The Strategic Formation of Bilateral and Multilateral Trade Agreements

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Abstract

A growing literature during the last two decades has examined the impact of preferential trade agreements (PTAs) on multilateral liberalization and vice versa. In a political economy model, Krishna (1998) shows that the formation of a two country preferential trade agreement reduces the incentives of trading partners for multilateral liberalization with third countries. We address the question as to whether the results of Krishna hold, if we consider strategic link formation of countries, where countries can form bilateral PTAs as well as a multilateral agreement (e.g. GATT). We use a network formation approach to make bilateral as well as multilateral link formation endogenous. We introduce the notion of multilateral stability and observe that multilateral liberalization can coexist with PTAs. We allow for heterogeneity of countries and find that a PTA between small countries is a stable structure. In order to investigate multilateral link formation that is in the spirit of the GATT, we introduce the structure of hypergraphs.

Keywords: PTAs, multilateral stability, efficiency, network formation.

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1 Introduction

A growing literature during the last two decades has examined the impact of preferential trade agreements (PTAs) on multilateral liberalization and vice versa\(^2\). Krishna (1998) shows that the formation of a PTA lowers countries’ incentives for multilateral tariff reduction. Two authors, Freund (2000) and Andriamananjara (2002), address the question on how multilateralism can impact bilateralism. Freund (2000) observes that multilateral tariff reduction can affect the formation of PTAs, and therefore shows that multilateral trade liberalization enhances the incentives to form PTAs. The conclusion is that many of the current PTAs may be a result of the success of the GATT in lowering tariffs. Andriamananjara (2002) addresses the question as to whether the current wave of regionalism and bilateralism is going to lead to a division of the world into a number of competing inward-looking blocs or to a more open multilateral trading system. This paper uses a multi-country political economy model to investigate the possibility of achieving global free trade through the expansion of preferential trade arrangements (PTAs).

The importance of this issue motivates an examination of the incentives of the players to form such agreements and of the strategic stability of different structures when linking decisions depend on players’ payoffs. A simple way to analyze stable network structures is to study networks in which individuals do not benefit from altering the structure by single deviations.

We use a network formation approach to model the formation of trade agreements. Goyal and Joshi (2006) have been the first to analyze the formation of trade agreements as a network formation game. By assuming that countries are symmetric with respect to market size, they obtain that a network formation process in which players are allowed to form bilateral links leads either to a complete network or to an almost complete network. Furusawa and Konishi (2007) use a similar approach but with a differentiated product market. They show that the complete network in which every pair of players is bilaterally linked is stable. Whereas the latter two papers investigating whether the bilateral formation of PTAs alone achieves global free trade, they put aside whether the formation of PTA lowers incentives for multilateral tariff reduction within the scope of the GATT. Thus, their approach differs significantly from our model, since we allow players to form multi-

lateral trade agreements in addition to bilateral links. In our model countries are assumed to be heterogenous with respect to market size, in order to investigate the question as to whether countries of different market size differ in their incentive to form multilateral links.

To examine the stability of different trading structures we introduce the notion of multilateral stability, which is an extension of the pairwise stability concept for bilateral link formation introduced by Jackson and Wolinsky (1996). The idea of multilateral stability is that players can form bilateral links and hyperlinks, which include more than two players. The formation of any of these links needs the consent of all players included but deletion can be done unilaterally.

Ethier (2004) pointed out that PTAs may have negative effects on third countries, as the market of one of the trading partners gets less attractive to third countries’ firms. This effect is called “concession diversion”. We investigate to what extend this effect occurs, when we introduce multilateral link formation between countries. We show that, depending on countries’ market size, third countries can have an incentive to delete the multilateral link and thus multilateral liberalization cannot be supported.

One characteristic of the GATT agreement is that due to the non-discrimination principle a member party does not have the opportunity to delete the connection to a single other member within the multilateral agreement. In our model a multilateral link is represented by means of a hyperlink\(^3\), whereas PTA are modeled by means of non-directed bilateral links. In Figure 1 country A, B and C are members of the WTO and agreed upon common tariff reduction. A hyperlink does not allow one member to delete trading arrangements with single members within the GATT, instead bilateral link formation allows players to delete single links. But this is not in the spirit of the GATT agreement. Figure 1 shows the difference between modelling networks by graphs and hypergraphs. In the real world, the number of GATT members increases and they all agree on reducing tariffs. However, due to Article XXIV of the GATT members can still form additional bilateral preferential trade agreements in which tariffs have to be reduced to zero. In our model countries forming a bilateral link reduce their tariffs to zero, whereas the tariff for all countries in a multilateral link is set to some value \( t > 0 \).

\(^3\)Two papers that introduce network formation and communication games on a fixed hypergraph structure are Durieu et al. (2005) and van den Nouweland et al. (1992).
We can observe that during the last two decades the tariff reduction as negotiated within the GATT does not satisfy the requested trade conditions and that countries like the U.S. started to form additional bilateral preferential trade agreements. Starting with an exogenously given multilateralized world, the existing literature on PTAs investigated what incentives countries have to form additional bilateral links and tried to answer the question if additional PTAs increase individual and global welfare. The literature on trading blocs investigates whether the effect of regional integration is positive or negative. Furthermore, the time path approach formulated by Bhagwati (1993) investigates whether regionalism leads to multilateral free trade for all, through continued expansion of the regional blocs. These questions motivate to introduce the endogenous formation of preferential trade agreements in international trade models. However, Baldwin (2006) observed, and as stated earlier in Deardorff and Stern (1997, p. 27), that regional and bilateral tariff reduction went hand in hand with multilateral liberalization and preferential trade agreements coexisted with multilateralism from the start. In our model countries can simultaneously form bilateral PTAs and multilateral links. What structures will emerge when countries choose bilateral and multilateral links simultaneously? Will the increasing number of PTAs lead to a more open multilateral trading system, when we consider strategic link formation of countries?

To answer these questions we introduce a three-country setting and an imperfect competitively produced good, that is traded among the three countries. In each country there is a single firm competing as a Cournot oligopolist in each market. Markets in different countries are assumed to be perfectly segmented as in Krishna (1998), so that each firm regards each country as a separate market. Welfare gains from trade stem from the

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additional competition in the domestic country. If two countries have signed a bilateral PTA, each of them offers the other one a zero tariff access to its respective domestic market. If two countries are connected via a multilateral link, they offer each other access to their respective domestic market at a medium tariff. If a trade agreement among a pair of players does not exist, each of them imposes a high tariff on the imports of the other one.

Goyal and Joshi (2006) also investigate strategic stability of trading structures in a segmented market Cournot competition model where countries can also obtain welfare gains from increased competition that is generated by free trade agreements. In their model they consider bilateral link formation of homogenous countries, in which tariffs are set to zero if two players have a link. Furthermore they show that a network with an isolated country and the rest having a free trade agreement is a stable state. In the case of three countries, this implies that the situation in which one country is isolated and the other two having a free trade agreement is stable. In contrast to Goyal and Joshi (2006), we find, that due to country heterogeneity, a PTA between two countries with relatively small market size is stable but a bilateral link between two countries of different market size cannot be stable. Moreover, we obtain that the global network in which all countries share a multilateral link cannot be stable whereas the global network with a PTA between every pair of players and the complete network are stable. When we consider endogenous tariff formation, we obtain a considerably lower number of possible stable states. When countries choose their welfare maximizing tariffs that they levy on foreign countries, we find that the network with a global link and a PTA between a single pair of players is the only stable structure. The complete network without an additional multilateral agreement can no longer be achieved, which implies that the GATT agreement stabilizes the trading structure. We also observe that a conflict between overall welfare efficiency and stability can occur in a heterogenous country model.

The results are driven by three different welfare effects that a trade agreement has on an importing country. The first effect is that an importing country gains from increased competition and increased consumer surplus in the domestic market. The welfare of a country also increases since the domestic firm gets greater access to the foreign market, but due to increased competition, the domestic firm gets a lower profit in its own market. We will see that the additional welfare gains from free trade are very high for countries with small market size, since a PTA allows small countries’ firms to access the market of
the large country and thus increases the profit of the domestic firm.

The paper is organized as follows: In Section 2, we define a notion for stability and efficiency of hypergraph structures, which is modeled as an extension of the pairwise stability concept introduced by Jackson and Wolinsky (1996). In Section 3, we introduce the model of international trade as a three country setting. The stable and efficient trading structures are analyzed in Section 4 in which tariffs are exogenously given. Section 5 extends the framework of section 4 in which countries choose their optimal non-cooperative tariffs. Section 6 provides the conclusion.

2 Hypergraph Networks

Consider a finite number of players $N = \{1, ..., n\}$ and assume $n \geq 3$. In what follows the set of player will be a set of countries.

In our setting we concentrate on undirected links as in Jackson and Wolinsky (1996), which means that a link between players needs the consent of all players involved in that link.

We introduce the structure of hypergraphs to model multilateral agreements between players.

**Definition 2.1.** Let $N = \{1, ..., n\}$ be a finite set of nodes. A pair $(N, \mathcal{L})$, where $\mathcal{L} = \{L_1, ..., L_m\}$ is a set of links, $\mathcal{L} \subseteq 2^N$, is called a hypergraph on $N$.

In the following the set of nodes will represent the set of players and the term network will be used as a synonym for the word hypergraph. A link as a subset of countries represents a trading agreement between these countries.

Since each player is linked with himself we restrict our attention to hypergraphs $(N, \mathcal{L})$ with $\mathcal{L} \subseteq \{L \in 2^N | |L| \geq 2\}$.

The set of all possible hypergraphs, that satisfy the conditions mentioned above is denoted by $\mathcal{H}$.

Whenever the hypergraph contains only one group of players with $\mathcal{L} = \{N\}$, the hypergraph is called global and is denoted with $\mathcal{L}^G$.

The complete graph $\mathcal{L}^N$ is the family of subsets of $N$ with $\mathcal{L}^N = \{L \in 2^N | |L| = 2\}$.

The star network, that we denote by $\mathcal{L}^S_i$, has only bilateral links from the central player $i$ to each of the other players with $\mathcal{L}^S_i = \{L \in 2^N | |L| = 2 \text{ and } i \in L\}$.

Let $\mathcal{L}^e$ denote the empty network.
Furthermore let \( N_i(\mathcal{L}) \) denote the set of players that are directly linked with player \( i \) in network \( \mathcal{L} \) and \( \eta_i(\mathcal{L}) = |N_i(\mathcal{L})| \).

The value of a hypergraph is represented by a real valued function \( v : \mathcal{H} \to \mathbb{R} \), which specifies for each hypergraph \( \mathcal{L} \in \mathcal{H} \) the total value \( v(\mathcal{L}) \) generated by \( \mathcal{L} \) and will be the aggregate of individual payoffs given by a hypergraph structure.

A payoff function \( Y_i : \mathcal{H} \to \mathbb{R} \) with \( \sum_{i \in N} Y_i(\mathcal{L}) = v(\mathcal{L}) \) assigns a payoff to each player \( i \in N \) and depends on the network structure. In what follows \( v(\mathcal{L}) \) will denote the aggregate welfare generated by network \( \mathcal{L} \) and \( Y_i \) will denote country \( i \)'s welfare.

**Stability**

We introduce some notations:

- For \( L \not\in \mathcal{L}, \mathcal{L} \cup \{L\} \) is the network we obtain from \( \mathcal{L} \) by adding the link \( L \).
- For \( L \in \mathcal{L}, \mathcal{L}\setminus\{L\} \) is the network we obtain from \( \mathcal{L} \) by deleting the link \( L \), if \( L \in \mathcal{L} \).

The formation of a link requires the consent of all parties involved, but severance can be done unilaterally. We introduce the following stability concept:

**Definition 2.2.** A hypergraph \( (N, \mathcal{L}) \in \mathcal{H} \) with \( \mathcal{L} = \{L_1, ..., L_m\} \) is called multilaterally stable, if

\[
(i) \quad Y_i(\mathcal{L}) \geq Y_i(\mathcal{L}\setminus\{L\}) \quad \forall L \in \mathcal{L}, \quad \forall i \in L \quad \text{and} \\
(ii) \quad Y_i(\mathcal{L} \cup \{L\}) > Y_i(\mathcal{L}) \Rightarrow \exists j \in L, \\
\text{such that } Y_j(\mathcal{L} \cup \{L\}) < Y_j(\mathcal{L}) \quad \forall L \not\in \mathcal{L}
\]

The above definition describes a situation in which no country has an incentive to delete any of his existing link and no subset of countries wants to form an additional agreement.

**Efficiency**

In order to analyze efficient hypergraphs, we need to consider global welfare, which is given by the sum of all countries’ payoffs.

**Definition 2.3.** A hypergraph \( (N, \mathcal{L}^*) \in \mathcal{H} \) is said to be efficient, if \( v(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}) \leq v(\mathcal{L}^*) = \sum_{i \in N} Y_i(\mathcal{L}^*), \quad \forall (N, \mathcal{L}) \in \mathcal{H} \).
3 The Model

We introduce a three country setting with \( i \in N = \{A, B, C\} \). Countries can agree to form a multilateral link and they can also form bilateral PTAs. A multilateral link between all three countries is called a global link. In the following a complete network is a trading structure in which each pair of players has formed a bilateral PTA. Two papers that focus on the decision to form PTAs are Grossman and Helpman (1995) and Krishna (1998), whereas the latter showed in a three-country setting that the formation of a two country free trade agreement reduces the trading partners’ incentives for multilateral liberalization with the third country, which represents the rest of the world. This model relies on a Cournot oligopoly model that will be adapted in the following \(^5\). We address the following question: Will the results of Krishna hold, if we consider strategic link formation of countries, where countries in addition to bilateral links can form a multilateral trading agreement that is in the spirit of the GATT agreement?

In each country there is a firm producing a homogenous good with marginal cost of production \( c \). The aggregate utility in country \( i \) is given by

\[
    u_i = q_0 + \alpha_i q_i - \frac{1}{2} q_i^2 ,
\]

where \( q_0 \) denotes the consumption of the competitively produced numeraire good. With \( q_i^j \) we denote the output produced by firm \( j \) in country \( i \) and \( q_i = \sum_j q_i^j \) denotes the total sales of all firms in country \( i \). Therefore the price of the good in country \( i \)'s market is given by a linear function:

\[
    P_i = \alpha_i - q_i .
\]

We assume that firms compete as Cournot oligopolists in each country such that each firm maximizes its profit in each country separately as in Krishna (1998). Furthermore, we introduce the following exogenous tariff structure:

\[
    t^{ij}(\mathcal{L}) = \begin{cases} 
    0 & \text{if } i \text{ and } j \text{ have a PTA}, \\
    T & \text{if there is no trade agreement between } i \text{ and } j, \\
    t & \text{otherwise}
    \end{cases}
\]

where \( t^{ij}(\mathcal{L}) \) denotes the tariff faced by firm \( i \) in country \( j \) for each quantity supplied in network \( \mathcal{L} \), whereas \( T > t > 0 \).

Multilateral tariff reduction means an equal reduction for all three countries to the same

\(^5\)Our model is a variant of the model used by Krishna (1998).
tariffs due to the GATT principles of reciprocity and non-discrimination. The assumption that two countries that are involved in a PTA face a tariff of zero is supported by the GATT Article XXIV that permit preferential trade agreements (PTAs) if tariffs are set to zero.

Under the Cournot assumption firms are assumed to maximize profits, taking other firms’ outputs as given, with all firms choosing their quantities simultaneously. First we assume that $T > \alpha_i \forall i$ to make sure that firms will only sell in another country if there exists at least a multilateral link between two countries. Therefore, a firm $j$ faces the following optimization problem to choose the quantity that it supplies in country $i$:

$$
\max_{q_i^j} \pi_i^j = (\alpha_i - q_i^j) \cdot q_i^j - c \cdot q_i^j - t_i^j \cdot q_i^j.
$$

Thus the equilibrium quantity that firm $j$ supplies in country $i$ is given by:

$$
q_i^j = \frac{(\alpha_i - c)}{(\eta_i(\mathcal{L}) + 1)} + \frac{\sum_{k \in N_i} t_k^i(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)} - t_i^j(\mathcal{L}),
$$

where $\eta_i(\mathcal{L})$ denotes the number of firms active in country $i$ and $k = A, B, C$. We restrict the parameter $t$ to be $0 < t < \frac{2\alpha_i - c}{3} \forall i$ to concentrate on the case in which there is a positive quantity traded between two countries that share a multilateral agreement.

A firm $j$’s profit in country $i$ with $j \in N_i(\mathcal{L})$ can be calculated as:

$$
\pi_i^j = (q_i^j)^2.
$$

We define country $i$’s welfare function as the sum of consumer surplus, producer surplus and tariff revenue. Governments choose the tariffs as well as the linking decision with respect to maximizing social welfare. The objective function is:

$$
Y_i(\mathcal{L}) = \frac{1}{2} q_i^2(\mathcal{L}) + [(P_i(\mathcal{L}) - c)q_i^1(\mathcal{L}) + \sum_{j \neq i} (P_j(\mathcal{L}) - c - t_j^i(\mathcal{L}))q_j^i(\mathcal{L})] + \sum_{j \neq i} t_j^i(\mathcal{L})q_j^i(\mathcal{L})
$$

The first term represents consumer surplus in country $i$. The second and third term are firm $i$’s profit in its own market and in the foreign markets, respectively. The last term is country $i$’s tariff revenue. This formulation of social welfare places equal weight on consumer surplus and the firm’s profit.

The total profit of a firm $j$ is given by the sum of all the profits the firm $j$ makes in all countries:

$$
\Pi^j = \sum_{i=1}^n \pi_i^j = \sum_{i=1}^n \frac{(\alpha_i - c)}{(\eta_i(\mathcal{L}) + 1)} + \frac{\sum_{k} t_k^i(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)} - t_i^j(\mathcal{L}))^2.
$$
Since countries only trade and supply in another country when a trade agreement exists, the social welfare function reduces to:

\[ Y_i(L) = \frac{1}{2} q_i^2(L) + \sum_{j \in N_i(L)} (\alpha_j - c - q_j(L) - t_j(L)) \cdot q_j(L) + \sum_{j \in N_i(L)} t_j(L) \cdot q_j(L). \] (9)

\section{Stability of Trading Structures and Market Size Asymmetries}

We assume asymmetry with respect to countries’ market size, which is expressed by different values of the parameter \( \alpha_i \) to answer the question how the variation of market size across countries affects their incentives for establishing trade agreements and whether this will lead to different stable trading structures.

\subsection{The Symmetric Model}

First we investigate possible stable structures with \( \alpha_A = \alpha_B = \alpha_C = \alpha \) and observe that we are in line with the results in Goyal and Joshi (2006) in which the complete network is the unique stable structure. Introducing multilateral link formation we obtain:

\textbf{Proposition 4.1.} \textit{The complete network and the global network with a PTA between each pair of players are the unique stable structures.}

The proof can be found in the appendix. The star cannot be stable since the two players that have only one link have an incentive to link to each other. The same argument counts for a star network with an additional global link. The empty network cannot be stable since a pair of players always gains from a PTA.

\subsection{Two Small Countries and One Large Country}

In the following we investigate whether the result will change when we consider asymmetries among countries.

We start with the assumption that \( \alpha_A > \alpha_B = \alpha_C = \alpha \) which implies that country \( A \)’s market size is relatively large compared to country \( B \)’s and \( C \)’s market size.
The analysis of possible stable structures leads to the next result that supports the observation that countries tend to form additional PTAs.

Lemma 4.1. The global network without any PTA cannot be stable.

The global network without any PTA can never be stable, since player $B$ and $C$ will both gain from an additional PTA between each other for the following reason: Consumer surplus in both countries increases due to increased competition and lower prices. We can also observe that a PTA leads to an increase in firm $B$’s profit in country $C$’s market and vice versa. However there is a small negative effect on countries’ welfare, since the domestic firm’s profit in its own market decreases. Given that the two positive effects are higher than the negative effect, the overall welfare effect of an additional PTA is positive in both countries and thus $B$ and $C$ will deviate. The intuition is shown in Figure 2 in which the green area represents consumer surplus, the red area is domestic firm’s profit and the blue area is tariff revenue.

The next result provides a full description of possible stable trading structures.

Proposition 4.2. We have three possible stable structures for values $t < \frac{\alpha - c}{3}$:

(i) For any parameter values the global network with a PTA between every pair of players
Figure 3: Stable Networks. a) The global network with a PTA between B and C, b) PTA between B and C and c) the global network with a PTA between every pair of players.

and the complete network are stable states, [Figure 3c].

(ii) For values $4(\alpha_A - c)^2 > 6(\alpha - c)^2$ and $12(\alpha_A - c)t + 4(\alpha - c)^2 + 16t^2 < 24(\alpha - c)t + (\alpha_A - c)^2$ a PTA between B and C is stable, [Figure 3b].

(iii) For values $12(\alpha_A - c)t + 4(\alpha - c)^2 + 16t^2 > 24(\alpha - c)t + (\alpha_A - c)^2$ the global network with a PTA between B and C can be stable, [Figure 3a].

Figure 3 illustrates the structures that can be stable.

To give an intuition of the result, note that there are three direct effects at work when two countries sign a preferential trade agreement with zero import tariffs: First, the domestic firm faces greater competition from a foreign firm in the domestic market. Second, the domestic firm gets greater access to the foreign market. Third, domestic consumers benefit from greater competition, in terms of lower prices. Therefore, the empty and the global network cannot be stable since B and C will form a PTA. The welfare effect in both countries from a PTA is positive.

From condition ii) and iii) we can observe that there exists a threshold for which country A will deviate from the situation described in Figure 2 a) which is due to the effect of “concession diversion”. To understand country A’s linking decision we start with a global network in which country A’s firm earns $(\frac{\alpha_A - c}{4} - \frac{3t}{4})^2$ from its operations in B, respectively C. Since country B and C increase domestic welfare by joining a PTA the profit of country A’s firm reduces to $(\frac{\alpha_A - c}{4} - \frac{t}{2})^2$ and thus the foreign markets are less attractive to firm A. As a consequence country A wants to delete the multilateral link with B and C if its welfare effect is positive. This is the case for $12(a_A - c)t + 4(a - c)^2 + 16t^2 < 24(a - c)t + (a_A - c)^2$. In this case a PTA between B and C is stable if A has no incentive to form an additional
PTA with $B$ or $C$ which is the case for $4(\alpha_A - c)^2 > 6(\alpha - c)^2$.

Proposition 4.2 gives a full characterization of multilateral stable networks, when country $B$ and $C$’s markets are relatively small compared to country $A$. However, the following question arises: Will we obtain the same results, when we define country $B$ and $C$ as countries with large markets and $A$ as a country with a relatively small market?

### 4.3 One Small Country and Two Large Countries

In the following, we analyze which networks are multilaterally stable, when $C$ and $B$ are the countries with the largest market (thus $\alpha_A < \alpha_B = \alpha_C = \alpha$). In particular, we are interested in the question as to whether country $B$ and $C$ still have incentives to keep their PTA or will other structures emerge in equilibrium. One possibility one might think of is that player $A$’s firm obtains additional gains from a PTA as the additional market demand from country $B$ (respectively $C$) is higher than the loss in market demand in its own market and therefore the PTA between $B$ and $C$ cannot be stable anymore. In contrast to Proposition 4.2 we make the following observation:

**Lemma 4.2.** The global network where country $B$ and $C$ have a PTA cannot be stable. A single PTA between country $B$ and $C$ cannot be stable.

To get an idea of the result, consider country $B$ and $C$ being involved in a PTA. Intuitively country $A$ and country $B$ may have an incentive to deviate by forming an additional PTA. As country $B$ is the central player, this seems plausible, but why does country $A$ benefit? The argument is similar as in Lemma 4.1: Due to increased competition in the domestic market, consumer surplus in $A$ increases. Since we consider the case of a large market $B$, firm $A$ obtains access to the large market of country $B$ and therefore the additional firm’s profit in $B$ is higher than the reduced profit in $A$’s domestic market. This intuition can as well be verified in Figure 4 which shows that country $A$ obtains a Consumer surplus of 32 on the right side, whereas without a PTA country $A$’s surplus is $\frac{1}{2}(\frac{31}{4})^2$. The additional profit in market $B$ is given by $(\frac{15}{4})^2 - 9 = 5,0625$. The result is in line with Ethier (2004), who addressed the question of why small countries tend to form trade agreements with large market countries. However, what are the stable states in this case?

We obtain a complete characterization of all possible stable states:

**Proposition 4.3.** For values $t < \frac{\alpha_A - c}{3}$ the global network with a PTA between every pair of players and the complete network are the unique stable states.
Figure 4: Demand function in country A; example with $\alpha_A = 12$, $c = t = 1$. Left side: global link with PTA between $B$ and $C$. $\pi_A^C = \pi_A^B = (\frac{9}{4})^2$ and $\pi_A^A = (\frac{13}{4})^2$. Right side: The global network with a star network with center player $B$. Here we obtain that $\pi_A^A = \pi_A^B = 9$ and $\pi_A^C = 4$.

The result differs from Proposition 4.2, where we obtain a large variety of possible stable structures. Lemma 4.2 already shows that a PTA between $B$ and $C$ cannot be stable anymore. The reason that a PTA between country $A$ and country $B$ (respectively $C$) is not stable is that $B$ and $C$ have an incentive to form a PTA, since $Y_C(L^B_C) > Y_C(\{A, B\})$ and $Y_B(L^B_C) > Y_B(\{A, B\})$. The sum of the increase of consumer surplus in country $C$ and an increase of firm $C$’s profit generated in market $B$ exceeds the decrease of firm $C$’s profit in its domestic market. Hence the overall welfare effect on country $C$ is positive. A similar argument explains why a global network with a PTA between $A$ and $B$ is not stable either.

4.4 Asymmetric Case

In the following we will describe that under total asymmetry with $\alpha_A > \alpha_B > \alpha_C$ it is much more complicated to analyze stable structures. To understand the difficulty in characterizing stable network structures we provide some examples.

Consider first the empty network and a pair of players $i$ and $j$ considers forming a bilateral link. Therefore it has to be fulfilled that $Y_i(L) - Y_i(\{i, j\}) = \frac{3}{8}(\alpha_i - c)^2 - \frac{1}{3}(\alpha_i - c)^2 - \frac{1}{9}(\alpha_j - c)^2 < 0$ and $Y_j(L) - Y_j(\{i, j\}) = \frac{3}{8}(\alpha_j - c)^2 - \frac{1}{3}(\alpha_j - c)^2 - \frac{1}{9}(\alpha_i - c)^2 < 0$. Since
\[ \alpha_i \neq \alpha_j \forall i \neq j \] we obtain that for the smaller market, let's say market \( j \), will always want to form a bilateral link whereas \( i \) only deviates if \( \frac{(\alpha_i - c)^2}{24} > \frac{(\alpha_j - c)^2}{9} \). The underlying intuition is that for market \( i \) the additional profit made in market \( j \) would be too small when compared to the profit loss in the home market.

We can exclude a set of structures that can never be stable. One structure that cannot be stable is a single PTA between \( B \) and \( A \). To see this consider that the small country \( C \) always has an incentive to form a PTA with a larger market country whereas the larger market always has an incentive to be the center player in a star network. The same counts for a PTA between \( C \) and \( A \), since \( B \) will want to form a PTA with \( A \). We can further exclude each star network, because for both edge players we have:

One stable network is the complete network since, as shown above, no star network can be stable, and since the global network with a PTA between each pair of players levies the same payoffs to each player. We can further elaborate that the global network with a PTA between each pair of players is not necessarily stable, since player \( A \) can increase his welfare by deleting his bilateral link with \( C \). Therefore we obtain:

**Proposition 4.4.** The complete network is stable.

We will present the complete proof in the appendix.

To see why the global network with a PTA between each pair of players is not necessarily stable, consider that player \( i \) might have an incentive to delete his bilateral link to any player \( j \) with \( \alpha_i > \alpha_j \) if \( Y_i(L^G \cup L^N) - Y_i(L^G \cup L^S_k) < 0 \) which is equivalent to \( 2 \cdot t(\alpha_i - c) > 4 \cdot t(\alpha_j - c) + \frac{t^2}{3} \). Therefore we cannot guarantee the stability of the global network which has a PTA between each pair of players. Intuitively we can argue again, that a PTA with a smaller market country might increase country \( i \)'s firm profit in the foreign market less than the reduction of firm \( i \)'s profit in the home market. And this will induce country \( i \) to delete the link with market \( j \).

### 4.5 Efficiency

We next examine the structure of the efficient network. We thus have to evaluate total welfare of different trading structures. With \( t < \frac{\alpha - c}{3} \) we obtain the result.

**Proposition 4.5.** For any parameter values of \( \alpha_i \) and \( c \) the global network with a PTA between every pair of players is an efficient structure. The complete network is efficient.

Proposition 4.5 can easily be verified if we compare different welfare levels as calculated in Appendix B. We can elaborate that total welfare in the complete network is
always larger than in any arbitrary network. We obtain two different trading structures that maximize the overall welfare level.

In contrast to stability there are just two efficient networks that produce the same total output. Hence, we observe a conflict between efficiency and stability with Proposition 4.2, whereas there is no conflict when all countries are symmetric.

5 Generalizations

The results from above motivate an examination of a more generalize setting. We generalize the model into three directions. First we allow for a more general social welfare measure with arbitrary weights on consumer surplus, profits and tariff revenue. Second we extend the model such that the tariffs are chosen exogenous by the countries. Third we want to give some implications on what is going to happen in the situation of countries with different market size, when we allow an arbitrary number of countries to form bilateral links.

5.1 Generalized Social Welfare Function

First we want to allow arbitrary weights on consumer surplus, firms’ profit and tariff revenue and define a more general social welfare function from (9) with:

\[ Y_i(L) = \beta \left( \frac{1}{2} q_i^2(L) \right) + \gamma \left( \sum_{j \in N_i(L)} (\alpha_j - c - q_j(L) - t^i_j(L) \cdot q_j^i(L)) \right) + \delta \left( \sum_{j \in N_i(L)} t^i_j(L) \cdot q_j^i(L) \right). \] (10)

In the framework of section 4 the welfare function places equal weight on profit, consumer surplus and tariff revenue with \( \beta = \gamma = \delta = 1 \). In a political economy context we might be interested in what structures will emerge, when the objective function depends only on firm’ profit which implies \( \gamma = 1 \) and \( \beta = \delta = 0 \). Henceforth we obtain for social welfare:

\[ Y_i(L) = \sum_{j \in N_i(L)} (\alpha_j - c - q_j(L) - t^i_j(L)) \cdot q_j^i(L). \] (11)

One observation that can be made is that firm j’s profit in market i decreases with the number of firms that are active in market i. Let’s assume i and j have a PTA and market i form a PTA with k. This reduces country j’s welfare since welfare is given exclusively by firm profit. This observation provides the intuition for the next result.
Proposition 5.1. When countries only care about producer profit, the only stable networks are the complete and the empty network.

First we investigate the welfare in the empty network. Firm $i$’s profit is given by $\frac{(\alpha - c)^2}{4}$ since it only supplies in market $i$. When countries only care about producer profit it can be shown that starting with an empty network no set of players wants to form a link. This suggests that the empty network is stable.

What can also be observed is that under the complete network no country $i, i \in N$ is going to delete any of his links with player $j, j \neq i \in N$, since the reduction of profits in market $j$ is higher than the additional profit obtained due to lower competition in its own market. A complete proof is provided in the Appendix.

Next we investigate what structures can emerge, when welfare is given by consumer surplus such that $\beta = 1, \gamma = \delta = 0$. Country $i$’s welfare is now:

$$Y_i(L) = \frac{1}{2}\left[\sum_{j \in N \setminus \{i\}} \left(\frac{(\alpha - c)}{\eta_i(L) + 1} + \frac{\sum_k \tilde{t}_{ij}(L) - (\eta_i(L) + 1) \cdot t_{ij}}{\eta_i(L) + 1}\right)^2\right].$$  \hspace{1cm} (12)

When we consider network without the global link we obtain that for an arbitrary network $L$ consumer surplus from an additional PTA between country $i$ and country $j$ with $\{(i, j)\} \notin L$ is given by

$$Y_i(L \cup \{(i, j)\}) - Y_i(L) = \frac{1}{2}\left[\sum_{j \in N \setminus \{i\}} \left(\frac{(\alpha - c)}{\eta_i(L) + 2} - \frac{(\eta_i(L) + 1)^2 (\alpha - c)^2}{(\eta_i(L) + 1)^2}\right) > 0,$$

since $t_{ij} = T$ for any pair of players without trade agreement and $t_{ij} = 0$ with a PTA between $i$ and $j$. This implies that an additional bilateral link is always profitable and countries form as many links as possible.

Under GATT regime we obtain for social welfare:

$$Y_i(L) = \frac{1}{2}\left(\frac{N(\alpha - c)}{N + 1} - \frac{(N - \tilde{N}_i(L)) \cdot t_i}{(N + 1)^2}\right).$$  \hspace{1cm} (13)

The first derivative implies: $\frac{\partial Y_i}{\partial \tilde{N}_i(L)} = \frac{N(\alpha - c)}{(N + 1)} - \frac{(N - \tilde{N}_i(L)) \cdot t_i}{(N + 1)^2} > 0$ with $t < \frac{(\alpha - c)}{3}$. Countries under GATT regime always want to form as many PTA’s as possible. This proves the next result.

Proposition 5.2. When social welfare is given by consumer surplus, the only stable networks are the complete network and the global network with a PTA between each pair of players.
In the following we will assume that countries’ welfare is given by tariff revenue such that
\[ Y_i(L) = \sum_{j \in N_i(L)} t^j_i(L) \cdot q^j_i(L). \]  
(14)

Since the tariffs between players in a PTA are zero and between two players in a multilateral link is \( t \), with \( t > 0 \), we have
\[ Y_i(L) = (\eta_i(L) - N_i(L)) \cdot t \left( \frac{(\eta_i(L) - N_i(L))}{\eta_i(L)+1} - (\eta_i(L)+1) \cdot t \right), \]
where \( N_i(L) \) denotes the number of players that have a PTA with country \( i \), \( i \in N_i(L) \) in network \( (L) \). We observe that \( \frac{\partial Y_i(L)}{\partial N_i(L)} < 0 \) such that with the number of bilateral links the welfare decreases. It is therefore intuitive that the global network maximizes welfare and a proof can be omitted. The deletion of the global link results in the empty network and tariff revenue for each country is zero. Under these considerations we can conclude:

**Proposition 5.3.** When countries only care about tariff revenue, the only stable network is the global network.

Next we want to combine the analysis and allow arbitrary values for \( \beta, \gamma \) and \( \delta \). We obtain that under general welfare function as given in (10):

**Proposition 5.4.** Under general welfare as given by (10) the empty network, the global network and the global network with a PTA between each pair of countries can be stable.

We can proof the result by excluding step by step all the other asymmetric networks. Intuitively, for certain value of \( \delta \) countries tend to keep as few PTA as possible and maintain the global link to obtain as many tariffs as possible. The lower \( \delta \) and the higher \( \gamma \) countries prefer no links at all, since the additional competition will decrease firm’s profit and therefore countries will delete all their links. Intuitively, for high values of \( \beta \) additional competition in the markets is profitable for consumer surplus and therefore countries will form the complete network.

**5.2 Stability of Trading Structures and Endogenous Tariffs**

In the above section we considered the case in which tariffs in a PTA and the GATT tariff level are exogenously given. In reality countries negotiate tariffs and due to the MFN clause a country within the GATT levies the same tariffs on each GATT member. In the absence of the GATT agreement, each country non-cooperatively chooses its welfare maximizing bilateral tariff.

We introduce the following setting: countries choose their non-cooperative and welfare
maximizing Nash tariff level with respect to the foreign countries’ tariff levels. The optimal tariff can be different for each trading structure. Based on the optimal tariffs, countries obtain a welfare level which is calculated by plugging in the optimal quantities that the firms supply in each market. Comparison of different welfare levels given the non-cooperative tariffs results in possible stable trading structures.

Based on the linking decision, when a global link exists, countries choose their tariffs non-cooperatively on the other countries with \( t^i_j(L) = t^k_j(L) = t_j(L) \) due to the MFN clause, and bilateral tariffs are set to zero with respect to Article XXIV. Whereas in a network without a global link (and therefore no GATT rules) countries choose their optimal bilateral tariffs non-cooperatively.

We now investigate the following questions: what are the optimal tariffs, when countries choose their GATT tariffs with respect to maximizing welfare level, and what are the stable trading structures? Moreover, do countries raise tariffs against third countries as they form additional PTAs? Throughout the analysis we assume that countries are symmetric with respect to market size such that \( \alpha_i = \alpha \ \forall i \in N \).

As per Article I of the GATT we impose for the tariff of the global link:

\[
t^i_j(L) = t^k_j(L) = t_j(L), \ \forall i, k, \neq j,
\]

where \( t_j(L) \) is the tariff that country \( j \) imposes on foreign firms in network \( L \). When countries have a PTA within the GATT agreement, they have zero tariffs due to Article XXIV. For social welfare we obtain:

\[
Y_i(L) = \frac{1}{2} \left( \frac{N \cdot (\alpha - c) - (N - \tilde{N}_i(L)) \cdot t_i(L)}{(N + 1)} \right)^2 + \sum_{j \in \tilde{N}_i(L)} \left( \frac{(\alpha - c)}{(N + 1)} - \frac{(\tilde{N}_j(L) + 1) \cdot t_j}{(N + 1)} \right)^2
\]

\[
+ \sum_{k \in \tilde{N}_i(L)} \left( \frac{(\alpha - c) + (N - \tilde{N}_k(L)) \cdot t_k}{(N + 1)} \right)^2 + (N - \tilde{N}_i(L)) \cdot t_i(L) \left( \frac{(\alpha - c) - (\tilde{N}_i(L) + 1) \cdot t_i(L)}{N + 1} \right)
\]

whereas \( \tilde{N}_i(L) \) denotes the number of players that have a PTA with country \( i \) in network \( L \), with \( i \in \tilde{N}_i(L) \). We obtain country \( i \)’s optimal tariff with:

\[
t_i(L) = \frac{3(\alpha - c)}{(11 \cdot \tilde{N}_i(L) - 1)}
\]

We observe that the tariffs on third countries within the GATT decrease with the number of PTAs that country \( i \) has, which contradicts the result of Krishna (1998), who suggests
that PTAs lower countries’ incentives for multilateral liberalization. We will provide an explanation for this result later in the section.

When there is no GATT agreement, countries non-cooperatively choose an external tariff to levy on those countries with whom they are linked bilaterally. We obtain social welfare with

\[ Y_i(L) = \sum_{j \in N_i(L)} t_{ij}(L) \cdot \left( \frac{(\alpha - c)}{\eta_i(L) + 1} \right) + \sum_{k \in N_i(L)} t_{ik}(L) \cdot \left( \frac{(\alpha - c)}{\eta_i(L) + 1} \right) - t_i(L) \]

and therefore country \( i \)'s optimal bilateral tariffs on the other countries with whom they are linked bilaterally, whereas we obtain that due to country symmetry \( t_{ij}(L) = t_{ki}(L) = t_i(L) \) \( \forall j, k \in N_i(L) \):

\[ t^*_i(L) = \frac{3(\alpha - c)}{7 + \eta_i(L)} \]

This implies that when the number of bilateral trading partners increases for country \( i \) increases, the tariffs levied on these countries decreases.

With the optimal tariffs we are able to calculate welfare level for different trading structures and obtain a complete characterization of stable networks:

**Proposition 5.5.** The global network with a PTA between each pair of countries is stable. Each structure with a global link and a PTA between one pair of countries is stable.

Hence the complete network cannot be stable anymore. The complete proof of the result is shown in the appendix. We first calculate welfare level for each structure and then compare them with respect to our stability notion to obtain the stable structures.

Here we want to offer a short intuition for the result. First we investigate why the global link with a PTA between two countries can be stable, consider tariffs of each country within the global structure: \( t_i(L^G) = \frac{3(\alpha - c)}{10} > \frac{(\alpha - c)}{7} = t_i(L^G \cup \{B, C\}) \), \( i = B, C \), which is larger than a global link with an additional PTA between, say, \( B \) and \( C \). Tariffs on \( A \) decrease as \( B \) and \( C \) form a bilateral link. The negative relation between the number of PTAs and the tariffs on third countries is due to different effects that an increase in tariffs has on the welfare level. First we observe that with a higher number of PTAs an increase of tariffs generates a higher loss in consumer surplus. That means \( B \) and \( C \) prefer a lower \( t_B \) so that the loss in consumer surplus is lower. To see this we obtain the first derivative
of country B’s welfare with respect to tariffs:

\[
\frac{\partial Y_i(L)}{\partial t_i(L)} = - \frac{N - \tilde{N}_i(L)}{N + 1} \left[ N(\alpha - c) - (N - \tilde{N}_i(L))t_i(L) \right] \\
+ \frac{N - \tilde{N}_i(L)}{N + 1} \left[ 2(\alpha - c) + 2(N - \tilde{N}_i(L))t_i(L) \right] \\
+ \frac{N - \tilde{N}_i(L)}{N + 1} \left[ (\alpha - c) - 2(\tilde{N}_i(L) + 1)t_i(L) \right]
\]

Another effect of an increase in tariffs is on firm B’s profit in its own market. The positive effect of a rise in tariffs is higher, when the number of PTAs is lower, so that in the global network, countries tend more to high tariffs. This can be seen in the second line of the above equation. Thus, when B and C form an additional PTA, they have less incentives to raise tariffs on A.

When we calculate the impact of a PTA between B and C on the welfare level of A, we can observe that due to lower external tariffs, country A’s welfare increases.

\[
Y_A(L^N \cup \{B, C\}) - Y_A(L^N) = 2 \cdot \left( \frac{\alpha - c}{4} - \frac{3}{4}(\alpha - c) \right)^2 - 2 \cdot \left( \frac{\alpha - c}{4} - \frac{3}{4}(\alpha - c) \right)^2 > 0.
\]

We can observe that due to decreasing tariffs of B and C on A, firm A’s profit in market B and C increases. Thus the overall effect is positive.

We can also observe that country B and C improve by forming the bilateral link, since the free entry to the foreign market leads to an increase of both countries’ payoffs.

The reduction of tariffs on country A and the resulting increase in welfare induces country A to maintain its global link with B and C but an additional bilateral PTA with either of the two countries would reduce its welfare.

The complete network cannot be stable, since all countries are going to deviate and form a global link, because in the global network with a PTA between each pair of countries the GATT agreement reduces tariffs to zero such that all countries are better off and increase welfare.

Without the GATT agreement countries choose non-cooperatively external tariffs on the other countries and this leads to a mutual reduction of welfare, whereas the GATT stabilizes the underlying structure and Article XXIV leads to an increase in each country’s welfare, and due to the MFN clause countries have to offer each member the same tariffs.

The reduction of tariffs against third countries, after a PTA is in place, suggests that PTAs increase countries’ incentives for multilateral liberalization.

We can furthermore obtain a result on efficiency.
Proposition 5.6. The unique efficient network is the global network with a PTA between each pair of players.

For any network $L$ we can calculate the overall level under optimal tariffs $t^*_i(L)$. We can observe that the complete network is no longer efficient because tariffs in the complete network $t^*_i(L^N) = \frac{3(\alpha-c)}{10}$ against foreign firms lead to a reduction of firm profits and consumer surplus. The payoff for each country in a complete network is given by $\frac{21}{52}(\alpha-c)^2$ whereas in the efficient network total payoff for each country is given by $\frac{15}{32}(\alpha-c)^2$. With non-cooperative tariffs countries mutually reduce their welfare level.

5.3 Arbitrary number of countries

Let’s provide an insight into possible implications for stable network structures when we increase the number of countries and let’s assume that countries are symmetric with respect to market size. In this framework we allow the total set of players to form a multilateral link $L = \{1, ..., n\}$ and each pair of players to form a bilateral link. We further assume endogenous tariffs as calculated in Chapter 5.2 for an arbitrary number of countries.

We want to learn something about the implications under endogenous tariffs with respect to tariffs on third countries. Can the complete network still be stable or do endogenous tariffs induce that a multilateral link is essential for stability (cf. Proposition 5.5)?

With equation (14) we could observe that tariff on third countries decrease within GATT, when the number of PTA increases, whereas equation (15) suggests that an additional bilateral trade agreement between a pair of countries lowers tariffs on third country trading partner. These observation help to understand the implications for the general case with $n \geq 3$.

The network with a single multilateral link is again the global network with $L^G = \{\{1,2, ..., n\}\}$. First we show that the global network with a PTA between each pair of countries with an arbitrary number of countries is a stable state and that the complete network can not be stable.

Proposition 5.7. For an arbitrary number of countries, a global link with a bilateral PTA between each pair of players is a stable network structure.

The prove proceeds in the way that it first demonstrates that starting from the stable network a player decreases welfare under GATT regime when he deletes any of his bilateral links and second it shows that all players are worse off without GATT in the complete network and this completes the proof. Therefore it is obvious that starting from a complete
network all players want to form a multilateral link and the complete network is not stable. We can further conclude that for an arbitrary number of players neither the empty nor the global network can be stable. First we show that the empty network can not be stable since a pair of players will deviate and form a bilateral link with:

\[
Y_i(\{\{i, j\}\}) - Y_i(\mathcal{L}^e) = \frac{1}{2} \left( \frac{2(\alpha - c) - \frac{3}{2}(\alpha - c)}{3} \right)^2 \\
+ \left( \frac{\alpha - c + \frac{3}{2}(\alpha - c)}{3} \right)^2 + \left( \frac{\alpha - c + \frac{3}{2}(\alpha - c) - (\alpha - c)}{3} \right)^2 + \left( \frac{\alpha - c + \frac{3}{2}(\alpha - c)}{3} \right)^2 \\
+ \left( \frac{3(\alpha - c)}{9} \right) \left( \frac{\alpha - c + \frac{3}{2}(\alpha - c) - (\alpha - c)}{3} \right) - \frac{3}{8} (\alpha - c)^2 > 0.
\]

Two countries obtain a higher payoff when they form a trade agreement. Under GATT without any PTAs it can be shown that players have an incentive to delete the global link since:

\[
Y_i(\mathcal{L}^G \cup \{\{i, j\}\}) - Y_i(\mathcal{L}^G) = \frac{1}{2} \cdot \frac{N \cdot (\alpha - c) - (N - 2) \cdot \left( \frac{(\alpha - c)}{7} \right)}{(N + 1)}^2 + (N - 2) \cdot \left( \frac{\alpha - c - 2 \cdot \frac{3(\alpha - c)}{10}}{(N + 1)} \right)^2 \\
+ 2 \cdot \left( \frac{\alpha - c + \frac{3}{2}(\alpha - c)}{(N + 1)} \right)^2 + (N - 2) \left( \frac{(\alpha - c) - \left( \frac{3(\alpha - c)}{7} \right)}{(N + 1)} \right) \\
- \frac{1}{2} \cdot \frac{N(\alpha - c) - (N - 1) \frac{3}{10}(\alpha - c)}{(N + 1)}^2 - (N - 1) \left( \frac{\alpha - c - \frac{3}{5}(\alpha - c)}{(N + 1)} \right)^2 \\
- \left( \frac{\alpha - c + (N - 1) \frac{3}{10}(\alpha - c)}{(N + 1)} \right)^2 - (N - 1) \frac{3}{10} \left( \frac{\alpha - c + \frac{3}{5}(\alpha - c)}{(N + 1)} \right) > 0.
\]

When countries are linked multilaterally they tend to form additional PTAs and a global network without PTAs cannot be stable.

It is difficult to characterize the full set of stable networks but we can observe two more features to narrow the set of possible stable states:

- Under GATT the network in which each pair of players has a bilateral PTA is the unique stable network in the class of symmetric networks.

- Without GATT there exists no stable network in the class of symmetric networks.

Recall that we denote a network to be symmetric if each player has the same number of bilateral links such that \( \eta_i = \eta_j \) \( \forall i, j \in N \).

The proof of the first observation is similar to the proof of Proposition 8 in Goyal and Joshi (2006) and can be omitted. The proof of the second observation is shown in Appendix B.
and proceeds in the way that it shows that an additional PTA among two players always increases welfare for both countries, when the number of existing PTAs is equal in both countries. With this in mind we can conclude:

**Proposition 5.8.** *In the class of symmetric networks the unique stable network is a global link with a bilateral PTA between each pair of players.*

This of course confirms the observation that neither the empty nor the global network can be stable.

6 Conclusion

What structures will emerge, when countries have the opportunity to form multilateral and bilateral trade agreements? Will the increasing number of PTAs lead to a more open multilateral trading system, when we consider strategic link formation of countries? We used a network formation approach to answer these questions and introduced the notion of multilateral stability to investigate trading structures, where players can form multilateral as well as bilateral links.

The idea of multilateral stability is that players can form bilateral links and hyperlinks, which include more than two players. The formation of any of these links needs the consent of all players included but deletion can be done unilaterally.

We used a three-country model of imperfect competition, with a single firm in each country producing a homogenous good. Each firm competes as a Cournot oligopolist in each market and markets in the different countries are assumed to be perfectly segmented. Welfare gains from trade are obtained by increased competition in the countries. We have shown that if countries are asymmetric with respect to market size, a complete network with a multilateral link is multilaterally stable. This result suggests that PTAs can coexist with multilateral liberalization. We investigate different cases with respect to market size, to analyze what effect heterogeneity can have on countries’ linking strategies. However, the complete global free trade network is always an efficient network. When we introduce endogenous tariffs, we find that the complete network in which each pair of players is linked bilaterally cannot be stable, since countries choose their tariffs non-cooperatively whereas due to Article XXIV of the GATT countries have to reduce their tariffs to zero. This increases countries’ welfare level and the GATT agreement leads to stability.

Although this is a simple three-country setting, we think that it provides valuable insights in countries’ decisions to form trade agreements. There are three different welfare effects
that a trade agreement has on an importing country and by investigating the welfare effects of different trading agreements, we can obtain a full description of stable states. We also observe that “concession diversion” due to bilateral PTAs occurs and that it influences third countries’ linking decisions with respect to multilateralism.

References


A Appendix

First we report the welfare levels of countries $i, j, k \in \{A, B, C\}$ with $i \neq j \neq k$ and overall welfare level of different trading structures.

The empty network

$$Y_i(\mathcal{L}^e) = \frac{3(\alpha_i - c)^2}{8}$$

The star network with center player $i$

$$Y_i(\mathcal{L}^S_i) = \frac{11(\alpha_i - c)^2}{32} + \frac{3(\alpha_i - c)}{16} + \frac{(\alpha_k - c)^2}{16}$$

$$Y_j(\mathcal{L}^S_i) = \frac{(\alpha_j - c)^2}{3} + \frac{(\alpha_i - c)^2}{16}$$

$$Y_k(\mathcal{L}^S_i) = \frac{(\alpha_k - c)^2}{3} + \frac{(\alpha_i - c)^2}{16}$$

The global network without PTAs

$$Y_i(\mathcal{L}^G) = \frac{11(\alpha_i - c)^2}{32} + \frac{3(\alpha_i - c)}{8} + \frac{(\alpha_j - c)^2}{16} + \frac{(\alpha_k - c)^2}{16} - \frac{(\alpha_j - c)\cdot t}{4} - \frac{(\alpha_k - c)\cdot t}{4} - \frac{t^2}{8}$$

The complete network

$$Y_i(\mathcal{L}^N) = \frac{11(\alpha_i - c)^2}{32} + \frac{(\alpha_j - c)^2}{16} + \frac{(\alpha_k - c)^2}{16}$$
PTA between \( i \) and \( j \)

\[
Y_i(\{(i, j)\}) = \frac{(\alpha_i - c)^2}{3} + \frac{(\alpha_j - c)^2}{9} \quad Y_j(\{(i, j)\}) = \frac{(\alpha_j - c)^2}{3} + \frac{(\alpha_i - c)^2}{9}
\]

\[
Y_k(\{(i, j)\}) = 3 \frac{(\alpha_k - c)^2}{8}
\]

Global network with a PTA between \( i \) and \( j \)

\[
Y_k(\mathcal{L}^G \cup \{(i, j)\}) = \frac{(\alpha_i - c)^2}{16} + \frac{(\alpha_j - c)^2}{16} + \frac{11(\alpha_k - c)^2}{32} + \frac{3(\alpha_k - c) - t}{8} - \frac{3(\alpha_i - c)t}{8} - \frac{3(\alpha_j - c)t}{8} + \frac{t^2}{8}
\]

\[
Y_j(\mathcal{L}^G \cup \{(i, j)\}) = \frac{11(\alpha_j - c)^2}{32} + \frac{(\alpha_i - c)^2}{16} + \frac{(\alpha_k - c)^2}{16} - \frac{(\alpha_j - c)t}{4} + \frac{4(\alpha_i - c)t}{32} - \frac{8(\alpha_k - c)t}{32} - \frac{11t^2}{32}
\]

Global network with a star network with center player \( i \)

\[
Y_i(\mathcal{L}^G \cup \mathcal{L}^S_i) = \frac{(\alpha_i - c)^2}{16} + \frac{(\alpha_k - c)^2}{16} + \frac{11(\alpha_i - c)^2}{32} + \frac{2(\alpha_i - c) - t}{16} + \frac{2(\alpha_k - c)t}{16} + \frac{4t^2}{32}
\]

\[
Y_j(\mathcal{L}^G \cup \mathcal{L}^S_i) = \frac{11(\alpha_j - c)^2}{32} + \frac{(\alpha_i - c)^2}{16} + \frac{(\alpha_k - c)^2}{16} + \frac{6(\alpha_k - c)t}{32} - \frac{3(\alpha_i - c)t}{8} - \frac{3t^2}{32}
\]

\[
Y_k(\mathcal{L}^G \cup \mathcal{L}^S_i) = \frac{11(\alpha_k - c)^2}{32} + \frac{(\alpha_i - c)^2}{16} + \frac{(\alpha_j - c)^2}{16} + \frac{3(\alpha_k - c)t}{16} - \frac{3(\alpha_i - c)t}{8} - \frac{3t^2}{32}
\]

B Appendix

Proof of Proposition 4.1

For the proof of Proposition 4.1 consider that in this case \( \alpha_i = \alpha \ \forall i \). We obtain with countries’ welfare levels as calculated in Appendix A:

The empty network cannot be stable, since each pair of players has an incentive to deviate and with

\[
Y_i(\mathcal{L}^e) = \frac{3(\alpha_i - c)^2}{8} < Y_i(\{(i, j)\}) = \frac{4(\alpha_i - c)^2}{9} < \frac{163(\alpha_i - c)^2}{288} = Y_i(\{(i, j), (i, k)\})
\]

and with

\[
Y_k(\{(i, j), (i, k)\}) > Y_k(\{(i, j)\}) \quad \forall i, j, k \ i \neq j \neq k
\]

the star with center player \( i \) is formed.

The complete network is formed since

\[
Y_k(\{(i, j), \{i, k\}\}) < Y_k(\mathcal{L}^N) = \frac{15(\alpha_i - c)^2}{32} \quad \text{and}
\]

\[
Y_j(\mathcal{L}^N) = \frac{15(\alpha_i - c)^2}{32}
\]

The global network with a PTA between each pair of players each country obtains the same payoff as in the complete network and it is stable since no player wants to delete any of his bilateral links. Thus the complete network and the global network with a PTA between each pair of players are stable. To show that the global link with a PTA between \( B \) and \( C \) is not stable consider that player \( A \) does not want to form an additional PTA with either \( B \) or \( C \) if \( 19t > 4(\alpha - c) \). Furthermore for
player A to keep the global link requires $3(\alpha - c)^2 + 16t^2 > 12t(\alpha - c)$. And for player B (respectively C) to maintain the global link requires: $7(\alpha - c)^2 + 18(\alpha - c)t > 99t^2$ such that $10(\alpha - c)^2 + 6(\alpha - c)t > 83t^2$ has to be fulfilled. But this contradicts the condition that A doesn’t want to form the PTA with B (respectively C) which proofs that the global link with a PTA between B and C is not stable. The global link is not stable since at least one pair of players improves by forming a PTA.

**Proof of Lemma 4.1**

Compare country B’s and country C’s welfare level in a global network with the welfare they obtain when they form an additional bilateral link. To prove that the global network is not stable we have to show that $Y_i(L^G) < Y_i(L^G \cup \{i,B\})$ for all $i \in \{B, C\}$. This induces:

$$\frac{4(\alpha - c) \cdot t}{32} + \frac{11 \cdot t^2}{32} - \frac{10(\alpha - c) \cdot t}{32} - \frac{4 \cdot t^2}{32} < 0 \quad \iff \quad 7 \cdot t < 6(\alpha - c),$$

since $\left(\frac{\alpha - c}{3}\right) > t$ this equation is always fulfilled. Hence B and C have an incentive to deviate and form an additional PTA.

**Proof of Proposition 4.2**

As shown in Lemma 4.2 a global network cannot be stable. The empty network is not stable since, as in the global structure, country B and C have again an inventive to deviate and form a PTA with:

$$Y_B(L^e) = Y_C(L^e) = \frac{3(\alpha - c)^2}{8} < \frac{4(\alpha - c)^2}{9} = Y_B(\{B, C\}) = Y_C(\{B, C\}).$$

Country A will not form an additional PTA with B if

$$Y_A(\{B, C\}) > Y_A(\{B, C\}, \{A, B\}) \quad \iff \quad 4(\alpha_A - c)^2 > 6(\alpha - c)^2,$$

whereas A will not want to form a global link with players C and B if:

$$Y_A(\{B, C\}) > Y_A(L^G \cup \{A, B\}) \quad \iff \quad 12(\alpha_A - c) \cdot t + 4(\alpha - c)^2 + 16 \cdot t^2 < 24(\alpha - c) \cdot t + (\alpha_A - c)^2.$$

If the reverse is true, then country A would like to form the global link. Under this condition the global network with a PTA between B and C can be stable.

If A also wants to form a PTA with B, that is $4(\alpha_A - c)^2 < 6(\alpha - c)^2$, A and C will
also form a PTA and the global network with a PTA between every pair of players is reached. This is stable, since no player will want to delete any of his bilateral links and the deletion of \( L = \{A, B, C\} \) will not make any of the players better off. We have \( Y_i(L^N) = Y_i(L^N \cup L^G) \quad \forall \ i. \)

**Proof of Lemma 4.2**

For a PTA between \( B \) and \( C \) to be stable we need the condition that country \( A \) and \( B \) would not want to form an additional PTA. In the following we will demonstrate that both players will gain from an additional PTA between them. With \( \alpha_A < \alpha \) we obtain:

\[
Y_A = \frac{3(\alpha_A - c)^2}{8} < \frac{(\alpha_A - c)^2}{3} + \frac{(\alpha - c)^2}{16},
\]

For country \( B \) we obtain:

\[
Y_B = \frac{4(\alpha - c)^2}{9} < \frac{11(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{9} + \frac{(\alpha - c)^2}{9},
\]

such that \( A \) and \( B \) have an incentive to form an additional PTA.

For the global network with a PTA between \( B \) and \( C \) to be stable, the following four conditions have to be fulfilled:

(i) \( 6(\alpha - c) \cdot t > 15 \cdot t^2 + 12(\alpha_A - c) \cdot t \)

(ii) \( 18(\alpha_A - c)^2 + 90(\alpha - c) \cdot t > 11(\alpha - c)^2 + 72(\alpha_A - c) \cdot t + 99 \cdot t^2 \)

(iii) \( 6(\alpha_A - c) \cdot t + 19 \cdot t^2 > 12(\alpha - c) \cdot t \)

(iv) \( 12(\alpha_A - c) \cdot t + 4(\alpha - c)^2 + 16 \cdot t^2 > 24(\alpha - c) \cdot t + (\alpha_A - c)^2 \)

We obtain from condition (i) and (iii) that:

\[
6(\alpha_A - c) \cdot t + 19 \cdot t^2 > 12(\alpha - c) \cdot t > 30 \cdot t^2 + 24(\alpha_A - c) \cdot t,
\]

which results in a contradiction. Therefore this can not be a stable structure.

**Proof of Proposition 4.3**

We start with an empty network which cannot be stable since \( B \) and \( C \) have an incentive to deviate and form a PTA. As shown in Lemma 4.2 this cannot be stable either and thus an additional PTA between \( B \) and \( A \) is formed. A PTA between \( B \) and \( C \) and \( B \) and \( A \)
can not be stable. We show that $A$ and $C$ have an incentive to form a link:

$$Y_C(\mathcal{L}^N) = \frac{13(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{16} > \frac{19(\alpha - c)^2}{48}$$

and

$$Y_A(\mathcal{L}^N) = \frac{11(\alpha_A - c)^2}{32} + \frac{(\alpha - c)^2}{8} > \frac{(\alpha_A - c)^2}{3} + \frac{(\alpha - c)^2}{16}$$

and thus the complete network is formed, whereas $Y_i(\mathcal{L}^N) = Y_i(\mathcal{L}^N \cup \mathcal{L}^G) \forall i$. None of the players wants to delete one of his links and we thus obtain that this is a stable structure.

The global network cannot be stable as shown in Lemma 4.1. With

$$\frac{(\alpha - c)^2}{3} + \frac{(\alpha_A - c)^2}{9} < Y_B(\mathcal{L}^N_B) = \frac{11(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{9} + \frac{(\alpha - c)^2}{9}$$

and

$$\frac{3(\alpha - c)^2}{8} < Y_C(\mathcal{L}^N_B) = \frac{19(\alpha - c)^2}{48}$$

a PTA between $B$ and $A$ (respectively $C$ and $A$) cannot be stable. This completes the proof.

**Proof of Proposition 4.4**

Condition (ii) of Definition 2.2. is trivially satisfied, since adding the global link makes no player better off. Condition (i) is satisfied, since the deletion of any of the existing links will result in a star network, whereas the payoff for any of the two edge players $i$ is given by $\frac{1}{3}(\alpha_i - c)^2 + \frac{1}{16}(\alpha_j - c)^2$ with center player $j$, which is smaller than $Y_i(\mathcal{L}^N) = \frac{11}{32}(\alpha_A - c)^2 + \frac{1}{16}(\alpha_j - c)^2 + \frac{1}{16}(\alpha_k - c)^2 \forall i \neq j \neq k$ such that the complete network is stable.

**Proof of Proposition 4.5**

Total welfare of the complete network is given by:

$$v(\mathcal{L}^N) = \sum_{i \in N} Y_i(\mathcal{L}^N) = \sum_{i \in N} \frac{1}{2} \left( \frac{(\alpha_i - c)}{N + 1} \right)^2 + \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \left( \frac{(\alpha_j - c)}{(N + 1)} \right)^2.$$  (19)
By comparison, in an arbitrary network the total welfare is given by:

\[
v(\mathcal{L}) = \sum_{i \in N} \left[ \frac{1}{2} \frac{(\alpha_i - c)\eta_i(\mathcal{L})}{\eta_i(\mathcal{L}) + 1} + \left( \sum_{j \in N} \frac{t^i_j(\mathcal{L}) - (\eta_i(\mathcal{L}) + 1)t^i_j(\mathcal{L})}{\eta_i(\mathcal{L}) + 1} \right) \right] \\
+ 2 \sum_{j \in N} \frac{(\alpha_j - c)(\sum_{k \in N} t^i_k(\mathcal{L}) - (\eta_j(\mathcal{L}) + 1)t^i_j(\mathcal{L}))}{\eta_j(\mathcal{L}) + 1} \\
+ \sum_{j \in N(\mathcal{L})} \left[ \left( \frac{(\alpha_j - c)^2}{(\eta_j(\mathcal{L}) + 1)} + \sum_{k \in N} t^i_k(\mathcal{L}) - (\eta_j(\mathcal{L}) + 1)t^i_j(\mathcal{L}) \right) \right] \\
+ \sum_{j \in N(\mathcal{L})} t^i_j \left( \frac{(\alpha_j - c) + \sum_{k \in N} t^i_k(\mathcal{L}) - (\eta_j(\mathcal{L}) + 1)t^i_j(\mathcal{L})}{\eta_j(\mathcal{L}) + 1} \right)
\]

With \( \sum_{i \in N} \sum_{j \in N(\mathcal{L})} (\sum_{k \in N} t^i_k(\mathcal{L}) - (\eta_j(\mathcal{L}) + 1)t^i_j(\mathcal{L})) \leq 0 \) for an arbitrary network \( \mathcal{L} \) we obtain that the complete network maximizes total welfare.

**Proof of Proposition 5.1**

The welfare in the empty network is given by: \( Y_i(\mathcal{L}^e) = \frac{(\alpha - c)^2}{4} \). Addition of a PTA between any pair of countries leaves: \( Y_i(\mathcal{L}^e) - Y_i(\{i, j\}) = \frac{(\alpha - c)^2}{4} - \frac{2(\alpha - c)^2}{9} > 0 \) such that no bilateral link is formed. For the global link we obtain: \( Y_i(\mathcal{L}^e) - Y_i(\mathcal{L}^G) = \frac{(\alpha - c)^2}{4} - \frac{3(\alpha - c)^2}{16} - \frac{3(\alpha - c)^2}{4} > 0 \) with \( t < \frac{(\alpha - c)}{4} \). This implies that the empty network is stable.

Now consider a star network with center player A. Players B and C’s payoffs are given by \( \frac{(\alpha - c)^2}{16} + \frac{2(\alpha - c)^2}{9} \). In the complete network each player obtains a payoff of \( \frac{3(\alpha - c)}{16} \). In the global network with a star network with center player A, country B and C’s payoffs are given by \( \frac{3(\alpha - c)^2}{16} + \frac{5}{8} \cdot t^2 - \frac{t(\alpha - c)}{4} < \frac{3(\alpha - c)^2}{16} \) such that the complete network with a PTA between each pair of players is formed.

A global link with a PTA between a pair of countries is not stable since the country without any bilateral link has an incentive to form a PTA with \( \frac{3(\alpha - c)^2}{16} + \frac{22}{16} \cdot t^2 - \frac{t(\alpha - c)}{2} < \frac{3(\alpha - c)^2}{16} + \frac{5}{8} \cdot t^2 - \frac{t(\alpha - c)}{4} \) since \( t < \frac{(\alpha - c)}{3} \).

**Proof of Proposition 5.5**

In order to calculate each country’s welfare level for a given network structure we can insert the optimal tariff level under GATT (equation(16)) and the optimal tariff level without GATT (equation (18)) into formsular (15) and (17), respectively. With \( t_i(\mathcal{L}^N) = \frac{3(\alpha - c)}{24} \)
we obtain that $Y_i(\mathcal{L}^N) = \frac{24}{49}(a - c)^2$ for all $i$ whereas tariffs are zero in the global link with a PTA between each pair of players and $Y_i(\mathcal{L}^G \cup \mathcal{L}^N) = \frac{15}{32}(a - c)^2$ for all players. Furthermore, we obtain that $Y_i(\mathcal{L}^G \cup \{B, C\}) = \frac{2199}{9800}(a - c)^2$ for each of his PTA’s. With $\alpha$ we obtain that countries in the complete network have an incentive to deviate.

The empty network is not stable since any arbitrary pair of countries has an incentive to form a multilateral link. Therefore the complete network is not stable. With $\delta = 0$ and $\delta_i^2 = T \forall i \neq j$.

$$Y_i(\mathcal{L}^N) = \frac{1}{2} N(a - c) - \frac{(N - 1) \cdot T}{N + 1}^2 + (N - 1) \cdot \frac{(a - c)}{N + 1}^2 + \frac{2 \cdot T}{N + 1} \cdot \frac{(a - c)}{N + 1}^2$$

We can therefore show that

$$Y_i(\mathcal{L}^G \cup \mathcal{L}^N) - Y_i(\mathcal{L}^N) = \frac{1}{2} \frac{(N - 1)^2 \cdot T^2}{(N + 1)^2} + \frac{(N - 1)(a - c) \cdot T}{(N + 1)^2} > 0.$$
The first part of the proof is to show that under GATT regime in a symmetric network two players who share no PTA improve by forming a link. This result can be shown similar to the second part of the proof in Goyal and Joshi (2006) Proposition 8.

We next demonstrate that without GATT two countries also gain by forming a link:

Consider any symmetric network \( \mathcal{L} \) with \( \eta_i(\mathcal{L}) = \eta_j(\mathcal{L}) \) \( \forall i, j \in N \) whereas the link \( L = \{ i, j \} \notin \mathcal{L} \). From consumer surplus we obtain:

\[
\frac{1}{2} \left( \frac{(\eta_i(\mathcal{L}) + 1)(\alpha - c) - (\eta_i(\mathcal{L})) \cdot (\frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) + 1)})}{(\eta_i(\mathcal{L}) + 2)} \right)^2 - \frac{1}{2} \left( \frac{(\eta_i(\mathcal{L}))((\alpha - c) - (\eta_i(\mathcal{L}) - 1) \cdot (\frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) - 1)})}{(\eta_i(\mathcal{L}) + 1)} \right)^2
\]

\[
= 4(\alpha - c)^2 \eta_i(\mathcal{L})^2 + 11 \cdot \eta_i(\mathcal{L}) + 26 \left( \frac{7 + \eta_i(\mathcal{L})}{7 + \eta_i(\mathcal{L})} \right)^2 (8 + \eta_i(\mathcal{L}))^2
\]

(20)

For firm \( i \)'s profit in market \( i \) we have:

\[
\left( \frac{(\alpha - c) + (\eta_i(\mathcal{L})) \cdot (\frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) + 1)})}{(\eta_i(\mathcal{L}) + 2)} \right)^2 - \left( \frac{(\alpha - c) + (\eta_i(\mathcal{L}) - 1) \cdot (\frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) - 1)})}{(\eta_i(\mathcal{L}) + 1)} \right)^2
\]

\[-16(\alpha - c)^2 \left( \frac{15 + 2\eta_i(\mathcal{L})}{(7 + \eta_i(\mathcal{L}))^2 (8 + \eta_i(\mathcal{L}))^2} \right)\]

(21)

The sum of firm \( i \)'s profit in market \( j \) plus the change in tariff revenue is:

\[
\left( \frac{(\alpha - c) - 2 \cdot (\frac{3(\alpha - c)}{7 + \eta_i(\mathcal{L}) + 1})}{(\eta_i(\mathcal{L}) + 2)} \right)^2 + \left( \frac{(\alpha - c) - 2 \cdot (\frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) - 1)})}{\eta_i(\mathcal{L}) + 1} \right) \cdot \frac{3(\alpha - c)}{7 + (\eta_i(\mathcal{L}) + 1)} \cdot \eta_i(\mathcal{L})
\]

(22)

From equation (20), (21) and (22) we obtain:

\[
Y_i(\mathcal{L} \cup \{ i, j \}) - Y_i(\mathcal{L}) = (\alpha - c)^2 \frac{15 + 2\eta_i(\mathcal{L})}{(7 + \eta_i(\mathcal{L}))^2 (8 + \eta_i(\mathcal{L}))^2} > 0 \text{ with } \eta_i(\mathcal{L}) > 0,
\]

hence player \( i \) and \( j \) will deviate and without a multilateral link and under non-cooperative tariffs there exists no stable symmetric network.