A Case of Skilled Labor Migration and Complementary Inputs: The Impact of Emigrant Farmers on Brazilian Agriculture

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Abstract

This paper proposes a framework for explaining international skilled labor migration in the 21st century. Evidence based on the migration of American farmers to Brazil and the “Polish Plumber” phenomenon motivate this study. The paper examines the motives for skilled labor migration with a particular focus on small entrepreneurs. An ample array of possible reasons leading to the international migration of skilled workers is presented. Next, a case study is addressed: A theoretical model is developed to investigate why American farmers move their business to Brazil and to analyze the impact of the entry of American migrants on the Brazilian market for agriculture. The model shows that American farmers only migrate if they are relatively more than or as productive as Brazilian farmers when both types farm in Brazil, and how changes on farmer productivity in Brazil affect equilibrium farm-size, capital usage and number of farmers in the Brazilian agricultural market. The simulation results confirm the model’s predictions.
1 Introduction

“Two months ago Mr. Carroll, of Carthage, IL, bought a soybean farm on the outskirts of [Luís Eduardo Magalhães,] 350 miles northeast of Brasília [, Brazil]. He joined more than a dozen other Americans who have recently begun farming [there]”\(^1\). In the last five years, at least 200 American farmers\(^2\) have migrated to Brazil. Like Mr. Carroll, they seem to believe that Brazil offers more and better farming opportunities than the Midwestern United States. In Europe, France’s referendum in 2005 rejected the contemporanean text of the European Constitution, mainly due to fear of the “Polish plumber.” This expression is based on the high numbers of Polish small entrepreneurs – plumbers, carpenters, and electricians – who have immigrated to the United Kingdom, and represents the immigration of skilled workers from the new Eastern European member countries to the Western members. The contemporanean text of the E.U. constitution was rejected because a proposed amendment for the Directive on Services\(^3\) “confirms the right of service providers to provide a service in a Member State other than the one where he is established.”\(^4\)” In other words, the constitution allows for the free movement of workers within all member countries and France was afraid it would be flooded by cheap skilled labor and the unemployment of French citizens would raise above the already high rate of 10.2% in May 2005\(^5\).

The above events are examples of skilled workers who migrate internationally seeking higher returns to their skills. This paper investigates the motives for skilled labor migration with a particular focus on small entrepreneurs. “Skilled labor” and “small entrepreneur” describe, in this paper, workers who are self-employed and know a specific trade – like carpentering, plumbing or farming – instead of workers who have several years of formal education. As a case study, section 4 of this paper examines why American farmers migrate to Brazil. The formation of free trade areas worldwide has raised the question of whether people should also move freely within these economic

\(^2\)Halcomb, Ruth. (Nov. 2006)  
\(^3\)Official name of the amendment to the directive: Amended proposal for a Directive of the European Parliament and of the Council on services in the internal market.  
\(^4\)Amended proposal for a Directive of the European Parliament and of the Council on services in the internal market, article 3.4.  
\(^5\)Frost, Laurence (May 2005).
blocks. As observed by the above examples, even the European countries – pioneers on the free trade block formation – are hesitant to open their border to all European citizens. This issue is of great importance due to its impact on the income distribution and production structure of both origin and host countries. Exit or entrance of skilled workers will lead to wage redistribution, driving it up in the origin country and down in the host country. In addition, the arrival of skilled labor can fill in skill gaps in industries where skilled workers were scarce and consequently increase industry output.

Trade theory based on factor-proportions and factor-endowments tells us that the returns to skilled labor are higher where it is scarce and lower where it is abundant. However, the above examples imply the opposite. In the case of American farmers, skilled labor is abundant and expensive in the United States; in the case of the “Polish plumber” skilled labor is scarce and cheap in Poland.

The Heckscher-Ohlin (H-O) model of trade states that the pattern and content of trade between countries are defined by each country’s relative factor endowment and relative factor proportions, as it assumes “identical technologies, constant returns to scale and common tastes”\(^6\). One of the main implications of the H-O model is that a country will export the good that uses the abundant factor intensively because this good is temporarily more expensive abroad. This means that the good that uses the abundant factor intensively is cheaper domestically, and therefore so is the abundant factor. In other words, the model states that factors are cheap where they are abundant and expensive where they are scarce. In the above examples, we observe the opposite phenomenon taking place: Skilled labor migration from skilled scarce to skill abundant countries.

It becomes therefore important to understand what forces are actually driving this migration trend. This paper draws from four major literatures to explain the reasons of such exodus: Complementarity of factors, high-prices of non-traded services in high-income countries, Ricardian trade theory and the theory of the multinational firm.

A first central cause of skilled labor migration is complementarity of factors, which simply states that skilled workers need factors that are complementary to their skills in order to realize their potential productivity and earn corresponding (higher) wages. The “Polish plumber” falls under this category. The explanation is that skilled workers cannot realize their potential productivity due to

the absence of complementary factors, and thus are paid low wages even though they are scarce in their home countries. These skilled workers will then migrate to countries where their skills are more abundant but, as complementary factors are relatively more common there, will earn higher wages than at home.

The second trend is that skilled workers that earn high returns where they are abundant migrate. The American farmers fall under this category. The idea is that these farmers earn high returns for their skills due to the high individual human capital accumulated in their home country. Based on Ricardian trade theory, farmers in each country have a different set of skills, which implies that an American farmer is not a substitute for a Brazilian farmer, as would be the case under the H-O assumptions. These skilled workers will migrate and still earn high returns for their unique skills, as they will have different productivity levels than the local farmers. Higher relative productivity abroad combined with cheaper complementary factors will yield lower production costs and higher profits abroad. As previously mentioned, this specific case is explored in detail in Section 4.

A third explanation for the international migration of skilled workers is that high income countries have a high price level due to the higher prices of non-tradable goods. Countries with higher productivity on the manufacturing of tradable goods face higher input prices. Because workers in high-income countries are more productive in making traded goods and are paid their marginal products, they earn higher wages than workers in low-income countries. As wages equalize within industries in each country, wages in the non-traded goods sector of the high-income country will be higher than wages in the same sector in other countries. Consequently, the high income country has higher wages and rent and has more expensive non-traded goods.

The theoretical model developed below is based on several ideas and assumptions introduced by the multinational enterprise literature. First and foremost, small entrepreneurs can be viewed as firms facing the decision of how to supply their goods to domestic and foreign customers. Secondly, if a relative is now supervising Mr. Carroll’s old farm in the United States, by migrating, buying land and producing soybeans in Brazil one can draw an analogy and say that Mr. Carroll built a new plan abroad and thus performed horizontal FDI.
This paper adds to the literature in two important ways. First, the model produces predictions as to whether the differences in productivity between U.S. and Brazilian farmers are mainly due to differences in initial levels of human capital. The partial equilibrium model implemented in this paper allows for the identification of equilibrium allocations, which is not attainable when using target-input models\textsuperscript{7}, for example. This paper also adds to the body of literature concerned with explaining whether and how differences in human capital and total factor productivity across countries contribute to cross-country national income differences\textsuperscript{8}. Allowing productivity to vary according to the adaptability of a farmer’s skill to a new environment enables one to derive and compare farmers by their relative productivities. This feature in the present model provides theoretical evidence in favor of the hypothesis that differences in human capital cause disparities in G.D.P. across nations.

Lastly, when modeling production, the theoretical model developed in this paper considers three factor inputs, when the multinational enterprise literature usually simplifies production to require only one factor, labor.

This paper is structured as follows. Section 3 addresses the literature explored in the paper. Section 4 analyses a case study, where a theoretical model is developed to investigate why American farmers migrate to Brazil and the effect of the entry of migrants on Brazilian agriculture. Section 5 concludes.

2 Literature Review

Trade theory based on factor-proportions and factor-endowments tells us that the returns to skilled labor are higher where it is scarce and lower where it is abundant. However, the above examples imply the opposite. In the case of American farmers, skilled labor is abundant and expensive in the United States; in the case of the “Polish plumber” skilled labor is scarce and cheap in Poland.

The Heckscher-Ohlin (H-O) model of trade states that the pattern and content of trade between countries are defined by each country’s relative factor endowment and relative factor proportions, as

\textsuperscript{7}Foster, Rosenzweig (1995).
\textsuperscript{8}Hendricks (2002).
it assumes “identical technologies, constant returns to scale and common tastes”\textsuperscript{9}. One of the main implications of the H-O model is that a country will export the good that uses the abundant factor intensively because this good is temporarily more expensive abroad. This means that the good that uses the abundant factor intensively is cheaper domestically, and therefore so is the abundant factor. In other words, the model states that factors are cheap where they are abundant and expensive where they are scarce. In the above examples, we observe the opposite phenomenon taking place: Skilled labor migration from skill scarce to skill abundant countries.

A first central cause of skilled labor migration is complementarity of factors, which simply claims that skilled workers need factors that are complementary to their skills in order to realize their potential productivity and earn corresponding (higher) wages. Therefore, the “Polish plumber” is an example of a skilled worker who will earn higher wages if he migrates to the Western member countries because there factors that are complementary with his skills are relatively more abundant.

In the late 1960’s and early 1970’s it became evident that while the number of educated workers increased, wages remained constant over time instead of decreasing. Initially it was thought that wages remained constant due to the substitutability of educated for non-educated workers\textsuperscript{10}. However, the observation that there is a positive correlation between the capital-to-unskilled-labor ratio and the skilled-to-unskilled-labor ratio\textsuperscript{11} could imply that skilled labor becomes more valuable as the stock of capital raises. This phenomenon motivated the study of the capital-skill complementarity hypothesis, i.e., the idea that “capital is more complementary with skilled than unskilled labor,\textsuperscript{12}” which has been empirically proved by many works, such as Griliches (1969), Fallon and Layard (1975), Goldin and Katz (1998).

Other studies support particular aspects of this paper. Krussel, Ohanian, Ríos-Rull and Violante (2000) use in their empirical study an efficiency index to calculate the skilled and unskilled labor inputs. Each labor input is measured in efficiency units, i.e., the product of the efficiency index and the raw number of labor hours. The authors state that this efficiency index can have different interpretations: “[H]uman capital, accumulated by the agent or it can represent a skill-specific technology

\textsuperscript{9}Markusen, Melvin, Kaempfer and Maskus (1995), p. 100.
\textsuperscript{10}Cross-sectional data on countries (Bowles 1970) and on U.S. states (Dougherty 1972).
\textsuperscript{11}Fallon, Layard (1975).
\textsuperscript{12}Papageorgiou, Chmelarova (2005), p. 59.
level” (p. 1035). This efficiency index supports the above idea that ‘skilled worker’ can also denote a small entrepreneur with a specific expertise, not only a worker with many years of education.

Regarding the degree of skill complementarity with capital and technology, Papageorgiou and Chmelarova (2005) find evidence that countries with moderate initial per capita income and low initial education (literacy rates) have higher capital-skill complementarity (p. 59). The authors thus suggest that in developed countries skill is more complementary with technology and that in developing countries skill is more complementary with capital (p. 80).

A second explanation for the international migration of skilled workers is that high income countries have a high price level due to the high prices of non-tradable services. Therefore, a skilled worker seeking higher wages would choose to migrate to a developed country because there prices, and therefore wages, are higher than in transition economies. The Harrod-Balassa-Samuelson\textsuperscript{13} effect explains why high income countries have higher price levels. It states that “countries with higher productivity in tradables compared with non-tradables [have a tendency] to have higher prices” (Obstfeld, Rogoff (1996), p. 210). It is intuitive to think that the wealthy countries of today became rich largely due to their higher productivity in tradable goods, like manufacturing and agriculture. Hence, we can conclude that price levels tend to rise with per capita income\textsuperscript{14}.

The implication that the high price level in developed countries is due to high prices of non-tradable services was analyzed by Gregorio, Giovannini and Wolf (1993). The authors investigate if inflation in non-tradable goods is greater than inflation in tradables in the OECD countries during the period 1970 to 1985. More specifically, Gregorio, Giovannini and Wolf “examine the time series and cross-sectional behavior of the price of non-tradables in terms of tradables” by first looking at the supply side approach – changes in relative productivity – then analyzing the demand side shifts (p. 1). They conclude that the trend of growth in income contributed to the increase in the relative price of non-tradables in the OECD countries. Therefore, we can claim that skilled workers would migrate specifically to high-income countries due to higher returns to their knowledge, especially if this worker provides a service.

The trade literature has vastly explored, theoretically and empirically, what determines a firm’s decision to relocate to a foreign country. The idea is that firms have three ways of supplying domestic and foreign customers with their product: By exporting, by producing the same product in plants located domestically and abroad – horizontal foreign direct investment (FDI), by moving part of the production process abroad – vertical FDI\(^{15}\) and lastly by contracting foreign firms to produce their product\(^{16}\). Firms that perform one of the last three production methods are considered multinational enterprises. The present literature review will focus on the horizontal FDI field of study, as vertical FDI applies to firms that brake up production across countries and thus is not applicable to the framework developed here.

The theoretical model developed below is based on several ideas and assumptions that were introduced by the multinational enterprise literature. First and foremost, farmers and small entrepreneurs can be viewed as firms facing the decision of how to supply their goods to domestic and foreign customers. Secondly, if a relative is now supervising Mr. Carroll’s old farm in the United States, by migrating, buying land and producing soybeans in Brazil one can draw an analogy and say that Mr. Carroll built a new plan abroad and thus performed horizontal FDI. Next, it has been shown by Melitz (2003) and Helpman, Melitz and Yeaple (2004) that only the most productive firms perform FDI, which is in consonance with the theoretical model derived below. More specifically, I take inspiration from Melitz’s (2003) idea of ordering firms by productivity and deriving a productivity cutoff such that firms with productivity higher than the cutoff “export and increase both their market share and profits” (Melitz (2003), p. 1714). Instead of ordering firms by productivity, the model below order firms by fixed set up costs (FC) and thus only those with a FC low enough migrate and enter the Brazilian agricultural market.

Even though the present paper has several similarities to the multinational enterprise literature, not all assumptions apply to the framework developed here. For instance, many works introduce production of differentiated products using increasing-returns-to-scale technology\(^{17}\). In agriculture,

\(^{15}\)See Helpman (1984, 1985), Markusen (2002, Ch. 9), and Gordon Hanson et al. (2002) for the analysis of vertical FDI.

\(^{16}\)Helpman, Melitz and Yeaple (2004), p. 300.

goods – soybean or corn for example – are homogeneous and different farmers do not produce different varieties of these grains. In addition, farmers have no market power in determining the price of soybean, which is decided by price indexes, such as the one calculated by the Chicago Board of Trade (CBOT). Also, the present model does not address firm-size as one of the determinants of FDI, as done by Helpman, Melitz and Yeaple (2004).

3 The Theoretical Model

3.1 Model Structure

The theoretical framework described below explains why American farmers move their businesses to Brazil, if those who migrate are more productive than their Brazilian counterparts, and how the migrants affect the dynamics of the partial equilibrium results in the Brazilian market for agriculture.

The model assumes the existence of two countries, United States (us) and Brazil (br) producing only one product X, representing agricultural goods. There are three factors of production, skill (S), capital (K), and land (T), with the latter two being internationally immobile. Subscripts (i, j) will be used to denote the countries (us, br). The production function of good X in country i is CRS and identical in both countries. It is intuitive to think that when a farmer doubles the amount of land and inputs used in production, the output level of X will also double. The production function of X in each country i is:

\[ X_i(S_i, K_i, T_i) = S_i^\alpha K_i^\beta T_i^\gamma, \quad i = \text{us, br}; \quad \gamma = 1 - \alpha - \beta \]  

Farmers produce X in a perfectly competitive market so that there is free entry and exit in the long-run and farmers take the price of X as given, which is normalized to 1. Let \( q_i \) represent the price of land and \( r_i \) the price of capital in each country. Farmers earn profits due to the returns to their skills only.

Farmers are endowed with a skill set acquired in their respective countries of origin that can also be used abroad in the event that a farmer migrates. Therefore, I assume that if a farmer relocates - for example, if an American farmer moves to Brazil - he takes his skill set with him, which may yield
higher or lower productivity in Brazil due to the differences in land and weather. With this structure, there are four possible productivity levels conditional on the farmer’s country of origin and where he is applying his skills. Let \( S_{\text{country of origin}} \) denote a farmer’s productivity given his country of origin and the country of actual production. Table 1 describes the four possible productivity levels:

<table>
<thead>
<tr>
<th>Country of Production</th>
<th>Country of Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States (us)</td>
<td>( S_{us} )</td>
</tr>
<tr>
<td>Brazil (br)</td>
<td>( S_{us} )</td>
</tr>
<tr>
<td>United States (us)</td>
<td>( S_{us} )</td>
</tr>
<tr>
<td>Brazil (br)</td>
<td>( S_{us} )</td>
</tr>
</tbody>
</table>

3.2 Decision to Migrate and Relative Productivity

The analysis performed in this section investigates (1) why American farmers move their business to Brazil and (2) if American farmers who migrated to Brazil are more productive than their Brazilian counterparts. Input prices are exogenously determined in this section.

Farmers in both countries are profit maximizers, so that each farmer in their respective country of origin:

\[
\max_{K_i, T_i} X_i^i - r_i K_i - q_i T_i \quad i = \text{us, br} \tag{2}
\]

where \( X_i^i \) denotes \( X_{\text{country of origin}, \text{country of production}} \), i.e., the output produced by an American farmer in the United States \( (X_{us}^i) \) or the output produced by a Brazilian farmer in Brazil \( (X_{br}^i) \). The amount of land and capital are defined the same way.

The first order conditions yield the following equilibrium factor demand functions:

\[
K_i^* = S_i^i \left( \frac{\beta}{r_i} \right)^{\frac{\alpha + \beta}{\alpha}} \left( \frac{\gamma}{q_i} \right)^{\frac{\gamma}{\alpha}} \quad i = \text{us, br} \tag{3}
\]

\[
T_i^* = S_i^i \left( \frac{\beta}{r_i} \right)^{\frac{\beta}{\alpha}} \left( \frac{\gamma}{q_i} \right)^{\frac{(1 - \beta)}{\alpha}} \quad i = \text{us, br} \tag{4}
\]
Consequently, profits are:

\[
\pi_i^{\text{max}} = \alpha S_i \left( \frac{1}{q_i} \right)^{\frac{\gamma}{\beta}} \left( \frac{\beta}{r_i} \right)^{\frac{\beta}{\alpha}} \implies \pi_i^{\text{max}} > 0 \text{ iff } \begin{cases} \alpha, \beta, q_i, r_i, S_i^j > 0 \\ 1 > \alpha + \beta \end{cases} \quad i = \text{us, br} \tag{5}
\]

Positive profits will lead to entry in the market abroad. There is a fixed migration costs \( c \) such that a farmer from country \( i \) will move to country \( j \) if and only if, after incurring the migration costs, his profits abroad are greater than his profits in his country of origin, as described by equation (6) below:

\[
\pi_j^i - c > \pi_i^i \text{ for } i \neq j; i = \text{us, br}; j = \text{us, br} \tag{6}
\]

where \( \pi_j^i \) denotes \( \pi_{\text{country of origin}}^{\text{country of production}} \), i.e., profits earned by U.S. farmers when farming in Brazil (\( \pi_{\text{us}}^{\text{br}} \)) or profits earned by Brazilian farmers when farming in the United States (\( \pi_{\text{br}}^{\text{us}} \)).

For simplicity, assume \( c = 0 \). Now, solving for the ratio of the productivities we can conclude that a farmer will move if:

\[
\frac{S_j^i}{S_i^i} > \left( \frac{q_j}{q_i} \right)^{\frac{1-\alpha-\beta}{\alpha}} i \neq j; i = \text{us, br}; j = \text{us, br} \tag{7}
\]

Equation (7) tells us that a U.S. farmer will move to Brazil under two conditions. First, a farmer will relocate if his relative productivity in Brazil increases sufficiently to realize greater relative profits. Alternatively, in the case when he is equally productive in both countries, an American farmer will only move if production costs are low enough to result in higher relative profits. This would only be possible if inputs to production (land) are cheaper in Brazil.

Regarding the Brazilian farmer’s decision to move to the United States, equation (7) implies that such a farmer will only move if he is absolutely more productive in the U.S. than in Brazil, independent of the input prices (land) in each country.

### 3.2.1 Comparative Analysis

It is now relevant to investigate how different land prices will affect the productivity ratio, and therefore the decision-to-migrate rule (equation (7)) for each farmer type. Examining simulation
values for the productivity ratio and their implications can help understand the model inferences discussed above.

Based on anecdotal evidence, land in Brazil is cheaper than in the United States. According to Baumel, Wisner, Duffy, and Hofstrand, in 2000, an acre of land in Iowa cost USD 140.00, while in Mato Grosso, B.R., the same acre cost USD 23.00. The simulations below, while using arbitrary values, illustrate the situation described above. In addition, assume that the productivity share in production is 75% ($\alpha = 0.75$), the capital share is 15% ($\beta = 0.15$), so that the share of land in production is 10%.

First, consider an American farmer’s decision to migrate to Brazil. He will move if his relative productivity in Brazil assumes the following values:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{br}$</td>
<td>$q_{us}$</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

Assuming arbitrary values for $q_{us}$ and $q_{br}$ such that $q_{us} = q_{br}$ initially, we can see that an American farmer will migrate to Brazil if his relative productivity in Brazil is greater than 1, meaning that he will only move if he is strictly more productive in Brazil than in the United States, as described in row 1. But if land in Brazil is cheaper than in the United States, as illustrated in row 3, the productivity ratio falls below 1, meaning that an American farmer would migrate even if he were less productive in Brazil because his production costs would be low enough to result in higher relative profits. Therefore, according to the model, American farmers who migrate and are now operating in Brazil are equally if not more productive than their Brazilian counterparts.

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Now consider a Brazilian farmer’s decision to move to the United States. Using the same example, he will move if his relative productivity assumes the following values:

Table 3–Brazilian Farmer’s Decision-to-Migrate Rule: Cheaper Land in Brazil

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>1.09</td>
</tr>
<tr>
<td>25</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Similarly to the previous results, when assuming that land prices are equalized across countries a Brazilian farmer will only migrate if he is relatively more productive in the United States. In the case when land is cheaper in Brazil, as illustrated in row 3, the productivity ratio becomes greater than 1, which tells us that a Brazilian farmer will only move if he is considerably more productive in the U.S. than in Brazil. This implies that the cheaper the land in Brazil the more productive a Brazilian farmer needs to be in the United States in order for him to move.

3.3 Partial Equilibrium in the Brazilian Agricultural Goods Market

3.3.1 Theoretical Framework

We now focus the analysis on the Brazilian market for agriculture, with the presence of both American and Brazilian farmers. The goal of this section is to study the equilibrium amount of land and capital used by each type of farmer, the endogenously determined price of land in Brazil and the equilibrium number of American and Brazilian farmers operating in Brazil.

The partial equilibrium model developed here assumes (a) free entry into the Brazilian market of X, so that the price of Brazilian land is determined endogenously by the model. This assumption implies that farmers enter the market for agricultural goods until their profit equals their fixed set up cost (FC). Fixed set up costs can be interpreted as the outside opportunity cost for both American and Brazilian farmers, and therefore are different than the migration cost c previously defined. (b)
Following Melitz’s (2003) model of heterogeneous firms, farmers are ordered by fixed set up costs in each country. For simplification, FC is a linear function of the number of farmers in the sector, such that only farmers with a FC low enough will enter the agricultural goods market, i.e.,

\[ FC_i > \alpha_i n_i, \quad i = \text{us, br} \]  

where \( \alpha_i > 0 \) is the slope coefficient in each country and \( n_i \) denotes the number of farmers from country \( i \) operating in Brazil. Endogeneity of FC is imposed so that not all American farmers migrate to Brazil. (c) Since the current analysis focuses on the Brazilian market for agricultural goods, it seems reasonable to assume that Brazilian farmers have lower fixed set up costs than American farmers. (d) American farmers are identical in the sense that they all have the same productivity in Brazil, \( S_{\text{us}}^i \); likewise, Brazilian farmers are identical as they also have the same productivity when operating in Brazil, \( S_{\text{br}}^i \). (f) The supply of land in Brazil is inversely determined by the total amount of land bought by each farmer type present in the market in equilibrium, i.e.,

\[ q > d \left( n_{\text{us}} T_{\text{us}} + n_{\text{br}} T_{\text{br}} \right), \quad d > 0 \]  

where \( d \) is the slope coefficient. (g) Only U.S. farmers can borrow in the U.S. market and thus pay an interest rate \( r_{\text{us}} \), and only Brazilian farmers can borrow in the Brazilian market and thus pay an interest rate \( r_{\text{br}} \), (d) which are assumed to be exogenously determined.

Following the derivations from the previous section, both American and Brazilian farmers maximize profits in the Brazilian market of agricultural goods:

\[ \max_{K_{\text{us}}^i, T_{\text{us}}^i} X_{\text{us}}^i - r_{\text{us}} K_{\text{us}}^i - q T_{\text{us}}^i, \quad \text{us, br} \]  

where \( q \) is the endogenous price of land in Brazil.

American farmers face the following equilibrium factor demands:

\[ T_{\text{us}}^i = S_{\text{us}}^i \left( \frac{\beta}{r_{\text{us}}} \right)^{\frac{1}{\alpha}} \left( \frac{\gamma}{q} \right)^{\frac{(1-\beta)}{\alpha}} \]  

where \( \text{us, br} \).
Similarly, Brazilian farmers face the equilibrium factor demands described below:

\[ T_{br} = S_{br} \left( \frac{\beta}{r_{br}} \right)^{\frac{\beta}{\alpha}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}} \]  
\[ (13) \]

\[ K_{br} = S_{br} \left( \frac{\beta}{r_{br}} \right)^{\frac{\alpha+\beta}{\alpha}} \left( \frac{\gamma}{q} \right)^{\frac{\alpha}{\alpha}} \]  
\[ (14) \]

This yields profits:

\[ \pi_{br}^{\text{max}} = \alpha S_{br} \left( \frac{\gamma}{q} \right)^{\frac{\alpha}{\alpha}} \left( \frac{\beta}{r_{i}} \right)^{\frac{\beta}{\alpha}} i = \text{us, br} \]  
\[ (15) \]

In the long run, farmers will enter the Brazilian market of agricultural goods until profits equal their opportunity cost (FC):

\[ \pi_{br}^{\text{long-run}} = X_{br}^{i*} - qT_{br}^{i*} - r_iK_{br}^{i*} - FC = 0 i = \text{us, br} \]  
\[ (16) \]

which in turn yields the following endogenously determined land price in Brazil:

\[ q^{*} = \gamma \left( \frac{\beta}{r_{i}} \right)^{\frac{\beta}{\gamma}} \left( \frac{\alpha S_{br}^{i}}{FC_{i}} \right)^{\frac{\alpha}{\gamma}} i = \text{us, br} \]  
\[ (17) \]

The partial equilibrium in the Brazilian market of agriculture goods is then characterized by 9 equations: (8) and (17) (one for each country), (9), (11), (12), (13), (14) and 9 unknowns: \( T_{us}^{br}, T_{br}^{br}, K_{us}^{br}, K_{br}^{br}, q, n_{us}, n_{br}, FC_{br}, FC_{us} \).

The above framework has valuable equilibrium implications. Regarding country differences in productivity, relative-high-productivity farmers will own more land and capital. More specifically, if American farmers are more productive than Brazilian farmers, they will own larger, more capital intensive plants. As a consequence of higher demand for land, prices increase. The equilibrium number of farmers will also depend on productivity, as the predominant farmer type will be that of the relatively more productive farmer. Concerning differences in fixed set up costs, the country
with higher relative fixed costs will exhibit a lower number of farmers in the Brazilian market. When looking at the impact of changes on the interest rates, if capital is relatively more expensive in Brazil, then, in equilibrium there will be fewer Brazilian farmers in the market, and those who stay will have a smaller plant-size than the American farmers. Lastly, the price per hectare of Brazilian land goes up as more American farmers enter the Brazilian land market.

### 3.3.2 Simulation - Comparative Statics

In order to better understand the direction and magnitude of the implications described above, simulations using MPS/GE, a subsystem of General Algebraic Modeling System (GAMS) were performed\(^\text{19}\). An increase on the parameters \(S_{us}^{br}\), \(S_{br}^{br}\), and \(r_{br}\) was imposed and their respective impact on the equilibrium outcome was analyzed. Overall, the simulations confirm the model implications.

The following parameter values were used in the simulations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(S_{us}^{br})</th>
<th>(S_{br}^{br})</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(r_{us})</th>
<th>(r_{br})</th>
<th>(a_{br})</th>
<th>(a_{us})</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>19.866</td>
<td>19.866</td>
<td>0.75</td>
<td>0.15</td>
<td>0.10</td>
<td>1</td>
<td>1</td>
<td>7.5</td>
<td>7.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\(^{19}\)For a list of the inequalities, corresponding complementary variables, the GAMS code, and the analytical solution for each variable, refer to the appendix.
Increase on American Farmer Productivity in Brazil \( (S_{br}^{us}) \)  As the model predicts, a consistent increase on the productivity level of American farmers in Brazil will, in equilibrium, increase both land and capital usage by American farmers, decrease both land and capital usage by Brazilian farmers, drive up the price of Brazilian land, increase the number of American farmers and lower the number of Brazilian farmers in the Brazilian agricultural goods market. Table 5 provides the benchmark and different equilibrium values corresponding to each productivity level:

Table 5 - Impact of Higher American Farmer Productivity on Inputs, Land Price, Number of Farmers and Fixed Costs.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark Value ( S_{br}^{us} = 19.886 )</th>
<th>( S_{br}^{us} = 39.73 )</th>
<th>( S_{br}^{us} = 59.60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{br}^{us} )</td>
<td>1.0</td>
<td>1.26</td>
<td>1.34</td>
</tr>
<tr>
<td>( K_{br}^{us} )</td>
<td>1.5</td>
<td>2.84</td>
<td>4.09</td>
</tr>
<tr>
<td>( T_{br}^{br} )</td>
<td>1.0</td>
<td>0.63</td>
<td>0.45</td>
</tr>
<tr>
<td>( K_{br}^{br} )</td>
<td>1.5</td>
<td>1.42</td>
<td>1.36</td>
</tr>
<tr>
<td>( q )</td>
<td>1.0</td>
<td>1.5</td>
<td>2.03</td>
</tr>
<tr>
<td>( n_{us} )</td>
<td>1.0</td>
<td>1.9</td>
<td>2.73</td>
</tr>
<tr>
<td>( n_{br} )</td>
<td>1.0</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td>( FC_{us} )</td>
<td>7.5</td>
<td>14.21</td>
<td>20.47</td>
</tr>
<tr>
<td>( FC_{br} )</td>
<td>7.5</td>
<td>7.11</td>
<td>6.82</td>
</tr>
<tr>
<td>( X )</td>
<td>2.0</td>
<td>2.97</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Increase on the Brazilian Farmer Productivity in Brazil \( (S_{br}) \). Again, the simulations confirm the model’s implications. Increases in Brazilian productivity lead to, in equilibrium, a greater usage of capital by both types of farmers, while only Brazilian farmers become more land intensive in the production of agricultural goods. The price of land increases as more land is demanded. In addition, as Brazilian productivity increases, American farmers exit the market while Brazilian farmers enter the Brazilian market for agricultural goods.

Table 6 - Impact of Higher Brazilian Farmer Productivity on Inputs, Land Price, Number of Farmers and Fixed Costs.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark Value ( S_{br} = 19.886 )</th>
<th>( S_{br} = 39.73 )</th>
<th>( S_{br} = 59.60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{us}^{br} )</td>
<td>1.0</td>
<td>0.63</td>
<td>0.45</td>
</tr>
<tr>
<td>( K_{us}^{br} )</td>
<td>1.5</td>
<td>1.42</td>
<td>1.36</td>
</tr>
<tr>
<td>( T_{br}^{us} )</td>
<td>1.0</td>
<td>1.26</td>
<td>1.34</td>
</tr>
<tr>
<td>( K_{br}^{br} )</td>
<td>1.5</td>
<td>2.84</td>
<td>4.09</td>
</tr>
<tr>
<td>( q )</td>
<td>1.0</td>
<td>1.5</td>
<td>2.03</td>
</tr>
<tr>
<td>( n_{us} )</td>
<td>1.0</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td>( n_{br} )</td>
<td>1.0</td>
<td>1.9</td>
<td>2.73</td>
</tr>
<tr>
<td>( FC_{us} )</td>
<td>7.5</td>
<td>7.11</td>
<td>6.82</td>
</tr>
<tr>
<td>( FC_{br} )</td>
<td>7.5</td>
<td>14.21</td>
<td>20.47</td>
</tr>
<tr>
<td>( X )</td>
<td>2.0</td>
<td>2.97</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Increase on the Interest Rate in Brazil \((r_{br})\) An increase of the Brazilian interest rate will make capital more expensive relative to land to Brazilian farmers only, as American farmers can borrow in the United States. As a consequence, American farmers will use more of both inputs in production; whereas Brazilian farmers will use fewer inputs in the production of X. In addition, the equilibrium price of land falls. Lastly, more Brazilian farmers exit the market, while American farmers enter the Brazilian market for agricultural goods. Due to the higher production costs, total output falls.

Table 7 - Impact of Higher Interest Rates in Brazil on Inputs, Land Price, Number of Farmers and Fixed Costs.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark Value (r_{br} = 1)</th>
<th>(r_{br} = 2)</th>
<th>(r_{br} = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T^u_{br})</td>
<td>1.0</td>
<td>1.067</td>
<td>1.103</td>
</tr>
<tr>
<td>(K_{br}^u)</td>
<td>1.5</td>
<td>1.511</td>
<td>1.517</td>
</tr>
<tr>
<td>(T^b_{br})</td>
<td>1.0</td>
<td>0.929</td>
<td>0.885</td>
</tr>
<tr>
<td>(K_{br}^b)</td>
<td>1.5</td>
<td>0.658</td>
<td>0.406</td>
</tr>
<tr>
<td>(q)</td>
<td>1.0</td>
<td>0.945</td>
<td>0.917</td>
</tr>
<tr>
<td>(n_u)</td>
<td>1.0</td>
<td>1.008</td>
<td>1.012</td>
</tr>
<tr>
<td>(n_{br})</td>
<td>1.0</td>
<td>0.877</td>
<td>0.812</td>
</tr>
<tr>
<td>(FC_{us})</td>
<td>7.5</td>
<td>7.557</td>
<td>7.587</td>
</tr>
<tr>
<td>(FC_{br})</td>
<td>7.5</td>
<td>6.579</td>
<td>6.09</td>
</tr>
<tr>
<td>X</td>
<td>2.0</td>
<td>1.888</td>
<td>1.832</td>
</tr>
</tbody>
</table>
Summary of Results  The following table displays a summary of the results presented above. The arrows represent the direction of change as American farmer productivity, Brazilian farmer productivity and the Brazilian interest rate increase\(^{20}\). A close analysis of the analytical solutions presented in part b of the appendix also support the results in table 8.

Table 8: Summary of Simulation Results

<table>
<thead>
<tr>
<th>MODIFIED PARAMETERS</th>
<th>U.S. farmer productivity in BR</th>
<th>BR farmer productivity in BR</th>
<th>Brazilian interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Land</strong></td>
<td>U.S. farmer</td>
<td>▲</td>
<td>▼</td>
</tr>
<tr>
<td></td>
<td>Brazilian farmer</td>
<td>▼</td>
<td>▲</td>
</tr>
<tr>
<td><strong>Capital</strong></td>
<td>U.S. farmer</td>
<td>▲</td>
<td>▼</td>
</tr>
<tr>
<td></td>
<td>Brazilian farmer</td>
<td>▼</td>
<td>▲</td>
</tr>
<tr>
<td><strong>Number of farmers</strong></td>
<td>U.S. farmer</td>
<td>▲</td>
<td>▼</td>
</tr>
<tr>
<td></td>
<td>Brazilian farmer</td>
<td>▼</td>
<td>▲</td>
</tr>
<tr>
<td><strong>Price of land in Brazil</strong></td>
<td>▲</td>
<td>▼</td>
<td>▲</td>
</tr>
<tr>
<td><strong>X output</strong></td>
<td>▲</td>
<td>▼</td>
<td>▲</td>
</tr>
</tbody>
</table>

As described in table 8, increases in American and Brazilian farmers’ productivity have symmetric but opposite results. An increase in one farmer’s productivity will lead to higher input usage by this farmer type, lower input usage by the other farmer type, while land prices and output always increase. Regarding increases on the Brazilian interest rate, American farmers use more of both inputs; Brazilian farmers use less of both inputs, especially capital, and the price of land and output fall.

\(^{20}\)Simulations using different input shares in the production function were performed. The direction of the changes remains as described above independently of the shares used.
4 Conclusion

This paper proposes a framework for explaining international skilled labor migration in the 21st century. The formation of free trade areas worldwide has raised the question of whether people should also move freely within these economic blocks. As observed by the above examples, even the European countries – pioneers on the free trade block formation – are hesitant to open their border to all European citizens. Evidence based on the migration of American farmers to Brazil and the “Polish Plumber” phenomenon motivate this study. More specifically, the paper investigates the motives for skilled labor migration with a particular focus on small entrepreneurs. A broad picture of possible reasons leading to the international migration of skilled workers is presented.

The present study proposes a novel framework for the analysis of migration of skilled labor. To do so, it combines characteristics and implications of the complementary factors, high-prices of non-traded services in high-income countries, Ricardian trade theory and the theory of the multinational firm literatures.

Next, a theoretical model is developed to examine a case study: Why American farmers move their business to Brazil, and also analyses the impact of the entry of American migrants on the Brazilian market for agriculture. The model shows that American farmers only migrate if they are relatively more than or as productive as Brazilian farmers when both types farm in Brazil, and how changes on farmer productivity in Brazil affect equilibrium farm-size, capital usage and number of farmers in the Brazilian agricultural market.

Simulations using the General Algebraic Modeling System (GAMS) confirm the model’s predictions that an increase in the American farmer productivity will cause an increase in both land and capital usage by American farmers, a decrease in both land and capital usage by Brazilian farmers, will drive up the price of Brazilian land, increase the number of American farmers and lower the number of Brazilian farmers in the Brazilian agricultural goods market. Similar but opposite results are obtained when increasing the Brazilian farmer’s productivity. Lastly, an increase on the Brazilian interest rate will decrease the amount of capital used by Brazilian farmers and increase the amount of capital used by American farmers. In addition, both types of farmers will use less land, where Brazilian farmers will use relatively less land than American farmers. The equilibrium price of land
goes up as the Brazilian interest rate goes up. Lastly, more Brazilian farmers enter the market, while American farmers exit the Brazilian market for agricultural goods.
5 Appendix

5.1 Inequalities and Complementary Variables

Table 9 - Description of Inequalities and Their Respective Complementary Variables

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Complementary Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1A) DEMANDTus_br</td>
<td>$T_{us} \geq S_{us}^{br} \left( \frac{\beta}{r_{us}} \right)^{\frac{\alpha}{\pi}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}}$</td>
</tr>
<tr>
<td>(2A) DEMANDTbr_br</td>
<td>$T_{br} \geq S_{br}^{br} \left( \frac{\beta}{r_{br}} \right)^{\frac{\alpha+\beta}{\pi}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}}$</td>
</tr>
<tr>
<td>(3A) DEMANDKus_br</td>
<td>$K_{us}^{br} \geq S_{us}^{br} \left( \frac{\beta}{r_{us}} \right)^{\frac{\alpha+\beta}{\pi}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}}$</td>
</tr>
<tr>
<td>(4A) DEMANDKbr_br</td>
<td>$K_{br}^{br} \geq S_{br}^{br} \left( \frac{\beta}{r_{br}} \right)^{\frac{\alpha+\beta}{\pi}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}}$</td>
</tr>
<tr>
<td>(5A) ZEROPROFITus</td>
<td>$FC_{us} \geq \alpha S_{us}^{br} \left( \frac{\beta}{r_{us}} \right)^{\frac{\alpha}{\pi}} \left( \frac{\gamma}{a} \right)^{\frac{\alpha}{\pi}}$</td>
</tr>
<tr>
<td>(6A) ZEROPROFITbr</td>
<td>$FC_{br} \geq \alpha S_{br}^{br} \left( \frac{\beta}{r_{br}} \right)^{\frac{\alpha+\beta}{\pi}} \left( \frac{\gamma}{q} \right)^{\frac{1-\beta}{\alpha}}$</td>
</tr>
<tr>
<td>(7A) INVERSESELANDSUPPLY</td>
<td>$q \geq (dn_{us} T_{us}^{br} + n_{br} T_{br}^{br})$</td>
</tr>
<tr>
<td>(8A) FIXEDCOSTbr</td>
<td>$FC_{br} \geq a_{br} n_{br}$</td>
</tr>
<tr>
<td>(9A) FIXEDCOSTus</td>
<td>$FC_{us} \geq a_{us} n_{us}$</td>
</tr>
</tbody>
</table>

Unknowns:

$T_{us}, T_{br}, K_{us}, K_{br}, q, n_{us}, n_{br}, FC_{br}, FC_{us}$.

Parameters:

$S_{us}^{br}$: American farmer’s productivity when farming in Brazil

$S_{br}^{br}$: Brazilian farmer’s productivity when farming in Brazil

$\alpha$: share of productivity and or skill in production

$\beta$: share of capital in production

$r_{us}$: Interest rate for US farmers

$r_{br}$: Interest rate for Brazilian farmers

$a_{br}$: slope of fixed cost equation for Brazilian farmers

$a_{us}$: slope of fixed cost equation for US farmers

$d$: slope coefficient of land inverse supply
5.2 Analytical Solution to the Complementary Variables

In this session I present the analytical solution to the complementary variables described above, as well as provide a brief explanation of how these solutions were derived.

The analytical solutions to the complementary variables are:

\[ q = \beta^{\frac{\alpha}{1-\beta}} (\alpha d)^{\frac{2-\alpha-2\beta}{2(1-\beta)}} \left( \frac{S_{us}^2}{a_{us} r_{us}^{\alpha \gamma}} + \frac{S_{br}^2}{a_{br} r_{br}^{\alpha \gamma}} \right) \]  

\[ T_{us} = \left( \frac{\gamma}{\alpha d} \right)^{\frac{1}{2}} \left[ \frac{S_{us}^2}{S_{br}^2} \left( \frac{r_{us}}{a_{br} r_{br}^{\alpha \gamma}} \right)^{\frac{2}{\alpha \gamma}} + \frac{1}{\alpha_{us}} \right]^{\frac{1}{2}} \]  

\[ T_{br} = \left( \frac{\gamma}{\alpha d} \right)^{\frac{1}{2}} \left[ \frac{1}{\alpha_{br}} + \frac{S_{us}^2}{S_{br}^2} \left( \frac{r_{br}}{a_{us} r_{us}^{\alpha \gamma}} \right)^{\frac{2}{\alpha \gamma}} \right]^{\frac{1}{2}} \]  

\[ K_{us} = \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{\alpha d} \right)^{\frac{2\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{us}^{\alpha \gamma}} + \frac{r_{us}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]  

\[ K_{br} = \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{\alpha d} \right)^{\frac{2\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{br}^{\alpha \gamma}} + \frac{r_{br}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]  

\[ FC_{us} = \alpha^{\frac{1+\alpha-\beta}{2(1-\beta)}} \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{d} \right)^{\frac{\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{us}^{\alpha \gamma}} + \frac{r_{us}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]  

\[ FC_{br} = \alpha^{\frac{1+\alpha-\beta}{2(1-\beta)}} \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{d} \right)^{\frac{\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{br}^{\alpha \gamma}} + \frac{r_{br}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]  

\[ n_{br} = \alpha^{\frac{1+\alpha-\beta}{2(1-\beta)}} \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{d} \right)^{\frac{\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{us}^{\alpha \gamma}} + \frac{r_{us}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]  

\[ n_{us} = \alpha^{\frac{1+\alpha-\beta}{2(1-\beta)}} \beta^{\frac{1}{1-\beta}} \left( \frac{\gamma}{d} \right)^{\frac{\gamma}{2(1-\beta)}} \left[ \frac{S_{us}^2}{r_{us}^{\alpha \gamma}} + \frac{r_{us}^{2\gamma}}{S_{br}^2} \right]^{-\frac{\gamma}{2(1-\beta)}} \]
The above solutions were derived using the inequalities in table 9 as equalities. The steps taken to derive the above equations were as follows. First, the price of land, \( q \), was obtained. To do so, solve equations (8A) and (9A) for \( n_{br} \) and \( n_{us} \), respectively, generating equations (27) and (28) below:

\[
\frac{FC_{br}}{a_{br}} = n_{br}
\]  

(27)

\[
\frac{FC_{us}}{a_{us}} = n_{us}
\]

(28)

Next, substitute (6A) into (27) for \( FC_{br} \) and (5A) into (28) for \( FC_{us} \) and solve for the respective number of farmers, obtaining equations (29) and (30) below:

\[
n_{br} = \frac{\alpha S_{br}^{\beta}}{\alpha a_{br}} \left( \frac{\beta}{r_{br}} \right)^{\frac{\beta}{\alpha}} \left( \frac{\gamma}{q} \right)^{\frac{\gamma}{\alpha}}
\]

(29)

\[
n_{us} = \frac{\alpha S_{us}^{\beta}}{\alpha a_{us}} \left( \frac{\beta}{r_{us}} \right)^{\frac{\beta}{\alpha}} \left( \frac{\gamma}{q} \right)^{\frac{\gamma}{\alpha}}
\]

(30)

Then, substitute into equation (7A) the following equations: (29) and (30) for the respective number of farmers and (1A) and (2A) for the respective size of land. After simplifying one obtains the following analytical value of the price of land:

\[
q = \beta^{\frac{2}{2-\beta}} (\alpha d)^{\frac{\alpha}{2-\beta}} \gamma^{\frac{2\alpha - 2\beta}{2(1-\beta)}} \left( \frac{\gamma S_{us}^{2}}{a_{us}\tau_{us}} + \frac{\gamma S_{br}^{2}}{a_{br}\tau_{br}} \right)^{2(1-\beta)}
\]

(18)

After obtaining \( q \), substitute it into equations (1A) through (6A). This yields equations (19) through (24) listed above. Finally, the solution to the number of Brazilian and American farmers is obtained by substituting (23) and (24) into (8A) and (9A), respectively, equations (25) and (26).
5.3 GAMS Code

**M05_PELandinBR May 6, 2007**

SETS
S /1*3/

PARAMETERS
Sus_br American farmer’s productivity when farming in Brazil
Sbr_br Brazilian farmer’s productivity when farming in Brazil
a Alpha share of productivity and or skill in production
b Beta share of capital in production
rus Interest rate for US farmers
rbr Interest rate for Brazilian farmers
abr Slope of fixed cost equation for Brazilian farmers
aus Slope of fixed cost equation for US farmers
d Slope coefficient of land inverse supply
Landus_br(S) Amount of land used by American farmer in Brazil
Landbr_br(S) Amount of land used by Brazilian farmer in Brazil
CapitaTus_br(S) Amount of capital used by American farmer in Brazil
CapitaTbr_br(S) Amount of capital used by Brazilian farmer in Brazil
Numberfarmersus(S) Number of American farmers in the Brazilian market of land
Numberfarmersbr(S) Number of American farmers in the Brazilian market of land
PriceLand(S) Price of land in Brazil
PriceCapitalus(S) Interest rate in the United States
PriceCapitalbr(S) Interest rate in Brazil
FixedSetCostus(S) Fixed set up cost for American farmers
FixedSetCostbr(S) Fixed set up cost for Brazilian farmers
Producus_br(S) Productivity of American farmer in Brazil
Producbr_br(S) Productivity of Brazilian farmer in Brazil
ProductionX(S) Amount of output X produced in equilibrium
RES1 (*,S) Simulation results for an increase in American farmer productivity see Excel file "compexam.xls"
RES2 (*,S) Simulation results for an increase in Brazilian farmer productivity see Excel file "compexam.xls"
RES3 (*,S) Simulation results for an increase in Brazilian interest rate see Excel file "compexam.xls"

POSITIVE VARIABLES
Tus_br Equilibrium amount of land used by American farmer in Brazil
Tbr_br Equilibrium amount of land used by Brazilian farmer in Brazil
Kus_br Equilibrium amount of capital used by American farmer in Brazil
Kbr_br Equilibrium amount of capital used by Brazilian farmer in Brazil
q Equilibrium endogenous price of land in Brazil
nus Equilibrium number of American farmers in the Brazilian market of land
nbr Equilibrium number of Brazilian farmers in the Brazilian market of land
FCbr Fixed set up cost for Brazilian farmer
FCus Fixed set up cost for American farmer
X Equilibrium amount of X produced in market

;
EQUATIONS
DEMANDTus_br
DEMANDTbr_br
DEMANDKus_br
DEMANDKbr_br
ZEROPROFITus
ZEROPROFITbr
INVERSELANSUPPLY
FIXEDCOSTus
FIXEDCOSTbr
XPRODUCTION;

*Defining the equations:
DEMANDTus_br - Tus_br = \( G = S_{us} \times (b/r_{us})^{(b/a)} \times ((1-a-b)/q)^{(1-b/a)} \);
DEMANDTbr_br - Tbr_br = \( G = S_{br} \times (b/r_{br})^{(b/a)} \times ((1-a-b)/q)^{(1-b/a)} \);
DEMANDKus_br - Kus_br = \( G = S_{us} \times (b/r_{us})^{(a+b/a)} \times ((1-a-b)/q)^{(1-a-b)/a} \);
DEMANDKbr_br - Kbr_br = \( G = S_{br} \times (b/r_{br})^{(a+b/a)} \times ((1-a-b)/q)^{(1-a-b)/a} \);
ZEROPROFITus - FCus = \( a \times S_{us} \times (b/r_{us})^{(b/a)} \times ((1-a-b)/q)^{(1-a-b)/a} \);
ZEROPROFITbr - FCbr = \( a \times S_{br} \times (b/r_{br})^{(b/a)} \times ((1-a-b)/q)^{(1-a-b)/a} \);
INVERSELANSUPPLY - q = \( d \times (n_{us} \times Tus_{br} + n_{br} \times Tbr_{br}) \);
FIXEDCOSTbr - FCbr = \( a \times b \times r_{br} \);
FIXEDCOSTus - FCus = \( a \times b \times r_{us} \);
XPRODUCTION - X = \( G = S_{us} \times a \times K_{us} \times b \times Tus_{br}^{(1-a-b)} + S_{br} \times a \times K_{br} \times b \times Tbr_{br}^{(1-a-b)} \);
MODEL MO5PELandinBR /
DEMANDTus_br.Tus_br,
DEMANDTbr_br.Tbr_br,
DEMANDKus_br.Kus_br,
DEMANDKbr_br.Kbr_br,
ZEROPROFITus.nus,
ZEROPROFITbr.abr,
INVERSELANSUPPLY.q,
FIXEDCOSTbr.FCbr,
FIXEDCOSTus.FCus
XPRODUCTION.X /
/

*Set initial values of variables and set parameter values:
q.l = 1;
Tus_br.l = 1;
Tbr_br.l = 1;
nus.l = 1;
nbr.l = 1;
a = 0.75;
b = 0.15;
rus = 1;
rbr = 1;
d = 0.5;
Sus_br = \left(\frac{q_l}{1-a-b}\right)^{\frac{1-b}{a}} \left(\frac{rus}{b}\right)^{\frac{b}{a}};\\
Sbr_br = \left(\frac{q_l}{1-a-b}\right)^{\frac{1-b}{a}} \left(\frac{rbr}{b}\right)^{\frac{b}{a}};\\
K_{us\ br\ l} = \frac{b q_l}{(1-a-b) rus};\\
K_{br\ br\ l} = \frac{b q_l}{(1-a-b) rbr};\\
FC_{br\ l} = \frac{a q_l}{1-a-b};\\
FC_{us\ l} = \frac{a q_l}{1-a-b};\\
abr = \frac{a q_l}{1-a-b};\\
aus = \frac{a q_l}{1-a-b};\\
X_l = \text{Sus}_{\ br\ l}^a \text{K}_{us\ br\ l}^b \text{Tus}_{\ br\ l}^{1-a-b} + \text{Sbr}_{\ br\ l}^a \text{K}_{br\ br\ l}^b \text{Tbr}_{\ br\ l}^{1-a-b};\\
\text{DISPLAY } \text{Sus}_{\ br}, \text{Sbr}_{\ br}, \text{abr}, \text{aus}, d, X_l;\\
\text{SOLVE M05PELandinBR USING MCP;}

*COMPARATIVE STATICS SIMULATIONS:

*Increase on the productivity of American farmers in Brazil:

\text{LOOP}(S, \text{Sus}_{\ br} = 39.7324 * \text{ORD}(S) - 19.8662 * \text{ORD}(S));

\text{SOLVE M05PELandinBR USING MCP;}

\text{Produce}_{\ br}(S) = \text{Sus}_{\ br};\\
\text{Producebr}_{\ br}(S) = \text{Sbr}_{\ br};\\
\text{Landus}_{\ br}(S) = \text{Tus}_{\ br\ l};\\
\text{Landbr}_{\ br}(S) = \text{Tbr}_{\ br\ l};\\
\text{CapitaTus}_{\ br}(S) = \text{Kus}_{\ br\ l};\\
\text{CapitaTbr}_{\ br}(S) = \text{Kbr}_{\ br\ l};\\
\text{Numberfarmersus}(S) = \text{nus}\ l;\\
\text{Numberfarmersbr}(S) = \text{nbr}\ l;\\
\text{PriceLand}(S) = q_l;\\
\text{PriceCapitalus}(S) = rus;\\
\text{PriceCapitalbr}(S) = rbr;\\
\text{FixedSetCostus}(S) = F\text{Cu}\text{s}\ l;\\
\text{FixedSetCostbr}(S) = F\text{Cbr}\ l;\\
\text{ProductionX}(S) = X_l;\\
\text{RES1(\"Productivityus\ br\",S) = Produce}_{\ br}(S);\\
\text{RES1(\"Productivitybr\ br\",S) = Producebr}_{\ br}(S);\\
\text{RES1(\"Landus\ br\",S) = Landus}_{\ br}(S);\\
\text{RES1(\"CapitaTus\ br\",S) = CapitaTus}_{\ br}(S);\\
\text{RES1(\"Landbr\ br\",S) = Landbr}_{\ br}(S);\\
\text{RES1(\"CapitaTbr\ br\",S) = CapitaTbr}_{\ br}(S);\\
\text{RES1(\"PriceLand\",S) = PriceLand}(S);\\
\text{RES1(\"Numberfarmersus\",S) = Numberfarmersus}(S);\\
\text{RES1(\"Numberfarmersbr\",S) = Numberfarmersbr}(S);\\
\text{RES1(\"FixedSetCostus\",S) = FixedSetCostus}(S);\\
\text{RES1(\"FixedSetCostbr\",S) = FixedSetCostbr}(S);\\
\text{RES1(\"ProductionX\",S) = ProductionX}(S);\\
\text{RES1(\"PriceCapitalus\",S) = PriceCapitalus}(S);\\
\text{RES1(\"PriceCapitalbr\",S) = PriceCapitalbr}(S);\\
});

$\text{LIBINCLUE XLDUMP RES1 COMPEXAM_5_6_07.XLS sheet1}$

Sus_br = 19.866;
*An increase on the productivity of Brazilian farmers in Brazil:

LOOP(S, Sbr_br = 39.7324*ORD(S) - 19.8662*ORD(S);

SOLVE M05PELandinBR USING MCP;

Producus_br(S) = Sus_br;
Produse_br(S) = Sbr_br;
Landus_br(S) = Tus_br.l;
Landbr_br(S) = Tbr_br.l;
CapitaTus_br(S) = Kus_br.l;
CapitaTbr_br(S) = Kbr_br.l;
Numberfarmersus(S) = nus.l;
Numberfarmersbr(S) = nbr.l;
PriceLand(S) = q.l;
PriceCapitalus(S) = rus;
PriceCapitalbr(S) = rbr;
FixedSetCostus(S) = FCur.l;
FixedSetCostbr(S) = FChl.l;
ProductionX(S) = X.l;
RES2("Productivityus_br",S) = Producus_br(S);
RES2("Productivitybr_br",S) = Produse_br(S);
RES2("Landus_br",S) = Landus_br(S);
RES2("CapitaTus_br",S) = CapitaTus_br(S);
RES2("Landbr_br",S) = Landbr_br(S);
RES2("CapitaTbr_br",S) = CapitaTbr_br(S);
RES2("PriceLand",S) = PriceLand(S);
RES2("Numberfarmersus",S) = Numberfarmersus(S);
RES2("Numberfarmersbr",S) = Numberfarmersbr(S);
RES2("FixedSetCostus",S) = FixedSetCostus(S);
RES2("FixedSetCostbr",S) = FixedSetCostbr(S);
RES2("ProductionX",S) = ProductionX(S);
RES2("PriceCapitalus",S) = PriceCapitalus(S);
RES2("PriceCapitalbr",S) = PriceCapitalbr(S);
);

$LIBINCLUDE XLDUMP RES2 COMPEXAM_5_6_07.XLS sheet2

Sbr_br = 19.866;

*An increase in rbr:

LOOP(S, rbr = ORD(S);

SOLVE M05PELandinBR USING MCP;

Producus_br(S) = Sus_br;
Produse_br(S) = Sbr_br;
Landus_br(S) = Tus_br.l;
Landbr_br(S) = Tbr_br.l;
CapitaTus_br(S) = Kus_br.l;
CapitaTbr_br(S) = Kbr_br.l;
Numberfarmersus(S) = nus.l;
Numberfarmersbr(S) = nbr.l;
PriceLand(S) = q.l;
$\text{PriceCapitalus}(S) = \text{rus};$
$\text{PriceCapitalbr}(S) = \text{rbr};$
$\text{FixedSetCostus}(S) = \text{FCus};$
$\text{FixedSetCostbr}(S) = \text{FCbr};$
$\text{ProductionX}(S) = \text{X};$
$\text{RES3("Productivityus_br",S) = Productus_br}(S);$
$\text{RES3("Productivitybr_br",S) = Productbr_br}(S);$
$\text{RES3("Landus_br",S) = Landus_br}(S);$
$\text{RES3("CapitaTus_br",S) = CapitaTus_br}(S);$
$\text{RES3("Landbr_br",S) = Landbr_br}(S);$
$\text{RES3("CapitaTbr_br",S) = CapitaTbr_br}(S);$
$\text{RES3("PriceLand",S) = PriceLand}(S);$
$\text{RES3("Numberfarmersus",S) = Numberfarmersus}(S);$
$\text{RES3("Numberfarmersbr",S) = Numberfarmersbr}(S);$
$\text{RES3("FixedSetCostus",S) = FixedSetCostus}(S);$
$\text{RES3("FixedSetCostbr",S) = FixedSetCostbr}(S);$
$\text{RES3("ProductionX",S) = ProductionX}(S);$
$\text{RES3("PriceCapitalus",S) = PriceCapitalus}(S);$
$\text{RES3("PriceCapitalbr",S) = PriceCapitalbr}(S);$
$);$
$\$\text{LIB INCLUDE XLDUMP RES3 COMPEXAM_5_6_07.XLS sheet3}$
$rbr = 1;$
$\text{SOLVE M05PELandinBR USING MCP;}$

References


