Registration taxes on cars inducing international price
discrimination: an optimal tariff approach

Switgard Feuerstein
University of Nottingham and University of Erfurt
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Abstract:

The EU car market is characterized by large price differences. Pre-tax prices are particularly low in countries with a high registration tax on cars (e.g. Denmark and Finland), that do not have an own car production meaning that an import tariff and a specific tax are equivalent. The European Commission aims at reducing the car price differences by facilitating commercial arbitrage, albeit up to now with very limited success. The paper studies what effects it would have if this EU policy was successful.

When the tax rate is exogenous, a fall in the maximum feasible price differences leads to higher prices in the importing country, which reduces the volume of trade, and to lower prices in the producing country. When the tax optimally in the importing country, a change in arbitrage costs has additional effects by altering the tax rate. The tax-inclusive price and thus the volume of imports depends non-monotonically on arbitrage costs. Moreover, a corner solution becomes relevant in which the price in the producing country is unaffected when the scope to price discriminate becomes larger.

Welfare effects also depend on whether the tax rate is endogenous or exogenous. With an exogenous tax rate, price discrimination increases welfare although total quantity does not increase, which stands in contrast to the standard welfare result of Varian (1985). In contrast, welfare falls for a wide range of parameter values when the tax is endogenous, as the optimal tax rate increases in response to a rising maximum price difference.
1 Introduction

Car prices within the European Union differ substantially, and the price difference for an identical new car may amount to several thousands euros. Pre-tax prices are particularly low in countries with high registration taxes on cars, which do not have an own car production. In the EU, car prices are the lowest in Denmark that has a registration tax of more that 100%. In Finland, which also has low pre-tax car prices, the tax amounts to around 30%. For instance, in May 2006 the net price of an Opel Vectra was 21000 euros in Germany to which only VAT has to be added. In contrast, the pre-tax price was only 15800 euros in Denmark, but including VAT and the registration tax, a Danish consumer has to pay more than 40000 euros for the car (see table 1).

For arbitrage within the European internal market, the pre-tax prices are relevant, as the tax rates of the country where the car is registered apply. Thus although a Danish consumer pays a much higher tax-inclusive price, German consumers would like to buy their car in Denmark - which often means a reimport - and not vice versa.

Note that without domestic production, a specific tax cannot be distinguished from the tariff. Like a tariff, the tax may lower the import price and may thereby increase domestic welfare, and the ideas of optimal tariff theory apply.

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Table 1: Pre-tax and tax-inclusive prices for an Opel Vectra, May 2006.
(Source: European Commission 2006)

The European Commission aims at reducing the large price differences, as they run counter the idea of the internal market, and wants to foster (commercial) arbitrage, and it has for instance introduced new rules for distribution systems of cars, as the contracts with the car dealers were thought of enabling car producers to segment markets. However, up to now price differences have only decreased slightly (European Commission 2006).

1 Other countries with a substantial registration tax include the Netherlands, Ireland and Spain. Annual taxes on cars may have similar effects as taxes that have to be paid for new cars. Empirical studies on European car prices include Goldberg and Verboven (2001), Ginsburgh (1985), Lutz (2004) and Verboven (2005).
In this paper, it is analysed what effects it would have if the policy of the European Commission was successful and the high price differences could not be sustained. The focus is on price discrimination that is induced by the registration taxes. Thus in the theoretical model, the tax is the only incentive for the producers to set different prices. When the tax rate is exogenous, a larger scope to price discriminate leads to a price increase in the high price country and to a price decrease in the low price country. In particular, this means that a further market integration that lowers the maximum feasible price difference, increases the price in the importing country and thus the volume of trade falls.

Moreover, price discrimination increases welfare, although total quantity remains constant. This result stands in contrast to Varian’s (1985) well-known result of, that price discrimination causes a misallocation among consumers and can therefore only improve welfare if the total quantity increases. However, the result but can be explained by the same intuition. Due to the tax, the producer price is low in the importing country, but the consumer price is high. The price discrimination thus mitigates the misallocation from the tax.

When the tax rate set optimally by the government of the importing country (and is thus endogenous) there are in addition indirect effects of better arbitrage possibilities that have to be considered as the optimal tax will change. The maximum feasible price difference (assuming that a higher price difference would trigger commercial arbitrage) is not only a restriction for the price setting of the firms, but also limits the scope of the government to induce a price difference. The lower the maximum feasible price difference is, the lower is the optimal tax and the higher is the pre-tax price in in the importing country.

When the government maximizes welfare taking into account that markets are not segmented but are linked by the possibility of arbitrage, two kinds of solutions occur depending on the parameter values. In an interior solution, the arbitrage condition remains binding if the tax rate is marginally changes. In this case, the tax has an impact also on the price in the producing country, where no registration tax is levied. In the corner solution, it is optimal to set the tax that high, that the maximum possible price difference is induced and the arbitrage condition just becomes binding without affecting the price of the producing country. In this case, the price in the producing country is independent of the scope to price differentiate. The corner solution is particularly relevant when the importing country is small and has only little impact on the world market.

Due to these two cases, the tax-inclusive price and thus the volume of imports depends non-monotonically on the maximum feasible price difference. When markets become less
integrated, the tax-inclusive price decreases in the interior solution and imports rise, but it increases in the corner solution. Moreover, when the scope to price discriminate becomes larger, total quantity falls (as the optimal tax rises), welfare in the importing country increases and welfare in the producing country decreases, and the latter effect usually dominates.

The welfare effects of the policy of the European Commission thus hinge on the question, whether the registration tax is exogenous or whether it is itself a function of the degree of market segmentation. When the tax is exogenous, a further market integration lowering the maximum price difference lowers decreases welfare. In contrast, welfare increases when the maximum feasible price difference decreases, when the registration tax is set optimally, because the tax rate will be decreased.

The paper is organized as follows. Section 2 sets out the model. Section 3 briefly discusses the case of an exogenous tax rate, whereas the main part of the paper is in section 4 where the interaction of price discrimination and optimal taxation is analysed. Section 5 concludes.

2 The Model

There are two countries, in one of which the good is produced whereas the other country imports it. It is assumed that the good is supplied by a monopolist as the basic effects do not depend on this assumption. Marginal costs $c$ are constant. The importing country levies a consumption tax (for instance a registration tax) on the good that induces an incentive for price discrimination. Demand in the producing country equals

$$X = a - bP, \quad (1)$$

where $X$ denotes quantity and $P$ the price. In the importing country, the quantity is denoted by $x$ and the producer price by $p$. Demand depends, however, on the tax-inclusive price $\rho$

$$x = \alpha(a - \rho b). \quad (2)$$

2It can be shown that the qualitative effects are the same when there are two symmetric firms with Bertrand competition in differentiated products as in Holmes (1989). Price differentiation may have fundamentally different effects in oligopolies when there is best response asymmetry, i.e. when the strong market of one firm is the weak market of the other one. This is not the case in the discussion of registration taxes, as all firms would like to set the lower price in the same country. A survey can be found in Stole (2007).
Note that the parameters in the demand function are the same in both countries, apart from the parameter $\alpha$, that represents the relative size of the importing country.

Thus, without the tax, the monopolist would set the same price in both countries even if he could set the prices separately. The only incentive for price discrimination comes from the tax.

In the model, the tax is assumed to be levied per unit, i.e. $\rho = p + t$, where $t > 0$ is the tax rate.\(^3\)

Price discrimination may be limited by the possibility of arbitrage, for which the pre-tax prices are relevant. Arbitrage is possible at costs $s$, which thereby is the maximum feasible price difference the firm can sustain. If the price difference is higher than $s$, arbitrage would take place and the firm would not sell anything at the high price, which cannot be profit-maximizing behavior. Arbitrage costs $s$ should be understood as the cost-equivalent of all barriers to arbitrage.\(^4\)

The government of the importing country sets the tax rate in an initial stage and aims at maximizing welfare $w$ measured as the sum of consumer surplus plus tax revenue, which equals gross consumer surplus minus the import receipt

$$w = \int_0^z \rho(z)dz - \rho x + tx = \int_0^z \left(\frac{\alpha a - \alpha b}{\alpha b}\right)dz - px$$

$$= \frac{1}{2} \alpha b a^2 - \frac{1}{2} \alpha b \rho^2 - p\alpha(a - b\rho).$$

The welfare of the producing country equals consumer surplus plus profits of the firm, and thus gross consumer surplus plus revenue from exports minus the costs of production\(^5\)

$$W = \frac{1}{2} a^2 - \frac{1}{2} bP^2 + p\alpha(a - b\rho) - c(a - bP + \alpha(a - b\rho)).$$

\(^3\)With ad valorem taxation, the model is not tractable analytically any more, but the simulation of a number of numerical example give rise to the expectation that the qualitative results derived for per unit taxation also hold in the case of ad valorem taxation.

\(^4\)The price difference can be higher than actual transportation costs and reflects all obstacles for arbitrage. For instance, consumers may have preferences to buy at their local car dealer, because they might want to establish a good basis for the long-term relation of after sales services. It may also be easier to make warranty claims at the car dealer where the car was bought, although the warranty of the car producers must be valid for the whole EU).

\(^5\)The monopolist’s profits are included in producing country’s welfare, i.e. it is assumed that the firm is domestically owned.
Note that aggregate welfare equals the sum of gross consumer surplus in both countries minus production costs
\[ w + W = \frac{1}{2} \alpha b a^2 - \frac{1}{2} \alpha b \rho^2 + \frac{1}{2} b a^2 - \frac{1}{2} b P^2 - c(a - bP + \alpha(a - b\rho)). \] (6)

For the interpretation of some results, total welfare will alternatively be divided into
\[ \tilde{w} = \frac{1}{2} \alpha b a^2 - \frac{1}{2} \alpha b \rho^2 - c\alpha(a - b\rho) \] (7)
and \[ \tilde{W} = \frac{1}{2} b a^2 - \frac{1}{2} b P^2 - c(a - bP), \] (8)

where \( \tilde{w} \) and \( \tilde{W} \) equal gross consumer surplus minus the production costs for the respective quantity in the individual countries. Note that \( \tilde{w} + \tilde{W} = w + W \), and that \( \tilde{W} \) is the welfare of the producing country without the profits from exporting, which are instead added to the importing country’s welfare. As long as the price is above marginal costs \( c \), the welfare aggregates \( \tilde{w} \) and \( \tilde{W} \) increase when the respective price \( \rho \) or \( P \) falls, and vice versa.

3 Exogenous tax rate

Segmented markets

When markets are segmented, the monopolist can set prices in the two countries independently.

In the producing country, the profit equals
\[ \Pi = (P - c)(a - bP) \] (9)
resulting in the well-known result of a profit-maximizing price of
\[ P_{seg}^* = \frac{1}{2} a + bc. \] (10)

The monopolist’s profit in the importing country is
\[ \pi = (p - c)(a - b(p + t)) \] (11)
resulting in a pre-tax price of
\[ p_{seg}^* = \frac{1}{2} a + bc - \frac{1}{2} t \] (12)
and a tax-inclusive price of
\[ \rho_{seg}^* = \frac{1}{2} a + bc + \frac{1}{2} t. \] (13)
Thus half of the tax is born by the consumers in the importing country and the other half by the monopolist, resulting in a price difference of pre-tax prices of

\[ P_{\text{seg}}^* - p_{\text{seg}}^* = \frac{1}{2} t. \] (14)

Note that all prices are independent of the size of the importing country \( \alpha \) and the equilibrium price in the producing country \( P_{\text{seg}}^* \) does not depend on the tax rate \( t \).

The quantity in the producing country is unchanged, whereas sales in the importing country fall in response to the tax. It is straightforward to see, that the welfare in the producing country \( W \) falls when a tax is introduced or increased, as \( \tilde{W} \) remains unchanged and profits from exporting fall (as both the pre-tax price and the volume of exports fall). Aggregate welfare \( w + W = \tilde{w} + \tilde{W} \) also depends negatively on the tax, as \( \tilde{W} \) remains unchanged and \( \tilde{w} \) falls with the increase in the tax-inclusive price \( \rho \). The impact of a tax increase on the welfare of the importing country is ambiguous, depending on whether the tax rate is below or above the optimal tax rate (see below section 4). These are the usual considerations and results of (optimal) tariff theory.

**Arbitrage limiting price discrimination**

Assume now, that arbitrage costs \( s \) are lower than the induced price difference \( \frac{1}{2} t \). In this case, the monopolist is restricted in setting its prices, and the maximum feasible price difference equals \( s \). As the arbitrage condition is binding, the firm actually only sets one price and \( P = p + s \), leading to profits of

\[ \Pi^{\text{tot}} = (p + s - c)X + (p - c)x \]

\[ = (p + s - c)(a - b(p + s)) + (p - c)(\alpha(a - b(p + t))) \] (16)

and resulting in the equilibrium pretax price in the importing country

\[ p_{\text{seg}}^* = \frac{1}{2} \frac{a + bc}{b} - s \frac{1}{1 + \alpha} - \frac{1}{2} \frac{t}{1 + \alpha} \]

\[ = p_{\text{seg}}^* + \frac{1}{2} \frac{t - 2s}{1 + \alpha}. \] (17)

This means for the price in the producing country

\[ P_{\text{seg}}^* = p + s = \frac{1}{2} \frac{a + bc}{b} + s \frac{\alpha}{1 + \alpha} - \frac{1}{2} \frac{t}{1 + \alpha} \]

\[ = P_{\text{seg}}^* - \frac{\alpha}{2} \frac{t - 2s}{1 + \alpha}. \] (18)
and for the tax-inclusive price in the importing country

\[ \rho^*_s = p + t = \frac{1}{2} \frac{a + bc}{b} - s \left( \frac{1}{1 + \alpha} + \frac{1}{2} \frac{\alpha + 2}{1 + \alpha} \right). \]

**Proposition 1:** Assume that the tax rate is exogenous and the arbitrage condition \( P - p \leq s \) is binding.

When the scope to price discriminate increases, i.e. when \( s \) rises,

(i) both the pretax and the tax inclusive price \( p^*_s \) and \( \rho^*_s \) in the importing country fall whereas the price in the producing country \( P^*_s \) rises

(ii) the sales in the importing country rises, while the quantity in the producing country falls and total quantity remains constant,

(iii) welfare in the importing country and aggregate welfare increases. The change in welfare in the producing country is ambiguous.

**Proof:** (i) follows directly from equation 12 to 13, and (ii) is a straightforward calculation inserting the expressions for the prices into the demand functions.

(iii) Express welfare in the importing country as a function of \( \rho \),

\[ w = \frac{1}{2} \frac{1}{b} a^2 - \frac{1}{2} \alpha b^2 - \alpha(\rho - t)(a - b\rho). \]

Thus

\[ \frac{dw}{ds} = \frac{dw}{d\rho} \frac{d\rho}{ds} = -\alpha (a - b\rho + bt) \frac{d\rho}{ds} > 0, \]

as \( a - b\rho = x \geq 0 \) in the relevant region and \( \frac{d\rho}{ds} < 0 \) by (i).

Aggregate welfare is the sum of both countries’ gross consumer surplus minus production costs,

\[ W + w = \frac{1}{2} \frac{1}{b} a^2 - \frac{1}{2} bP^2 + \frac{1}{2} \frac{1}{b} a^2 - \frac{1}{2} \alpha b^2 - c(x + X) \]

and thus

\[ \frac{d(W + w)}{ds} = bP \frac{dP}{ds} - abP \frac{d\rho}{ds} - c \frac{d(x + X)}{ds} \]

\[ = -bP \frac{\alpha}{1 + \alpha} + \alpha b\rho \frac{1}{1 + \alpha} \quad \text{(by ii)} \]

\[ = b\alpha \frac{\rho - P}{1 + \alpha} > 0 \quad \text{(as \( \rho > P \))} \]
For the welfare of the producing country $W$, the calculation is given in the appendix. □

The corresponding results hold, when the results for segmented markets are compared to the the case of a binding arbitrage condition (or the case of a uniform price that corresponds to $s = 0$). For instance, on segmented markets, prices in the importing country are lower, the price in the producing country is higher and total welfare is higher, than if price discrimination is limited by arbitrage (i.e. $p_s^* - p_{seg}^* = \rho_s^* - \rho_{seg}^* > 0$ and $P_s^* - P_{seg}^* < 0$). This can be shown by direct computation, but it is also clear from interpreting the transition to segmented markets as increasing the maximum feasible price difference until it is not binding any more.

The results on prices and quantities correspond to the usual results on price discrimination. When the firm has more scope for price discrimination, it decreases its lower price on its weak market, and increases its higher price on its strong market, with the quantities on the individual markets reacting accordingly. Due to the assumption of linear demand, total quantity remains constant. The size of the effects depends on the size of the importing country $\alpha$. The larger the relative size of a country is, the smaller is its share of price adjustment to a change in $s$.

In the importing country, (more) price discrimination lowers the price, and both the consumer surplus and the tax revenue - and thus welfare - increases. Consumers in the producing country lose due to the higher price, but the firm gains, as it has more scope to move its prices towards the monopoly prices of the individual markets. It is ambiguous, which effect dominates.

The result on total welfare is surprising at first glance. Aggregate welfare unambiguously increases, although total quantity is constant. This result stands in contrast to the well-known result of Varian (1985) that price discrimination can only increase welfare if total quantity increases. The puzzle is explained by the fact that the importing country has the low pre-tax price that is relevant for arbitrage and for price discrimination, but the high tax-inclusive price that is relevant for consumers. Allowing for price discrimination makes cars in the importing country cheaper and redistributes the quantity from consumers with a lower to consumers with a higher willingness to pay.

Note however that the combination of the results that price discrimination increases welfare and that the tax induces price discrimination does not mean that introducing a tax is welfare improving. Allowing for price discrimination increases welfare if the tax is exogenously given. Introducing or increasing a tax decreases total quantity.
\[ x^*_s + X^*_s = a - bP^*_s + \alpha(a - \beta^*_s) \]  
\[ = \frac{1}{2} (1 + \alpha) (a - cb) - \frac{1}{2} b \alpha t \]  

and also decreases total welfare, as can be shown by straightforward, albeit tedious, computation using equations 18 to 21 and 4 on welfare and prices.

4 Endogenous tax rate

In this subsection, the tax rate is not considered as exogenous, but it assumed instead that the government of the importing country sets the tax rate in order to maximize domestic welfare, taking into account how the firm will react with its prices to the tax rate chosen. In particular, it is analysed how the arbitrage condition affects the optimal tax policy and what effects this has on prices, quantities and welfare. As a benchmark, the case of segmented markets is again discussed first.

Segmented markets

Using equation 12 and 13 on the equilibrium prices, the welfare of the importing country can be expressed as a function of the tax rate

\[ w = \frac{1}{2} \frac{\alpha}{b} a^2 - \frac{1}{2} \alpha b \rho^2 - p (\alpha(a - \beta)) \]  
\[ = -\frac{1}{8} \frac{\alpha}{b} (-a^2 + 2abc - 2abt - b^2 c^2 + 2b^2 ct + 3b^2 t^2) \]  

implying the welfare-optimizing tax rate

\[ \hat{t}_{seg} = \frac{a - bc}{3b} \]  

and equilibrium prices

\[ \hat{P}_{seg}^* = \frac{1}{2} \frac{a + bc}{b} - \frac{1}{2} \frac{t_{opt}}{b} = \frac{1}{3} \frac{a + 2bc}{b} \]  
\[ \hat{\beta}_{seg}^* = \frac{1}{2} \frac{a + bc}{b} + \frac{1}{2} \frac{t_{opt}}{b} = \frac{2}{3} \frac{a + bc}{b} \]  

As markets are segmented, the price in the producing country is independent of the tax rate, it continues to be

\[ \hat{P}_{seg}^* = P_{seg}^* = \frac{1}{2} \frac{a + bc}{b}, \]
and the difference of pre-tax prices, that is relevant for arbitrage, is

$$P_{seg}^* - \hat{p}_{seg}^* = \frac{1}{2} a + \frac{1}{3} a + \frac{2}{3} bc - \frac{a}{b} = \frac{1}{6} a - \frac{bc}{b}. \quad (33)$$

**Arbitrage limiting price discrimination**

When the possibility of arbitrage limits the firm’s scope to price discriminate, the above result on the optimal tax rate does not hold any more. When setting the tax, the government of the importing country has to take into account that the price difference that it can induce cannot exceed the arbitrage costs, i.e. $P - p \leq s$. As soon as $s < \frac{1}{6} a - \frac{bc}{b}$, the tax rate $\hat{t}_{seg}$ will not be optimal any more, but a lower rate will be chosen, as determined in the following proposition.

**Proposition 2:** (Optimal tax rate)

Assume that the government of the importing country sets the tax rate in order to maximize the country’s welfare.

(a) The optimal tax rate $\hat{t}$ equals

(i) (interior solution)

if $s \leq \frac{(a - bc)}{2b(3\alpha + 4)}$

$$\hat{t} = \frac{a - bc}{b} \frac{\alpha (1 + \alpha)}{(\alpha + 2)(3\alpha + 2)} + \frac{2s\alpha}{(\alpha + 2)(3\alpha + 2)}; \quad (34)$$

(ii) (corner solution)

if $\frac{(a - bc)}{2b(3\alpha + 4)} < s < \frac{a - bc}{6b}$

$$\hat{t} = 2s \quad (35)$$

(iii) (segmented markets)

if $s \geq \frac{a - bc}{6b}$

$$\hat{t} = \hat{t}_{seg} = \frac{1}{3} \frac{a - bc}{b} \quad (36)$$

**Proof:**

(i) Assume that the arbitrage condition is binding and thus $P = p + s$. To get the welfare of the importing country as a function of the tax rate, insert the equilibrium prices for
a given tax rate (equation 12 and 13) into equation 4. Straightforward maximization results in the optimal tax rate $\hat{t}$ stated. It remains to check, whether for this tax rate, the arbitrage condition limiting price discrimination is indeed binding. On segmented markets, a tax rate $\tilde{t}$ induces a price difference of $\frac{1}{2}\tilde{t}$ (equation 14), and thus the firm will only choose prices such that $P = p + s$ as long as $s \leq \frac{1}{2}\tilde{t}$. Solving the inequality $s < \frac{1}{2}\tilde{t} = \frac{1}{2}\alpha \left(1 + \alpha\right) \frac{a - bc}{b(\alpha + 2)(3\alpha + 2)} + \frac{s\alpha}{(\alpha + 2)(3\alpha + 2)}$ for $s$ results in $s < (a - bc)\frac{\alpha}{2b(3\alpha + 4)}$, giving the range of parameters for which the solution for the optimal tax rate is consistent.

(iii) When markets are segmented, the optimal tax rate is $\frac{1}{3}(a - bc)$ and it induces a price difference of $\frac{a - bc}{6b}$. As long as the maximum feasible price difference $s$ is at least as large (i.e. as long as $s \geq \frac{a - bc}{6b}$), the arbitrage condition is irrelevant and the solution for segmented markets continues to hold.

(ii) Note that $(a - bc)\frac{\alpha}{2b(3\alpha + 4)} < \frac{a - bc}{6b}$, thus for $s$ between these two boundaries, neither of the two solutions applies. It is then optimal for the government to set the tax rate such that it is just not binding. If the firm is unrestricted in its price setting, a tax of $\hat{t} = 2s$ induces a price difference of $s$, the maximum feasible one. A lower tax rate is not optimal, as $s$ would not be binding for the firm and thus increasing the tax rate would increase welfare (as it moves in the direction of the optimal tax rate in case of unrestricted pricing). On the other hand, a tax rate higher than $\hat{t} = 2s$ also cannot be optimal, as in this case $s$ would be binding and lowering $t$ marginally would increase the welfare of the
importing country (as \( t \) would move in the direction of the optimal tax rate under the condition that \( P = p + s \)). □

The regions corresponding to the three cases of the theorem are illustrated in figure 1 showing the thresholds for \( s \) as a function of the size of the importing country \( \alpha \).\(^6\)

When arbitrage costs \( s \) are large, the maximum price difference is not binding and the same result as for segmented markets holds (case iii). For \( s \) below the threshold \( \frac{a - bc}{6b} \), the arbitrage condition becomes binding when the optimal tax for segmented markets is set, and the government of the importing country has to take this fact into account when it determines the optimal tax rate. For small \( s \) (as defined in case i), there is an interior solution for the optimal tax rate under the restriction of a binding arbitrage condition. In this case, the prices in the two countries are linked by the condition \( P = p + s \), and the tax does not only have an impact on the prices in the importing country, but on the price in the producing country, too (see proposition 2 below). However, when the size of the importing country \( \alpha \) is too small or \( s \) is too large, the tax rate derived in this way may be so low that the price difference induced if markets were segmented would be smaller than \( s \) and the condition \( P = p + s \) would not hold. Facing this tax rate, the firm would not fully make use of its scope to price differentiate - but it would do so if the optimal tax rate for segmented markets, that is higher, was set. In this case, the corner solution for the optimal tax rate applies. The government sets the tax such that the arbitrage condition \( P - p \leq s \) is just not binding for the firm, i.e. that the price difference induced if markets were segmented is exactly \( s \).

Note that when markets become more integrated and thus the arbitrage condition becomes binding as \( s \) falls, there is no direct transition from the case of segmented markets to the case of an interior solution for the optimal tax rate. There is always an intermediate interval where the corner solution applies. The smaller the size of the importing country \( \alpha \), the larger is the range of parameter values for \( s \) for which the corner solution, in which the tax of the importing country does not affect the price in the producing country, and the interior solution for the optimal tax rate may apply only for a small range of parameter values \( s \).

\(^6\)The parameters \( a, b \) and \( c \) only alter the scale. For the plot, the parameters \( a = 10, b = 1 \) and \( c = 4 \) where chosen.
The following corollary summarizes how the optimal tax rate depends on the maximum feasible price difference $s$ and on the size of the importing country $\alpha$.

**Corollary:**

(i) For $s \leq \frac{a - bc}{6b}$, the optimal tax rate $\hat{t}$ depends positively on arbitrage costs $s$. In the corner solution, $\frac{d\hat{t}}{ds} = 2$ holds, whereas in the region of the interior solution, $\frac{d\hat{t}}{ds} < 0.14$.

(ii) In the interior solution, the optimal tax rate $\hat{t}$ depends positively on the size of the importing country $\alpha$, whereas in the corner solution, $\hat{t}$ is independent of $\alpha$.

(iii) As long as $s > 0$ or $\alpha > 0$, the optimal tax rate is positive.

**Proof:** Most of the corollary follows directly from proposition 2. (i) In the interior solution, $\frac{d\hat{t}}{ds} = \frac{2s\alpha}{(\alpha + 2)(3\alpha + 2)}$. It is straightforward to show, that the maximum of $\frac{d\hat{t}}{ds}$ for $\alpha \geq 0$ is smaller than 0.14.

(ii) In the interior solution,

$$\frac{d\hat{t}}{d\alpha} = \frac{(4 + 8\alpha + 5\alpha^2)(a - bc) + 2bs(4 - 3\alpha^2)}{b(\alpha + 2)^2(3\alpha + 2)^2}.$$

The first term of the numerator is positive and the second term is positive as long as $s < \sqrt{\frac{4}{3}}$. When $s > \sqrt{\frac{4}{3}}$, the second term is negative, but using $s < \frac{a - bc}{6b}$, a positive lower boundary can be found.

As a next step, the equilibrium prices when the arbitrage condition is binding and the government of the importing country sets the optimal tax rate $\hat{t}$ are considered.

In the case of an interior solution for the optimal tax rate (as defined in proposition 2(i)), the monopolist will set the following prices in the importing and the producing country

$$\hat{p}_s = \frac{1}{2} \frac{a + bc}{b} - \frac{1}{2} \frac{(a - bc) \alpha^2}{b(\alpha + 2)(3\alpha + 2)} - s \frac{4(1 + \alpha)}{(\alpha + 2)(3\alpha + 2)}$$

and the tax-inclusive price in the importing country is

$$\hat{\rho}_s = \hat{p}_s + t = \frac{1}{2} \frac{a + bc}{b} + \frac{1}{2} \frac{(a - bc) \alpha}{(3\alpha + 2) b} - s \frac{2}{(3\alpha + 2)}.$$
These expressions are derived by inserting \( \hat{t} \) into equations 18 to 21.

In the \textit{corner solution}, (as defined in proposition 1 iii), the equilibrium prices equal the respective prices for segmented markets (equations 12 and 13) for the tax rate \( \hat{t} = 2s \). In the producing country, the price is unaffected by the possibility of arbitrage,

\[
\hat{P}_s^* = P_{seg}^* = \frac{1}{2} \frac{a + bc}{b}
\]  

whereas the pre-tax and the tax-inclusive price in the importing country equal

\[
\hat{p}_s^* = \frac{1}{2} \frac{a + bc}{b} - \frac{1}{2} \hat{t} = \frac{1}{2} \frac{a + bc}{b} - s \quad (41) \\
\hat{\rho}_s^* = \frac{1}{2} \frac{a + bc}{b} + \frac{1}{2} \hat{t} = \frac{1}{2} \frac{a + bc}{b} + s. \quad (42)
\]

From these expressions, the following proposition on the effects of a change in arbitrage possibilities on prices, quantities and welfare can be derived. Figure 2 shows the equilibrium prices as a function of arbitrage costs \( s \). The cases of an interior solution for the optimal tax (corresponding to ”small values of \( s \” \) ) and of the corner solution (corresponding to ”intermediate values of \( s \” \) ) are defined as in proposition 2.

**Proposition 3:** Assume that arbitrage limits price discrimination in the sense that the maximum feasible price difference \( s \) is binding if the optimal tax for segmented markets is set (i.e. \( s < \frac{a - bc}{6b} \)). If the tax is set optimally, the following holds.
(i) The pre-tax price in the importing country $\hat{p}_s^*$ depends negatively on $s$. The effect of $s$ on $\hat{p}_s^*$ is larger in the corner solution than in case of an interior solution for the optimal tax rate.

(ii) The tax inclusive price $\hat{\rho}_s^*$ depends non-monotonically on $s$, as it depends negatively on $s$ in case of an interior solution, but positively in case of the corner solution. Accordingly, the volume of imports depends non-monotonically on $s$.

However, for $\hat{\rho}_{s=0} < \hat{\rho}_{seg}$, i.e. if a uniform price has to be set, the tax inclusive price is lower and thus the quantity imported is higher than on segmented markets.

(iii) $\hat{P}_s^*$ depends positively on $s$ in case of an interior solution and is unaffected by $s$ in the corner solution. Thus in case of an interior solution, the quantity sold in the producing country falls when $s$ increases, and it is independent of $s$ in the corner solution.

(iv) As $s$ rises, total quantity falls.

(v) As $s$ rises, welfare in the importing country rises and welfare in the producing country falls. In the corner solution, total welfare unambiguously falls. In contrast, the change in welfare is ambiguous in case of an interior solution. It rises, when $s$ is close to 0, and it increases, when $s$ is close to the threshold $(a - bc)\frac{\alpha}{2b(3\alpha + 4)}$ between the cases of interior and corner solutions.

In the following, these results will interpreted and discussed. The formal proof is given in the appendix.

With an endogenous tax rate, a change in the scope to price discriminate has two effects – the direct effect of an increase in $s$ and the indirect one by raising the optimal tax rate. For a given tax rate, an increase in $s$ lowers both the pre-tax price $p$ and and the tax inclusive price $\rho$, whereas the rise in the tax rate itself lowers $p$ and increases $\rho$. For $p$ these two effects reinforce each other, while for $\rho$ they act in opposite directions. Note that the tax rate reacts much less to changes in $s$ in the interior solution than in the corner solution, which explains, why in the former case, the direct effect dominates and $\rho$ falls in response to an increase of $s$, and in the latter case, the indirect effect of the tax increase dominates and $\rho$ increases. In particular, the volume of imports $x$ depends non-monotonically on the scope to price discriminate.

The direct effect of an increase in $s$ increases the price in the producing country $P$, whereas the indirect effect of the tax increase lowers it. In the corner solution for the optimal tax
rate, these two effects exactly offset each other and $P$ is independent of $s$. In fact, in the
corner solution, the tax rate is chosen as the highest tax rate that does not affect $P$. In
case of an interior solution, the direct effect of the increase in $s$ dominates and $P$ rises
and thus the quantity sold $X$ in the producing country falls. There is no direct effect of
a change in $s$ on total quantity $x + X$, but it falls due to the tax increase.

Finally, welfare is considered. As the binding arbitrage condition does not only restrict
the monopolist in setting its prices, but also constitutes a condition for the government of
the importing country on setting its tax to influence the price setting of the monopolist,
the welfare of the importing country increases when $s$ increases. In contrast, welfare in
the producing country falls. In case of a corner solution, the price $P$ is unchanged, and
the result of the falling welfare is due to the decrease in profits from exports. In case of
an interior solution, the price $P$ increases and thus $\tilde{W}$, the welfare minus the profit from
exports, falls.

In case of a corner solution, the aggregation leads to an unambiguous result: A larger
scope to price discriminate decreases total welfare. This can be easily seen considering,
that $P$ and therefore $\tilde{W}$ remain unchanged, whereas $\rho$ rises and thus $\tilde{\omega}$ falls. In case of an
interior solution, the change in total welfare is ambiguous. For $s = 0$, a marginal increase
in $s$ increases total welfare. The intuition is that there nevertheless is a positive optimal
tax, and – similar to the case of an exogenous tax in proposition 1 – allowing for some
price discrimination redistributes the good to the market with the higher tax inclusive
consumer price and thus the higher (marginal) willingness to pay for the good. When
$s$ is larger, the effect of a falling total quantity dominates and total welfare falls, when
arbitrage costs $s$ – and therefore the scope for price discrimination – rise.

5 Conclusion

One reason for the large price differences observed on the European car market are high
registration taxes in some countries without own car production, where pre-tax prices
are particularly low. These taxes act like import tariffs and may increase the welfare of
the country levying the tax at the expense of the producing countries. The European
Commission aims at further market integration by facilitating commercial arbitrage and
making price differences smaller. The paper discusses what effects would occur if this
policy was successful in a model in which the tax is the only reason for price discrimination.

When the tax rate is exogenous, a decrease in the maximum feasible price difference
decreases the high producer price in the producing country and increases both the pre-
tax and the tax-inclusive price in the importing country, thereby decreasing the volume of trade. Welfare falls, as the price discrimination partially offsets the misallocation arising from the registration tax.

When the government of the importing country chooses the tax rate optimally, facilitating arbitrage has additional indirect effects by lowering the optimal tax rate. Starting from segmented markets and lowering the maximum feasible price difference until the arbitrage condition becomes binding, first the corner solution applies in which the tax rate is set at a height such that the maximum price difference is just (not) binding and the price in the producing country is not affected. The pre-tax price increases, thereby lowering the price difference relative for arbitrage, but in the corner solution the optimal tax rate reacts so strongly to changes in arbitrage costs, that the tax-inclusive price falls and the volume of trade increases. Welfare in the importing country falls, as its scope to set an optimal tax is limited, but total quantity and total welfare rise in the course of market integration.

When the maximum feasible price difference falls further, an interior solution for the optimal tax rate applies and the smaller scope to price discriminate also affects the price in the producing country, which falls. In the importing country, both the pre-tax and the tax-inclusive price increase, and the volume of trade falls as in case of an exogenous tax rate. Thus the consumer price in the importing country and the volume of trade depend non-monotonically on arbitrage costs. Nevertheless, as in the corner solution total welfare increases, except when the maximum price difference is very small and the situation is close to the case of a uniform price.

Thus the effects of the policy of the European Union that aims at lowering the price differences on the car market, substantially depend on whether the tax rate should be considered as exogenous or whether (and to what extent) the high registration taxes will be lowered in reaction to import price increases when price discrimination is further restricted by falling arbitrage costs.
Appendix:

Proof of proposition 1, (iii)

It remains to show, that the sign of $\frac{dW}{ds}$ is ambiguous.

$$W = \frac{1}{2} b a^2 - \frac{1}{2} b P^2 - c (a - b P) + (p - c) (\alpha (a - b \rho))$$

and

$$\frac{dW}{ds} = \frac{1}{2} \alpha - (1 + \alpha) (a - b c) + b (\alpha + 2) (t - 2 s)$$

The first term of the numerator is negative, whereas the second term is positive (as $s < \frac{1}{2} t$). If $2 s - t \equiv 0$ (which is possible), then $\frac{dW}{ds} < 0$.

For $s = 0$ and a high (exogenous) tax rate, the expression is positive. It can be checked that no inconsistencies occur (as e.g. negative expressions for prices or quantities) occur for a tax rate that makes the numerator marginally positive. □

Proof of proposition 3

(i) It follows directly from the expressions for $\hat{p}_s$ that in the interior solution $\frac{d\hat{p}_s}{ds} = -\frac{4 (1 + \alpha)}{(\alpha + 2)(3\alpha + 2)}$ and in the corner solution $\frac{d\hat{p}_s}{ds} = -1$. Moreover, $\frac{4 (1 + \alpha)}{(\alpha + 2)(3\alpha + 2)} = \frac{4 + 8\alpha + 3\alpha^2}{4 + 8\alpha + 3\alpha^2} < 1$ for $\alpha > 0$, thus in the corner solution, the absolute value of the derivative is larger.

(ii) The directions of change of $\hat{\rho}_s^*$ follows directly from the solutions given in the text. Moreover

$$\hat{\rho}_{s=0} - \hat{\rho}_{seg}^* = \frac{1}{2} \frac{a + bc}{b} - \frac{1}{2} \frac{(a - bc)}{b (\alpha + 2)(3\alpha + 2)} - \left(\frac{12a + bc}{3} \frac{1}{b}\right)$$

(iii) is obvious from from the expressions for $\hat{P}_s^*$.

(iv) The reaction of total quantity $\hat{x}_s^* + X_s^*$ can be derived by inserting the expressions for the prices into the demand functions. It can also be shown by the consideration, that
the direct effect of \( s \) on total quantity is 0 and the effect of the tax increase is negative, as can be seen from

\[
x_s^* + X_s^* = a - bP_s^* + \alpha (a - b\rho_s^*)
\]
\[
= \frac{1}{2} (1 + \alpha) (a - cb) - \frac{1}{2} b\alpha t
\]

(v) The effects on welfare are derived by inserting the respective equilibrium prices into the expressions for the welfare and differentiating.

a) interior solution

Welfare in the importing country:

\[
\frac{dw}{ds} = 2 \left( (1 + \alpha) (a - bc) + 2bs \right) \frac{\alpha}{(\alpha + 2) (3\alpha + 2)} > 0
\]

Welfare in the producing country:

\[
\frac{dW}{ds} = -\alpha \left( \frac{8 + 22\alpha + 20\alpha^2 + 5\alpha^3}{(\alpha + 2)^2 (3\alpha + 2)^2} \right) (a - bc) + bs (32 + 64\alpha + 40\alpha^2 + 9\alpha^3) < 0
\]

Total welfare:

\[
\frac{d(w + W)}{ds} = \alpha \left( 2 + 2\alpha + \alpha^2 \right) \frac{(a - bc) - bs (16 + 32\alpha + 28\alpha^2 + 9\alpha^3)}{(\alpha + 2)^2 (3\alpha + 2)^2} > 0
\]

\[
\frac{d(w + W)}{ds} \text{ is positive for } s = 0. \text{ For the upper boundary of the range of the interior solution } s = \frac{(a - bc)}{2b(3\alpha + 4)} \frac{\alpha}{2b(3\alpha + 4)}
\]

\[
\frac{d(w + W)}{ds} = -\frac{1}{2} \left( \frac{(a - bc) \alpha^3}{(3\alpha + 2) (\alpha + 2) (3\alpha + 4)} \right) < 0.
\]

b) For the corner solution:

Welfare in the importing country:

\[
\frac{dw}{ds} = \frac{1}{2} \alpha (a - bc - 6bs) > 0
\]

because \( s < \frac{a - bc}{6b} \).

Welfare in the producing country:

\[
\frac{dW}{ds} = -\alpha (a - bc - 2bs) < 0
\]

Total welfare:

\[
\frac{d(w + W)}{ds} = -\frac{1}{2} \alpha (a - bc + 2bs) < 0.
\]
References


