Endogenous Quality and Non-Homothetic Production in a Monopolistic Competition Model of Trade

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Abstract
Within detailed product categories goods produced in high income countries and capital and skill-abundant countries display higher unit values. This result of recent empirical work is at odds with the standard heterogeneous productivity monopolistic competition model. In this model productivity is defined as the inverse of marginal costs. So, with fixed markups more productive firms charge lower instead of higher prices. Also, prices are unrelated to factor abundance. The goal of this paper is to extend the heterogeneous productivity model with endogenous quality and non-homothetic production to bring it in line with the mentioned empirical results. Two models are proposed: a within country and a between country differences model. In the within country differences model quality is endogenous. Each firm has a different productivity to produce quality. More productive firms invest more in fixed product development costs raising the quality of the good. As marginal costs also rise in quality, the price of a good relates positively to the quality of it. In the between country differences model production is non-homothetic in skilled and unskilled labor. Higher quality goods require more skilled labor. The implication is that countries that are relatively more skill-abundant produce higher quality goods with higher prices.

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1 Introduction

Empirical work by Schott (2004) shows that unit values of export goods within detailed product categories are related to exporting country characteristics. In particular, goods originating from richer and more capital and skill-abundant countries display higher unit values. The empirical results pose problems for the standard heterogeneous productivity monopolistic competition model, as introduced by Melitz (2003). With income likely to be strongly related to productivity, goods from higher productivity countries should have higher prices. In Melitz (2003) productivity is defined as the inverse of marginal cost. With a fixed markup, more productive firms charge lower instead of higher prices. Also, in the Melitz model or one of its numerous extensions there is no relation of the market price with skill or capital abundance.

The present paper proposes modifications of the heterogeneous productivity monopolistic competition model that can account for the empirical regularities found by Schott (2004). Two different modifications are put forward. In a within country differences model, firms have a different productivity to produce quality. More productive firms produce higher quality goods charging higher prices. In a between country differences model, production is non-homothetic in quality. Higher quality goods require more skilled labor implying that skill abundant countries produce higher quality goods.

The within country differences model is in the spirit of Melitz’ heterogeneous productivity model. Utility is CES with a CES quality parameter specific for each variety. So, the CES parameter measures the quality of a good. Firms can enter a market by drawing a productivity to produce quality parameter. This parameter can be seen as the natural appeal of a variety. The quality of a good can be increased by more investments in product development, product branding and marketing. The productivity to produce quality parameter determines the effectiveness of these investments. A larger quality also requires larger marginal costs. The within country differences model abstracts from between country differences: there are two identical countries trading with each other.

The model produces outcomes that can account for Schott’s (2004) empirical findings. More productive firms have a larger quality and charge higher prices. Furthermore, only more productive firms and thus firms with a higher quality can export. Exporting firms are on average bigger. Trade creates a reallocation effect, more productive firms gain market share at the expense of less productive ones. As a result, average quality of goods rises in the economy. Also trade liberalization reduces the average productivity and quality of exports, because also the less productive firms can export.

The second model of between country differences assumes homogenous firms. There is one sector, two countries and two production factors, skilled and unskilled labor. Production is non-homothetic: a larger quality good requires relatively more labor. The model finds that more skill abundant countries produce higher quality goods. This result accounts for Schott’s (2004) empirical findings that within product categories skill abundant countries export higher quality goods. The model is a Hekscher-Ohlin factor abundance model. However, factor abundance does not determine in which sectors a country has a comparative advantage but in which quality segment a country has a comparative advantage.
The models in this paper are related to the strand of literature on vertical product differentiation and trade. The first model is similar to Helble and Okubo (2006), a modification of Melitz taking into account quality differences. More productive firms produce as well higher quality goods and charge larger prices. But their model differs from the one in this paper in several respects. They do not model an entry/exit steady state of firms. There is no reallocation effect from trade, because the distribution of firms is given. Productivity is defined in a different way: more productive firms have larger marginal costs, but they need to invest less in marketing instead of more in this paper. This produces the counterfactual outcome that more productive exporting firms are smaller than domestic producing firms due to the higher prices they charge. Baldwin and Harrigan (2007) also adapt Melitz to account for the empirical finding that export unit values increase with distance. Marginal costs rise with quality in their model. But quality is not an endogenous choice variable in their paper and quality as it is here. Furthermore, quality is not related to fixed costs. Hummels and Klenow (2002) contains a model of between country differences and quality that is similar to the second model of this paper. In their model it is assumed that there are differences in the productivity to produce quality between countries. The present model relates productivity differences to factor abundance. So, Hummels and Klenow is a Ricardian model whereas the model in this paper is a Hekscher-Ohlin model.

The two models can be combined in an integrated model. An integrated model could generate interesting results, in particular on the effect of trade liberalization on wage inequality. Solving the combined model requires numerics, which is left for future work. In the next section the first model on within country differences is pointed out. In section 3 the model of between country differences is presented. Section 4 contains a discussion of the models and a comparison with other models. Section 5 concludes.

2 Within Country Productivity Differences

This section proposes a model of monopolistic competition with heterogeneous productivity, where more productive firms produce goods of larger quality and with larger prices. The model is based on CES-preferences with firm specific CES-weights, their taste parameters. All firms have a productivity to produce quality. Firms can produce a higher level of quality reflected in their taste parameter when their productivity to produce quality is bigger. A larger taste parameter requires more investment in product development, branding and marketing. The price of a good is dependent on its quality, because the marginal cost of production varies with quality.

The beneficial effect of international trade works in the same way as in Melitz. Due to iceberg trade costs and fixed trade costs, only more productive firms can export. International trade raises the expected profit of entry, induces more entry leading to higher real wages that squeeze the least productive firms out of the market. Section 2.1 points out the closed economy model and section 2.2 outlines the open economy model. A discussion of the model is deferred to a separate section 4.
2.1 Closed Economy Model

A representative consumer has CES utility with taste parameter $\alpha_v$ for variety $v$:

$$U = \left[ \int_{v \in V} \alpha_v x_v^{-\frac{1}{\sigma-1}} \frac{\sigma}{\sigma-1} \, dv \right]^\frac{\sigma}{\sigma-1}$$  (1)

There are $L$ consumers in the economy. Normalizing the wage at 1, the market demand facing a firm is given by:

$$x_v = \alpha_v^\sigma p_v^{-\sigma} P^{\sigma-1} L$$  (2)

The price index $P$ is defined as:

$$P = \left[ \int_{v \in V} \alpha_v^\sigma p_v^{1-\sigma} \, dv \right]^\frac{1}{1-\sigma}$$  (3)

The cost function of a firm is given by:

$$C(x_v) = a_v(\alpha_v)x_v + f + f_{\text{pbm},v}$$  (4)

Labor is the only production factor. Firms have a marginal cost $a_v$ that is dependent on the quality of the good $\alpha_v$ produced. In the remainder it is assumed that this relation is linear. The parameter restrictions to get a solution when a non-linear relation is chosen will be discussed below.

$$a_v = \alpha_v$$  (5)

There are two types of fixed cost, a regular fixed cost $f$ that consists of for example overhead costs and fixed costs of product development, branding and marketing $f_{\text{pbm}}$. The last type of fixed costs affects the quality of the good. There are several papers in industrial organization that make a similar assumption with fixed costs rising in quality, i.e. Mussa and Rosen (1978), Shaked and Sutton (1983), Gal-Or (1983) and Motta (1993).

The taste parameter of a variety depends on the amount of product development, branding and marketing fixed costs spent $f_{\text{pbm}}$ and a ‘quality productivity’ parameter $\beta_v$. The relation is defined by the following expression:

$$\alpha_v = \beta_v f_{\text{pbm},v}^{\kappa}, \quad 0 < \kappa < 1$$  (6)

So, a firm can raise the attractiveness of its good by increasing the amount of product development and marketing expenditures. The effectiveness of these expenditures is
larger for firms that have drawn a larger productivity to produce quality parameter, $\beta_v$. $\kappa < 1$ is a necessary assumption for equilibrium. This assumption is realistic as long as the equilibrium is in the range of product development, branding and marketing costs, where there are decreasing returns.

A firm can enter the market by incurring sunk entry costs $f_e$. After payment of the sunk entry costs a firm can draw the quality productivity parameter $\beta_v$. So, the sunk entry costs should be seen as a kind of market exploration costs, necessary for a firm to get to know how popular its variety can become. After a firm knows $\beta_v$ it either starts to produce or leaves the market immediately. When it starts to produce a firm has to incur product development costs and costs of marketing. There is a fixed death probability $\delta$ of a certain variety.

The product development, branding and marketing costs $f_{p_{bm}}$ are partly sunk and partly fixed in nature. Because the amount of sunk investments in product development has to be decided on after the uncertainty of the quality productivity is released, the sunk part of $f_{p_{bm}}$ can be expressed as per period amortized costs using the probability of death parameter $\delta$. So, $f_{p_{bm}}$ can be seen as a combination of costs that are more sunk like product development costs and marketing costs that are more fixed in nature.

In this model a firm has two choice variables, the price $p_v$ and the amount of product development, branding and marketing fixed costs $f_{p_{bm},v}$. The pricing equation is the familiar CES-markup equation:

$$p_v = \frac{\sigma}{\sigma - 1} \alpha_v$$

(7)

So, the price is rising in the endogenously determined quality level $\alpha_v$.

Substituting equations (6) and (7), the profit of the firm is given by:

$$\pi_v = p_v x_v - a_v x_v - f - f_{p_{bm},v}$$

$$\pi_v = \beta_v f_{p_{bm},v} \kappa \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^{\sigma-1} L - f - f_{p_{bm},v}$$

(8)

The first order condition of equation (8) with respect to $f_{p_{bm},v}$ is given by:

$$\kappa \beta_v f_{p_{bm},v} \kappa^{1-\sigma} p^{\sigma-1} L \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} = 1$$

(9)

The optimal $f_{p_{bm},v}$ is:
\[ f_{p bm, v} = \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma^{-1} \frac{1}{1-\kappa}} P^{\frac{1}{1-\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-\kappa}} (\kappa \beta_v)^{\frac{1}{1-\kappa}} \]  

(10)

Substituting this back into the taste parameter and the profit function one gets:

\[ \alpha_v = \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma^{-1} \frac{1}{1-\kappa}} P^{\frac{1}{1-\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-\kappa}} \beta_v^{\frac{1}{1-\kappa}} \]  

(11)

\[ \pi_v = \left( \frac{\kappa}{\kappa^{\frac{1}{1-\kappa}} - \kappa^{\frac{1}{1-\kappa}}} \right) \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma^{-1} \frac{1}{1-\kappa}} P^{\frac{1}{1-\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-\kappa}} \beta_v^{\frac{1}{1-\kappa}} - f \]  

(12)

The price is given by:

\[ p_v = \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma^{-1} \frac{1}{1-\kappa}} P^{\frac{1}{1-\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-\kappa}} \beta_v^{\frac{1}{1-\kappa}} \]  

(13)

The above derivations give rise to the following observation:

**Observation 1**
*The market price \( p_v \) of an individual firm is rising in quality \( \alpha_v \) and in the productivity to produce quality \( \beta_v \).*

Equation (7) shows that the market price is rising in quality and from equation (13) it is clear that the market price is rising in the productivity to produce quality. The empirical critique on the heterogeneous productivity monopolistic competition model discussed in the introduction was that more productive firms charge lower prices, whereas in reality more productive firms charge higher prices. Observation 1 shows that one can modify the monopolistic competition model to confront this critique. In the present model productivity is reformulated in terms of the ability to produce quality. In this way more productive firms charge higher prices, because they choose a higher level of quality.

Equilibrium in the economy can be found by combining a free entry condition and a zero cutoff profit condition. The free entry (FE) condition is given by:

\[ \tilde{\pi} = \frac{\delta f_v}{1 - G(\beta^*)} \]  

(14)

\( \tilde{\pi} \) is average profit and \( \beta^* \) is the cutoff quality productivity. The zero cutoff profit condition (ZCP) is given by:
\[ \pi(\beta^*) = \left( \kappa^{\frac{1}{1-k}} - \kappa^{\frac{1}{1-k}} \right) \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-k}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-k}} - f = 0 \]

\[ \beta^* = f^{\frac{1}{1-k}} \left( \kappa^{\frac{1}{1-k}} - \kappa^{\frac{1}{1-k}} \right) \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-k}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-k}} \]

(15)

Writing average profit as an integral over quality productivities one can express the ZCP as a function of average profit and the cutoff quality productivity. Average profit is given by:

\[ \tilde{\pi} = \left( \kappa^{\frac{1}{1-k}} - \kappa^{\frac{1}{1-k}} \right) \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1-k}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-k}} \int_{\beta^*}^{\infty} \frac{1}{1-G(\beta^*)} g(\beta) d\beta - f \]

(16)

The ZCP can now be rewritten as:

\[ \tilde{\pi} = f \left( \frac{1}{\beta^*} \right)^{\frac{1}{1-k}} \int_{\beta^*}^{\infty} \frac{1}{1-G(\beta^*)} g(\beta) d\beta - 1 \]

(17)

Or more concisely:

\[ \tilde{\pi} = f \left( \frac{\tilde{\beta}}{\beta} \right)^{-1} \]

(18)

With

\[ \tilde{\beta} = \left[ \int_{\beta^*}^{\infty} \frac{1}{1-G(\beta^*)} g(\beta) d\beta \right]^{1-k} \]

(19)

Equations (14) and (18) together determine the cutoff quality productivity and average profit. It can be shown that (14) and (18) together yield a unique equilibrium in the same way as in appendix B of Melitz (2003). The proof is in appendix A. From the cutoff quality productivity and the average profit one can determine the average quality productivity, the average quality, the number of firms and all other endogenous variables. The number of firms can be derived from the condition for steady state of entry and exit. This leads to the following expression (derivations in appendix B):

\[ N = \frac{L}{\sigma(\tilde{\pi} + f)} \]

(20)
The only variable left to determine is the price index. Writing the price index as an integral over the quality productivity gives the following solution for the price index as a function of the cutoff productivity, the number of firms and parameters of the model:

\[
P = \frac{\sigma}{\sigma - 1} \left( \frac{1}{N} \right)^{\frac{1-\kappa}{\sigma-1}} \left( \frac{\sigma}{L\kappa} \right)^{\frac{\kappa}{\sigma-1}} \beta^{1-\sigma}
\]

(21)

The price index declines in the productivity to produce quality. At first sight this seems at odds with observation 1. But the price index is not only dependent on prices of individual firms, but also on their quality levels. Higher productivity leads to higher quality leading to a lower price index. This effect through quality dominates the effect through prices.

The model solves as follows: the FE and ZCP in equations (14) and (18) together determine the cutoff quality productivity and average profit. (20) can be used to determine the number of firms and (21) to calculate the price index.

In equation (5) it was assumed that the relation between marginal costs and the taste parameter is linear. This can be generalized to a non-linear relation:

\[
a_v = \alpha_v^\mu
\]

(22)

Appendix C shows that one finds a positive solution for \( f_{pbm,v} \) that satisfies the second order condition of the firm when:

\[
0 < \sigma(1-\mu) + \mu < \frac{1}{\kappa}
\]

(23)

Appendix A shows that \( \kappa \) should be smaller than 1 to find an equilibrium for the cutoff quality productivity. So, when \( \mu \) is 1 as in the standard model, the SOC is satisfied. A larger \( \mu \) makes satisfaction of the SOC easier but can lead to a negative optimum. So, \( \mu \) cannot be too large. For \( \sigma = 2 \), \( \mu \) should be smaller than 2 for example to generate a positive optimum. The condition on \( \mu \) is a restriction on the model. To generate large price differences between firms, the taste parameters also have to display large differences.

2.2 Open Economy Model

International trade is introduced in a standard way. There are two countries of equal size and with an equal quality productivity distribution, so as to guarantee equal wage levels. There are per unit iceberg trade costs \( \tau \) and a firm has to incur separate product development, branding and marketing costs for the exporting market, \( f_{pbm,x,v} \). There are also fixed costs of exporting \( f_x \) which can be seen as well as sunk entry exporting
costs as there is no exporting uncertainty. It is assumed that the productivity to produce quality is equal to the quality productivity at home.\(^2\)

The profit from exporting (substituting the CES pricing equation) is defined as:

\[
\pi_{v,x} = \tau^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \beta_v \left( f_{p,m} \right)^{\kappa} P^{\sigma-1} \frac{L}{\sigma} - f_{p_{bm},x,v} - f_x
\]

The equal countries assumption allows focusing on one country only. Country subscripts will be omitted therefore. The subscripts \(d, x\) indicate whether a good is for the domestic or exporting market. The first order condition generates the following solution for \(f_{p_{bm},x,v}\):

\[
f_{p_{bm},x,v} = \left( \frac{1}{\tau} \right)^{\frac{\sigma-1}{1-\kappa}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma-1}{1-\kappa}} \frac{L}{\sigma} \left( \beta_v \right)^{\frac{1}{1-\kappa}} P^{\sigma-1} \frac{L}{\sigma} \left( \beta_v \right)^{\frac{1}{1-\kappa}} - f_x
\] (24)

Substituting this back into the profit equation leads to:

\[
\pi_{v,x} = \left( \frac{1}{\tau} \right)^{\frac{\sigma-1}{1-\kappa}} \kappa^{\frac{1}{1-\kappa}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma-1}{1-\kappa}} \left( \frac{L}{\sigma} \right)^{\frac{1}{1-\kappa}} \left( \beta_v \right)^{\frac{1}{1-\kappa}} - f_x
\] (25)

This implies for the exporting ZCP:

\[
\beta_v^* = f_x \tau^{1-\kappa} \left( \frac{\kappa}{\kappa^{1-\kappa}} - \frac{1}{\kappa^{1-\kappa}} \right)^{\kappa-1} \left( \frac{\sigma - 1}{\sigma} \right)^{1-\sigma} \frac{P^{\sigma-1} \sigma}{L}
\] (26)

Firms have to be more productive to export than for domestic production when:

\[
\beta_v^* > \beta^* : \quad f_x \tau^{\sigma-1} > f
\] (27)

The partition of firms between domestic producing and exporting firms works in the same way as in Melitz (2003). Equation (27) has the following implications:

**Observation 2**

*Only more productive firms can export and firms producing a larger quality can export. Hence, the average quality of exported goods is larger than the average quality of all goods in the economy.*

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\(^2\) It can also be assumed that the quality productivity abroad is lower than in the home market. The motivation would be that it is more difficult to make a product appealing in a foreign market than in the domestic market. The literature on cultural proximity lends support for this approach. Making such an assumption creates problems for the proof of a unique equilibrium in the model where no specific distribution of productivities is assumed.
The partition of firms in equation (27) implies that only more productive firms can export. So, on average exporting firms are more productive. From observation 1 it is known that more productive firms have a larger quality. The size of a firm is equal to:

\[
r_v = p_v x_v = \alpha v \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^\sigma L
\]  

(28)

The following observation follows from equation (28):

**Observation 3**

*Firms with higher quality are bigger. Hence, exporting firms are on average bigger than domestic producing firms.*

The exporting and domestic ZCP can be combined to find one relation between average profit and the two cutoff quality productivities:

\[
\tilde{\pi} = f \left[ \left( \frac{\beta}{\beta^*} \right)^{1-x} - 1 \right] + f_s \left[ \left( \frac{\beta_s}{\beta^*_s} \right)^{1-x} - 1 \right]
\]  

(29)

With:

\[
\tilde{\beta}_s = \left[ \frac{1}{1-G(\beta^*_s)} \int_G^{\beta^*_s} \frac{1}{\beta^{1-x}} g(\beta) d\beta \right]^{1-x}
\]  

(30)

The free entry condition remains as in (14). The open economy model can be solved by combining the FE in (14), the ZCP in (29) and the relation between the two cutoff quality productivities:

\[
\beta^*_s = \left( \frac{f^*}{f} \right)^{1-x} \beta^*
\]  

(31)

Equations (14), (29) and (31) yield solutions for average profit \( \tilde{\pi} \) and the domestic and exporting cutoff productivities, \( \beta^* \) and \( \beta^*_s \). Appendix A shows that one can find a unique equilibrium for the cutoff productivities from these equations. The number of firms can be derived as in the closed economy (see appendix B):

\[
N = \frac{L}{\sigma(\tilde{\pi} + f + p_x f_x)}
\]

\( p_x \) is the probability of exporting, \( p_x = \frac{1-G(\beta^*_s)}{1-G(\beta^*)} \). The expression for the price index becomes:
\[
P = \frac{\sigma}{\sigma - 1} \left( \frac{1}{N} \right)^{\frac{1-\kappa}{\sigma - 1}} \left( \frac{\sigma}{L \kappa} \right)^{\frac{\kappa}{\sigma - 1}} \left[ \beta^{\frac{1}{1-\kappa}} + \frac{1-\sigma}{1-G(\beta_x^*)} \beta_x^{\frac{1}{1-\kappa}} \right]^{\frac{1-\kappa}{1-\sigma}} \tag{32}
\]

The following observations can be made on the effect of trade liberalization in this model:

**Observation 4**
*Trade liberalization increases the average productivity to produce quality and the average quality in the economy.*

**Observation 5**
*Trade liberalization decreases the average productivity to produce quality of exporting firms and decreases their average quality.*

Appendix D proofs that the domestic cutoff productivity $\beta^*$ rises and the exporting cutoff productivity $\tilde{\beta}_x$ declines with trade liberalization. Average productivity $\bar{\beta}$ and average exporting productivity $\tilde{\beta}_x$, defined in equation (19) and equation (30) respectively, are rising in their respective cutoff productivities. This proofs that average productivity rises and average exporting productivity declines with trade liberalization. It can be shown that average quality and average exporting quality rise in average productivity and average productivity of exporting firms respectively, which proofs the last parts of observation 4 and 5.

The reallocation effects are familiar from Melitz (2003). The least productive firms are squeezed out of the market with trade liberalization increasing average productivity in the economy. Trade liberalization enables more firms to export, so also firms with a lower productivity and quality can start to export. Observations 4 and 5 are empirically testable. The predictions are that the economy wide average quality rises with trade liberalization, but the average quality of exporting declines with trade liberalization. Section 4 contains a discussion of the model presented in this section. Before that, in the next section the between country differences model is pointed out.

### 3 Between Country Productivity Differences

Productivity differences between countries can be modeled in Ricardian fashion and a factor abundance fashion. In the former approach there are productivity differences between countries by assumption. In the latter approach productivity differences are related to factor abundance differences. A Dixit-Stiglitz monopolistic competition model with Ricardian differences in productivity is proposed by Hummels and Klenow (2002). Schott (2004) shows that quality differences between countries are related to factor abundance. Therefore, here a monopolistic competition model with productivity differences between countries based on factor abundance will be considered. The
novelty is that production is non-homothetic in factors of production. Higher quality goods require relatively more high skilled labor. The model reverts to equal firms within countries to keep the model analytically tractable. Section 3.1 points out the closed economy model and section 3.2 goes into the open economy model. Section 3.3 contains a discussion of the properties of the model and a comparison with other, similar models.

3.1 Closed Economy Model

This section considers the closed economy. Utility, the demand facing a firm and the price index are the same as in the previous section and given by equations (1), (2) and (3) respectively. Production is increasing returns with a fixed cost of production. There are two factors of production, high-skilled labor and low-skilled labor. The cost function is a non-homothetic CES function described for example in Shimomura (1999). The cost function is non-homothetic in quality. It is given by:

\[ C(\alpha, w_s, w_U) = (\alpha x + f) \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_U^{1-\theta} \right)^{\frac{1}{1-\theta}} \]  

\( w_s \) and \( w_U \) are the wage levels of skilled and unskilled labor, respectively. \( \theta \) is the substitution elasticity between skilled and unskilled labor and \( \gamma(\alpha) \) is the CES-weight dependent on quality, constituting the non-homotheticity. So, in this specification costs depend on quality through the marginal costs and through the non-homotheticity in skilled and unskilled labor. The effect through fixed product development, branding and marketing costs present in the within country heterogeneity model is omitted.

The price of a good is given by the familiar markup equation:

\[ p = \frac{\sigma}{\sigma - 1} \alpha \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_U^{1-\theta} \right)^{\frac{1}{1-\theta}} \]  

(34)

Substituting the CES-pricing equation, profit becomes:

\[ \pi = \alpha \left[ \frac{\sigma}{\sigma - 1} \left( \gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_U^{1-\theta} \right)^{\frac{1}{1-\theta}} \right]^{1-\sigma} P^{\sigma-1} \frac{I}{\sigma} \]

\[ - f(\gamma(\alpha) w_s^{1-\theta} + (1 - \gamma(\alpha)) w_U^{1-\theta})^{\frac{1}{1-\theta}} \]  

(35)

Taking the FOC towards quality gives:
\[
\frac{p_x}{\sigma}\left[1 - \frac{\sigma-1}{\theta-1}\frac{\gamma'(\alpha)(w_U^{1-\theta} - w_S^{1-\theta})}{\gamma(\alpha)w_S^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta}}\right] \\
-\frac{f}{\theta-1}\left(\gamma(\alpha)w_S^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta}\right)^{\frac{1}{1-\theta}} \frac{\gamma'(\alpha)(w_U^{1-\theta} - w_S^{1-\theta})}{\gamma(\alpha)w_S^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta}} = 0
\]  

(36)

A zero profit equation can be added to the model:

\[
\frac{p_x}{\sigma} = f\left(\gamma(\alpha)w_S^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta}\right)^{\frac{1}{1-\theta}}
\]  

(37)

Substituting the zero profit equation (37) into the FOC, equation (36), one finds:

\[
\frac{p_x}{\sigma}\left[1 - \frac{\sigma}{\theta-1}\frac{\gamma'(\alpha)(w_U^{1-\theta} - w_S^{1-\theta})}{\gamma(\alpha)w_S^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta}}\right] = 0
\]  

(38)

The first term between brackets measures the marginal benefit of larger quality, due to larger sales. The second term measures the loss due to a larger required use of skilled labor when quality is larger.

To solve for \(\alpha\), an explicit function for \(\gamma(\alpha)\) should be chosen that is rising in \(\alpha\) and bounded between 0 and 1. The following function is chosen:

\[
\gamma(\alpha) = \alpha^\eta; \quad \eta > 1
\]  

(39)

The condition on \(\eta\) is needed to satisfy the second order condition. This \(\gamma(\alpha)\) yields the following solution for \(\alpha\):

\[
\alpha = \left[\frac{\theta - 1}{\theta - 1 + \sigma\eta}\right]^{\frac{1}{\eta}} \left\{\frac{1}{1 - \left(\frac{w_U}{w_S}\right)^{\theta-1}}\right\}^{\frac{1}{\eta}}
\]  

(40)

Equation (40) shows that quality is rising in the relative wage of unskilled over skilled workers, \(\frac{w_U}{w_S}\). Logdifferentiating equation (40) gives:

\[\alpha \text{ is smaller than 1 as long as } \frac{w_U}{w_S} < \left(\frac{\sigma\eta}{\theta - 1 + \sigma\eta}\right)^{\frac{1}{\theta-1}}. \text{ So, it is assumed that } \theta \text{ is not too large relative to } \sigma \text{ and } \eta. \text{ Unfortunately it is not possible to give a general condition for } \alpha < 1 \text{ depending on parameters and exogenous variables only.}\]
Unit wage costs of production are equal to:

\[
\hat{w}_s \equiv \frac{-1}{\theta-1} \left( \frac{w_s}{w_U} \right)^{\theta-1} \hat{\gamma}
\]  

(41)

Normalizing the unskilled wage at 1, this expression can be logdifferentiated as:

\[
\hat{UC} = \frac{1}{\theta-1} \left( \frac{\gamma w_s^{1-\theta}}{\gamma w_s^{1-\theta} + 1 - \gamma} \right) \hat{\gamma} + \frac{\gamma w_s^{1-\theta}}{\gamma w_s^{1-\theta} + 1 - \gamma} \hat{w}_s
\]  

(43)

Substituting the logdifferentiation in (41) the relative change of unit wage costs in (43) becomes zero. A larger skilled wage \( w_s \) leads to lower quality. The net effect is that unit costs do not change in response to a change in the relative wage because of the endogenous reaction in quality. This result is due to the specification chosen for \( \gamma(\alpha) \).

The following observation can be made:

**Observation 6**

*Unit wage costs do not change when relative wages change. An increase in the relative wage of skilled labor decreases the quality of goods implying that unit wage costs do not rise.*

To solve for the endogenous variables in the model, two labor market equilibrium equations can be added. Using Shepard’s lemma, one finds:

\[
L_s = N(\alpha x + f)(\gamma(\alpha)w_s^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta})^{\theta} \gamma(\alpha)w_s^{-\theta}
\]  

(44)

\[
L_U = N(\alpha x + f)(\gamma(\alpha)w_s^{1-\theta} + (1 - \gamma(\alpha))w_U^{1-\theta})^{\theta} (1 - \gamma(\alpha))w_U^{-\theta}
\]  

(45)

Dividing the labor market equations (44) and (45) leads to:

\[
\frac{L_s}{L_U} = \frac{\gamma(\alpha)}{1 - \gamma(\alpha)} \left( \frac{w_U}{w_s} \right)^{\theta}
\]  

(46)

Combining equations (40) and (46) generates an implicit relation between \( \gamma \) (or \( \alpha \)) and the relative factor abundance:
\[
\frac{L_s}{L_u} = \gamma(\alpha) \left[ \frac{((\theta-1)+\sigma\eta)\gamma(\alpha)-\theta-1}{(\theta-1+\sigma\eta)\gamma(\alpha)} \right]^{\alpha/(\theta-1)}
\]  
(47)

Logdifferentiating equation (47) shows that the RHS (47) rises monotonically in \(\gamma\):

\[
\frac{\hat{L_s}}{L_u} = \frac{1}{1-\gamma} \hat{\gamma} + \frac{\theta}{\gamma(\theta-1+\sigma\eta)-\theta-1} \hat{\gamma}
\]  
(48)

Using (48) it can be shown that the RHS of (47) rises monotonically from \(-\frac{\theta-1}{\theta-1+\sigma\eta}\) to \(\infty\) for \(\gamma \in [0,1]\). So, there is a unique positive equilibrium for \(\gamma\).

Equation (48) also shows that quality rises in relative factor abundance of high skilled labor. So, \(\alpha\) and \(\gamma\) rise when an economy becomes more skill abundant. Therefore, equation (48) has the following implication.

**Observation 7**

*The quality of goods produced rise in the skill abundance of the economy.*

Equation (48) can be rewritten as:

\[
\hat{\gamma} = \frac{(1-\gamma)\gamma\sigma\eta - (1-\gamma)^2(\theta-1)}{1-\gamma+\sigma\eta} \frac{\hat{L_s}}{L_u}
\]  
(49)

Equation (49) shows that the effect of skill abundance on quality declines when the elasticity of substitution between skilled and unskilled labor is larger, as to be expected. Logdifferentiating the market price equation (34) and using observation 6 that the unit cost does not change one finds:

\[
\hat{p} = \frac{1}{\gamma} \frac{(1-\gamma)\gamma\sigma\eta - (1-\gamma)^2(\theta-1)}{1-\gamma+\sigma\eta} \frac{\hat{L_s}}{L_u}
\]  
(50)

The implication is that the market price only changes in reaction to changes in skill abundance through the changed marginal cost that depends on quality and not through the unit wage costs. From equation (50) the following observation can be made.

**Observation 8**

*The market price rises in the skill abundance of the economy.*

The model presented in this subsection contains a link between skill abundance, quality and market price. The relations are congruent with the findings by Schott (2004). More
high skilled labor leads to higher quality and larger prices. The next subsection introduces trade in the model.

3.2 Open Economy Model

There are two countries, \( k, l = H, F \). There are per unit iceberg trade costs \( \tau \). There are also fixed costs of exporting \( f_x \). Venables (1994), Jean (2002) and Medin (2003) include fixed export costs in their models as well, but their motivation is not grounded on firm empirical evidence. When there is no uncertainty, the fixed export costs can also be seen as per period equivalents of sunk export costs for which there is ample empirical evidence. The cost function of a firm in country \( k \) is given by:

\[
C_k = (\alpha_{k}x_{k} + f_x) (\gamma(\alpha_{k}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{k})) w_{Uk}^{1-\theta})^{1/\theta} + (\alpha_{k}x_{k} + f_x) (\gamma(\alpha_{k}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{k})) w_{Uk}^{1-\theta})^{1/\theta}
\]

Country subscript \( k, l = H, F \) indicate the country of origin and the subscripts \( d, x \) indicate whether a good is for the domestic or exporting market. Substituting the markup pricing equations, the profit of a firm in \( k \) can be written as:

\[
\pi_k = \alpha_{kd} \left[ \frac{\sigma - 1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Ud}^{1-\theta})^{1/\theta} \right]^{\gamma - 1} \left[ \frac{\sigma - 1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta})^{1/\theta} \right]^{\gamma - 1} \pi_k^{\gamma - 1} \tau_k
\]

The first order conditions towards quality in the domestic and exporting market are given by:

\[
\frac{P_{kd}x_{kd}}{\sigma} \left[ \frac{1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Ud}^{1-\theta} - w_{Sk}^{1-\theta}) \right]^{\gamma - 1} \left[ \frac{1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta}) \right]^{\gamma - 1} \pi_k^{\gamma - 1} \tau_k = 0
\]

\[
\frac{P_{kd}x_{kd}}{\sigma} \left[ \frac{1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Ud}^{1-\theta} - w_{Sk}^{1-\theta}) \right]^{\gamma - 1} \left[ \frac{1}{\sigma - 1} (\gamma(\alpha_{kd}) w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta}) \right]^{\gamma - 1} \pi_k^{\gamma - 1} \tau_k = 0
\]
Zero profit equations can be added to the model. There are two zero profit equations, for domestic and exporting production. This can be motivated as follows. Suppose profits were negative in one of the two markets. Then firms would leave that market. If profits were positive in one of the markets, firms would enter that market. The domestic and exporting market can be seen as separate markets, because there are fixed exporting costs besides regular fixed costs. One can also interpret both fixed costs as beachhead costs to enter the respective markets. Entering the domestic market and paying the domestic beachhead cost does not imply that one can also export as in the standard Krugman (1980) model without fixed export costs. Considering the domestic and exporting markets as separate markets leading to two zero profit conditions implies that the number of firms in the two markets can be different. The zero profit conditions are given by:

\[ \frac{p_{kd}x_{kd}}{\sigma} = f\left(\gamma(\alpha_{kd})w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd}))w_{Uk}^{1-\theta}\right)^{\frac{1}{\eta}} \]  

(55)

\[ \frac{p_{ks}x_{ks}}{\sigma} = f\left(\gamma(\alpha_{ks})w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{ks}))w_{Uk}^{1-\theta}\right)^{\frac{1}{\eta}} \]  

(56)

Combining the zero profit conditions with the first order conditions leads to:

\[ \frac{p_{kd}x_{kd}}{\sigma} \left[ \frac{1}{\alpha_{kd}} - \frac{\sigma}{\theta - 1} \frac{\gamma'(\alpha_{kd})}{\gamma(\alpha_{kd})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta} \right] = 0 \]  

(57)

\[ \frac{p_{ks}x_{ks}}{\sigma} \left[ \frac{1}{\alpha_{ks}} - \frac{\sigma}{\theta - 1} \frac{\gamma'(\alpha_{ks})}{\gamma(\alpha_{ks})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{ks})) w_{Uk}^{1-\theta} \right] = 0 \]  

(58)

Using the explicit expression for \( \gamma(\alpha) \) in equation (39), the solutions for the domestic and exporting quality level become:

\[ \alpha_{kd} = \left[ \frac{\theta - 1}{\theta - 1 + \sigma \eta} \right]^{\frac{1}{\eta}} \left[ \frac{1}{1 - \left(\frac{w_{Uk}}{w_{Sk}}\right)^{\theta-1}} \right] \]  

(59)

\[ \alpha_{ks} = \left[ \frac{\theta - 1}{\theta - 1 + \sigma \eta} \right]^{\frac{1}{\eta}} \left[ \frac{1}{1 - \left(\frac{w_{Uk}}{w_{Sk}}\right)^{\theta-1}} \right] \]  

(60)

Comparing equation (59) and (60) shows that quality for the domestic market and the exporting market are equal, which can be used later on.
As market size effects are not the focus of the present paper, it is assumed that the two countries are of equal size, i.e. $L_{Sk} + L_{Uk} = L_{Si} + L_{Ui}$. This implies that market size effects are small.\footnote{A precise analytic condition for the absence of market size effects dependent only on the parameters of the model and factor endowments cannot be given.} Abstracting from market size effects one can concentrate on differences in relative factor endowments instead of absolute factor endowments. The analysis continues by focusing on the effect of relative factor abundance differences between countries.

The labor market equilibrium equations can be added to solve for the endogenous variables in the model:

\[
L_{Sk} = N_{kd} (\alpha_{kd} x_{kd} + f)^{\gamma(\alpha_{kd})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta}) \frac{\theta}{\beta} \gamma(\alpha_{kd}) w_{Sk}^{-\theta}
\]
\[
+ N_{ks} (\alpha_{ks} x_{ks} + f)^{\gamma(\alpha_{ks})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{ks})) w_{Uk}^{1-\theta}) \frac{\theta}{\beta} \gamma(\alpha_{ks}) w_{Sk}^{-\theta}
\]
\[
L_{Uk} = N_{kd} (\alpha_{kd} x_{kd} + f)^{\gamma(\alpha_{kd})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{kd})) w_{Uk}^{1-\theta}) \frac{\theta}{\beta} (1 - \gamma(\alpha_{kd})) w_{Uk}^{-\theta}
\]
\[
+ N_{ks} (\alpha_{ks} x_{ks} + f)^{\gamma(\alpha_{ks})} w_{Sk}^{1-\theta} + (1 - \gamma(\alpha_{ks})) w_{Uk}^{1-\theta}) \frac{\theta}{\beta} (1 - \gamma(\alpha_{ks})) w_{Uk}^{-\theta}
\]

From equations (59) and (60) it is clear that $\alpha_{kd}$ and $\alpha_{ks}$ are equal. Dividing the labor market equations, using as well the zero profit conditions, $\sigma x = \sigma f$, leads to:

\[
\frac{L_{Sk}}{L_{Uk}} = \frac{\gamma(\alpha_k)}{1 - \gamma(\alpha_k)} \left( \frac{w_{Uk}}{w_{Sk}} \right)^{\theta}
\]

(63)

So, one arrives at the same expression as in the closed economy. Logdifferentiating towards relative factor abundance and quality generates:

\[
\hat{a}_d = \hat{a}_s = \frac{1}{\eta} (1 - \gamma) \sigma \eta - \frac{(1 - \gamma)^2}{\theta - 1} \frac{L_{Sk}}{L_{Uk}}
\]

(64)

The market price rises in the quality of a good like in the closed economy model. So, observations 6 and 7 also hold in the open economy model:

**Observation 9**

*Export goods from relatively more skill abundant countries have a larger quality and a higher price.*
4 Discussion

The within country differences model in section 2 created a link between the amount of resources invested in product development, branding and marketing and the quality of a good. A larger quality of a good in turn implies a larger marginal cost and thus a larger price. This brings the heterogeneous productivity model more in line with the empirical finding that goods from high income countries display higher unit values. The within differences model also generates some outcomes that are empirically testable. Exporting firms are more productive, produce a higher quality and are bigger. Trade liberalization increases the average quality in the economy, but it decreases the average quality of exports, because more firms can export.

A possible critique to the presented model is that the definition of productivity is unnatural. There are two replies. First, empirically productivity is measured as value added divided by the value of inputs. So, also in the standard monopolistic competition model theoretical productivity is not well related to measured productivity. Second, there is a natural interpretation of the productivity to produce quality applied in this paper. It is a measure for the inherent appeal of a product variety, so it can be seen as the productivity of a variety.

Another point of critique could be that the model features regular fixed costs besides fixed costs in product development, branding and marketing. These regular fixed costs are needed to ensure that only a part of the entering firms can produce profitably. The regular fixed costs can be motivated referring to standard fixed costs like those of overhead. Also in exporting regular fixed costs are required besides fixed costs in product development, branding and marketing to ensure that only some firms can export profitably. There is wide empirical support for the importance of sunk export costs (See for example Roberts and Tybout, 1997 and Das, Roberts and Tybout, 2007). This paper only assumes that these sunk export costs can be split up in a part that does affect the quality of the good like product development costs and a part that does not like setting up distribution channels and costs to comply with local regulations.

Three models that are similar to the present model are discussed now. The first features in Hummels and Klenow (2002). In one of the models in their paper costs are also dependent on quality in a CES-framework. They also introduce a productivity to produce quality parameter and firms can choose the amount of resources invested in product quality. But in their model all firms in a country have the same quality productivity. So, it is a model of between country differences. Furthermore, quality is only related to marginal costs. Choosing Hummels and Klenow’s (2002) framework to model within country differences does not lead to sensible results. The price rises in their model only in quality, because firms in a country with higher quality productivity demand more labor leading to higher wages. So, the quality differences between countries are caused by differences in wage levels. Hence, one cannot generate within country differences in prices and quality with their modeling framework.⁵

⁵ Hummels and Klenow’s model with quality only related to marginal costs can generate within country differences in quality and prices, when the marginal cost function would be made very complex. But then the model would not be solvable anymore and for example the price index can not be written as an integral over the productivities. Besides the computational problems involved in making the quality productivity only dependent on marginal costs, the setup in the present model has
A second paper comes closest to the within country differences model. Helble & Okubo (2006) also model vertical product differentiation in a Melitz type monopolistic competition model. More productive firms produce goods with higher prices, which reflects larger quality. They use a different way to achieve this result than in the present paper. ‘More productive’ firms have a higher marginal cost and therefore charge larger prices. Firms with higher marginal costs are nonetheless more productive, because they need less marketing costs. The model by Helble & Okubo (2006) contains some predictions that are congruent with empirics. Exports have a more than average quality; trade liberalization decreases the average quality of exports and more trading partners leads to a higher product quality.

There are several differences between Helble & Okubo (2006) and the present model. First, Helble & Okubo don’t model a steady state of entry and exit. A distribution of firms is given and there is no entry or exit. Second, because the distribution of firms is given, there is no reallocation effect. In the present paper there is a reallocation effect: trade liberalization squeezes the least productive firms out of the market and raises average product quality. Third, ‘more productive’ firms are smaller, because they charge higher prices due to the larger marginal costs. This implies that exporting firms are on average smaller, which is at odds with empirical findings. In the present paper, more productive and exporting firms do have a larger size. Fourth, Helble & Okubo assume that more productive firms need to incur less marketing costs. It seems more intuitive that more productive firms spend more on marketing, because it generates a larger payoff for them like in the present model.

A third similar model is proposed in Baldwin and Harrigan (2007). They adapt the Melitz model introducing quality in the utility function through the CES-weights and making marginal costs dependent on quality. In this way higher quality goods display higher prices. The present model contains the same elements as Baldwin and Harrigan with marginal costs rising in quality, but has a richer setup. First, quality is a choice variable for firms in the present model, whereas in Baldwin and Harrigan firms just draw their level of quality and corresponding marginal costs. Second, the quality of goods also depends on fixed costs of product development in the present model.

The between country differences model can account for a different empirical finding, the fact that goods from more skill abundant countries display higher unit values. Production is non-homothetic in quality in the between country differences model. The larger the quality of a good, the more skilled labor is needed. This model setup implies that more skill abundant countries produce higher quality goods. As marginal costs also rise in quality, higher quality goods also have a larger price.

Hummels & Klenow (2002) also model between country differences in the productivity to produce quality. They just assume that countries have different productivities. The present paper relates the capability to produce quality to relative factor abundance and skill abundance in particular. Therefore, Hummels and Klenow’s model is Ricardian, whereas the present model is of the Heksch-Olhin-type. Hummels and Klenow’s model can be linked to the empirical finding that goods from higher income countries and so from more productive countries display higher unit values, whereas the model in this paper accounts for the empirical finding that goods from more skill abundant countries display higher unit values.

more intuitive appeal. The quality of a good can be increased by investing more in product development and marketing which are costs that are fixed and sunk in nature.
Verhoogen (2007) introduces a model with heterogeneous firms and quality differentiation where more productive firms pay higher wages to attract the most qualified workers. Verhoogen’s (2007) model contains the same building blocks as the present model, but the setup is basically different. The demand system is different and moreover the model is partial equilibrium with given higher average wages for skilled workers.

5 Concluding Remarks

The present paper provides a theoretical reply to recent empirical findings on exporting unit values, in particular in Schott (2004). It is shown that the heterogeneous productivity monopolistic competition model can be adapted to account for empirical findings that seemed to be at odds with this standard model. In the within country differences model the heterogeneous productivity Melitz model is modified in such a way that more productive firms produce higher quality goods and charge larger prices instead of lower prices. This is in line with empirical work showing that high income countries export goods with higher unit values. The CES-model with quality weights is used, a productivity to produce quality is introduced and product quality is dependent on product development, branding and marketing expenditures.

In the between country differences model the monopolistic competition Krugman model is modified such that productivity differences within sectors relate to factor abundance. Production is non-homothetic with higher quality goods requiring relatively more skilled labor. This implies that more skill abundant countries produce higher quality goods with higher unit values, congruent with empirical findings. Combining the two models using simulations would make it possible to study the effect of trade liberalization on the skill premium. This is left for future work. The present paper’s main goal was to show that the heterogeneous productivity monopolistic competition model can be modified in a tractable way to bring this standard model in line with recent empirical findings on exporting unit values.
References


Hummels, David and Peter J. Klenow (2002), ‘The Variety and Quality of a Nation's Trade,’ NBER Working Papers 8712


Appendix A Uniqueness of the Closed and Open Economy Equilibrium of the Within Productivity Differences Model

To prove that the FE and ZCP, equations (14) and (18) yield a unique equilibrium for the cutoff quality productivity, one can proceed analogous to Melitz (2003) in his appendix B. Equations (14) and (18) can be combined to get the following equation:

\[
(1 - G(\beta^*)) \left( \frac{\tilde{\theta}(\beta^*)}{\beta^*} \right)^{\frac{1}{1-x}} - 1 = \frac{\partial f}{\partial f}
\]  

(A1)

It will be shown that the LHS of (A1) is monotonically decreasing from \(\infty\) to 0 on \((0, \infty)\), which is sufficient to show that there is a unique equilibrium.

Define \(k(\beta)\) as:

\[
k(\beta) = \left[ \left( \frac{\tilde{\beta}(\beta)}{\beta} \right)^{\frac{1}{1-x}} - 1 \right]
\]  

(A2)

Differentiating \(k(\beta)\) towards \(\beta\) gives:

\[
k'(\beta) = \frac{g(\beta)}{1-G(\beta)} \left[ \left( \frac{\tilde{\beta}(\beta)}{\beta} \right)^{\frac{1}{1-x}} - 1 \right] - \frac{1}{1-\kappa} \left( \frac{\tilde{\beta}(\beta)}{\beta} \right)^{\frac{1}{1-x}} \frac{1}{\beta}
\]  

(A3)

\(j(\beta)\) is defined as:

\[
j(\beta) = (1 - G(\beta))k(\beta)
\]  

(A4)

\(j(\beta)\) is equal to the LHS of A1. Therefore \(j'(\beta)\) can be computed as:

\[
j'(\beta_v) = -\frac{1}{1-\kappa} \left( \frac{\tilde{\beta}_v(\beta_v)}{\beta_v} \right)^{\frac{1}{1-x}} \frac{1}{\beta_v} < 0
\]  

(A5)

It is easy to see that \(\lim_{\beta \to \infty} j(\beta) = \infty\) and \(\lim_{\beta \to \infty} j(\beta) = 0\). Hence, \(j(\beta)\) is monotonically decreasing from \(\infty\) to 0 on \((0, \infty)\).

The equilibrium of the open economy model is constituted by equations (14), (29) and (31). Combining those leads to the following equilibrium equation:
\[ f_j(\beta^*) + f_x j(\beta_x^*) = \partial \theta \]  

(A6)

Equation (A5) showed that \( j(\beta) \) is monotonically decreasing from \( \infty \) to 0 on \((0, \infty)\). Given that \( \beta_x^* \) is monotonically increasing in \( \beta^* \), the LHS of A6 is also decreasing from \( \infty \) to 0 on \((0, \infty)\) and hence there is a unique equilibrium.

Appendix B Number of Firms in Within Differences Model

First the number of firms in the closed economy is derived and then in the open economy. In the closed economy the steady state of entry and exit dictates that the number of entering firms should be equal to the number of exiting firms:

\[ (1 - G(\beta^*)) N_e = \delta N \]  

(B1)

\( N_e \) is the number of all successful and unsuccessful entrants. Labor market equilibrium in the production sector and innovation sector are given by:

\[ L_p = R - \Pi = N \tilde{r} - N \tilde{\pi} \]  

(B2)

\[ L_e = N_e f_e = \frac{\delta N}{1 - G(\beta_e^*)} f_e = N \tilde{\pi} \]  

(B3)

\( L_p \) and \( L_e \) are the amount of labor used in the production and innovation sector, respectively. Adding the two labor market equilibriums leads to:

\[ L = L_p + L_e = N \tilde{r} \]  

(B4)

(B4) can be rewritten to find the number of firms as a function of average profit:

\[ N = \frac{L}{\sigma(\tilde{\pi} + f)} \]  

(B5)

In the open economy the derivation is similar. Labor can be allocated in four different ways, in domestic production \( L_p \), in domestic innovation \( L_e \), in exporting production \( L_{p,x} \) or in exporting innovation \( L_{e,x} \):

\[ L = L_p + L_{p,x} + L_e + L_{e,x} \]  

(B6)

The expression for domestic innovation is equal to the closed economy one, given in equation (B3). The others become:

\[ L_p = N \tilde{r}_d - N \tilde{\pi}_d \]  

(B7)
\[ L_{p,x} = N_x \tilde{r}_x - N_x \tilde{\pi}_x - N_x f_x \quad (B8) \]

\[ L_{e,x} = f_{e,x} N_{e,x} = f_{e,x} \delta N_x = f_x N_x \quad (B9) \]

\( N_x \) is the number of firms producing for the exporting market and \( N_{e,x} \) is the number of firms entering the export market. So, adding the labor market allocations gives:

\[ L = N (\tilde{r}_d + p_x \tilde{r}_x) \quad (B10) \]

The expression for the number of firms becomes:

\[ N = \frac{L}{\sigma (\tilde{\pi} + f + p_x f_x)} \quad (B11) \]

Appendix C More General Marginal Cost Function in the Within Productivity Differences Model

When the marginal cost function is given by (22) the profit of the firm becomes:

\[
\pi_v = \alpha_v \sigma^{(1-\mu)\mu} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^{\sigma-1} \frac{L}{\sigma} - f_{pm,v} - f \]

\[
= \left( \beta_v f_{pm,v}^{\kappa} \right)^{\sigma^{(1-\mu)\mu}} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^{\sigma-1} \frac{L}{\sigma} - f_{pm,v} - f \quad (C1) \]

The first order condition is:

\[
\frac{\partial \pi_v}{\partial f_{pm,v}} = \kappa (\sigma (1-\mu) + \mu) \left( \beta_v f_{pm,v}^{\kappa} \right)^{\sigma^{(1-\mu)\mu}-1} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p^{\sigma-1} \frac{L}{\sigma} = 1
\]

So, the solution for \( f_{pm,v} \) is:

\[
f_{pm,v} = \left[ \kappa (\sigma (1-\mu) + \mu) p^{\sigma-1} \frac{L}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma - 1} \right]^{1-\kappa (\sigma (1-\mu) + \mu)} \beta_v^{\sigma^{(1-\mu)\mu}} \quad (C2) \]

So, a positive solution for \( f_{pm,v} \) is found when:

\[
\sigma (1-\mu) + \mu > 0 \quad (C3) \]

The second order condition is satisfied when:
\[
\sigma (1 - \mu) + \mu < \frac{1}{\kappa}
\]  \tag{C4}

Appendix D The Effects of Trade Liberalization in the Within Productivity Differences Model on Cutoff Productivities

This appendix shows that the domestic cutoff productivity to produce quality \( \beta^* \) rises with trade liberalization and that the exporting cutoff productivity \( \beta_x^* \) declines with trade liberalization. Equation (A6) is the equilibrium equation of the open economy model. Totally differentiating this equation towards the cutoff productivity and trade costs \( \tau \) taking into account equation (31) for the relation between the domestic and exporting cutoff productivities, generates:

\[
\frac{d\beta^*}{d\tau} = -\frac{f_x j'\left(\beta^*_x\right)(\sigma - 1) \left(\frac{f_x}{f}\right)^{1-\kappa} \tau^{\sigma-2} \beta^*}{f'\left(\beta^*_x\right) + f_x j'\left(\beta^*_x\right) \left(\frac{f_x}{f}\right)^{1-\kappa} \tau^{\sigma-1}} < 0 \tag{D1}
\]

The RHS of (C1) is negative as \( j'(\beta) \) is negative for all values of \( \beta \). Hence (D1) proofs that lower trade costs lead to a higher cutoff productivity. A declining exporting cutoff productivity level as a result of trade liberalization can be proved by differentiating equation (31) towards trade costs \( \tau \) :

\[
\frac{\partial \beta^*_x}{\partial \tau} = \left(\frac{f_x}{f}\right)^{\frac{1}{1-\kappa}} \tau^{\sigma-1} \left[ (\sigma - 1) \frac{\beta^*_x}{\tau} + \frac{d\beta^*}{d\tau} \right] \tag{D2}
\]

Substituting equation (D1) leads to:

\[
\frac{\partial \beta^*_x}{\partial \tau} = \left(\frac{f_x}{f}\right)^{\frac{1}{1-\kappa}} \tau^{\sigma-1} \left[ 1 - \frac{f_x j'\left(\beta^*_x\right)(\sigma - 1) \left(\frac{f_x}{f}\right)^{1-\kappa} \tau^{\sigma-1}}{f'\left(\beta^*_x\right) + f_x j'\left(\beta^*_x\right) \left(\frac{f_x}{f}\right)^{1-\kappa} \tau^{\sigma-1}} \right] > 0 \tag{D3}
\]

The second term between brackets of the LHS of (D3) is smaller than 1. Therefore, the exporting cutoff productivity \( \beta_x^* \) rises in trade costs \( \tau \), implying that also average exporting productivity.