Endogenous Product Differentiation and International Competition*

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- Work in Progress -

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Abstract

Firms face competition from international producers. Can they reduce the competitive pressure? We will develop a two stage duopoly model with endogenous product differentiation. In the first stage firms can invest in R&D to differentiate their output from its rivals. In the second stage firms play a quantity game. We will find for the investment game, that there exist situations with no-, multiple- or unique equilibria. Focusing on unique equilibria, we find, that it is an optimal strategy to differentiate the products. Further we show, that if support by a policy maker is possible, it is optimal to subsidize R&D costs if the market expansion effect is small. If the market expansion effect is large the optimal strategy is a tax for a policy maker. This result is at odds with Brander and Spencer (1985).

Keywords: Trade; Subsidies; R&D; Duopoly; Product differentiation;
JEL Classification: F12;F13;F15;L10

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1 Introduction

"The ranking of the top [competitiveness] priorities has remained..., with the low prices continuing to be rated as the lowest priority." LABS 2006

In this paper we will look at two aspects of product differentiation in international markets. Firstly, we will consider the strategic decision of a firm to differentiate its own product. If a firm differentiates the good that it sells, it eases the competitive pressure on its output decision. Secondly, we will explore the possibility of strategic behavior of a policy maker in the investment game when firms compete internationally for a third market. The question we will look at is whether it is optimal for a government to support the respective domestic firm by subsidizing or taxing R&D investment in product differentiation. We show that either a subsidy or a tax is optimal depending on the particular set of circumstances. The latter results from what we call the market expansion effect.

In recent years, firms have faced stronger competition due to international markets. Firms have many possibilities to respond to that increase in competition. One avenue is to reduce costs in order to meet lower prices by competitors. This indeed is an important strategy given the increased importance of offshoring to low costs locations. However, another important strategy is to differentiate the firm’s product from its rivals. In a survey by the City of London, London based firms rated performance strategies which are associated with product differentiation more highly than cost reduction strategies. This is emphasized in table 1, which reproduces the figures in the report. Priorities that are most strongly associated with product differentiation are in bold.

Priorities associated with product differentiation are more highly ranked as cost related priorities, with the exception of marketing. 'Low prices' are actually ranked lowest. Furthermore, manufacturing firms on average rated the differentiation of their product as being more important than other sectors did. The stylized facts presented here support the idea that product differentiation is an important strategy for firms competing in international markets.

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1London Annual Business Survey 2006; page 79.
2or be able to set lower prices to gain a larger market share.
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<th>Priorities</th>
<th>Mean Value</th>
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Priorities range from 1 to 5

Source: London Annual Business Survey 2006; page 79

Table 1: Strategic Priorities of Firms

By differentiating their product, manufacturers distinguish their product from their competitors. It is assumed that consumers value differentiated products. This results in market power for final good producers. In a simple two-country one-firm model each firm is able to invest in differentiating product. With more differentiation, the profits of each firm increases. Hence a firm has an incentive to invest. Furthermore, the investment generates an externality for the competitor. If one firm invests in differentiation, it stops producing the common variety on the market and makes accessible a new part of the market for itself. At the same time, its rival is left with less competition on its variety. This leads to a spillover from R&D investment. We show that in a strategic environment, no firm has the incentive not to invest in product differentiation and free-ride on its rival’s investment.

Further, the incentive to invest depends on the market expansion effect. With a higher degree of product differentiation, consumers are willing to increase their spending in the relevant industry. For example, if a different variety comes into existence, consumers value the new variety. Accordingly, they spend more money in the overall market by redirecting some of their expenditures on other goods. Unlike in the love-of-variety approach where consumers spread their expenditures
over the varieties, as in Krugman (1980), we allow for the expansion of the market. In the current paper, firms can systematically exploit the market expansion effect by investing in product differentiation.

We show that the strategic nature of the investments depends on the degree of market expansion. With a strong market expansion effect, the investments are complements from a strategic point of view. This implies that the investments are reinforcing each other. If one firm does an investment, the increase in the market improves the return on investment of the other firm. Therefore the rival has an incentive to increase its investment. This also indicates that both firms invest in product differentiation and that there is no complete free riding.

With a weak market expansion effect, the investments are strategic substitutes. This implies that the investment of one firm reduces the investment of its rivals. Here the free riding incentive dominates the effect of a larger market. Hence the firms try to exploit the externality of the investment.

In the second part of the paper we look at government intervention. As we will argue, firms compete for the investments in the R&D process. Can a policy maker influence this investment competition in favor of its domestic firm? Similar to Brander and Spencer (1985), we will look at governments that have the option to subsidize or tax the investment in R&D by the domestic firm. By doing so, the policy maker directly influences the decision of the respective domestic firm to invest. For example by offering a subsidy, the domestic firm will increase its investments and thus increase its market power. According to the nature of product differentiation looked at, the subsidy imposes an externality on the investment of the foreign firm.

We show that the optimal strategy depends on the market expansion effect. If the latter is strong, firms enforce each other’s investment. This leads to wasteful investment from the point of view of the policy maker. Hence, a tax of the R&D investment is optimal to reduce the investment. On the other hand, with a weak market expansion effect, the optimal strategy is to subsidize the investment to avoid underinvestment by the firms.

In the trade literature, the concept of product differentiation is widely used. However, to the knowledge of the author, little has been done on endogenous investment on product differentiation. The focus on R&D has been primarily on process R&D. The work that is closest to investment game in the model is by Motta and Polo (1998). In their paper, two firms endogenously choose the degree
of product differentiation. The investment, however, does not lead to an increase in total market size. Accordingly the investment are strategic substitutes, whereas in the current work they can be strategic complements. In a paper by Leahy and Neary (2001), the authors conclude that "...a positive investment subsidy is once again optimal" when looking at investments in market expansion. The primary aim of their paper is to shed light on whether a subsidy is optimal under general conditions. In the work presented here, we show that when looking at product differentiation, the conjecture does not hold if the market expansion effect is strong.

On the trade side, we review the argument by Brander and Spencer (1985): do governments have an incentive to subsidize (or tax) the investment of a domestic firm? As with the latter authors, we only look at the profit of the firm as domestic welfare. Haaland and Kind (2007) remark that neglecting domestic consumers leads to excessive R&D. Yet we use this simplification to focus on the strategic effect of product differentiation.

2 A Simple Model

2.1 Demand

In this section we discuss the underlying utility function and the resulting demand functions. The basic setup is a duopoly with a foreign and a home firm competing for a third market. The home firm is labeled as $i$ whereas $j$ indicates the foreign firm. Consumers view the output of each firm as differentiated. The utility function takes the form

$$U = a(q_i + q_j) - b(1 + \sigma(1 - \theta))(\frac{q_i^2}{2} - \frac{q_j^2}{2}) - b\theta q_i q_j + m$$

where $\theta$ is the degree of horizontal product differentiation. The parameter $m$ summarizes all other products in the economy and $p_m$ is chosen as the numeraire and normalized to one. By choosing this functional form there are no income effects. Consumers optimize their consumption of goods $i$ and $j$ and spend the rest of their income on the numeraire good. A crucial element of using quasilinear preferences is, that with an increasing degree of product differentiation the consumer directs income towards the differentiated sector. The parameter $\sigma$ we call the market expansion effect of product differentiation. With product differentiation the market

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3 This is one of the special cases they look at in their paper. The setup differs in that they use Bertrand competition and firms invest in market expansion.
might expand. For example, consumer increase their expenditures on the market as the product become less similar. However, this effect might vary in magnitude or even disappear fully. We assume that $\sigma \in [0, 1]$, where the upper boundary corresponds to no market expansion effect.

The resulting inverse demand function for good $i$ is

$$p_i = a - b\alpha q_i - b\theta q_j,$$

where $\alpha \equiv (1 + \sigma(1 - \theta))$. For $\theta \in [0, 1]$ the goods are substitutes. A lower $\theta$ is indicating a lower substitutability. For $\theta < 0$ the goods become complements which we ruled out by the above assumption.

### 2.2 Firm Behaviour

The home and the foreign firm are symmetrical, especially with regard to the marginal costs. The demand is given by

$$p_i = \begin{cases} 1 - \alpha b q_i - b\theta q_j & \text{if } \theta \geq 0 \\ 1 - b(1 + \sigma) q_i & \text{if } \theta < 0 \end{cases}$$

where we assumed that with $\theta < 0$ the firms are monopolies. The degree of product differentiation can vary according to $0 \leq \theta \leq 1$. If $\theta$ is close to zero, the goods are highly differentiated and thus the firms are close to the monopoly case. On the other hand with a $\theta$ close to one there is a high degree of substitutability between the goods. The degree of differentiation is determined by the investment of each firm and assumed to be $\theta = 1 - x_1 - x_2$, where $1$ is the current level of product differentiation in the market. If no firm makes an investment the goods are homogeneous. Each firm can make an investment in differentiating the goods from its rival. This investment is costly with a cost function $g_i(x_i) = \gamma x_i^2$. The parameter $\gamma \in [0, 1]$ indicates the efficiency of a firms investment. We will assume that both firms face the same $\gamma$.

The game is one of complete but imperfect information. The structure of the game and the profit functions of each firm are common knowledge. Further, decision become common knowledge as soon as they are implemented. However, at each point in time firms move simultaneously. The timing of the game is as follows: (1) the firms make an investment to differentiate their product; (2) the firms play the Cournot quantity game. The game is solved backwards. At each stage of the
game the firms play subgame perfect strategies. After the firms have chosen their investment the latter is treated as a fixed cost. To circumvent discounting problems, we assume that the fixed costs are appropriately discounted which is incorporated in the investment costs.

In the second stage of the game each firm maximizes (net) profits\(^4\) with respect to the output taking the degree of differentiation as given which results in the following reaction functions

\[ q_i = \begin{cases} \frac{a-c}{2(1+\sigma)} - \frac{\theta}{2\sigma}q_j & \text{if } \theta \geq 0 \\ \frac{a-c}{2(1+\sigma)} & \text{if } \theta < 0. \end{cases} \tag{3} \]

The output response of each firm is increasing in the degree of product differentiation; the higher \(\theta\), the less competitive pressure there is for its good and hence a higher output is set. In the extreme if \(\theta = 0\) each firm will set the monopoly output. However, if both firms engage in a sufficiently high enough investment such that \(\theta < 0\) the output is the monopoly output and hence independent of the other firm. We will focus on the case \(\theta \geq 0\).

To obtain the optimal output of a firm if \(\theta \geq 0\), we note that both firms are symmetric and hence the outputs must be equal. Accordingly we get

\[ q_i = \frac{a-c}{b(2(1+\sigma) + \theta(1-2\sigma))}. \tag{4} \]

Both optimal outputs are increasing in the degree of product differentiation. A higher degree of differentiation gives the firms stronger market power and thus firms set output closer to the monopoly outcome. The optimal outputs lead to the following profit functions

\[ \pi_i = \frac{(1+\sigma(1-\theta))}{b} \left( \frac{a-c}{2(1+\sigma) + \theta(1-2\sigma)} \right)^2 - \gamma x_i^2. \tag{5} \]

In deciding how much to invest in product differentiation a firm invests up to the point where marginal net profits equals marginal costs of investment. Net profits are increasing in the degree of product differentiation. The first order condition for the optimal investment is

\[ \frac{2(1+\sigma(1-\theta))(a-c)^2(1-2\sigma)}{b(2(1+\sigma) + \theta(1-2\sigma))^3} + \frac{\sigma(a-c)^2}{b(2(1+\sigma) + \theta(1-2\sigma))^2} - 2\gamma x_i = 0 \tag{6} \]

which is an implicit reaction function (RF) for the investment of firm \(i\). The reaction function is denoted by \(x_i^*(x_j)\). The shape of the RF depends on the parameter values

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\(^4\)Net profits are profits less the investment costs.
of $c$ and $\gamma$. For example lower investment costs (lower $\gamma$) reduce the marginal cost of investment and therefore making a lower $\theta$ due to lower marginal net profits optimal. There is a negative correlation between $c$ and $\gamma$. To see this assume that $\gamma$ increases. For the FOC to remain constant the marginal cost must fall.

At this point we should recall the timing of the game and which parameter influences the decision of the firm at what point in time. At the output setting stage, the investment is sunk and therefore the decision of the firm is independent of $\gamma$. The quantity a firm sets thus depends on the marginal costs. If the latter are low, the firm can play a more aggressive strategy and set a higher output, given the degree of product differentiation. However, the latter influences the output decision as well (lower $\theta$ means less competition) and in turn is influenced by the first stage investment game. If the investment costs are low, firms make a higher investment in the first stage of the game. Accordingly, the first stage of the game sets the environment the firms operate in in the second stage.

The first order condition is not valid for all values of $(x_i, x_j)$. Imagine the situation where firm $j$ chooses an investment of $x_i + x_j > 1$. In this case it is optimal for firm $i$ to leave its FOC and choose an investment such that $x_i + x_j = 1$, where the latter is the full differentiation line. It is optimal for firm $i$ to do that, because a cumulated investment that exceeds one does not bring any additional gains in terms of product differentiation. Therefore firms anticipate this behavior and the reaction function has a discontinuity. Accordingly we can write the reaction function as

$$R_i = \begin{cases} x_i^*(x_j) & \text{if } x_i - x_j > 1 \\ 1 - x_j & \text{if } x_i + x_j \leq 1 \end{cases} \quad (7)$$

Before discussing the properties of the RF we will investigate whether optimizing profits is a maximization problem. The results will be useful in a later stage of the discussion. We define the FOC to be $F_i(x_i, x_j) \equiv \frac{\partial \Pi_i}{\partial x_i}$. Accordingly, if $\frac{\partial F_i(x_i, x_j)}{\partial x_i} < 0$ holds the profit function is concave and we have a maximum.

**Lemma 1.** The investment $x_i^*(x_j)$ maximizes profits for

1. $\sigma \geq \frac{1}{2}$
2. $\sigma < \frac{1}{2}$ if $\frac{b\gamma}{(a-c)^2} > \frac{(1-2\sigma)(3+\sigma)(1-2\sigma)(1-\theta)}{(2(1+\sigma)+\theta(1-2\sigma))^2}$

**Proof.** To get to the results we rearrange $F'$ such that $\frac{b\gamma}{(a-c)^2} > \frac{(1-2\sigma)(3+\sigma)(1-2\sigma)(1-\theta)}{(2(1+\sigma)+\theta(1-2\sigma))^2}$. First note, that the right hand side is always positive. Second, the denominator of
the left hand side will always be positive as well. Therefore we have to determine
the sign of the numerator. Obviously the latter is 0 if \( \sigma = \frac{1}{2} \). For \( \sigma > \frac{1}{2} \) it is nega-
tive as the first term in the nominator is negative and the second remains positive.
To see the latter we have to examine the second term of the numerator which will
always be positive for the given parameter restrictions. The condition for \( \sigma < \frac{1}{2} \) is
given in the proposition.

To get a better understanding, especially for \( \sigma < \frac{1}{2} \), we will look at the limit case
of \( \sigma = 0 \). In this case, differentiating products results in a full scale expansion of the
market. The condition for a maximum reduces to \( \frac{b_2}{(a-c)^2} > \frac{3}{12+\theta} \). Therefore, we
have a maximum if the productivity of the investment is not too high (a low \( \gamma \)) and if
marginal production costs are not too low. Violation of the former condition would
result in firms investing as much as possible in product differentiation as profits
increase in the latter. This is quite similar to the second condition of reasonably
high marginal costs. If the latter are too low, firms generate high profits which
makes differentiation even more attractive.

On the other hand if \( \sigma \geq \frac{1}{2} \) the market expansion effect is weak. This implies
that firms have a reduced incentive to invest in product differentiation. Accordingly,
firm\s would not engage in over investment. Therefore the investment problem is a
maximization.

**Proposition 1.** The reaction function is upwards sloping for \( \sigma < \frac{1}{2} \) if the profit
function has a maximum. Accordingly the investments are strategic complements.
If \( \sigma = \frac{1}{2} \) then the reaction function is vertical.

A proof of the proposition is found in the appendix. What are the economic
implications for the reaction function? Firstly, with strategic complements\(^5\) a higher
investment by one firm increases the incentive of the other firm to invest. This is the
core result of the investment game. Each firm is exposed to two strategic effects; an
incentive to free-ride on the rivals investment and to increase the size of the market.
In the case under consideration the market expansion effect is strong and dominates
the free-riding incentive. This leads to a mutual reinforcement of the investments.

**Proposition 2.** The reaction function is downwards sloping for \( \sigma < \frac{1}{2} \) if the profit
function has a maximum. Accordingly the investments are strategic substitutes.

\(^5\)The definition we use for strategic complements is found in Bulow et al. (1985), who define
strategic complements (substitutes) as \( \frac{\partial^2 \pi_i}{\partial S_i \partial S_j} > 0(< 0) \) where \( S_i \) is the strategy of firm \( i. \)
Similarly, with strategic substitutes a higher investment by one firm decreases the incentive of the other firm to invest in product differentiation. In this case the free-riding incentive is dominant due to a weak market expansion effect.

![Figure 1: First Order Conditions](image)

To illustrate the previous lemmas graphically we have drawn FOCs for different values of the market expansion effect in figure 1. We see a summary of the previous propositions; depending on the market expansion effect, the investments are strategic substitutes or complements. Additionally what can be seen in the figure, that the total investment is higher with strategic complements than with strategic substitutes. This is implied by the nature of the latter. For example with strategic complements the investment of one firm has a positive effect on the incentive of the other firm which leads to a higher investment.

At this point we should pay attention to the shape of the RF for $\sigma < \frac{1}{2}$. As can be seen from figure 1 the slope of the strategic complements is declining and eventually there is a turning point at which the RF is negatively sloped. This must be because the second order condition for a profit maximum turns positive. This implies, that the profit function is convex at this point and hence we do not look at a maximization problem. This will become important again later when we look at subsidies.

**Lemma 2.** Each firm chooses the same level of investment if a profit maximum exists.
Proof. The proof is relatively straight forward. We have to compute the first order condition for each firm $F_i(x_i, x_j)$ and set $F_i(.) = F_j(.)$. The only term the both FOCs are different is the investment costs. Rearranging the term yields $x_i = x_j$. □

This lemma is a result of the symmetry. It shows that no firm has an incentive to not invest in product differentiation and completely free-ride on the other firm’s investment.

3 Subsidies

In this section we investigate how the decision to invest in product differentiation differs if the government gives subsidies to a firm. We will look at two types of subsidies: (i) a R&D subsidy which lowers the costs of investment in product differentiation and (ii) a subsidy of marginal costs.

Introducing a policy maker we add a further stage to the game. We assume that the policy maker announces a policy schedule $s_i$ for firm $i$ before the firms play the game of the previous section. The policy is to subsidize or tax the investment costs of the domestic firm. The policy schedule is not restricted to be positive. In the case of $s_i < 0$ firms would pay a tax whereas if $s_i > 0$ would correspond to a subsidy. We assume that the policy maker can credibly announce the policy schedule. Due to the subgame perfection a policy maker can anticipate the behaviour of the firms. It is a quite strong assumption that the policy maker knows the rule of the game. However, we will not introduce further complications to the model and focus on the results obtained. To sum timing of the policy game up: (1) the government makes a credible announcement of a subsidy; (2) the home firm chooses the investment in product differentiation; (3) firms choose their output.

In the remainder we will use 'subsidy' instead of policy schedule. If the subsidy is negative it is actually a tax.

3.1 R&D Subsidy

The assumption about the subsidy is, that the investment subsidy is proportional to the investment. We denote the subsidy by $\lambda_i$, where the subscript indicates the country. A R&D subsidy does not directly change the output game. It indirectly influences the output decision by a firm by altering the decision to invest in product differentiation. Hence, we can work with the quantities given in equation (4).
Therefore the profits of firm $i$ are

$$\Pi_i = \frac{(1 + \sigma(1 - \theta))(a - c)}{b} \left( \frac{a - c}{2(1 + \sigma) + \theta(1 - 2\sigma)} \right)^2 - \gamma x_i^2 + \lambda_i x_i. \quad (8)$$

To obtain the optimal investment, firm $i$ would set the marginal net profit equal to marginal investment costs which includes the subsidy. Therefore, the FOC of firm $i$'s investment decision yields

$$\frac{\sigma(x_i + x_j)(a - c)}{bA^2} + \frac{2(1 + \sigma(x_i + x_j)(a - c)^2(1 - 2\sigma)}{bA^3} = 2\gamma x_i - \lambda_i, \quad (9)$$

where $A \equiv 2(1 + \sigma) + \theta(1 - 2\sigma)$. This implicitly yields $x_i^*$ which is the optimal investment. Note, that the R&D subsidy reduces the marginal costs of investment by a fixed amount which increases the incentive of the firm to invest.

How does the investment of a firm change with a subsidy? If we look again at equation (9) where we have no subsidy and compare it to the first order condition with a subsidy in equation (9) there is the additive term that captures the subsidy. In other words the RF of the firm's investment shifts. To see this more formally, we have to look at the derivative of the investment with respect to the subsidy

$$\frac{d}{d\lambda_i} = -\frac{\gamma x_i}{\pi_{x_i} x_i}. \quad \text{The right side of the equation is obtained by the implicit function theorem. The derivative of the FOC with respect to the subsidy is constant, 1.}$$

Hence, the investment is increasing in the subsidy iff the underlying problem is a maximization. This implies that the profit function is concave in the investment which in turn implies $\pi_{x_i} x_i < 0$.

The shift of the optimal responses is graphically illustrated in figure 2. The Reaction functions (RF) are depicted for the case of $\sigma = 0$. The results hold for a smaller market expansion effect as well. We can see, that the RF shifts up with a positive subsidy without changing the slope of the RF.

From the graph we can also see what would happen if one policy maker announces a higher subsidy. The RF of the respective firm shifts out without changing the RF of the other firm. Accordingly, the firm with the higher subsidy will expand its investment. The other firm will expand its investment as well, however, by relatively less. This shows that a subsidy has a spillover effect on the other firm.

The shifting of the RF is similar to the model in Brander and Spencer (1985). In their model a policy maker is able to make the threat of a higher investment credible by announcing an investment subsidy. This shifts out the RF.
Lemma 3. The investment response of a firm to a respective change in the subsidy is the same over countries, \( \frac{dx_i}{d\lambda_i} = \frac{dx_j}{d\lambda_j} \).

Proof. This lemma follows from the symmetry of the firms. However it will be useful to derive some of the results. We employ the implicit function theorem to derive \( \frac{dx_i}{d\lambda_i} = -\frac{\pi_{x_i,\lambda_i}}{\pi_{x_i,\pi_i}} \). It is straightforward to show that \( \pi_{x_j,\lambda_i} = \pi_{x_i,\pi_j} \). Further, \( \pi_{x_i,\lambda_i} = 1 \) for \( i = j \), which can be seen from the FOCs.

To provide some intuition for this result remember, that the investments enter additively in the degree of product differentiation. Further, the profit functions are symmetric and the firms share the same parameters. Due to the additivity, the return on investment is the same for both firms which leads to the result in the lemma.

Which firm will make the higher investment? To answer this question we have to look at the FOCs of each firm. The left hand side of the respective FOCs are equal to each other due to the additivity in the investment. Therefore, the investment difference is

\[
x_i - x_j = \frac{\lambda_i - \lambda_j}{2\gamma}
\]

To interpret this, we note that if the firm in country \( i \) has a higher subsidy per unit of investment it also has a higher investment. Further, if the subsidies are equally high in each country, there is no investment differential. Additionally note, that the investment difference is inversely related to the efficiency of investment,
\[ \gamma. \] Therefore, a subsidy has a stronger effect on investment differences the more efficient the firms are in R&D.

Taking this into account the policy maker announces a subsidy schedule that maximizes \[ W = \Pi_i(x_i^*) - \lambda x_i^*. \] From the expression of the welfare it is apparent, that the subsidy costs cancel with the gain by the subsidy in profits. Thus similar to Brander and Spencer, the subsidy alters the strategy space of the firm by making a new strategy credible.

We will now investigate, whether the policy maker has an incentive to set a positive subsidy. To see this we look at the first order derivative of the welfare function.

\[ \frac{\partial W_i}{\partial \lambda_i} = \frac{(2 \frac{dx_i}{d\lambda_i} - \frac{1}{2}c)}{bB^3} \left( \delta B - 2(1 - 2\delta) \left( 1 + \delta 2x_i - \frac{\lambda_i - \lambda_j}{2\gamma} \right) \right) - 2\gamma x_i \frac{dx_i}{d\lambda_i} \] (11)

where \( B \equiv 2(1 + \sigma + (1 - 2x_i)(1 - 2\sigma)) \). Unfortunately we can not solve for a optimal subsidy. The first term of the derivative is positive as long as the derivative of the investment with respect to the subsidy is sufficiently large\(^6\). Note, that the FOC of a welfare maximum is similar to the FOC of the firm in equation (??). The crucial difference is, that there is an additional term which increases the marginal return of investment.

**Lemma 4.** The FOCs’ of the welfare maximization are the same for each country, which implies \( \lambda_i = \lambda_j \).

**Proof.** This lemma is, again, follows from the symmetry. Again, it will prove useful later. We will sketch the proof here to get an intuitive understanding. However, the actual proof will be in the appendix. To start we have to look at the FOC of welfare for each country. Then we can use the result, that each firm reacts to a change in the respective subsidy in the same way. Further we can use, that we know the investment difference. In order to obtain a welfare maximum in each country we need that \( \frac{\partial W_i}{\partial \lambda_i} = 0 \). This holds only for both countries if \( \lambda_i = \lambda_j \), which is the statement in the lemma.

The latter lemma proves to be useful for the graphical analysis of welfare. In figure 3 we did plot the welfare function for different degrees of the market expan-

\[^6\] To see this assume that the bracket with the derivative is positive and that note that \( x_i < 1 \).
sion effect $\sigma$. To obtain those graphs we had to rely on numerical simulations\(^7\).

![Welfare function graph](image)

Figure 3: Welfare function (for blue=$\sigma = 0.25$ and black=$\sigma = 0.75$)

Welfare is a concave function in the subsidy as can be seen from figure 3 for the given parameterization. To make some comparisons to the robustness of the welfare function to change in the market expansion effect we plotted welfare for $\sigma = 0.25$ and $\sigma = 0.75$. In other words, we look at welfare with a large and small market expansion effect respectively. In table 2 we report some of the results of the numerical simulation. What we should mention at that point is, that if the market expansion effect is very large (close to $\sigma = 0$) then the welfare function has a local minimum within the range of possible values. However, for the rest of the discussion we consider only the maxima\(^8\).

In the first three columns are the results for collusive behavior of the governments. We can see, that even for a small market expansion effect the subsidy is positive. The numbers reported in the table are relatively robust to changes in other parameters. For example an decrease in $\gamma$ results in an increase in both, the investment and the subsidy. This seems to be right; as the investment becomes more productive the incentive to invest increases. Additionally, the value of $a - c$ should not be too high otherwise the profits get large and make an investment larger than one optimal which we ruled out.

\(^7\)The simulation files are available on request from the author.

\(^8\)If $\sigma$ is small enough the investment tax actually makes the investment negative.
The picture with respect to the subsidy changes if we look at the Nash subsidy game. Here we see, that the governments actually tax the respective firms if the market expansion effect is large ($\sigma > 0.5$). On the other hand, for a small market expansion effect the subsidy is positive. This can be explained by the strategic nature of the investments. Firstly, if the investments are strategic complements ($\sigma > 0.5$) the incentive through market expansion dominates the free-riding incentive. Accordingly, the firms push each other to invest more. A government that is able to alter this incentive will reduce the latter by an investment tax. This occurs due to an over investment by the firms. The policy makers, however, try to exploit the free riding on the foreign firms investment and thus announce a tax.

If the market expansion effect is small there is a smaller effect of free riding and governments subsidize investments of their respective firms. The reason for subsidizing more than under collusion is in the Nash game. Both firms rival subsidizing the investments and thus produce a too high investment.

At this point we should mention, that for a $\sigma$ close to zero the investment gets negative. This occurs due to incentive for the government to set a high tax. This high tax makes it optimal for the firm to disinvest. However, we have ruled this out by assuming that the investment must be positive. Therefore, with a very large market expansion effect we get a corner solution.

A positive subsidy is in line with Brander and Spencer (1985). In their model, a policy maker has an incentive to announce a subsidy to increase the R&D investments of the home firm. We find a similar result for a small market expansion effect. However, with a large market expansion effect we find, that a tax is optimal. This

<table>
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<th>$\sigma$</th>
<th>Welfare$^C$</th>
<th>Investment$^C$</th>
<th>Subsidy$^C$</th>
<th>Investment$^N$</th>
<th>Subsidy$^N$</th>
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</thead>
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<td>0</td>
<td>0.1906</td>
<td>0.1888</td>
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<td>-0.0177</td>
<td>~ −0.14</td>
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<td>0.1279</td>
<td>0.115</td>
<td>0.0085</td>
<td>~ −0.09</td>
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<tr>
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<td>0.0945</td>
<td>0.085</td>
<td>0.0532</td>
<td>0</td>
</tr>
<tr>
<td>0.75</td>
<td>0.1751</td>
<td>0.0686</td>
<td>0.060</td>
<td>0.1112</td>
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<td>0.0506</td>
<td>0.045</td>
<td>0.1617</td>
<td>~ 0.23</td>
</tr>
</tbody>
</table>

Parameter values: $a = 2$, $b = 2$, $c = 0.25$, $\gamma = 0.8$

C: Collusion; N: Nash

Table 2: Numerical Solutions
is at odds with Brander and Spencer. Further, Leahy and Neary (2001) in their important contribution show that under general conditions a subsidy is optimal. The results found in this paper show that, in a different set up, this proposition might not hold. The reason is the strategic nature of the investments.

4 Conclusion

In this paper we looked at strategic product differentiation. Two firms compete in a Cournot environment and are able to invest in product differentiation. A policy maker announces a policy schedule to either subsidize or tax the investment by the respective domestic firm. We find that, depending on what we call a market expansion effect, a tax or a subsidy are optimal. A strong market expansion effect implies that the investments are strategic substitutes. This makes a tax the optimal instrument due to over investment by the firms. On the other hand, if the market expansion effect is weak, the investments are strategic complements. Therefore a subsidy is optimal due to underinvestment by the firms. A possible policy implication is, that it seems important to look at the strategic nature of the investments.

References


Appendix

A

Proof of the Slope of the Reactions Function

Here we will proof proposition 1. The proof for proposition 2 is written in parenthesis.

Proof. To proof the above result we utilize the implicit function theorem. From the latter we know that \( \frac{dx_i}{dx_j} = -\frac{\Pi_{x_i}}{\Pi_{x_j}} \), where the subscript denote the derivatives. In proposition 1 we established the condition under which the profit function has a maximum, \( \Pi(x_i, x_j) < 0 \). As mentioned in the proposition we restrict our view on maximizations only. We now have to show, that the cross derivative is positive (negative) in order to show the positive (negative) slope of the RF. Computing \( \Pi_{x_i x_j} \) and rearranging the expression yields \( (1 - 2\sigma)(3 + \sigma(1 - 2\sigma)(1 - \theta)) \). The second term is always positive given any value of \( \sigma \) and \( \theta \). To see this remember that \( \sigma, \theta \in [0, 1] \). Thus the sign of the cross derivative depends on the term \( 1 - 2\sigma \) which has a positive (negative) value if \( \sigma > \frac{1}{2} \) (\( \sigma < \frac{1}{2} \)). This as well establishes that the investment are strategic complements (substitutes). If \( \sigma = \frac{1}{2} \) the cross derivative is equal to zero. Therefore the reaction function is vertical. \( \square \)
Proof of lemma 4

Proof. Note that $\frac{\partial W_i}{\partial \lambda_i} = 0 = \frac{\partial W_j}{\partial \lambda_j}$. We can rewrite this as

$$\frac{\partial W_i}{\partial \lambda_i} - \frac{\partial W_j}{\partial \lambda_j} = 0.$$  \hspace{1cm} (12)

Using the result form lemma 3 and the investment difference in equation (10) the difference in equation (12) equals zero iff $\lambda_i = \lambda_j$ which is the result states in lemma 4. \hfill \Box