 “Improvement in processing and marketing of agri-food products”). According to the ex-ante evaluation of the Polish SAPARD programme, the meat sector absorbed almost 50% of the financial assistance paid under measure 1, equalling about 266.2 million euro (IERiGZ-PIB, 2007). Overall, the share of the SAPARD funds however covered only about 25% of the total investment in the Polish meat sector. Mainly large Polish meat firms benefited from the SAPARD funds that the EU15 and Poland co-financed. Relatively fewer small and medium firms obtained support by the SAPARD programme and invested less due to their particular financial constraints, amongst other factors. As already mentioned, the large number of small and medium firms had great difficulties to apply the EU food safety and hygiene standards, and compliance continues to determine the Polish meat firms’ access to the EU market.

**Figure 1: Development of meat trade and investment in the Polish meat sector**

Source: Eurostat.
With regard to the aforementioned directives, Rau and van Tongeren (2007) elaborate on the state of compliance in Polish meat sector and the resulting market opportunities for firms in the accession year 2004. Polish meat firms that comply are approved to export to the other EU member states and receive an EU export licence, whereas non-complying firms can sell their produce on the domestic market only. The production capacity marks the dividing line between complying and non-complying Polish meat firms. In 2004, almost 70% of the Polish meat firms did not meet the EU requirements, and they were mainly small and medium enterprises with considerably lower production capacities than large Polish meat firms. Due to their tremendous difficulties to adopt the EU standards, small Polish meat firms have been granted special provisions, mainly relating to the detailed documentation requirements and record keeping that allow them to continue their production for the domestic market without fully meeting the EU requirements.

The Polish meat sector comprising the production, processing, and preserving of meat and meat products can generally be characterised by its very asymmetric structure. With many small firms and only a few large firms the size distribution of Polish meat firms is extremely skewed. In 2002, 72% of the total of 4271 Polish meat firms registered by Eurostat employed between 1 and 9 persons, and only 10% had more than 50 employees. By 2004 the number of meat firms declined to 3881. Most of the industry exit occurred in the smallest size class whose share accounted for 69% of meat firms in 2004. Table 1 presents the asymmetric structure in the Polish meat sector by some indicators of inequality estimated for the time period 2000-2004. For comparison, the respective indicators for the EU15 are included.

<table>
<thead>
<tr>
<th></th>
<th>Gini-coefficient</th>
<th>C5-ratio</th>
<th>C10-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poland employment</td>
<td>0.62</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>turnover</td>
<td>0.64</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>value-added</td>
<td>0.59</td>
<td>0.19</td>
<td>0.22</td>
</tr>
<tr>
<td>EU15 employment</td>
<td>0.65</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>turnover</td>
<td>0.68</td>
<td>0.18</td>
<td>0.22</td>
</tr>
<tr>
<td>value-added</td>
<td>0.69</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Source: own calculation based on Eurostat.

The inequality indicators are calculated in the basis of estimated Pareto size distributions using grouped Eurostat data; see Appendix 1 for details.
Both markets were rather concentrated, with a considerable share of employment, turnover and value-added generated by the larger firms. The Gini-coefficients estimated point to a slightly lower inequality in the structure of the Polish meat sector compared to the EU15. Gini-coefficients are generally higher for larger countries. On the other hand, the concentration ratios are higher for Poland, indicating the relative economic dominance of large firms in the sector. According to the C5-ratio, the five largest firms generated 23% of the total turnover in the Polish meat industry; adding the next five largest firms increases the ratio to 27%. In the substantially larger EU15 market, the five largest firms are estimated to account for 18% of the total turnover.

4. Application of the model

4.1. Parameter estimation using full information least squares

In our application to meat trade between Poland and the EU15 (two countries/regions case), the model features a total of 15 parameters that have to be determined. The $2 \times 2$ Pareto shape and location parameters ($\alpha_s, \beta_s$) are estimated outside the model (see appendix A1). This leaves 11 parameters to be determined: $c_s, f_{3r}, \tau_{3r}$ and $\sigma$. Since $\tau_{30}=1$, the number of parameters to be estimated reduces to $n_r(1+2n_r)-1=9$. The estimation method is inspired by Balistreri et al. (2007) who propose a full information least squares method to estimate the unobservable trade cost parameters by using cross section data on bilateral trade and imposing exogenous estimates of the variety substitution elasticity, the Pareto location parameter and the domestic set-up costs. Our estimation however relies on (short) time series, instead of a cross-section for one year, and estimates all parameters consistently with the model equations.

Let $\gamma$ denote the vector of parameters, $x$ the vector of endogenous variables generated by the model, $\hat{x}$ a vector of a subset of endogenous variables and $x^0$ the corresponding observations of these variables. Writing the equations of the model as $F(x, \gamma)$, the estimation method finds the values of the subset of parameters $\hat{\gamma}$ that minimizes the sum of squares:

$$\min_{\hat{\gamma}} (\hat{x} - x^0)^T (\hat{x} - x^0)$$

subject to: $\hat{x} = F(x, \gamma)$

$$\gamma = k$$

$$\hat{\gamma} \leq a$$

where $\gamma = k$ is the value of Pareto parameters estimated outside the model, and the last inequality represents possible restrictions on the parameters.
The theory of the model puts one important restriction on the variety elasticity of substitution: \( \sigma < \alpha + 1 \). The data \( x^0 \) are time series of bilateral trade in meat products between Poland and the EU15 for the years 2002, 2003 and 2004, yielding 12 observations. In addition, we use six observations on the total number of firms (\( M_x \)) for each year. Table 2 presents the estimates obtained by this method as well as those of the Pareto parameters of the Pareto productivity distribution that have been obtained through OLS outside the model (see appendix A1). A remarkable feature of the estimates is that they imply that the variable and fixed trade costs are higher for firms in the EU15 than for their Polish counterparts. It appears to be far less costly for Polish firms to enter the EU market, and once they have entered, the variable trade costs are also smaller. Our estimates of fixed costs implicitly take the effects of standards and compliance subsidies into account. By ‘back casting’ the model to a situation without subsidies, we shall explore the effects of subsidies on trade and markets. As shown in table 2, the domestic set-up cost are also much smaller in Poland, which is consistent with the size distribution of firms in Poland being dominated by many small firms with low productivity. On average, EU15 firms are larger and even the smallest have a productivity that is 4.5 times greater than that of the smallest Polish firm. The estimated unit price for variable inputs (\( c_s \)) is slightly smaller in Poland, but the difference is far less pronounced than the extraordinary difference in the other parameter estimates. Finally, the estimate of the elasticity of substitution between varieties (\( \sigma \)) is found to be on its upper bound, dictated by the value of the Pareto shape parameter for Poland.

Table 2: Parameter estimates used in the model.

<table>
<thead>
<tr>
<th>Parameter estimates, full information least squares</th>
<th>Poland</th>
<th>EU15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iceberg trade cost ( \tau_{rc} ):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>1.000</td>
<td>3.608</td>
</tr>
<tr>
<td>EU15</td>
<td>4.553</td>
<td>1.000</td>
</tr>
<tr>
<td>Fixed market entry cost ( f_{sr} ) (1000 euro):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>22.553</td>
<td>14.891</td>
</tr>
<tr>
<td>EU15</td>
<td>2031.772</td>
<td>585.307</td>
</tr>
<tr>
<td>Unit price of variable inputs ( c_s ):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>0.731</td>
<td>0.759</td>
</tr>
<tr>
<td>EU15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Substitution elasticity of varieties ( \sigma ):</td>
<td>3.237</td>
<td>3.237</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimates of Pareto productivity distribution</th>
<th>Poland</th>
<th>EU15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto shape parameter ( \alpha_s ):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polish</td>
<td>2.337</td>
<td>3.993</td>
</tr>
<tr>
<td>standard error</td>
<td>0.099</td>
<td>0.193</td>
</tr>
<tr>
<td>Pareto location parameter ( \beta_s ) (minimum productivity):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polish</td>
<td>0.0263</td>
<td>0.117</td>
</tr>
<tr>
<td>standard error</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.963</td>
<td>0.953</td>
</tr>
</tbody>
</table>

Source: own estimations.
Unfortunately the reliability of the parameter estimates cannot be judged on the basis of standard test statistics. First, it is not obvious how to obtain covariance matrices of the parameters, and second we do not know the distributions that govern the parameters. The latter implies that we cannot rely on test statistics that assume normality. Some insights into the distribution of the parameters can be gained by Monte Carlo simulation.

_Monte Carlo simulation:_

Given the parameter estimates and observations on exogenous variables, we generate a new dataset of trade values $\hat{x}^s$ by adding random disturbances to the model simulated trade values: $\hat{x}^s = F( x, \hat{\phi}) + e$. The random disturbances $e$ are assumed to be iid $N(0,s)$, and the standard deviation is determined from the residuals of the original least squares estimation. Subsequently, the least-squares estimation is repeated with $\hat{x}^s$ assuming the role of $x^0$. After repeating this process 100 times a reasonably large sample of parameter estimates is obtained. This sample of estimates reveals that the parameters are certainly not normally distributed, as seen by comparing the mean and median values presented in table 3 for example and hence the usual normality-based test statistics do not apply.

### Table 3: Summary statistics of Monte Carlo simulated estimations, $n=100$.  

<table>
<thead>
<tr>
<th></th>
<th>Unit price $c$</th>
<th>Variable trade costs $\tau$</th>
<th>Fixed costs $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Domestic market</td>
<td>Foreign market</td>
</tr>
<tr>
<td></td>
<td>PL-EU15</td>
<td>EU15-PL</td>
<td>PL-EU15</td>
</tr>
<tr>
<td>Mean</td>
<td>2.97</td>
<td>0.61</td>
<td>4.63</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.42</td>
<td>0.32</td>
<td>2.40</td>
</tr>
<tr>
<td>Standard error (mean)</td>
<td>0.04</td>
<td>0.03</td>
<td>0.24</td>
</tr>
<tr>
<td>Skewedness</td>
<td>-1.43</td>
<td>1.06</td>
<td>4.34</td>
</tr>
<tr>
<td>Median</td>
<td>3.24</td>
<td>0.55</td>
<td>3.73</td>
</tr>
<tr>
<td>96% confidence interval of median*</td>
<td>3.15</td>
<td>0.46</td>
<td>3.71</td>
</tr>
<tr>
<td></td>
<td>3.15</td>
<td>0.65</td>
<td>3.83</td>
</tr>
</tbody>
</table>

* Calculated by using the binomial distribution to determine the 96% probability of observations falling in the interval around the median: $\text{Prob}[x_i < \text{median} < x_j] = \sum_{l=i}^{j-1} \binom{n}{l} \left( \frac{1}{2} \right)^N$.

In practice some of the Monte Carlo simulated bilateral trade values do not yield feasible solutions to the estimation problem and those observations were discarded. The optimal sample size is not a priori known, but could be approximated by using a non-parametric test statistic on the moments of key parameters.
Kernel density estimation

The non-parametric kernel density estimation allows us to empirically construct a density function on the basis of the sample of observations generated through the Monte Carlo simulations (Silverman, 1986). The results of the kernel density estimates are presented in figure 2. As shown, the estimates of the substitution elasticity (σ) are very much clustered near its theoretical upper bound, while some estimates also yield lower values. In comparison to other estimates, the unit price of variable inputs (c) is most symmetric around the mean. The kernel density estimates suggest that there is indeed a difference in c, with both the median and mean values for the EU15 being larger than for Poland. In contrast, the estimated distributions for trade cost are all extremely left-skewed. The median value of “iceberg trade costs” (τ) for Poland is below the EU15 median, and this appears to indicate a consistent difference in variable trade cost. The densities of fixed cost of domestic market entry are sufficiently far apart to be able to conclude that there is a significant difference between the two markets. Likewise, the fixed costs of entry into the export market are orders of magnitudes apart, although the density estimates reveal a similar shape.

The results presented in this section indicate that consistent estimation of the parameters for the heterogeneous firm model is possible with limited information, but the estimates cannot be assumed to be normally distributed. We have been able to estimate the unobservable trade cost by imposing a huge theoretical structure on the estimation problem. The highly non-linear characteristics of the model in combination with theoretical bounds on parameters and a very pronounced size asymmetry between the two markets makes parameter estimation a perilous task.
Figure 2: Kernel density estimates of model parameters, Poland and EU15.

Source: own simulations
4.2. Scenarios and simulation results

In our simulations, we explore the effects of subsidizing compliance costs, like the EU SAPARD programme did in the Polish meat sector. With the calibration the simulations take the accession year 2004 as the base, and our parameter estimates implicitly include the effects of both strict standards and SAPARD funds in Poland. We therefore “backcast” the model in the first set of simulations that consequently reflect scenarios where standards are in place but the subsidies to comply with them are reduced. This allows us to investigate the effects of supporting firms in their compliance with standards.

More specifically, we increase the fixed entry costs for both the domestic and the EU15 market in the first simulation experiment (S1) in order to reflect the fact that the SAPARD programme was not specifically targeted at exporting firms. Given the information about the SAPARD programme, the size of the shock amounts to 25% and approximates the full removal of the subsidy paid to the meat firms in Poland. In the second simulation (S2), we refer to the considerable test and documentation requirements for exporting to the EU market, and thus increase the variable “iceberg trade costs” for Polish meat firms. Without representative information, we also apply a 25% shock. The third simulation is a combination of S1 and S2. The forth simulation (S4) relates to the upgrading of the Polish meat sector to the EU standards that may lead to more dynamic productivity gains. Whether standards bring about an overall increase in productivity through technological improvements is first and foremost an empirical question. However, since the Polish meat sector can generally be considered as rather traditional we assume that the investments undertaken to meet the EU requirements had a positive effect on the level of productivity of Polish meat firms. In the simulation, we mimic this by raising the minimum productivity level determined by the Pareto location parameter $\beta$.

Focusing on the perspective of Polish meat firms, table 4 summarizes key simulation results. The 25% rise in fixed entry costs following an assumed withdrawal of the SAPARD programme leads to an increase of the average productivity of Polish meat exporters (10.6%). This is not due to technological improvements, but the result of less productive firms being driven out of the EU15 export market. In other words, without the financial assistance they cannot bear the higher market entry cost. Only those firms that have at least a 10.1% higher productivity enter the EU15 market without the subsidy, and the number of exporters consequently falls. The change in the number of firms is an indication for a shrinking extensive trade margin: in the model each firm is producing one variety, and hence the fewer firms there are, the less varieties are exported. The intensive margin, on the other hand, relates to the amount of exports by existing firms to existing destinations. In the simulation of higher
fixed market entry cost, the extensive margin shrinks (less varieties) while the intensive margin grows (expansion of exports of surviving firms).

### Table 4: Selected simulation results.

<table>
<thead>
<tr>
<th></th>
<th>BASE</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n/a</td>
<td>Increase fixed entry cost PL</td>
<td>Increase variable trade cost PL</td>
<td>Increase fixed entry and variable trade cost PL</td>
<td>Increase of minimum productivity level PL</td>
</tr>
<tr>
<td><strong>% change</strong></td>
<td></td>
<td>% change</td>
<td>% change</td>
<td>% change</td>
<td>% change</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Polish domestic market:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed trade cost, $f_{PL,PL}$</td>
<td>22.553</td>
<td>25.0</td>
<td>0.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Variable trade cost, $\tau_{PL,PL}$</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>EU15 export market:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed trade cost, $f_{PL,EU15}$</td>
<td>14.891</td>
<td>25.0</td>
<td>0.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Variable trade cost, $\tau_{PL,EU15}$</td>
<td>3.608</td>
<td>0.0</td>
<td>25.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Simulation results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average productivity exporters, $\theta_{PL,EU15}$</td>
<td>0.322</td>
<td>10.6</td>
<td>25.2</td>
<td>38.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Cut-off productivity exporters, $\theta^*_{PL,EU15}$</td>
<td>0.079</td>
<td>10.1</td>
<td>24.1</td>
<td>38.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Number of exporters, $N_{PL,EU15}$</td>
<td>291</td>
<td>-21.0</td>
<td>-40.5</td>
<td>-52.9</td>
<td>24.7</td>
</tr>
<tr>
<td>Export value, (million EUR)</td>
<td>331.904</td>
<td>-0.1</td>
<td>-40.6</td>
<td>-40.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Volume average firm supply/demand, $\bar{q}_{PL,PL}$</td>
<td>172.458</td>
<td>37.6</td>
<td>0.0</td>
<td>37.6</td>
<td>9.5</td>
</tr>
<tr>
<td>Volume average firm supply/Demand, $\bar{q}_{PL,EU15}$</td>
<td>95.120</td>
<td>38.1</td>
<td>0.0</td>
<td>38.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Consumer utility index*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>-</td>
<td>-5.1</td>
<td>0.0</td>
<td>-5.1</td>
<td>120.4</td>
</tr>
<tr>
<td>EU15</td>
<td>-</td>
<td>-0.6</td>
<td>-26.3</td>
<td>-26.6</td>
<td>16.1</td>
</tr>
</tbody>
</table>

Source: own calculations.

* The consumer utility index refers to the indirect utility relating to expenditure and price, see equation (I).

The results are different in case where the variable “iceberg trade costs” are increased (S2). Although the average productivity of exporters increases, the number of exporters considerably falls and export values decline. The reason for the different effects between fixed and variable trade costs is that the former are sunk and are not taken into account once the firm has entered the market. In contrast, the variable trade costs play a role in the firm’s pricing decisions. At any given level of market entry costs, higher variable trade costs lead to higher prices for exports to the EU15, reducing export demand for existing firms (the
intensive margin shrinks) and driving firms out of the EU15 export market (the extensive margin shrinks). Note that the quantity supplied by the average exporting firm does not change, even though the average productivity of exporters increases by some 25%. The reason is that the change in average productivity (and the cut-off productivity) is as large as the change in variable trade cost (25%) under the current parameterization of the model, so that the two effects cancel out in the optimal pricing equation (equation II). The price set by the average firm does not change, and hence export demand (equation III) remains constant. Combining higher fixed entry costs with higher variable trade cost reinforces the previous effects: drop in the number of exporting firms and a higher average productivity to cover market entry cost and variable trade costs.

The simulation results also give the consumer utility index. In the case of higher fixed entry cost (S1), Polish consumers are affected by higher prices, but this does not significantly spill over to EU15 consumers. Polish meat has a very small market share on the EU15 market, and consequently the supply changes do not have big market effects in the EU15. In contrast, the higher variable trade costs in S2 negatively affect EU15 consumers’ welfare since higher prices and less variety reduce welfare. Overall, the welfare of consumers in the EU15 decreases in all scenarios, and thus the subsidy of compliance is beneficial from the consumers’ consumption point of view.

The sets of simulation experiments (S1, S2 and S3) presented indicate that the subsidy to support Polish meat firms in meeting the fixed and variable costs of compliance with the EU standards may have slowed down the exit of smaller and less productive firms.

Producing according to standards can be expected to lead to productivity upgrades, which will be reinforced if compliance costs are subsidized. To capture this more dynamic effect, the last column in table 4 shows the effects of a 10% increase in the minimum productivity level (\(\beta\)) in the Polish meat sector. This is a very modest increase considering that the economy wide labour productivity growth in Poland between 2002-2004 amounted to about 4% per year. The change in minimum productivity shifts the lower end of the productivity distribution to the right, but it does not affect the export market entry decisions. While Polish meat firms that were able to export before the productivity upgrades continue to do so, some of the firms that previously only produce for the domestic Polish market reach the unchanged cut-off productivity for exporting and thus start supplying the EU15 market. The simulation results reveal that such a productivity increase would have been sufficient to compensate the export drop that results from increased fixed cost of compliance without the subsidy.
4.3. Sensitivity analysis

It has been shown above that there is some uncertainty regarding the estimates of model parameters. A sensitivity analysis can shed some light on the importance of that uncertainty. Since we already perturb the market entry cost and variable trade cost parameters in the simulations the effects of changes in these parameters, we concentrate here on one remaining key parameter in the model, the elasticity of substitution between varieties ($\sigma$). The important impact of $\sigma$ on trade effects in a model with heterogeneous firms has been underscored by Chaney (2005), amongst others. For the sensitivity analysis, we take the estimated median value and vary it within its estimated 96% confidence interval (see table 3). The simulations reported in the previous section are subsequently repeated three times with $\sigma$ taking a low, median and high value.\(^7\) Figure 3 shows the trade effects, including the effects on imports into Poland from the EU15, following the variations in market entry parameters and minimum productivity.

Figure 3: Sensitivity of trade results to the variety elasticity of substitution $\sigma$

Figure 3 shows that a low elasticity of substitution coincides with greater effects of increasing trade costs: exports from Poland contract more and imports into Poland expand more, the lower the elasticity of substitution. This contradicts the usual model results in which a lower $\sigma$ implies that cost changes are not directly translated into changes in market shares. In standard trade models, a higher elasticity of substitution implies more competition between varieties.

\(^7\) Alternatively, we could have performed a Monte Carlo analysis using random draws from the estimated kernel density distribution for $\sigma$. This would yield some information on the distribution of endogenous variables, following variations in $\sigma$. However, we feel that the three simulations presented here, concentrating on the extreme but plausible values provide adequate information for our sensitivity analysis.
and more pronounced effects of changes in trade costs. Without firm heterogeneity, all firms expand their exports following a fall in trade cost: the intensive margin increases and this effect is more pronounced the higher the elasticity of substitution. In contrast, in the heterogeneous firm model a low elasticity of substitution means that increasing (falling) trade costs drive out (attracts) relatively productive and large firms. In other words, the lower the elasticity of substitution $\sigma$ the more concentrated the exports amongst the more productive and larger firms. Consequently, higher trade cost will lead to a more pronounced decrease in the quantity supplied, as relatively large firms are driven out of the market. As Chaney (2005) formulates, the extensive margin is more sensitive to changes in trade cost for lower values of $\sigma$. The simulation results of the Polish meat case presented indicate that the extensive margin dominates: the lower the elasticity of substitution the larger the effect on trade.

5. Conclusion

The heterogeneous firm model we develop recognizes the asymmetric situation in the Polish meat sector, where complying and non-complying firms co-exist but supply different markets. The model features are characterised by firm-level monopolistic competition, and we estimate the model parameters by using limited data and full information least squares method to match observed patterns of trade. The estimates of the size and productivity distribution of Polish and EU15 meat firms and the approximated compliance costs reflect the heterogeneity in the sector. Our analysis examines the effects of standards on indicators of market performance as well as trade patterns.

While this paper investigates the case of Polish meat trade in the context of EU enlargement, some more general conclusions can be drawn. Our analysis shows that financial assistance programmes to help firms adopt standards lowers the firms’ productivity threshold to meet standards and to qualify for exporting. Since smaller and less productive firms that already operate in the industry can continue to exist structural changes in the industry of the exporting country tend to be dampened. In order to present a more complete picture, the analysis combines the direct cost-increasing effects of standards with productivity upgrades. Although the data do not allow us to exactly pin down the size of the productivity gains, the simulation analysis shows that a modest 10% increase of the minimum productivity level in the industry would be more than sufficient to compensate for the export drops following a 25% increase in fixed costs of compliance.
References

Balistreri, E. J., Hillberry, R. H. and T. F. Rutherford (2007), Structural estimation and solution of international trade models with heterogeneous firms, paper contribution, 10th GTAP Conference, June 7-9, Purdue/West-Lafayette.


Chaney, T. (2005), Distorted gravity: heterogeneous firms, market structure and the geography of international trade, mimeo, Massachusetts Institute of Technology (MIT), Cambridge/Massachusetts.


Appendix

A1: Estimating the Pareto distribution with grouped data.

The problem

Ideally, one would use firm-level data to estimate the parameters of the Pareto density function that we assume to describe the distribution of productivity in the model, but typically these data are not readily available. But data are regularly published in aggregate form by Eurostat that give the number of firms per size class and the corresponding share of the total market, plus other variables of interest grouped into these size classes. Size classes are defined in terms of employment. Such grouped data, naturally, lead to a loss of information compared to individual firm-level data, but can nevertheless be employed for estimation purposes. After presenting the estimation of the parameters of the Pareto distribution, this annex discusses the derivation of additional indicators of market structures, notably indexes of inequality economic variables such as employment, revenues and value added.

<table>
<thead>
<tr>
<th>Employment size class</th>
<th>Absolute frequency, number of firms</th>
<th>Employment (1000 employees)</th>
<th>Turnover (million Euro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between 1 and 9</td>
<td>2804</td>
<td>12570</td>
<td>573</td>
</tr>
<tr>
<td>Between 10 and 19</td>
<td>327</td>
<td>4710</td>
<td>380</td>
</tr>
<tr>
<td>Between 20 and 49</td>
<td>405</td>
<td>12743</td>
<td>874</td>
</tr>
<tr>
<td>Between 50 and 99</td>
<td>188</td>
<td>12974</td>
<td>889</td>
</tr>
<tr>
<td>Between 100 and 249</td>
<td>121</td>
<td>18755</td>
<td>1135</td>
</tr>
<tr>
<td>Between 250 and 499</td>
<td>41</td>
<td>14625</td>
<td>1042</td>
</tr>
<tr>
<td>Between 500 and 999</td>
<td>24</td>
<td>15694</td>
<td>907</td>
</tr>
<tr>
<td>1000 or more</td>
<td>9</td>
<td>14132</td>
<td>933</td>
</tr>
</tbody>
</table>

Source: Eurostat

The skewed pattern revealed in Table A.1 is typical of firm size distributions, and various functional forms have been estimated (see for example Simon and Bonini (1958) and Clarke 1979). The Pareto distribution is amongst the simpler forms.

Estimating the parameters of the Pareto distribution

Recall the Pareto probability density function \( f(x) \) and the cumulative density \( F(x) \):

\[
f(x) = \frac{\alpha}{\beta} \left(\frac{\beta}{x}\right)^{\alpha+1} \quad ; \quad F(x) = 1 - \left(\frac{\beta}{x}\right)^\alpha ; \beta \leq x \leq \infty; \alpha > 0; \beta > 0
\]

The ‘Pareto-\(\alpha\)’ parameter \((\alpha>0)\) is a measure of inequality: the larger \(\alpha\), the flatter, and hence the less unequal, the distribution. The parameter \(\beta \) \((\beta >0)\) is the location parameter, and defines the low end of the support of the probability density function (pdf). Rearranging \(F(x)\) and taking logarithms:

\[
\ln(1 - F(x)) = \alpha \cdot \ln(\beta) - \alpha \cdot \ln(x) + \varepsilon \quad \text{where} \quad \varepsilon \text{ denotes the error term.}
\]

It is easily seen from this transformation that the logarithm of one minus the cumulative density follows a straight line with a negative slope equal to \(\alpha\) and an intercept equal to \(\alpha \cdot \ln(\beta)\). Both parameters \(\beta\) and \(\alpha\) can be retrieved, but OLS will be unable to find the true value of the location (cut-off) parameter \(\beta\). Fortunately, this parameter in itself is not very interesting, except for its impact on the expected value, it merely acts as shifting the log-linear
curve up or down. The shape parameter $\alpha$ is of prime interest, as it determines the distributional characteristics, and it is the one that figures as a key parameter in the trade model. By transforming the observations on $x$ we can eliminate $\beta$ from the estimation equation. Letting $x' = x/\beta$, we obtain:

$$\ln(1 - F(x')) = \alpha \cdot \ln(x') + \epsilon'$$

This can be estimated using OLS without a constant term. Since $\beta$ determines the low end of the support of the pdf, it is equal to the lowest observed value of $x$, in our case the lowest observed productivity level. A final point to be noted is that observations in one size class have to be dropped from the estimation since frequencies sum to one.

A proxy for productivity is obtained by using the information such as presented in Table A.1 to calculate turnover per person employed. This vector can simply be sorted from lowest to highest, and the relative frequency $f(x)$ with which each observation occurs is taken to be the share of firms. This amounts to assuming that productivity is uniformly distributed within each size class. By using data for the years 2001-2004, we obtain a reasonably large number of observations, $n=22$. Estimation results are presented in Table A.2 below. The estimated equation appears to fit the data very well, according to the standard errors and the proportion of variance explained. Note that the smaller $\alpha$ indicates that productivity in the Polish meat sector is more unequally distributed than in the EU15. At the same time the average productivity is more than 3 times higher in the EU15: a turnover of 156 000 euro per person in the EU15 versus 46 000 in Poland.

### Table A.2: Estimates of productivity distribution in the meat sector.

<table>
<thead>
<tr>
<th></th>
<th>Poland</th>
<th>EU15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.337</td>
<td>3.993</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>$\beta$ (million euro per employee)</td>
<td>0.0263</td>
<td>0.1172</td>
</tr>
<tr>
<td>mean productivity, (million euro per employee)</td>
<td>0.0460</td>
<td>0.1564</td>
</tr>
</tbody>
</table>

Source: own estimation

Note: standard errors in parentheses. Mean productivity is calculated from the expected value of the Pareto distribution: $E(x) = \alpha \beta/(\alpha-1)$; $\beta$ is equal to the lowest observed productivity.

### Additional variables and market concentration indicators

Data on the firm size distribution measured by employment are often supplemented with additional economic variables, such as revenues, by employment class. This is also a type of distributional information, but since it involves two variable, e.g. employment and revenue, it is not evidently to be interpreted as a size distribution of the economic variable of interest. We can however use this information to calculate indicators of market concentration. A popular measure of market concentration is the C$k$ index, where $k$ is a positive integer. The C5 index, for example, shows the combined market share of the five largest firms. McLoughan and Abounoori (2003) derive the following expression for the C$k$ index:

$$Ck = 1 - F(1) \left( F^{-1}(1 - \frac{k}{n}) \right)$$

where $n$ denotes the total number of firms, and $F(1)$ is the first moment distribution function:
with \( \bar{x} \) denoting a minimum level of employment and \( E(x) \) is the expected value of \( X \).

In case of the Pareto size distribution the expression for the \( C_k \) index becomes:

\[
C_k = \left( \frac{k}{n} \right)^{\frac{\alpha-1}{\alpha}}
\]

The Lorenz curve of revenue plots the share of the firm population, ranked from lowest revenue firm to the highest, against its share in total revenues. The cumulative revenue share of the fraction \( F \) of the firm population can be written in terms of the pdf \( f(x) \) as:

\[
L(F) = \frac{\int_{x(F)}^{\infty} xf(x)dx}{\int_{\beta}^{\infty} xf(x)dx}
\]

where \( x(F) \) denotes the inverse of the cumulative density function (cdf), i.e. the value of \( x \) corresponding to a given cumulative frequency \( F \). Using the Pareto pdf and considering that the denominator of the Lorenz curve is just the expected value, one obtains:

\[
L(F) = 1 - \left( 1 - F \right)^{\frac{\alpha-1}{\alpha}}
\]

The Gini-coefficient is calculated as the ratio of the area that is enclosed by between the line of perfect equality (45°) and the Lorenz curve and the total area under the line of perfect equality as follows:

\[
G = 0.5 - \int L(F) dF = 1 - 2\int L(F) dF = \frac{1}{2\alpha - 1}
\]

The Gini-coefficient lies between zero (complete equality for \( \alpha = \infty \)) and one (complete inequality for \( \alpha = 1 \)).