Network effects and inward FDI policy

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Abstract

I outline the effect of business networks on trade, FDI and welfare in a two-country, two-firm duopoly. The network effect, following Greaney (2002), is modelled as a marginal cost disadvantage facing a firm from Foreign in selling to Home. Unlike traditional trade costs, this cost cannot be avoided by investing in Home. My main addition is a Nash game between governments in which they subsidise the fixed costs of inward FDI. While the network effect is shown to lead to favourable outcomes for the Home firm, I show that once government subsidies to the fixed costs of FDI are included and welfare functions analysed, the network effect leads to asymmetric outcomes unfavourable to Home. This result can help inform the debate on countries’ (in particular Japan’s) international trade and investment relations.

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1 Introduction

The existence of business networks, and the extent to which they can be indicted as informal trade barriers, has been the topic of intense academic and policy debate for many years. This debate has focused to a large degree on the vertical relationships within Japanese corporate groups known as keiretsu. These relationships are characterized by long-term relationships between final good producers and their suppliers or subcontractors. Suppliers make relationship-specific investments in return for a long-term commitment from the final good producer, which leads to a “locking in” effect. These relationships have been accused of acting as barriers to trade, with the US auto industry’s persistent bilateral trade deficit with Japan a frequently studied example. Along with the supplier-producer case, keiretsu relationships have also been shown to exist in the distribution network in Japan, as studied in Flath (2002). He shows that industries with a high incidence of distribution keiretsu have lower import penetration, and to the extent that import penetration is lower, have a higher incidence of Foreign Direct Investment (FDI). This indicates the degree to which foreign firms have difficulty getting their goods onto Japanese shelves; if they do decide to go ahead with exporting to Japan, they are often forced to invest in the distribution network in order to sell.

The debate on keiretsu’s effect on Japan-US trade relations has usually focused on the degree to which it harms US industry. The US-Japan bilateral trade deficit, amounting to an average of $72bn a year between 1998 and 2000\(^1\), and given as $90bn for 2006 in Wakasugi et al (2008) is often cited as evidence of the harmful nature of Japan’s business networks. Less often cited, but of equal interest to this paper, is Japan’s inward FDI deficit with the US, which is reported as averaging $13bn a year between 1998-2000 in Greaney (2002)

The network effect leads to a trade surplus and an inward FDI deficit for the Home country in my model. Japan is the best-known example of a country with these characteristics. According to Paprzycki and Fukao (2005), in 2003 Japan accounted for 12 percent of global GDP, but only 5 percent of worldwide merchandise imports and only one percent of global inward FDI. They also show that Japanese outward FDI outweighs inward FDI by a ratio of four to one. While I posit that network effects are a causal factor, I must note at this point that geography, a historical antipathy to outside involvement in economic affairs, a rigid labour market\(^2\) and comparative advantage also certainly play a role in explaining Japan’s trade and FDI balances.

\(^1\)Source: Greaney(2002)

\(^2\)See Haaland and Wooton (2000) for further evidence of the effect of rigid labour markets on MNEs’ investment decisions
The previous literature on business and social networks, which includes, *inter alia*, Casella and Rauch (2002), Spencer and Qiu (2001), McLaren (1999), and Rauch (1996), deals specifically with the link between networks and international trade. Greaney (2002) claims to be the first paper to consider how networks affect FDI as well as trade balances. Aside from the Greaney paper, Spencer and Qiu (2001) is the most relevant to this paper in that it deals with the US-Japan trade friction induced by Japanese firms’ *keiretsu* relationships. The remaining mentioned studies deal with the effect of overseas emigrant networks on trade.

I argue in this paper that focusing solely on the effects of Japanese networks on it’s trade and FDI balances is perhaps to look at the issue through too narrow a lens. EU Trade Commissioner Peter Mandelson has drawn attention to a similar asymmetry in the context of the EU-Japan relationship. In a speech to a business and government audience in Tokyo on 21/04/08, Mandelson cited a Japanese trade surplus of $30bn and FDI deficit of $49.5bn as causes for concern. The interesting aspect of this speech is that Mandelson claimed that Japan has as much to lose as the EU does from these asymmetries, an attitude that seems less widespread in the US-Japan debate of the last two decades. In this paper I approach the issue more in this light. By including consumer surplus, returns to local labour and firm profits in a national welfare function, I propose a broader framework in which to analyse the effect of business networks on the Home and Foreign countries. By looking at this version of welfare and modelling governments competing in a Nash game offering subsidies to FDI, I show that there is a larger parameter range for welfare improvement for the Foreign government, in an equilibrium in which both governments offer subsidies to inward FDI. The negative effect of networks in Home on Home welfare seems intuitive if we consider that Home exists in a world of economic globalisation. In this world, Foreign firms sell to the Home market. When the Foreign firm suffers from a network effect, there is a negative effect on Home Consumer Surplus. Furthermore, if firms from Foreign invest in Home in order to sell into that market, the network effect leads to decreesd rents for Home workers who are employed by the Foreign affiliate. These results can help motivate arguments such as those made by Mr. Mandelson.

I design a model to explain Japan’s trade surplus and inward FDI deficit with other developed economies by building on Greaney (2002), simplifying from her multi-product firms to single-product firms. She explains the US-Japan trade and FDI asymmetry using network effects in a two-country, two-firm international oligopoly. The network effect exists in Home, and leads to a marginal cost disadvantage to Foreign firms attempting to sell into the Home market. The network effect can be explained as it is in the Greaney paper: “...the added cost may
reflect search costs involved in locating buyers, distribution costs and/or information costs that are assumed to be higher for “outsiders” in some markets.” This disadvantage exists whether the Foreign firm sells via export or FDI. The single-product firm approach allows my main contribution, a section on government policy towards inward FDI, to be added in a tractable fashion. The result of this addition is that the network effect which is shown to lead to a favourable outcome for Home firms in Greaney’s paper, in fact leads to an unfavourable asymmetric outcome for the Home government when both governments attract FDI in a subsidy offering game. These results are arrived at in a three-stage game solved by backward induction. In the final stage, firms decide outputs, after having made their location decision in Stage 2. Stage 1 involves the governments competing on subsidies to the fixed cost of inward FDI.

I follow the method of Greaney in specifying four possible equilibria with different combinations of multinational and national firm activity in a two-country, two-firm, partial equilibrium duopoly model with homogenous goods and single-product firms. The firms compete on quantities a la Cournot. The two countries are considered identical in their wage and cost levels, and similarly-endowed in labour, capital, knowledge, technology etc. This assumption ensures that it is “North-North” trade and FDI that is being modelled here. If FDI exists in this model it is “market-seeking” or horizontal FDI, in which the firm invests abroad to serve similar markets from close proximity. This ensures that the results arrived at credibly represent Japan’s trade and FDI balances with the US and EU. Despite the undoubted increase in North-South FDI, and the possible positive implications for economic development, it must still be acknowledged that the bulk of global FDI is not of the North-South form. Blonigen (2005), using 1999 BEA data on US affiliate sales, shows that 67 percent of these sales are in the host country of the affiliate, which gives an indication of the importance of “market-seeking” FDI in total FDI from rich countries. Reinforcing the fact that the US figures are a good indicator for North FDI activity, UNCTAD’s World Investment Report 2007 records developed countries as accounting for 84 percent of global FDI outflows and 66 percent of global FDI inflows.

In deciding how to serve the overseas market, firms’ decisions are based on the trade-off between the fixed costs of a second plant and the trade costs associated with exporting 3. It is certainly the case, as outlined in Baldwin and Ottaviano (2001), that trade and FDI are often complements. The literature finds instances both of complementarity and substitutability. Blonigen (2001), using product-level data for Japanese FDI in and exports to the US, finds that when FDI

3 see for instance Markusen and Venables (1998)
and exports are substitutes, (which is found to be the case for final automobile and consumer goods production at the product level) FDI replaces trade in a large one-time shift, rather than in a gradual fashion. This adds legitimacy to the modelling structure adopted here.

In modelling a Nash game in subsidies to the fixed costs of FDI in Stage 1, optimal subsidy levels for both governments are arrived at, and welfare levels are calculated. I show that in the parameter range which induces multinationality as the Nash Equilibrium outcome for both firms and both governments, the Foreign government has a wider range of welfare improvement than Home. This result can help inform the debate on the trade and investment relations of Japan with its developed country partners, which have normally focused on the detrimental effect of Japanese networks on foreign businesses.

The remainder of the paper is organised as follows: In Section 2, I outline characteristics and assumptions of the model. In Section 3, I specify the Nash Equilibrium outputs for the four equilibria, for both the case including and excluding network effects. In Section 4, I model the location decisions of the firms. In Section 5, I examine the effect of policy on the equilibrium outcome and subsequent national welfare. Section 6 concludes.
2 Model

There are two countries, Home (H) and Foreign (F). There are two firms, 1 and 2. Firm 1 is based in H and Firm 2 is based in F. Good $Y$ is produced with constant returns by a competitive industry in both countries. Good $X$ is produced by one firm in each country, both of which compete in international duopoly. Both goods are tradable. Markets are perfectly separated. The $Y$ good is numeraire, with the ratio of the price of good $X$ to $Y$ in country $i$ given by $p_i$. Production is of the nature that one unit of labour leads to one unit of output. Consumers have quasi-linear preferences, giving national utility in country $i$ as

$$U_i = a(X_{ii} + X_{ji}) - \frac{1}{2}(X_{ii} + X_{ji})^2 + Y_{ii} + Y_{ji} (1)$$

where $Y_{ji}$ is consumption of good $Y$ produced by a firm $j$, $j \in (1, 2)$, by consumers in country $i$, $i \in (H, F)$.

The national budget constraint requires that the value of the labour endowment plus profits, minus subsidies paid out by the government equal what is consumed.

$$wZ_i + wL_i + \pi_k - s_i = p_i(X_{ii} + X_{ji}) + (Y_{ii} + Y_{ji}) (2)$$

where $w$ is the rent extracted by labour in industry $X$ and $Z_i$ is production of the good on the soil of country $i$. This will differ from $(X_{ii} + X_{ji})$ in all equilibria except for M due to one or both firms selling by export in the $N$, $A_1$ and $A_2$ equilibria. In industry $X$, $w = w_X - \bar{w}$, where $w_X$ is the wage paid by firms in industry $X$ and $\bar{w}$ is the reservation wage that could be earned elsewhere in the economy. $w$ does not depend on the number of firms producing in country $i$ due to the assumption that there is sufficient supply of workers in the $Y$ sector willing to move to earn the wage premium on offer in the $X$ sector. $L_i$ is the total labour endowment in country $i$. The left hand side of the national budget constraint can also be written as

$$w_X Z_i + \bar{w} Y_i = w Z_i + \bar{w} L_i$$

I assume that workers in industry $Y$ are paid the reservation wage, $\bar{w}$ so that in this industry $w = 0$. $wZ_i$ will thus from here on be referred to as labour compensation in the industry $X$ in country $i$.

Substituting from the national budget constraint (2) into the national utility function (1) gives the following

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4The origin of the difference in wages between the X and Y sectors is outlined in Appendix 1.
$$Max U_i = a(X_{ii} + X_{ji}) - \frac{1}{2}(X_{ii} + X_{ji})^2 + wZ_i + \bar{w}L_i + \pi_k - p_i(X_{ii} + X_{ji}) - s_i \quad (3)$$

Optimization of the above national welfare function $U_i$ with respect to national consumption $(X_{ii} + X_{ji})$, leads to the following:

$$a_i - (X_{ii} + X_{ji}) - p_i = 0.$$  

Segmented markets as in Brander (1981) and Helpman (1984), lead to the following linear inverse demand curve for $X$

$$p_i = a - (X_{1i} + X_{2i}) \quad (4)$$

Where,

$X_{1H}$ is Firm 1 domestic sales in the Home market.

$X_{2H}$ is Firm 2 sales in the Home market, which can be either through export or through a Firm 2 affiliate in Home.

$X_{1F}$ is Firm 1 sales in the Foreign market, which can be either through export or a Firm 1 affiliate in Foreign

$X_{2F}$ is Firm 2 domestic sales in the Foreign market.

(4) is then put into (3) above, yielding

$$U_i = a(X_{ii} + X_{ji}) - \frac{1}{2}(X_{ii} + X_{ji})^2 + wZ_i + wL_i + \pi_k - a(X_{ii} + X_{ji}) + (X_{ii} + X_{ji})^2 \quad (5)$$

So that we end with national welfare

$$U_i = \frac{1}{2}(X_{ii} + X_{ji})^2 + wZ_i + \bar{w}L_i + \pi_k - s_i \quad (6)$$

Where the first term on the RHS is Consumer Surplus of country $i$. I consider the employment structure, wage levels, trade costs and miscellaneous marginal costs, excluding network costs, as identical in the two countries. I also assume them identically endowed in terms of labour, technology etc.

A domestic parent plant already exists in each country. The fixed costs of this plant are already incurred. For this reason, along with the fact that marginal cost savings can never be had from FDI, a firm never moves all its production to another similarly-endowed country. Given the existence of a parent plant, with a homogenous good and in a North-North setting, a firm will never serve its own market by investing abroad, incurring both setup and export costs, and “reverse importing”. 


Export costs, $g$, are identical for both countries.

There are network costs, $n$, which are modelled as an additional cost for Firm 2 in selling to the Home market. This advantage for Firm 1 can be considered to exist due to exogenous cultural, historical, institutional or language reasons, as well as relationship-specific investments by intermediate and final good producers\(^5\). It is possible and almost certainly the case that there will be a “network effect” in both countries, but for tractability the network effect in Foreign is normalised to zero, with the difference in strength between Home and Foreign network effects given by $n$.

The demand intercept is given by $a$, with other non-wage marginal costs given by $c$.

For ease of notation, I define $\alpha = a - c - w_X$, given that the latter two variables are constant for both firms across all potential equilibria.

$V$ denotes the fixed cost of FDI.

In stage one, government $i$ offers a subsidy to the fixed costs of FDI of the firm native to country $j$. In stage two, there are four possible equilibria. The titles given to these equilibria are borrowed from Greaney (2002). The first equilibrium is an N-type equilibrium, in which neither firm engages in FDI, and is referred to as a national firm. The second is an M-type equilibrium, in which both become multinational, i.e. both set up affiliates in the other country. The remaining two types are asymmetric equilibria, referred to as A-type equilibria, where one of the firms becomes multinational and the other remains national.

Given that the game is solved by backward induction, I start with Stage 3, which gives firm profit functions and equilibrium outputs in the presence of network effects. The special case where $n = 0$ is outlined in brief at the end of the section.

\(^5\)This latter example is the nature of the *keiretsu* network analysed in Spencer and Qiu (2001).
3 The Game - Stage 3

In this section I outline Stage 3 of the game, in which firms compete in a Cournot-Nash game. I first give profit functions for the four possible equilibria, then solve for equilibrium outputs in the absence of any government policy. I include the subsidy levels, \( s_i \) for use in later sections, but in this section it will always be the case that \( s_i = 0 \). Outputs are solved in standard Cournot-Nash fashion via the maximisation of a profit function where firms have output as their choice variable. The focus is on the case with network effects, followed by a brief description of the case where \( n = 0 \).

3.1 Profit functions

\( N \)-type equilibrium with network effects

For the equilibrium in which both firms remain national, the profit functions are given as follows:

\[
\Pi_1^N = (P_H - w_X - c)X_{1H} + (P_F - w_X - c - g)X_{1F} \\
\Pi_2^N = (P_H - w_X - c - g - n)X_{2H} + (P_H - w_X - c)X_{2F}
\]

Where \( g \) represents export costs, \( n \) represents network effect costs and \( c \) represents non-wage marginal costs.

\( M \)-type equilibrium with network effects

In the equilibrium in which both firms invest abroad, neither faces export costs. Firm 2 however still faces network costs and will thus be at a disadvantage. The possibility of government altering firm behaviour in earlier stages is accounted for by the inclusion of the \( s_i \) terms.

\[
\Pi_1^M = (P_H - w_X - c)X_{1H} + (P_F - w_X - c)X_{1F} - (V - s_F) \\
\Pi_2^M = (P_H - w_X - c - n)X_{2H} + (P_H - w_X - c)X_{2F} - (V - s_H)
\]

\( A_1 \)-type equilibrium with network effects

In the asymmetric equilibrium in which only Firm 1 engages in FDI, the profit functions look at follows:
\[ \Pi_{11}^A = (P_H - w_X - c)X_{1H} + (P_F - w_X - c)X_{1F} - (V - s_F) \]
\[ \Pi_{21}^A = (P_H - w_X - c - g - n)X_{2H} + (P_H - w_X - c)X_{2F} \]

\( A_2 \)-type equilibrium with network effects

In the equilibrium in which only Firm 2 invests abroad, the profit functions are as follows:
\[ \Pi_{12}^A = (P_H - w_X - c - g)X_{1H} + (P_H - w_X - c - g)X_{1F} \]
\[ \Pi_{22}^A = (P_H - w_X - c - n)X_{2H} + (P_H - w_X - c)X_{2F} - (V - s_H) \]

3.2 Equilibrium Outputs

The table below gives the equilibrium outputs that emerge for each equilibrium outlined above. These outputs are arrived at following a Cournot-Nash game in which each firm maximises its profit taking account of the actions of the other firm.

<table>
<thead>
<tr>
<th></th>
<th>( X_{1H} )</th>
<th>( X_{1F} )</th>
<th>( X_{2F} )</th>
<th>( X_{2H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>( \frac{\alpha + g + n}{3} )</td>
<td>( \frac{\alpha - 2g}{3} )</td>
<td>( \frac{\alpha + g}{3} )</td>
<td>( \frac{\alpha - 2g - 2n}{3} )</td>
</tr>
<tr>
<td>M</td>
<td>( \frac{\alpha + n}{3} )</td>
<td>( \frac{\alpha - g}{3} )</td>
<td>( \frac{\alpha}{3} )</td>
<td>( \frac{\alpha - 2n}{3} )</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( \frac{\alpha + g + n}{3} )</td>
<td>( \frac{\alpha}{3} )</td>
<td>( \frac{\alpha + g}{3} )</td>
<td>( \frac{\alpha - 2g - 2n}{3} )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( \frac{\alpha + n}{3} )</td>
<td>( \frac{\alpha - g}{3} )</td>
<td>( \frac{\alpha}{3} )</td>
<td>( \frac{\alpha - 2n}{3} )</td>
</tr>
</tbody>
</table>

*Table 1: Equilibrium Outputs in the presence of network effects*

We can see from the above table that Firm 1’s domestic output will be the highest-ranking in the first three equilibria mentioned due to the network advantage, and can be so in the \( A_2 \) equilibrium if network costs outweigh export costs. In all instances, the network effect drives an asymmetric advantage in Firm 1’s favour. These results help to explain how network effects in Home drive a trade surplus in Home’s favour.

**Condition 1** Given all the above outputs, the condition for an interior solution to exist can be given by \( \alpha > 2(g + n) \)

I include the equilibrium outputs in the hypothetical situation in which no network effects exist \(^6\) in either country in Appendix 2. Comparing this with Table 1, it is clear that the network effect is driving the trade surplus in favour of Firm 1.

\(^6\)Alternatively, the network effect could be of identical strength in both countries
4 Stage 2: Location Decision

Firms decide their location contingent on the Fixed Costs of FDI and output for both themselves and their rival. The locational choices of the firms are inextricably linked to the notion of the subgame perfection of a given equilibrium. For an equilibrium to be subgame perfect, the profit for each firm must be simultaneously greater than the profit to each firm from deviating. The conditions for each of the four equilibria to be subgame prefect are outlined in turn. It is worth recalling at this stage that $V$, the fixed cost of FDI, is identical in both countries. Profits are of the form $\pi = X^2$. See Appendix 3 for a derivation.

$N$ subgame perfect

For the equilibrium in which both firms remain National to be subgame perfect (SGP), the following two conditions must simultaneously be met:

\[ \pi_1^N > \pi_1^{A1}, \text{ leading to } (V - s_F) > \frac{4g(\alpha - g)}{9} \]
\[ \pi_2^N > \pi_2^{A2}, \text{ leading to } (V - s_H) > \frac{4g(\alpha - g - 2n)}{9} \]

Once the Fixed Cost of FDI is above the larger of these thresholds, N is a unique subgame perfect equilibrium.

For the propositions put forward in this section I focus for now on the “laissez-faire” case, where $s_H = s_F = 0$. Note that in Section 5 this will not be the case as government intervention is considered.

In the absence of government action, N is subgame perfect if $V > \frac{4g(\alpha - g)}{9}$

$M$ subgame perfect

\[ \pi_1^M > \pi_1^{A2}, \text{ leading to } (V - s_F) < \frac{4g(\alpha - g)}{9} \]
\[ \pi_2^M > \pi_2^{A1}, \text{ leading to } (V - s_H) < \frac{4g(\alpha - g - 2n)}{9} \]

M will be SGP if the Fixed Costs of FDI are less than the smaller of these two thresholds. Thus in the absence of government policy, M is subgame perfect if $V < \frac{4g(\alpha - g - 2n)}{9}$

$A_1$ subgame perfect

The asymmetric equilibrium in which Firm 1 invests in Foreign while Firm 2 remains a National firm is SGP if the following hold:
\[ \pi_1^{A1} > \pi_1^N, \] leading to \((V - s_F) < \frac{4g(\alpha - g)}{9}\)

\[ \pi_2^{A1} > \pi_2^M, \] leading to \((V - s_H) > \frac{4g(\alpha - g - 2n)}{9}\)

In the absence of any policy, this asymmetric equilibrium is SGP if \(\frac{4g(\alpha - g - 2n)}{9} < V < \frac{4g(\alpha - g)}{9}\)

**\(A_2\) subgame perfect**

For the asymmetric equilibrium in which Firm 2 invests in Home while Firm 1 remains a National firm to be SGP, the following must hold:

\[ \pi_1^{A2} > \pi_1^M, \] leading to \((V - s_F) > \frac{4g(\alpha - g)}{9}\)

\[ \pi_2^{A2} > \pi_2^N, \] leading to \((V - s_H) < \frac{4g(\alpha - g - 2n)}{9}\)

These two conditions lead to the following range for to be SGP:

\(\frac{4g(\alpha - g)}{9} < V < \frac{4g(\alpha - g - 2n)}{9}\)

For a positive \(n\), this can never be a subgame perfect equilibrium, in the absence of government intervention, as the above condition can never hold.

The “laissez-faire” Stage 2 in the presence of network effects can be summarized as follows:

**Proposition 1**

With the existence of network effects, in the “laissez-faire” case, for a range of high \(V\), the \(N\)-type equilibrium is SGP. In an intermediate range of \(V\), the \(A_1\) equilibrium is SGP, while for low \(V\), \(M\) is SGP. \(A_2\) can never be SGP in the absence of government intervention.

The results arrived at in Stage 2 illustrate how network effects lead to a larger range in which Firm 1 will engage in FDI, which helps explain how network effects drive a Home FDI deficit. The results of this section coupled with the Home trade surplus of Stage 3, combine to explain the stylized facts given in the introduction concerning Japan’s trade and FDI balances with the EU and US. It is the network effect in Home, introduced into this simple model, that drives Home firms to export more than their Foreign rivals, and in turn leads to a higher likelihood of Home firms investing abroad. In policy terms, this will it will take
larger subsidies to encourage EU/US firms to invest in Japan than vice-versa.

4.1 Stage 2 in the absence of network effects

In the special case of $n = 0$, there are only two equilibria which can be SGP in the absence of government action. The conditions for an SGP equilibrium are laid out exactly as in Section 4.1, leading to the following propositions:

Proposition 2
In the “laissez-faire” case, in the absence of network effects, there is a clear threshold level of Fixed Costs of FDI, below which we have an $M$ equilibrium and above which we have a $N$ equilibrium.
5 Stage 1: Introducing government policy

In this section I examine the effect that government policy can have on firm behaviour and subsequent national welfare. The policy in question is a subsidy to the Fixed Cost of FDI. Examples of such subsidies may include the provision of infrastructure, the cost of which would otherwise have been borne by the MNC; direct subsidies to the plant setup costs of MNCs; removal of lump-sum regulatory fees. Chor (2007) states that subsidies to Variable Costs can induce larger welfare gains than those to Fixed Costs, for the same total subsidy bill. This is so because a Variable Cost subsidy alters both entry and production decisions of firms whereas Fixed Cost subsidies only affect the former. He uses a heterogeneous firm, monopolistic competition modelling structure, similar to Helpman, Melitz and Yeaple (2004) and shows that a subsidy induces only the most productive exporters to switch to FDI. Given that the welfare analysed in Chor’s paper consists solely of a consumption measure, I argue that the welfare effects of subsidising inward FDI are not explored to a large enough extent. He claims “the consumption gains are perhaps the most direct benefit: The relocation of production lowers the prices that MNCs charge in their host country’s market, due to the savings on cross-border transport costs and also possibly labour costs”. I argue here that a fuller definition of national welfare is required to analyse the conditions under which inward FDI is beneficial to the host country. Chor does go on to acknowledge that (his approach) “puts aside other potential benefits such as technological spillovers, agglomeration effects, or an increased demand for local labour”. While the two former effects cannot be modelled in this oligopoly setting, the latter is accounted for here, along with another arguably important effect of inward FDI, that on native host-country firms, along with a measure of Consumer Surplus.

The Welfare Function specified below is similar to that in Collie and Vandenbussche (2001) and Zhao (1995). One major difference in Collie and Vandenbussche is that labour compensation, referred to as “union rents” in their paper, can be zero in a given country under certain regimes, due to the fact that they assume that a firm can shift all production abroad and serve its own market through “reverse imports”, even under the assumption of homogenous goods. In their model, FDI can take the North-South form, modelled through wage differentials between the two countries. As mentioned already, with a domestic parent plant already in place, identical wage structures and homogenous goods, a firm will never fully leave its domestic country in my model. The “lump-sum” nature of Fixed Cost subsidies means that governments can only initiate once-off discontinuities in the Welfare.

\footnote{The importance of the “market-seeking FDI” assumption is emphasised here. If reverse imports or export-platform FDI were possible here, a portion of Consumer Surplus resulting from inward FDI would accrue to third-party consumers rather than those in the host country. This issue is raised in Chor (2007)}
function, also referred to as a regime shift.

5.1 Home government policy

In this section I assume initially that governments are acting unilaterally, in that the other government is not offering a subsidy. I remind the reader of the national welfare function derived in Section 2. From Equation 6, $\bar{L}_i$ is not included in welfare analysis as it is identical across equilibria. $L_i$ is assumed constant, so that only the division of labour between the two sectors can differ depending on the equilibrium in question. In the case of Home, Equation 6 becomes

$$U_H = CS_H + \pi_1 + wZ_H - s_H$$

Where
- $U_H$ is national welfare
- $CS_H$ is Home Consumer Surplus
- $Z_H$ is production on Home soil. This can differ from sales, depending on the equilibrium in question.
- $wZ_H$ is referred to as Labour Rent.
- $s_H$ is the subsidy level which will be given to the Fixed Costs of inward FDI by the government of country $H$.

The inclusion of network effects in the model leads to an asymmetry in the countries’ welfare functions. The Home Welfare Function under each of the four possible equilibria is first detailed in Table 3.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$M$</th>
<th>$A_1$</th>
<th>$A_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}(\alpha^2 - \gamma - n)^2 + (\frac{\alpha + g + n}{3})^2 + (w^2\frac{\alpha^2 - \gamma - n}{3})^2 + (w^2\alpha^2 + \gamma + n) - S_H$</td>
<td>$\frac{1}{3}(\alpha^2 - \gamma - n)^2 + (\frac{\alpha + g + n}{3})^2 + (\frac{w^2\alpha^2 - \gamma - n}{3})^2 + (w^2\alpha^2 + \gamma + n)$</td>
<td>$\frac{1}{3}(\alpha^2 - \gamma - n)^2 + (\frac{\alpha + g + n}{3})^2 + (w^2\frac{\alpha^2 - \gamma - n}{3})^2 + (w^2\alpha^2 + \gamma + n) - S_H$</td>
<td>$\frac{1}{3}(\alpha^2 - \gamma - n)^2 + (\frac{\alpha + g + n}{3})^2 + (w^2\frac{\alpha^2 - \gamma - n}{3})^2 + (w^2\alpha^2 + \gamma + n)$</td>
</tr>
</tbody>
</table>

Table 3: Home Welfare

<table>
<thead>
<tr>
<th>Consumers</th>
<th>Firm</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>(-)</td>
<td>+</td>
</tr>
<tr>
<td>$M$</td>
<td>(-)</td>
<td>+</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(-)</td>
<td>+</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(-)</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4: Change in return to each section of economy relative to $n = 0$ case
We see from Table 4 that network effects are certainly not beneficial to everyone in the Home economy. Consumers are worse off in all equilibria, due to the fact that Firm 2 will always sell to that market, by export or FDI, and will always suffer the network cost, regardless of the form of sale. Labour is worse off in an equilibrium in which Firm 2 invests in Home (namely $M$ and $A_2$), due to the fact that Firm 2 is paying the wages of some of the Home workers in these equilibria, and wages suffer an adverse effect caused by the network effect.

Inward Home government policy can initiate two regime shifts. The first to be detailed is that from the $N$-type to $A_2$-type equilibrium. The changes in Welfare resulting from this shift are as follows:

\[
CS_H: +\frac{1}{2} \left( \frac{4ag-g^2-nq}{9} \right) \\
\pi_1: -\left( \frac{2ag+g^2+nq}{9} \right) \\
LR_H: +w\left( \frac{a-g-2n}{9} \right)
\]

Consumers are better off because Firm 2 is now selling goods at a closer proximity to the Home market. Firm 1 is worse off as it suffers more intense competition from Firm 2. Labour is better off as there is now an extra production unit located in Home, without the loss of any of Firm 1’s production in this asymmetric equilibrium. We can see from the above that network effects have a negative impact on the welfare of each section of the Home economy. For the parameter range in which welfare rises for the $N$ to $A_2$ shift, network effects cause it to rise by a smaller amount.

For the move from $N$ to $A_2$ to be Welfare-improving, the sum of the terms above must be greater than zero. Define $\hat{s}_H$ as the subsidy level below which Welfare will improve for the shift from $N$ to $A_2$. The range of Welfare-improving subsidy is given by:

\[
s_H \leq \hat{s}_H = \frac{w(a-g-2n)}{3} - \left( \frac{2g^2+3ng}{9} \right)
\]

If we were to compare the above subsidy level with the $n = 0$ case, we see that the inclusion of network effects results in a smaller range of welfare-improving subsidy levels for Home. This is an interesting result in that it shows that despite the undoubted benefits of networks such as keiretsu to the Home economy, the range in which welfare improves as a result of the attraction of FDI is lower when these networks exits.

The second shift that the Home government can initiate through policy is that
from an $A_1$ to $M$ type equilibrium. As in the case without network effects, the encouragement of investment from the rival Firm into the domestic economy leads to the exact same welfare shift, no matter which equilibrium switch is in question. The welfare-improving subsidy threshold is the same as that for the $N$ to $A_2$ shift, with the same disparities when compared to the analogous shift in the absence of network effects.

In calculating the optimal subsidy level, it must be taken into account that there is no guarantee that the welfare-improving level should coincide with a level that will entice the overseas firm to invest. For this I calculate $\bar{s}$, the subsidy level above which the regime shift is profitable for the overseas firm. As in the welfare case, the threshold for the $N$ to $A_2$ shift is identical to that for $A_1$ to the $M$ shift. The subsidy from the Home government that will induce Firm 2 to invest is arrived at by simultaneously solving the following:

\[ V > \frac{4g(\alpha-g)}{9}, \text{ which is the condition for Firm 1 to remain national in N or A2 and Firm 2 to remain national in N.} \]

\[ (V - S_H) < \frac{4g(\alpha-g-2n)}{9}, \text{ which is the condition for Firm 2 to become multinational in an } A_2 \text{ or } M \text{ type equilibrium.} \]

These conditions lead to the following threshold subsidy level:

\[ s_H \geq \bar{s} = V - \frac{4g(\alpha-g-2n)}{9} \]

Now that I have defined and calculated $\bar{s}$ and $\hat{s}$, I am in a position to identify the optimal level of unilateral government subsidy.

Denote the optimal subsidy level $s^*$.

For the parameter range $\bar{s} > \hat{s}$, the optimal subsidy level is always zero. This is because in this range, the subsidy offered to induce multinationality will automatically lead to a welfare reduction.

The relationship between subsidies and welfare can be graphed. From this graph the optimal subsidy level will be apparent. In this analysis I start with prohibitively high Fixed Costs, leaving the world in an $N$-type equilibrium in the absence of policy.

**Figure 1**: Optimal unilateral Home subsidy with network effects included.

In the graph we see that until $\bar{s}$, the welfare curve is flat. Any subsidy level below
\( \bar{s} \) will not attract Firm 2, so that we will remain in the \( N \) Equilibrium. Once \( \bar{s} \) is reached, there is a once-off jump in welfare (provided the parameters satisfy \( \bar{s} < \hat{s} \)), to the Welfare level associated with the \( A_2 \) equilibrium. Welfare then drops one for one with the subsidy level, as any subsidy payment beyond the level that just attracts Firm 2 is a deadweight contribution to Firm 2 profits. \( \hat{s} \) is as before the subsidy level at which welfare is the same in the \( N \) and \( A_2 \) equilibria.

\[ s^* = \bar{s} \]

The range in which the optimal subsidy must lie, \( \bar{s} < s^* < \hat{s} \), corresponds to the following threshold range of Fixed Cost\(^8\).

\[ V < V^{NASH}_H = \frac{3w(a-g-2n)+4ag-\frac{11}{9}g^2-11ng}{9} \]

Saying that \( V < V^{NASH}_H \) is analagous to saying \( \hat{s} > \bar{s} \). If \( V > V^{NASH}_H \), the move from \( N \) to \( A_2 \) will not take place and \( N \) will be the Nash Equilibrium in the absence of government policy.

The optimal subsidy level, \( s^* \) is given by the level which maximises welfare. In this case the optimal level is that which just entices Firm 2 to invest in Home.

The \( A_2 \) Equilibrium arrived at by the offering of the optimal Home subsidy is a Nash Equilibrium, as neither firm has an incentive to deviate from it. Firm 1 does not have an incentive as its fixed costs of FDI remain above the threshold that kept it in the \( N \) Equilibrium, with no offering of a subsidy from the Foreign government in this unilateral case.

**Proposition 3**

In the appropriate parameter range, given by \( V < V^{NASH}_H \), the optimal Home subsidy with the inclusion of network effects is

\[ S^*_H = \bar{s} = V - \frac{4g(a-g-2n)}{9} \]

**Proposition 4**

The threshold level of Fixed Cost below which there will be a welfare-improving range of optimal Home subsidy, is lower once network effects are included, implying a smaller potential Welfare-improving range.

\(^8\)This is denoted \( V^{NASH}_H \) as it is the same threshold that will be used to determine Nash Equilibrium in the next section.
5.2 Foreign government policy

The policy of the Foreign government in the case where it acts unilaterally is now outlined. Welfare under the four regimes is given below.

\[
\begin{align*}
N & : \frac{1}{2} \left( \frac{2\alpha - g}{3} \right)^2 + \left( \frac{\alpha + g}{3} \right)^2 + \left( \frac{\alpha - 2g - 2n}{3} \right)^2 + (w \frac{2\alpha - g - 2n}{3})^2 \\
M & : \frac{1}{2} \left( \frac{2\alpha}{3} \right)^2 + \left( \frac{g}{3} \right)^2 + \left( \frac{\alpha - 2n}{3} \right)^2 - V + (w \frac{2\alpha}{3}) - s_F \\
A_1 & : \frac{1}{2} \left( \frac{2\alpha}{3} \right)^2 + \left( \frac{g}{3} \right)^2 + \left( \frac{\alpha - 2g - 2n}{3} \right)^2 + (w \frac{3\alpha - 2g - 2n}{3}) - s_F \\
A_2 & : \frac{1}{2} \left( \frac{2\alpha - g}{3} \right)^2 + \left( \frac{\alpha - 2n}{3} \right)^2 + \left( \frac{\alpha + g}{3} \right)^2 - V + (w \frac{3\alpha + g}{3})
\end{align*}
\]

Table 5: Foreign Welfare

<table>
<thead>
<tr>
<th></th>
<th>Consumers</th>
<th>Firm</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0</td>
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<td>(-)</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>(-)</td>
<td>0</td>
</tr>
<tr>
<td>A_1</td>
<td>0</td>
<td>(-)</td>
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</tr>
<tr>
<td>A_2</td>
<td>0</td>
<td>(-)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6: Change in return to each section of economy relative to \( n = 0 \) case

The regime shifts that can be initiated by the Foreign government are \( N \) to \( A_1 \) and \( A_2 \) to \( M \). \( \hat{s}_F \), the subsidy level below which welfare will improve for a given regime shift, is calculated for the shift as

\[
s_F < \hat{s}_F = \frac{w(\alpha - g)}{3} - \frac{3}{2} \left( \frac{g^2}{9} \right)
\]

This is larger than that for the Home government and identical to the \( n = 0 \) case. The subsidy level that will attract Firm 1 to invest in Foreign is given as follows:

\[
s_F > \bar{s}_F = V - \frac{4g(\alpha - g)}{9}
\]

Again these are identical regardless of the existence of network effects due to the fact that the network effect does not affect sales in the Foreign market. It is also smaller than that required to induce Firm 2 to invest in Home as Firm 1 earns higher profits from its overseas sales than does Firm 2.

As before, in the parameter range \( \bar{s} > \hat{s} \) the optimal subsidy is zero.

The optimal subsidy will be non-zero if \( \bar{s} < s^* < \hat{s} \). The existence of this range is given in terms of the threshold Fixed Cost level below which the optimal subsidy can exist, \( V_{NASH} \)

\[
V < V_{NASH}^F = \frac{3w(\alpha - g) + 4ag - \frac{11}{2}g^2}{9}
\]
This is larger than that for Home, indicating that there is a larger parameter range in which the Foreign government can improve welfare from the unilateral offering of a subsidy to the Fixed Cost of FDI.

The optimal unilateral subsidy is again the subsidy which just entices Firm 2 to invest in Home. This is summed up in Proposition 5. The resulting $A_1$ Equilibrium is Nash as Firm 2 has no incentive to invest in Foreign in the absence of a subsidy, with its fixed costs still the same as those which kept it in the $N$ Equilibrium.

**Proposition 5**
In the appropriate parameter range, given by $V < V^{NASH}$, the optimal Foreign government subsidy in the presence of network effects is

$$s^*_F = \bar{s}_F = V - \frac{4g(\alpha-g)}{9}$$

### 5.3 Retaliatory case

Given that in the previous subsections the optimal unilateral subsidy levels for Home and Foreign are calculated, an analysis of the case in which subsidy levels affect each other is a logical extension. If we assume as in Section 4.1 that in the absence of policy the analysis begins with prohibitively high fixed costs and neither firm investing abroad (i.e. where $V > \frac{4g(\alpha-g)}{9}$), then in the range $\bar{s} < \hat{s}$, the dominant strategy for both governments is to offer $s^* = \bar{s}$. In this case with network effects, this leads to a Nash equilibrium subsidy pair of

$$(s_H, s_F) = (\frac{4g(\alpha-g-2n)}{9}, \frac{4g(\alpha-g)}{9})^{10}$$. This subsidy pair leaves the game in an M-type equilibrium, with both Firms investing abroad. When fixed costs are prohibitively high, i.e. $\bar{s} < \hat{s}$ or when $V > V^{NASH}$, the Nash equilibrium is always of the N-type, with $(s_H, s_F) = (0, 0)$

**Proposition 6**
In the appropriate parameter space and with the subsidy levels that lead to M being a Nash Equilibrium, both firms would be better off in the N equilibrium than they are in the M equilibrium. This result is proven in Appendix 4.

We see from Appendix 4 that government rivalry does not enhance the fortunes of native firms, once governments pursue a wider welfare agenda. The existence of network effects does not change this fact for either firm. In the M equilibrium, variable costs are lower for each firm due to their avoiding export costs. They

---

9In this case, $\bar{s} < \hat{s}$ holds if $V < V^{NASH}$

10For proof that this is indeed a Nash Equilibrium, see Appendix 5
now face a fixed cost of FDI, however, which government subsidy does not fully offset. This is because each government offers the subsidy that would entice the overseas firm to invest in the unilateral case, not that which would entice the firm in an M equilibrium. This is the prisoner’s dilemma aspect of the game for firms.

The question of whether or not the M-type Nash Equilibrium is welfare-improving must now be asked. To answer this question welfare under the $M$ and $N$ equilibria must be compared.

In order for the $M$-type Nash Equilibrium to be welfare-improving the condition $U^M_i > U^N_i$ must be satisfied.

I begin the analysis with the Home case. For Home, the above condition $U^M_i > U^N_i$ simplifies to

$$V_H < V^{M\succ N}_H = \frac{3w(g-2n)-\left(\frac{4}{9}g^2+3ng-4\alpha g\right)9 + s_F - s_H}{9}$$

Substituting the two optimal subsidy levels into the above gives

$$V_H < V^{M\succ N}_H = \frac{3w(g-2n)+4ag-\frac{11}{2}g^2-11ng}{9}$$

This must be compared with $V^{NASH}_H$. If $V^{M\succ N}_H - V^{NASH}_H > 0$, the shift from $N$ to $M$ is always welfare-improving.

Substituting for these two values gives:

$$\frac{3w(g-2n)+4ag-\frac{11}{2}g^2-(3w(a-g-2n)+4ag-\frac{11}{2}g^2-11gn)}{9} > 0$$

Which simplifies to

$$3w(2g - \alpha > 0)$$

It is always the case that $V^{M\succ N}_H < V^{NASH}_H$, as the above can never hold due to Condition 1. This implies that, in the $M$-type Nash Equilibrium resulting from Nash government behaviour in the parameter range $\hat{s} < \bar{s}$, there are two distinct parameter ranges, summarized in Proposition 7

**Proposition 7**

In the parameter range in which the Home government will attract investment from Firm 2 ($V < V^{NASH}_H$), the move from $N$ to $M$ results in a welfare reduction

\[11\alpha > 2(g + n)\]
in the range $V^{M\succ N} < V < V^{NASH}$, and results in a welfare improvement in the range $V < V^{M\succ N}$.

For the Foreign government, the results are different. The threshold fixed cost level below which there will be a welfare improvement is given by

$$V_F^{M\succ N} < \frac{3w(2n+g) - \left(\frac{11}{9}g^2 + 8ng - 4ag\right)}{g} + s_H - s_F$$

Substituting the Equilibrium subsidy pair from above gives

$$V_H^{M\succ N} < \frac{3w(2n+g) + 4ag - \frac{11}{2}g^2}{g}$$

As above, this must be compared with $V_F^{NASH}$

If $V_F^{M\succ N} - V_F^{NASH} > 0$, the shift from $N$ to $M$ is always welfare-improving. Substituting for these two values leaves the following condition:

$$3w(2g + 2n - \alpha > 0)$$

It is again the case that the above never holds, due to Condition 1. As for Home, it is always the case that $V_H^{M\succ N} - V_H^{NASH} < 0$.

**Proposition 8**
In the parameter range in which the Foreign government will attract investment from Firm 1 ($V < V^{NASH}$), the move from $N$ to $M$ results in a welfare reduction in the range $V^{M\succ N} < V < V^{NASH}$, and results in a welfare improvement in the range $V < V^{M\succ N}$.

**Proposition 9**
The parameter range in which welfare improves, $0 < V < V^{NASH}$, is larger for Foreign. That is, when the $M$ Equilibrium is Nash, there is a greater range of $V$ over which the Foreign government experiences a welfare increase.

In the $n = 0$ case, the results are identical for both countries to those obtained for Home above. When symmetric governments arrive at the M equilibrium, there is always a range for both in which the Nash equilibrium has the properties of a welfare-reducing prisoner’s dilemma, along with a range in which welfare is improved. The addition of network effects does not change this fact, but does lead to an asymmetric outcome that is favourable to the Foreign government, in that the range of $V$ over which welfare improves is larger.
An interesting finding of the Nash game among governments is that the network effect, which in Section 3 was shown to benefit firms native to Home, does not in fact give the Home government advantage over its Foreign counterpart when governments compete to attract inward FDI. This finding can shed some light on the conundrum surrounding business linkages in countries such as Japan. Looking at Tables 4 and 6, it is clear that the network effect does still lead to lower returns to firms and labour in Foreign. While also leading to trade and FDI imbalances which are favourable to Home, the suggestion here is that in the context of FDI attraction, when consumers and workers are taken into account, these business “network effects” may lead to a disadvantage relative to the Foreign rival.

6 Conclusion

I have modelled a homogenous-good version of Greaney’s (2002) model, allowing for asymmetric trade and FDI outcomes in a two country, two firm duopoly. The factor driving the asymmetric outcome in this model is a “network effect”. The network effect is an added to cost to Foreign firms selling to Home, which may reflect “search costs involved in locating buyers, distribution costs and/or information costs that are assumed to be higher for ‘outsiders’ in some markets”. I show that the firm native to the country with the network effect fairs better in all equilibria than it did in the absence of network effects.

The homogenous good modelling strategy employed here allows for the potential impact of government policy to be included in the network effect model. The policy analysed here is a subsidy to the Fixed Cost of FDI, applied only to inward FDI. National Welfare Functions are specified for both governments. As subsidy levels rise, the firms pass through different “regimes” or equilibria, based on which of them are engaged in multinational activity. Government chooses the optimal subsidy level which maximises welfare for the regime shift in question. In the case without network effects, in the appropriate parameter space, Nash behaviour drives both governments to offer their optimal subsidy, leaving the game in the M equilibrium with both firms engaging in foreign investment. It is shown that in this parameter space there is a range of Fixed Costs that leads to an overall welfare loss from this Nash behaviour.

An interesting finding of the paper is that with the inclusion of network effects, it is more likely that the Foreign government will realise a Welfare improvement when governments offer their optimal subsidies. The conclusion drawn is thus that network effects, although good for the Home firm, may not be beneficial to Home as a whole. This can help inform the debate on the real losers from the trade and FDI asymmetries which the EU and US suffer with Japan. Furthermore, I show
that strategic interaction between governments pursuing a wide Welfare agenda is likely to lead to losses for firms in both countries, regardless of the existence of network effects.

A Appendix 1

Wages are determined in a framework similar to that in Shapiro and Stiglitz (1984). I assume that firms in the X sector do not have perfectly observable effort, i.e. its workforce must be monitored to ensure shirking is not occurring. This can be due to either a technological endowment or some other firm or industry characteristic.

With imperfect monitoring, there is a probability $q$ that a worker will be caught shirking. Thus, a firm in the X sector, in order to ensure that its workers are not shirking, will pay a premium $p$ on top of the reservation wage in the economy. The extent of this premium is decreasing in $q$ (as we get closer to perfect monitoring we approach the reservation wage) and increasing in $b$, the rate of exogenous turnover in the industry (in a cyclical industry in which workers are likely to be fired anyway, they have more incentive to shirk, and must thus be paid a higher wage in order to avoid shirking). It is also increasing in the cost of shirking to the firm, which means that firms with more productive workers should pay them more to ensure they do not shirk as the losses from this shirking are larger than for firms with less productive workers.

In my model I am assuming that the X sector is characterised by low levels of $q$, which means we observe $w_X > \bar{w}$, i.e. workers in this industry are paid a premium. In the Y sector, on the other hand, there is perfect monitoring, yielding $q = 1$ and $w_Y = \bar{w}$. The sector houses firms which can avoid shirking without having to pay any wage premium at all, so that wages in this sector are equal to the reservation wage.

B Appendix 2

Equilibrium Outputs in the absence of network effects

This is simply a special case of the above Section 3.2 with $n = 0$. I present this case as it gives an indication of the way in which business networks in the Home country affect the trade balance between the countries. The outputs calculated here will also be used to show the effect of networks on the FDI balance and welfare in sections 4 and 5.

This special case gives the following outputs:
As we can see, when there is no advantage for Firm 1, all equilibrium outcomes are perfectly symmetric. The outputs for Firm 1 in the \( A_1 \) Equilibrium are identical to those for Firm 2 in the \( A_2 \) Equilibrium. These results show that in the absence of the network effect in this simple model, there is balanced trade between Home and Foreign.

### C Appendix 3

Profits for each equilibrium are calculated as follows:

Operating profits in each market are equal to the square of output from the first order conditions for output.

The profit function is derived as follows:
\[
\pi = (p - c)X
\]
Here, \( c \) can comprise any combination of network, export, wage and other marginal costs.
\[
\frac{\delta \pi}{\delta X} = (p - c) + p'X
\]
\( p' = -1 \) in this instance

Profit Maximisation requires \( \frac{\delta \pi}{\delta X} = 0 \)
Which gives
\[
p - c = X
\]
Therefore
\[
\pi = X^2
\]

### D Appendix 4

The M Equilibrium induced by subsidy retaliation results in lower profits for Firms than the N Equilibrium in the absence of policy.

Assume that both governments, in the M Equilibrium are offering their optimal subsidies
\[
(s_H, s_F) = \left( \frac{4\alpha(\alpha-g-2n)}{9}, \frac{4\alpha(\alpha-g)}{9} \right)
\]

Firm 1
\[ \pi^1_N = \left(\frac{\alpha + g + n}{3}\right)^2 + \left(\frac{\alpha - 2g}{3}\right)^2 \]

\[ \pi^1_M = \left(\frac{\alpha + n}{3}\right)^2 + \left(\frac{\alpha}{3}\right)^2 - (V - s_F) \]

**N \succ M** if \( \pi^1_N > \pi^1_M \)

i.e. \( \frac{4g(\alpha - g)}{9} > g(2\alpha - 5g - 2n) \)

Which holds if

\[ 2\alpha + g + n > 0 \]

Which is always the case if Condition 1 holds.

---

**Firm 2**

\[ \pi^2_N = \left(\frac{\alpha + g}{3}\right)^2 + \left(\frac{\alpha - 2g - 2n}{3}\right)^2 \]

\[ \pi^2_M = \left(\frac{\alpha - 2n}{3}\right)^2 + \left(\frac{\alpha}{3}\right)^2 - (V - s_H) \]

**N** if \( \pi^2_N > \pi^2_M \)

i.e. \( \frac{4g(\alpha - g)}{9} > g(2\alpha - 5g) \)

Which holds if

\[ 2\alpha + g > 0 \]

Which is always the case if Condition 1 holds.

---

### E Appendix 5

Discussion of the conditions under which the M equilibrium is Nash.

In all below, I refer only to the case in which parameters satisfy \( \hat{s} < \bar{s} \). When this is not the case, the only Nash Equilibrium is N, in which no firms invest abroad.

I show in Section 5.1.2 and 5.1.3 that for any subsidy level below \( \bar{s} \), the actual subsidy offered is zero, as it is not sufficient to attract the overseas firm. For any subsidy level above \( \bar{s} \), the government will offer exactly \( \bar{s} \), as anything larger than this would simply amount to a deadweight contribution to the overseas firm’s profits.

Given these facts, it can be said that each government has two ***potentially un-dominated*** strategies available to it at any point, given as (0, \( \bar{s} \)).

The game can be represented in payoff matrix form

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>( \bar{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( U_N^N ), ( U_F^N )</td>
<td>( U_N^{A1} ), ( U_F^{A1} )</td>
</tr>
<tr>
<td>( \bar{s} )</td>
<td>( U_N^{A2} ), ( U_F^{A2} )</td>
<td>( U_N^{M*} ), ( U_F^{M*} )</td>
</tr>
</tbody>
</table>

I mark with an asterisk the strategy which is the best reply for each government,
given the other government’s actions. It can be seen from this that the Nash Equilibrium is M, with both governments offering subsidies. Below I prove that each strategy marked with an asterisk is indeed a best reply.

1) \( n = 0 \) case.

Column 1 and Row 1 (due to symmetry): If the Foreign government offers no subsidy, the Home government faces the choice between \( U^N_H \) and \( U^A_2 \). This is exactly the setting analysed in Section 5.1.3. There we see that the offering of leads to a Welfare improvement, in the specified parameter range \( \bar{s} < \hat{s} \), meaning that in this range, \( U^{A_1}_H > U^N_H \) holds. Given that the case is symmetric, it can be said that \( U^{A_1}_F > U^N_F \). This explains the asterisks in the top right and bottom left boxes above.

Column 2 and Row 1 (due to symmetry): If the Foreign government offers a subsidy \( \bar{s} \), the Home government is left with the choice between \( U^{A_1}_H \) and \( U^M_H \). The Welfare levels in both instances are

\[
U^{A_1}_H = \frac{1}{2} \left( \frac{2 \alpha - g}{3} \right)^2 + \left( \frac{g}{3} \right)^2 + \left( \frac{\alpha + g}{3} \right)^2 - (V - s_F) + w \frac{2 \alpha + g}{3} \\
U^M_H = \frac{1}{2} \left( \frac{2 \alpha}{3} \right)^2 + \left( \frac{g}{3} \right)^2 + \left( \frac{\alpha}{3} \right)^2 - (V - s_F) + w \frac{2 \alpha}{3} - S_H
\]

Subtracting one from the other, Welfare increases if \( s_H < \hat{s} = \frac{w(\alpha - g)}{3} - \left( \frac{3}{2} \right) g^2 \).

Firm 2 will invest in Home if \( s_H > \bar{s}_H = V - \frac{4 g (\alpha - g)}{9} \). These are the same two conditions that exist for the shift from \( N \) to \( A_2 \). So in the range \( \bar{s} < \hat{s} \), Welfare will be improved by offering \( \bar{s} \).

Due to symmetry, the above results are identical for the Foreign government when it compares \( A_2 \) to \( M \). This explains both asterisks in the bottom right hand corner of the payoff matrix, which gives us \( M \) as the only Nash Equilibrium in the parameter range \( \bar{s} < \hat{s} \).

2) Network effects included

The exact same approach as above is adopted, and the resulting payoff matrix is identical, leaving \( M \) as the only Nash equilibrium when \( \hat{s} < \bar{s} \).

Column 1 and Row 1: When the other government sets a subsidy equal to zero, Section 5.2.1 and 5.2.2 prove that welfare will be increased by offering \( \bar{s} \). The
thresholds are different for the two governments in this instance, but the result holds in both cases. This explains the asterisks in the top right and bottom left hand corners.

**Column 2: When the Foreign government sets $\bar{s}_F$**

Home has the choice between $U^A_H$ and $U^M_H$. Welfare levels in these two equilibria are

\[
U^A_H = \frac{1}{2}(\frac{2\alpha-g-n}{3})^2 + \left(\frac{\alpha}{3}\right)^2 + \left(\frac{\alpha+g+n}{3}\right)^2 - \left(V - s_H\right) + (w\alpha + g + n)
\]

\[
U^M_H = \frac{1}{2}(\frac{2\alpha-n}{3})^2 + \left(\frac{\alpha+g+n}{3}\right)^2 - \left(V - s_H\right) + (w\frac{2\alpha-n}{3}) - S_H
\]

Subtracting these, the threshold subsidy level below which welfare will be increased is

\[
s_H < \hat{s} = \frac{w(\alpha-g-2n)}{3} - (\frac{\frac{\alpha}{3} + 3n g}{9}).
\]

The level required to induce Firm 2 to invest, leading from the $A_1$ to the $M$ equilibrium, is

\[
s_H > \bar{s}_H = V - \frac{4g(\alpha-g-2n)}{9}.
\]

These two thresholds are identical to those for the move from $N$ to $A_2$ outlined in section 5.2.1 and 5.2.2.

In the range $\bar{s} < \hat{s}$, it is therefore shown that offering $s^* = \hat{s}$ will lead to a welfare improvement for Home. Thus $M$ dominates $A_2$.

**Row 2: When the Home government sets $\bar{s}_H$,**

Foreign has the choice between $U^A_F$ and $U^M_F$. Welfare levels in these two equilibria are

\[
U^A_F = \frac{1}{2}(\frac{2\alpha-g}{3})^2 + \left(\frac{\alpha-2n}{3}\right)^2 + \left(\frac{\alpha+g}{3}\right)^2 - \left(V - s_F\right) + (w\alpha + g)
\]

\[
U^M_F = \frac{1}{2}(\frac{2\alpha}{3})^2 + \left(\frac{\alpha-2n}{3}\right)^2 + \left(\frac{\alpha}{3}\right)^2 - \left(V - s_F\right) + (w\frac{2\alpha}{3}) - S_F
\]

Subtracting these, the threshold subsidy level below which welfare will be increased is

\[
s_F < \hat{s}_F = \frac{w(\alpha-g)}{3} - (\frac{\frac{\alpha}{3} + g^2}{9}).
\]

The level required to induce Firm 2 to invest, leading from the $A_1$ to the $M$ equilibrium, is $s_F > \bar{s}_F = V - \frac{4g(\alpha-g)}{9}$.

These two thresholds are identical to the figures for both countries when $n = 0$. 

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In the range $\bar{s} < \hat{s}$, it is therefore shown that offering $s^* = \hat{s}$ will lead to a welfare improvement for Home.

From the above results it is proven that if $\bar{s} < \hat{s}$, the best response of the Foreign government to the Home government setting $\bar{s}_H$ is to set $\bar{s}_F$. $M$ therefore dominates $A_1$ for Foreign.

These last two results explain the asterisks in the bottom right hand corner of the payoff matrix. The only unique Nash Equilibrium is the $M$ equilibrium in which both governments offer $s^* = \bar{s}$. 