Competition for firms in an oligopolistic industry:
Do firms or countries have to pay?*

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1 Introduction

There are two strands in the literature. One focuses on the interests that potential host countries have in attracting internationally mobile firms. Reasons why countries (or states) may be willing to pay subsidies to firms are scale economies in the provision of public goods and services, in conjunction with a mobile workforce (Black and Hoyt, 1989), the existence of unemployment in the host jurisdiction (Haaparanta, 1996; Barros and Cabral, 2000) or savings in transportation costs (Haufler and Wooton, 1999). On the other hand, firms may earn location rents in a particular jurisdiction. These rents can then be taxed by the host country without causing the firm to leave the country. This line of argument is stressed in the new economic geography literature, where location rents for firms arise from industry agglomeration (Kind et al., 2000; Baldwin and Krugman, 2004; Borck and Pfüger, 2006). This literature suggests that competition between countries may lead to a ‘race to the top’, rather than a ‘race to the bottom’. The incentive for governments to raise taxes are particularly strong when the profits accrue to foreigners (Huizinga and Nielsen, 1997; Fuest, 2005). In contrast, when all rents are earned by domestic residents, then equilibrium taxes will typically be negative, despite the existence of location rents (Ottaviano and van Ypersele, 2005).

In this paper we combine these two strands in the literature by combining the desire of national governments to attract internationally mobile firms with a location rent of firms that can be taxed by host governments. The incentive to tax profits arises in our model because profits either accrue to foreigners, or because governments discount profits accruing to domestic capitalists by a sufficiently high factor. A special feature of our model is that location rents accrue to firms, even if countries are fully symmetric. Intuitively location rents arise because a firm that moves to the other country will face more competitors, and therefore a lower mark-up, in this market. Our analysis builds on Ferrett and Wooton (2005), who analyse tax competition for a duopoly.

1 Location rents are further increased when firms are heterogeneous. See Burbidge et al. (2006).

2 This contrasts with models where asymmetries between countries are the core reason for location rents. Such rents can arise from differences in country size (Haufler and Wooton, 1999), technology differences between countries (Fumagalli, 2003), or different market structures in the two countries (Bjorvatn and Eckel, 2006).
we extend this setting by considering an oligopolistic industry with \( k \) identical firms. Importantly, the analysis of a generalised oligopoly framework allows us to use standard methods of calculus, rather than compare discrete locational equilibria.

\section{The model}

\subsection{Consumers}

We consider two countries \( i \in \{a, b\} \), which compete for the attraction of a fixed number of firms. These firms produce a homogeneous good, labelled \( x \), in an oligopolistic industry. A second private good, the numeraire commodity \( z \), is produced under conditions of perfect competition. Consumers in both countries have the same preferences for the two goods, given by

\[ u_i = \alpha x_i - \frac{\beta}{2} x_i^2 + z \quad \forall i \in \{a, b\}. \tag{1} \]

The two countries potentially differ in size. The world population is normalised to unity and, without loss of generality, we take country \( a \) to be the larger of the two countries. Hence there are \( n \geq 0.5 \) consumers in country \( a \) and \( (1 - n) \) consumers in country \( b \). Each household exogenously supplies one unit of labour. In each country the wage rate \( w \) is fixed in the numeraire industry, which uses labour as the only input. Free trade in the numeraire good will then equalise wages across countries. Moreover, total income from the business tax (as detailed below), denoted by \( T_i \), is redistributed equally and in a lump-sum fashion to the consumers in each country. The budget constraint for a representative consumer in each of the two countries is then

\[ w + \frac{T_a}{n} = z + p_a x_a, \quad w + \frac{T_b}{1-n} = z + p_b x_b, \tag{2} \]

where \( p_h \) is the price of good \( x \) in country \( h \). Utility maximization leads to the inverse demand curves

\[ \alpha - \beta x_i = p_i \quad \forall i. \tag{3} \]

Aggregating the demand for good \( x \) over all consumers yields market demand curves in each country, denoted by \( X_i \)

\[ X_a = \frac{n(\alpha - p_a)}{\beta}, \quad X_b = \frac{(1-n)(\alpha - p_b)}{\beta}. \tag{4} \]
Hence, the market demand curve of the larger country \( a \) is flatter than that of country \( b \). In this sense market \( a \) is the more profitable location for firms, as we will see below.

### 2.2 Firms

Each firm in the imperfectly competitive industry \( x \) requires one unit of capital to produce any output. There is a total of \( k \) units of capital in the world economy, implying that a maximum of \( k \) firms can engage in production.\(^3\) These firms are assumed to be identical except with respect to the location of their production facilities. Location matters because, while all firms can sell their products in both countries, there are trade costs associated with exports to a firm’s foreign market. Thus each country may be served by both “local” firms that produce domestically and “foreign” firms that are based in the other country.

We assume that labour is the only variable input with marginal cost of \( w \).\(^4\) The cost of exporting each unit of output is \( \tau \), which effectively raises the marginal cost of serving the foreign market to \((w + \tau)\). Firms are assumed to behave as Cournot competitors and are able to segment their market, choosing the quantities to sell on their domestic and export markets independently.\(^5\)

Capital owners seek to maximise the total profits of each firm (which equal the return to capital) by choosing output in each market:

\[
\begin{align*}
\pi_a &= (p_a - w) x_{aa} + (p_b - w - \tau) x_{ba}, \\
\pi_b &= (p_a - w - \tau) x_{ab} + (p_b - w) x_{bb},
\end{align*}
\]

where \( \pi_j \) are the pre-tax profits of a firm based in country \( j \) and \( x_{ij} \) represents sales in country \( i \) by a firm based in \( j \), \((i, j \in \{a, b\})\). Given that the marginal cost of exports

\(^3\)The capital owners can either reside in one of the two countries \( a \) and \( b \), or in a third country. There is no need to specify the ownership pattern in our model, as the firms’ profits do not enter the government objective function (cf. section 2.3).

\(^4\)Since wage costs are equalised between the two countries, they do not enter the location decision of firms in our model. For a recent analysis on tax competition for foreign direct investment which focuses on production cost differentials, see Davies (2005).

\(^5\)In equilibrium, firms will receive a lower producer price for their exports than for goods destined for the domestic market. The trade structure is simply a generalisation of the “reciprocal dumping” model of Brander and Krugman (1983).
is relatively higher than for domestic sales, a firm’s perceived marginal revenue in its export market must be comparably larger. In equilibrium, this will arise when each foreign firm has a smaller market share than the share going to a domestic firm (as perceived marginal revenue is inversely related to market share). Essentially, a firm is at a cost disadvantage in its export markets and will sell less than an indigenous rival.

Suppose that \( m \) firms are located in country \( a \) and the remaining \((k-m)\) firms produce in country \( b \). Maximising (5) taking into account demand (4), yields firm output levels:

\[
x_{aa} = \frac{n[\alpha - w + (k - m) \tau]}{\beta (k + 1)}; \quad x_{ba} = \frac{(1-n)[\alpha - w - (1 + k - m) \tau]}{\beta (k + 1)};
\]

\[
x_{ab} = \frac{n[\alpha - w - (1 + m) \tau]}{\beta (k + 1)}; \quad x_{bb} = \frac{(1-n)[\alpha - w + m \tau]}{\beta (k + 1)}.
\]

Note that \( x_{ab}/x_{aa}, x_{ba}/x_{bb} < 1 \), confirming the assertion that a foreign firm’s share of a market is always less than that of a local firm whenever there are trade costs \((\tau > 0)\).\(^6\)

The corresponding equilibrium market outputs and prices are:

\[
X_a = \frac{n[k(\alpha - w) - (k - m) \tau]}{\beta (k + 1)}; \quad X_b = \frac{(1-n)[k(\alpha - w) - m \tau]}{\beta (k + 1)};
\]

\[
p_a = \frac{\alpha + kw + k - m \tau}{k + 1}; \quad p_b = \frac{\alpha + kw + m \tau}{k + 1}.
\]

Notice that consumer prices in both countries fall when the total number of firms \( k \) increases and competition in the oligopolistic industry is thus intensified. Moreover, for any given level of \( k \), increasing \( m \), the number of firms that locates in country \( a \), reduces the consumer price of good \( x \) in country \( a \), but increase it in country \( b \). This is a consequence of the changes in the aggregate level of transportation costs that follows from the relocation of firms.

Substituting (6) and (7) into (5) yields the pre-tax profits arising to firms located in each country:

\[
\pi_a = \frac{n[\alpha - w + (k - m) \tau]^2}{\beta (k + 1)^2} + \frac{(1-n)[\alpha - w - (1 + k - m) \tau]^2}{\beta (k + 1)^2};
\]

\[
\pi_b = \frac{n[\alpha - w - (1 + m) \tau]^2}{\beta (k + 1)^2} + \frac{(1-n)[\alpha - w + m \tau]^2}{\beta (k + 1)^2}.
\]

\(^6\)As our model allows for differences in country size, this is not inconsistent with the possibility that a firm may sell more in its export market than at home.
We assume that profits are taxed at source by the host countries of the firms. Let $t_h$ be the lump-sum tax imposed on each firm by country $h$. The tax differential between countries is $\Delta \equiv t_a - t_b$. In deciding upon where to invest, firms will compare profits net of taxes and locate in the more profitable country. The locational equilibrium for the industry will be characterised by $\pi_a - t_a = \pi_b - t_b$. Substituting (8) gives the equilibrium number of firms to locate in country $a$:

$$m^* = \frac{k}{2} + \frac{(2n - 1) [2 (\alpha - w) - \tau]}{2 \tau} - \frac{\Delta \beta (k + 1)}{2 \tau^2}. \quad (9)$$

Suppose, initially, that each country charges the same tax, that is $\Delta = 0$. If the countries were the same size ($n = 0.5$), it is clear from (9) that $m^* = k/2$, the firms would be evenly split between the two locations. In the absence of trade costs, neither country has a locational advantage and $m$ is undefined. When trade is costly and country $a$ is relatively large ($n > 0.5$), it attracts more than half of the firms. Differences in taxes will further affect the location of firms such that, if country $a$ were to tax firms more heavily than country $b$ (that is, $\Delta > 0$), its share of the firms would fall.

How is the international distribution of firms affected by changes in relative country size, trade costs, and tax differences? Partial differentiation of (9) yields:

$$\frac{dm^*}{dn} = \frac{2 (\alpha - w) - \tau}{\tau} > 0;$$
$$\frac{dm^*}{d\tau} = -\frac{(2n - 1) (\alpha - w)}{\tau^2} + \frac{\Delta \beta (k + 1)}{\tau^3} \leq 0;$$
$$\frac{dm^*}{d\Delta} = \frac{dm^*}{dt_a} = \frac{-\beta (k + 1)}{2 \tau^2} = -\frac{dm^*}{dt_b} < 0. \quad (10)$$

Assume that profit taxes are the same in both countries ($\Delta = 0$) and that country $a$ is the larger country initially ($n > 0.5$). From (10), as the asymmetry in country size increases, the country $a$ will attract a greater share of the firms, while a rise in the trade cost increases the international dispersion of firms, moving some firms to the smaller country. When $\Delta > 0$, eq. (9) reveals a negative impact on $m^*$ of a relatively high

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7There is a general agreement in the literature that international company taxation closely follows the source principle. This principle applies directly, if countries avoid international double taxation by exempting foreign-earned income from domestic tax. If countries grant an international tax credit instead, source taxation is still effective in many cases, because crediting applies only after profits have been repatriated, and because countries do not rebate ‘excess’ taxes paid abroad.
tax in country $a$. However, (10) shows that this effect is mitigated both when $n$ and $\tau$ increase. This reinforces the concentration of industry in country $a$ resulting from increases in $n$ but makes the effect of increasing trade costs on firm location ambiguous. Lastly, increasing a country’s tax clearly creates a disincentive for firms to locate there.

2.3 Governments

The governments of countries $a$ and $b$ impose a lump-sum tax on each firm that locates within their respective jurisdiction. Importantly, these taxes can become negative and turn into location subsidies. If revenues from the business tax are positive, then these revenues are redistributed equally and in a lump-sum way to the residents in each country; whereas negative revenues from the business tax are covered by lump-sum taxes imposed on consumers. Moreover recall that, despite their lump-sum character, business taxes distort the location decision of internationally mobile firms [eq. (9)].

In addition to tax revenue, governments care about the total consumer surplus in their jurisdiction. Substituting the equilibrium market prices for good $x$ [see (7)] into the linear market demand curves (4), total consumer surplus, denoted $S_i$, in the two countries is given by

$$S_a = \frac{n[k(\alpha - w) - (k - m)\tau]^2}{2 : \beta(k + 1)^2}, \quad S_b = \frac{(1 - n)[k(\alpha - w) - m : \tau]^2}{2 : \beta(k + 1)^2}.$$  

(11)

It is straightforward to show that consumer surplus in both countries is rising in the total number of firms $k$, as this will intensify competition and reduce producer prices in both countries [cf. eq. (7)]. Moreover, a rise in $m$, which increases the share of firms that is located in country $a$, raises consumer surplus in this country, but lowers it in country $b$. This effect derives from changes in the aggregate level of transport costs and gives each country an incentive to attract firms to its home jurisdiction.

We assume that the objective of each government is to maximise the sum of consumer surplus and tax revenue collected from locally producing firms. In contrast, firms’ profits do not enter the government objective. One interpretation for this last assumption is that all firms are owned by residents of a third country. In this case our specification implies that the governments of countries $a$ and $b$ maximise aggregate national welfare. Alternatively, profits accrue at least partly to domestic capital owners, but governments
put a very low weight on the welfare of capitalists, as opposed to consumer-workers. With these assumptions the objective functions of the two governments are given by

\[ W_i = S_i + T_i \quad \text{where} \quad T_a = t_a m \quad \text{and} \quad T_b = t_b (k - m). \]  \hfill (12)

We substitute (11) in (12) and differentiate with respect to \( t_i \), using the derivative properties of the function \( m(t_a, t_b) \) in (10). This gives the following first-order conditions for each country’s tax rate

\[ t^*_a = \frac{\tau \{ \tau m^* [2(k+1) - n] - nk : (\alpha - w - \tau) \}}{\beta(k+1)^2} \],

\[ t^*_b = \frac{\tau \{ \tau : (k - m^*)[2(k+1) - (1 - n)] - k : (\alpha - w - \tau) \}}{\beta(k+1)^2}. \]  \hfill (13)

To arrive at reduced-form expressions for the Nash equilibrium tax rates the division of firms across the two countries, as given by \( m^* \) [see eq. (9)] must still be substituted into the set of first-order conditions (13). To develop an intuition for our results we will first perform this analysis in the benchmark case of symmetric countries \( (n = 1) \). In a second step, we then move to the more general case where countries differ in size.

3 The benchmark: Symmetric countries

To analyze the case where countries are symmetric in all respects, we evaluate the first-order conditions for the two countries’ optimal tax rates (13) at \( n = 0.5 \) and substitute the equilibrium allocation of firms (9), again using this restriction. This yields symmetric best response function for the two countries

\[ t^*_i(t_j) = \frac{k : \tau \{ 4\tau : (k + 1) - [2(\alpha - w) - \tau] \}}{\beta(k+1)[8(k+1) - 1]} + t_j \frac{[4(k+1) - 1]}{[8(k+1) - 1]} \quad \forall \, i, j, \, i \neq j. \]  \hfill (14)

Best response functions are positive sloped, implying that an increase in one country’s business tax rate will also raise that of the other country.

Solving the set of simultaneous equations yields closed-form solutions for the common Nash equilibrium tax rate \( t^s \), where the superscript \( s \) stands for the symmetric case

\[ t^s_a = t^s_b = \frac{k : \tau^2}{\beta(k+1)} - \frac{k : \tau [2(\alpha - w) - \tau]}{4\beta(k+1)^2} \equiv t^s. \]  \hfill (15)

\(^8\)In this last interpretation it must also be assumed that there are only few capital owners in each country, which do not significantly affect the size of each country’s (working) population.
The equilibrium tax rates (15) reflect the two fundamental effects that are at work in the present model. The positive first term is due to the fact that each firm earns a location rent in a symmetric equilibrium, where both consumers and firms are evenly split between the two countries. Intuitively, starting from a symmetric equilibrium, moving one firm from country \( b \) to country \( a \) implies that each firm in country \( a \) will now have to export a larger fraction of its total sales, for which a lower price can be charged. The opposite change occurs in country \( b \). This increases the gross profits of each remaining firm in country \( b \) while reducing gross profits for each firm in country \( a \). The resulting profit differential represents a location rent that can be taxed by the two governments without causing firms to move to the other jurisdiction.\(^9\)

On the other hand, the negative second effect results from the desire of governments to attract firms in order to reduce consumer prices. When an additional firm enters the country consumer prices fall due to a reduced level of aggregate transportation costs that must be borne by consumers. This isolated effect gives each country an incentive to grant location subsidies to firms in equilibrium. Depending on which of the counteracting effects dominates, equilibrium taxes may therefore be either positive or negative. Note finally, that both of the effects described above result from the existence of transport costs and disappear when \( \tau = 0 \). In this case Nash equilibrium taxes are zero as both governments and firms are indifferent with respect to the equilibrium pattern of firm location.\(^10\)

How are the symmetric Nash equilibrium tax rates affected by a change in transport costs? Differentiating (15) with respect to \( \tau \) gives

\[
\frac{\partial t^s}{\partial \tau} = \frac{k[4\tau(k + 1) - (\alpha - w - \tau)]}{2\beta(k + 1)^2} \quad (16)
\]

which may be positive or negative, in general. However, a comparison of (15) and (16) shows that if the tax is positive in the initial equilibrium, then it must further rise in response to an increase in transport costs. Turning this result around, economic

\(^9\)Algebraically, this effect can be shown by differentiating the gross profits that each firm earns in a symmetric equilibrium [eq. (8)] with respect to \( m \). The resulting profit differential between a firm that is located in country \( b \) and one that is located in \( a \) is just equal to the first term in (15).

\(^10\)This special case is related to the analysis by Janeba (1998), who introduces firm mobility to the standard model of strategic tax policy but does not include transport costs. The core result of his analysis is that equilibrium taxes will be zero in both countries in this case.
integration, as commonly interpreted by a reduction in transport costs, will reduce the ability of symmetric countries to raise positive business taxes in equilibrium. In the opposite case where countries are willing to subsidize firms, the effects of reduced transport costs are ambiguous, even in the benchmark case of symmetric countries. This asymmetry is due to the fact that firm’s location rents, and hence the ability of governments to tax these rents, are rising more steeply in $\tau$ than the increase in consumer surplus that can be achieved by attracting an additional firm.

In contrast, raising the total number of firms in the economy will unambiguously raise the Nash equilibrium tax rates. This is seen from

$$\frac{\partial t^*}{\partial k} = \frac{\tau \{4\tau (k+1) + (k-1) : [2(\alpha - w) - \tau]\}}{4\beta (k+1)^3} > 0.$$ \hspace{1cm} (17)

Intuitively, an increase in the total number of firms lowers the costs that are perceived by each country from losing one of the firms to the other region. This changes the trade-off faced by each government between rent extraction and low consumer prices in the direction of higher business taxes.

Note, however, that taxes can only be raised as long as post-tax profits remain non-negative; otherwise firms would not produce at all. To derive this constraint for tax policy, we subtract the tax rate in the symmetric Nash equilibrium (15) from the firms’ pre-tax profit expression (8). Setting the difference equal to zero and solving for the critical value of trade costs that is compatible with non-negative after-tax profits yields

$$\tau \leq \bar{\tau} = \frac{(\alpha - w) [k - 2 + \sqrt{13k^2 + 8k - 4}]}{(3k^2 + 3k - 2)}.$$ \hspace{1cm} (18)

It is easily checked that equation (18) defines a negative relationship between the total number of firms in the market, and the critical (maximum) level of transport costs $\bar{\tau}$. For any given level of $(\alpha - w)$, a high level of $k$ reduces gross profits that can be earned by each firm in equilibrium, whereas a high level of $\tau$ increases the ability of a country to set higher taxes when the latter are positive in the initial equilibrium.\(^{11}\)

Finally, we derive a condition under which positive levels of trade will occur in equilibrium. With symmetry, this condition is the same for firms located in either country $a$

\(^{11}\)Note that the constraint (18) can only be binding when business taxes are positive in equilibrium. In this case we know from (16) that $\partial t^*/\partial \tau > 0$ must hold.
or b. From (8) and using \( m = k/2 \), exporting to the other market is profitable as long as

\[
\tau < \tau^{NT} = \frac{2(\alpha - w)}{2 + k}.
\]

(19)

If \( \tau > \tau^{NT} \), then there will be one firm in each market in equilibrium, which does not export to the respective other country. Comparing the critical values in (18) and (19) shows that \( \bar{\tau} \leq \tau^{NT} \) for all \( k \geq 2 \). Hence, apart from the case of a monopoly firm, eq. (18) is the binding constraint in our model. When this constraint is met, and after-tax profits are non-negative, then there will also be international trade in equilibrium.

### 4 Differences in country size

Having discussed the basic working of our model in the special case of symmetric countries, we now turn to the general case where countries differ in size. Substituting \( m \) from (9) into (13) we first obtain the two countries’ best response functions, which are relegated to the appendix [see eq. (A.1)]. From these we obtain closed-form solutions for the asymmetric Nash equilibrium tax rates:

\[
t^*_a = \mu + \frac{\tau \mu(2n - 1)(3k + 2)[2(k + 1) - n]}{2\beta : (k + 1)^2 : [6(k + 1) - 1]} \equiv \mu + \varepsilon_a,
\]

\[
t^*_b = \mu - \frac{\tau \mu(2n - 1)[2(k + 1) + n(3k + 2)]}{2\beta : (k + 1)^2 : [6(k + 1) - 1]} \equiv \mu + \varepsilon_b,
\]

(20)

where \( \mu \equiv [2(\alpha - w) - \tau] > 0 \) and \( \varepsilon_a > 0 > \varepsilon_b \).

The first term, \( \mu \), in (20) is common to the larger and the smaller country. This term is identical to \( t^s \) for the case of symmetric countries.\(^{12}\) The remaining terms describe how the overall “bargaining position” of countries vis-a-vis individual firms is modified by the existence of differences in country size. These terms are of opposite signs in the two countries, raising the tax rate in the larger country \( a \) and lowering it in the smaller country \( b \). Hence it can be directly inferred from (20) that the larger country levies the higher tax rate in the asymmetric Nash equilibrium. The equilibrium tax differential is given by

\[
\Delta^s \equiv t^*_a - t^*_b = \frac{3\tau(2n - 1)\mu}{\beta[6(k + 1) - 1]}.
\]

(21)

\(^{12}\)It is easily checked that these two terms reduce to eq. (15) for the symmetric case \( n = 0.5 \).
Equation (21) shows that the international tax differential is rising in the size difference between the two countries, but falls when the number of firms $k$ is increased. Moreover, the tax differential is unambiguously increasing in the transport cost parameter $\tau$:

$$\frac{\partial \Delta^*}{\partial \tau} = \frac{6(2n-1)(\alpha - w - \tau)}{\beta[6(k+1)-1]} > 0. \tag{22}$$

The intuition for these results is that a rise in $k$ reduces the profits of firms and hence also the location rent in the larger country. The same is true when $\tau$ is reduced and location accordingly matters less. To look more closely at the effects of economic integration (a reduction in $\tau$), differentiating the tax levels in each country [eq. (20)] and re-substituting their values in the initial equilibrium yields

$$\frac{\partial t^*_i}{\partial \tau} = \frac{2(\alpha - w - \tau)}{2(\alpha - w) - \tau} t^*_i + \frac{2(\alpha - w)}{\tau(\alpha - w - \tau)} \quad \forall \; i \in \{a,b\}. \tag{23}$$

Hence, for either of the two countries, a positive tax rate in the initial equilibrium is a sufficient condition (but not a necessary one) to ensure that a fall in transport costs reduces the optimal tax rate on firms. We have already obtained this result for the symmetric Nash equilibrium [eq. (16)], and it is now seen to carry over to the more general case with differences in country size.

## 5 Policy coordination

How does the outcome of asymmetric tax competition compare with a situation where the two countries are able to coordinate their policies? To answer this question we assume that a central planner in the region can decide on the optimal coordinated tax rates in each of countries $a$ and $b$, and can also allocate the $k$ firms to one of the two countries. The central planner therefore maximises the regional welfare function

$$W_a + W_b = S_a + S_b + t_a m + t_b (k - m) \tag{24}$$

This problem can be solved in two steps. For any given allocation of firms (i.e., for any given level of $m$), tax coordination will imply that each country fully taxes away the pre-tax profits of firms earned in its jurisdiction. As profits do not enter the countries’ objective function, this will maximise regional surplus in (24). At the same time this
situation constitutes an equilibrium as firms earn zero after-tax profits in both coun-
tries. Hence, given a pre-determined allocation of firms $m$, the coordinated tax rates are

$$t_a^c = \pi_a, \quad t_b^c = \pi_b$$  \hspace{1cm} (25)

where $\pi_a$ and $\pi_b$ are given in (8).

The result that countries can extract all pre-tax profits from firms when they coordinate their tax policies is not surprising in the present setting where firms cannot locate outside the region.\(^{13}\) However, it remains to determine which allocation of firms maximises regional surplus. Substituting the coordinated tax rates (25) [and hence $\pi_b$ from (8)] into the regional welfare expression (24) shows that the latter can equivalently be expressed as the sum of consumer surplus and profits. Maximising this expression with respect to $m$ yields the Pareto optimal allocation of firms (denoted by a superscript $PO$):

$$m^{PO} = \frac{k}{2} + \frac{(2n - 1)(k + 2)\mu}{2\tau(2k + 3)}.$$  \hspace{1cm} (26)

We can now address the question whether tax competition will lead to too much or too little concentration of firms in the larger country, relative to the Pareto optimal benchmark.\(^{14}\) Substituting (21) into (9) gives the allocation of firms in the Nash equilibrium (superscript $N$):

$$m^N = \frac{k}{2} + \frac{(2n - 1)(3k + 2)\mu}{2\tau[6(k + 1) - 1]}.$$  \hspace{1cm} (27)

Finally, setting $\Delta = 0$ in (9) gives the market equilibrium allocation of firms in the absence of taxes (superscript $ME$):

$$m^{ME} = \frac{k}{2} + \frac{(2n - 1)\mu}{2\tau}.$$  \hspace{1cm} (28)

Comparing (26)–(28) shows that $m^{ME} > m^{PO} > m^N$. Hence the market equilibrium in the absence of taxes features too much agglomeration of firms in the larger market, relative to the Pareto optimal benchmark, whereas the equilibrium with tax competition exhibits a suboptimally low degree of agglomeration. Hence, starting from the market equilibrium, tax competition acts as a strong ‘dispersion force’ that leads to an ‘undershooting’ of the Pareto efficient allocation of firms.

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\(^{13}\)See Haufler and Wooton (2006) for an analysis of regional tax coordination when firms can also locate outside the union.

\(^{14}\)Recall, however, that not only $m$, but also $t_b$ changes in this comparison.
6 Discussion

Our analysis can be related to previous work that has considered specific market structures of monopoly (Haufler and Wooton, 1999) and duopoly (Ferrett and Wooton, 2005). Note, however, that our use of calculus in deriving optimal tax rates implicitly assumes that the number of firms changes continuously in response to tax changes. This assumption is not correct when the number of firms is small so that there is a discrete change in the equilibrium allocation when taxes reach a critical threshold. These discrete changes in equilibrium are analysed in detail in the above-mentioned literature. Nevertheless, our analysis reaches qualitatively similar results and can therefore be seen as an approximation of the ‘true’ Nash equilibria even when the number of firms is small.

The monopoly case is particularly simple because, with symmetric countries, neither country has the ability to extract rents from the monopolist and, in equilibrium, both countries will always offer a location subsidy to the monopolist. The same is also true in our model. If $k = 1$ the symmetric Nash equilibrium tax in (15) reduces to $t^*_a = t^*_b = -\frac{\tau \mu}{8\beta} < 0$. If countries differ in size, then the firm obtains a rent from locating in the larger country, and this rent can be taxed. As a result, the larger country will levy the higher tax rate and the equilibrium tax may be positive tax rate, if the size differential is sufficiently large (Haufler and Wooton, 1999). Inspection of (20) shows that the same is true in the general (asymmetric) case of the present model.

In the duopoly case, in contrast, the rent extraction effect is strong because, starting from an equilibrium where one firm is located in each market, competition between firms increases discretely when one of the firms is driven to the other market. This effect is the stronger, the more markets are effectively separated by high transport costs. Hence, Ferrett and Wooton (2005) obtain the result that for sufficiently high transport costs, both countries will be able to set positive taxes in a symmetric equilibrium, which completely exhaust the firms’ pre-tax profits. In the present model, setting $k = 2$ in (15) the symmetric Nash equilibrium taxes for the duopoly case reduce to $t^*_a = t^*_b = \frac{\tau (12\tau - \mu)}{9\beta}$, which can clearly be positive for sufficiently high levels of $\tau$. Setting $k = 2$ in (18) gives a critical level $\bar{\tau} = (\alpha - w)/2$, where all pre-tax profits of firms are taxed away in equilibrium. In our analysis, this critical level will just eliminate all trade
when there are only two firms [cf. eq. (19)]. For a larger number of firms, however, a comparison of (18) and (19) shows that a symmetric equilibrium where international trade occurs but all pre-tax profits are taxed away by host governments is a possible outcome in our model.

Our results for the agglomeration of firms in section 5 coincide with those in Ottaviano and van Ypersele (2005), even though equilibrium tax rates are negative in their analysis. The role of tax competition as a ‘dispersion force’ is also known from Andersson and Forslid (2003). (...)

**Appendix**

**Derivation of equation set (20)**

Substituting $m$ from (9) into (13) and rearranging gives each country’s best response function

$$t_a = \frac{2\tau^2k}{\beta [4(k + 1) - n]} + \frac{\tau \mu \{(2n - 1)[2(k + 1) - n] - nk\}}{\beta(k + 1)[4(k + 1) - n]} + t_b \frac{2(k + 1) - n}{4(k + 1) - n},$$

$$t_b = \frac{2\tau^2k}{\beta [4(k + 1) - u]} - \frac{\tau \mu \{(2n - 1)[2(k + 1) - u] + uk\}}{\beta(k + 1)[4(k + 1) - u]} + t_a \frac{2(k + 1) - u}{4(k + 1) - u} \tag{A.1}$$

where $\mu \equiv [2(\alpha - w) - \tau] > 0$ and $u \equiv (1 - n)$. Solving the set of simultaneous equations (A.1) yields the reduced-form expressions for the Nash equilibrium taxes in the asymmetric equilibrium, given in (20).
References


