Knowledge spillover externality and the home market effect
Preliminary version

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Warsaw (Poland) 11-13 Sep 08

Summary 1 This paper analyzes the international transmission of knowledge externality and its effects in “home market effect”. In the context of monopolistic competition where international trade is subject to transaction costs, according to the home market effect, a change in demand for domestically produced goods raises the number of varieties more than proportionally, and raises domestic factor prices. So, our theoretical model extend the pattern of intra-industry trade shown by Krugman’s (1980) and Weder’s (1995) analysis by introducing knowledge spillover externality in production function. This allows us to generate productivity growth, implications of competition between domestic and foreign firms and changes in market size. The magnitude of these effects is sensitive to the spillovers which vary under different market structures, trading costs, elasticity of labour supply and taste for variety.

1 Introduction

In this paper we will combine the two concepts in the title: Home market effect and knowledge spillover externality, and identify their meeting point.

For this, we begin by contextualising both subjects:

On one hand, the works of Fujita, Krugman or Venables, among others, have given rise, since the early 1990s, to a new strain in the literature on localisation known as the New Economic Geography (NEG). One of the theoretical inferences deduced from this is the so-called Home Market Effect (HME) or effect of increased demand. The HME concept is one of the cornerstones of the New International Trade Theory and of the NEG, and with different nuances is generally accepted as one of the typical outcomes of the models which present increasing returns to scale (IRS) and transport costs. However, on the theoretical level, there is a strain of literature which specifies under which circumstances this is produced or not produced, depending on the different initial hypotheses. The theoretical foundations of HME are found in Krugman (1980)[4] and Weder (1995)[7] which in turn have inspired both theoretical discussion and diverse empirical tests.

The principal groundbreaking innovation of Krugman is to show how the introduction of transportation costs alongside the presence of increasing returns imply that an increased demand for a certain product within a country will drive a more-than-proportional increase in production of that product in that country,
a phenomenon known as HME. More recently, Weder (1995) introduces the distinction between absolute and relative differences in the dimensions of domestic demand of nations. He considers two countries as completely identical except for the absolute and relative size of their domestic demand for each type of differentiated product. Market size does not affect the scale of production, only the number of varieties of the product such that, when international transportation costs are reduced, competition between national and foreign companies increases, which implies that small differences in market sizes can have a great effect on the relative number of varieties produced in the two countries. In a nutshell, one country may be smaller than another and yet be a net exporter when its relative demand is greater. To summarise, the principal contribution of Weder (1995) is the introduction of the idea of a greater relative demand and the study of its consequences on production.

On the other hand, an essential reason for the geographic concentration of an industry is the existence of increasing returns in the individual production function, besides which, the Marshallian tradition of economic analysis has emphasised the positive effects of externalities on the productivity of companies. One type of externalities are known as economies of agglomeration, which mainly occur with expensive transport costs, and begin a trend towards grouping new companies in the nearest geographical area, leading to a self-reinforcing process of industrial localisation (Hoover 1937[3]; Weber 1929[6]). Agglomeration is fuelled by geographical proximity and the flow of knowledge, so important spillovers should be expected between neighbouring regions. Definitively, externalities give rise to agglomeration processes whose mechanisms have obvious effects on industry productivity.

Meeting point: Advantages in productivity produced by these externalities will have an effect on the HME of the country where they are produced, and we propose to analyse this and identify if this effect is really produced.

More recently and as an example of the joining of these two ideas in the literature, Corsetti, et al. (2007)[1] analyse how when externalities between countries (international externalities) are increased, competition between home and foreign companies increases, and this implies that small differences between market sizes have a greater effect on the relative number of varieties produced in the two countries. Externalities on one hand generate gains in productivity, as they provoke changes in market sizes when they reduce entry costs for new companies or decrease marginal costs, and on the other hand, provoke changes in product differentiation, creating new varieties. Meanwhile, spillovers vary under different market structures, trade costs, elasticity in the job supply and love of variety.

Thus, the contribution of our paper will be the extension of Weder’s analysis by introducing externalities in the production function. It also deals with a generalisation of the production function in terms of two productive factors, capital and labour, and not just in work units as in most studies.

The rest of the paper is structured as follows: the model is presented in section 2. In part 3 the pattern of production of equilibrium in the open economy is derived. In 4 the different propositions about the trade pattern generated are
established. The work ends with our conclusions.

2 The Model

We adopt a similar model to Weder’s (1995): Two completely identical countries (home and foreign) except regarding the absolute and relative size of the domestic demand for each class of differentiated products (two classes, A and B, of horizontally differentiated products, with varieties within each class with some common characteristics, as for example, small and big cars); in other words, a monopolistic competition structure.

**Domestic Demand** We use the Marshallian demand function of the representative consumer [véase Helpman y Krugman (1985)] to obtain the aggregate demand for each variety and the aggregate expenditure of the home country’s residents for a home-produced class variety and an imported foreign variety, in order to make the important distinction between absolute and relative differences in demand which will help us to determine the relative number of varieties produced by both countries.

**Producción: Función de costes y beneficios** We consider that knowledge spillovers affect only home productivity, i.e., that they are not symmetrical, so our objective will be to see what happens to HME when one of the two countries (home) has an advantage in its productivity; in this case the externality producing this advantage is the relative size of the job market: \( \frac{L}{L^*} \). Starting with the following Cobb Douglas home and foreign production functions, respectively:

\[
q_{Ai} = AK_{Ai}^{\alpha}L_{Ai}^{1-\alpha}Z; \quad q_{Ai}^* = AK_{Ai}^{\alpha}L_{Ai}^{1-\alpha}Z^* \tag{1}
\]

where \( Ai \) is the \( i \)th good in group differentiated class \( A \) product, \( K \) capital, \( L \) trabajo, \(*\) indicates the foreign country and \( Z^* = \zeta \left( \frac{L^*}{L_{Ai}} \right) = \left( \frac{L^*}{L_{Ai}} \right)^{\gamma} \) is the externality.

The production function has constant returns in the primary factors \( K \) and \( L \) (\( 0 < \alpha < 1 \)) but with the externality increasing returns appear (\( \gamma > 0 \)). The effect we expect from this external effect is that while the technology is the same, joint productivity rises because of its influence, so that we obtain increasing returns in the country with the bigger relative market size (if \( L = L^* \) the effect of the externality is null). This gives home a comparative advantage which will make its domestic demand increase because it will sell more cheaply. The countries specialise in the group of products for which there is a larger internal market. Transport costs permit both countries to produce both classes of differentiated products (\( A \) and \( B \)) and let the bigger country be a net exporter, thus producing the HME.

\^1See Lucas (1988)[5]
Thus, we determine the production balance minimizing the cost function
\[ C(q) = rK + wL \]
subject to the production function according to formula (1), where \( r \) is the price of the capital factor and \( w \) of the labor factor.

Each level of production of \( q \) determines a single value of \( L \), the equation only has one solution depending on \( q \), a quantity which will depend on \( K \) if the same constant is assumed:

\[
L_{Ai} = \left( \frac{q_{Ai}}{AK_{Ai}^\alpha Z^\gamma} \right)^{\frac{1}{\gamma - \alpha}}; \quad L^*_A = \left( \frac{q^*_{Ai}}{A^*K_{Ai}^\alpha} \right)^{\frac{1}{\gamma - \alpha}}
\]

such that the cost function will be:

\[
C(q) = rK + w \left( \frac{q_{Ai}}{AK_{Ai}^\alpha Z^\gamma} \right)^{\frac{1}{\gamma - \alpha}}; \quad \text{(3)}
\]

\[
C^*(q^*) = r^*K^* + w^* \left( \frac{q^*_{Ai}}{A^*K_{Ai}^\alpha} \right)^{\frac{1}{\gamma - \alpha}} \quad \text{(4)}
\]

Next, according to Krugman (1980) we obtain the profit-maximizing price for each differentiated class \( A \) product produced in the home and the foreign country, replacing the cost function with the benefit function for home and foreign, respectively:

\[
\pi_{Ai} = p_{Ai}q_{Ai} - \left( rK + w \left( \frac{q_{Ai}}{AK_{Ai}^\alpha Z^\gamma} \right)^{\frac{1}{\gamma - \alpha}} \right) \quad \text{(5)}
\]

\[
\pi^*_{Ai} = p^*_{Ai}q^*_{Ai} - \left( r^*K^* + w^* \left( \frac{q^*_{Ai}}{A^*K_{Ai}^\alpha} \right)^{\frac{1}{\gamma - \alpha}} \right) \quad \text{(6)}
\]

where differentiating regarding price and thus determining the first order equation:

\[
p_{Ai} = \left[ \frac{w \cdot \text{con} \cdot \frac{r}{ \gamma - \alpha \rho (1 - \alpha)}}{(AK^\alpha Z^\gamma)^{\frac{1}{\gamma - \alpha}}} \right]^{\frac{1 - \alpha}{\alpha (\gamma - 1)}} \quad \text{(7)}
\]

\[
p^*_{Ai} = \left[ \frac{w^* \cdot \text{con}^* \cdot \frac{r^*}{ \gamma - \alpha \rho (1 - \alpha)}}{(A^*K^* \cdot \alpha)^{\frac{1}{\gamma - \alpha}}} \right]^{\frac{1 - \alpha}{\alpha (\gamma - 1)}} \quad \text{(8)}
\]

where \( \text{con} \) is a constant representing a simplification of a part of the demand for a variety (expenditure in manufacture and price index for the other varieties) and which in turn implies that the elasticity of the demand price for a variety is

\[2\] A convex function increasing over \( q \), which begins with value \( rk \) when \( q = 0 \) (cost of the fixed factor) and growing over the price of both factors.
constant and equals parameter $\varepsilon > 1$. The parameter $\rho$ represents the love-of-variety effect of consumers, so that $\varepsilon \equiv \frac{1}{1-\rho}$. We need $\varepsilon > 1$ and $\rho > 0$ to ensure that the individual varieties are substitutes (and not complements) for each other, which enables price-setting behavior based on monopolistic competition power.

Thus the price depends mainly on three variables: salaries, substitution elasticity and the externality. Because if salaries are higher prices will be higher to cover costs, greater elasticity of substitution means lower prices due to greater competition, and last, the existence of the externality compared to its absence means that prices fall, assuming greater productivity, and the greater this positive external effect, the cheaper they are compared to the rest of the world, as the price difference will increase.

### 3 Pattern of Production in the Open-Economy Equilibrium

In this section we obtain the number of differentiated varieties in groups $A$ and $B$ produced by the two countries in the open-economy equilibrium. First, we must adopt a series of simplifying suppositions in order to carry out the analysis. First, $A = A^*$ are exogenous parameters which we consider to be equal, as they represent the state of technology in each country, i.e., we suppose the same technological level. Second, the variable $\text{con} = \text{con}^*$, as we understand that the functional form of demand is the same for home as for foreign: this demand function behaves in such a way that the more a consumer spends on manufactured goods, the more he spends on each variety, and meanwhile, the demand for a variety depends on the general (mean) price index of all the varieties competing with it (variety 1); if this increases, demand for variety 1 increases. We consider this behaviour to be the same for home and for foreign. Finally, $K = K^*$, the short term capital for home and foreign is given, so that the cost function is short term (fixed K). Also, in the long term, we consider work only as a variable factor in the cost function, in order to make the model simpler.

Starting from the following market-clearing conditions for home and foreign produced class A products, setting supply equal to demand yields [Weder (1995) y Helpman y Krugman (1985)]. We want to determine the pattern of production, so that in the following expressions we have the expenditure on goods in an industry as a sum of domestic residents’ and foreigners’ expenditures on the goods:

\[ pnq_A = \frac{n}{n + n^*} hwL + \frac{nc^*}{n + nc^*} f w^* L^*; \]  \hspace{1cm} (9)  

\[ p^* n^* q_A^* = \frac{n^*}{n^* + nc} f w^* L^* + \frac{n^* c^*}{n + n^* c} hwL \]  \hspace{1cm} (10)  

where $n$ and $n^*$ are the number of class A varieties produced in home and foreign, respectively. The term $c$ is defined as the home country’s ratio of the
value of an imported variety per unit of value spent on the domestically produced variety. The proportion of individuals buying each product class $A$ in home and foreign we name $h$ and $f$, respectively.

Replacing $p$ and $p^*$ obtained in equations (7) and (8) above and dividing by $n$ and $n^*$ on both sides of the equation, we obtain the following two conditions for equilibrium of the open economy:

$$ q = \frac{1}{n + n^*c}w + \frac{c^*}{n^* + n^*c}f w^* L^*; $$

\hspace{1cm} (11)

$$ (w^*)^{\frac{1-\alpha}{\alpha(\varepsilon-1)+1}} q^* = \frac{1}{n^* + n^*c}f w^* L^* + \frac{c}{n + n^*c}hw L $\hspace{1cm} (12)

These are two equations in $n$ and $n^*$. We assume that $n, n^* > 0$ and solve this equation system for $\frac{n}{n^*}$. Thus,

$$ \frac{n}{n^*} = \frac{h - c\beta}{\beta - c^* h} f, \text{ where } \beta = \nu L^* q \left[ \frac{w}{(Z^*)^{\frac{1-\alpha}{\alpha(\varepsilon-1)+1}}} \right] - q^* c^* (w^*)^{\frac{1-\alpha}{\alpha(\varepsilon-1)+1}} q^* (w^*)^{\frac{1-\alpha}{\alpha(\varepsilon-1)+1}} - cq \left[ \frac{w}{(Z^*)^{\frac{1-\alpha}{\alpha(\varepsilon-1)+1}}} \right] $$

\hspace{1cm} (13)

being $\nu = \frac{w}{w^*}$. Equation (13) indicate the pattern of production for class $A$ goods $(\frac{n}{n^*})$ as a function of relative demand $(\frac{f}{h})$ and of the externality $Z^*$. First, we can see the effect of the absence of externality: when the country’s relative domestic demand for class $A$ varieties $(\frac{f}{h})$ rises, it produces an increase of the relative number of class $A$ products produced by the home country $(\frac{n}{n^*})$, an effect already analysed in other works. Second, the existence of externality $Z^*$, our contribution in this paper, makes $\beta$ less, and thus, the relative number of varieties in class $A$ $(\frac{n}{n^*})$ more. That is, the externality increases specialisation, which will be magnified when the external effect is greater. Thus, we can conclude that the rank of incomplete specialisation is lower when transport costs are lower and when the relative market size of the countries is greater (measured through relative employment). This externality gives home a comparative advantage which will make a larger internal market available to it, leading it to be a net exporter, thus producing the HME.
4 Home-Market Advantage and the Pattern of Trade

In this section we adapt the analysis of the pioneering works mentioned above, regarding the relationship between differences in demand and the pattern of trade. Specifically, we will show how the existence of an externality and its growth affects the differences between absolute home-market advantage and comparative home-market advantage, as only the relative differences in domestic demand determine the pattern of trade, while the absolute differences are reflected in wage rates.

Absolute Advantage as a Determinant of the Range of the Wage Rate

Aim: To test that when wages are equal in the two countries, there cannot be equilibrium, due to the larger country accumulating a surplus in its trade balance. Balanced trade exists between the two countries if, and only if, the two countries are the same size.

Let us assume that the home country has an absolute home-market advantage in both groups of goods $A$ and $B$. To test our objective, we calculate the home country’s trade balance for class $A$ and $B$ differentiated products, $T_A$ and $T_B$, taking into account the results obtained in equations (9), (10) and (13).

\[
T_A = f wL^*c \left( \frac{n - n^*}{n^* + nc} \right) \left( \frac{q^*(w^*)}{(\gamma^*)} \right)^{1-\alpha} - q^*(w^*)^{1-\alpha} \left( \frac{w}{(\gamma^*)} \right)^{1-\alpha} \frac{1-\alpha}{2} \left( \frac{q}{(\gamma^*)} \right)^{1-\alpha} \left( \frac{w}{(\gamma^*)} \right)^{1-\alpha} \cdot (14)
\]

\[
T_B = (1-f) wL^*c \left( \frac{m - m^*}{m^* + mc} \right) \left( \frac{q^*(w^*)}{(\gamma^*)} \right)^{1-\alpha} - q^*(w^*)^{1-\alpha} \left( \frac{w}{(\gamma^*)} \right)^{1-\alpha} \frac{1-\alpha}{2} \left( \frac{q}{(\gamma^*)} \right)^{1-\alpha} \left( \frac{w}{(\gamma^*)} \right)^{1-\alpha} \cdot (15)
\]

$T_A$ is positive only if $n > n^*$, and similarly for $T_B$.

\[
T = T_A + T_B;
\]

\[
T = \frac{wwc}{q^*(w^*)^{1-\alpha} - cq^(*)^{1-\alpha}} \cdot (16)
\]
\[
Lq^* (w^*)^{\frac{1-n}{(x-1)\pi+1}} - L^* \frac{w}{(Z^*)^{\frac{1-n}{x-1}} - q^* (w^*)^{\frac{1-n}{(x-1)\pi+1}}} \bigg( \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} - cq \bigg) \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \]  

(16)

Through the trade balance we see a country’s specialisation, who exports or imports more, if both countries produce both types of goods. We must test that if \( L > L^* \) and \( w = w^* \), then, in turn, that if we add the existence of \( Z^* \) or its increase, this also leads to the same conclusion. Equation (16) shows that the home country accumulates a trade surplus in its total trade balance if it is larger than the foreign country and the existence of externality magnifies this effect.

This conclusion obtained makes it clear that two countries of unequal size cannot have equilibrium if there is nothing to compensate the disadvantage of the small country. This “something” is the relative wage rate which will be adjusted so that the trade balance between the two countries is equalized. Thus a country with an absolute home-market advantage (disadvantage) in both groups of goods will have a higher (lower) wage rate [the other country will have to lower wages to compensate for this absolute advantage for home (or absolute disadvantage for foreign) and maintain employment levels, thus reducing production costs and the price of the product, increasing the import ratio of home]. Our aim now is to find the relationship between relative wage and the relative size of the countries, adding as always the effect which the appearance and increase of the externality has on these variables. For this, we reduce to zero the equation (14) and (15): \( T = 0 \), and substituting \( \beta \) defined in (13) we have:

\[
\frac{L}{L^*} = \frac{\left( q \left[ \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \right] - q^* (w^*)^{\frac{1-n}{(x-1)\pi+1}} \bigg) c^* \bigg( \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} - cq \bigg) \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \right) c^* wq^* (w^*)^{\frac{1-n}{(x-1)\pi+1}}}{\left( q^* (w^*)^{\frac{1-n}{(x-1)\pi+1}} - cq \left[ \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \right] \right) \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \bigg) \frac{w}{(Z^*)^{\frac{1-n}{x-1}}} \bigg)}
\]

(17)

This relationship is only possible if \( L > L^* \) as, on one hand, if \( L = L^* \) the effect of the externality is null \( \left( \left( \frac{L}{L^*} \right)^{1/\gamma} = 0 \right) \), as is the case in Weber, which is not the object of our study; and on the other hand, if \( L < L^* \) this leads to absurdity, because the externality would be negative. In fact, the appearance and growth of the externality give home a comparative as well as an absolute advantage, magnifying even more the effect of the former, as can be shown by equation (17). Greater size in home \( (L > L^*) \) implies \( (w > w^*) \) which will be magnified by the effect of the positive externality \( \left( \frac{L}{L^*} \right)^{1/\gamma} > 0 \), i.e., the final wage in home is still higher than in foreign to compensate for this country’s comparative disadvantage.
Comparative Advantage as a Determinant of the Pattern of Trade

The aim now is to link relative demand to a positive final trade balance which indicates the specialisation of a country, as it is precisely this relative demand which causes the HME. The basis is that in an open economy in equilibrium, each country is a net exporter of the differentiated group of goods for which it has a comparative home-market advantage, when it has a higher proportion of demand for one class than for another. For this we will also take into account the effect of introducing the externality which has modified this trade balance and generated a magnification of this specialisation. That is, we will find a relationship between relative demand \( h_f \), the relative market size \( L \), relative wages \( \frac{w}{w^*} \) and the externality \( (Z^*) \). Thus we reduce the equation to zero (14), \( T_A = 0 \), substituting \( \beta \) defined in (13) and clearing \( \frac{h}{f} \) we have:

\[
\frac{h}{f} \geq \frac{L^*_L}{L} = \frac{L^*_L}{L} \ln \left[ q \left( \frac{w}{(Z^*)^{\frac{1-\alpha}{\alpha}}} \right)^{\frac{1-\alpha}{\alpha}} - q^* e^* \left( \frac{w^*}{(Z^*)^{\frac{1-\alpha}{\alpha}}} \right)^{\frac{1-\alpha}{\alpha}} + cw^* + e^* w q^* \left( \frac{w^*}{(Z^*)^{\frac{1-\alpha}{\alpha}}} \right)^{\frac{1-\alpha}{\alpha}} \right]
\]

From equation (18) we can show that \( \frac{h}{f} \geq \frac{L^*_L}{L} \frac{f}{L^*_f} \) where \( \frac{f}{L^*_f} \) comes from equation (17), i.e., simplifying \( T_A \geq 0 \) iff \( \frac{h}{f} \geq 1 \) and \( T_B \geq 0 \) iff \( \frac{1-h}{f} \geq 1 \). To which we must add that the effect of the externality, as we have shown, implies a greater trade balance for home. The compensation between sizes and wages is always magnified by \( Z^* \), in fact, the relative market size and relative wages correspond to a greater relative demand.

5 Conclusion

This paper examines the effects of externalities on absolute and relative differences in demand between two countries in the context of intra-industrial trade. We have attempted to combine and join two crucial concepts of the New Economic Geography, the home market effect and knowledge spillover externality. The contribution of our paper was the extension of Weder’s analysis by introducing externalities in a Cobb Douglas production function. Also, we have covered two production functions, capital and labour, in this production function, and not only the labour resource, as in the best-known papers. Our main
conclusion is that the introduction of the externality in the analysis means that specialisation increases, which is magnified when the external effect is greater. This happens when transport costs are lower and the relative market size of the countries is higher (measured by relative employment). The home market effect is produced when a comparative advantage appears in a country, due to the effect of the externality on productivity, which will make a larger market available to it, leading it to become a net exporter.

References


