Enlarging Integration’, Regional Convergence, and the Exchange Rate Dynamics

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Abstract. This paper extends Zee’s [1987] two-country OLG model by introducing disparities into Home following from the ‘enlarging integration’ of a less-developed region. It studies the effects of intra-Home convergence on the dynamics of the real exchange rate between Home and Foreign. It is found that in the long run the real exchange rate of enlarged Home depreciates by exactly that percentage by which the domestic population increases. Along the transition towards full convergence the exchange rate and the capital intensity of the less-developed region monotonically increase while the capital intensities of the more developed region and of Foreign temporarily decrease. JEL Classification: F15, F36, F41.

Introduction

Recent decades have seen the enlargement of economic regions embracing both highly and less developed nations.¹ As a consequence, the population weight of the enlarged region relative to the rest of the world has increased. In politics the population size of a country apparently matters, in neoclassical growth theory however, it does not matter at all. The question of whether the enlargement of an economic region by the rise of the number of participants in its commodity and capital markets has any economic effects at all provides the main motivation for this paper.

In particular, it investigates the dynamic effects of the increase in the population size of an economic region following from the ‘enlarging integration’² of less-developed countries on intra-regional convergence and on the international competitiveness of the enlarged region measured by its external terms of trade. In contrast to neoclassical growth theory, in our modified neoclassical model population size matters: the terms of trade of the enlarged region deteriorate while capital accumulation (growth) of both the enlarged region and of another large open economy connected with the former through trade in commodities and financial capital temporarily decrease.

¹ Cf. the unification of Western and Eastern Germany and the Eastern Enlargement of the European Union.
² The term ‘enlarging integration’ borrowed from Walz [1998, 298] is set in quotes because we are aware that the term makes really only sense if the integration between three countries and not only between two is considered.
The related literature on the economic effects of (political) unification of economically heterogeneous countries or the economic impacts of the enlargement of economic unions by integrating less-developed countries has used either closed-economy or small-open economy models. As regards the former, Funke and Strulik [2000] study the dynamic effects of interregional fiscal transfers on the growth and regional convergence of unified Germany. Among the latter approach are macro-econometric and CGE models studying the welfare effects of the last EU Enlargement two years ago. Neither the former nor the latter model approach is capable of eliciting the dynamic impacts of the ‘enlarging integration’ and related regional convergence on the international competitiveness of the enlarged region. Nor are they capable of investigating the impacts of the dynamics of the real exchange rate (= inverse of the external terms of trade) on the capital accumulation (growth) of the enlarged region and on the capital accumulation (growth) of the other, not enlarged economic region.

This paper intends to examine these questions within a two-country overlapping generations (OLG) model originally developed to study the international interdependence of fiscal policies of large open economies [Zee 1987, Lin 1994]. However, in such models countries are assumed to be homogeneous and of equal, unchanging size. To depict the integration of market participants of less-developed countries into the commodity and capital markets of the highly developed regions, we adapt the basic two-country OLG model by introducing regional disparities into one of the countries and allow for a change in its population size.

Zee [1987] and Lin [1994] employ different definitions of the real exchange rate. While Lin [1994, 95] defines the real exchange rate as the ratio of the foreign wage rate to the domestic real wage, Zee [1987, 608] sets the real exchange equal to the ratio of the price of the foreign commodity basket to the price of the domestic basket. We adopt Zee’s definition of the real exchange rate since we presume that the enlarged region has introduced a common currency differing from the currency used in the second country of the two-country model.

As is well-known, the convergence of per-capita incomes of less developed countries towards those of high-income countries proceeds rather slowly while the equalization of regional rates of return on real capital within an economic region takes much less time. It is plausible to assume that within one period of a two-period OLG model (25-30 calendar years) real interest parity holds in the enlarged region. Both capital market integration within one period and the more lengthy process of convergence cannot be introduced into Zee’s [1987] two-country OLG model without modifications. Zee’s model exhibits either real interest parity between the regions and immediate convergence, or real interest parity does not apply and convergence extends over several periods.

The solution of this specification problem consists of integrating Funke’s and Strulik’s [2000] two-region version of Barro’s [1990] model of government spending and (endogenous) growth into Zee’s [1987] two-country model. To avoid inconsistencies between Barro’s scale model of endogenous growth and Zee’s non-scale model of

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3 For an example of a macro-econometric model and a survey of related CGE models of EU Enlargement see Breuss [2002].
An early study with regard to the welfare effects of the Eastern EU enlargement is Baldwin et al. [1997], more recent studies are provided by Heidra et al. [2004] as well as by Lejour et al. [2004].
exogenous growth, we modify Barro’s approach by assuming that private and public capital do not exhibit constant, but rather decreasing returns to scale.

The present paper contributes to the existing literature in several respects. First, it provides a theoretical answer to the new question of how the ‘enlarging integration’ between heterogeneous regions and the subsequent regional convergence influences the international competitiveness of the country measured by the real exchange rate. Second, Zee’s two-country model is extended to a two-country, two-region OLG model with non-instantaneous convergence. It is true that in applying Farmer’s [1998] log-linear Cobb-Douglas version of Zee’s [1987] model we have to admit some loss of generality. However, this loss of generality might be seen as the cost of the higher complexity of our two-country, two region model. In addition, to check the stability of the complex model dynamics we have to resort to a numerical specification of the model parameters. Within these confines, we extend Zee [1987] so as to calculate numerically both the transitional dynamics of the real exchange rate and the private as well as public capital intensities after the shock of integrating the population of the less developed region into the world economy.

The paper is organized as follows. In the next section the main structure of the two-country, two-region model is presented, leaving the description of its intertemporal equilibrium dynamics and the steady-state solution to the following section. In the fourth section we investigate numerically the stability of the steady-state solutions and look analytically at the long-run effects of ‘enlarging integration’. In the fifth section the algebra and the economic rationale behind the instantaneous jump of the real exchange rate in response to the integration shock are dealt with. In the sixth section the convergence process after integration and the dynamic transition of the real exchange rate as well as of private and public capital intensities are numerically explored. The last section concludes.

**The two-country, two-region model**


The present model portrays a world economy with two interdependent countries, Home and Foreign (e.g. the EU and the US). Home and Foreign are assumed to be identical with respect to consumer preferences and production technologies. One of the two countries, Home is composed of two regions $r = R_1, R_2$, whereby $R_2$ is the less, and $R_1$ is the more developed region. The two regions differ in terms of private capital intensities (per-capita-incomes)

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4 The assumption of log-linear utility implies that changing interest rates do not impact upon savings behavior. In view of the fact that we are unable to provide a complete analytical solution of the model, the log-linearity assumption seems to be unduly restrictive. On the other hand, since we do not intend to offer a full-blown Computable General Equilibrium (CGE) analysis but only a numerical illustration of the transitional dynamics for typical parameter values the assumption is less restrictive than it appears.
and (per-capita-) public infrastructure. Each country produces a specific composite commodity, which can be used for consumption as well as for public and private investment. The domestically produced commodity is designated by \( x \) and the foreign-produced commodity by \( y^* \). Time is discrete.

**Firms**

A large number of identical firms operate under perfect competition in both regions of Home as well as in Foreign. Their technology is specified according to a Cobb-Douglas production function. In region \( r \) of Home, private capital services \( K^*_r \) together with labor services \( N^*_r \) are employed to produce total output \( X^*_r \) under constant returns to scale:

\[
X^*_r = A^*_r \left( K^*_r \right)^{\alpha} \left( N^*_r \right)^{1-\alpha}, \quad r = R1, R2. \tag{1}
\]

Since firms operate in a fully competitive environment, the production elasticity of capital (labor) services, \( \alpha \) \((1-\alpha)\), represents the capital (labor) income share.

\( A^*_r \) follows Barro’s [1990] specification of the productivity effects of public infrastructure:

\[
A^*_r = A \left( G^*_r / L^*_r \right)^{\eta}, \quad A > 0, \quad \eta < 1-\alpha, \tag{2}
\]

whereas \( A \) is an exogenously fixed level parameter of total factor productivity, equal for both regions. \( G^*_r \) denotes the aggregate stock of infrastructure (public capital) of region \( r \) in period \( t \). “Infrastructure is a common external input to each firm’s production function and is publicly provided” (Glomm and Ravikumar [1994, 1176]). In order to avoid unwanted scale effects, it is assumed that the infrastructure stock per capita determines total factor productivity of each firm (Funke and Strulik [2000, 366]). As mentioned above, to make Barro’s technological progress compatible with Zee’s international OLG model with exogenous long-run growth rates, diminishing returns to scale of accumulating factors are assumed \((\eta < 1-\alpha)\).

Profit maximization implies:

\[
g^*_r = \alpha A^*_r \left( K^*_r / N^*_r \right)^{\alpha-1}, \tag{3}
\]

\[
w^*_r = (1-\alpha) A^*_r \left( K^*_r / N^*_r \right)^{\alpha}. \tag{4}
\]

Denoting real investment of private capital in region \( r \) by \( I^*_r \), and real investment of public capital in region \( r \) by \( I^*_g \), private and public capital accumulate over time as follows:

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5 In line with the macroeconomic nature of Zee’s two-country model, the commodity-specialization pattern of each country remains open. It also implies that all commodities of each country are tradable albeit to less than 100%. The magnitude of the percentage is governed by the utility elasticity with regard to the consumption of foreign goods.

6 Henceforth all variables, refering to Foreign are denoted by an asterisk.

7 To simplify we assume that the growth rate of total factor productivity is zero.
\[ K_{r,t+1}^c = I_t^c, \]  
\[ G_{r,t+1}^c = I_t^c. \]  
(5)  
(6)

It is apparent that both capital stocks depreciate completely within one period.

For Foreign, the corresponding equations are:

\[ Y_t^F = A_t^F (K_t^F)^{α} (N_t^F)^{1-α}, \]  
(1.1)
\[ A_t^F = A \left( G_t^F / L_t^F \right)^{0}, \]  
(2.1)
\[ q_t^F = \alpha A_t^F (K_t^F / N_t^F)^{α-1}, \]  
(3.1)
\[ w_t^F = (1-\alpha) A_t^F (K_t^F / N_t^F)^{0}, \]  
(4.1)
\[ K_{r,t+1}^F = I_t^F, \]  
(5.1)
\[ G_{r,t+1}^F = I_t^F. \]  
(6.1)

**Households**

Two generations overlap in each period \( t \) as in the standard overlapping generations (OLG) framework. Each generation lives for two periods, working during the first when young and retiring in the second when old. Each member of the young generation supplies one unit of labor inelastically to firms. There is no labor-leisure choice. In the following, the superscript \( n = 1, 2 \) refers to the first, respectively to the second, period of life.

In Home, households are differentiated by their region of residence \( r = R1, R2 \). We denote the young population residing in region \( r \) by \( L_t^r \). In each period \( t \), the population of each region grows according to an exogenously fixed factor \( G_t^r \). The population does not migrate between regions.\(^8\) The young population of the less-developed region, \( L_t^{R2} \), is defined as a fixed share \( ξ \) of the young population of region \( R1 \), hence: \( L_t^{R2} = ξ L_t^{R1}, \xi \geq 0.\) \(^9\) The (young) population of the enlarged Home is therefore given by:

\[ L_t = (1 + \xi) L_t^{R1}. \]  
(7)

In each period of life, domestic households choose between consumption of domestic commodities, \( x_{t}^{rn} \) and of foreign commodities, \( y_{t}^{rn}. \)\(^{10}\) Following Zee \(1987, 605\) we assume that only “the domestically produced commodity can be purchased and stored by domestic residents as capital to be used in home-country production in the following period.” Capital is therefore internationally immobile.

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\(^8\) This assumption is motivated by differences between the regions originating in culture, language, etc.

\(^9\) This specification follows Farmer and Wendner \(1999^7, 281\).

\(^10\) Lower-case consumption quantities indicate consumption per capita. Henceforth, all lower-case quantity variables represent per-capita quantities.
The budget constraint (in real and per-capita terms) of the household living in Home, when young is
\[ x^y_t + e_t y^z_t + K^{R1}_{t+1}/L_t + K^{R2}_{t+1}/L_t + B^{R1}_{t+1}/L_t + B^{R2}_{t+1}/L_t + e_t B^*_{t+1}/L_t = (1 - \tau^y) w^y_t + z^y_t, \]
whereby \( s^y_t = K^{R1}_{t+1}/L_t + K^{R2}_{t+1}/L_t + B^{R1}_{t+1}/L_t + B^{R2}_{t+1}/L_t + e_t B^*_{t+1}/L_t \)
and when old is
\[ x^o_t + e_t y^z_t = (1 + i_o)\left( K^{R1}_{t}/L_t + K^{R2}_{t}/L_t + B^{R1}_{t}/L_t + B^{R2}_{t}/L_t \right) + e_o \left( 1 + i^*_o \right) B^*_{o}/L_t, \]
where \( e_t \) is the real exchange rate (units of the domestic good per unit of the foreign good), \( w^y_t \) is the real wage of region \( r \) in period \( t \) (units of the domestic good per one unit of labor), \( i_o = (1 - \varphi^o) q^o_r - 1 \) is the domestic and \( i^*_o = (1 - \varphi^*_o) q^*_o r - 1 \) is the foreign real interest rate, \( \tau^y_r \) is the domestic wage (capital income) tax rate, \( \varphi^o_r \) is the foreign capital income tax rate and \( z^o_r \) denotes region \( r \)'s real per-capita transfers from the government. In addition, \( K^{R1}_{t+1} \) (\( K^{R2}_{t+1} \)) indicates the capital stock of region \( R1 \) (\( R2 \)) which the household of region \( r \) plans to hold at the beginning of period \( t + 1 \) while \( B^*_{t+1} \) (\( B^*_{o} \)) denotes the stock of domestic (foreign) government bonds which the household of region \( r \) plans to hold at the beginning of period \( t + 1 \). \( s^y_t \) denotes per-capita savings.

Home households’ preferences are represented by the following intertemporal log-linear utility function:
\[ U^y_t = \zeta \ln x^y_t + (1 - \zeta) \ln y^z_t + \beta \left[ \zeta \ln x^y_{t+1} + (1 - \zeta) \ln y^z_{t+1} \right], \]
where \( \beta \) denotes the time preference factor of the young generation and \( \zeta \) \((1 - \zeta)\) is the expenditure share for domestic (foreign) commodities. Each household maximizes the utility function (10) subject to the budget constraints defined by equations (8) and (9). The optimal consumption and savings quantities of the domestic household are given in the appendix (see equations (A.1) to (A.7)).

The corresponding budget constraints and utility function for the foreign household are:
\[ (1/e_t) x^o_t + y^z_t + K^*_{t+1}/L_t + (1/e_t)\left( B^{R1}_{t+1}/L_t + B^{R2}_{t+1}/L_t \right) + B^*_{t+1}/L_t = (1 - \tau^o) w^o_t + z^o_t, \]
whereby \( s^o_t = K^*_{t+1}/L_t + (1/e_t)\left( B^{R1}_{t+1}/L_t + B^{R2}_{t+1}/L_t \right) + B^*_{t+1}/L_t \)
\[ (1/e_o) x^o_{t+1} + y^z_{t+1} = (1 + i^*_o)\left( K^*_{t+1}/L_t + B^*_{t+1}/L_t \right) + (1/e_o)(1 + i_o)\left( B^{R1}_{t+1}/L_t + B^{R2}_{t+1}/L_t \right), \]
and
\[ U^o_t = \zeta \ln x^o_t + (1 - \zeta) \ln y^z_t + \beta \left[ \zeta \ln x^o_{t+1} + (1 - \zeta) \ln y^z_{t+1} \right]. \]

It is assumed that domestic productive capital is fully mobile across regions.\(^\text{11}\) Because of initial regional disparities within enlarged Home, capital spontaneously flows towards the region with the higher marginal product.

\(^\text{11}\) Complete immobility of real capital between Home and Foreign is an extreme assumption motivated by the fact that capital mobility costs within a country are lower than between countries on account of lower distance costs.
of private capital (i.e. to region \( R_2 \)). These intra-period capital movements between \( R_1 \) and \( R_2 \) come to an end if the following real interest parity condition holds between \( R_1 \) and \( R_2 \):

\[
q^{R_1} = q^{R_2}. \quad (11)
\]

Since government bonds are perfectly mobile across Home and Foreign, an international interest parity condition holds between Home and Foreign:

\[
1 + i_{\text{int}} = (e_{\text{int}}/e)(1 + i_{\text{int}}). \quad (12)
\]

The public sector

The government of Home partly finances its expenses through the emission of public debt \( B^{R_1} + B^{R_2} \). Public revenues include labor and capital income taxes. Total public expenditures of region \( r \) are defined as a fixed, common share \( \Gamma \) \((0 < \Gamma < 1)\) of regional GDP \( X_r^r \). Region-specific expenditures are allocated to transfers to private households and to expenditures for investment in public capital in region \( r \) as follows:

\[
I^r = \gamma^r \Gamma X^r, \quad (13)
\]

\[
L^r = (1 - \gamma^r) \Gamma X^r, \quad (14)
\]

whereby \( 0 < \gamma^r < 1, r = R_1, R_2 \) is the share of total expenditures spent on public capital accumulation in region \( r \).

The budget constraint for the domestic government is therefore

\[
B^{R_1}_{t+1} + B^{R_2}_{t+1} + \tau_r \left( w^{R_1} L^{R_1} + w^{R_2} L^{R_2} \right) + \phi_r \left( q^{R_1} K^{R_1} + q^{R_2} K^{R_2} \right) = (1 + i_t)^r \left( B^{R_1}_t + B^{R_2}_t \right) + \Gamma \left( X^{R_1}_t + X^{R_2}_t \right). \quad (15)
\]

Likewise, foreign public investment expenditures, total transfers, and the budget constraint of the foreign government are as follows:

\[
\[\text{Similarly, for } r = R_1, R_2 \text{ the share of total expenditures spent on public capital accumulation in region } r \text{ is defined as } \gamma^r \text{ and the budget constraint for the domestic government is:}
\]

\[
I^r = \gamma^r \Gamma Y^r, \quad (13.1)
\]

\[
L^r = (1 - \gamma^r) \Gamma Y^r, \quad (14.1)
\]

\[
B^{R_1}_{t+1} + \tau^*_r w^r L^r + \phi^*_r q^r K^r = (1 + i^*_r) B^r_t + \Gamma Y^r. \quad (15.1)
\]

The convergence process within Home

A new variable, the convergence indicator \( \theta_i \), is introduced to measure regional disparities within Home after the 'enlarging integration': \(^{13}\)

\[
\theta_i = x^R_i / x^R_i, \text{ with } x^R_i = X^R_i / L^R_i, \quad (16)
\]

whereby \( x^r, r = R_1, R_2 \) denotes per-capita income in region \( r \).

Combining the production function (1) and the real interest parity condition between region \( R_1 \) and region \( R_2 \) (equation (11)), yields:

\[\text{Note that one period in the model comprises 25 to 30 years!}\]

\[\text{The concept of } \theta_i \text{ is borrowed from Funke and Strulik [2000, 367].}\]
\[ \theta_1 = \frac{x_{i}^{R2}}{x_{i}^{R1}} = k_{i}^{R2}/k_{i}^{R1} = \left( \frac{g_{i}^{R2}/g_{i}^{R1}}{\eta^{\alpha}} \right)^{\frac{\eta}{\alpha}}. \]  

Equation (17) reveals that “interregional mobile private capital ties down [R2’s] relative income per capita to its relative stock of infrastructure per capita” (Funke and Strulik [2000, 367]).

Recalling (6), and considering (17), we obtain:

\[ \theta_{i} = \left( \frac{\gamma_{i}^{R2}}{\gamma_{i}^{R1}} \right)^{\frac{\eta}{\alpha}} \left( \theta_{1} \right)^{\frac{\eta}{\alpha}}. \]  

This equation depicts the motion of the convergence indicator in Home. It shows that the convergence is governed by the ratio of the shares of investment expenditures on public infrastructure \( \gamma_{i}^{R2}/\gamma_{i}^{R1} \) and by the production elasticities of private capital \( \alpha \) and public infrastructure \( \eta \).

**Intertemporal market equilibrium**

In this section, the main market clearing conditions for all periods \( t = t_0, t_0 + 1, t_0 + 2, \ldots \) are briefly described.

Market clearing of the regional labor markets in Home requires:

\[ N_{i}^{r} = L_{i}, \quad r = R1, R2, \]  

while in Foreign

\[ N_{i}^{r} = L_{i} \]  

holds.

In Home, full integration of the internal market for private capital services implies:

\[ K_{i}^{R1} + K_{i}^{R2} = K_{i}^{R1,R1} + K_{i}^{R1,R2} + K_{i}^{R2,R1} + K_{i}^{R2,R2}. \]  

The product market clearing condition of Home reads as follows:

\[ x_{i}^{R1} + \xi x_{i}^{R2} = x_{i}^{R1,1} + \left( \frac{1}{G^{L}} \right) x_{i}^{R1,2} + \xi x_{i}^{R2,1} + \left( \frac{1}{G^{L}} \right) x_{i}^{R2,2} + G^{L} g_{i}^{R1} + G^{L} g_{i}^{R2} + \xi G^{L} g_{i}^{R1} + \xi G^{L} g_{i}^{R2} + \xi x_{i}^{R1,1} + \xi x_{i}^{R2,2}, \]  

while foreign product market clearing demands:

\[ y_{i}^{r} = y_{i}^{R1} + \left( \frac{1}{G^{L}} \right) y_{i}^{R2} + G^{L} k_{i+1} + G^{L} g_{i+1} + y_{i+1}^{R1} + \left( \frac{1}{G^{L}} \right) y_{i+1}^{R2} + \xi y_{i+1}^{R1} + \xi \left( \frac{1}{G^{L}} \right) y_{i+1}^{R2}. \]  

The world market for Home bonds clears according to:

\[ B_{i}^{R1} + B_{i}^{R2} = \left( B_{i}^{R1,R1} + B_{i}^{R1,R2} + B_{i}^{R1,R2} \right) + \left( B_{i}^{R2,R1} + B_{i}^{R2,R2} + B_{i}^{R2,R2} \right) \]  

and symmetrically for Foreign bonds we have:

\[ B_{i}^{r} = B_{i}^{R1,R1} + B_{i}^{R1,R2} + B_{i}^{R1,R2} \]  

14 We assume that \( \gamma_{i}^{R2} = \gamma_{i}^{R1} \) and both are exogenously fixed. Alternatively, we could have introduced a “policy function”
like Funke and Strulik [2000, 373] which specifies that \( \gamma_{i}^{R2} \) exceeds \( \gamma_{i}^{R1} \) as long as the convergence indicator is less than one.
The world capital market clearing condition requires that the total amount of savings in the world equals the total world demand:

\[ s_{t}^{R1} + \xi s_{t}^{R2} + e_{t} s_{t}^{*} = G_{t}^{L} \left[ h_{t}^{R1} + \xi k_{t}^{R1} + (1 + \xi) b_{t}^{R1} + e_{t} k_{t}^{*} + e_{t} b_{t}^{*} \right]. \] (23)

The intertemporal equilibrium dynamics and the steady state

This section is devoted to presenting the intertemporal equilibrium dynamics of the two-country, two-region model described in the previous Section and to studying the existence and uniqueness of its steady states. In order to simplify the complex algebra, we make the following assumption:

\[ L_{t} = L^{*}. \] (24)

which does not alter the qualitative nature of the results.\(^{15}\)

From the international interest parity condition (12) together with (1)-(3), the equation of motion of the real exchange rate follows:

\[ e_{t+1} = e_{t} \left( 1 - \varphi \right) \left( 1 - \varphi^{*} \right) \left[ \left( g_{t+1}^{*} \right)^{\eta} \left( k_{t+1}^{*} \right)^{\eta-1} \right] / \left[ \left( g_{t}^{*} \right)^{\eta} \left( k_{t}^{*} \right)^{\eta-1} \right]. \] (25)

Next, similarly as in Diamond [1965, 1137], it is assumed that Home and Foreign governments hold the stocks of public debt per capita constant over time: \( b_{t} = b^{*}, b_{t}^{*} = b^{*}, \forall t \). In addition, to ensure perfect convergence in Home in the long run, \( b = b^{R1} = b^{R2} \) is assumed. The budget constraints of Home and Foreign governments (see (15) and (15.1)) imply that tax rates become endogenous. To simplify the algebra, we assume that capital income tax rates \( \varphi, \varphi^{*} \) are exogenously fixed by Home and Foreign governments, while wage tax rates \( \tau_{t}, \tau_{t}^{*} \) are determined as follows:

\[ \tau_{t} = \left( A / G_{t}^{L} \right) \left( g_{t}^{*} \right)^{\eta} \left( g_{t} \right)^{\eta} \left[ \left( 1 - \varphi \right) \left( 1 + \xi \right) \left( b^{*} / k^{*} \right) + \left( 1 - \alpha \varphi \right) \left( 1 + \xi \theta \right) \right] / \left( 1 - \alpha \right) \left( 1 + \xi \theta \right) \left( G_{t}^{L} \right) \left( g_{t}^{*} \right)^{\eta} \left( k_{t}^{*} \right)^{\eta-1}, \] (26)

\[ \tau_{t}^{*} = \left( A / G_{t}^{L} \right) \left( g_{t}^{*} \right)^{\eta} \left( g_{t} \right)^{\eta} \left[ \left( 1 - \varphi \right) \left( b^{*} / k^{*} \right) + \left( 1 - \alpha \varphi \right) \left( 1 + \xi \theta \right) \right] / \left( 1 - \alpha \right) \left( G_{t}^{L} \right) \left( g_{t}^{*} \right)^{\eta} \left( k_{t}^{*} \right)^{\eta}. \] (26.1)

By inserting the optimal domestic savings function (A.7) and the corresponding foreign savings function into the world capital market clearing condition (23) and respecting profit maximizing conditions (3) and (3.1), the following difference equation is obtained:

\(^{15}\) To simplify the cumbersome notation note that in the dynamical equations the superscripts indicating region R1 are henceforth deleted.
From the equation of motion for public capital (6), the following difference equation can be easily derived:

$$
g(k_i+1) = \left( \frac{A\gamma}{G^i} \right) \left( \frac{g^*}{k^*} \right)^{\gamma}(k_i)^{1-\gamma}. \quad (28)
$$

A further reduction in the dimensions of the dynamic system is obtained if the foreign government keeps the foreign per-capita stock of public capital $g^*$ constant over time and fixed at the level $g^*$. The foreign share of investment expenditures on public infrastructure would thus become endogenous and it follows from 6.1 that

$$
g^* = \left( \frac{G^i}{A} \right) \left[ \left( \frac{g^*}{k^*} \right)^{\gamma} \right]. \quad (29)
$$

From the two national product market clearing conditions (21) and (21.1), the fourth dynamic equation is obtained:

$$
(1+\xi_{ii})k_i+1 + e_i k_{i+1} + e_i b^* = \\
= \left( \frac{A\sigma}{G^i} \right) \left[ \left( (1-\tau) (1-\gamma) \right) \left( 1+\xi_{ii} \right) \right] (g_i)^{\gamma}(k_i)^{1-\gamma} + \\
+ e_i \left( \frac{A\sigma}{G^i} \right) \left[ (1-\tau^*) (1-\gamma^*) \right] (g_i^*)^{\gamma} (k_i^*)^{1-\gamma}. \quad (27)
$$

(27)

Equations (18), (25), (27)-(28) and (30) represent the five-dimensional dynamic system of the two-country, two-region model.

Under the presumption of parameter sets which ensure the existence of at least one non-trivial\(^{16}\) steady state, the dynamic equations can be reduced onto a system of two equations (A.8) and (A.9) to determine the endogenous variables $k$ and $e$. Following Zee [1987, 613], equation (A.8) is interpreted as the geometrical locus of all pairs $(k,e)$ which assures the international capital market equilibrium condition. The equilibrium locus of this market in the $k-e$ space is labeled as the KK-curve. Equation (A.9) represents the equilibrium condition in the two national commodity markets taken together and is called the GG-locus.\(^{17}\)

In Figure 1 two typical configurations of KK- and GG-lines are illustrated.\(^{18}\) In Figure 1a Home is a net creditor of Foreign, while Figure 1b depicts the case of Home being a net debtor of Foreign (see Zee [1987, 613]). The KK-

---

\(^{16}\) In fact, there is also a trivial steady-state solution. However, in contrast to intuition, the existence of a trivial steady state cannot be inferred from an inspection of the steady-state lines. To see how the existence of a trivial steady-state solution can be demonstrated in a similar formal context, see Farmer and Wendner [2003, 780].

\(^{17}\) The economic reasoning which Zee [1987, 613] provides for the slopes of the KK- and GG-curve applies to our curves as well. In contrast to Zee, our steady-lines rely on the specific numerical parameters given in Table 1. However, there is a broad range of admissible and plausible parameter values which generate qualitatively similar KK- and GG-curves.

\(^{18}\) A mathematical analysis of the existence of non-trivial steady-state solutions (i.e. the intersection of the KK- and GG-curve in the first orthant) is available from the authors upon request. It shows that there is a sufficiently broad range of plausible
locus is in general double-branched: if Home is a net creditor (debtor), the KK-locus to the left of $k$ is positively (negatively) sloped, while right of $k$ the KK-locus is negatively (positively) sloped. In both panels, the GG-locus is (slightly) positively sloped.\textsuperscript{19}

\textbf{Stability of steady states and the long-run effects of ‘enlarging integration’}

Being assured of the existence of two distinct steady-state solutions, it is natural to ask about the local stability of the steady states. To this end, the equilibrium dynamics is linearly approximated in a small neighborhood of each of the two steady states. Due to the algebraic complexity of the Jacobian of the equilibrium dynamics around the steady states,\textsuperscript{20} the stability of the steady state can be verified only numerically. To remain within the theoretical focus of this paper, the parameters of the two-country, two-region model are numerically specified such that stylized facts of the world economy are roughly replicated. In the following, Home can be identified with the EU-25, Foreign with the US,\textsuperscript{21} Region $R_1$ corresponds then to the former EU-15, region $R_2$ represents the CEECs.

Following EUROSTAT (2004) and CENSUS (2003), the yearly growth rate of the world active population is around 0.54\%. The corresponding growth factor, $G^c$ over a 25 years’ period is 1.1441. The utility discount factor, $\beta$ is set equal to 0.6. This figure is equivalent to a subjective time preference rate of 0.02 per year (see Auerbach and Kotlikoff [1987, 51] for a similar figure). The scale parameter $A$ is set equal to 5.0 (Auerbach and Kotlikoff [1998, 260] assume $A = 10.0$ for a two-generations model of a closed economy\textsuperscript{22}). Under perfect competition, the production elasticity of labor $1 - \alpha$ corresponds to the wage share, which is roughly equal to two thirds in developed countries. Therefore, $\alpha = 0.35$ is chosen. The production elasticity of public capital $\eta$ is set equal to 0.3 as in

\begin{center}
\textbf{Figure 1a about here}
\end{center}

\begin{center}
\textbf{Figure 1b about here}
\end{center}

\textsuperscript{19} The slope of the GG-curve depends on the magnitudes of domestic relative to foreign fiscal parameters. If the former are “larger” (‘smaller”) than the latter, the GG-curve is positively (negatively) sloped. A glance at the GG-curve (equation A.9) reveals that the slope of the GG-curve is zero if domestic and foreign per-capita public capital stocks are equal $(e = (1 + \xi)(1 - \zeta)/\xi)$.\textsuperscript{11}

\textsuperscript{20} The Jacobian of the dynamical system is available from the authors upon request.

\textsuperscript{21} Note since we do not intend to provide a CGE analysis the application of the two-country, two-region model to EU-25 and US is not meant literally.

\textsuperscript{22} It is worthwhile to note that steady-state solutions with larger values of $A$ exist if the fiscal parameters of Home and Foreign get similar.
Funke and Strulik [2000, 372]. The complete parameter set, including the fiscal policy parameters for the two economies, is summarized in table 1.23

Table 1 about here

For this parameter set, the calculation of the eigenvalues of the Jacobian matrix of the dynamic system in the two steady states indicates that in the steady state with the smaller capital intensity in region $R_1$ of Home (see Figure 1) two eigenvalues are larger than unity and three are less than one, while in the steady state with the larger capital intensity in region $R_1$ four eigenvalues are less than unity, and one is larger than unity. Hence24, the former steady state is saddle-path instable, while the latter is saddle-path stable.25

Knowing which steady state qualifies as being locally stable, we can turn now to the investigation of the long-run effects of ‘enlarging integration’ on the main variables of our two-country, two-region model. In the long run, full convergence between both regions of Home is achieved ($\theta = 1$). We compare the steady-state solution of our model without integration of region $R_2$ ($\xi = 0$) with the steady-state when region $R_2$ is fully integrated into the world economy ($\xi > 0$). Intuitively, one would expect that per-capita variables in the long run do not respond to the increase of Home population while the real exchange rate between Home and Foreign would depreciate since the relative scarcity of the foreign product would rise with a larger domestic population.

An inspection of the following equation (31), obtained by equating (A.8) and (A.9), and a glance at equation (32) taken from the appendix (see equation (A.9))

\[
\begin{align*}
\left\{ k - (A/G^i) \sigma \left[ 1 - \alpha (1 - \varphi) \left( 1 + b/k \right) - \gamma \Gamma \right] \left( g^* \right)^\gamma \left( k^* \right)^\nu + (1 - \sigma) b \right/ \\
\left[ \left( k^* - (A/G^i) \sigma \left[ 1 - \alpha (1 - \varphi) \left( 1 + b^*/k^* \right) \right] \left( g^* \right)^\gamma \left( k^* \right)^\nu + \sigma g^* + (1 - \sigma) b^* \right] \right\}
\end{align*}
\]

(31)

\[
e = (1 + \xi) (1 - \xi)^{\nu} \left[ \left( 1 - \gamma \Gamma \right) \left( A/G^i \right) \left( g^* \right)^\gamma \left( k^* \right)^\nu - k \right] \sqrt{ \left( A/G^i \right) \left( g^* \right)^\gamma \left( k^* \right)^\nu - g^* - k^* },
\]

(32)

reveal that the intuition is right. Equation (31) shows that the steady-state capital intensity of region $R_1$ is independent of the parameter $\xi$. Equation (32) tells us that the real exchange rate after the enlargement is $1 + \xi$

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23 Some values of the fiscal parameters are taken from the parameter set of Mendoza and Tesar [1998, 233].

24 If two eigenvalues are larger than one, we would need two jump variables to ensure saddle-path stability. However, there is only one dynamic variable which qualifies as being a jump variable: the real exchange rate.

25 In contrast to Zee [1987], neither steady state is asymptotically stable. This is true for a broad range of plausible parameter sets. In a still unpublished paper we have shown that for equal capital tax rates and equal per-capita public capital stocks in Home and Foreign neither steady-state is asymptotically stable for all admissible parameter sets.
times larger than before. Hence, the long-run effects of the ‘enlarging integration’ of region $R_2$ can be summarized as follows:

1. The real exchange rate is the only variable which changes in the new steady state. Its percentage increase equals the percentage rise of the population of Home following the ‘enlarging integration’ of region $R_2$.

2. Neither the private nor the public capital intensities of domestic region $R_1$ and Foreign are affected by the ‘enlarging integration’.

The steady-state effects of ‘enlarging integration’ are clear-cut since in the long run no convergence effect arises. This is not the case, however, when the effects of the integration of a less-developed region into the world economy are considered. In this case an initial non-steady state situation is compared with a full-convergence steady state and a transition process arises. The enlarging of Home has thus two distinct effects, an integration effect and an accompanying convergence effect. As we already know, the former includes the effects of the integration of the capital and good markets of region $R_2$ into the respective markets of region $R_1$ (see equations (20) and (21)) as well as the integration of the capital market of region $R_2$ into the world capital market (see equation (23)). The latter effect consists of faster growth of the economy of region $R_2$ due to free capital movements.

In the next section we investigate both the integration and convergence effects of the ‘enlarging integration’, as reflected by the instantaneous jump of the real exchange rate onto the stable arm of the equilibrium dynamics.

**Enlarging integration and the instantaneous jump of the real exchange rate**

The investigation of the transitional dynamics following the ‘enlarging integration’ requires that the type and the timing of the shock are specified. In order to keep the analysis as simple as possible, we assume that the shock of the ‘enlarging integration’ is permanent, unannounced and occurs at the beginning of the transition period. As mentioned above, the value of the parameter $\xi$ increases suddenly from zero to a strictly positive value. The period in which the shock is introduced is denoted by $t = t_0$ and it is called the enlargement period.

It makes a difference whether the ‘enlarging integration’ takes place after the conclusion of the convergence process or before. In the first case, the exchange rate jumps instantaneously to its new steady-state value. Only in the latter case, does the exchange rate move along a transition path towards the new steady state.

Along the transition path of the exchange rate, two phases can be distinguished: first its instantaneous jump onto the stable arm of the transitional dynamics and then its motion along the stable arm towards the steady state of full convergence.

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26 From this point of view, our enlarging integration is a typical regional integration shock, accounting for three of the four Single Market effects (that of the free movement of people is not taken into consideration, see Breuss [2002]).

27 This assumption also allows us to model partial globalization effects (see Breuss [1997]) of the enlargement.

28 In reality, announcement effects may occur!

29 In view of the last EU-enlargement we take a value of $\xi = 0.18$ which approximately represents the share of the CEEC population with respect to that of the EU-15.
To analyze the instantaneous jump of the exchange rate, two different but complementary approaches are taken in this section. First, in the next subsection we use the equilibrium manifold of the real exchange rate derived in the appendix to gain some insight concerning the nature and the magnitude of the exchange rate jump. Second, in the subsequent subsection we provide an economic rationale for the upward jump of the exchange rate.

The equilibrium manifold of the real exchange rate

From the previous section it is known that one characteristic value of the Jacobian in the steady state with the larger Home capital intensity is larger than one. To prevent explosive equilibrium dynamics in the neighborhood of the steady state, this characteristic value must not be included within the general solution of the system of linear difference equations representing the linear approximation of the equilibrium dynamics around the steady state (equations (A.10)-(A.14) in the appendix). From the general theory of linear difference equations under rational expectations (Blanchard and Kahn [1980, 1305-1312]) it is known that one of the dynamic variables in the two-country, two-region model has to be a jump variable and that the initial value of the jump variable is to be located on an equilibrium manifold, called the stable arm, in the \((e_t, k_t, k_t^*, g_t, \theta_t)\)-space. Since all dynamic variables with the exception of the real exchange rate are sluggish, it follows immediately that the real exchange rate represents the jump variable.

Being a jump variable also implies that in the enlargement period the real exchange rate is the only dynamic variable to be affected by the shock of a larger \(\xi\). This can also be seen formally by inspecting the real exchange rate equilibrium manifold which is derived in the appendix and reproduced in the following equation (33):

\[
e_{t_e} = e + \rho^\xi (k_t - k) + \rho^* (k_t^* - k^*) + \rho^g (g_t - g) + \rho^e (\theta_t - 1),
\]

whereby the constants \(\rho^\xi, \rho^*, \rho^g, \rho^e\) represent complex combinations of the eigenvectors of the Jacobian (see (A.15) in the appendix) associated with the four stable eigenvalues (i.e. those less than one).

Equation (33) shows that the real exchange rate in the period of ‘enlarging integration’ \(t_e\) is determined by the steady state and by the initial values of all dynamic variables. From the previous section it is known that the increase of \(\xi\) raises the steady-state value of the real exchange rate by the same magnitude whereas the capital intensities do not change at all. A glance at equation (33) shows that the jump of the real exchange rate \(e_{t_e}\) can be attributed to the increase of the steady-state value of the exchange rate \(e\) and to the difference between the long-run value and the initial value of the convergence indicator \(\theta_t\). At the time of the enlargement the convergence indicator is less than unity\(^{30}\), hence the difference \(\theta_t - 1\) is negative.

To determine how (much) the exchange rate \(e_{t_e}\) differs from its new steady-state value, the sign and the magnitude of \(\rho^\theta\) must be known. Due to the complexity of the Jacobian, \(\rho^\theta\) can only be calculated numerically, a

\(^{30}\) At the time of the last EU-enlargement the convergence indicator is calculated as \(\theta_t = 0.46\) (EUROSTAT [2004]).
calculation which shows that $\rho^\theta$ is positive. Thus, equation (33) corroborates that the short-term exchange rate does not “overshoot” (Dornbusch [1976]) the new steady-state value since the rise of steady-state value of the exchange rate is damped by the negative term $\rho^\theta (\theta_\epsilon - 1)$. In addition, the equilibrium manifold (33) tells us why only the integration of a less developed region $R2 (\theta_\epsilon < 1)$ into the world economy generates a transitional dynamics of the real exchange rate. If region $R2$ were equally developed ($\theta_\epsilon = 1$), no transitional dynamics would take place.

The economic rationale behind the upward jump of the exchange rate

To explain the economic rationale behind the immediate increase of the exchange rate in the enlargement period, we assume provisionally that the exchange rate does not respond to the enlargement shock. Under this proviso imbalances in the domestic and foreign current account in the enlargement period will occur which are inconsistent with an intertemporal general equilibrium.31

The (per-capita) current account of (enlarged) Home and Foreign, respectively, in the enlargement period read as follows:

$$ca_{t_0} = x^{*1}_{t_0} + \left(1/G^L\right)x^{*2}_{t_0} - e_{t_0} + \left(1/G^L\right)y^{R1,1}_{t_0} + \xi j^{R1,1}_{t_0} + \xi \left(1/G^L\right)y^{R2,1}_{t_0} + \xi \left(1/G^L\right)y^{R2,2}_{t_0} - e_{t_0}^{\epsilon} h^{R1,1}_{t_0} - i_{t_0} h^{R1,*}_{t_0} = 0 .$$

(3.4)

The first step is to show that under a fixed exchange rate the current account of enlarged Home gets into disequilibrium necessitating a change of the real exchange rate. The reason is that Home’s exports do not react to the enlargement shock while Home’s imports increase due to the fact that after enlargement more households in Home demand foreign goods (see equation (34)). Note, however, that this increase of Home’s imports is unambiguous only if the per-capita consumption of foreign goods by (young) households of both domestic regions does not decline in response to the enlargement shock. This is true under the assumptions of this subsection since a zero public debt stock in Home implies that the domestic wage tax rate is independent of $\xi$ (see equation (26)) and hence the optimal consumption of young households in both regions of Home remains unaltered. The same applies to the per-capita consumption of the old households in both regions of Home (see their optimal consumption functions in the appendix).

The next step is to elaborate whether an upward jump of the real exchange rate (real depreciation of Home currency) restores an equilibrium in the current account of Home (and Foreign) which means that Home’s exports increase and Home’s imports decrease. Home’s exports which are equal to the per-capita consumption of domestic goods by young and old foreign households increase ceteris paribus with a rising exchange rate (see budget constraints (8.1)). On the other hand, Home’s imported quantities which are equal to the sum of per-capita

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31 For ease of exposition we will assume in this subsection that public bonds per capita in Home vanish. The numerical calculation shows that the qualitative results obtained in this subsections hold mutatis mutandis in the general case of a non-vanishing domestic public debt per capita.
consumptions of foreign goods by young and old households in both regions of Home decrease with a rising real exchange rate (see the optimal consumption functions in the appendix). However, the real value of Home’s imports will not necessarily decrease since the rise of the real exchange rate might exceed the decline of imported quantities in response to a larger exchange rate. This is certainly not the case in our model because the export and import functions are unit-elastic and hence the Marshall-Lerner conditions apply. In sum, the current account equilibrium disturbed by the enlargement shock is restored by an upward jumping real exchange rate.

**Home’s regional convergence and the transitional dynamics of the world economy**

This section focuses on the description of the convergence effects after the enlargement period as obtained by the numerical calculations. While the economic integration of region $R_2$ into the world economy is achieved within the enlargement period, the convergence process within Home is perfectly completed only after 17 periods. In practice, the convergence is roughly completed after 4-5 periods (approximately 80-100 years). Convergence proceeds monotonically (see Figure 2).

![Figure 2](about here)

The international effects of the convergence process are reflected by the transitional dynamics of the real exchange rate after its instantaneous upward jump and by the dynamics of private and public capital intensities in Home and Foreign. We start with an analysis of the exchange rate dynamics.

To explain the increase of the exchange rate in the period after the enlargement, consider the dynamic version of the international interest parity condition (25):

$$ e_{i+1} = e_i \left[ \frac{(1-\varphi)}{(1-\varphi')} \right] \left( g_{k_{i+1}} \right)^{\alpha_{i+1}} \left( k_{i+1} \right)^{\alpha_{i+1}} + \left( g^* \right)^{\alpha_{i+1}} \left( k^*_{i+1} \right)^{\alpha_{i+1}}. $$

$g_{k_{i+1}}$ does not respond to the enlargement shock. The private capital intensities $k_{i+1}$ and $k^*_{i+1}$ respond to the enlargement shock directly (under the proviso of a fixed real exchange rate) as well as indirectly when the exchange rate $e_i$ reacts to the enlargement shock. As regards the direct response, the capital intensity $k_{i+1}$ increases since in the domestic product market (see equation (21)) the supply increase (by the number of suppliers from region $R_2$) is always larger than the demand increase (by the number of demanders from region $R_2$), a consequence of the fact that a portion of region’s $R_2$ product (income) must be left over for the purchase of foreign commodities. The direct response of the foreign capital intensity $k^*_{i+1}$ is a strong decrease since foreign private capital is crowded out by the larger number of Home consumers of foreign goods. When the exchange rate increases the private capital intensity in Home $k_{i+1}$ decreases, while the foreign capital intensity increases. The former reaction follows from the rise of Home’s exports in response to the real depreciation which crowds out private capital, the latter follows from crowding in of private capital through declining foreign exports. The effects of the exchange rate increase on the capital intensities reverse the immediate adaptations of the capital intensities to the enlargement shock. Summing up the effects of the enlargement shock as well as of the exchange rate increase on the capital intensities, a decline of
both capital intensities can be numerically stated. The decrease of the capital intensity of the domestic region $R1$ is larger than the decrease of the foreign capital intensity.

From this, it follows that in the first period after the enlargement:

$$\left[\frac{1 - \varphi}{1 - \varphi^*}\right] \left[(g_{t_0})^\gamma (k_{t_0})^{\varphi-1}\right] / \left[(g^*)^\gamma (k^*)^{\varphi-1}\right] > 1.$$ 

Therefore, $e_{n+1}$ is in fact larger than $e_n$. The exchange rate after period $t_0+1$, keeps on growing monotonically towards the steady state of full convergence as illustrated in figure 3.

In period $t_0+1$, the (per-capita) product of region $R1$, $x_{n+1}$, is also lower. On the other hand, the wage tax rate increases. As a consequence, the disposable incomes of young households decrease. The consumption demand of young as well as old households in region $R1$ decreases. The same holds true for Home savings. Similar considerations apply to the foreign economy, albeit with changes of much smaller magnitude. In the converging region $R2$, private and public capital intensities are higher than in the enlargement period, allowing higher (per-capita) output levels. This fact reverberates positively on the wage income, letting consumption and savings increase accordingly.

In the product market in Foreign, the overall change of the aggregate consumption demand is, in general, indeterminate. The numerical calculations show that it declines. The aggregate supply in Foreign decreases as well. The private investment demand also declines because the supply decrease exceeds the demand decrease.

In the product market in Home, the overall supply further increase due to the strong increase of the per-capita output in region $R2$. On the demand side, in contrast to the foreign product market, the aggregate consumption demand rises. In addition, public and private investment expenditures in region $R2$ also increase. Since the rise of the aggregate supply is larger than the demand increase, private capital investment in region $R1$ rises. Herewith, the initial decline of the private capital intensity in region $R1$ is reversed.

Beginning with period $t_0 + 2$ ($t_0 + 3$ for the net debtor case), the private capital intensity in $R1$ keeps on growing monotonically towards the steady state (see figure 4).

32 As equation (26) shows, the lower per-capita income has multiple, counteracting effects on the wage tax rate. From the numerical analysis it emerges that the overall effect is negative.
The dynamics of the private capital intensity in region $R_2$ follows the law of motion of both the private capital intensity in region $R_1$ and of the convergence indicator (see equation (17)). However, in contrast to the capital intensity in region $R_1$, there is no decline of $R_2$ capital intensity in $t_0+1$ because the increase of the convergence indicator exceeds the decline of the capital intensity in region $R_1$. Hence, the private capital intensity in $R_2$ increases monotonically from the beginning towards the steady state (see figure 5).

Figure 5 about here

As evident from figure 5, two periods after enlargement, $R_2$ capital intensity has almost doubled, thus lowering marginal productivity and hence the real rate of return on capital in $R_2$, and decreasing the incentive to invest in region $R_2$.

The transition path of the capital intensity in Foreign is affected differently to that of the domestic region $R_1$. As mentioned above, the capital intensity in Foreign decreases less than in $R_1$, but the fall persists longer, as illustrated by figure 6. By the fourth period, foreign capital intensity starts growing again. The transition path of the foreign capital intensities is shown in figure 6.

Figure 6 about here

Conclusions

This paper presents an extension of Zee’s [1987] two-country OLG model by splitting up Home into two regions with initially hugely differing per-capita incomes and by introducing a two-region version of Barro’s [1990] endogenous growth engine. Although highly stylized, our model is still capable of accounting for the effects of ‘enlarging integration’ of a less-developed region and the effects of the subsequent convergence process on the economic relations between enlarged region (Home) and another large open economy named Foreign. The analysis focuses on the dynamics of the real exchange rate between Home and Foreign, the main competitiveness indicator of the enlarged Home.

The intertemporal equilibrium dynamics of the two-country, two-region model is analytically presented and solved for steady-state solutions. Due to the numerous dimension of the dynamic system, the existence, uniqueness and local stability of the steady state solutions are not investigated analytically, but numerically, using a plausible parameter set for the two-country, two-good OLG economy. Being assured of the existence of a saddle-path stable steady state, the long-run and transitional dynamics effects of the integration of the less developed region $R_2$ into the more developed two-country world economy are then algebraically and numerically explored.

In the long run, only the real exchange rate and the per-capita variables of the less developed region of Home are affected by ‘enlarging integration’, while the pre-integration Home and Foreign capital intensities remain unaltered. In the medium run, the enlargement brings about integration as well as convergence effects. Along the transition path of the convergence of the domestic regions the per-capita income of the less developed region $R_2$ increases...
monotonically from its initial level (whereby the income of region $R_2$ is supposed to be less than 50% of that of region $R_1$) to full convergence, which is fully completed after 17 periods, with most of the convergence being achieved 4-5 periods after the enlargement.

The real exchange rate between Home and Foreign, being the sole jump variable of the dynamic system, adapts to the shock of integrating a $\xi$ percentage of region $R_1$’s population into Home by rising significantly. The exchange rate continues to rise during the entire catching-up process of region $R_2$.

Private capital intensity in region $R_1$ experiences a sharp decline in the first period after the shock and starts increasing from the second period onwards. $R_2$’s capital intensity faces a monotonic upward trend over the whole convergence period, until meeting the long-term level of region $R_1$. In Foreign, capital intensity initially decreases slightly but for a longer time span. Afterwards, it starts increasing again towards its steady-state level.

The complexity of the theoretical framework presented in this paper prevented a thorough analytical solution. On the other hand, to perform a full-blown CGE analysis the theoretical structure (utility and production functions) is too restrictive. In addition, we did not calibrate the model parameters to reliable empirical data. This sets the agenda for future research: (1) Development of an analytically tractable version of the above two-country, two region model; (2) Specification of a two-country, two region CGE model with e.g. CES utility and production function and calibration of the parameters of the CGE model to empirical data sets.

References


Appendix

Optimal consumption and savings of domestic households

\[ x_{i}^{t} = \left[ \zeta / (1 + \beta \tau_i) \right] \left[ (1 - \tau_i) w_i^{t} + z_i^{t} \right] \]  
(A.1)

\[ y_{i}^{t} = \left[ (1 - \zeta)/ (1 + \beta \tau_i) \right] \left[ (1 - \tau_i) w_i^{t} + z_i^{t} \right] \]  
(A.2)

\[ x_{i}^{R1,1} = \zeta / L_{i}^{R1} \left[ (1 - \phi) (K_{i}^{R1,R1} + K_{i}^{R2,R1}) q_{i}^{R1} + (1 + i_e) (B_{i}^{R1,R1} + B_{i}^{R2,R1}) + (1 + \zeta) e_i B_{i}^{R1,R1} \right] \]  
(A.3)

\[ x_{i}^{R2,2} = \zeta / L_{i}^{R2} \left[ (1 - \phi) (K_{i}^{R1,R2} + K_{i}^{R2,R2}) q_{i}^{R2} + (1 + i_e) (B_{i}^{R1,R2} + B_{i}^{R2,R2}) + (1 + \zeta) e_i B_{i}^{R2,R2} \right] \]  
(A.4)

\[ y_{i}^{R1,1} = (1 - \zeta)/ L_{i}^{R1} \left[ (1 - \phi) (K_{i}^{R1,R1} + K_{i}^{R2,R1}) q_{i}^{R1} / e_i + (1 + i_e) / e_i (B_{i}^{R1,R1} + B_{i}^{R2,R1}) + (1 + \zeta) e_i B_{i}^{R1,R1} \right] \]  
(A.5)

\[ y_{i}^{R2,2} = (1 - \zeta)/ L_{i}^{R2} \left[ (1 - \phi) (K_{i}^{R1,R2} + K_{i}^{R2,R2}) q_{i}^{R2} / e_i + (1 + i_e) / e_i (B_{i}^{R1,R2} + B_{i}^{R2,R2}) + (1 + \zeta) e_i B_{i}^{R2,R2} \right] \]  
(A.6)

\[ s_i^{t} = \sigma \left[ (1 - \tau_i) w_i^{t} + z_i^{t} \right] \]  
(A.7)

The KK-curve and the GG-curve

\[ e = -(1 + \zeta) \frac{k}{k - (A/G^{c})} \left[ 1 - \alpha(1 - \phi)(1 + b/k) - \gamma \Gamma \right] \left( g^{c} \right)^{(k - 1)}/(1 - \sigma b)^{1} \]  
(A.8)

\[ e = (1 + \zeta) \frac{k}{k - (A/G^{c})} \left[ 1 - \alpha(1 - \phi)(1 + b/k) - \gamma \Gamma \right] \left( g^{c} \right)^{(k - 1)}/(1 - \sigma b)^{1} \]  
(A.9)

with \( k = \left[ (1 - \phi)/(1 - \phi) \right]^{(n-1)}/(A^{c} \Gamma / G^{c})^{(n-1)}/(1 - \sigma b)^{1} \)

The equilibrium dynamics

Denote the four eigenvalues of the Jacobian \( J \) with elements \( j_{ij}, \ i, j = 1, \ldots, 5 \) which are less than unity by \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \). The equilibrium dynamics in the neighborhood of the steady state can be linearly approximated as follows:

\[ e_i = e + \kappa_1 \nu_i \left( \lambda_1 \right)^{\nu_i} + \kappa_2 \nu_i \left( \lambda_2 \right)^{\nu_i} + \kappa_3 \nu_i \left( \lambda_3 \right)^{\nu_i} + \kappa_4 \nu_i \left( \lambda_4 \right)^{\nu_i} \]  
(A.10)

\[ k_i = k + \kappa_1 \nu_i \left( \lambda_1 \right)^{\nu_i} + \kappa_2 \nu_i \left( \lambda_2 \right)^{\nu_i} + \kappa_3 \nu_i \left( \lambda_3 \right)^{\nu_i} + \kappa_4 \nu_i \left( \lambda_4 \right)^{\nu_i} \]  
(A.11)

\[ k_i^{*} = k^{*} + \kappa_1 \nu_i \left( \lambda_1 \right)^{\nu_i} + \kappa_2 \nu_i \left( \lambda_2 \right)^{\nu_i} + \kappa_3 \nu_i \left( \lambda_3 \right)^{\nu_i} + \kappa_4 \nu_i \left( \lambda_4 \right)^{\nu_i} \]  
(A.12)

\[ g_i = g + \kappa_1 \nu_i \left( \lambda_1 \right)^{\nu_i} + \kappa_2 \nu_i \left( \lambda_2 \right)^{\nu_i} + \kappa_3 \nu_i \left( \lambda_3 \right)^{\nu_i} + \kappa_4 \nu_i \left( \lambda_4 \right)^{\nu_i} \]  
(A.13)

\[ \theta_i = \theta + \kappa_1 \left( \lambda_1 \right)^{\nu_i} + \kappa_2 \left( \lambda_2 \right)^{\nu_i} + \kappa_3 \left( \lambda_3 \right)^{\nu_i} + \kappa_4 \left( \lambda_4 \right)^{\nu_i} \]  
(A.14)
In equations (A.10)-(A.14), the vectors \( \left( v_i^*, v_i^1, v_i^2, v_i^3, v_i^4, 1 \right) \), \( i = 1, 2, 3, 4 \) represent the transposes of the normalized eigenvectors associated with the stable eigenvalues \( \lambda_i, i = 1, \ldots, 4 \). As is well-known, the eigenvectors are calculated as follows:

\[
\begin{bmatrix}
 j_{i1} & j_{i2} & j_{i3} & j_{i4} & j_{i5} \\
 j_{i3} & j_{i3} & j_{i3} & j_{i3} & j_{i3} \\
 j_{i3} & j_{i3} & j_{i3} & j_{i3} & j_{i3} \\
 0 & 0 & j_{i3} & j_{i4} & 0 \\
 0 & 0 & 0 & 0 & j_{i5}
\end{bmatrix}
\begin{bmatrix}
 v_i^* \\
v_i^1 \\
v_i^2 \\
v_i^3 \\
v_i^4
\end{bmatrix} = \lambda_i
\begin{bmatrix}
 v_i^* \\
v_i^1 \\
v_i^2 \\
v_i^3 \\
v_i^4
\end{bmatrix}, \quad i = 1, \ldots, 4.
\] (A.15)

Elimination of \( \kappa \lambda_i^*, i = 1, \ldots, 4 \) from (A.10)-(A.13) and insertion of the results into (A.10) gives the equilibrium manifold of the real exchange rate (33) in \( \left( e, k_i^*, k_i^1, g_i^*, \theta^* \right) \)-space. The constants \( \rho^i, \rho^*, \rho^\circ, \rho^\circ \) represent combinations of the eigenvectors obtained from solving (A.15). The constants \( \kappa_i \) are determined from (A.11)-(A.14) evaluated at \( t = 0 \).

Tables

Tables to be placed in the text

**TABLE 1**

The typical parameter set

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Population and preference parameters</strong></td>
<td></td>
<td><strong>Production parameters</strong></td>
<td></td>
</tr>
<tr>
<td>population growth factor</td>
<td>( G^\circ )</td>
<td>level parameter of total factor productivity</td>
<td>( A )</td>
</tr>
<tr>
<td>time preference factor</td>
<td>( \beta )</td>
<td>production elasticity of private capital</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>expenditure share of domestic</td>
<td>( \zeta )</td>
<td>production elasticity of public capital</td>
<td>( \eta )</td>
</tr>
<tr>
<td>consumption</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fiscal policy parameters - Home</strong></td>
<td></td>
<td><strong>Fiscal policy parameters - Foreign</strong></td>
<td></td>
</tr>
<tr>
<td>capital income tax rate</td>
<td>( \varphi )</td>
<td>capital income tax rate</td>
<td>( \varphi^* )</td>
</tr>
<tr>
<td>public expenditures as share of GDP</td>
<td>( \Gamma )</td>
<td>public expenditures as share of GDP</td>
<td>( \Gamma^* )</td>
</tr>
<tr>
<td>public investment as share of total</td>
<td>( \gamma )</td>
<td>per capita stock of public capital</td>
<td>( g^* )</td>
</tr>
<tr>
<td>public expenditures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>per-capita bond stock</td>
<td>( b )</td>
<td>per-capita bond stock</td>
<td>( b^* )</td>
</tr>
</tbody>
</table>

°Home is a net creditor; * Home is a net debtor.
FIGURE 1a
Home is net creditor

FIGURE 1b
Home is net debtor

FIGURE 2
The dynamics of the convergence indicator
FIGURE 3
The dynamics of the real exchange rate

FIGURE 4
The dynamics of private capital intensity in region R1
**FIGURE 5**
The dynamics of private capital intensity in region R2

![Graph showing the dynamics of private capital intensity in region R2](image)

**FIGURE 6**
The dynamics of private capital intensity in Foreign

![Graph showing the dynamics of private capital intensity in Foreign](image)