How to improve international competitiveness of Portuguese economy*

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1 Introduction

Last developments of European process of integration, as the enlargement to east, created new questions related to the Portuguese economy performance and is future development. Many politicians have been saying that Portuguese firms must be more competitive in international markets, especially now that new member-states have the same accessibility to the European market than Portugal and some advantages on the attraction of new investments. It has been said also that the improvement of Portuguese economy is only possible if labour become more productive. Thus, the improvement of productivity in Portugal is being considered one important key issue to future success of the economy.

In this work we do not pretend to study how to improve the productivity, or which policy should be adopted by government, or even which incentive should be given to Portuguese firms. Instead, it is our aims to identify the type of labour (skilled or unskilled) which productivity should be improved and in which sectors should that happen.

The importance of these questions relates to the costs of productivity improvements and also to the diverse importance of different sectors on exports. Thus, it is expected that the same increase in productivity, but in different sectors, will lead to different effects on the improvement of exports and trade balance.

2 Static General Equilibrium Model for Portugal

The aim of this framework is to model the Portuguese economy. To this end we consider 5 agents and 2 markets. Foreign currencies are not considered because data are expressed in the same monetary unit.

<table>
<thead>
<tr>
<th>Agents</th>
<th>Markets</th>
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<tbody>
<tr>
<td>Families</td>
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<td>Firms</td>
<td>Factors</td>
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<td>Banks (Investment)</td>
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<tr>
<td>Government</td>
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<tr>
<td>Rest of the World</td>
<td></td>
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The adopted aggregation used for sectors is that used in Brucker (1998), in his classification of competitiveness factors.\footnote{See appendix A with the relationship between the GTAP Data Base and the Brucker (1998) proposal.}

Table 2: Sectors of activity
In geographical terms four regions are considered. Portugal plus three regions with which our economy has commercial relationships - EU 14 (older member countries, excluding Portugal)\(^2\), EU10 (the newly acceded member countries)\(^3\) and ROW (the Rest Of the World).

For reasons of technical simplification, it is considered that Portugal, EU14, EU10 and ROW, have the same behaviour structure, therefore presenting identical functions for all agents and markets. However, since the initial statistical data are obviously distinct, both the calibrated parameters and the effects from a similar economic policy in Portugal will be different in each region.

Given the current scenario of increasing globalisation of economic activities, and knowing that a significant part of production and trade within the EU is controlled by multinational corporations, it is acceptable that this simplification, though a source of bias in the results, is not considered excessively limiting. It is also evident the increased similarity of consumption patterns in the different EU countries. Therefore, it is believed that considering the differences in production patterns among regions as more related to production specialisation and economic rationalisation, rather than to different behaviour patterns, will not, by itself, diminish the interest of the results obtained with our model.

### 2.1 Firm Behaviour and Foreign Trade

The productive sector in this model of the Portuguese economy is characterised by the existence of 6 profit maximiser sectors that produce 6 goods and supply in accordance to a nested production function with capital (a composite factor that aggregates capital, natural resources and land), labour (qualified and non-qualified), and intermediate goods (also a composite good). At the first level a Leontief technology is used with the added value and intermediate goods as factors of production. At the second level we have, on the one hand, the added value as a CES (Constant Elasticity of Substitution) function with constant returns to scale, with capital and labour as factors of production, and, on the other hand, the intermediate goods as a Leontief technology function.\(^4\)

The elasticities of substitution between capital and labour, between domestic production for domestic market and exports, and between domestic production for domestic market and imports are exogenous in the model.

---

\(2\)Germany, Austria, Belgium, Denmark, Spain, Finland, France, Greece, Netherlands, Italy, Ireland, Luxembourg, United Kingdom, Sweden.

\(3\)Cyprus, Slovenia, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Czech Republic, Slovakia.

\(4\)See appendix D with the productive structure of the economy.
Returns on capital and wages are equal across sectors since it is considered that there is perfect domestic factors’ mobility.

Firms pay taxes for the use of resources (capital and labour) as well as for the use of intermediate goods.

The behaviour of each firm may be generalised in two groups of decisions on how and how much to produce. In the first group the producer should choose the optimal combination of primary and intermediate resources that are needed to produce, i.e. the best way of obtaining goods or services. At the second group the agent’s decisions determine how much will be distributed in the domestic market along with imported goods, and how much will go to the foreign market, i.e. the optimum level of production.

2.1.1 How to Produce?

At a first level the firm chooses the basket of intermediate goods and the basket of primary factors by means of a Leontief production function. This type of function assumes that the production of each sector is done with minimum fixed amounts from the baskets of intermediate goods and factors of production, i.e. fixed coefficients. This mean that it is not possible the substitution between them, i.e. is not possible to produce only with intermediate goods or only with primary factors, since they are perfect complementary in this function.\(^5\)

At this level the generic production function \((g)\), where \(XD_{\text{reg,sec}}\) is the domestic production, \(VA_{\text{reg,sec}}\) is the value added by each sector (or the basket of capital and labour necessary in each sector) and \(IO_{\text{reg,sec}}\) is the basket of intermediate goods used by each sector.\(^6\)

\[
XD_{\text{reg,sec}} = g(VA_{\text{reg,sec}}, IO_{\text{reg,sec}}) \tag{1}
\]

Both \(VA_{\text{reg,sec}}\) and \(IO_{\text{reg,sec}}\) are fixed parts of \(XD_{\text{reg,sec}}\),

\[
VA_{\text{reg,sec}} = b_{\text{reg,sec}} \times XD_{\text{reg,sec}} \tag{2}
\]

\[
IO_{\text{reg,sec}} = (1 - b_{\text{reg,sec}}) \times XD_{\text{reg,sec}} \tag{3}
\]

where \(b_{\text{reg,sec}}\) is the fixed coefficients that relates the basket of productive factors with the production, in each region. Therefore, \(XD_{\text{reg,sec}}\) can be written as

\[
XD_{\text{reg,sec}} = \min \left\{ \frac{1}{b_{\text{reg,sec}}} VA_{\text{reg,sec}}, \frac{1}{(1 - b_{\text{reg,sec}})} IO_{\text{reg,sec}} \right\} \tag{4}
\]

The cost minimisation implicit in the rational behaviour of the producer allows the presentation of the former equation in the following way,

\(^5\)See Silberberg and Suen (2001) for specific issues about Leontief and CES functions .
\(^6\)The subscript "reg" and "sec" means that the variable is disaggregated by regions and sectors.
\(^7\)We can consider \(VA_{\text{reg,sec}}\) and \(IO_{\text{reg,sec}}\) as generic functions with distinct factors of production, \(VA_{\text{reg,sec}} = f(K_{\text{reg,sec}}, L_{\text{reg,sec}})\) and \(IO_{\text{reg,sec}} = z(X_{\text{reg,sec,sec}})\), respectively.
\[ XD_{reg,sec} = \frac{1}{b_{reg,sec}} VA_{reg,sec} = \frac{1}{(1 - b_{reg,sec})} IO_{reg,sec} \] (5)

At the second level, the determination of the optimum combination of the different intermediate goods is equally done with a Leontief function,

\[ IO_{reg,secc,sec} = d_{reg,secc} * IO_{reg,sec} \] (6)

where \( IO_{reg,secc,sec} \) is the intermediate consumption of the goods produced by sector \( secc \) and used in sector \( sec \).

From equations (3) and (6) we know that,

\[ IO_{reg,secc,sec} = d_{reg,secc} * (1 - b_{reg,sec}) * XD_{reg,sec} \] (7)

If we consider the technical coefficients equal to \( io_{reg,secc,sec} = (1 - b_{reg,sec}) \) \( d_{reg,secc} \), than we may define the demand for intermediate goods as a function of the domestic final demand,

\[ IO_{reg,secc,sec} = io_{reg,secc,sec} * XD_{reg,sec} \] (8)

Still at this level, the optimum combination of capital and labour by sector \( (VA_{reg,sec}) \) is the aggregation of \( K_{reg,sec} \), \( LQ_{reg,sec} \) and \( LU_{reg,sec} \) following a CES function,\(^9\)

\[
VA_{reg,sec} = aP_{reg,sec}[\gamma Fk_{reg,sec} * K_{reg,sec}^{1-\rho_{F,sec}} + \gamma Fq_{reg,sec} * LQ_{reg,sec}^{1-\rho_{F,sec}} + \\
+ \gamma Fu_{reg,sec} * LU_{reg,sec}^{1-\rho_{F,sec}} - \frac{1}{\rho_{F,sec}}]
\] (9)

where \( aP_{reg,sec} \) is the efficiency parameter, \( \gamma Fk_{reg,sec} \), \( \gamma Fq_{reg,sec} \) and \( \gamma Fu_{reg,sec} \) are the factors distribution parameter. The function is homogeneous of degree 1 since it has constant scale economies.

If \( \frac{aP_{reg,sec}}{b_{reg,sec}} = aF_{reg,sec} \) and knowing that \( VA_{reg,sec} = XD_{reg,sec} * b_{reg,sec} \), than,

\[
XD_{reg,sec} = aF_{reg,sec}[\gamma Fk_{reg,sec} * K_{reg,sec}^{1-\rho_{F,sec}} + \gamma Fq_{reg,sec} * LQ_{reg,sec}^{1-\rho_{F,sec}} + \\
+ \gamma Fu_{reg,sec} * LU_{reg,sec}^{1-\rho_{F,sec}} - \frac{1}{\rho_{F,sec}}]
\] (10)

The cost function may be represented by

---

\(^8\)The subscript \( secc \) and \( sec \) have the same meaning - sectors, which allows the differentiation between the sector of origin and the sector of destiny of the flow. The subscript \( reg \) \( regg \) also have the same meaning - regions.

\(^9\)It is possible to prove that a CES production function become a Cobb-Douglas production function when \( \rho_{F,sec} \to 0 \) \( (\sigma F_{sec} \to \infty) \), and become a LES production function when \( \rho_{F,sec} \to \infty \) \( (\sigma F_{sec} \to 0) \).
\[
\text{Cost}_\text{reg,sec} (K_{\text{reg,sec}}, LQ_{\text{reg,sec}}, LU_{\text{reg,sec}}) = (1 + tk_{\text{reg,sec}}) \cdot pk_{\text{reg}} \cdot K_{\text{reg,sec}} + \\
+ (1 + tlq_{\text{reg,sec}}) \cdot plq_{\text{reg}} \cdot LQ_{\text{reg,sec}} + \\
+ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \cdot LU_{\text{reg,sec}} 
\] 

(11)

where \( pk_{\text{reg}} \) and \( tk_{\text{reg,sec}} \) are the price of capital and the weight of the tax on its use, by regions, and \( plq_{\text{reg}}, tlq_{\text{reg,sec}}, plw_{\text{reg}} \) and \( tlw_{\text{reg,sec}} \) represent the same for the two types of labour, respectively.

Minimizing the cost function, subject to the restriction \( XD_{\text{reg,sec}} \), equation (10), we obtain the demand functions for \( K_{\text{reg,sec}}, LQ_{\text{reg,sec}} \) and \( LU_{\text{reg,sec}} \) in each sector,

\[
K_{\text{reg,sec}} = \frac{XD_{\text{reg,sec}}}{\sigma F_{\text{reg,sec}}} \cdot \left[ \frac{\gamma F_{k_{\text{reg,sec}}}}{(1 + tk_{\text{reg,sec}}) \cdot pk_{\text{reg}}} \right]^{\frac{1}{\sigma F_{\text{reg,sec}}}} \\
\quad \times \left[ (1 + tk_{\text{reg,sec}}) \cdot pk_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{k_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlq_{\text{reg,sec}}) \cdot plq_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lq_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lw_{\text{reg,sec}}} 
\] 

(12)

\[
LQ_{\text{reg,sec}} = \frac{XD_{\text{reg,sec}}}{\sigma F_{\text{reg,sec}}} \cdot \left[ \frac{\gamma F_{q_{\text{reg,sec}}}}{(1 + tlq_{\text{reg,sec}}) \cdot plq_{\text{reg}}} \right]^{\frac{1}{\sigma F_{\text{reg,sec}}}} \\
\quad \times \left[ (1 + tlq_{\text{reg,sec}}) \cdot plq_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{q_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lw_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lw_{\text{reg,sec}}} 
\] 

(13)

\[
LU_{\text{reg,sec}} = \frac{XD_{\text{reg,sec}}}{\sigma F_{\text{reg,sec}}} \cdot \left[ \frac{\gamma F_{u_{\text{reg,sec}}}}{(1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}}} \right]^{\frac{1}{\sigma F_{\text{reg,sec}}}} \\
\quad \times \left[ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{u_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlq_{\text{reg,sec}}) \cdot plq_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lq_{\text{reg,sec}}} \\
\quad + \left[ (1 + tlw_{\text{reg,sec}}) \cdot plw_{\text{reg}} \right]^{1-\sigma F_{\text{reg,sec}}} \cdot \gamma F_{lw_{\text{reg,sec}}} 
\] 

(14)

where \( \sigma F_{\text{reg,sec}} \) represents Allen’s partial elasticity of substitution (from now on referred to as elasticity of substitution) between factors and determined by

\[
\sigma F_{\text{reg,sec}} = \frac{1}{1 + \sigma F_{\text{reg,sec}}^{10}}
\]

10 In Cobb-Douglas functions \( \sigma F_{\text{reg,sec}} = 1 \) and in Leontief functions \( \sigma F_{\text{reg,sec}} = 0 \).
In perfect competitive markets profits does not exist. Than, the condition of zero profits for the Leontief and the CES functions is represented by the equalisation of the returns on the domestic production and the respective costs,

\[
(1 - txd_{reg,sec}) \cdot pd_{reg,sec} \cdot XD_{reg,sec} = (1 + tk_{reg,sec}) \cdot pk_{reg} \cdot K_{reg,sec} + \\
+ (1 + tlq_{reg,sec}) \cdot plq_{reg} \cdot LQ_{reg,sec} + (1 + tl\mu_{reg,sec}) \cdot pl\mu_{reg} \cdot LU_{reg,sec} + \\
+ \sum_{secc} \left[ i\nu_{reg,secc,sec} \cdot XD_{reg,sec} \cdot p_{reg,secc} \cdot (1 + tcf_{reg,secc,sec}) \right] \tag{15}
\]

where \( txd_{reg,sec} \) is the weight of the tax on the production of each sector (liquid of subsidies) in each region, \( p_{reg,secc} \) is the price of the composite good \( X_{reg,secc} \) (corresponding to the amount of offered goods in each domestic market, composed by domestic production and imports) and \( tcf_{reg,secc} \) is the weight of taxes on the consumption of intermediate goods.

### 2.1.2 Productivity of primary factors

Starting with the value added equation (equation (9)),

\[
VA_{reg,sec} = aP_{reg,sec} \cdot F_{reg,sec} \cdot K_{reg,sec}^{-\frac{1-\sigma_{F_{reg,sec}}}{\sigma_{F_{reg,sec}}}} + F_{q_{reg,sec}} \cdot LQ_{reg,sec}^{-\frac{1-\sigma_{F_{q_{reg,sec}}}}{\sigma_{F_{q_{reg,sec}}}}} + \\
+ F_{u_{reg,sec}} \cdot LU_{reg,sec}^{-\frac{1-\sigma_{F_{u_{reg,sec}}}}{\sigma_{F_{u_{reg,sec}}}}}
\]

we can determine the expression of average productivity of each factor dividing last equation by the respective factor of production.

\[
\begin{align*}
\pi k_{reg,sec} &= \frac{VA_{reg,sec}}{K_{reg,sec}} = aP_{reg,sec} \cdot F_{reg,sec} \cdot K_{reg,sec}^{-\frac{1-\sigma_{F_{reg,sec}}}{\sigma_{F_{reg,sec}}}} + \\
&+ F_{q_{reg,sec}} \cdot LQ_{reg,sec}^{-\frac{1-\sigma_{F_{q_{reg,sec}}}}{\sigma_{F_{q_{reg,sec}}}}} + F_{u_{reg,sec}} \cdot LU_{reg,sec}^{-\frac{1-\sigma_{F_{u_{reg,sec}}}}{\sigma_{F_{u_{reg,sec}}}}}, \\
\pi q_{reg,sec} &= \frac{VA_{reg,sec}}{LQ_{reg,sec}} = aP_{reg,sec} \cdot F_{reg,sec} \cdot K_{reg,sec}^{-\frac{1-\sigma_{F_{reg,sec}}}{\sigma_{F_{reg,sec}}}} + \\
&+ F_{q_{reg,sec}} \cdot LQ_{reg,sec}^{-\frac{1-\sigma_{F_{q_{reg,sec}}}}{\sigma_{F_{q_{reg,sec}}}}} + F_{u_{reg,sec}} \cdot LU_{reg,sec}^{-\frac{1-\sigma_{F_{u_{reg,sec}}}}{\sigma_{F_{u_{reg,sec}}}}}, \\
\pi u_{reg,sec} &= \frac{VA_{reg,sec}}{LU_{reg,sec}} = aP_{reg,sec} \cdot F_{reg,sec} \cdot K_{reg,sec}^{-\frac{1-\sigma_{F_{reg,sec}}}{\sigma_{F_{reg,sec}}}} + \\
&+ F_{q_{reg,sec}} \cdot LQ_{reg,sec}^{-\frac{1-\sigma_{F_{q_{reg,sec}}}}{\sigma_{F_{q_{reg,sec}}}}} + F_{u_{reg,sec}} \cdot LU_{reg,sec}^{-\frac{1-\sigma_{F_{u_{reg,sec}}}}{\sigma_{F_{u_{reg,sec}}}}}
\end{align*}
\]

It is evident that factor productivity depends on different elements. In addition to the amount of all factors, the efficiency \((aP_{reg,sec})\) and distribution \((\gamma F_{q_{reg,sec}}, \gamma F_{q_{reg,sec}}\) and \(\gamma F_{u_{reg,sec}}\)) parameters of value added function also have an important role. Than, assuming exogenous elasticity of substitution and being endogenous the quantities of factors used \((K_{reg,sec}, LQ_{reg,sec}\) and \(LU_{reg,sec}\)), the elements that can incorporate external changes in the production process, i.e. the exogenous improvement of productivity, are the efficiency and
distribution parameters. Changes on the first parameter are reflected equally on all factors of production and changes on distribution parameters can affect specific factors. Examples of this last group are professional specific formation, higher education in special fields or sciences, or financial incentives linked to labour productivity.

To evaluate the effects of this type of economic policy we can calculate the impact of a change (for example, of 10%) on these parameters and compare different scenarios. Naturally, the final effect on factor productivity will be different from the initial change since the economy adjusts with changes of endogenous variables.

2.1.3 How much to Produce?

This second group of decisions is divided in two parts. On one hand, the producer determines the share of its production that will be distributed in the domestic market and in foreign markets. On the other hand, the agent quantifies the composite good $X_{reg, sec}$, using the amount offered in the domestic market and the imported amount, which will be subject to the intermediate and final demand of the market.

At the first part, the rational agent will maximise revenues as a function of the demand location,

$$
REV_{reg, sec}(X_{DD, reg, sec}, E_{reg, regg, sec}) = pdd_{reg, sec} \times X_{DD, reg, sec} +
+ \sum_{regg} pe_{reg, regg, sec} \times E_{reg, regg, sec} +
+ pw_{reg, regg, sec} \times \text{MARGE}_{reg, sec}
$$

under the restriction of a transformation function with constant elasticity (CET function), which reveals a limited substitution between the domestic distribution and exports, by sector,

$$
XD_{reg, sec} = aT_{reg, sec} \times \left[ \sum_{regg} \gamma_{reg, regg, sec} \times E_{reg, regg, sec}^{-\sigma_{T, reg, sec}} \right]^{\gamma_{reg, regg, sec}} X_{DD, reg, sec}^{-\sigma_{T, reg, sec}} \times \frac{1}{T_{reg, sec}}
$$

where $aT_{reg, sec}$ is the efficiency coefficient and $\gamma_{reg, regg, sec}$ assumes the values of the parameters of export distribution in the different locations. $\sigma_{T, reg, sec}$ is the domestic production’s elasticity of substitution between exports for the different regions and domestic distribution, and may be calculated by $\frac{1}{1 + \gamma_{T, reg, sec}}$. $X_{DD, reg, sec}$ represents domestic production offered in each domestic market, $E_{reg, regg, sec}$ are exports of domestically produced goods to the regions considered in this work (EU14, EU10 and ROW), $pdd_{reg, sec}$ represents prices of
domestic goods in the domestic market, \( p_{reg,regg,sec} \) are domestic prices of exports for the different destinations (market prices of exports). \( pw_{reg,regg,sec} \) * \( MARGE_{reg,sec} \) is the value of transport services associated with exports valued at export prices. Since \( MARGE_{sec} \) is not discriminated by regions of destiny is considered valued at the currency of the major trade partner in each sector. Than, for sector "sca" will be used \( pw_{reg,"UE14",sec} \) and for sector "non" will be used \( pw_{reg,"row",sec} \). All parameters and variables, are differentiated according to the sector considered.

It is important to note that the used database (GTAP, version 6) considers customs taxes but also a set of selected non-tariff barriers, besides anti-dumping rights (used in Canada, USA and the EU). On the one hand, non-tariff barriers that increase the prices of foreign goods in domestic markets are aggregated as customs taxes. On the other hand, those non-tariff barriers that change the domestic prices of domestically produced goods are added to production taxes. Finally, those non-tariff barriers that change the domestic prices of exports are added to export taxes. One disadvantage of this distribution is that all these taxes are accounted for as a government revenue or expenditure, even though some trade barriers are gains for importers only.

The optimum distribution for domestic production in the different destination markets is represented by the following expressions:

\[
XDD_{reg,sec} = \frac{XD_{reg,sec}}{aT_{reg,sec}} \left( 1 - \sum_{regg} \gamma_{T_{reg,regg,sec}} \frac{\sigma_{T_{reg,sec}}}{pdd_{reg,sec}} \right) \left( 1 - \sum_{regg} \gamma_{T_{reg,regg,sec}} \frac{\sigma_{T_{reg,sec}}}{pdd_{reg,sec}} \right)
\]

\[
E_{reg,regg,sec} = \frac{XD_{reg,sec}}{aT_{reg,sec}} \left( 1 - \sum_{regg} \gamma_{T_{reg,regg,sec}} \frac{\sigma_{T_{reg,sec}}}{pdd_{reg,sec}} \right) \left( 1 - \sum_{regg} \gamma_{T_{reg,regg,sec}} \frac{\sigma_{T_{reg,sec}}}{pdd_{reg,sec}} \right)
\]

The zero profit condition of this optimization problem is supplied by the following expression, where \( pd_{reg,sec} \) is the price of the good produced by each sector,
\[ pd_{\text{reg},\text{sec}} \times XD_{\text{reg},\text{sec}} = pdd_{\text{reg},\text{sec}} \times XDD_{\text{reg},\text{sec}} + \]
\[ + \sum_{\text{regg}} (pe_{\text{reg},\text{regg},\text{sec}} \times E_{\text{reg},\text{regg},\text{sec}} ) + \]
\[ + pwe_{\text{reg},\text{regg},\text{sec}} \times MARGE_{\text{reg},\text{sec}} \]  \hspace{1cm} (20)

At the second part of this structure of production 'distribution' the producer supplies the market with a composite good \( X_{\text{reg},\text{sec}} \) consisting of domestic and foreign goods, which he himself produces and imports. This is expressed by means of a linearly homogeneous CES production function, revealing the existence of limited or imperfect substitution between national and foreign goods. Usually such an assumption is referred to as an Armington assumption, following the first author to consider that consumers use \( X_{\text{reg},\text{sec}} \) in accordance to this type of function.\(^{11}\)

\[ X_{\text{reg},\text{sec}} = a_{\text{reg},\text{sec}} \times \left[ \sum_{\text{regg}} \left( \gamma A_{\text{reg},\text{regg},\text{sec}} \times M_{\text{reg},\text{regg},\text{sec}} \right) \right]^{\frac{1}{1-\gamma}} \]  \hspace{1cm} (21)

In order to maximise profits, each firm has to minimise the cost of this composite good, under the restriction of equation (21),

\[ Cost_{\text{reg},\text{sec}}(M_{\text{reg},\text{regg},\text{sec}}, XDD_{\text{reg},\text{sec}}) = \sum_{\text{regg}} (pm_{\text{reg},\text{regg},\text{sec}} \times M_{\text{reg},\text{regg},\text{sec}}) + \]
\[ + pdd_{\text{reg},\text{sec}} \times XDD_{\text{reg},\text{sec}} \]  \hspace{1cm} (22)

\( a_{\text{reg},\text{sec}} \) is the efficiency parameter, \( \gamma A_{\text{reg},\text{regg},\text{sec}} \) assumes the values of the distribution parameters, and \( \sigma A_{\text{reg},\text{sec}} = \frac{1}{1+\rho A_{\text{reg},\text{sec}}} \) is the elasticity of substitution between the domestic good and goods imported from the distinct regions. \( M_{\text{reg},\text{regg},\text{sec}} \) is the imported amount from each region. The prices, in domestic currency, of such imported goods are represented by \( pm_{\text{reg},\text{regg},\text{sec}} \).

This optimization problem generates the domestic consumer demand functions to all goods contained in the composite good \( X_{\text{reg},\text{sec}} \).\(^{12}\)

\(^{11}\)See Armington (1969).
\(^{12}\)The model assumes that \( XDD_{\text{reg},\text{sec}} \) is the same in the equations (18) and (23), which means that the demand equals the supply.
\[ XXD_{\text{reg,sec}} = \frac{X_{\text{reg,sec}}}{a_{\text{reg,sec}}} \left( 1 - \sum_{\text{regg}} \frac{\gamma A_{\text{reg,regg,sec}}}{p_{\text{dd,reg,sec}}} \right)^{\sigma A_{\text{reg,sec}}} \]

\[ \times \left( \sum_{\text{regg}} \left( \gamma A_{\text{reg,regg,sec}} \ast pm_{\text{reg,regg,sec}}^{1 - \sigma A_{\text{reg,sec}}} \right) + \right) \]

\[ + \left( 1 - \sum_{\text{regg}} \gamma A_{\text{reg,regg,sec}} \right)^{\sigma A_{\text{reg,sec}}} \ast p_{\text{dd,reg,sec}}^{1 - \sigma A_{\text{reg,sec}}} \left( \frac{\sigma A_{\text{reg,sec}}}{\gamma A_{\text{reg,regg,sec}}} \right) \]

(23)

\[ M_{\text{reg,regg,sec}} = \frac{X_{\text{reg,sec}}}{a_{\text{reg,sec}}} \left( \frac{\gamma A_{\text{reg,regg,sec}}}{pm_{\text{reg,regg,sec}}} \right)^{\sigma A_{\text{reg,sec}}} \]

\[ \times \left( \sum_{\text{regg}} \left( \gamma A_{\text{reg,regg,sec}} \ast pm_{\text{reg,regg,sec}}^{1 - \sigma A_{\text{reg,sec}}} \right) + \right) \]

\[ + \left( 1 - \sum_{\text{regg}} \gamma A_{\text{reg,regg,sec}} \right)^{\sigma A_{\text{reg,sec}}} \ast p_{\text{dd,reg,sec}}^{1 - \sigma A_{\text{reg,sec}}} \left( \frac{\sigma A_{\text{reg,sec}}}{\gamma A_{\text{reg,regg,sec}}} \right) \]

(24)

The zero profit condition of this optimization problem, which reveals that the market is competitive, is given by the following expression, where \( P_{\text{reg,sec}} \) is the price of the composite good offered by each firm,

\[ p_{\text{reg,sec}} \ast X_{\text{reg,sec}} = p_{\text{dd,reg,sec}} \ast XXD_{\text{reg,sec}} + \sum_{\text{regg}} \left( pm_{\text{reg,regg,sec}} \ast M_{\text{reg,regg,sec}} \right) \]

(25)

### 2.1.4 International trade as a connection between economies

In this general equilibrium model, international trade is the link between different regions. It is explicitly considered that Portuguese exports to the distinct partners correspond to imports from Portugal in those countries. The introduction of linkages between countries must be done with caution since trade flows are valued in different ways. In fact, the exports of each countries are valued at f.o.b. prices and their imports are valued at c.i.f. prices, over which are applied tariffs.\(^{13}\)

If we consider that \( pw_{\text{reg,regg,sec}} \) is the f.o.b. (Free on Board) price of exports it may be verified that the difference between the f.o.b. prices of exports

\(^{13}\)In GTAP-6 data base, the difference between c.i.f. prices and f.o.b. prices are the "transport margins on imports". This means that we have also the imports valued at f.o.b. (\( M_{\text{fob,regg}} \)).
and their market prices, in domestic currency, are the weight of taxes (or of subsidies if their values are negative) on exports ($t_{reg, regg, sec}$).

$$p_{reg, regg, sec} = p_{we, reg, regg, sec} \times (1 - t_{reg, regg, sec}) \quad (26)$$

On the other hand, if the weight of customs taxes, discriminated by region, is $t_{m, reg, regg, sec}$ and the c.i.f. (Cost, Insurance and Freight) price of imports is $p_{wm, reg, regg, sec}$, the difference between these prices will be the taxes applied by the region to imported goods according to their origin,

$$pm_{reg, regg, sec} = (1 + t_{m, reg, regg, sec}) \times p_{wm, reg, regg, sec} \quad (27)$$

In a multinational model the relations between countries are modelled for each couple of trade partners, and the international import prices correspond to international export prices of each partner. Then, the last equation can be written as

$$pm_{reg, regg, sec} = (1 + t_{m, reg, regg, sec}) \times p_{we, regg, regg, sec} \quad (28)$$

Lets use an example with the rest of the world imports from Portugal of one product. $DA$ is the tariffs value, $MARG$ is the value of transport margins on imports, $VM_{fob}$ and $VE_{fob}$ are the value of imports and exports, valued at f.o.b. prices, and $M$ and $E$ are the quantities imported and exported.

$$pm_{row, prt} \times M_{row, prt} =
DA_{row, prt} + MARG_{row, prt} + VM_{row, prt} =
DA_{row, prt} + MARG_{row, prt} + VE_{fob}^{prt, row} =
(1 + t_{m, row, prt}) \times [MARG_{row, prt} + VE_{fob}^{prt, row}] =
(1 + t_{m, row, prt}) \times [MARG_{row, prt} + p_{we, row, row} \times E_{prt, row}] \quad (29)$$

If we consider that transport margins are proportional to imports quantities, this last expression will be,

$$pm_{row, prt} \times M_{row, prt} = (1 + t_{m, row, prt}) \times p_{we, row, row} \times (1 + m_{g, row, prt}) \times E_{prt, row} \quad (30)$$

where $m_{g, row, prt}$ is the weight of margins on imports.

Than, the import prices are,

$$pm_{row, prt} = (1 + t_{m, row, prt}) \times p_{we, row, row} \quad (31)$$

and quantities

$$M_{row, prt} = (1 + m_{g, row, prt}) \times E_{prt, row} \quad (32)$$
In the equation (30) it is possible to see the influence of each country on each partner.

In the context of this model, the Balance of Payments is represented by the liquid flow of goods and services.\(^{14}\)

\[
\sum_{\text{sec}}^{n} \left( pwe_{\text{reg,reg,sec}} \times M_{\text{reg,reg,sec}} \right) = \sum_{\text{sec}}^{n} \left( pwe_{\text{reg,reg,sec}} \times E_{\text{reg,reg,sec}} \right) + SF_{\text{reg,reg}} \tag{33}
\]

being \(SF_{\text{reg,reg}}\) the foreign savings, i.e. the surplus of the Portuguese economy if negative, or deficit if positive.

### 2.2 The Behaviour of the representative family

In this study a representative family is used as a proxy for all consumers. It is an assumption that erases social diversity, a characteristic of all economies, but is justified by the fact that the objective of the model is the measurement of the effects of economic policies in countries’ external competitiveness and not at the level of income distribution among consumers.

It is considered that the representative family is the owner of all production factors and that the capital and labour endowments are exogenous, i.e. it is assumed that there is an external immobility of such factors. Unemployment is allowed in the model.

The representative family maximises a non-homogeneous Stone-Gary utility function, which produces a linear system of expenses (known as LES function), subject to a budget constraint.

Family income is obtained with the selling of productive resources to firms (capital, skilled and unskilled labour), with the payment of unemployment subsidies and of other government transfers,

\[
Y_{\text{reg}} = p_{k_{\text{reg}}} \times (1 - d_{\text{reg}}) \times KS_{\text{reg}} + p_{lq_{\text{reg}}} \times (LQS_{\text{reg}} - UNEMPQ_{\text{reg}}) + plu_{\text{reg}} \times (LUS_{\text{reg}} - UNEMPU_{\text{reg}}) + TRF_{\text{reg}} \tag{34}
\]

\(Y_{\text{reg}}\) represents the family total income, \(KS_{\text{reg}}, LUS_{\text{reg}}\) and \(LQS_{\text{reg}}\) are capital and labour endowments, \(UNEMPQ_{\text{reg}}\) and \(UNEMPU_{\text{reg}}\) are the unemployment of skilled and unskilled labour,\(^{15}\) \(TRF_{\text{reg}}\) is the total amount of family’s government transfers, and \(d_{\text{reg}}\) is the rate of depreciation of capital.

---

\(^{14}\)Capital flows are not included in the equation because it is assumed that all factors are immobile between countries. It is also not considered the transfers form Brussels, like structural funds and other European funds. These issues are left for future developments of the model.

\(^{15}\)It is possible to use endogenous unemployment by means of a "Phillips' curve". This curve relates the rate of change of the wage rate and the rate of change of the unemployment rate. During this stage of the investigation, the unemployment is exogenous.
Families’ expenses are allocated to income taxes (\(ty_{reg}\)), savings (\(SH_{reg}\)) and goods and services consumption (\(C_{reg,sec}\)).

Savings are a fixed share of the income, which means that the marginal propensity to save (\(mps_{reg}\)) is constant, after deducting taxes paid to the government,

\[
SH_{reg} = mps_{reg} \times [YH_{reg} - ty_{reg} \times (YH_{reg} - TRF_{reg})]
\]  

(35)

and allow the calculation of the income available to consumption (\(CBUD_{reg}\)),

\[
CBUD_{reg} = YH_{reg} - ty_{reg} \times (YH_{reg} - TRF_{reg}) - SH_{reg}
\]  

(36)

The consumer optimum choice is determined through the maximisation of his LES utility function (\(UH_{reg}(C_{reg,sec})\), subject to the budgetary constraint that relates the income available to consumption with the value of expenses,

\[
UH_{reg}(C_{reg,sec}) = \prod_{sec}(C_{reg,sec} - \mu_{H_{reg,sec}})^{\alpha H_{reg,sec}}
\]  

(37)

where \(\sum_{sec} \alpha H_{reg,sec} = 1\) and \(C_{reg,sec} > \mu_{H_{reg,sec}} \geq 0\)

s.t. \(CBUD_{reg} = \sum_{sec} [(1 + tc_{reg,sec}) \times p_{reg,sec} \times C_{reg,sec}]\)

(38)

where \(\mu_{H_{reg,sec}}\) represents the minimum amount of family consumption for each good, and \(p_{reg,sec}\) is the price of the goods sold in the domestic market (domestic and imported goods).\(^{16}\) The final private demand for goods and services is represented by,

\[
C_{reg,sec} = \mu_{H_{reg,sec}} + \alpha_{H_{reg,sec}} \times [(1 + tc_{reg,sec}) \times p_{reg,sec}]^{-1} \times \left\{CBUD_{reg} - \sum_{secc} [(1 + tc_{reg,secc}) \times p_{reg,secc} \times \mu_{H_{reg,secc}}]\right\}
\]  

(39)

i.e.,

\[
(1 + tc_{reg,sec}) \times p_{reg,sec} \times C_{reg,sec} = (1 + tc_{reg,sec}) \times p_{reg,sec} \times \mu_{H_{reg,sec}} + \\
+ \alpha_{H_{reg,sec}} \times \left\{CBUD_{reg} - \sum_{secc} [(1 + tc_{reg,secc}) \times p_{reg,secc} \times \mu_{H_{reg,secc}}]\right\}
\]  

(40)

\(^{16}\)When \(\mu_{H_{reg,sec}} = 0\), \(\forall sec\), the LES function is transformed into a Cobb-Douglas function, which is homogenous of degree 1 (linear homogenous) if \(\sum_{sec} \alpha_{H_{sec}} = 1\). Therefore, LES functions are a generalization of Cobb-Douglas functions, and let the elasticity of substitution to be different from 1. So, these functions may be more reasonable to study the consumer behaviour.
It is interesting to note that \((1 + t_{\text{reg}, \text{sec}}) \cdot p_{\text{reg}, \text{sec}} \cdot \mu H_{\text{reg}, \text{sec}}\) is the family expense that allows the attainment of the minimum level of consumption explicit in the utility function. \(\alpha H_{\text{reg}, \text{sec}}[CBUD_{\text{reg}} - \sum_{\text{sec}}(1 + t_{\text{reg}, \text{sec}}) \cdot p_{\text{reg}, \text{sec}} \cdot \mu H_{\text{reg}, \text{sec}}]\) is the part of the available income that remains, after assuring the minimum level of consumption (residual income), and is expended in goods and in services in fixed parts for each sector, according to the parameters \(\alpha H_{\text{reg}, \text{sec}}\).

### 2.3 Government Behaviour

In what concerns the behaviour of the economic agent 'government', it is considered that it is responsible for tax collection and transfers' payments to families, namely unemployment subsidies and other transfers (such as pensions or health related transfers). The considered taxes are those on consumption \((t_{\text{reg}, \text{sec}}, t_{\text{cg}, \text{reg}, \text{sec}}, t_{\text{ci}, \text{reg}, \text{sec}}, t_{\text{cf}, \text{reg}, \text{sec}})\), on the use of capital \((t_{\text{K}, \text{reg}, \text{sec}})\) and labour \((t_{LQ, \text{reg}, \text{sec}}\) and \(t_{LU, \text{reg}, \text{sec}}\)), on income \((t_{y_{\text{reg}}})\), on imports \((t_{m_{\text{reg}, \text{reg}g, \text{sec}}})\) and on exports \((t_{e_{\text{reg}, \text{reg}g, \text{sec}}})\), and on production \((t_{xD_{\text{reg}, \text{sec}}})\). All these taxes are in proportion to the taxable basis.

It is assumed that the government maximises a Cobb-Douglas utility function \((UG_{\text{reg}}(CG_{\text{reg}, \text{sec}}))\) subject to an initially balanced budget. It is possible to assume a value for the budget deficit or to bound it (e.g. 3% of GDP) by the endogeneization of taxes. This option introduces a great complexity in the model that should be avoided if the European economies are not really bounded by that assumption of Stability and Growth Plan.

Total government revenues consist of total tax revenues \((TAXR_{\text{reg}})\) since the productive activities of the government are included in the activity of firms. This is due to the fact that government behaviour in what concerns production decisions should be similar to that of private agents and also because the number of totally public firms are decreasing in number.

\[
TAXR_{\text{reg}} = t_{y_{\text{reg}}} \cdot (Y H_{\text{reg}} - TRF_{\text{reg}}) + \sum_{\text{sec}}[p_{\text{reg}, \text{sec}} \cdot (t_{\text{reg}, \text{sec}} \cdot C_{\text{reg}, \text{sec}} + t_{\text{cg}, \text{reg}, \text{sec}} \cdot CG_{\text{reg}, \text{sec}} + t_{\text{ci}, \text{reg}, \text{sec}} \cdot I_{\text{reg}, \text{sec}} + \sum_{\text{sec}}(t_{\text{cf}, \text{reg}, \text{secc, sec}} \cdot X_{\text{reg}, \text{sec}}) + t_{\text{K}, \text{reg}, \text{sec}} \cdot K_{\text{reg}, \text{sec}} + t_{LQ, \text{reg}, \text{sec}} \cdot LQ_{\text{reg}, \text{sec}} + t_{LU, \text{reg}, \text{sec}} \cdot LU_{\text{reg}, \text{sec}} + \sum_{\text{regg}}(t_{m_{\text{reg}, \text{reg}g, \text{sec}} \cdot p_{\text{w}, \text{reg}g, \text{reg}, \text{sec}} \cdot E_{\text{reg}, \text{reg}g, \text{sec}} + t_{xD_{\text{reg}, \text{sec}} \cdot pd_{\text{reg}, \text{sec}} \cdot XD_{\text{reg}, \text{sec}})]
\]

Government pays unemployment subsidies at a rate \(t_{rep_{\text{reg}}}\) as a share of the average wage and other transfers, such as pensions and health subsidies, that are constant in real terms and transformed into nominal variables using a Laspeyres price index \((pc_{\text{index}})\).
\[ pcindex_{reg} = \sum_{sec} \left( \frac{(1 + t_{reg,sec}) \cdot p_{reg,sec} \cdot C_{reg,sec}}{(1 + t_{reg,sec}) \cdot p_{reg,sec} \cdot C_{reg,sec}} \right) \] (42)

where \( t \) is the moment in time (0 for values before the scenario simulation and 1 for values after the scenario simulation).

Total transfers \((TRF_{reg})\) are expressed by the equation,

\[
TRF_{reg} = tre_{reg} \cdot (plq_{reg} \cdot UEMPQ_{reg} + plu_{reg} \cdot UEMPU_{reg}) + TRO_{reg} 
\]

(43)

It is expected that government consumption decisions are also a result of the maximisation of a linearly homogeneous Cobb-Douglas utility function, subject to the budget constraint, where \( \alpha CG_{reg,sec} \) is the income elasticity of the government demand of goods and services and \( CG_{reg,sec} \) the referred demand,

\[
UG_{reg}(CG_{reg,sec}) = \prod_{sec} CG_{reg,sec}^{\alpha CG_{reg,sec}} \quad \text{being} \quad \sum_{sec} \alpha CG_{reg,sec} = 1 \] (44)

s.t.

\[
TAX_{reg} - TRF_{reg} - pcindex_{reg} \cdot SG_{reg} = \sum_{sec} (1 + tc_{reg,sec}) \cdot p_{reg,sec} \cdot CG_{reg,sec} \] (45)

where \( pcindex_{reg} \cdot SG_{reg} \) is the budget balance, being \( SG_{reg} \) the real government saving and \( tc_{reg,sec} \) is the tax weight on public consumption.

Government’s demand for goods and services, obtained via the optimization, as the following expression,

\[
(1 + tc_{reg,sec}) \cdot p_{reg,sec} \cdot CG_{reg,sec} = \alpha CG_{reg,sec} \]

(46)

\[
\text{where } (TAX_{reg} - TRF_{reg} - pcindex_{reg} \cdot SG_{reg}) \]

As it would be expected, government’s nominal expenditure in each good and service is a fixed share of its revenues. If the demand functions are added, across sectors, the result is the government’s budget constraint, which puts in evidence the existence of constant scale returns linked to the homogeneity of this agent’s utility function.

### 2.4 Investment Demand

The demand for investment will be included in the model in a very simple way, considering investment as investment goods, i.e. goods and services identical to those demanded by firms and consumers, valued at market prices (including
It is considered that there is an entity that allocates savings across investment goods, in all sectors, in accordance to a Cobb-Douglas utility function \( UI_{reg}(I_{reg,sec}) \), where \( I_{reg,sec} \) is the amount of investment goods and \( \alpha I_{reg,sec} \) is the income elasticity of the investment good demand.

\[
UI_{reg}(I_{reg,sec}) = \prod_{sec} I_{reg,sec}^{\alpha_{I_{reg,sec}}} \quad \text{where} \quad \sum_{sec=1}^{n} \alpha_{I_{reg,sec}} = 1 \tag{47}
\]

The demand expression is determined by the maximisation of this utility function, subject to the constraint of total savings \( S_{reg} \) where \( tci_{reg,sec} \) is the weight of taxes on the consumption of the goods used as investment goods,

\[
S_{reg} = \sum_{sec} I_{reg,sec} \cdot p_{reg,sec} \cdot (1 + tci_{reg,sec}) \tag{48}
\]

being total savings equal to the following identity,

\[
S_{reg} = SH_{reg} + p_{cindex_{reg}} \cdot SG_{reg} + \sum_{regg} (SF_{reg,regg}) + \sum_{sec} d_{reg} \cdot p_{k_{reg}} \cdot K_{reg,sec} - MARGE_{reg,sec} \cdot pw_{reg,regg,sec} \tag{49}
\]

The solution of the maximisation problem is

\[
(1 + tci_{reg,sec}) \cdot p_{reg,sec} \cdot I_{reg,sec} = \alpha I_{reg,sec} \cdot S_{reg} \tag{50}
\]

i.e.,

\[
I_{reg,sec} = \alpha I_{reg,sec} \cdot S_{reg} \cdot [(1 + tci_{reg,sec}) \cdot p_{reg,sec}]^{-1} \tag{51}
\]

### 2.5 General equilibrium in the economy

The general equilibrium in the economy implies the equality between supply and demand in all markets (goods and services, capital and labour). Thus, in labour market the demand must equal the supply of the two types of labour (LUS for the unskilled labour and LQS for the skilled), liquid of unemployment,

\[
\sum_{sec} LQ_{reg,sec} = LQS_{reg} - UNEMPQ_{reg} \tag{52}
\]

\[
\sum_{sec} LU_{reg,sec} = LUS_{reg} - UNEMP_{reg} \tag{53}
\]

The same should occur in the capital market, where it is considered that there are no unemployed resources,
\[
\sum_{sec} K_{\text{reg,sec}} = KS_{\text{reg}} \tag{54}
\]

as well as in goods and services market,

\[
X_{\text{reg,sec}} = C_{\text{reg,sec}} + I_{\text{reg,sec}} + \sum_{secc} (io_{\text{reg,sec,secc}} \times XD_{\text{reg,secc}}) + CG_{\text{reg,sec}} \tag{55}
\]

As in all general equilibrium models the Walras Law must be satisfied. This law say briefly that in an economy with \(m\) markets, if \(m - 1\) markets are in equilibrium, than the last market will be necessarily in equilibrium. This implies that we must not consider one of the last four equations during the solution.\(^\text{17}\)

Finally, to close the model the numeraire will be the price of one of the factors of production (\(pk_{\text{reg}}, plq_{\text{reg}}\) or \(plu_{\text{reg}}\)).

The factor endowments in the economy are exogenous (\(KS_{\text{reg}}, LQS_{\text{reg}}\) and \(LUS_{\text{reg}}\)), as well as the unemployment levels (\(UNEMPQ_{\text{reg}}\) and \(UNEMP\bar{U}_{\text{reg}}\)), other government transfers for the households (\(TRO_{\text{reg}}\)), transport services related to exports (\(MARGE_{\text{reg,sec}}\)), government savings (\(SG_{\text{reg}}\)) and foreign savings between countries of the region "rest of the world" (\(SF_{\text{reg,regg}}\)). This last closure equation exist since walras law is satisfied.

### 2.6 Data base

In this version of the model is used the GTAP (version 6) data base for major variables, except for unemployment levels (\(UNEMPQ_{\text{reg}}\) and \(UNEMP\bar{U}_{\text{reg}}\)), rates of unemployment subsidy (\(trep_{\text{reg}}\)), other government transfers for the households (\(TRO_{\text{reg}}\)), transport services related to exports (\(MARGE_{\text{reg,sec}}\)), government savings (\(SG_{\text{reg}}\)) and foreign savings between countries of the region "rest of the world" (\(SF_{\text{reg,regg}}\)). The 87 regions of the data base are aggregated into 4 regions (Portugal, EU14, EU10 and ROW). In respect to sectors, the 57 sectors of the data base are aggregated into 6 already reported. The great advantage of this data base is the possibility of direct comparison of different input-output matrixes, and its easy accessibility. In what concerns to the parameters that not exist in this data base, the statistical sources are different.

For the unemployment level it will be used the rates of National Statistic Institute (INE), for Portuguese levels and the rates of EUROSTAT for remaining regions.\(^\text{18}\)

The parameter \(trep_{\text{reg}}\) is calculated with EUROSTAT data, as the weight of unemployment subsidy \textit{per} unemployed person, in each region, relatively to the nominal compensation \textit{per} employee.\(^\text{19}\)

\(^{17}\)It will be ignored the equation of the market which price will be the numeraire.


\(^{19}\)See "Out-of-work income maintenance and support" and "Nominal compensation per employee" in http://epp.eurostat.ec.europa.eu/.
The source of other government transfers for the households \((TRO_{reg})\) is National Accounts of INE, and EUROSTAT.\footnote{See "Quadro de Contas Económicas Integradas", INE (2004) and http://epp.eurostat.ec.europa.eu/} To avoid any incompatibility between this statistics sources and GTAP data base, this parameter is introduced as a percentage of households consumption at current prices.

The unknown parameter of the household utility function \((\mu H_{reg,sec})\) is very subjective because it depends mainly on household preferences. Since we have only one representative household in each region, it is almost a random choice. The option made is the average consumption in the beginning of ninety decade, when started the accession negotiations to EU.\footnote{Calculas are based on national statistics (INE and Portuguese Bank) and european statistics (http://epp.eurostat.ec.europa.eu/).} So, is assumed that households considers the consumption level before the start of European enlargement process as the minimum acceptable.

With respect to substitution and transformation elasticities the sources are different. For elasticities of substitution between production factors \((\sigma F_{reg,sec})\), the values generated by the general equilibrium program "RunGTAP - Version 5" of GTAP data base are used, considering the same sectorial and regional aggregation as in this model. For elasticities of substitution between domestic and imported goods \((\sigma A_{reg,sec})\), the values considered by OECD in a tariff trade simulator (the most used in international literature) are used.\footnote{See} The discrimination between regions are made using the respective weights of each product in each sector.

Finally, for transformation elasticities \((\sigma T_{reg,sec})\), an approximation are calculated, using total flows in Portugal, and for the other regions are applied the values used in DART model.\footnote{See}

### 3 Alternative Scenarios and Results

The purpose of this paper is the identification of which type of labour (skilled or unskilled) permits the greater improvement of Portuguese competitiveness when its productivity increase, and in which sector that happen. Naturally, to get the final decision we need to know the cost of each alternative of policy to increase the labour productivity.

Since each sector has a different weight on both intermediate and final consumption (domestic and foreigner), the increase of labour productivity also has a different impact on the promotion of exports and imports. Thus, it is important to test the effect of changes in productivity of both skilled and unskilled labour, in different sectors and in different combinations. We will test a 10\% change in distribution parameters related with labour \((\gamma F_{q,sec}^p and \gamma F_{u,sec}^p)\).

The following scenarios will be tested:
Table 3: Scenarios (increases of 10%)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Increase of $F_q$ and $F_u$ in all sectors</td>
</tr>
<tr>
<td>C2</td>
<td>Increase of $F_q$ in all sectors</td>
</tr>
<tr>
<td>C3</td>
<td>Increase of $F_u$ in all sectors</td>
</tr>
<tr>
<td>C4</td>
<td>Increase of $F_q$ in sector &quot;res&quot;</td>
</tr>
<tr>
<td>C5</td>
<td>Increase of $F_q$ and $F_u$ in sector &quot;lab&quot;</td>
</tr>
<tr>
<td>C6</td>
<td>Increase of $F_q$ in sector &quot;spe&quot;</td>
</tr>
<tr>
<td>C7</td>
<td>Increase of $F_q$ and $F_u$ in sector &quot;sca&quot;</td>
</tr>
<tr>
<td>C8</td>
<td>Increase of $F_q$ and $F_u$ in sector &quot;rd&quot;</td>
</tr>
<tr>
<td>C9</td>
<td>Increase of $F_q$ and $F_u$ in sector &quot;non&quot;</td>
</tr>
<tr>
<td>C10</td>
<td>Increase of $F_q$ and $F_u$ in sectors &quot;lab&quot;, &quot;spe&quot;, &quot;sca&quot; and &quot;rd&quot;</td>
</tr>
<tr>
<td>C11</td>
<td>Increase of $F_u$ in all sectors and of $F_q$ in sectors &quot;res&quot; and &quot;lab&quot;</td>
</tr>
<tr>
<td>C12</td>
<td>Increase of $F_u$ in all sectors and of $F_q$ in sectors &quot;res&quot;, &quot;lab&quot;, and &quot;non&quot;</td>
</tr>
<tr>
<td>C13</td>
<td>Increase of $F_u$ in all sectors and of $F_q$ in sectors &quot;res&quot;, &quot;lab&quot;, &quot;non&quot; and &quot;sca&quot;</td>
</tr>
</tbody>
</table>

Since it is being used a multi-national and multi-sector general equilibrium model, it is possible to evaluate changes in all trade flows of all regions. The only closure condition that is acting on the results is the fact that foreigner saving between countries in "rest of the world" region. Notwithstanding, it is not supposed that changes in Portuguese labour productivity will, in fact, make any difference in these region. So, it is not a refraining condition.

The effects can be seen in terms of relative changes of all variables. However, in this paper will be only presented the results for exports, agents utility and equivalent variation index.

It is important to notice that different size of the regions and the sectors implies a careful analyses.

We can see that trade relations between Portugal and EU10 are insignificant both in exports and in imports (see tables 4 and 5). This insignificance is also valid for sector "rd" ("research and development").

Table 4: Weight of each sector exports on total exports, by partner (%)

<table>
<thead>
<tr>
<th>Partner</th>
<th>Total|</th>
</tr>
</thead>
<tbody>
<tr>
<td>UE14</td>
<td>13,81</td>
</tr>
<tr>
<td>UE10</td>
<td>6,50</td>
</tr>
<tr>
<td>ROW</td>
<td>14,17</td>
</tr>
</tbody>
</table>

Table 5: Weight of each sector imports on total imports, by partner (%)

<table>
<thead>
<tr>
<th>Partner</th>
<th>Total|</th>
</tr>
</thead>
<tbody>
<tr>
<td>UE14</td>
<td>13,81</td>
</tr>
<tr>
<td>UE10</td>
<td>6,50</td>
</tr>
<tr>
<td>ROW</td>
<td>14,17</td>
</tr>
</tbody>
</table>

\|$-$ monetary units.
In table 6 we can see that the same exogenous improvement of skilled and unskilled labour productivity (changes on $\gamma Fq_{prt,sec}$ and $\gamma Fu_{prt,sec}$) have different results on exports, especially on sectors "res", "lab" and "spe". The next 6 scenarios show that the sector where the labour is improved have also different impacts on exports. If labour productivity is improved in sector "non", the results are reflected in all other sectors. These changes are not observed in the same dimension on the other scenarios. However, this happens because in this particular sector there exist all services that are important for all other sectors. The labour intensive sector ("lab") is also one important sector. In this case the reason is the weight of this sector on Portuguese exports. The improvement of labour productivity decrease the exports only if it is applied just on the very small sector "rd".
Table 6: Changes in exports, by sectors, by regions of destiny and by scenario (%)

<table>
<thead>
<tr>
<th></th>
<th>res</th>
<th>lab</th>
<th>spe</th>
<th>sca</th>
<th>rd</th>
<th>non</th>
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<tr>
<td>PRT→UE14</td>
<td>47,40</td>
<td>41,86</td>
<td>40,51</td>
<td>29,53</td>
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<td>C1 PRT→UE10</td>
<td>47,53</td>
<td>42,08</td>
<td>40,63</td>
<td>32,48</td>
<td>28,55</td>
<td>46,22</td>
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<td>47,55</td>
<td>42,12</td>
<td>40,77</td>
<td>32,57</td>
<td>28,55</td>
<td>46,23</td>
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<tr>
<td>PRT→UE14</td>
<td>16,78</td>
<td>12,90</td>
<td>15,48</td>
<td>11,04</td>
<td>10,58</td>
<td>20,41</td>
</tr>
<tr>
<td>C2 PRT→UE10</td>
<td>16,82</td>
<td>12,96</td>
<td>15,52</td>
<td>12,05</td>
<td>10,64</td>
<td>20,42</td>
</tr>
<tr>
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<td>16,82</td>
<td>12,96</td>
<td>15,57</td>
<td>12,08</td>
<td>10,64</td>
<td>20,42</td>
</tr>
<tr>
<td>PRT→UE14</td>
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<td>12,90</td>
<td>15,48</td>
<td>11,04</td>
<td>10,58</td>
<td>20,41</td>
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<td>15,52</td>
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<td>16,82</td>
<td>12,96</td>
<td>15,57</td>
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<td>10,64</td>
<td>20,42</td>
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<tr>
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<td>22,33</td>
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<tr>
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<td>17,05</td>
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<tr>
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<td>16,78</td>
<td>22,33</td>
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<td>22,71</td>
<td>17,05</td>
<td>16,70</td>
<td>22,31</td>
</tr>
<tr>
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<td>16,78</td>
<td>22,33</td>
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<td>26,35</td>
<td>22,83</td>
<td>18,71</td>
<td>16,78</td>
<td>22,33</td>
</tr>
<tr>
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<td>22,71</td>
<td>17,05</td>
<td>16,70</td>
<td>22,31</td>
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<td>22,33</td>
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</tr>
<tr>
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<td>22,71</td>
<td>17,05</td>
<td>16,70</td>
<td>22,31</td>
</tr>
<tr>
<td>C13 PRT→UE10</td>
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<td>22,76</td>
<td>18,66</td>
<td>16,78</td>
<td>22,33</td>
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<tr>
<td>PRT→ROW</td>
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<td>26,35</td>
<td>22,83</td>
<td>18,71</td>
<td>16,78</td>
<td>22,33</td>
</tr>
</tbody>
</table>

To have a more accurate analyses is important to look also for imports and for domestic production. Nevertheless, it is already evident that the choose of
both labour type and sectors is crucial for the improvement of international competitiveness of Portuguese economy.

In table 8 is also evident that this type of policy have different impacts on the well-being of all agents.

Table 8: Changes of agents utility in Portugal (%), by scenarios

<table>
<thead>
<tr>
<th>EV</th>
<th>$UH(C_{sec})$</th>
<th>$UG(CG_{sec})$</th>
<th>$UI(I_{sec})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>21,047</td>
<td>68.08</td>
<td>-6.48</td>
</tr>
<tr>
<td>C2</td>
<td>8,511</td>
<td>27.85</td>
<td>-0.65</td>
</tr>
<tr>
<td>C3</td>
<td>11,549</td>
<td>37.39</td>
<td>-4.74</td>
</tr>
<tr>
<td>C4</td>
<td>2,301</td>
<td>7.80</td>
<td>-2.67</td>
</tr>
<tr>
<td>C5</td>
<td>1,081</td>
<td>3.15</td>
<td>-2.40</td>
</tr>
<tr>
<td>C6</td>
<td>0,339</td>
<td>0.60</td>
<td>-1.45</td>
</tr>
<tr>
<td>C7</td>
<td>1,303</td>
<td>3.71</td>
<td>-2.35</td>
</tr>
<tr>
<td>C8</td>
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<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>C9</td>
<td>13,265</td>
<td>44.17</td>
<td>6.32</td>
</tr>
<tr>
<td>C10</td>
<td>2,807</td>
<td>7.67</td>
<td>-6.54</td>
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<tr>
<td>C11</td>
<td>12,471</td>
<td>40.37</td>
<td>-6.04</td>
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<tr>
<td>C12</td>
<td>20,830</td>
<td>67.63</td>
<td>-5.50</td>
</tr>
<tr>
<td>C13</td>
<td>20,209</td>
<td>65.81</td>
<td>-4.37</td>
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</tbody>
</table>

In table 8 we can see that scenarios C1, C12 and C13 are the most favourable to household and government.

From this preliminary analysis it is evident that the choose of labour factor and sectors, that must be considered with priority when the option is to improve the productivity, is crucial for the improvement of international competitiveness of Portugal. This choose should depend on the interest and capacity of each industry, on the government interests and their political orientation but essentially must have in consideration the relations between sectors.

References


$^{23}$EV is the abbreviation for “Equivalente Variation”, index to measure social effects from each policies.


## A Used Aggregation

### Table 1: Sector Aggregation

<table>
<thead>
<tr>
<th>Brucker Aggregation by Factor of Competitiveness</th>
<th>GTAP Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resource Intensive</strong></td>
<td>cmt - Cattle Meat</td>
</tr>
<tr>
<td></td>
<td>omt - Other Meat</td>
</tr>
<tr>
<td></td>
<td>vol - Vegetable Oils</td>
</tr>
<tr>
<td></td>
<td>mil - Milk</td>
</tr>
<tr>
<td></td>
<td>per - Processed Rice</td>
</tr>
<tr>
<td></td>
<td>sgr - Sugar</td>
</tr>
<tr>
<td></td>
<td>ofd - Other Food</td>
</tr>
<tr>
<td></td>
<td>b_t - Beverages &amp; Tobacco</td>
</tr>
<tr>
<td></td>
<td>p_c - Petroleum &amp; Coke</td>
</tr>
<tr>
<td></td>
<td>nmm - Non-Metallic Minerals</td>
</tr>
<tr>
<td></td>
<td>nfm - Non-Ferrous Metals</td>
</tr>
<tr>
<td><strong>Labour Intensive</strong></td>
<td>wol - Wool</td>
</tr>
<tr>
<td></td>
<td>tex - Textiles</td>
</tr>
<tr>
<td></td>
<td>wap - Wearing Apparel</td>
</tr>
<tr>
<td></td>
<td>lea - Leather</td>
</tr>
<tr>
<td></td>
<td>fmp - Fabricated Metal Products</td>
</tr>
<tr>
<td></td>
<td>omf - Other Manufacturing</td>
</tr>
<tr>
<td><strong>Specialised Suppliers</strong></td>
<td>lum - Lumber</td>
</tr>
<tr>
<td><strong>Scale and Capital Intensive</strong></td>
<td>ppp - Paper &amp; Paper Products</td>
</tr>
<tr>
<td></td>
<td>crp - Chemical Rubber Products</td>
</tr>
<tr>
<td></td>
<td>i_s - Iron &amp; Steel</td>
</tr>
<tr>
<td></td>
<td>mvh - Motor Vehicles</td>
</tr>
<tr>
<td></td>
<td>otn - Other Transport Equipment</td>
</tr>
<tr>
<td></td>
<td>otp - Other Transport</td>
</tr>
<tr>
<td><strong>R&amp;D Intensive</strong></td>
<td>ele - Electronic Equipment</td>
</tr>
<tr>
<td></td>
<td>ome - Other Machinery &amp; Equipment</td>
</tr>
<tr>
<td><strong>Non-Industrial and Non-Classified Goods</strong></td>
<td>pdr - Paddy Rice</td>
</tr>
<tr>
<td></td>
<td>wht - Wheat</td>
</tr>
<tr>
<td></td>
<td>gro - Other Grains</td>
</tr>
<tr>
<td></td>
<td>v_f - Veg &amp; Fruit</td>
</tr>
<tr>
<td></td>
<td>osd - Oil Seeds</td>
</tr>
<tr>
<td></td>
<td>c_b - Cane &amp; Beet</td>
</tr>
<tr>
<td></td>
<td>pfb - Plant Fibres</td>
</tr>
<tr>
<td></td>
<td>ocr - Other Crops</td>
</tr>
<tr>
<td></td>
<td>ctl - Cattle</td>
</tr>
<tr>
<td></td>
<td>oap - Other Animal Products</td>
</tr>
<tr>
<td></td>
<td>rmk - Raw Milk</td>
</tr>
<tr>
<td></td>
<td>for - Forestry</td>
</tr>
<tr>
<td></td>
<td>fsh - Fishing</td>
</tr>
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<td></td>
<td>col - Coal</td>
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<td>Brucker Aggregation by Factor of Competitiveness</td>
<td>GTAP Sectors</td>
</tr>
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<td>-----------------------------------------------</td>
<td>--------------</td>
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<td>Non-Industrial and Non-Classified Goods (cont)</td>
<td>oil - Oil</td>
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<tr>
<td></td>
<td>omn - Other Mining</td>
</tr>
<tr>
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<td>ely - Electricity</td>
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<td>gdt - Gas Distribution</td>
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<td>wtr - Water</td>
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<td>trd - Trade</td>
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<td>atp - Air Transport</td>
</tr>
<tr>
<td></td>
<td>cnn - Communications</td>
</tr>
<tr>
<td></td>
<td>ofi - Other Financial Intermediation</td>
</tr>
<tr>
<td></td>
<td>isr - Insurance</td>
</tr>
<tr>
<td></td>
<td>obs - Other Business Services</td>
</tr>
<tr>
<td></td>
<td>ros - Recreation &amp; Other Services</td>
</tr>
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<td>osg - Other Services (Government)</td>
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<td>dwe - Dwellings</td>
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B Variables List

B.1 Endogenous variables

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<th>Description</th>
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<td>price of skilled labour</td>
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<td>plu_{reg}</td>
<td>price of unskilled labour</td>
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</tr>
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<td>pd_{reg,sec}</td>
<td>price of the domestic production (XD_{sec})</td>
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<td>price of exports</td>
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<tr>
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<td>M_{reg,regg,sec}</td>
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B.2 Exogenous variables

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</tr>
<tr>
<td>LUS_{reg}</td>
<td>unskilled labour endowment</td>
</tr>
<tr>
<td>MARGE_{reg,sec}</td>
<td>transport services related to exports</td>
</tr>
<tr>
<td>UNEMPQ_{reg}</td>
<td>unemployment of skilled labour</td>
</tr>
<tr>
<td>UNEMP\bar{Q}_{reg}</td>
<td>unemployment of unskilled labour</td>
</tr>
<tr>
<td>SG_{reg}</td>
<td>government savings</td>
</tr>
<tr>
<td>SF_{row,row}</td>
<td>foreign savings between countries in &quot;row&quot; region</td>
</tr>
<tr>
<td>TRO_{reg}</td>
<td>other transfers</td>
</tr>
</tbody>
</table>
B.3 Parameters

\( \pi_{k,reg,sec} \)  average productivity of capital
\( \pi_{q,reg,sec} \)  average productivity of skilled labour
\( \pi_{u,reg,sec} \)  average productivity of unskilled labour
\( t_{y,reg} \)  weight of taxes on income
\( t_{xd,reg,sec} \)  weight of taxes on production
\( t_{c,reg,sec} \)  weight of taxes on household consumption
\( t_{cf,reg,sec,sec} \)  weight of taxes on intermediate consumption
\( t_{ci,reg,sec} \)  weight of taxes on the consumption of investment goods
\( t_{c,reg,sec} \)  weight of taxes on government consumption
\( t_{k,reg,sec} \)  weight of taxes on the use of capital
\( t_{lq,reg,sec} \)  weight of taxes on the use of skilled labour
\( t_{lu,reg,sec} \)  weight of taxes on the use of unskilled labour
\( t_{m,reg,regg,sec} \)  weight of taxes on imports
\( t_{e,reg,regg,sec} \)  weight of taxes or subsidies on exports
\( a_{d,reg} \)  rate of depreciation of capital
\( a_{F,reg,sec} \)  efficiency parameter of production function
\( \gamma_{F,k,reg,sec} \)  capital distribution parameter
\( \gamma_{F,q,reg,sec} \)  skilled labour distribution parameter
\( \gamma_{F,u,reg,sec} \)  unskilled labour distribution parameter
\( \sigma_{F,reg,sec} \)  elasticity of substitution between production factors
\( a_{T,reg,sec} \)  efficiency parameter of CET function
\( \gamma_{T,reg,regg,sec} \)  distribution parameter of CET function
\( \sigma_{T,reg,sec} \)  elasticity of substitution between domestic production for domestic market and exports
\( a_{A,reg,sec} \)  efficiency parameter of Armington function
\( \gamma_{A,reg,sec} \)  distribution parameter of Armington function
\( \sigma_{A,reg,sec} \)  elasticity of substitution between domestic production for domestic market and imports
\( \alpha_{H,reg,sec} \)  income elasticity of household demand
\( \alpha_{CG,reg,sec} \)  income-elasticity of government demand
\( \alpha_{I,reg,sec} \)  income-elasticity of investment goods demand
\( \mu_{H,reg,sec} \)  minimum level of consumption
\( mps_{reg} \)  marginal propensity to save
\( io_{reg,secc,sec} \)  technical coefficients
\( t_{re,reg} \)  weight of unemployment subsidies on average wages
C Equation list

Household demand for goods and services

\[(1 + tc_{reg,sec}) * p_{reg,sec} * C_{reg,sec} = (1 + tc_{reg,sec}) * p_{reg,sec} * \mu_{H_{reg,sec}} + \alpha H_{reg,sec} * \{ CBU_{reg} - \sum_{secc} [(1 + tc_{reg,secc}) * p_{reg,secc} * \mu_{H_{reg,secc}}] \} \]

Household savings

\[SH_{reg} = mps_{reg} * [YH_{reg} - ty_{reg} * (YH_{reg} - TRF_{reg})] \]

Household income

\[YH_{reg} = pk_{reg} * (1 - d_{reg}) * KS_{reg} + plq_{reg} * (LQS_{reg} - UNEMPQ_{reg}) + plq_{reg} * (LUS_{reg} - UNEMPQU_{reg} + TRF_{reg}) \]

Household consumption budget

\[CBU_{reg} = YH_{reg} - ty_{reg} * (YH_{reg} - TRF_{reg}) - SH_{reg} \]

Demand for capital

\[K_{reg,sec} = \left[ XD_{reg,sec} \right]^{\frac{\gamma_{Fk_{reg,sec}}}{\sigma_{Fk_{reg,sec}}}} * \left[ \frac{\gamma_{Fk_{reg,sec}}}{(1 + tk_{reg,sec}) * pk_{reg}} \right]^{\sigma_{Fk_{reg,sec}}} * \left\{ \left[ (1 + tk_{reg,sec}) * pk_{reg} \right]^{1 - \sigma_{Fk_{reg,sec}}} \right\} \]

Demand for skilled labour

\[LQ_{reg,sec} = \left[ XD_{reg,sec} \right]^{\frac{\gamma_{Fq_{reg,sec}}}{\sigma_{Fq_{reg,sec}}}} * \left[ \frac{\gamma_{Fq_{reg,sec}}}{(1 + tlq_{reg,sec}) * plq_{reg}} \right]^{\sigma_{Fq_{reg,sec}}} * \left\{ \left[ (1 + tlq_{reg,sec}) * plq_{reg} \right]^{1 - \sigma_{Fq_{reg,sec}}} \right\} \]

Demand for unskilled labour

\[LU_{reg,sec} = \left[ XD_{reg,sec} \right]^{\frac{\gamma_{Fu_{reg,sec}}}{\sigma_{Fu_{reg,sec}}}} * \left[ \frac{\gamma_{Fu_{reg,sec}}}{(1 + tlq_{reg,sec}) * plq_{reg}} \right]^{\sigma_{Fu_{reg,sec}}} * \left\{ \left[ (1 + tlq_{reg,sec}) * plq_{reg} \right]^{1 - \sigma_{Fu_{reg,sec}}} \right\} \]

Profits

\[(1 - txd_{reg,sec}) * pd_{reg,sec} * XD_{reg,sec} = (1 + tk_{reg,sec}) * pk_{reg} * K_{reg,sec} + (1 + tlq_{reg,sec}) * plq_{reg} * LQ_{reg,sec} + (1 + tlq_{reg,sec}) * plq_{reg} * LU_{reg,sec} + \sum_{secc} [io_{reg,secc,sec} * XD_{reg,sec} * p_{reg,secc} * (1 + tc_{f_{reg,secc,sec}})] \]
Total savings

\[ S_{\text{reg}} = S H_{\text{reg}} + p \text{index}_{\text{reg}} \times S G_{\text{reg}} + \sum_{\text{reg}} (S F_{\text{reg}, \text{regg}}) + \sum_{\text{sec}} d_{\text{reg}} \times p k_{\text{reg}} \times K_{\text{reg,sec}} - M A R G E_{\text{reg,sec}} \times p w e_{\text{reg,sec}} \]

Demand for investment goods

\[ (1 + t c i_{\text{reg,sec}}) \times p r e g, \text{sec} \times I_{\text{reg,sec}} = \alpha I_{\text{reg,sec}} \times S_{\text{reg}} \]

Government demand for goods and services

\[ (1 + t c g_{\text{reg,sec}}) \times p r e g, \text{sec} \times C G_{\text{reg,sec}} = \alpha C G_{\text{reg,sec}} \times (T A X R_{\text{reg}} - T R F_{\text{reg}} - p \text{index}_{\text{reg}} \times S G_{\text{reg}}) \]

Government revenues

\[ T A X R_{\text{reg}} = t y_{\text{reg}} \times (Y H_{\text{reg}} - T R F_{\text{reg}}) + \sum_{\text{sec}} [p r e g, \text{sec} \times (t c r e g, \text{sec} \times C r e g, \text{sec} + t c g r e g, \text{sec} \times C G r e g, \text{sec} + t c i_{\text{reg,sec}} \times I_{\text{reg,sec}}) + \sum_{\text{sec}} (t c f_{\text{reg,sec,sec,sec}} \times \sum_{\text{sec}} d_{\text{reg}} \times p k_{\text{reg}} \times K_{\text{reg,sec}} + t l l_{\text{reg,sec}} \times p l q_{\text{reg,sec}} \times L Q_{\text{reg,sec}} + t l u_{\text{reg,sec}} \times p l u_{\text{reg,sec}} \times L U_{\text{reg,sec}} + \sum_{\text{regg}} (t m_{\text{reg,regg,sec}} \times p w e_{\text{reg,regg,sec}} \times M_{\text{reg,regg,sec}} + t r o_{\text{regg,sec}} \times p w e_{\text{reg,regg,sec}} \times E_{\text{reg,regg,sec}} + t x d_{\text{reg,sec}} \times p d_{\text{reg,sec}} \times X D_{\text{reg,sec}}) ] \]

Total transfers

\[ T R F_{\text{reg}} = t r e p_{\text{reg}} \times (p l q_{\text{reg}} \times U N E M P Q_{\text{reg}} + p l u_{\text{reg}} \times U N E M P U_{\text{reg}}) + T R O_{\text{reg}} \times p \text{index}_{\text{reg}} \]

Laspeyres price index

\[ p \text{index}_{\text{reg}} = \sum_{\text{sec}} \left( \frac{(1 + t c l_{\text{reg,sec}}) \times p l q_{\text{reg,sec}} \times C_{\text{reg,sec}}}{(1 + t c l_{\text{reg,sec}}) \times p l q_{\text{reg,sec}} \times C_{\text{reg,sec}}} \right) \]

Exports prices

\[ p e_{\text{reg,regg,sec}} = p w e_{\text{reg,regg,sec}} \times (1 - t e_{\text{reg,regg,sec}}) \]

Exports supply

\[ E_{\text{reg,regg,sec}} = \frac{X D_{\text{reg,sec}}}{a T_{\text{reg,sec}}} \times \left( \frac{p e_{\text{reg,regg,sec}}}{p e_{\text{reg,regg,sec}}} \right)^{T_{\text{reg,sec}}} \times \left( \sum_{\text{regg}} (g T_{\text{reg,regg,sec}} \times p e_{\text{reg,regg,sec}})^{1 - \sigma_{T_{\text{reg,sec}}}} \right) + \left( 1 - \sum_{\text{regg}} g T_{\text{reg,regg,sec}} \right)^{1 - \sigma_{T_{\text{reg,sec}}}} \times p d d_{\text{reg,sec}} \]

Supply of domestic goods for domestic market
Zero profit condition for CET function

\[
p_{\text{reg}, \text{sec}} \times XD_{\text{reg}, \text{sec}} = pdd_{\text{reg}, \text{sec}} \times XD_{\text{reg}, \text{sec}} + \sum_{\text{reg}} \left( p_{\text{reg}, \text{reg}, \text{sec}} * E_{\text{reg}, \text{reg}, \text{sec}} \right) + p\text{w}_{\text{reg}, \text{"regg"}, \text{sec}} * MARGE_{\text{reg}, \text{sec}}
\]

Demand for domestic goods and services in the domestic market

\[
XD_{\text{sec}} = \frac{X_{\text{sec}}}{\sigma A_{\text{sec}}} \left( \frac{1 - \sum_{\text{reg}} \gamma A_{\text{reg}, \text{sec}}}{pdd_{\text{sec}}} \right) \sigma A_{\text{sec}} * \left( \sum_{\text{reg}} \left( \gamma A_{\text{reg}, \text{sec}} * p_{\text{reg}, \text{reg}, \text{sec}} \right) \right) + \left( 1 - \sum_{\text{reg}} \gamma A_{\text{reg}, \text{sec}} \right) * pdd_{\text{sec}} \left( 1 - \alpha A_{\text{sec}} \right) \left( \frac{\sigma A_{\text{sec}}}{1 - \alpha A_{\text{sec}}} \right)
\]

Imports prices

\[
p_{m_{\text{reg}, \text{reg}, \text{sec}}} = (1 + t_{m_{\text{reg}, \text{reg}, \text{sec}}}) * p\text{w}_{\text{reg}, \text{reg}, \text{sec}}
\]

Relation between imports and exports

\[
M_{\text{reg}, \text{reg}, \text{sec}} = (1 + m_{\text{reg}, \text{reg}, \text{sec}}) * E_{\text{reg}, \text{reg}, \text{sec}}
\]

Demand for imports in domestic market

\[
M_{\text{reg}, \text{reg}, \text{sec}} = \frac{X_{\text{reg}, \text{sec}}}{\sigma A_{\text{reg}, \text{sec}}} \left( \frac{\gamma A_{\text{reg}, \text{reg}, \text{sec}}}{p_{m_{\text{reg}, \text{reg}, \text{sec}}}} \right) \sigma A_{\text{reg}, \text{sec}} * \left( \sum_{\text{reg}} \left( \gamma A_{\text{reg}, \text{reg}, \text{sec}} * p_{m_{\text{reg}, \text{reg}, \text{sec}}} \right) \right) + \left( 1 - \sum_{\text{reg}} \gamma A_{\text{reg}, \text{reg}, \text{sec}} \right) * pdd_{\text{reg}, \text{sec}} \left( 1 - \alpha A_{\text{reg}, \text{sec}} \right) \left( \frac{\sigma A_{\text{reg}, \text{sec}}}{1 - \alpha A_{\text{reg}, \text{sec}}} \right)
\]

Demand for domestic goods in domestic market

\[
XD_{\text{reg}, \text{sec}} = \frac{X_{\text{reg}, \text{sec}}}{\sigma A_{\text{reg}, \text{sec}}} \left( \frac{1 - \sum_{\text{reg}} \gamma A_{\text{reg}, \text{reg}, \text{sec}}}{pdd_{\text{reg}, \text{sec}}} \right) \sigma A_{\text{reg}, \text{sec}} * \left( \sum_{\text{reg}} \left( \gamma A_{\text{reg}, \text{reg}, \text{sec}} * p_{m_{\text{reg}, \text{reg}, \text{sec}}} \right) \right) + \left( 1 - \sum_{\text{reg}} \gamma A_{\text{reg}, \text{reg}, \text{sec}} \right) * pdd_{\text{reg}, \text{sec}} \left( 1 - \alpha A_{\text{reg}, \text{sec}} \right) \left( \frac{\sigma A_{\text{reg}, \text{sec}}}{1 - \alpha A_{\text{reg}, \text{sec}}} \right)
\]

Zero profit conditions for Armington function

\[
p_{\text{reg}, \text{sec}} \times X_{\text{reg}, \text{sec}} = pdd_{\text{reg}, \text{sec}} \times XD_{\text{reg}, \text{sec}} + \sum_{\text{reg}} \left( p_{m_{\text{reg}, \text{reg}, \text{sec}}} * M_{\text{reg}, \text{reg}, \text{sec}} \right)
\]

Trade Balance
\[
\sum_{sec} (pwe_{reg,sec} \times M_{reg,sec}) = \sum_{sec} (pwe_{reg,sec} \times E_{reg,sec}) + SF_{reg,sec}
\]

Equilibrium in skilled labour market

\[
\sum_{sec} LQ_{reg,sec} = LQS_{reg} - UNEMP_{reg}
\]

Equilibrium in unskilled labour market

\[
\sum_{sec} LU_{reg,sec} = LUS_{reg} - UNEMP_{reg}
\]

Equilibrium in capital market

\[
\sum_{sec} K_{reg,sec} = KS_{reg}
\]

Equilibrium in goods and services market

\[
X_{reg,sec} = C_{reg,sec} + I_{reg,sec} + \sum_{secc} (i_{0,reg,secc} \times XD_{reg,secc}) + CG_{reg,sec}
\]