Trade Liberalization, Foreign Aid, and Welfare under Unemployment and Cross-border Pollution

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Abstract

We construct a two-country general equilibrium model of foreign aid, with unemployment in the recipient country and cross-border pollution from this country to the donor. Pollution is entirely abated by the recipient’s private and public sectors. We examine the optimal response of the two countries in terms of the size and use of aid, and in terms of environmental policies in reaction to two exogenous shocks. First, an improvement in the terms of trade as a proxy of freer trade and globalization. Second, a decrease in the recipient’s minimum wage as a reform proxy of its labor market. We examine cases where such changes may lead to welfare improvement in both countries, accounting for the optimal adjustments of their policy instruments.

Keywords: Foreign Aid, Cross-border Pollution, Pollution Abatement, Unemployment, Minimum Wage.

J.E.L Classification: F35, H41, J64, Q28

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1. Introduction

Throughout the past decades country officials, international organizations and policy makers have strongly argued that international transfers (i.e., foreign aid), both monetary and non-monetary, from developed to developing economies can alleviate acute global and national economic, and often, non-economic problems. For example, it has been maintained, and ample empirical evidence confirms, that untied or tied to specific objectives provision of funds is made from developed to developing countries for purposes of consumption, of financing capital accumulation, for provision of public goods and public inputs, of environmental preservation, or of promoting structural reforms in commodity and factor (e.g., labor) markets.\(^1\) Such transfers primarily aim at improving welfare in the recipient countries, but often can benefit, either directly or indirectly, the donor countries as well.\(^2\)

Moreover, during the long history of foreign aid between donor and recipient countries, there has been frequent and prolonged variability both in the volume of aid offered by the donor countries, and in the way aid has been used by the recipients. On the one hand, issues such as “aid fatigue”, or “appropriate” use of foreign aid by the recipient countries, or their compliance to the terms of receiving (tied) foreign aid, have been among the reasons invoked by developed-donor countries to increase or decrease aid to the developing world. On the other hand, recipient countries opted to use foreign aid in ways most fitting their own economic and non-economic objectives regardless of clauses imposed on them by the donors for the disbursement of the received aid. Aside of the above, however, changes in the global economic environment, e.g., freer international trade or greater degree of markets globalization, or economic conditions in the donor or the recipient countries, may account for changes in both the size and use of foreign aid,

\(^1\) Burnside and Dollar (2000) provide an extensive review of economic motivation regarding the donation of foreign aid.
\(^2\) Although the logic of aid is to benefit the recipient countries, quite frequently aid has been used as a policy instrument to affect the terms of trade between donor and recipient countries in favor of the former. This issue has been the subject matter of a large literature on the so-called “transfers paradox” (e.g., see Cassen 1988, Kemp 1992). The objective of the present analysis, however, is not on this issue.
and in other policy instruments. Such changes may entail important welfare implications for both the donor and recipient countries.

Here, motivated by such considerations, we examine how, in the presence of economic distortions, exogenous shocks can affect welfare and the optimal behavior of donor and recipient countries towards aid, and other economic policy instruments. Two distortions, capturing certain economic stylized facts, are introduced in the analysis. The first is production related pollution emissions that adversely affect utility in both the emitting country (-ies) and in neighboring (non-) emitting country (-ies). This distortion captures a major “global” consideration for which nowadays foreign aid is offered. For example, the U.S., the European Union, but also independently developed countries such as Germany, the UK, Sweden, and Japan are among the primary providers of extensive foreign aid to developing economies (e.g., Central and Eastern European countries) for financing pollution abatement activities (OECD, 2003). The second distortion is unemployment, due to the existence of a minimum wage, in the recipient country. This distortion attempts to capture an important structural feature of developing economies, and whose reform may constitute a pre-condition for aid.\(^3\) For the purposes of our analysis two exogenous shocks are considered. First, an improvement in the terms of trade as a proxy of freer trade (or globalization) due to reduced transportation costs or due to reduced tariffs and non-tariff barriers, and second, a decrease in the recipient’s minimum wage as a reform proxy of its labor market. In reaction to these shocks, we examine (i) the optimal response of the two countries in terms of foreign aid and environmental policy, and (ii) the induced welfare effects for the two countries accounting for the optimal policy adjustments.

2. Literature Review

Various aspects of international trade and environmental policy are related to the present study, e.g., foreign aid, changes in terms of trade, minimum wage, private and public abatement of cross-border pollution. To the best of our knowledge these aspects are not yet synthesized in a single study. This synthesis, motivated by the stylized facts

\(^3\) Deregulation of labor markets and the removal of barriers in commodity markets (e.g., tariffs and other import barriers) are among the factors which determine how donors allocate aid among multi-recipient countries (e.g., see, Lahiri and Raymondos-Möller 1997, and Lahiri et al., 2002).
outlined in the introduction, constitutes the primary contribution of the present chapter. For the purposes of our analysis we classify the relevant international trade literature in two strands. The first focuses on the links between foreign aid and environmental protection. The second focuses on the so-called double-dividend of environmental policy reforms (i.e., raising employment and reducing pollution), in open economies with pollution externalities and unemployment due to exogenous (i.e., minimum) or non-competitive wage determination.  

Regarding the first strand, a number of studies have examined the interaction among foreign aid, pollution and welfare, under the existence or not of cross-border pollution. Recently, for example, in a model with endogenous terms of trade and without cross-border pollution Chao and Yu (1999) examine the welfare effects of aid tied to public abatement activities in a pollution generating aid-recipient country. Hatzipanayotou et al. (2002) show that even in the absence of international cooperation, cross-border pollution may reduce the total amount of pollution emission by inducing more foreign aid from the developed to developing countries. Naito (2003), in a model with endogenous terms of trade, transboundary pollution and without environmental regulation, shows that untied aid is Pareto-improving in terms of welfare if the marginal propensity to consume in the donor country is sufficiently larger than that in the recipient. Turunen-Red and Woodland (2004), among others, examine a variety of Pareto-improving multilateral environmental reforms when compensating international lump-sum transfers are assumed or tariff reform is combined with the emission tax reforms. Lastly, Schweinberger and Woodland (2006) in a model of tied aid, cross-border pollution and simultaneous provision of private and public pollution abatement investigate cases where tied foreign aid may not be effective in reducing pollution.

The second strand of the literature examines whether an environmental tax reform provides the double-dividend i.e., an improvement of the environmental quality and an alleviation of the unemployment problem (e.g., see, Bovenberg and van der Ploeg 1996 and 1998, Hoel 1997, Koskela and Schöb 1999). According to this literature

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4 A sizeable literature on international economic aspects of cross-border pollution is not of direct relevance here (e.g., Ludema and Wooton, 1994, Copeland and Taylor 1995, and Copeland, 1996). Copeland (1994) examines the welfare implications of environmental and trade policy reforms in a full employment small open economy.
environmental tax revenue is used to cut taxes on labor or other distortionary taxes. For example, Bayindir-Upmann and Raith (2003) argue that in the high tax countries a revenue-neutral environmental tax reform under non-competitive labor markets yields a positive employment effect while the environmental dividend is hardly attainable.

3. The Model

We consider a two small open economies general equilibrium model, featuring foreign aid from a donor developed to a recipient developing country, cross-border pollution and unemployment in the recipient country due to the existence of a minimum wage. Hereon, asterisks indicate all variables of the donor country. Both countries produce, under perfectly competitive conditions, two goods. For analytical simplicity of the results we call, good 1 the numeraire exported commodity and good 2 the non-numeraire imported one. We assume that both economies are small in world commodity markets so that the international relative price of good 2, \( p \), is exogenous. An analytical implication of this assumption is that both countries can export and import the same commodities vis-à-vis the rest of the world. Moreover, with constant world commodity prices foreign aid and cross-border pollution constitute the international linkages between the two countries. Details regarding this feature of the model are given later on in the analysis. These goods are freely traded in international markets, and several primary factors are used in their production. The endowments of these factors in both countries are fixed and factor markets are perfectly competitive. Full employment of all factors is ensured in the developed donor country, while in the developing country involuntary unemployment of labor occurs due to the existence of a minimum wage (\( \bar{w} \)) above the market-clearing wage level. This minimum wage is assumed to be binding throughout the analysis, and therefore, labor supply (\( L \)) in the recipient country is infinitely elastic and smaller than the economy’s labor endowment (\( \bar{L} \)) at the prevailing

\[ 5 \] The present model can depict, inter alia, the case of two countries in the European Union, an “old” developed member-country (e.g., Germany, France) and a “new” post-accession, less-developed, country (e.g., Slovakia, Czech Republic) affected by cross-border pollution, with free trade between them and constant international commodity prices. In this context, it is plausible to assume that these countries are price takers in a much bigger world trading system. From the viewpoint of the chapter, this simplification (i.e., constant terms of trade) facilitates the analysis whose primary objective is not to examine international trade issues.
wage ($\bar{w}$). Pollution is generated in the recipient developing economy as a by-product of production and is transmitted to the developed donor country in the form of cross-border pollution. All pollution abatement is undertaken in the recipient country by private producers and the public sector, in a matter to be explicitly defined soon after. The donor country neither generates nor does it abate pollution. Both countries suffer disutility from local or cross-border pollution. We first lay out the general equilibrium model of the recipient developing economy with unemployment and production generated pollution and then the model of the donor developed country.

As noted above, pollution in the recipient country is partly abated by the private sector in response to an emissions tax ($t$) imposed by the government, and partly by the public sector. The level of public sector pollution abatement is denoted by $g$ imported by the country’s government at a constant world commodity price ($p_g$), and financed through emission tax revenue and a fraction of the received aid. Following Neary (1985), in the presence of factor price rigidities, the production side of the aid recipient economy can be described by the restricted revenue function defined as: $^6$

$$\bar{R}(p, t, \nu, \bar{w}) = R(p, t, \nu, L) - \bar{w}L, \quad (1)$$

where $\bar{R}(p, t, \nu, \bar{w}) = \max \{ p'x - tr - \bar{w}L : (x, z, L) \in \Phi(\nu) \}$, $\nu$ denotes the economy’s vector of the *flexprice* factor endowments. Since these endowments are assumed fixed, vector $\nu$ is omitted from the rest of the analysis. $\Phi(\nu)$ is the country’s aggregate technology set including both production and pollution abatement technologies by the private sector, $^7$ and $x$ is the vector of net outputs. The function $\bar{R}(p, t, \nu, \bar{w})$ is the income of the flexible-price factors and adding to it, the income of the *fix-price* factor labor i.e., $\bar{w}L$, gives $R(p, t, \nu, L)$, the unconstrained revenue function capturing the economy’s total value of production (GDP), or total factor income. From Hotteling’s Lemma, the partial derivative of the unconstrained revenue function with respect to ($t$), i.e., $^6$

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$^6$ For the general properties of this function and related issues see Neary (1985). Kreickemeier (2005) also follows the Neary approach in characterizing the supply side of an economy with unemployment due to a minimum wage.

$^7$ For simplicity we consider only one type of pollution generated in one or more sectors.
\(-R_t(p,t,L)\) denotes the level of pollution emissions net of private abatement (see Copeland 1994). The function \(R\) is assumed strictly convex in the emission tax rate, \(i.e., R_u > 0\), and when accounting for changes in employment \(\hat{R}_u = R_u - R_{ul}R_{Ll}^{-1}R_{Ll} > 0\). Its partial derivative with respect to \(L\) gives the level of the minimum wage, \(i.e.,\)
\[
  R_L(p,t,L) = \bar{w}. \quad (2)
\]
The above condition determines the level of employment of the fix-price factor, labor.

The function \(R\) is assumed strictly concave in \(L\), \(i.e., R_{Ll} < 0\).

Taking into account both private and public sector pollution abatement, the net emission of pollution is defined as:
\[
z = -R_t(p,t,L) - g. \quad (3)
\]
The following assumptions are maintained throughout in order to facilitate the analysis:

\((A1)\): \(R_{ul} < 0\). In view of (3) this assumption implies that an increase in the level of employment raises outputs and the level of production generated pollution.

\((A2)\): \(R_{tp} < 0\), and when accounting for changes in employment \(\hat{R}_{tp} = R_{tp} - R_{ul}R_{Ll}^{-1}R_{Lp} < 0\).

Accounting for \((A1)\) and \((A2)\) it follows that the imported good, is the polluting commodity\(^8\) and it is labor intensive \(i.e., R_{tp} > 0\). Given the two assumptions, an improvement in the terms of trade or a decrease in the minimum wage always causes a trade off between unemployment and the aggregate level of pollution. On the one hand, a terms of trade improvement reduces pollution emissions by inducing the polluting sector to contract, and on the other hand aggravates unemployment. Similarly, a decrease in the minimum wage alleviates unemployment and increases pollution emissions.

\(^8\) Otherwise if the exported good is the polluting good then \(R_{tp} > 0\).
Since both economies are considered small in world commodity markets, both can be considered exporters or importers of the same commodity to or from the rest of the world. Clearly this assumption is not valid in the presence of terms of trade effects. Here, we assume that both countries are importers of the non-numeraire good and thus, an improvement in the terms of trade of both the donor and the recipient countries is stated in the following assumption.

\((A3)\): \(dp < 0\) indicates an improvement in the terms of trade of both the donor and recipient countries.

As for the public sector, we assume that the government finances the cost of publicly provided pollution abatement, \(i.e., \ p_g g\), through the emission tax revenue, \(^9\) \(i.e.,\ -tR_i(p,t,L)\), and by allocating a fraction \(\lambda T\) of the aid received. The remaining fraction of aid \((1 - \lambda)T\) is assumed to be lump-sum distributed to the country’s households. In the present context, we do not require for the recipient country to provide matching funds for the aid received. That is \(\lambda \leq 1\). Thus, the government’s budget constraint is written as:

\[
p_g g = -tR_i(p,t,L) + \lambda T.\tag{4}
\]

Turning to the demand side of the economy, utility, as previously noted, is adversely affected by locally generated net pollution \(z\). The behaviour of the household sector is summarized by the expenditure function \(e(p,z,u)\) which denotes the minimum expenditure required to achieve a given level of utility \(u\) at constant commodity prices \(p\) when the level of net pollution is \(z\). The partial derivative of the expenditure function with respect to \((u)\), \(i.e., e_u\), denotes the reciprocal of the marginal utility of income. Since pollution adversely affects household utility, the partial derivative of the expenditure

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\(^9\) Brett and Keen (2000) report that in the US it is common for environmental taxes to be earmarked for specific environmentally related public expenditure, e.g., the financing of clean-up activities or of road construction and of public transport networks. The 2000 OECD report of the Task Force for the implementation of the environmental action programmes for the Central and Eastern Europe, reports that the Kazakhstan’s Law of Environmental Protection in 1999, stipulates that “... It should be prohibited to spend money from Environmental Protection Funds for purposes not connected with environmental protection...”. Revenues for these environmental funds are listed to come primarily from pollution charges, non-compliance fines and international assistance (aid) for environmental protection.
function with respect to \( z \), i.e., \( e_z \), is positive and denotes the households’ marginal willingness to pay for a reduction in pollution (e.g., see Chao and Yu, 1999). That is, a higher level of net pollution requires a higher level of spending on private goods to mitigate its detrimental effects so that a constant level of utility is maintained. Also, \( e_{pz} > (\leq) 0 \) assuming that goods and clean environment are substitutes (complements) in consumption.\(^{10}\) The expenditure function is assumed strictly convex in \( z \), i.e., \( e_z > 0 \). That is, a higher level of net pollution raises the households’ marginal willingness to pay for its reduction. It is also assumed that \( e_{zu} = 0 \) implying that all income effects pertain to the numeraire good (see Lahiri and Raimondos-Moller 1998).

The budget constrained of the representative consumer requires that private spending, i.e., \( e(z, p, u) \) is equal to total factors income, i.e., \( \bar{R}(p, t, \bar{w}) + \bar{w}L \), plus the fraction of aid lump-sum distributed, i.e., \( (1 - \lambda)T \). Thus, the recipient country’s budget constraint is written as:

\[
e(u, r, p) = \bar{R}(p, t, \bar{w}) + \bar{w}L + (1 - \lambda)T.
\]

For the donor country we assume that all factors are fully employed. Moreover, as previously noted, we assume that the country neither generates nor does it abate pollution. Denoting by \( \theta' \) the rate of cross-border pollution from the recipient country, the welfare of the donor is adversely affected by the recipient’s level of pollution i.e., \( z^* = \theta' z \). The donor’s expenditure and the revenue functions are thus given by \( e^*(p, z^*, u^*) \) and \( R^*(p) \) respectively. The country’s income-expenditure identity is given by:

\[
e^*(p, z^*, u^*) = R^*(p) - T.
\]

Equations (2), (4), (5) and (6) constitute a system of four equations in four unknowns, namely \( u, u^*, g \) and \( L \). The model contains three policy instruments: the amount \( T \) of aid

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\(^{10}\) Standard properties of the \( e(p, z, u) \) function are that \( e_p \) is the (vector) of compensated demand functions, and that \( e_{pp} \) is non-positive.
for the donor country, and for the recipient, the emissions tax rate $t$ and the fraction $\lambda$ of aid allocated to public pollution abatement. The exogenous parameters are the terms of trade $p$ and the recipient country’s minimum wage rate $\bar{w}$. Following Copeland and Taylor (2003, ch. 4), an improvement in the terms of trade can be interpreted as a decrease in trade costs of the iceberg type, thus representing trade liberalization.

Appendix A lays out the complete comparative statics of the reduced form system.

3.1 Pollution and Welfare Effects in the Donor and Recipient Countries

We conclude this section by examining how changes in the policy and exogenous parameters affect the level of pollution and welfare in the two countries. Changes in the level of pollution are given by differentiating equation (3) and using the results from Appendix A to obtain:

\[
\frac{\partial z}{\partial t} = -TP_g^{-1}d\lambda - \lambda p_g^{-1}dT + \left[ (t - p_g)\tilde{R}_p + R_t \right] p_g^{-1}dt + (t - p_g)R_t R_{tt} p_g^{-1}d\bar{w} \\
+ \tilde{R}_p p_g^{-1}(t - p_g)dp \\
\]

(7)

An increase either in the amount of aid, i.e., $dT > 0$ or in the fraction of foreign aid allocated to public pollution abatement, i.e., $d\lambda > 0$ unambiguously reduces the level of production generated pollution in the recipient country. An increase in the recipient’s emission tax rate i.e., $t$, on one hand, reduces pollution emissions by the private sector, i.e., $R_t$. On the other hand, this reduction in the level of pollution emissions reduces the tax base for the provision of public abatement which exacerbates the level of pollution. A priori, the effect of a higher emission tax on the level of pollution is ambiguous. However, a higher emissions tax reduces the level of pollution if the emission tax rate ($t$) is lower than the unit cost of public abatement ($p_g$). That is, if $t - p_g < 0$. A decrease in the recipient’s minimum wage has an ambiguous effect on the level of pollution. On the one hand, the lower $\bar{w}$ raises production in the recipient country and thus it raises pollution emissions. On the other hand, higher production implies a larger tax base, higher emission tax revenue for financing public abatement and therefore lower level of pollution. A sufficient condition for the lower $\bar{w}$ to lower (raise) the net pollution level in
the recipient country is that \( t - p_g > 0 (<0) \). In the case where the emission tax rate equals the unit cost of public abatement \( i.e., t = p_g \), then a change in minimum wage has no direct effect on the level of pollution. An exogenous improvement in terms of trade \( i.e., dp < 0 \) reduces the pollution level in the recipient country if the emission tax rate is lower than the unit cost of public abatement \( i.e., t - p_g < 0 \) and under (A2). The level of pollution in the recipient country worsens if \( t - p_g < 0 \) and the country exports the polluting good, \( i.e., \tilde{R}_p > 0 \). In the case where the emission tax rate equals the unit cost of public abatement then a change in the terms of trade has no direct effect on the level of pollution. As previously noted, since the donor country neither generates nor it abates pollution, \( i.e., z^* = 0 \zeta \), then, as a result of policy induced or parameter changes, changes in the level of cross-border pollution affecting the donor follow directly the induced changes in \( z \).

Using the reduced form system of Appendix A we obtain the induced changes in the level of countries welfare as follows:

\[
\Delta d u = A_T dT + A_{\lambda} d\lambda + A_t dt + A_{\pi} d\pi + A_p dp , \quad \text{and} \quad (8)
\]

\[
\Delta d u^* = B_T dT + B_{\lambda} d\lambda + B_t dt + B_{\pi} d\pi + B_p dp , \quad (9)
\]

where the expressions and the sign of the coefficients in the above equations are given in Appendix B. Also recall from Appendix A that \( \Delta (= e_n e_n^*, p_g R_{LL}) < 0 \).

From equations (8) and (9) we conclude the following. An increase in the amount of foreign aid to the recipient unambiguously increases welfare in that country, while it may also increase the welfare of the donor. Since in the recipient country there is no aid co-financing, increased aid raises households’ real incomes and public pollution abatement. Both effects exert a positive impact on the recipient country’s welfare. Thus, \( A_T \) in equation (8) is of negative sign. On the other hand, \( B_T \) in equation (9) has an ambiguous sign. In the absence of cross-border pollution, \( i.e., \theta^* = 0 \), higher aid to the recipient country unambiguously reduces the donor country’s welfare. However, in the
presence of cross-border pollution, *i.e.*, \( \theta^* > 0 \) and under the assumption that part of the aid to the recipient country is used for public pollution abatement, increased aid to that country may reverse the negative direct negative impact on the donor’s welfare. An increase in the fraction of aid allocated to public pollution abatement raises the recipient country’s welfare if the households’ marginal willingness to pay for pollution abatement exceeds the unit cost of \( g \), that is if \( e_z > p_g \).\(^{11}\) In such a case we say that there is social under-provision of public abatement, and \( A_\lambda \) in equation (8) is of negative sign. On the other hand, because of cross-border pollution, an increase in \( \lambda \) unambiguously raises the donor country’s welfare, *i.e.*, \( B_\lambda < 0 \). An increase in the recipient’s emission tax rate, according to the expression \( A_t \) of Appendix B, affects the country’s welfare through three channels. First, the term \( \Delta^{-1} e_u^* R_{LL} p_g R_t (= e_u^{-1} R_t) \) denotes a negative impact on welfare capturing a transfer of resources from the private to the public sector, and thus a loss in private real incomes. The second term \( -\Delta^{-1} e_u^* e_z p_g ([t - p_g]) R_t + R_t ] R_{LL} \) captures the welfare impact of the higher emission tax through its induced impact on the country’s level of pollution emissions, *i.e.*, \( -\Delta^{-1} e_u^* e_z p_g (dz/ dt) R_{LL} \). If, following the discussion of equation (6), the higher tax lowers the level of pollution, then through this term it also exerts a positive impact on welfare. Finally, the last term \( -\Delta^{-1} e_u^* p_g \bar{w} R_{Lt} \), following Assumption (A1), captures a negative tax induced employment-effect on the country’s welfare. That is, assuming that the higher \( t \) reduces the marginal revenue product of labor (*i.e.*, \( R_{Lt} < 0 \)), which in turn reduces employment and labor incomes and it affects negatively the representative household’s welfare in the recipient country. Expression \( B_t \) shows that the higher emission tax unambiguously improves the donor’s welfare in that country as long as it reduces pollution emissions in the recipient, *i.e.*, \( du^* = -\theta^* e_z^* e_u e_u^{-1} (dz/ dt) \). Clearly in the absence of cross-border pollution, changes in \( t \) do not affect the donor country’s welfare. A sufficient condition for the lower

\(^{11}\) When \( e_z > p_g \) then there is social under-provision of public abatement (\( g \)).
minimum wage to raise the recipient country’s welfare is that \( t - p_g > 0 \). In this case, however, the lower \( \bar{w} \) unambiguously increases the donor country’s welfare. If \( t - p_g < 0 \), then the lower \( \bar{w} \) may reduce the recipient’s welfare, while it unambiguously reduces the welfare of the donor. The expression \( A_p \) in Appendix B gives the welfare impact of an improvement in the terms of trade (i.e., \( dp < 0 \)) in the recipient country. The first terms captures the standard welfare effect due to a terms of trade improvement (i.e., \( -M \)). The second term, due to the existence of unemployment, indicates that via this term, the decrease in \( p \) affects negatively the recipient’s welfare by reducing employment and labor incomes in the country (i.e., \( -p_g \bar{w}R_{Ll}^{-1}R_{lp} > 0 \)). Finally, the last term \( -e_z(t - p_g)\bar{R}_{lp} \) is the environmental effect of the terms of trade improvement on the recipient’s welfare. Following the relevant discussion of equation (8), this term exerts a positive impact on the country’s welfare if \( t - p_g < 0 \), and under (A1) and (A2). By observing expression \( B_p \) of Appendix B, an analogous discussion can be carried out regarding the impact of \( dp < 0 \) on the donor’s welfare.

4. Nash Equilibrium

Having accounted for the pollution and welfare equations, we now characterize the non-cooperative Nash equilibrium values of the policy instruments. That is we determine the levels of the policy instruments, \((\hat{t}, \hat{\lambda})\) for the recipient and \((\hat{T})\) for the donor country, respectively, assuming that the two countries simultaneously maximizes their own welfare, treating the other country’s policy parameters as given. The symbol \((^\wedge)\) over a policy variable indicates its Nash equilibrium value. The first order conditions are given by:

\[
\Delta \left( \frac{du}{dt} \right) = A_t = 0 ,
\]

12 Using equation (7) in equation (8) we can write \( du = \Delta^{-1}e_{u}^{*}p_g[\bar{w} - e_zR_{Ll}(dz/d\bar{w})]d\bar{w} \). Then \( du > 0 \) iff \( (dz/d\bar{w}) > 0 \), which requires \( t - p_g > 0 \).
\[ \Delta \left( \frac{du}{d\lambda} \right) = A_\lambda = 0 , \quad \text{and} \]
\[ \Delta \left( \frac{du^*}{dT} \right) = B_T = 0 . \]

Equations (10)-(12) give the best-response functions and simultaneously determine the Nash equilibrium values of the policy instruments in the two countries. Manipulating these equilibrium conditions, we obtain the following Nash equilibrium conditions:

\[ e_z = p_g , \quad \text{(13)} \]
\[ \hat{t} = p_g - \bar{w}(R_{Lt} \bar{R}_{Lt})^{-1} R_{Lt} , \quad \text{and} \quad \text{(14)} \]
\[ p_g = \lambda \theta^* e_z^* . \quad \text{(15)} \]

Recognizing that pollution is a public bad and its abatement is a public good, Nash equilibrium condition (13) represents the Samuelson rule for the optimal provision of public goods in a closed economy without distortionary taxation. That is, the households’ marginal willingness to pay for public pollution abatement is equal to the marginal cost of producing it. Condition (14) can be viewed as a modified, in the presence of unemployment, Pigouvian rule for the optimal provision of public goods. First, in the absence of unemployment, condition (14) combined with condition (13), reduces to the exact Pigouvian rule for optimal public good provision. That is, the households’ marginal willingness to pay for public abatement is equal to the emission tax rate, i.e., \( t = e_z = p_g \). However, as condition (14) indicates, in the presence of unemployment, this rule is no longer valid. In particular, under (A1), i.e., \( R_{Lt} < 0 \), in the present case we have that the Nash equilibrium emission tax rate \( \hat{t} \) is lower than the corresponding Pigouvian rate (i.e., \( t = p_g \)) in order to account for its detrimental impact on employment. Given the existence of cross-border pollution, and the assumption that a fraction \( \lambda \) of aid to the recipient country is used by the latter country to finance the provision of public abatement, the condition (15) can be viewed as a modified Samuelson rule of optimal aid.
5. Exogenous Shocks, Optimal Policy Response and Welfare Effects

Having described the Nash equilibrium of this two-country model, as given by equations (10)-(12), we proceed to evaluate how the Nash equilibrium values of the policy instruments change due to changes in exogenous variables. That is, we examine how does an improvement in the terms of trade, a proxy for trade liberalization or economic globalization,\textsuperscript{13} or a decrease in the recipient country’s minimum wage (\textit{i.e.}, $d\bar{w} < 0$), a proxy for a labor market reform, affect the values of $\hat{\tau}, \hat{\lambda}$ and $\hat{T}$. We also examine the welfare effects of these exogenous shocks accounting for the adjustments in the two countries Nash values of their policy instruments. In doing so we examine three different cases:

\textit{Case 1:} One of the two countries is inactive. That is, either the recipient country adjusts the Nash values of its policy instruments $(t, \lambda)$, while the donor does not adjust the Nash value of aid. Or, \textit{vice-versa}.

\textit{Case 2:} The recipient country adjusts only the Nash value of $(\lambda)$ and the donor the Nash value of $(T)$.

\textit{Case 3:} The recipient country adjusts only the Nash value of $(t)$ and the donor the Nash value of $(T)$\textsuperscript{14}.

\textit{Case 4:} The recipient country adjusts the Nash values of its policy instruments $(t, \lambda)$ and the donor the Nash value of $(T)$.

In each of the cases above, we assume that the initial values of the policy instruments are at their Nash level. The reduced form systems and the coefficients of the unknowns for each of the above cases are given in Appendix C.\textsuperscript{15}

5.1. \textit{Case 1: One of the two countries remains inactive.}

\textsuperscript{13} Copeland and Taylor (2003) consider the notion of an improvement in the terms of trade as a notion of trade liberalization. The same notion is followed, in a different context by Hatzipanayotou \textit{et al.} (2005).

\textsuperscript{14} In cases 2 and 3 it is as if assuming that the recipient country is free to use the other instrument to achieve selfish interests.

\textsuperscript{15} Hereon, for simplicity and tractability of the results, we assume that all third order derivatives are zero. The authors provide upon request detailed calculations of the results.
Consider first the case where the recipient country adjusts the Nash values of its policy instruments \((t, \lambda)\), while the donor remains inactive. Then, differentiating (10) and (11) we obtain:

\[
A_{\lambda t} dt + A_{\lambda \lambda} d\lambda = -A_{m} d\overline{w} - A_{y} dp, \quad \text{and} \\
A_{\lambda t} dt + A_{\lambda \lambda} d\lambda = -A_{x\lambda} d\overline{w} - A_{x\mu} dp.
\]

(16)

(17)

The effect of a decrease in the minimum wage on the recipient’s values of \((\hat{t}, \hat{\lambda})\) are summarized in the following Corollary:

**Corollary 1:** A decrease in the recipient country’s minimum wage raises the country’s Nash value of the emissions tax rate and it reduces the Nash value of the fraction of aid allocated to public pollution abatement.

**Discussion:** Using equations (C.1) we obtain:

\[
\frac{dt}{d\overline{w}} = -(R_{tt} \hat{R}_{tt}^{-1} R_{t\mu}) < 0, \quad \text{and} \quad \frac{\hat{\lambda}}{d\overline{w}} = -(TR_{t\lambda} \hat{R}_{\lambda\lambda}^{-1} R_{t\lambda} R_{\lambda} > 0.
\]

(18)

The lower minimum wage raises labor’s marginal revenue product, and thus employment. As a result production and production-generated pollution emissions rise. Consequently, the Nash value of the emission tax rate increases. On the other hand, the lower minimum wage reduces the Nash value of the fraction of aid allocated to financing the provision of public abatement. As previously noted the lower minimum wage by increasing production and pollution emissions it raises the government’s emission tax revenue used for financing public abatement. Thus, the Nash fraction \(\hat{\lambda}\) of foreign aid allocated by the recipient to finance a given level of public abatement falls.

Turning to the welfare effects of the decrease in \(\overline{w}\), accounting for the adjustments in \(\hat{\lambda}\) and \(\hat{t}\) we proceed as follows. Using equations (8), (9) and (18), the conditions (14) and (15), and recalling that all terms are evaluated at Nash equilibrium, e.g., \(A_t = A_{\lambda} = 0\), we obtain:
\[ \frac{du}{d\bar{w}} = \Delta^{-1} A_{\pi} = e_u^{-1} \bar{w} R_u^{-1} R_u^{-1} R_u < 0, \quad \text{and} \]
\[ \Delta \frac{du^*}{d\bar{w}} = B_{\pi} + B_{\lambda} \frac{d\hat{\lambda}}{d\bar{w}} + B_{t} \frac{d\hat{t}}{d\bar{w}} = 0, \]  

where the right-hand-side of equation (19) emerges after substituting the optimality conditions (13)-(15) into the expression for \( A_{\pi} \) as given in Appendix B. Also recall that \( B_{\lambda} < 0 \), and that when evaluated at Nash equilibrium both \( B_{\pi} \) and \( B_{t} \) are negative.

Intuitively, equation (19) can be explained as follows. A decrease in the minimum wage results in higher employment, outputs, and production generated pollution. Since the decrease in \( \bar{w} \) unambiguously raises the recipient country’s welfare, the induced positive employment-output effect dominates the negative effect due to higher pollution emissions. Equation (20) indicates that the lower \( \bar{w} \) does not affect the welfare of the donor country.\(^{16}\) The decrease in \( \bar{w} \) incurs a negative direct impact on the donor’s welfare due to higher cross-border pollution from the recipient country. Also, since the lower \( \bar{w} \) entails a decrease in \( \hat{\lambda} \), thus a decrease in public pollution abatement, more cross-border pollution results in an additional, indirect, negative effect on the donor’s welfare (\( i.e., \Delta^{-1} B_{\lambda} (d\hat{\lambda} / d\bar{w}) < 0 \)). Lastly, the induced increase in \( \hat{t} \) (\( i.e., \frac{d\hat{t}}{d\bar{w}} < 0 \)) implies more private pollution abatement, thus less cross-border pollution, which exerts a positive impact on the donor country’s welfare (\( i.e., \Delta^{-1} B_{t} (d\hat{t} / d\bar{w}) < 0 \)). The result in equation (20) indicates that these effects outweigh each other, resulting to a zero net effect of the lower \( \bar{w} \) on \( u^* \). Note that, qualitatively, the same result, \( i.e., (du^* / d\bar{w}) = 0, \) emerges when there is no cross-border pollution between the two countries.

The effect of an improvement in the terms of trade affects on Nash values \( (\hat{t}, \hat{\lambda}) \) are summarized in the following Corollary:

**Corollary 2: Under (A1) and (A2), an improvement in the terms of trade lowers the Nash value of the fraction of aid allocated to public pollution abatement if**

\(^{16}\) Straightforward calculations using the optimality conditions (13)-(15) and the relevant expressions in Appendix B lead to the result in equation (20).
clean environment and good 2 are substitutes in consumption. The terms of trade improvement leaves unaffected the Nash value of the recipient’s emissions tax rate ($\hat{t}$).

**Discussion:** Using equations (C.1) we obtain:

$$\frac{d\hat{\lambda}}{dp} = (T e_{zz})^{-1} e_{pz} \left[ e_{pz} - \bar{\omega} e_{zz} \left( p_{g} R_{tt} \tilde{R}_{tt} \right)^{-1} R_{tt} \tilde{R}_{tp} \right], \text{ and } \frac{d\hat{t}}{dp} = 0. \tag{21}$$

The improvement in the terms of trade leads the polluting sector of the economy to contract in the recipient country according to (A2). The terms of trade improvement also reduces the demand for clean environment if the latter and good 2 are substitutes in consumption, i.e., $e_{pz} > 0$. This effect exerts a negative impact on the Nash value $\hat{\lambda}$ of the fraction of aid allocated to public pollution abatement, i.e., $(T e_{zz})^{-1} e_{pz} e_{pz} > 0$. Moreover, since by (A1) and (A2) $R_{tt} < 0$ and $\tilde{R}_{tp} < 0$, respectively, then the second right-hand-side term of $(d\hat{\lambda} / dp)$ is also positive. Therefore, in the present case, sufficient, but not necessary, condition for an improvement in the terms of trade to reduce the Nash value of $\hat{\lambda}$ is that good 2 and clean environment are substitutes in consumption. If any of these assumptions does not hold, then the improvement in the terms of trade may raise the Nash value of $\hat{\lambda}$. A change in the terms of trade does not affect the Nash value of the recipient’s emission tax rate. An improvement in the terms of trade reduces the pollution emissions and thus it tends to reduce the Nash value of $\hat{t}$, whereas a decrease in the amount of foreign aid that is allocated to public abatement tends to increase $\hat{t}$ in a completely offsetting way to the initial increase.

Evaluating at the Nash equilibrium the welfare implications of the terms of trade improvement, after accounting for the induced policy adjustments, we obtain:

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17 Using equations (C.1) and equation (6) after some algebra we can write $\Omega_{t}(d\hat{t} / dp) = -A_{zz} A_{g} + A_{z} A_{g}A_{zz} T^{-1} p_{g} (\hat{\omega} / \partial t) - A_{z} A_{g} T^{-1} p_{g} (\hat{\omega} / \partial t)$, where $p_{g} (\hat{\omega} / \partial t) = -(\bar{\omega} R_{tt} \tilde{R}_{tt} - R_{t})$. 

18
\[ \Delta \frac{du}{dp} = A_p \Rightarrow \frac{du}{dp} = e_u^{-1}(-M - \bar{w}R_{tL}R_{tL}^{-1} + \bar{w}R_{tL}R_{tL}^{-1}R_{uL} \tilde{R}_{uL} \tilde{R}_{uL}^{-1}R_{uL} \tilde{R}_{uL}) , \quad (22) \]

\[ \Delta \frac{du^*}{dp} = B_p + B_{\lambda} \frac{d\lambda}{dp} + B_t \frac{dt}{dp} \Rightarrow \frac{du^*}{dp} = e^{*} e^{-1}(-M^* + \theta^* e_z^* e_{z\lambda} e_{z\lambda}) , \quad (23) \]

Under the Assumptions of the model equation (22) indicates that the improvement in the terms of trade improves the recipient country’s welfare if the beneficial impact on welfare due to lower pollution emissions (i.e., the term \(w_{tL}R_{tL}^{-1}R_{uL} \tilde{R}_{uL}^{-1}R_{uL} \tilde{R}_{uL} < 0\) captures the pollution effect of a decrease in \(p\)), dominates the negative employment effect (i.e., \(R_{tL}R_{tL}^{-1} \tilde{w} > 0\)). In the present context of unemployment due to a minimum wage, the standard beneficial impact (i.e., increase in real incomes) of the improvement in the terms of trade on the country’s welfare, i.e., \(-e^{-1}M\), is weakened due to the existence of unemployment and is enhanced due to lower pollution emissions. For the donor country, the standard beneficial terms of trade improvement effect, i.e., \(-e^* e_M^*\), is augmented by the additional term \(e^{*} e^{-1} \theta^* e_z^* e_{z\lambda} e_{z\lambda}\). The latter term also exerts a positive impact on the donor’s welfare, due to the improvement in the terms of trade, if in the recipient country the non-numeraire good and clean environment are complements in consumption, i.e., \(e_{z\lambda} < 0\).

Now, consider the case where it is the donor that adjusts its Nash value of aid, due to an increase in the recipient country’s minimum wage or due to an improvement in the terms of trade, while the recipient country remains inactive. The effects of a reduction in \(\tilde{w}\) and of an improvement in the terms of trade on the Nash size of foreign aid are summarized in the following Corollary:

**Corollary 3:** A reduction in the minimum wage unambiguously raises the Nash size of foreign aid. A sufficient, but not necessary, condition for an improvement

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18 The expression for \(A_p\) in equation (22) is obtained after substituting in it (see Appendix B) the optimality conditions (13)-(15).
in the terms of trade to reduce (may increase) \((\hat{T})\) is that clean environment and good 2 are substitutes (complements) in the donor’s consumption.

**Discussion:** Differentiating the equilibrium condition (12) i.e., \(B_T = 0\) and using equations (see Appendix C) we obtain:

\[
\frac{d\hat{T}}{d\bar{w}} = -B_T^{-1}B_{T\bar{w}} < 0, \quad \text{and} \quad \frac{d\hat{T}}{dp} = -B_T^{-1}B_{Tp}.
\] (24)

The decrease in minimum wage increases employment, production and pollution emissions in the recipient country. Given the Nash values \((\hat{i}, \hat{\lambda})\), the amount of cross border pollution rises, thus inducing the donor to raise the Nash size of foreign aid to the recipient. On the other hand, an improvement in donor’s terms of trade increases real incomes and decreases the levels of cross-border pollution. Since by (A1) and (A2) respectively, \(R_{L_T} < 0\) and \(\hat{R}_p < 0\), respectively, the terms of trade improvement induces the donor to reduce the Nash amount of foreign aid when the polluting good and clean environment are substitutes in consumption (i.e., \(e_{zp}^* > 0\)). In the absence of cross-border pollution, either exogenous shock, i.e., \(d\bar{w} > 0\) or \(dp < 0\), does not affect the donor’s Nash amount of aid.

The welfare implications of the lower \(\bar{w}\), when the donor country adjusts its Nash equilibrium value of aid, after some algebraic manipulations, are given as follows:

\[
\Delta \frac{du}{d\bar{w}} = A_{\pi} + A_r \frac{d\hat{T}}{d\bar{w}} \Rightarrow \frac{du}{d\bar{w}} = \bar{w}(e_{u}R_{LL}\hat{R}_u)^{-1}(R_u - \hat{\lambda}^{-1}R_{L}\hat{R}_{L}R_{LL}^*) < 0, \quad \text{and} \tag{25}
\]

\[
\Delta \frac{du^*}{d\bar{w}} = B_{\pi^*} = \theta^* e_{z^*}^* e_{z^*}^* (p_g - t)R_{L^*} > 0, \tag{26}
\]

where, evaluated at Nash equilibrium \(A_{\pi^*} = e_{u^*}^* p_g \bar{w} \hat{R}_u^* \hat{R}_u > 0\) and \(A_r = e_{u^*}^* p_g R_{LL} < 0\). The intuition of the result in equation (25) follows previous discussion (e.g., see equation (19)) where by the positive employment-output effect of the lower \(\bar{w}\) dominates the negative effect due to lower pollution emissions, and thus overall the lower minimum wage exerts a positive impact on the recipient country’s welfare. Equation (26) indicates
that, in the presence of cross-border pollution, the lower $\tilde{w}$ unambiguously worsens the donor’s welfare. Else, when $\theta^* = 0$, then, also $du^*/d\tilde{w} = 0$.

Turning to examine at Nash equilibrium the welfare implications of the improvement in the terms of trade, accounting for the adjustments in the two countries’ Nash values of policy instruments, we obtain:

$$\Delta \frac{du}{dp} = A_p + A_t \frac{d\hat{T}}{dp} = A_p - A_t B_{rr}^{-1} B_{tp}, \text{ and}$$

$$\Delta \frac{du^*}{dp} = B_p = -e_a p_p R_{tl} [M^* + \lambda^{-1}(t - p_p) \hat{R}_p],$$

Equation (27) indicates that, in general, the welfare effect of an improvement in the terms of trade is ambiguous. The improvement in the terms of trade increases the recipient country’s welfare if it leads to a rise of the amount of aid received i.e., $\frac{d\hat{T}}{dp} < 0$, and also, if the beneficial impact from lower pollution emissions outweighs the negative impact on welfare due to aggravation of unemployment. Equation (28) shows that under the assumptions of the model, an improvement in the donor’s terms of trade unambiguously improves its welfare. The first right hand term captures the standard positive direct effect of an improvement in a country’s terms of trade. The second term enhances the previous positive impact of the terms of trade due to lower level of cross border pollution.

The welfare implications of $d\tilde{w} > 0$ and $dp < 0$ accounting for the induced adjustments in the countries’ Nash policy are summarized in the following proposition.

**Proposition 1**: Consider the case of two small open economies with cross-border pollution, foreign aid, and public pollution abatement and unemployment in the recipient country.

a. Let, due to exogenous shocks, only the recipient country adjust the Nash values of its policy instruments, while the donor does not, then:

(i) a decrease in the recipient country’s minimum wage raises its welfare, and leaves the welfare of the donor unchanged,
(ii) an improvement in the terms of trade improves the recipient’s welfare if the pollution effect dominates the employment effect. It improves the donor’s welfare if good 2 and clean environment are complements in consumption (sufficient condition) in the recipient country.

b. Let only the donor country adjust the Nash amount of aid, while the recipient does not adjust the Nash values of its policy instruments, then:

(i) a decrease in the recipient’s minimum wage improves the country’s welfare and it worsens the welfare of the donor,

(ii) an improvement in the terms of trade improves, under the assumptions of the model, the donor’s welfare, and it may worsen the welfare of the recipient if good 2 is a substitute to clean environment in consumption in the donor country.

5.2. Case 2: The recipient country adjusts only the Nash value of \((\lambda)\) and the donor the Nash value of \((T)\).

In this case, differentiating (11) and (12) we obtain:

\[
A_{\lambda\lambda} d\lambda + A_{\lambda T} dT = -A_{\lambda \pi} d\tilde{w} - A_{\lambda p} dp, \quad \text{and} \quad (29)
\]
\[
B_{T\lambda} d\lambda + B_{T T} dT = -B_{T \pi} d\tilde{w} - B_{T p} dp, \quad (30)
\]

The effects of a reduction in the minimum wage on \((\hat{\lambda}, \hat{T})\) are given in the following:

Corollary 4: A reduction in \(\tilde{w}\) unambiguously raises the Nash size of foreign aid, while it leaves unaffected its fraction allocated by the recipient to public pollution abatement.

Discussion: Using equations (C.2), the effect of a decrease in the minimum wage on the Nash values \((\hat{\lambda}, \hat{T})\) are as follows:
The decrease in minimum wage increases cross-border pollution and thus induces the donor to raise the Nash size of foreign aid. An increase in the amount of foreign aid tends to reduce the fraction of aid allocated to public abatement, while an increase in the pollution emissions due to expansion of the polluting sector in the recipient country increases \( \hat{\lambda} \). Equation (31) suggests that these two effects cancel each other out. Thus, provided that the recipient country maintains a constant Nash emissions tax rate, i.e., \( \hat{d}\hat{t} = 0 \), while the donor raises its Nash equilibrium amount of aid, the recipient country, at the Nash equilibrium, does not change the fraction \( \hat{\lambda} \) of aid allocated to public pollution abatement.

The effect of the lower \( \bar{w} \) on the two countries welfare, accounting for the induced adjustments in the Nash equilibrium values of aid and fraction of aid, are given as follows:

\[
\Delta \frac{du}{d\bar{w}} = A_u + A_{\bar{w}} \frac{d\hat{T}}{d\bar{w}} \Rightarrow \frac{du}{d\bar{w}} = (e_u R_{uu} \tilde{R}_u)^{-1} \bar{w} [R_u - (\lambda R_{nu})^{-1} R_{w}^2] < 0 , \quad \text{and} \quad (32)
\]

\[
\Delta \frac{du^*}{d\bar{w}} = \theta^* e^* e^*_z (p^*_z - t) R_{u} > 0 \Rightarrow \frac{du^*}{d\bar{w}} = \bar{w} \theta^* (e^*_u p^*_z)^{-1} e^*_z R_{u} R_{u}^{-1} R_{u}^2 > 0 . \quad (33)
\]

The positive effect of the lower minimum wage on the recipient country’s welfare follows the intuition of the results in equations (19) and (25). Equation (33) indicates an unambiguous worsening in the donor’s welfare due to the reduction in the recipient country’s minimum wage.

An improvement in the terms of trade entails the following effects on the Nash values of \( \lambda \) and \( T \):

\[
\frac{d\hat{\lambda}}{dp} = \lambda (e^*_z e^*_z)^{-1} (\theta^* e^*_z e^*_z - e^*_z e^*_z) , \quad \text{and} \quad (34)
\]
Equations (34) and (35) indicate, by use of the optimality condition (15), that in the absence of cross-border pollution, changes in the terms of trade do not affect the Nash values of the two policy instruments. In the presence of cross-border pollution, however, equation (34) indicates that the sign of the effect of an improvement in the terms of trade on the Nash value of $\lambda$ depends on the relationship between good 2 and clean environment in the two countries’ consumption. For example, the sufficient and necessary condition for the improvement in the terms of trade to raise $\hat{\lambda}$, i.e., $d\hat{\lambda}/dp < 0$, is that $e_{x2}^* > 0$ and $e_{x2}^* > 0$, and it is $\frac{|e_{x2}^*|}{e_{x2}^*} < \theta^* \frac{|e_{x2}^*|}{e_{x2}^*}$ when good 2 and clean environment are complements in consumption. To better illustrate these results, the following Lemma considers a special case of the general analysis.

**Lemma 1:** Suppose the two countries have identical preferences. Then, a terms of trade improvement raises (lowers) the Nash value of $\lambda$ if good 2 and clean environment are substitutes (complements) in consumption (necessary and sufficient condition). It reduces (may increase) the Nash size of $T$ if good 2 and clean environment are substitutes (complements) in consumption (sufficient condition).

**Discussion:** Making use of the assumption of identical preferences, Nash equilibrium conditions (13) and (15) become $e_z = e_{x2}^* = p_x$, and $\lambda \theta^* = 1$ respectively. Substituting these into equations (34) and (35), respectively, we obtain:

$$\frac{d\hat{\lambda}}{dp} = -(1 - \theta^*) \lambda e_{x2}^* e_z^{-1},$$

(36)
\[ \frac{d \tilde{T}}{dp} = \left[ T(1 - \theta') e_{z, z}^{-1} + \lambda^{-1} (e_{z, z}^{-1} e_{z}^{-1} + (t - p_g) \tilde{R}_g) \right] \].

(37)

Then, Lemma 1 follows straightforwardly.

Turning to examine at Nash equilibrium the welfare implications of the improvement in the terms of trade, accounting for the adjustments in the two countries’ Nash values of policy instruments, and also assuming identical preferences in the two countries, we obtain:

\[ \Delta \frac{du}{dp} = A_p + A_T \frac{d \tilde{T}}{dp} \Rightarrow \]

\[ e_u \frac{du}{dp} = -M - R_{1,1}^{-1} R_{1, p} \bar{w} + (1 - \lambda / \lambda)(t - p_g) \tilde{R}_p + \left[ T e_{z, z}^{-1} (1 - \theta') + e_{z, z}^{-1} \lambda^{-1} \right] \]  

(38)

\[ \Delta \frac{du^*}{dp} = B_p + B_{\lambda} \frac{d \lambda}{dp} \Rightarrow \]

\[ e_u^* \frac{du^*}{dp} = -(M^* + \lambda^{-1} (t - p_g) \tilde{R}_p) - e_{z, z}^{-1} \lambda^{-1} (1 - \theta') T \].

(39)

Equation (38) indicates that, in general, the terms of trade improvement on the recipient’s welfare is ambiguous. The terms of trade improvement increases welfare due to the higher volume of imports, and due to the reduction in pollution emissions, while exacerbates unemployment problem according to the assumptions of the problem. The last term is the impact of the improved terms of trade on the recipient’s welfare through the induced adjustments in the Nash amount of foreign aid. Through this term, the improvement in the terms of trade exerts a negative impact to the recipient country’s welfare if good 2 and clean environment are substitutes in consumption. Furthermore, the improved terms of trade increase the donor’s welfare if good 2 and clean environment are substitutes in consumption. Finally, in the absence of cross-border pollution, adjustments in the Nash values of the policy instruments \( \lambda \) and \( T \), due to the terms of trade improvement, do not affect either country’s welfare.

The welfare implications of \( d \bar{w} > 0 \) and \( dp < 0 \) accounting for the induced adjustments in the countries’ Nash policy are summarized in the following proposition.
Proposition 2: Consider the case of two small open economies with cross-border pollution, foreign aid, and public pollution abatement and unemployment in the recipient country. Let, due to exogenous shocks, the recipient country adjust the Nash value of the fraction of aid allocated to public pollution abatement, and the donor country adjust the Nash amount of foreign aid. Then:

(i) a decrease in the recipient country’s minimum wage increases its welfare, and decreases the welfare of the donor country,

(ii) assuming identical preferences in the two countries, an improvement in the terms of trade increases the donor’s welfare and may reduce the recipient’s welfare if good 2 and clean environment are substitutes in consumption (sufficient condition).

5.3. Case 3: The recipient country adjusts only the Nash value of \((t)\) and the donor the Nash value of \((T)\).

In this case, differentiating (10) and (12) we obtain:

\[
A_{t}dt + A_{s}d\lambda = -A_{m}d\bar{w} - A_{q}dp , \quad \text{and} \quad (40)
\]
\[
B_{t}dt + B_{s}dT = -B_{m}d\bar{w} - B_{q}dp . \quad (41)
\]

The following Corollary states the impact of a reduction in the minimum wage on the Nash values \((\hat{t}, \hat{T})\).

**Corollary 5:** A reduction in \(\bar{w}\) unambiguously raises the Nash rate of the recipient’s emission tax and it reduces the Nash size of foreign aid.

**Discussion:** Using equations (C.3), the effect of a decrease in the minimum wage on the Nash values \((\hat{t}, \hat{T})\) are as follows:
\[
\frac{d\hat{t}}{d\hat{w}} = -(R_{ttL} \tilde{R}_u)^{-1} R_{tt} < 0, \quad \text{and} \quad \frac{d\hat{T}}{d\hat{w}} = -(\lambda R_{ttL} \tilde{R}_u)^{-1} R_{tt} R_t > 0. \tag{42}
\]

A reduction in the minimum wage raises employment, outputs and production generated pollution. As a result the recipient country’s government increases the Nash value \((\hat{t})\) of the emission tax rate. On the other hand, this increase in \((\hat{t})\) raises the recipient’s emission tax revenue that is used to finance the provision of public pollution abatement. As a result, pollution emissions and cross-border pollution fall resulting to a reduction in the Nash equilibrium amount of foreign aid by the donor.

Turning to the welfare effects of the lower \(\hat{w}\), accounting for the adjustments in the Nash values of the policy instruments we obtain:

\[
\Delta \frac{du}{d\hat{w}} = A_\pi + A_T \frac{d\hat{T}}{d\hat{w}} \Rightarrow \frac{du}{d\hat{w}} = (e_u^* R_{ttL} \tilde{R}_u)^{-1}(\hat{w} R_u - \lambda^{-1} R_{tt} R_t), \quad \text{and} \tag{43}
\]

\[
\Delta \frac{du^*}{d\hat{w}} = B_\pi + B_T \frac{d\hat{T}}{d\hat{w}} \Rightarrow \frac{du^*}{d\hat{w}} = (e_u^* R_{ttL} \tilde{R}_u)^{-1} R_{tt} R_t \lambda^{-1} < 0. \tag{44}
\]

The lower minimum wage entails an ambiguous effect, through changes in the level of pollution emissions, on the recipient country’s level of welfare. On the one hand, as shown by equations (42), the induced increase in \((\hat{t})\) results in lower levels of pollution and this exerts a positive impact on the country’s welfare. This effect is captured by the term \((e_u^* R_{ttL} \tilde{R}_u)^{-1} \hat{w} R_u\) of equation (43). At the same time, however, the lower \(\hat{w}\) entails a expansionary effect on employment and output, and thus on pollution emissions which affects the country’s welfare negatively. This is the \(- (\lambda e_u R_{ttL} \tilde{R}_u)^{-1} R_{tt} R_t\) term of equation (43). Depending on the relative size of these two effects, the lower \(\hat{w}\) may decrease or increase the recipient’s welfare. Observing equation (44) we infer that the lower \(\hat{w}\) entails a negative impact on the donor’s welfare due to the induced increase in pollution as a result of higher employment and production in the recipient country, \textit{i.e.}, \(\Delta^{-1} B_\pi > 0\), and a positive one due to the induced decrease in pollution as a result of the increase in the recipient’s Nash emission tax rate, \textit{i.e.}, \(\Delta^{-1} B_T(d\hat{t} / d\hat{w}) < 0\). Given that
\( (du^* / d\tilde{w}) < 0 \) the latter effect outweighs the former. Thus, in this case, the recipient’s lower minimum wage has a beneficial effect on the donor country’s welfare.

An improvement in the terms of trade entails the following effects on the Nash values of \( \lambda \) and \( T \):

\[
\frac{d\tilde{t}}{dp} = (\theta^* p_g e^*_z, \tilde{R}_g)^{-1} (\tilde{w}R_{zz}^{-1} R_{zz} - R_g) \left( \theta^* e^*_z - e^*_z e^*_p \right), \quad \text{and} \quad (45)
\]

\[
\frac{d\tilde{T}}{dp} = (\lambda \theta^* e^*_z, \tilde{R}_n)^{-1} \left[ p_g^{-1} (\tilde{w}R_{zz}^{-1} R_{zz} - R_n) \right] \left( e^*_z e^*_p - \theta^* e^*_z e^*_p \right)
+ \tilde{R}_n (e^*_z e^*_p - e^*_z, \tilde{w}\theta^* (R_{zz}^{-1} R_{zz}^{-1} R_{zz} \tilde{R}_n)). \quad (46)
\]

Equation (45) indicates that the sign of the effect of an improvement in the terms of trade on the Nash value of \( \lambda \) depends on the relationship between good 2 and clean environment in the two countries’ consumption. Following the discussion of equation (34), the sufficient and necessary condition for the improvement in the terms of trade to raise \( \tilde{t} \), i.e., \( d\tilde{t} / dp < 0 \), is that \( \frac{e^*_z}{e^*_p} > \frac{\theta^*}{e^*_z} \), assuming that the two are substitutes in consumption, i.e., \( e^*_p > 0 \) and \( e^*_z > 0 \), and it is \( \left| \frac{e^*_z}{e^*_p} \right| < \theta^* \left| \frac{e^*_p}{e^*_z} \right| \) when good 2 and clean environment are complements in consumption. Equation (46) indicates that in general the effect of the terms of trade improvement on the Nash amount of foreign aid is ambiguous.

Under the assumptions of the model, sufficient, but not necessary, conditions for \( \tilde{T} \) to increase as a result of the terms of trade improvement are: (i) good 2 and clean environment to be complements in consumption, (ii) \( \left| \frac{e^*_z}{e^*_z} \right| > \theta^* \left| \frac{e^*_z}{e^*_z} \right| \), and (iii) the numeraire good is the country’s polluting commodity, i.e., \( \tilde{R}_p > 0 \). Following the discussion of equations (34), (35) and of Lemma 1, Lemma 2 below states the results of equations (45) and (46) again for the special case where preferences in the two countries are identical.
Lemma 2: Suppose the two countries have identical preferences. Then, a terms of trade improvement lowers (raises) the Nash value of $t$ if good 2 and clean environment are complements (substitutes) in consumption (necessary and sufficient condition). It reduces (may increase) the Nash size of $T$ if good 2 and clean environment are substitutes (complements) in consumption (sufficient condition).

Finally, examining at Nash equilibrium the welfare implications of the improvement in the terms of trade, accounting for the adjustments in the two countries’ Nash values of policy instruments, we obtain:

$$
e_u \frac{du}{dp} = -M - R_{zz}^{-1} R_{zp} w + (1 - \lambda/\theta) (t - p_g) \tilde{R}_p + (R_{zy} p_g)^{-1} (\tilde{w} R_{zz}^{-1} R_{zy} - R_z)^2 e_g (1 - \theta') + e_{zy} e_z^{-1}
$$

$$
e_u^* \frac{du}{dp} = -M^* - \lambda^2 (t - p_g) \tilde{R}_p + e_g (1 - \theta') [(t - p_g) \tilde{R}_u + R_t] (\tilde{w} R_{zz}^{-1} R_{zy} - R_z) (p_g \tilde{R}_u)^{-1}.
$$

(47)

An improvement in terms of trade increases the donor’s welfare and may reduce recipient’s welfare if the good 2 and clean environment are substitutes in consumption.

The welfare implications of $d\tilde{w} > 0$ and $dp < 0$ accounting for the induced adjustments in the countries’ Nash policy are summarized in the following proposition.

**Proposition 3:** Consider the case of two small open economies with cross-border pollution, foreign aid, and public pollution abatement and unemployment in the recipient country. Let, due to exogenous shocks, the recipient country adjusts the Nash value of its emission tax rate, and the donor country adjusts the Nash amount of foreign aid. Then:

(i) a decrease in the recipient country’s minimum wage may worsen the country’s welfare if it also reduces the Nash amount of foreign aid to it, and it unambiguously improves the welfare of the donor country,

(ii) assuming identical preferences in the two countries, an improvement in the terms of trade improves the donor’s welfare and may reduce recipient’s welfare if the good 2 and the clean environment are substitutes in consumption (sufficient condition).
5.4. Case 4: The recipient country adjusts the Nash values of its policy instruments \((t, \lambda)\) and the donor the Nash value of \((T)\).

In this case, differentiating (10), (11) and (12) we obtain:

\[
A_{w} dt + A_{t} d\lambda = -A_{w} d\bar{w} - A_{y} dp, \tag{48}
\]

\[
A_{\lambda} d\lambda + A_{\lambda t} dT = -A_{\lambda} d\bar{w} - A_{\lambda t} dp, \quad \text{and} \tag{49}
\]

\[
B_{w} dt + B_{t} dT = -B_{w} d\bar{w} - B_{y} dp. \tag{50}
\]

Using equations (C.4), the effect of a decrease in the minimum wage on the Nash values \((\hat{\lambda}, \hat{t}, \hat{T})\) are as follows:

\[
\frac{dt}{d\bar{w}} = -R_{t1}^{-1}R_{w1}\bar{R}_{w1}^{-1} < 0, \quad \frac{d\lambda}{d\bar{w}} = 0, \quad \text{and} \quad \frac{dT}{d\bar{w}} = (-R_{t})R_{t1}^{-1}\bar{R}_{w1}^{-1}\lambda^{-1} > 0. \tag{51}
\]

A reduction in \(\bar{w}\) unambiguously raises the Nash rate of the recipient’s emission tax, it has no effect on its fraction of aid allocated by the recipient to public pollution abatement, while it reduces the Nash size of foreign aid.

Using equations (C.4) and assuming identical preferences in the two countries, an improvement in the terms of trade entails the following effects on the Nash values of \(\hat{\lambda}, \hat{t}, \hat{T}\):

\[
\frac{d\hat{T}}{dp} = \left[e_{z}^{-1}e_{z}^{-1}(1-\theta') + \lambda^{-1}(e_{t}e_{y}e_{z}^{-1} + (t - p_{g})\tilde{R}_{p})\right],
\]

\[
\frac{dt}{dp} = 0, \quad \text{and} \quad \frac{d\lambda}{dp} = -e_{z}^{-1}e_{z}\lambda(1-\theta'). \tag{52}
\]

An improvement in the terms of trade increases the Nash value of the fraction of aid allocated to public pollution abatement and reduces the Nash amount of foreign aid if clean environment and polluting good are substitutes in consumption, while it leaves unaffected the Nash rate of the emission tax.

Turning to the welfare effects from a decrease in the minimum wage and accounting for the adjustments in the Nash values of the policy instruments we obtain the
same results as in equations (43) and (44) and the analysis follows that of Proposition 3(i). Regarding the impact of a term of trade improvement on donor and recipient countries’ welfare the analysis follows that of Proposition 2(ii), and equations (38) and (39).

6. Conclusions

One of the mechanisms that the international organizations and developed countries apply for managing cross-border externalities (e.g. pollution, migration) is international transfers. Nevertheless, in the last decades there is a considerable public debate on issues such as “aid fatigue” and “aid effectiveness”. The interaction between cross-border externalities and foreign aid, and also the link between this interaction and the rationales for international transfers will be major problematic facing aid agencies.

Motivated by such considerations, we develop a two-country model with unemployment in the recipient country and cross border pollution from this country to the donor. Pollution is abated by both private and public sectors. Pollution tax revenue and a fraction of foreign aid are earmarked for the financing of public abatement activities. Within this framework we examine how labor market reforms and exogenous terms of trade shocks affect this link between cross border externalities and foreign aid. Moreover, we examine the optimal response of the two countries in reaction to two exogenous shocks, when the donor chooses the amount of aid and the recipient decide on emission tax rate and on the fraction of aid allocated to public abatement. Whether the impact of the exogenous shocks on countries’ welfare is positive or negative it depends on: i) the relative strength of the two opposing effects, unemployment and pollution effects, ii) the relationship between the clean environment and the polluting good in consumption, and iii) the endogeneity of foreign aid.
APPENDIX A: The Reduced Form System of Equations (1), (3), (4) and (5)

\[
\begin{bmatrix}
e_u & 0 & -e_z & -(\bar{w} + e_z R) \\
e_u^* & -\theta^* e_z & -\theta^* e_z R_{LL} \\
0 & p_g & tR_{LL} \\
0 & 0 & 0 & R_{LL}
\end{bmatrix}
\begin{bmatrix}
du \\
du^* \\
dg \\
DL
\end{bmatrix}
= \begin{bmatrix}
-M + e_z R_{tp} \\
-M^* + e_z^* \theta^* R_{tp} \\
tR_{tp} \\
-R_{tp}
\end{bmatrix}
dp + \begin{bmatrix}
R_t + e_z R_{nt} \\
e_z^* \theta^* R_{nt} \\
-(R_t + tR_{nt}) \\
-R_{Lt}
\end{bmatrix}
dt
\]

\[
(1 - \lambda)
\begin{bmatrix}
-1 \\
\lambda \\
0
\end{bmatrix}
dT + \begin{bmatrix}
-T \\
T \\
0
\end{bmatrix}
d\lambda + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
d\bar{w},
\]

where \( M(=e_p - R_p) \) is the recipient country’s imports-exports vector, and similarly is \( M^* \) defined for the donor country. The determinant of the matrix of coefficients of the unknowns is \( \Delta = e_u^* p_g R_{LL} \), which is negative.

APPENDIX B: Coefficients of Equations (5) and (6)

\[
A_t = e_u^* R_{LL} \left\{ p_g R_t - e_z \left[ (t - p_g) \tilde{R}_{tt} + R_t \right] - p_g \bar{w} R_{LL}^{-1} R_{LL} \right\} = \]

\[
e_u^* R_{LL} \left[ -(t - p_g) \tilde{R}_{tt} + (p_g - e_z) R_t - \bar{w} p_g R_{LL}^{-1} R_{LL} \right]
\]

\[
A_T = e_u^* R_{LL} \left[ p_g - \lambda (p_g - e_z) \right] < 0, \quad A_\lambda = -e_u^* R_{LL} T (p_g - e_z),
\]

\[
A_w = e_u^* \left[ p_g \bar{w} - e_z \left( t - p_g \right) R_{LL} \right],
\]

\[
A_p = -e_u^* R_{LL} \left[ p_g M + p_g R_{LL}^{-1} R_{tp} \bar{w} + e_z \left( t - p_g \right) \tilde{R}_{tp} \right],
\]

\[
B_t = -\theta^* e_u^* R_{LL} \left[ (t - p_g) \tilde{R}_{tt} + R_t \right], \quad B_T = -e_u^* R_{LL} \left( p_g - \lambda \theta^* e_z^* \right),
\]

\[
B_\lambda = \theta^* e_u^* R_{LL} T < 0, \quad B_w = \theta^* e_u^* (p_g - t) R_{LL},
\]

\[
B_p = e_u^* R_{LL} \left[ M^* p_g - \theta^* e_z^* \left( t - p_g \right) \tilde{R}_{tp} \right].
\]
APPENDIX C: Expressions of Coefficients in Section 4, evaluated at Nash equilibrium:

**Case 1:** The reduced form system is as follows:

\[
\begin{bmatrix}
A_u & A_{\lambda,t} \\
A_{\lambda,t} & A_{\lambda,\lambda}
\end{bmatrix}
\frac{d\hat{t}}{dt} = \begin{bmatrix}
-A_{\lambda,\pi} \\
-A_{\lambda,\pi}
\end{bmatrix} d\hat{w} + \begin{bmatrix}
-A_{\lambda,\pi} \\
-A_{\lambda,\pi}
\end{bmatrix} dp, \quad (C.1)
\]

where, \( \Omega_1 = \det \begin{bmatrix}
A_{\lambda,t} & A_{\lambda,\lambda} \\
A_{\lambda,\lambda} & A_{\lambda,\lambda}
\end{bmatrix} = (A_u A_{\lambda,\lambda} - A_{\lambda,t} A_{\lambda,\lambda}) = T^2 e_u^2 e_{zz} R_{LL}^2 \tilde{R}_{tt} > 0 ,
\]

\[ A_{\lambda,t} = -e_u^* R_{LL} \left[ e_{zz} p^{-1} (\tilde{w} R_{LL}^{-1} R_{tt} - R_{tt})^2 + e_{zz} \tilde{R}_{tt} \right] > 0 , \]

\[ A_{\lambda,\lambda} = -e_u^* e_{zz} R_{LL} p^{-1} T^2 > 0 , \]

\[ A_{\lambda,\pi} = e_u^* R_{LL} T \left[ e_{zp} - e_{zz} \tilde{w} (R_{LL} p \tilde{R}_{tt})^{-1} R_{tt} \tilde{R}_{tp} \right] , \]

\[ A_{\lambda,\pi} = -e_u^* R_{tt} \left[ e_{zp} - e_{zz} \tilde{w} (R_{LL} p \tilde{R}_{tt})^{-1} R_{tt} \tilde{R}_{tp} \right] , \]

\[ A_{\lambda,\pi} = -e_u^* e_{zz} T R_{tt}^2 \tilde{w} (p \tilde{R}_{tt})^{-1} > 0 , \]

\[ B_{\lambda,\pi} = \theta \lambda e_u R_{LL} \left[ e_{zp}^* - \theta \tilde{e}_{zz} (p \tilde{R}_{tt})^{-1} R_{tt} \tilde{R}_{tp} \right] , \]

\[ B_{\lambda,\pi} = -e_u^* e_{zz} R_{tt}^2 \tilde{w} (p \tilde{R}_{tt})^{-1} > 0 , \]

\[ B_{\lambda,\pi} = -p g^{-1} \theta \lambda e_u \tilde{e}_{zz} R_{tt}^2 \tilde{w} (p \tilde{R}_{tt})^{-1} > 0 . \]

**Case 2:** The reduced form system is as follows:

\[
\begin{bmatrix}
A_{\lambda,\lambda} & A_{\lambda,\lambda} \\
B_{\lambda,\lambda} & B_{\lambda,\lambda}
\end{bmatrix}
\frac{d\hat{T}}{dt} = \begin{bmatrix}
-A_{\lambda,\pi} \\
-B_{\lambda,\pi}
\end{bmatrix} d\hat{w} + \begin{bmatrix}
-A_{\lambda,\pi} \\
-B_{\lambda,\pi}
\end{bmatrix} dp, \quad (C.2)
\]

where, \( \Omega_2 = \det \begin{bmatrix}
A_{\lambda,\lambda} & A_{\lambda,\lambda} \\
B_{\lambda,\lambda} & B_{\lambda,\lambda}
\end{bmatrix} = (A_{\lambda,\lambda} B_{\lambda,\lambda} - B_{\lambda,\lambda} A_{\lambda,\lambda}) = e_u^* e_{zz} T R_{tt}^2 > 0 ,
\]

\[ A_{\lambda,\lambda} = -p g^{-1} \lambda e_u e_{zz} R_{tt} > 0 , \]

33
\[ B_{TT} = -p_g^{-1}\theta e_u R_{LL} e^{*}_{z^*} > 0 \]
\[ B_{T\lambda} = \theta e_u R_{LL} \left[ e^{*}_{z^*} - p_g^{-1}\theta T e^{*}_{z^*} \right] \]
\[ B_{T\pi} = -\theta^2 \lambda e_u e^{*}_{z^*} . R_{tt}^2 \bar{w} \left( p_g R_{LL} \tilde{R}_{tt} \right)^{-1} > 0 \]
\[ B_{T\rho} = \theta e_u R_{LL} \left[ e^{*}_{z^*} e^{*}_{p} - \theta \bar{w} e^{*}_{z^*} \left( p_g R_{LL} \tilde{R}_{tt} \right)^{-1} R_{tt} \tilde{R}_{tp} \right] \]

**Case 3:** The reduced form system is as follows:

\[
\begin{bmatrix}
A_u & A_{IT} \\
B_{TI} & B_{TT}
\end{bmatrix}
\begin{bmatrix}
d\bar{w} \\
dT
\end{bmatrix} = \begin{bmatrix}
-A_{\bar{w}} \\
-B_{\bar{w}}
\end{bmatrix} d\bar{w} + \begin{bmatrix}
-A_{\rho} \\
-B_{\rho}
\end{bmatrix} d\rho, \quad (C.3)
\]

where, \( \Omega_3 = \det \begin{bmatrix}
A_u & A_{IT} \\
B_{TI} & B_{TT}
\end{bmatrix} = (A_u B_{TT} - B_{TI} A_{IT}) = \theta^2 \lambda^2 e_u e^{*}_{z^*} . R_{LL}^2 \tilde{R}_{tt} > 0 \),
\[ A_{IT} = -p_g^{-1}\lambda e_u e^{*}_{z^*} R_{ll} \left( \bar{w} R_{ll}^{-1} R_{tt} - R_t \right) > 0 \],
\[ B_{TI} = -\theta^2 \lambda e_u e^{*}_{z^*} R_{ll} p_g^{-1} \left( \bar{w} R_{ll}^{-1} R_{tt} - R_t \right) > 0 \]

**Case 4:** The reduced form system is as follows:

\[
\begin{bmatrix}
A_{\lambda} & A_{\lambda\lambda} & A_{\lambda T} \\
A_{\lambda T} & A_{\lambda\lambda} & A_{\lambda T} \\
B_{T\lambda} & B_{TT} & B_{TT}
\end{bmatrix}
\begin{bmatrix}
d\lambda \\
dt \\
dT
\end{bmatrix} = \begin{bmatrix}
-A_{\lambda\lambda} \\
-A_{\lambda\lambda} \\
-B_{\lambda\lambda}
\end{bmatrix} d\lambda + \begin{bmatrix}
-A_{\lambda\rho} \\
-A_{\lambda\rho} \\
-B_{\lambda\rho}
\end{bmatrix} d\rho \quad (C.4)
\]

where, \( \Omega_4 = \det \begin{bmatrix}
A_{\lambda} & A_{\lambda\lambda} & A_{\lambda T} \\
A_{\lambda T} & A_{\lambda\lambda} & A_{\lambda T} \\
B_{T\lambda} & B_{TT} & B_{TT}
\end{bmatrix} = e^{*}_{z^*} \theta e_u A_{\lambda\lambda} R_{LL}^2 \tilde{R}_{tt} \bar{w} . T^{-1} \lambda e^{*}_{z^*} > 0 \).
References


----------, 1998, Consequences of environmental tax reform for unemployment and welfare, Environmental and Resource Economics 12, 137-150.


