International trade and the political economy of transport infrastructure investment

Gabriel J. Felbermayr†
University of Tübingen

July, 2006
PRELIMINARY, PLEASE DO NOT CITE

Abstract

This paper develops a simple model of intra- and international trade with endogenous transport infrastructure investment. Transportation costs between two addresses depend on cumulative infrastructure investment between these points. Consumers demand goods from all over the world, but independent governments decide on infrastructure projects without taking into account the externality that they exert on foreign consumers. In this setting world, infrastructure investment will be inefficiently low and its spatial variation excessively high. The setup provides a rationale for the strong border effects found in the literature. Tentative econometric evidence for within US trade flows corroborate the theoretical predictions.

JEL Codes: F1, R0, R3

Keywords: Political Economy, International Trade, Infrastructure Investment, Border Effect Puzzle.

*I am grateful to seminar participants at the European University Institute and at Tübingen University for comments. I thank Giancarlo Corsetti, Benny Jung, Wilhelm Kohler, Omar Licandro, Philip Sauré and Farid Toubal for stimulating discussions and helpful comments. Any remaining deficiencies are mine.

†Address: Nauklerstraße 47, 72074 Tübingen, Germany. Tel.: +49 7071 29 78183. Email: gabriel.felbermayr@uni-tuebingen.de
1 Introduction

In empirical models of bilateral trade, international trade volumes are by an order of magnitude smaller than intranational ones. Using 1993 data, McCallum (1995) compares trade flows within Canada to flows between Canadian provinces and U.S. states, controlling for distance and regional GDPs. Everything else equal, crossing the border reduces trade by a factor as high as 22. For Europe, Nitsch (2000) finds that on average intranational trade is about 10 times higher than international one. Nitsch arrives at his results after controlling for cultural proximity (language), along other conventional gravity covariates.¹ Obstfeld and Rogoff (2000) have cited this fact as a major puzzle in international macroeconomics.

More recently, Anderson and van Wincoop (2003) have proposed a solution to the border puzzle. They argue that bilateral trade flows are affected by average trade barriers with respect to third countries. Controlling for size, distance and multilateral resistance effects, Anderson and van Wincoop’s results imply that intra-Canadian trade is 10.5 times higher than cross border trade. The number for the US is lower, with intra-US trade 2.6 times higher than cross border trade (Feenstra, 2005, p. 160). The degree to which these results actually are a solution of the border puzzle is a matter of ongoing debate.

However, there are at least two reasons why research into a better understanding of border effects is important. First, sizeable border effects also appear in the context of interregional trade within the same country where explanations of the border effect based on home market bias in preferences, tax systems, formal and in-formal trade policies, linguistic, legal, or currency differences, do not help. Okubo (2004) shows its importance for trade between Japanese regions, Wolf (2004) for the US, while Combes et al. (2005) find a strong border effect for their sample of French departments. Second, even if the word ‘puzzle’ may be an exaggeration, estimated border effects are large in the sense that they reflect the importance of unmeasured or unmeasurable trade barriers between countries.² A better grasp of the economic factors behind the border effect may offer new

¹Wei (1996) constructs measures for imports of countries to themselves and compares this with imports from a statistically identical foreign country. He finds that the former magnitude is 2.5 times larger than the latter. Helliwell (1998) offers a comprehensive overview of the pre Anderson and van Wincoop state of the econometric literature. Evans (2003) decomposes cross-country price differences of traded goods into a component due to distortions and a component driven by consumer preferences. She demonstrates that the preferences effect is relatively important quantitatively.

²As with total factor productivity in the empiricals of economic growth, we may refer to the border effect
avenues for deeper trade integration.

This paper proposes a novel explanation for the border effect that applies on the international as well as on the interregional level. If countries (or regions') have some autonomy in deciding on the quantity and quality of infrastructure within their reach, they tend to concentrate investment in central regions, even if the geographical shape and the distribution of economic activity across space would not warrant such behavior.

The reasoning behind this key result is the following. Each region’s social planner cares about utility only of domestic agents. Consumers demand goods from all possible locations. Hence, investment at any location improves the utility of all consumers in the world. However, regional social planners behave in a non-cooperative way, failing to fully internalize the effect of their infrastructure investment decisions on consumers other than those residing in their political constituency. This behavior leads to global infrastructure under-investment. It also biases investment within countries towards central regions since the average domestic consumer benefits more from central than from peripheral investment. Our result is sufficient general to survive in a number of different political economy setups and economic environments. It follows from the fundamental separation between political and economic space, the former being regional, the latter global in scope.

Modeling endogenous infrastructure decisions in a spatial world is interesting in its own right, since it leads to interesting political issues relating to efficient provision of transport infrastructure. However, the excessive spatial variation of infrastructure investment that results from the political economy process may help towards an explanation of the border puzzle. Empirically, anecdotal and more rigorous econometric evidence both suggest that trade costs are larger whenever border regions are involved. For example, Combes and Lafourcade (2005) document a strong core-periphery pattern of real trade costs for France. This pattern cannot be explained by variation in topology alone, suggesting systematic core-periphery pattern in the quality and quantity of infrastructure investment.

The present paper is of course not the first that takes the border puzzle as a starting point. There are two main existing theoretical explanations: The most obvious one relates to home-market bias in preferences (Obstfeld and Rogoff, 2000). This explanation is plausible, but does not lend to straightforward empirical validation. The second explanation relates to trade costs. Measurable trade costs at borders are too small to justify the size

as to a ‘measure of ignorance’, since we find the effect important and yet do not understand what hides behind it.
of the border effect. Some researchers argue that informal (and hence unmeasured) trade costs go some way in explaining the border effect. Casella and Rauch (2003) focus on differences between domestic and foreign information costs, Anderson (2003) on contract enforcement. Combes et al. (2005) investigate the role of ethnic networks that facilitate trade creation. A smaller literature, exemplified by the recent paper of Rossi-Hansberg (2005) focus on small measurable border costs such as tariffs that lead to strong effects of bilateral trade flows.

The importance of trade within-countries for the pattern of international trade has first been emphasized by Courant and Deardorff (1992). Those authors have used a Heckscher-Ohlin framework to discuss the role of the intra-country distribution of production factors on the pattern of international trade. New economic geography models also discuss inter-country dynamics in the context of globalization. A recent example of such a paper is Rossi-Hansberg (2005), who studies the effects of small border costs on the regional distribution of workers within a country. He then assesses the implications of the equilibrium distribution on intra- versus international trade flows. However, his focus is not on infrastructure investment.

Formally, a close cousin of the present paper is the racetrack model of Fujita et al. (2001, ch. 6 and 17). In that model, space is continuous and organized along a circle, just as in the proposed setup. However, the two approaches differ in focus. The racetrack model endogenizes the distribution of manufacturing labor across space under conditions of increasing economies of scale. It does not say anything on the endogenous spatial distribution of the stock of transport infrastructure and how, nor on how that stock shapes transport costs. In the proposed setup, production technologies exhibit constant returns to scale, and the distribution of workers is exogenous. In turn, transport costs are endogenous. While the proposed model can be generalized to allow for increasing returns to scale and worker mobility, excluding these elements makes the model straightforward to analyze. Essentially, in the conventional economic geography setup it is worker mobility that drives regional differences, in the present framework it is the infrastructure investment decisions of central planners.

The paper contributes to the literature along three dimensions: First, while maintaining the basic iceberg trade cost assumption, we propose a plausible and tractable formulation of transportation costs as a function of cumulative infrastructure investment.

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3 The racetrack model is discussed also as the 'seamless world model' (Krugman and Venables, 1997).
This requires modeling regions (or countries) as having some spatial extension themselves.\textsuperscript{4} Second, governments implement their preferred infrastructure allocations across space separately for their specific spatial reach. Hence, transport costs are endogenous. Governments turn out to invest too little in border regions, therefore generating a border effect even in absence of formal and informal trade barriers. Finally, the paper presents tentative econometric evidence based on data for US states.

The structure of the paper is as follows. Section 2 presents some stylized facts on the transportation sector and proposes a mathematical formalization of transport costs, where infrastructure investment has an important role to play. Section 3 sets up the general equilibrium environment which motivates intra- and international trade. Section 4 discusses the optimal spatial distribution of infrastructure investment in an autarkic economy and shows that the main features of the distribution do not depend on the precise nature of the political economy process. Later on, it allows for international trade (along intranational one) and shows how a Nash equilibrium for a world with many symmetric countries can be constructed. Section 5 presents tentative econometric evidence in favor of the mechanism, and section 6 concludes.

\section{Modeling transportation costs}

\subsection{Stylized facts on the transport sector}

This section discusses a number of important empirical facts that have a bearing on the modeling of transportation costs. The evidence discussed below draws on US data, which is most easily available. However, similar empirical patterns tend to exist across Europe.

\textit{Fact 1: The public input into the provision of transportation services is important.} According to the Bureau of Economic Analysis, in the US, public gross investment plus government consumption spending on transportation goods amount to about 9.4 percent of total government spending (across all levels of government) or 1.8 percent of GDP in 2004. Private gross fixed investment in transport equipment (this excludes cars used

\textsuperscript{4}Much of the economic geography literature (Fujita et al., 1999) and almost all new trade models do not explicitly model the spatial extension of countries. The global economy is assumed to be a collection of countries each modeled as points without spatial extension of their own.
for private use) is 1.3 percent of GDP.\(^5\) Hence, albeit the fact that private and public investments into transportation goods differ dramatically in terms of their nature, they are both quantitatively significant and comparable in size.

Whether publicly provided transport infrastructure is financed through taxes or through user fees such as the road tolls does not matter for the present argument as long as planners have discretion on where to invest toll receipts; see below. Data collected by the Bureau of Transport Statistics indicates that toll revenue finances only a small fraction of total public transport infrastructure spending in the US.

**Fact 2:** Regional governments influence interregional infrastructure projects substantially. All levels of government contribute towards spending on transport infrastructure and equipment. However, in the US, spending falls predominantly on the local or state level. Table 1 shows that state and local government command about 99 percent of total spending on highways and about 3 percent on transit and railroad projects. Federal involvement is higher for air and water transportation, so that the lower levels of government account for about 86 percent of total spending on infrastructure. These data are only indicative; the federal government influences local and state infrastructure decisions indirectly, e.g., through sales of federal land.

In Europe, the allocation of infrastructure spending on different levels of government is usually less skewed than in the US. However, in Germany, all interregional highways are planned, financed and maintained by state governments. In France, the interregional network of highways is managed centrally, but even in this case, regional entities have considerable influence over total infrastructure spending. Local governments are involved in the planning of interregional infrastructure projects in all larger OECD countries.

**Fact 3:** Intracontinental trade flows are mostly land-borne. Table 2 shows for 2001 trade flows between the US and Canada/Mexico that truck and rail transport amounts to about 83 percent of total trade in terms of value, and 56.3 percent in terms of quantities. While air-borne trade has increased dramatically over the last 30 years, it still commands only 6.3% of total trade flows in terms of value. Not surprisingly, water transport is important only for goods with low unit values, commanding 43.4 of trade in terms of quantities and 5.0 percent in terms of values. This fact implies that the relevant mode of transportation in trade relationships which feature the border effect (US-Canadian trade,

\(^5\)NIPA tables 1.15 and 3.155.
intra-European trade) is land-borne transportation.\textsuperscript{6}

Table 1: US infrastructure spending on different levels of government

<table>
<thead>
<tr>
<th>USD bn, 2004</th>
<th>Federal</th>
<th>Local/State</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highways</td>
<td>1.6</td>
<td>153.0</td>
<td>154.6</td>
</tr>
<tr>
<td>Transit and Railroad</td>
<td>0.5</td>
<td>14.0</td>
<td>14.5</td>
</tr>
<tr>
<td>Air</td>
<td>17.3</td>
<td>9.0</td>
<td>26.3</td>
</tr>
<tr>
<td>Water</td>
<td>10.3</td>
<td>3.2</td>
<td>13.5</td>
</tr>
<tr>
<td>All</td>
<td>29.7</td>
<td>179.2</td>
<td>208.9</td>
</tr>
</tbody>
</table>

Source: BEA-NIPA Table 3.1505

Table 2: US trade with NAFTA countries by mode

<table>
<thead>
<tr>
<th>Percent of total merchandise trade, 2001</th>
<th>Value</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>67.3</td>
<td>36.6</td>
</tr>
<tr>
<td>Rail</td>
<td>15.8</td>
<td>19.7</td>
</tr>
<tr>
<td>Air</td>
<td>6.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Water</td>
<td>5.0</td>
<td>43.4</td>
</tr>
<tr>
<td>Other</td>
<td>5.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Pipeline transport excluded.
Source: Bureau of Transportation Statistics.

2.2 Generalized iceberg transport costs

Economic geography models, pioneered amongst others by Krugman (1991), bring together monopolistic competition with Samuelson’s (1952) iceberg trade costs assumption.\textsuperscript{7}

\textsuperscript{6}While there is data on the mode of transport in trade of European countries with non-Europe, we are not aware of similar data for intra-European trade.

\textsuperscript{7}Note that much of the new trade theory literature that discusses trade in differentiated goods under increasing returns to scale and monopolistic competition uses an essentially discrete formalization of trade costs. Associated empirical papers using the gravity equation do, however, resort to Krugman’s specification. For a model trade in continuous space see Krugman and Venables (1997).
The iceberg assumption has proved convenient, because it makes the introduction of a specific transportation sector redundant: during transportation, a distance-dependent share of the output shipped from the location of production gets lost, i.e., melts away. The implicit transportation sector production function uses the good being transported as the only input. Formally, assume that economic space \( S \) is unidimensional and continuous (e.g., the real line, or the circumference of a circle). For two arbitrary addresses \( x \) and \( z \) in \( S \), a Krugman-type transportation costs specification would be

\[
T(x, z) = e^{a|x-z|} \geq 1,
\]

where the coefficient \( a \) is the iceberg decay parameter. \( T(x, z) \) models the cost of delivering a good over the distance \( |x-z| \) as an \textit{ad-valorem} tax equivalent, where the tax income is lost in transit. In order to receive one unit of the good at \( x \), \( T(x, z) \) units of that good have to leave the factory at \( z \). A share \( 1 - T(x, z)^{-1} \) of the good ‘melts’ in transport; the share \( T(x, z)^{-1} \) arrives at \( x \) when one unit of the good is shipped at \( z \).

The iceberg formulation amounts to introducing a shadow transport sector which uses the share \( 1 - T(x, z)^{-1} \) of a good to be shipped from \( z \) to \( x \) as an input. Importantly, that input is just the good produced at location \( z \). One can say that the transportation service is produced at the producer’s location. Given the continuous space nature of our setup, one could alternatively posit that transportation services are produced at every point in the interval \([x, z]\) or that they are produced at the consumer’s location. In the specific economic environment proposed below, these differences do not matter, as f.o.b. prices any any location are independent of demand for the variety produced at those locations.

McCann (2005) provides a critical discussion of the iceberg assumption and discusses some implications, namely that the delivered price of a good transported from a producer to a consumer over some distance is (i) convex with respect to distance, (ii) directly proportional to the original price of the good, and (iii) freight cost ratios are independent of the quantity shipped. These properties rule out economies of distance of scale in the transportation of goods; a feature, that is much discussed in transportation economics.

McCann’s criticism applies in particular for air or sea-borne transport, where fixed costs (charging, uncharging) relative to the sum of variable costs (fuel) are high. This paper is about intra-continental and therefore mostly land-borne transport where the failure of the iceberg assumption is less obvious. Hence, both for the sake of simplicity and to keep in line with the literature, we stick to the iceberg assumption. However, transportation costs depend on the quantity and/or quality of public infrastructure available. The
only modification relative to (1) lies in assuming the existence of an essential publicly
provided input–infrastructure–that is different to the good being transported.

In the present paper public infrastructure investment refers to the process of investing
some resource at specific locations \( s \in S \) with the aim of reducing transportation costs.
Before being more explicit, we cite a number of important properties of the interplay be-
 tween infrastructure investment \( i(s) \) and transportation costs. Transport economists have
long discussed these properties (Winston, 1985, and Gramlich, 1994), without, however,
proposing a formal description for use in general equilibrium trade models.\(^8\)

Denote the distribution of infrastructure over space by \( i(s) \), and let \( s \in [x, z] \). The interaction between
\( i(s) \) and \( T(x, z) \) should respect the following properties: (i) The cost of transporting a
good over the interval \([x, z]\) should depend on cumulative infrastructure investment over
that interval. In other words, \( T(x, z) \) cannot be described by information of addresses
\( x \) and \( z \) alone, but requires data on the entire distribution of \( i(s) \) on the interval \([x, z]\). (ii) Investments at different points \( s', s'' \in [x, z] \) are imperfect substitutes. (iii) The public-
ly provided stock of infrastructure is an essential input. Hence, \( T(x, z) \to \infty \) for all
\( x, z \in S \), if \( i(s) = 0 \) at least at one address \( s \in [x, z] \). (iv) Under bounded availability of
resources, transportation is always costly, regardless the state of infrastructure. Hence,
\( T(x, z) > 1, z \neq x \), for all finite \( i(s) \in [x, z] \). We also require that \( \partial T(x, z) / \partial |x - z| > 0 \)
for all finite \( i(s) \in [x, z] \). In words, the private input is essential for overcoming distance;
infrastructure investment cannot completely undo the force of distance.

The proposed functional form that links transportation costs to infrastructure invest-
ment is as follows:

\[
T(x, z) = \exp \left[ \frac{1}{\delta - 1} \int_x^z i(s)^{1-\delta} ds \right], \delta > 1, x \leq z, \tag{2}
\]

where \( \delta > 1 \) is a constant technological parameter (which will have a precise economic
interpretation later). In (2) the address of the consumer \( x \) lies to the left of the address of
producer \( (x \leq z) \). The opposite case is considered by simply putting \( T(z, x) \). Note that
the specification (2) is formally similar to the usual formulation of spatial externalities
in urban economics, where the explanandum is the location of firms and workers relative
to each other (see, i.a., Fujita and Thisse (2002) and Lucas and Rossi-Hansberg (2002)).
The expression within square brackets is also well known from dynamic models, where

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\(^8\)Since the model proposed is static, we use the term infrastructure investment and stock of infrastruc-
ture interchangeably.
it sums consumption at different dates into an aggregate. Also note, that with \( x \) and \( z \) finite numbers, and \( i(s) > 0 \), the integral converges.\(^9\)

The share of output produced at address \( x \) and shipped to address \( z \) has the following important properties (2):

(i) \( T(x, z) \geq 1 \).

(ii) \( T(x, z) \) is increasing and strictly convex in distance if the elasticity of infrastructure investment with respect to distance is large enough.

(iii) The (interregional) elasticity of substitution between infrastructure investment at different locations is \( 0 < 1/\delta < 1 \). Moreover, \( T(x, z) \to \infty \) if \( i(s) = 0 \) at some \( s \in [x, z] \).

(iv) The marginal effect of investment on \( T(x, z) \) is negative but increasing. The marginal effect in(de)creases with distance to the address of investment \( k \) if \( i' (k) < (>0) \) and is constant otherwise.

(v) \( T(x, z) = \exp[a(z-x)] \) if \( i(s) \) is a constant \( i \) over the interval \([x, z]\), and \( a = i^{1-\delta}/(\delta-1) \).

(vi) If \( i(s) \) is a symmetric function over some interval \( \Gamma \), and \( x, z \in \Gamma \), then \( T(x, z) = T(x+k, z+k) \) where \( k > 0 \)

(vii) Investment smoothing property: \( i = \arg \min \{ T(x, z) | q \int_x^z i(s) ds \leq B \} \), where \( q \) is the spatially invariant price of infrastructure investment and \( B \) is the (exogenous) total investment spending.

Property (ii) implies that the private input is essential in the process of transportation. Leibniz’ rule implies that \( T(x, z) / \partial z > 0 \). Defining the elasticity of infrastructure investment with respect to distance as \( \iota(z) = (z-x)i'(z)/i(z) \), we have \( T^2(x, z) / \partial z^2 > 0 \) if and only if \( \iota(z) > -(z-x)i(z)^{1-\delta}/(1-\delta)^2 \). This condition is always fulfilled if \( \iota(z) \geq 0 \), e.g. in the Krugman case. Concavity of \( \tau(x, z) \) obtains if the distribution of \( i(z) \) does not decline too much at address \( z \), otherwise a marginal increase of distance leads to an overproportional increase in \( T(x, z) \). Property (iii) can be seen by noting that the technical rate of substitution between investments at to addresses

\(^9\)Note the similarity of (2) with the characterization of intertemporal utility in dynamics macro models.
$k, l \in [x, z]$ is $TRS = [i(k)/i(l)]^\delta$. It follows that the interregional elasticity of substitution $d\ln [i(k)/i(l)]/d\ln TRS$ is just $1/\delta$. Since $\delta > 1$, investment at different locations are gross complements. This property has a close counterpart in the theory of intertemporal consumption. Properties (iv) can be understood by noting that the elasticity of $\tau(x, z)$ relative to $i(k), k \in [x, z]$ is $i(k)^{1-\delta}$ and $\delta > 1$. Property (v) is obvious. It shows that our more general specification of transport costs collapses to the Krugman formulation when infrastructure investment is constant over space. Property (vi) Property (vii) is readily seen by solving the problem stated above. It depends crucially on the assumption of spatially invariant investment prices. Relaxing that assumption, we would have that $[i(k)/i(l)]^\delta = q(l)/q(k)$ for all $l, k \in [x, z]$. This property will be important when considering the efficient world distribution of investment. It is reminiscent of a similar result in the theory of optimal intertemporal consumption.

Note that in (2), we assume that transportation costs are unaffected by traffic volume. However, since the distribution of $i(s)$ will be ultimately endogenous and non-constant across space, the amount of infrastructure investment at point $s, i(s)$, will not be a constant proportion of transportation volume $X(s)$ through that point. There will be traffic congestion in the sense that at some points infrastructure investment will be inadequately low.

We make a further important assumptions in relation to (2): The resource used to produce infrastructure investment goods can be transported freely across space. This assumption can be relaxed only at the price of considerable complication. It is similar to the assumption of a costlessly tradable agricultural good in much of the economic geography literature.

Infrastructure at address $s$ is produced according to a linear production function $i(s) = b(s)/q(s)$ where $b(s)$ denotes the input of the resource used for infrastructure investment, and $1/q(s) > 0$ measures the rate at which that resource is transformed into infrastructure. The total economy-wide supply $B$ of the resource required for investment will be endogenously determined by government policy. Feasibility of an investment policy $i(s)$ implies that

$$\int_{s \in S} q(s)i(s) \leq B.$$  

(3)
3 Endowments, firms and consumers

This section embeds the transport technology described in (2) into a specific model of intra- and international trade. The model is static and features a single factor of production, labor. The economic environment combines spatial product differentiation with constant returns to scale production functions, with countries and locations modeled symmetrically.

3.1 Geographical space and goods space

Geographic space $S$ is understood as a continuum of locations (or: addresses) $s \in S$ organized along the circumference of a circle. We define regions (or: countries) as connected, ordered subset of $S$, with $S^j$ the collection of locations associated to region $j$. We let $j = 1, ..., J$, so that there are $J$ regions. Moreover, the partition of geographical space into regions is exogenous and perfectly symmetric. Hence, we may normalize the length of each region to unity. Clearly, $S = \bigcup_j S^j$ and the length of the world is just equal to $J$. Denote $\bar{x}_j$ as the address at the middle of each region. Figure 1 illustrates the situation.

At each location $s$ there is a representative household who inelastically supplies $m(s)$ units of labor. Households are immobile across space, so $m(s)$ is exogenous. We can leave the form of $m(s)$ open as long as $m(s) > 0$ for all $s \in S$ (no inhabitated locations) and $m(s)$ is twice continuously differentiable. Moreover, locations may differ with respect to the topological circumstances. Hence, we have a distribution of productivities $q(s)$ which
gives the rate at which resources are transformed into infrastructure investment goods. Again, we only restrict \( q(s) \) to be twice continuously differentiable. To simplify notation, we refrain from using regional indexes until entering into political economy considerations.

At each location \( s \), a homogeneous agricultural and a spatially differentiated industrial good can be produced. Both types are produced with linear production functions (no fixed costs) under conditions of perfect competition. Consumers consume both types, perceiving industrial goods produced at specific locations as imperfect substitutes. There are no costs of transporting the agricultural good. Moreover, the agricultural good serves as an input into infrastructure provision. Each location \( s \) is home of consumers and producers. We denote addresses of consumers by \( x \) and addresses of producers by \( z \).

The above assumptions have the advantage that they deliver factor price equalization across space as long as all locations produce both types of goods (which we assume). The existence of the agricultural good as an input in infrastructure production makes thinking about a transportation technology for transferring infrastructure production inputs from one region to the other redundant.

### 3.2 Consumer behavior

The utility function of the representative household at location \( x \) is a Cobb-Douglas aggregate over the homogeneous agricultural good and a Dixit-Stiglitz aggregate over industrial goods,

\[
U(x) = \left[ c^A(x) \right]^\alpha u(x)^{1-\alpha}, \alpha \in (0, 1),
\]

\( c^A(x) \) denotes the total quantity of the agricultural good consumed at address \( x \), and \( u(x) \) is the subutility index attributable to spatially differentiated goods. Let

\[
u(x) = \left[ \int_{z \in S} c^I(x, z)^\rho dz \right]^{\frac{1}{\rho}}, 0 < \rho < 1,
\]

where \( c^I(x, z) \) is the quantity of a good produced at address \( z \) and consumed at \( x \).

Let \( Y^n(x) \) denote household \( x \)'s net income in terms of a numéraire to be defined below. Then, the its budget constraint is

\[
Y^n(x) = c^A(x) p^A(x) + \int_{z \in S} c^I(x, z) p^I(x, z) dz,
\]

where \( p^A(x) \) is the price of the agricultural good at location \( x \) and \( p^I(x, z) \) is the price of a variety imported from location \( z \) and consumed at \( x \).
Since preferences are separable in category $A$ and $B$ goods, we may solve the consumer problem as a two-stage budgeting problem. The Marshallian demand functions for varieties of categories $A$ and $B$ are respectively

$$c^A(x) = \alpha Y^n(x) \frac{p^A(x)}{p^A(x)}$$

and

$$c^I(x, z) = (1 - \alpha) Y(x) \frac{p^I(x, z)^{-\sigma}}{P^I(x)^{1-\sigma}},$$

where $\sigma = 1/(1 - \rho)$ and

$$P^I(x) = \left[ \int_{z \in S} p^I(x, z)^{1-\sigma} dz \right]^{1/\sigma}$$

is the price index for industrial goods.

The indirect utility attainable at prices $p^A(x), p^I(x, z)$ and income $Y^n(x)$ can be written as

$$\tilde{V}(x) = \alpha^\alpha (1 - \alpha)^{1-\alpha} \left[p^A(x)\right]^{-\alpha} \left[P^I(x)\right]^{-\alpha} Y^n(x),$$

where the term $Q(x) = [p^A(x)]^\alpha [P^I(x)]^{1-\alpha}$ can be interpreted as the cost of living index in the economy.

### 3.3 Firm behavior

At each location $z \in S$, the agricultural and the industrial good are produced under conditions of perfect competition. The only input of production is labor. Production functions for the two types of goods are linear

$$y^A(z) = bl^A(z), y^I(z) = l^I(z),$$

where $b > 0$ is a productivity parameter. Output quantities are denoted by $y^A(z)$ and $y^I(z)$, and labor inputs by $l^A(z)$ and $l^I(z)$, respectively. We assume that workers are perfectly mobile across agricultural and industrial firms. Hence, optimal firm behavior implies that $p^I(z) = w(z)$ and $p^A(z) = w(z)/b$, where $w(z)$ is the wage rate (expressed in units of numéraire) at address $z$.

### 3.4 Equilibrium

Industrial goods bear transportation costs. Hence, the c.i.f. prices faced by consumers differ from the f.o.b. (ex-factory) prices. In particular, a consumer at $x$ faces the price
\[ p^A(x, z) = p^A(z) T(x, z) \] for a variety of good imported from location \( z \). There are no trade costs other than transportation costs. In particular, there are no formal or informal barrier to trade at borders. In contrast, agricultural goods can be transported freely. We impose a non-full-specification (NFS) assumption: there is always a strictly positive quantity of agricultural production at each location. The NFS assumption introduces factor price equalization in terms of the agricultural good.\(^\text{10}\) We may therefore choose the agricultural good as the numéraire and set \( p^A(z) = p^A(x) = 1 \) for all \( z \) and \( x \). Hence, \( Q(x) = P^I(x)^\alpha \). Since \( p^A(x) = 1 \), we drop the superscripts \( A \) and \( I \) in the following.

Profit maximizing firm behavior implies factor price equalization, i.e. \( w(z) = b \) at every location. Income at location \( x \) in terms of the numéraire is then \( Y(x) = bm(x) \). From our assumption on technology, \( p(z) = b \) so that c.i.f. prices for industrial goods are

\[ p(x, z) = bT(x, z). \quad (11) \]

The government imposes a lump-sum tax \( t \) which is assumed identical across addresses \( s \in S^c \) which fall into the reach of the government. In terms of the numéraire good, total tax income is \( B = bt \int_{s \in S^c} m(s) \). Hence, \( Y^n(x) = (1 - t) bm(x) \)

We may now rewrite indirect utility function (9) per member of the representative household:

\[ V(x) = \Omega \left( 1 - t \right)^{\frac{\sigma - 1}{\alpha}} m(x)^{\frac{\sigma - 1}{\alpha} - 1} \int_{z \in S} T(x, z)^{1 - \sigma} \, dz \]

where \( \Omega \equiv \left[ (1 - \alpha) b \right]^{\frac{(\sigma - 1)(1 - \alpha)}{\alpha}} \alpha^{\sigma - 1} \) is a function of exogenous parameters only and \( V(x) = \tilde{V}(x)^{\frac{\sigma - 1}{\alpha}} \) is a positive transformation of \( \tilde{V}(x) \).

In order to facilitate notation, we define the following sets of addresses:

\[ L(r) = \{ z \in S : z \leq r \}, \quad (12a) \]
\[ R(r) = \{ z \in S : z \geq r \}. \]

The set \( L(r) \) collects all addresses of producers that are relevant to utility and that lie to the the left of some specific location \( r \), while \( R(r) \) collects all such addresses that lie to the right of location \( r \) that

\(^{10}\)Relaxing the NFS assumption would allow to study the interaction between infrastructure investment policies and regional specialization patterns. This is an interesting issue that rises additional complications. It is therefore left to future research.
Using this notation, we may define $F_L(x) = \int_{z \in L(x)} T(z, x)^{1-\sigma} dz$, $F_{R}(x) = \int_{z \in R(x)} T(x, z)^{1-\sigma} dz$, and $F_L(x) + F_{R}(x) = F(x) = \int_{z \in S} T(x, z)^{1-\sigma} dz$. Clearly, $S = L(x) \cup R(x)$ and $L(x) \cap R(x) = \emptyset$.

Note that $F(x) = \left[ b/P(x) \right]^{\sigma-1}$, so $F(x)$ is negatively related to $P(x)$. Therefore, we interpret the terms $F_L(x)$, $F_{R}(x)$ and $F(x)$ as measures of gross indirect utility attained by agent $x$ over goods in $L(x)$, $R(x)$ and $S$, respectively. We rewrite indirect utility as

$$V(x) = \Omega(1-t)^{\frac{\sigma}{\sigma-1}} m(x)^{\frac{\sigma}{\sigma-1}} \left[ F_L(x) + F_{R}(x) \right]$$

(13)

Costless tradability of category $B$ varieties effectively equalizes wages in terms of the numéraire. It also pins down ex-factory (f.o.b.) prices $p(z)$ directly. Unlike in the standard Armingtonian setup, locations with a larger mass of workers do not suffer from lower terms of trade, since $p(x)/P(x)$ is independent from the distribution of economic activity across space $m(s)$. The reason is that a higher supply of workers is absorbed by an expansion of the agricultural sector at constant marginal value productivity.

4 The political economy of infrastructure investment

This section presents the core results of the model. As mentioned above, the world is modeled as a circle, while individual countries are segments on that circle. Alternatively, one may think of the circle as a relatively closed country, with segments representing autonomous regions within that country. The rationale behind many results in the following analysis is the discrepancy between the political and the economic reach of countries: while infrastructure investment decisions are limited to subsets $S^c$ of space, consumers demand imports from all countries and locations. Since countries decide in a non-cooperative way, they do not internalize the positive externality that their investment decisions exert on consumers in other countries. This leads to global underprovision of infrastructure. More importantly, it also leads to excessive spatial variation in infrastructure relative to the first best solution, since the externality has different size at different addresses within a region.

In the following, first the autarkic equilibrium is discussed. In that case, consumers cannot consume goods produced in countries other than their own. This is hence a situation where trade restrictions are prohibitive. Alternatively, one may interpret this case as the solution to the infrastructure provision problem if the decision maker acts for
Figure 2: Agents located in $[-b, s]$ benefit through goods imported from $[s, b]$ while agents in $[s, b]$ benefit through goods from $[-b, s]$. 

the entire world, but the world is a line instead of a circle. We will show that under the assumptions made in section 3 the median voter problem leads to qualitatively similar results than the central planner solution. Then, we present the solution to the political economy problem for a circular world which is fragmented into many countries. We compare the world-planner solution to the outcome that obtains if infrastructure decisions are undertaken by independent countries. We show that the qualitative shape of the world infrastructure distribution will be robust to the details of the political economy process and whether or not the political actors take into account how their decisions affect decisions of agents in other countries.

In the following sections we will characterize optimal policies $\{\hat{i}(s), \hat{t}\}_{s \in [0,1]}$ under different assumptions. More precisely, we distinguish between autarky outcomes obtained by a median voter and a social planner, as well as situations where countries are open to trade but fragmented politically. In that latter case we describe the world planner outcome and the result that materializes in a median voter and a social planner environment.

4.1 Political economy equilibrium under autarky

We let the relevant economic space be a line $[0,1]$. In contrast to the circular case, the linear geography exhibits a natural periphery in the sense that peripheral regions will have higher average transportation costs and hence lower indirect utility. Note, however, that even in that case, the assumptions made in section 3 allow us to set $p(z) = 1$. In the
following, a uniform spatial distribution of population is assumed, hence \( m(s) = 1 \) for all \( s \). Since the length of a country is normalized to unity, gross income in terms of the numéraire good is \( b \) in each country. It is straightforward, albeit cumbersome, to allow for variation in \( m(x) \) over space. The robustness of the main results of the paper to this specific assumption is discussed below.

4.1.1 Preferred policies across households

First, we need to establish each household’s preferred infrastructure distribution \( i^x(s) \) and total infrastructure spending \( t^x \). Note the superscript \( x \) which indicates the optimality from the perspective of a household at address \( x \) (referred to as household \( x \) for simplicity).

The program that household \( x \) solves is

\[
\{i^x(s), t^x\} = \arg \max \left\{ \Omega \left( 1 - t \right)^{\frac{1-\sigma}{\alpha}} F(x) \; \text{s.t.} \; tb \geq \int_0^1 q(s) i(s) ds \right\},
\]

(14)

where \( Y = b \) is total income available in the region, \( F'(x) = \int_{z \in L(x)} T(z,x)^{1-\sigma} dz + \int_{z \in R(x)} T(x,z)^{1-\sigma} dz \) and \( \tau(x,z) \) is defined in (2). Each agent balances the gain in utility of a marginal increase in the stock of transport infrastructure at location \( s \) against the associated cost.

The optimal allocation of infrastructure spending across space is implicitly determined by the continuum of first order conditions

\[
i^x(s)^k = \begin{cases} 
\alpha b \phi^L(x,s) \frac{1-t}{q(s)} & \text{if } s \in L(x) \\
\alpha b \phi^R(x,s) \frac{1-t}{q(s)} & \text{if } s \in R(x)
\end{cases}
\]

for all \( k \in [0, 1] \),

(15)

where the terms

\[
\phi^L(x,s) = \frac{F^L(s)(x)}{F(x)} \text{ and } \phi^R(x,s) = \frac{F^R(s)(x)}{F(x)}
\]

measure the share of gross indirect utility derived from goods that transit through the location \( s \) relative to total gross indirect utility \( F(x) \). A higher importance of industrial goods \( \alpha \) in the utility specification (5), or a higher productivity \( b \), lead to higher investment. Moreover, the larger the shares \( \phi^L \) and \( \phi^R \), the higher will optimal infrastructure investment be. Note that for agents with addresses \( 0 < x < 1 \), we have \( \phi^L(x,0) = \phi(x,1) = 0 \). Hence all of those agents prefer zero investment at the borders: \( i^x(0) = i^x(1) = 0 \). If \( x = 1 \) only the first line in (15) is relevant, and \( i^1(1) > 0 \) while \( i^1(0) = 0 \). Similarly, \( i^0(0) > 0 \) and \( i^0(1) = 0 \). One can totally differentiate that expression and find that a marginal increase in \( q(s) \) unambiguously leads to a reduction in the amount of investment.
at address \( s \). Also, an increase of \( \sigma \) beyond unity lowers \( i^x(s) \) as agents perceive varieties as closer substitutes.

We can now differentiate \( i^x(s) \) with respect to \( s \) and find the proportional growth rate of \( i^x \) in \( s \).

\[
g^x(s) \equiv \frac{i'^x(s)}{i^x(s)} = \begin{cases} \frac{1}{\delta} \left[ \frac{T(s,x)^{1-\sigma}}{F_L(s)(x)} - \frac{q'(s)}{q(s)} \right] & \text{if } s \in L(x) \\ -\frac{1}{\delta} \left[ \frac{T(x,s)^{1-\sigma}}{F_R(s)(x)} + \frac{q'(s)}{q(s)} \right] & \text{if } s \in R(x) \end{cases} \quad \text{for all } s \in [0,1]. \tag{16}
\]

The proportional rate at which the stock of infrastructure investment changes across space is a function of \( 1/\delta \), the elasticity of interregional substitution of infrastructure investment. The higher that elasticity, the faster will \( |g^x(s)| \) decline as we move away from agent \( x \)'s address. The reason for this result is straightforward: Consider a spending-neutral perturbation of some optimal investment schedule \( i^x(s) \) by \( di(s') > 0 \) and \( di(s'') < 0 \), such that \( q(s') = q(s'') \) and \( |x-s'| < |x-s''| \). A marginal increase in investment at the point close to agent \( x \), \( s' \), has the advantage that more shipments to \( x \) will transit through that point. Hence, the cumulative impact of \( di(s') \) will be larger than \( di(s'') \), so that the proposed perturbation increases \( x \)'s utility. However, thinning out investment at \( s'' \) comes with a cost, as higher investment at \( s' \) is an incomplete substitute for investment at \( s'' \). If \( 1/\delta = 0 \), i.e., investments at different points at perfect complements, then agent \( x \) desires a flat infrastructure distribution: higher investment close to the own location is useless if investment is not increased also at other locations. However, if \( 1/\delta = \infty \), investments at different points are perfect substitutes. Then, there are no costs from reallocating investment close to \( x \)'s address. The cumulative effect dominates, and agent \( x \) wants to invest exclusively in a neighbourhood of her own address.

Note that in the above expression, the address \( s \) describes the location from which goods are shipped to \( x \). Hence, the two cases \( s \leq x \) and \( s \geq x \) refer to imports of agent \( x \) from locations at her left and her right, respectively. Expression (16) allows a number of interesting expressions. First, the absolute value \( |g^x(s)| \) at which any agent wants to vary her preferred stock of infrastructure over time increases in the interregional elasticity of substitution, \( 1/\delta \).

Assuming that geographical space is perfectly flat on the interval \([0,1]\), i.e., that there are no natural obstacles (rivers, mountains) that would warrant variation of \( q(s) \) over space, we have \( q'(s) = 0 \) for all \( s \in [0,1] \). Then spatial variation in the desired level of infrastructure investment is governed only by variation in the terms \( T(s,x)^{1-\sigma}/F_L(s)(x) \), and \( T(x,s)^{1-\sigma}/F_R(s)(x) \) respectively. Focusing on the latter case (the former is perfectly
symmetric) it is easy to show that $\partial g^x (s) / \partial s < 0$, so that $i (s)$ is a decreasing and concave function of $s$. Moreover, $\partial g^x (s) / \partial x > 0$, so that the decline rate of the desired stock of infrastructure across space is less negative for households closer to $s$. This allows to trace the desired investment schedules, see figure 3.\textsuperscript{11}

The functions $i^x (s)$ are piecewise differentiable and continuous over the intervals $[0, x]$ and $[x, 1]$ and that are non-differentiable at the address $x$ and (except for the median agent) discontinuous. Note that the growth rate $g^x (x)$ is not defined; moreover, $i^x (x) = 0$ since varieties produced at the own address can be shipped with zero cost regardless of the stock of infrastructure investment at $x$.

Still holding $q^\prime (s) = 0$, total infrastructure spending desired by agent $x$ can be measured as the total area below the curves $i^x (k)$. It is straightforward to show that (see the appendix)

$$t^\bar{x} = \inf \{t^x \}, x \in [0, 1],$$  \hspace{1cm} (17)

where $\bar{x}$ denote the average (median) address (in the present case $\bar{x} = 1/2$).

In the more general case, where $q (s)$ is allowed to vary across space, we have to make explicit assumptions about the function $q (s)$. The simplest case is the one where $q (s)$ is a symmetric function over the interval $[0, 1]$, reaching an extremum at $\bar{x}$ and growing monotonically at a constant rate $q^\prime (s) = g_q$ at points different than the one where the

\textsuperscript{11}Note that we have characterized $i^x (s)$ through the partial differential equation (pde) given by $g^x (s)$. The plot in figure 3 is a plausible solution to that pde.
extremum is reached.

If \( q(s) \) reaches a minimum at \( \bar{x} \), the situation depicted for \( i^{\bar{x}}(s) \) in figure 3 holds a fortiori: the spatial variation in the cost of providing infrastructure investments adds to the natural tendency towards concentration at \( \bar{x} \), resulting in steeper slopes of \( i^{\bar{x}}(s) \).

For agents other than \( \bar{x} \) the situation is more complicated. Agents to the left of \( \bar{x} \) would choose a steeper slope of \( i^{\bar{x}}(s) \) for \( s < \bar{x} \), while reverting to an S-shaped curve with high slope close to \( s = \bar{x} \) and \( s = 1 \) for locations \( s > \bar{x} \).

If \( q(s) \) reaches a maximum at \( \bar{x} \), spatial variation in \( i^{\bar{x}}(s) \) is lower. Now, the sign of expressions (16) can change, giving rise to a U-shaped infrastructure distribution across space. It could also be that the growth rates \( g^{\bar{x}}(s) \) are exactly equal to zero due to the variation in \( q(s) \).

Finally, consider the case where geography is essentially flat, \( q(s) \) is constant for all \( s \) except one location \( s' \) where \( q(s') \) is very high. This situation could capture the effect of a river. Because agents want to smooth infrastructure investment over space, the local geographical disturbance at \( s' \) virtually leaves preferred total spending and the distribution of investment across space unchanged. This is an extremely important property that we will need later when we turn to the interpretation of the global political economy outcome.

All these examples suggest that the interior geography of countries is important to understand the allocation of infrastructure investment. However, geography has only a limited role to undo the central tendency that agents want to concentrate investment close to their home addresses and that peripheral agents tend to prefer larger total investment volumes than central ones. For that reason, we set \( q'(s) = 0 \) for the remainder of the theoretical discussion. In empirical applications, one would however take natural variation in the cost of investment into account.

### 4.1.2 Median voter outcome

We denote by \( V^{x'}(x'') \) the indirect utility obtained by household \( x' \) if the infrastructure allocation and tax rate preferred by agent \( x'' \) is implemented across the country. Clearly, \( \Delta = V^{x'}(x') - V^{x'}(x'') \geq 0 \), for all \( x', x'' \in [0, 1] \), since \( V^{x}(x) \) is a maximum value function.

\[\text{\footnotesize \[12\]However, in this context the additional variation in } i(x) \text{ is caused by geographic fundamentals and therefore need not be associated to a deterioration in welfare.}\]
It can be easily seen (and formally shown) from figure 1 that strict concavity of the \( i^x(s) \) functions implies that
\[
\frac{\partial \Delta (|x' - x''|)}{\partial |x' - x''|} > 0.
\] (18)
Hence, a larger geographical distance between \( x' \) and \( x'' \) implies that the utility loss suffered from adopting the other agent’s preferred policy is larger. Hence, preferences are single peaked over the continuum of policies preferred by agents residing at addresses \([0, 1]\). It follows that bilateral referenda would bring out the median voter’s preferred policy as the political equilibrium.

Hence, the equilibrium policy \( \{i_0(s), t_0\}_{s \in [0, 1]} \) obtained in a median voter setup under autarky is given by
\[
\hat{i}_0(s) = i^x(s), \hat{t}_0 = t^x.
\] (19)

### 4.1.3 Central planner problem

Next, we turn to the solution to the social planner problem for the case of autarky. The planner’s objective function is an unweighted aggregate of the indirect utility positions achieved at every address. Total welfare then is \( W = (1 - t) \int_0^1 F(x) \, dx \). We denote the optimal policy in that context as \( \{i^a_0(s), t^a_0\}_{s \in [0, 1]} \). Accordingly, the planner problem is
\[
\{i^a_0(s), t^a_0\} = \arg \max \left\{ \Omega (1 - t) \frac{a - 1}{\alpha} \int_0^1 F(x) \, dx \bigg| \begin{array}{l} bt \geq \int_0^1 q(s) i(s) \, ds \end{array} \right\}. \tag{20}
\]

The first order condition to this problem can be stated as
\[
\hat{i}^a_0(s)^\alpha = \frac{\alpha (1 - t^a_0) b}{q(s)} \left[ \phi^L(s) + \phi^R(s) \right],
\] (21)
where
\[
\phi^L(s) \equiv \int_s^1 \int_0^1 T(z, x)^{1-\sigma} \, dz \, dx \int_0^1 F(x) \, dx \quad \text{and} \quad \phi^R(s) \equiv \int_0^s \int_1^1 T(x, z)^{1-\sigma} \, dz \, dx \int_0^1 F(x) \, dx
\]
denote the share of gross welfare derived from industrial products delivered from places to the left and the right of location \( s \), respectively. Since \( \phi^L(1) = \phi^L(0) = \phi^R(1) = \phi^R(0) \), investment is zero at the borders of the region. It is higher the larger the sum \( \phi^L(s) + \phi^R(s) \), which measures the total gross welfare that stems from varieties that flow through point \( s \).

Note that, in contrast to individual preferred policies, the first order condition of the central planner is not discontinuous since the planner integrates over all individual utility
positions, thereby smoothing out the discontinuities. The evolution of $i(s)$ over space can now be shown to behave according to (see appendix)

$$g_0^a(s) = \frac{\hat{v}_0^a(s)}{\hat{v}_0^a(s)} = \left[ \frac{\sigma - 1}{\delta - 1} \frac{\Delta(s)}{\int_s^1 T(z,x)^{1-\sigma} dz dx} - \frac{q'(s)}{q(s)} \right]$$

where $\Delta(s) = \int_s^1 T(s,x)^{1-\sigma} dx - \int_s^0 T(x,s)^{1-\sigma} dx$.

In the case where $q'(s) = 0$, the sign of $g_0^a(s)$ depends only on the sign of $\Delta(s)$. It is easy to show that $\Delta(0) = \int_0^1 T(0,x)^{1-\sigma} dx > 0$ and $\Delta(1) = -\int_0^1 T(x,1)^{1-\sigma} dx < 0$, $\Delta'(s) < 0$, and $\Delta(1/2) = 0$. Hence, $g_0^a(s) > 0$ for $s \in [0,1/2)$, $g_0^a(1/2) = 0$, and $g_0^a(s) < 0$ for $s \in (1/2,1]$. Moreover, $g_0^a(0) \to \infty$ while $g_0^a(1) \to -\infty$ as the denominator in (22) tends to zero. One can show that $i_0^a(s)$ is strictly concave in $s$ (see the appendix). Figure 3 shows the social planner’s investment profile across space.

Hence, the distribution of infrastructure investment chosen by the social planner under autarky is qualitatively similar to the outcome obtained under the median voter scenario. One key difference is that $i_0^a(s)$ is now continuously differentiable at all locations. This reflects the investment smoothing property highlighted in section 2.2.

It is difficult to obtain clear comparisons of the social planner outcome $i_0^a(s)$ and the one chosen by the median voter. In general, the median voter solution displays an inefficient distribution of infrastructure across space, since it does not satisfy the investment smoothing property. However, whether the median voter over- or underinvests depends on model parameters. The higher the degree of substitution, $\sigma$, between varieties, the more likely is underinvestment. In the limiting case, where $\sigma \to 1$ (Cobb-Douglas utility), there is overinvestment in the median voter case. Hence, total spending in the central planner case strictly lies below that desired by any agent in the median voter case.

The reason for this is clear enough: if agents can impose tax rates for the entire economy, the will tend to chose high ones but since they have the power to decide on the spatial distribution of infrastructure investment, they will concentrate investment close to their home addresses.

4.2 Political economy equilibrium under international trade

We now turn the situation where the world economy is a collection of independent countries, each with its own government that decides on infrastructure investment in a non-cooperative way. However, consumers demand goods produced all over the world. We
have therefore a situation with ‘global market, regional politics’. With a uniform distribution of infrastructure investment, unlike in the autarky case discussed above, there would not be a natural periphery in spite of non-zero variable transportation costs.

If investment were to be decided on the world level, the median voter problem is not well-defined since all agents would opt for the same investment profile, which would again be a hump-shaped function with zero investment at the (zero-mass) point with maximum distance to the median voter’s address. Hence, we focus on the world central planner problem first and then compare it to the outcomes obtained. With single jurisdictions, the median voter problem is still well-defined and we present both, the median voter and the central planner investment solution. Notice that we will now focus on cases where \( q'(s) = 0 \) and remember that we are assuming a uniform distribution of economic activity on the circle.

### 4.2.1 World planner problem

In the world planner problem, the space across which individuals trade and over which infrastructure decisions are made coincide. Hence, we have one market and one jurisdiction that decides over infrastructure investments. The world planner problem can be written as

\[
\{i_0^*(s), t_0^*\} = \arg\max \left\{ \Omega (1-t) \frac{x-1}{\int_{x \in S} F(x) \, dx} \bigg| bt \geq \int_{x \in S} q(s) i(s) \, ds \right\}
\]

The first order condition to this problem can be stated as

\[
i^*(s) \delta = \frac{(1-t^*)}{\int_{s \in S} F(x) \, dx} \frac{1}{q(s)}.
\]

Clearly, with \( q'(s) = 0 \), the distribution of infrastructure investment across space is uniform, with its level depending only on the price of investment goods, the size of the world (i.e., the circle) and \( \delta \).

This result is quite intuitive. Perfect symmetry plus uniform distribution of activity implies that transit volume at every address is identical. Since the world planner does not discriminate between traffic directed towards different addresses, she will naturally chose \( i^*(s) = i \) for all \( s \in S \). Note also that this result is a natural application of the investment smoothing property of \( \tau \) discussed above.
4.2.2 Global economy, regional politics

In this section we discuss the solution of a game between different jurisdictions, which each set policies \( \{i^j(k), \mu^j\}_{k \in S^j} \), where \( j \in J \) now indicates the jurisdiction. For the sake of simplicity we assume that those policies are set by social planners. Note however, that the median voter setup would run into problems if we allow for strategic interactions between countries. The reason is that when contemplating their preferred policies voters would have to think about the strategic implications of their decisions. This would force them to make conjectures also about what agent’s policy would be implemented in any other country. They would have to conclude that it would be the median’s preferred policy. We avoid these complications by choosing the social planner setup.

Now, we have to make a basic dissociation between jurisdiction-specific policies, which are defined over the set \( S^j \), and optimal demand functions defined over export country locations in \( S \). We set \( q(s) = 1 \); hence, all jurisdictions are symmetric and we may therefore restrict the solution space to symmetric infrastructure distribution functions and tax rates.

Denote the optimal infrastructure policy in jurisdiction \( j \) is \( \tilde{i}^j : \bigcup M \to M \) where \( M \) is the (Banach) space of non-negative and continuous functions. Then, we may write the best response of the planner in region \( j \) as a function of

\[
\tilde{i}^j(s) = G\left( \left\{ \tilde{i}^k(s) \right\} \right) \equiv \arg\max \left\{ \int_{s \in S^j} F[i(s)](1 - \mu^j) \left\{ \tilde{i}^k(s) \right\}_{k \in S^j \setminus S^j} \right\}, \text{ for all } j \in J, \tag{25}
\]

where we stress the dependence of \( F \) on the world distribution of infrastructure investment.

We may now represent the symmetric equilibrium infrastructure allocation chosen by all planners as the solution to the fixed point problem

\[
G\left( \left\{ \tilde{i}(s) \right\} \right) = \tilde{i}(s), \tag{27}
\]

where the world equilibrium allocation of infrastructure is the collation \( \bigcup_{j \in J} \tilde{i}(s) \).

The proof of existence and unicity of \( \tilde{i}(s) \) follows Rossi-Hansberg (2003). In particular, existence is shown by an application of Schauder’s fixed point theorem, which requires that the functions \( F[i(s)](1 - \mu^j) \) are bounded, differentiable and continuous with respect
Figure 4: Equilibrium world infrastructure distribution.

to other jurisdictions’ policies, which they trivially are in the present context.\textsuperscript{13}

Unicity can be shown under the additional assumption that the equilibrium infrastructure distributions functions are continuous themselves.\textsuperscript{14} We can now summarize the properties of $\tilde{i}(s)$ Let $\bar{x}^j$ denote the mean (=median) address in jurisdiction $j$. First, $\tilde{i}'(\bar{x}^j) = 0$ for all $j \in J$. This means that as in the autarkic social planner problem, local maxima are attained at median addresses in each jurisdiction. Second, $\tilde{i}'(s) > 0$ for all $s \in [\bar{x}^j - 1/2, \bar{x}^j]$, $j \in J$ and $\tilde{i}'(s) < 0$ for all $s \in [\bar{x}^j, \bar{x}^j + 1/2]$, $j \in J$. Third, $\tilde{i}(\bar{x}^j - 1/2) = \tilde{i}(\bar{x}^j + 1/2) = \Gamma(\delta) > 0$, which shows that $\tilde{i}(s)$ is symmetric around the median address. We may therefore conclude that $\tilde{i}(s)$ is a perfectly symmetric wave function, $s \in S$, on the circle. Figure illustrates this finding.

\textsuperscript{13}See also Ok, Ch. J.

\textsuperscript{14}The assumptions used to show existence and uniqueness are unnecessarily strong, but are anyway satisfied under our functional assumptions.
5 Tentative empirical evidence

5.1 Testable implications from the model

The model has a range of predictions that can be put to an empirical test. In this section we discuss a number of those predictions without going into much detail. Appropriately discretized spatial data on intra- and international trade flows, associated to economic variables such as GDP, and direct information on the quantity and quality of transport infrastructure is difficult to construct and raises a number of important econometric issues, for example, related to spatial correlation of error terms.

The model also has interesting predictions relating to the effects of preferential trade liberalization. Infrastructure investment should be skewed towards that border which is economically permeable. Subsequent waves of EU enlargement could be used to check this hypothesis.

The model could also be brought to a calibration exercise. Standard new economic geography models predict economic inequality, but require the existence of natural peripheries. Our argument would allow economic inequality across space even in circumstances where no natural periphery exists, and borders have exclusively political significance. Calibration exercises of the standard models often lead to simulated inequalities statistics that are too low compared to the data. Allowing for the endogenous allocation of \( i(s) \) across space could help improve this fit.

Straightforward testing of the proposed model requires data on the quantity and quality of transport infrastructure across space. The actual distribution could then be compared to the distribution generated from a version of the model, where at least \( m(s) \) is taken from real data. However, continuous data on \( i(s) \), \( m(s) \) or \( q(s) \) is not available, hence, one has to discretize. This leads to a number of complications whose discussion we relegate to a full-fledged empirical analysis of the proposed model to future research.

In the present paper, we use a gravity-type estimation strategy to check whether easily available data for intra-US trade is consistent with our major result. While this ‘test’ is of limited reach, it nevertheless delivers results consistent with the model. However, any reasonable test of the model will have to relax the simplifying assumptions of a uniform distribution of population and investment prices adopted in the above analysis. We have already discussed what happens if \( q(s) \) is not uniformly distributed and concluded that spatial variation in \( q(s) \) needs to be very strong in order to undo the core results in
our model. Similar considerations hold if \( q(s) \) and \( m(s)^{-1} \) are distributed in the same non-uniform manner. In the extreme case, where all agents are concentrated at \( \bar{x} \) so that \( m(s) = 0 \) if \( s \neq \bar{x} \), the government would want to choose a infrastructure distribution much the same as individual agents would in the results presented in section , hence, \( i(s) \) would have a hump-shaped form if the concentration of the mass of agent does not occur exactly at a border. If agents would locate arbitrarily close to borders, we would observe a U-shaped distribution of \( i(s) \). However, it is clear that controlling for \( m(s) \), we would recover our results, as investment beyond the benchmark of a uniform distribution would again take a hump-shaped form.

5.2 A gravity equation application

In the following empirical exercise, we want to explore data on US internal trade. We can test the above model using this specific data, since data from foreign countries is not necessary to check some of the predictions of the model. We now turn to the gravity equation that emanates from the structure of the model. Starting from the demand function for industrial goods (7), substituting for \( p(x, z) \) and the price index by (8) and (11), and recognizing that \( Y^n = (1-t)m(x)b \), it is possible to derive an expression for the c.i.f. value of trade from \( z \) to \( x \):

\[
X(x, z) = (1-\alpha)(1-t)m(x)b^{1-\sigma}\frac{T(x, z)^{1-\sigma}}{\left[\int_{z\in S} T(x, z)^{1-\sigma} dz\right]^{1-\sigma}} \text{ for } z \geq x. \tag{28}
\]

The value of bilateral trade in industrial goods between \( x \) and \( z \) is a rather complicated function of the income of region \( x \), \( m(x)b \), and on the distribution of infrastructure in the interval \([x, z]\) which, in turn, shapes the cost factors \( T(x, z) \). However, the comparative statics with respect to distance is relatively simple. The bilateral trade volume (28) between \( x \) and \( z \) depends on distance only through the term \( T(x, z)^{1-\sigma} \). We may therefore write \( X(x, z) = K(x)T(x, z)^{1-\sigma} \). Denoting the elasticity of \( X \) with respect to distance by \( \varepsilon(x, z) \), we have

\[
\varepsilon(x, z) \equiv \frac{\partial X(x, z)}{\partial z} \frac{z}{X(x, z)} = -\frac{\sigma-1}{\delta-1}zi(z)^{1-\delta} < 0.
\]

Clearly,

\[
\frac{\partial \varepsilon(x, z)}{\partial i(z)} = (\sigma-1)zi(z)^{-\delta} > 0,
\]

so that the marginal effect of an increase in distance depends positively on the stock of infrastructure invested at the marginal location. The higher that stock, the smaller the
Figure 5: Border and central regions in the empirical setup.

\[ \begin{align*}
\text{Figure 5:} & \quad \text{Border and central regions in the empirical setup.}
\end{align*} \]

5.3 Data and empirical strategy

We use data on bilateral trade volumes between US states for 1993. This data has been used by Anderson & van Wincoop (2003) and is extensively discussed in that article. The

\[ \epsilon (x', x'') > \epsilon (x', z') \approx \epsilon (x', z'') \approx \epsilon (x'', z') \approx \epsilon (x'', z'') > \epsilon (z', z''). \]  

(29)

\[ \text{Note the role of the elasticity of substitution } \sigma \text{ between varieties of the industrial good. We have constrained } \sigma > 1 \text{ in (5) when we have specified the utility function. Usually, with monopolistic competition, this assumption is required to ensure that the monopolists' decision problems are well defined. With perfect competition, however, this is not necessary. However, we need the assumption } \sigma > 1 \text{ to ensure that the c.i.f. value of trade declines in iceberg costs.} \]
focus on the US follows from the fact that Canadian provinces are excessively large and have almost always a North–South extension that makes almost all of them border states.

In order to check a discretized version of (??), we run a standard gravity equation with importer/exporter fixed effects and usual covariates to explain intra US trade volumes. The model predicts that infrastructure investment is lower in border regions than in interior ones, hence trades involving border regions should be affected by a larger distance elasticity. For a formal empirical test, we interact the border-region dummy with the log of distance and see whether it turns out whether the absolute value of the elasticity of distance $\varepsilon(x, z)$ is indeed more negative for trades where either $x$ or $z$ or both belong to border regions. Note that the estimated elasticities are conditional on distance, regional GDPs, and the density of population in each state.

5.4 Results

Table 3 shows the results of our regression. Column (1) shows the results of a very standard gravity equation with importer and exporter fixed effects. These fixed effects are meant to capture all unobserved state specific variables that may be important for the determination of bilateral trade volumes. Most importantly for the economic environment at hand, they control for differences across states in infrastructure investment relating to non land-borne transit, e.g., air traffic. A bunch of control is included to control for geographical variation in economic activity and the price of infrastructure investment. GDP measures are country specific and would drop out of the equation if the panel were balanced. It is not, which explains why they show up in the table. However, their estimates take values close to unity, which is exactly what gravity theory would predict. Also the distance elasticity, which can be interpreted as an average over interior and border regions takes a value very close to what the literature finds. All covariates are estimated with substantial precision, delivering an overall $R^2$ statistics of 94 percent.

Column (2) shows that $\varepsilon(x', x'') = 0.884$ and $\varepsilon(z', z'' = 1.078$, with both estimates significantly different from each other. Hence, a marginal increase in distance hurts significantly more for trade relations that involve only border states than for trade relations...
involving only central locations. This finding is consistent with the proposed literature. Note, however, when the model is estimated without fixed effects (column (3)), the interaction term still takes the same sign and similar size, but is no longer statistically different from zero. This is expected, see the recent discussion on the correct specification of the gravity equation (Anderson and van Wincoop, 2003, or Feenstra, 2004, p. 161 ff). Columns (4) and (5) show that in line with theory, trade relations that involve only one border region have distance elasticities lying in the interval $[\varepsilon(x',x''), \varepsilon(z',z'')]$, namely 1,003 and 1,076. While the latter estimate is close to the one obtained for both partners being border regions, column (4) and (5) can still be interpreted as lending additional support to the theoretical model.

The presented econometric evidence is suggestive, but not more. There are a couple of problems that relate to the discretization of economic space into unevenly designed states. Ideally, one would run a model where the distance coefficients are interacted with a measure of distance to the border. Continuous space trade data being unavailable, the researcher has to make compromises. However, drawing on European NUTS or US county level data, one could expect to make more precise inference than using state level data.

6 Extensions, Conclusion

6.1 Extensions

Toll taxes. In reality, many countries operate toll taxes for freight traffic. The ratio of interregional highways subject to decentrally administered toll systems ranges from 6 percent to 52 percent in France.\textsuperscript{17} Countries such as Germany and Austria have centrally administered distance dependent road pricing for lorries. User fees may be contingent on a wide array of factors such as the situation of the environment (e.g., smog), the degree of congestion, the time at which a road is traveled (fees may be higher for travel during night or weekends), or whether an sensible region is crossed (e.g., some protected zone).

How a toll system affects the main argument in this paper depends very much on its specific design. Suppose, the government decides on the infrastructure allocation across space, but rather than taxing consumers through lump-sum taxes, it taxes road users without discriminating between home- or foreign-bound transport. This kind of taxation is of course distortionary, since it will affect goods prices of the regional and the world

\textsuperscript{17}Data from 1998 for France (Combes and Lafourcade, 2005), and 2006 for the US.
Table 3.
The border/distance interaction
Within US trade, 1993

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Robust standard errors in parentheses. $^a p < 0.01, ^b p < 0.05, ^c p < 0.1$
include constants and exporter/importer fixed effects; not shown.

economy. Further assume that the fee may vary continuously on the space of addresses and that it works just as our iceberg transportation costs, albeit with the shaved transported good not lost but transferred into the governments coffers. In that case, the government has two margins of action: it sets a distribution of fees, $f(s), s \in [0, 1]$ and decides on the infrastructure allocation, $i(s)$. Total income from fees will be $B = \int_0^1 f(s) X(s) ds$, where $X(s)$ is the value of goods, measured at ex-factory prices, that are transported through the point $s$. Note that $X(s)$ depends on the domestic and foreign distribution of infrastructure. If the government is free to spend $B$ on whatever infrastructure distribution it prefers,
and does not impose any additional tax, it will find it optimal to make fees dependent on the distance to the border, with higher fees the closer the border. Moreover, it will concentrate its spending as before in the central regions of the jurisdiction. The reason for this result is identical to the one discussed above: The government cares only about home welfare, thus taxing foreigners and transferring the receipts to home citizens is a welcome option.

If governments are not allowed to spend toll income on places other than those where the income has been generated, there will now be a direct link between infrastructure investment and transit volume, \( f(s) X(s) / q(s) = i(s) \). Governments will then set \( f(s) \) such that native welfare is maximized. By imposing high fees in border regions, governments tax foreign consumers more strongly than domestic ones; however, the implied high investment volumes are of little value for domestic consumers. By imposing high fees in the center, governments affect domestic consumers, but also achieve high utility for them. The implied distribution of infrastructure need not exhibit excessive spatial variation, but it still can; the exact outcome depending on underlying model parameters.

There are a number of institutional arrangements that involve the private sector into the construction and maintenance of transport infrastructure projects. Governments could sell exploitation licenses to private firms who construct roads and set fee structures \( f(s) \). While this is a choice of many governments, the economic modeling poses a number of problems, since one would have to decide whether the licenses are sold to a single provider, to a consortium, or to local monopolists, and whether those firms can commit to certain fee schedules and infrastructure investments when making their bids.

**Supranational entities.** Since decentralized transport infrastructure decisions generate positive externalities leading to a global underprovision and excessive spatial variation of infrastructure investment, there is a case for a supranational entity, such as the European Union (EU), to intervene. Indeed, the EU is involved in a large project, the Trans-European-Networks (TEN), that strives to coordinate and cofinance national infrastructure provision efforts. Member states have committed themselves to construct a number of road and rail links that link European regions. The EU, in turn, cofinances these projects. The degree of cofinancing is higher in peripheral regions than in central ones. Is this policy able to internalize the externalities highlighted in the present paper?

In principle, the answer is yes. In order to achieve the first-best infrastructure distribution, the EU should be allowed to tax citizens (or member state governments on their behalf) and subsidize the price of infrastructure. For example if \( q'(s) = 1 \) for all \( s \), so
that the efficient infrastructure schedule should exhibit no spatial variation, the EU could allow subsidize $q(s)$ such that the effective price of infrastructure projects paid by domestic governments declines as we move closer to the border. The optimal rate of decline should be related to the rate at which the the share of domestic beneficiaries falls, so that the growth rates of the stock of infrastructure investment is zero across space; see, e.g., equation (16).

The problem with the above subsidization principle is that it does not ensure that the overall quantity of infrastructure provision is efficient. To the extent that the domestic planner overinvests in the neighborhood of the median address $\bar{x}$, a policy geared towards a uniform spatial distribution of infrastructure leads to overinvestment.

**Internal labor mobility.** Instead of allowing goods prices to adjust to demand conditions so that an endogenous distribution $p(s)$ emerges but leave the distribution $m(s)$ uniform, one could also let $m(s)$ adjust endogenously so that indirect utilities are equalized across space. The governments would nevertheless propose uneven infrastructure investment schedules, since they still fail to take into account the positive externality that they exert on consumers in the rest of the world.

### 6.2 Conclusions

This paper develops a model where consumers demand goods from the entire world, but the world is fragmented into jurisdictions which set infrastructure investment schedules in a non-cooperative way. Governments caring only for their own welfare constituency will ignore the effects that their decisions have on foreign consumers; this basic externality leads to global underinvestment. The externality is stronger the more foreign-bound transit flows through an address, and the size of such transit is larger the closer national borders are to that address. Hence, infrastructure underinvestment is stronger in peripheral regions of jurisdictions rather than in central ones.

The local lack of infrastructure investment makes imports from other countries more expensive than imports from other regions from the same country, even if geographical distance or incomes of trading partners are the same. Our infrastructure story may therefore contribute towards unpacking trade costs and explaining the border puzzle highlighted by McCallum (1995).

The paper also presents some tentative empirical support for the theoretical results, drawing on trade data for US states. It turns out that trade relations that involve at least
one border region feature higher distance elasticities. Hence, transportation costs seem higher for the same distance when a border is crossed.
References


A Various proofs

A.1 Autarky: preferred policies

A.1.1 First order condition

The first order condition for agent $x$ has the form

$$\frac{1 - \sigma}{\alpha} \frac{\partial t}{\partial i(s)} F(x) + (1 - t) \frac{\partial F(x)}{\partial i(s)} = 0$$

where

$$\frac{\partial t}{\partial i(s)} = \frac{q(s)}{b}$$

for all $s$ from the government budget constraint, and

$$\frac{\partial F(x)}{\partial i(s)} = \begin{cases} i(s)^{-\delta} (\sigma - 1) \int_{z \in L(s)} T(z,x)^{1-\sigma} dz & \text{if } s \in L(x) \\ i(s)^{-\delta} (\sigma - 1) \int_{z \in R(s)} T(x,z)^{1-\sigma} dz & \text{if } s \in R(x) \end{cases}.$$

The limits of integration in the last expression follow from the fact that when $s \in L(x)$, only deliveries of goods from locations $z \in L(s)$ are affected by investment at $s$, the remaining deliveries are not affected; similarly for $s \in R(x)$.

Hence, the first order condition is

$$i(s)^{\delta} = \begin{cases} \alpha b \left[ (1 - t) / q(s) \right] \left[ F_{L}(s)(x) / F(x) \right] & \text{if } s \in L(x) \\ \alpha b \left[ (1 - t) / q(s) \right] \left[ F_{R}(s)(x) / F(x) \right] & \text{if } s \in R(x) \end{cases},$$

where we have used the notation introduced in (??). This latter expression corresponds to (15) in the text.

A.1.2 Growth rate of $i^\tau(s)$

Note that in $t$ and $F(x)$ the dependence on $s$ has been integrated out. Then, differentiating (15) for the case $s \in R(x)$ with respect to $s$, we obtain

$$i'(s) = -\frac{1}{\delta} \left[ \frac{\alpha b (1 - t)}{F(x)} \right]^{\frac{1}{\delta}} \left[ \frac{F_{R}(s)(x)}{F(x)} \right]^{\frac{1}{\delta}} \left[ \frac{T(x,s)^{1-\sigma} q(s) + q'(s) \int_{z} T(x,z)^{1-\sigma} dz}{q(s)^2} \right]$$

which can be straightforwardly used to compute (16) in the text. Similarly for the case $s \in L(x)$.  

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A.1.3 Strict concavity of $i^x(s)$

The growth rate of infrastructure investment at $s \in R(x)$ desired by agent $x$ is

$$g^x(s) = -\frac{1}{\delta} \frac{T(x, s)^{1-\sigma}}{\int_s^1 T(x, z)^{1-\sigma} \, dz}.$$ 

We have

$$\frac{\partial T(x, s)}{\partial s} = T(x, s) \frac{\dot{i}(s)^{1-\delta}}{\delta - 1} > 0.$$ 

Hence

$$\frac{\partial g^x(s)}{\partial s} = -\frac{1}{\delta} T(x, s)^{-\sigma} \left[ (1 - \sigma) \frac{\partial T(x, s)}{\partial x} \int_s^1 T(x, z)^{1-\sigma} \, dz + T(x, s)^{2-\sigma} \right] < 0 \quad \text{for all } s,$$

so the growth rate of $i(s)$ is negative and decreasing in $s$. It follows that $i(s)$ is strictly concave in $s$. That last expression corresponds to (2) in the text.

A.1.4 Shape of $i^x(s)$ for different $x$

Next, we may compute

$$\frac{\partial T(x, s)}{\partial x} = -T(x, s) \frac{\dot{i}(s)^{1-\delta}}{\delta - 1} < 0$$

and

$$\frac{\partial g^x(s)}{\partial x} = -\frac{1 - \sigma}{\delta} T(x, s)^{-\sigma} \left[ \frac{(1 - \sigma) \frac{\partial T(x, s)}{\partial x} \int_s^1 T(x, z)^{1-\sigma} \, dz + T(x, s)^{2-\sigma} \frac{\partial T(x, s)}{\partial x} \, dz}{\int_s^1 T(x, z)^{1-\sigma} \, dz} \right] > 0.$$ (30)

It follows that the desired growth rate $g^x(s)$ is less negative for a given $s$ for a household with a lower distance to the point of investment $s$.

A.1.5 Comparison of preferred tax rates $t^x$

We may use figure 3 to show that the tax rates preferred by an agent at address $x$ is larger than the one preferred by the median agent, i.e., $t^x > t^\bar{x}$, $x \neq \bar{x}$ and $\partial t^x / \partial |x - \bar{x}|$. 

First, note that the $t^\bar{x}$ is given by the area below the curve $i^\bar{x}(s)$, which is a continuous function of $s$. Now consider the preferred investment schedule of an agent residing at $\bar{x} + dx$. We still have $i^{\bar{x}+dx}(0) = i^{\bar{x}+dx}(1) = 0$, so that differences in the slopes $g^\bar{x}(s)$ and $g^{\bar{x}+dx}(s)$ suffice to make claims about $t^\bar{x}(s)$ and $t^{\bar{x}+dx}(s)$. In our figure, $dx > 0$. 

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Equation (30) shows that for points lying to the right of \( \bar{x} + dx \), we have \( g_{\bar{x} + dx}^s > g_{\bar{x}}^s \) while for points to the left, the contrary holds. Now, holding \( x \) constant, we would have

\[
\int_0^{\bar{x} + dx} i^s ds + \int_{\bar{x} + dx}^1 i^s ds = \int_0^{\bar{x} + dx} i_{\bar{x} + dx}^s ds + \int_{\bar{x} + dx}^1 i_{\bar{x} + dx}^s ds.
\]

Now, since \( i^s (s) < i_{\bar{x} + dx}^s (s) \) for all \( s < \bar{x} + dx \) and \( i^s (s) > i_{\bar{x} + dx}^s (s) \) for all \( s > \bar{x} + dx \) it holds that

\[
t_{\bar{x} + dx} = \int_0^{\bar{x} + dx} t_{\bar{x} + dx}^s ds + \int_{\bar{x} + dx}^1 t_{\bar{x} + dx}^s ds > t^\bar{x}.
\]

In other words, \( \partial t^x / \partial x > 0 \). This holds a fortiori for larger than infinitesimal deviations of \( x \) away from \( \bar{x} \).

### A.2 Autarky: Planner problem

#### A.2.1 First order condition

The first order condition to problem (20) is

\[
- \frac{\sigma - 1}{\alpha} \frac{q(s)}{(1-t)} b \int_0^1 F(x) dx + \frac{\partial}{\partial i(s)} \int_0^1 F(x) dx = 0.
\]  

where the relevant component of gross welfare affected by investment at \( s \) is

\[
\int_s^1 \int_0^1 T(z,x)^{1-\sigma} dzdx + \int_0^s \int_0^1 T(x,z)^{1-\sigma} dzdx.
\]

Moreover, the marginal effect of a change of \( i(s) \) on \( T(x,z)^{1-\sigma} \) is

\[
\frac{\partial}{\partial i(s)} T(x,z)^{1-\sigma} = (\sigma - 1) T(x,z)^{1-\sigma} i(s)^{-\delta} \geq 0.
\]
We may now write (32) the marginal effect of \(di(s)\) on social welfare as
\[
\frac{\partial}{\partial i(s)} \int_0^1 F(x) \, dx = (\sigma - 1) i(s)^{-\delta} \left[ \int_s^1 \int_0^s T(z, x)^{1-\sigma} \, dz \, dx + \int_0^s \int_s^1 T(x, z)^{1-\sigma} \, dz \, dx \right],
\]
which, inserted into the F.O.C. (31) delivers
\[
\frac{1}{\alpha (1 - \delta)} \frac{q(s)}{b} \int_0^1 F(x) \, dx = i(s)^{-\delta} \left[ \int_s^1 \int_0^s T(z, x)^{1-\sigma} \, dz \, dx + \int_0^s \int_s^1 T(x, z)^{1-\sigma} \, dz \, dx \right],
\]
which can be rewritten, using the definitions of \(\phi^L(s)\) and \(\phi^R(s)\) as equation (21) in the text.

\[
i(s) = [\alpha (1 - t) b]^{1/\delta} \left[ \frac{\phi^L(s) + \phi^R(s)}{q(s)} \right]^{1/\delta},
\]

where
\[
\phi^L(s) \equiv \frac{\int_s^1 \int_0^s T(z, x)^{1-\sigma} \, dz \, dx}{\int_0^1 F(x) \, dx} \quad \text{and} \quad \phi^R(s) \equiv \frac{\int_0^s \int_1^s T(x, z)^{1-\sigma} \, dz \, dx}{\int_0^1 F(x) \, dx}
\]

### A.2.2 Rate of change of \(i(s)\) across space

Differentiating (21) with respect to \(s\)
\[
\frac{i'(s)}{i(s)} = \frac{1}{\delta} \left[ \frac{\phi^L(s) + \phi^R(s)}{\phi^L(s) + \phi^R(s) - q'(s)} \right].
\]
The derivatives of \(\phi^L(s)\) and \(\phi^R(s)\) are, respectively
\[
\phi'^L(s) = \frac{1 - \sigma}{\int_0^1 F(x) \, dx} \left[ \int_s^1 T(s, x)^{-\sigma} \frac{\partial T(s, x)}{\partial s} \, dx + \int_0^s T(z, s)^{-\sigma} \frac{\partial T(z, s)}{\partial s} \, dz \right],
\]
\[
\phi'^R(s) = \frac{1 - \sigma}{\int_0^1 F(x) \, dx} \left[ \int_0^s T(x, s)^{-\sigma} \frac{\partial T(x, s)}{\partial s} \, dx + \int_s^1 T(s, z)^{-\sigma} \frac{\partial T(s, z)}{\partial s} \, dz \right].
\]

Substituting out the partial derivatives, \(\partial T(z, s)/\partial s = T(z, s) i(s)\), \(\partial T(s, z)/\partial s = -T(s, z) i(s)\), \(\partial T(s, x)/\partial s = -T(s, x) i(s)\) and \(\partial T(x, s)/\partial s = T(x, s) i(s)\), where \(i(s) \equiv i(s)^{1-\delta}/(\delta - 1)\), we have
\[
\phi'^L(s) = \frac{1 - \sigma}{\int_0^1 F(x) \, dx} i(s) \left[ -\int_s^1 T(s, x)^{1-\sigma} \, dx + \int_0^s T(z, s)^{1-\sigma} \, dz \right],
\]
\[
\phi'^R(s) = \frac{1 - \sigma}{\int_0^1 F(x) \, dx} i(s) \left[ \int_0^s T(x, s)^{1-\sigma} \, dx - \int_s^1 T(s, z)^{1-\sigma} \, dz \right].
\]

It follows that
\[
\frac{i'(s)}{i(s)} = \frac{1}{\delta} \left[ \left( 1 - \sigma \right) i(s) \left[ -\int_s^1 T(s, x)^{1-\sigma} \, dx + \int_0^s T(z, s)^{1-\sigma} \, dz + \int_0^s T(s, x)^{1-\sigma} \, dx - \int_s^1 T(z, s)^{1-\sigma} \, dz \right] \right]
\]
\[
\int_s^1 \int_0^s T(z, x)^{1-\sigma} \, dz \, dx + \int_0^s \int_1^s T(x, z)^{1-\sigma} \, dz \, dx
\]

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Exploiting the symmetry of trade costs, i.e., \( T(a, b) = T(b, a) \), defining \( \Delta(s) \equiv \int_s^1 T(s, x)^{1-\sigma} dx - \int_0^s T(x, s)^{1-\sigma} dx \), the expression simplifies to

\[
\frac{i'(s)}{i(s)} = \frac{1}{\delta} \left[ \frac{\sigma - 1}{\delta - 1} i(s)^{1-\delta} \Delta(s) \int_s^1 \int_0^s T(z, x)^{1-\sigma} dzdx - \frac{q'(s)}{q(s)} \right],
\]

which is equation (22) used in the text.

### A.3 International trade: linear world 2 country case

#### A.3.1 First order condition

Now, agent \( x \) decides on \( i(s) \), where \( s \in [0, 1] \), but consumes goods from \( s \in [0, 2] \). The given foreign infrastructure distribution is \( i^* (s) \), where \( s \in [1, 2] \). The F.O.C. of agent \( x \) remains essentially the same as before, with the only difference that the sets \( L(s) \), \( R(s) \) contain foreign addresses as well. Hence,

\[
i(s)^{\delta} = \begin{cases} \alpha b \left[ (1-t)/q(s) \right] \left[ F^L(s)(x)/F(x) \right] & \text{if } s \in L(x) \\ \alpha b \left[ (1-t)/q(s) \right] \left[ F^R(s)(x)/F(x) \right] & \text{if } s \in R(x) \end{cases}
\]

For the case of \( s \in R(x) \), we have

\[
\phi^{*R}(x, s) = \frac{\int_s^1 T(x, z)^{1-\sigma} dz + K}{\int_0^1 T(x, z)^{1-\sigma} dz + K}
\]

where \( K = \int_1^2 T(x, z)^{1-\sigma} dz \). \( \phi^R(x, s) \) under autarky is obtained by setting \( K = 0 \). Regardless of the exact distribution of \( i^* (s) \), we can note that \( \phi^{*R}(x, s) = \phi^R(x, s) \zeta + (1 - \zeta) \), where \( \zeta \equiv \int_0^1 T(x, z)^{1-\sigma} dz/ \left[ \int_0^1 T(x, z)^{1-\sigma} dz + K \right] \). Hence, \( \phi^{*R}(x, s) \geq \phi^R(x, s) \).

#### A.3.2 Growth rate of \( i^{*x}(s) \)

Also the growth rate of \( i^{*x}(s) \) is derived exactly as before.

\[
g^{Hx}(s) = -\frac{1}{\delta} \left[ \frac{T(x, s)^{1-\sigma}}{F^R(s)(x)} + \frac{q'(s)}{q(s)} \right].
\]

For the case where \( q'(s)/q(s) \) is finite, we now find that \( |g^{*x}(1)| \) is a finite number (possibly zero), too, and no longer infinite as under autarky.
A.3.3 Central planner Nash equilibrium

We focus on symmetric situations only. In that case, infrastructure investments in Home and Foreign, will be mirrored at about the border point $s = 1$, i.e. $i^H(s) = i^F(2 - s)$.

We define $H$ as the set of points $[0, 1]$ and $F$ as the set of points $[1, 2]$.

Indirect utility of an agent $x \in H$ is

$$
V^H(x) = \int_0^x T(z, x)^{1-\sigma} \, dz + \int_x^1 T(x, z)^{1-\sigma} \, dz
$$

while that of an agent $x' \in F$ is

$$
V^F(x') = \int_0^{x'} T(z, x')^{1-\sigma} \, dz + \int_{x'}^2 T(x', z)^{1-\sigma} \, dz
$$

The crucial thing now is that $L(x)$ contains addresses only from $H$, while $R(x)$ contains addresses from $F$ as well. Similarly, $L(x')$ contains points from $H$, while $R(x')$ does not.

The sets $R(x), L(x), R(x'), L(x')$ define the economic space, while $H$ and $F$ define the political space of the model.

Consider the problem of the planner in $H$. The sum of gross utilities affected by investment at $s \in H$ is

$$
\int_s^1 \int_0^s T(z, x)^{1-\sigma} \, dz \, dx
$$

We have that $T(x, z) = \exp \left[ \frac{1}{\lambda} \int_x^z i(s)^{1-\delta} \, ds \right]$ for $z \in H$, and $T(x, z) = \exp \left[ \frac{1}{\lambda} \int_x^1 i(s)^{1-\delta} \, ds + \frac{\lambda}{1-\lambda} \int_x^1 i(s) \, ds \right]$.

The planner in $H$ maximizes

$$
\int_{x \in H} V^H(x) \, dx \text{ s.t. } b t \leq \int_{s \in H} q(s) i^H(s) \, ds \text{ and given } i^F(s)
$$

A.4 International trade: multi-country world

A.4.1 Symmetric policies

We describe the symmetric spatial distribution of infrastructure by the wave function

$$
i(s) = a - b \cos(2\pi s), \ a/b \geq 1.
$$
The length of a country is unity, and borders are located at \( s = 0, 1, 2, \ldots \). That function takes minima of \( a - b \) at border locations and maxima of \( a + b \) at average locations \( 1/2, 3/2, 5/2, \ldots \). The average investment level over the entire interval is just \( a \). We require \( a/b \geq 1 \) so that \( i(s) \) is non-negative over the interval. Note that all cosine waves, regardless of \( a \) and \( b \) take the value \( a \) at \( 1/4, 3/4, 5/4, 7/4, \ldots \). We have \( i'(s) = 2\pi b \sin(2\pi s) \), so that the slope of the distribution is stronger in absolute value the larger \( b \) is.

Share of a variety coming from location \( z' \) in total spending of \( x \)

\[
s(x, z') \equiv \frac{p(x, z') c'(x, z')}{(1 - \alpha) Y^n(x)} = b^{-\sigma} \frac{T(x, z')^{1-\sigma}}{\left[ \int_{s \in S} T(x, z)^{1-\sigma} \, dz \right]^{1-\sigma}}
\]